# CRITICAL FLUCTUATIONS AND CORRELATIONS OF QUARK SPIN NEAR CEP

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arXiv: 2410.20704

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@第二十届全国中高能核物理大会

暨第十四届全国中高能核物理专题研讨会

# OUTLINE

- Introduction: rotational effects in heavy  ion collisions
- Motivation: a question comes up to our mind
- Technical details: NJL model calculation
- Results: Critical spin correlation near CEP

# ANGULAR MOMENTUM IN HEAVY-ION COLLISIONS

- Non-central heavy-ion collisions create fireballs with large global orbital angular momenta

$$\boldsymbol{L}_{\rm init} \sim 10^5 \hbar$$ F. Becattini, F. Piccinini, J. Rizzo, PRC 77 (2008) 024906

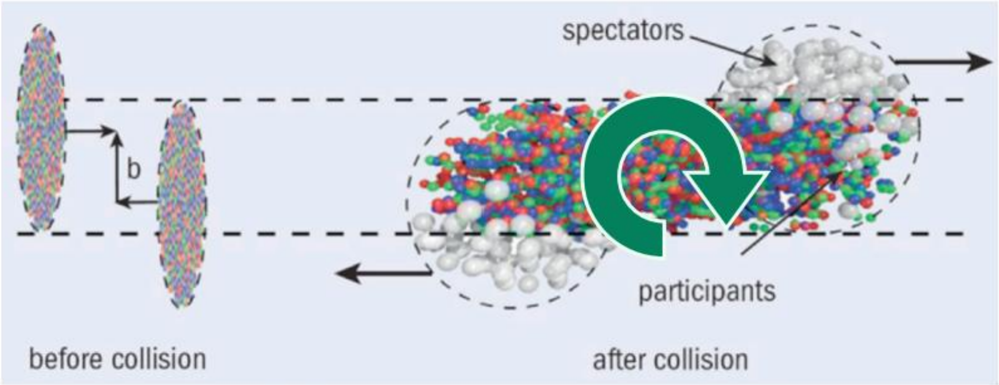
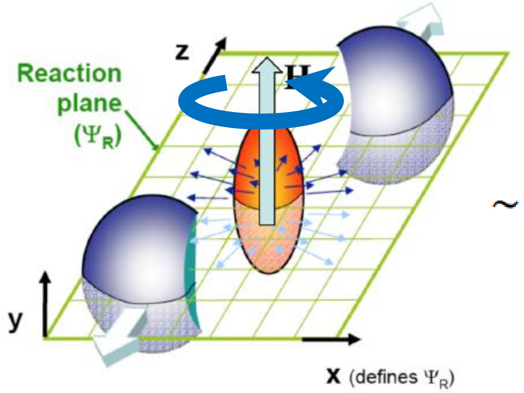



figure: M. Lisa, talk @ "Strangeness in Quark Matter 2016"

- angular momenta $\longrightarrow$ rotation (local/global)
- Rotational effects attract many attentions recently in several aspects

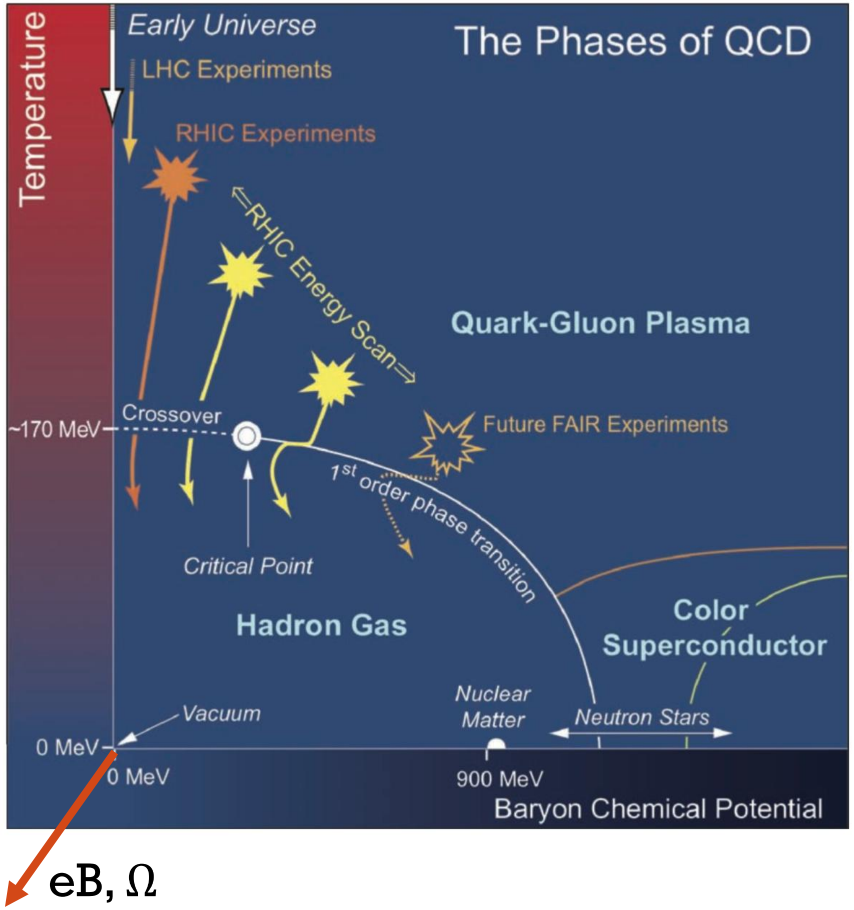
# ROTATION RELATED STUDIES




$\sim 10^{21}/s$ at RHIC

- Spin polarization/alignment


Phase transition




Anomalous transport

CVE, Vilenkin (1979)

$$J = \left(\frac{\mu^2}{4\pi^2} + \frac{T^2}{12}\right)\omega$$

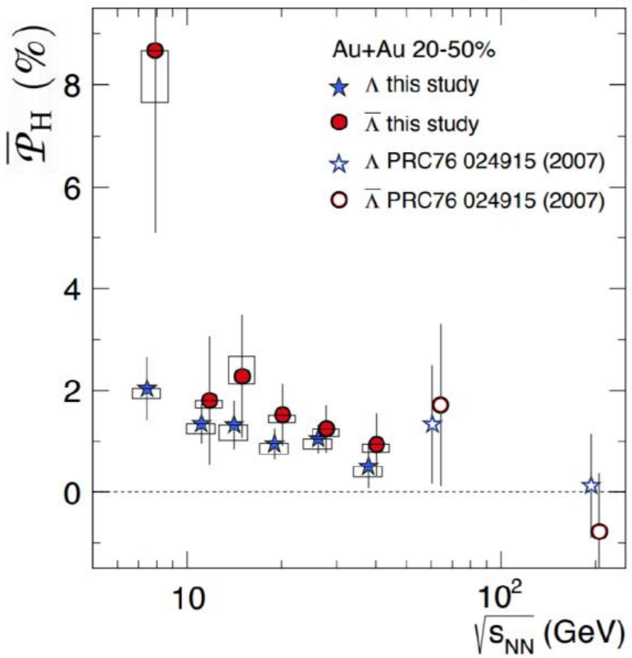
magneto-vortical transport, Hattori-Yin (2016)

$$J^0 = \frac{eB\omega}{4\pi^2}$$

# SPIN POLARIZATION AND ALIGNMENT

- Global $\Lambda$ and $\bar{\Lambda}$ spin polarization

L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65





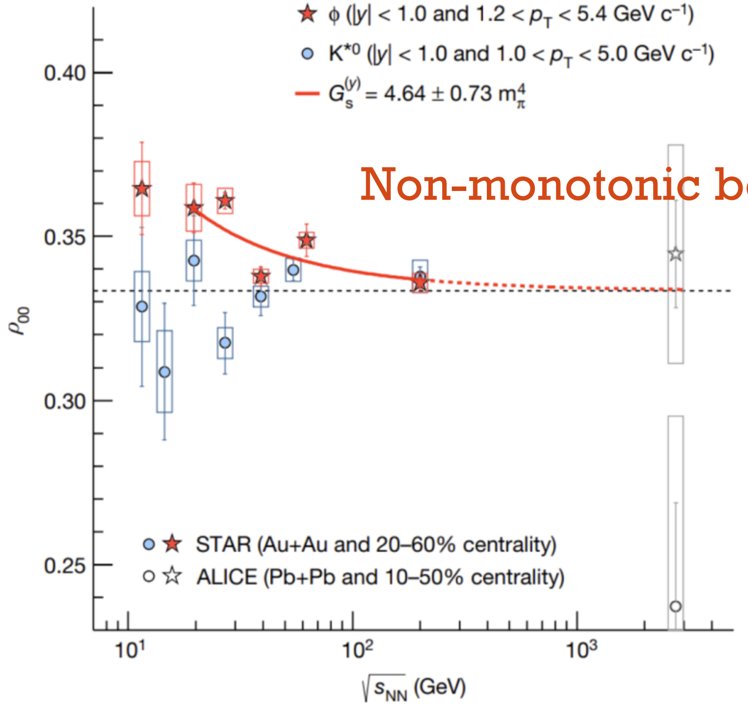
most vortical fluid produced in the laboratory .

$$\omega = (P_\Lambda + P_{\bar{\Lambda}}) k_B T/\hbar \sim 0.6 - 2.7 \times 10^{22}\ \mathrm{s}^{-1}$$

Local polarization

- $\phi$ meson spin alignment

STAR Collaboration, Nature 614 (2023) 7947.

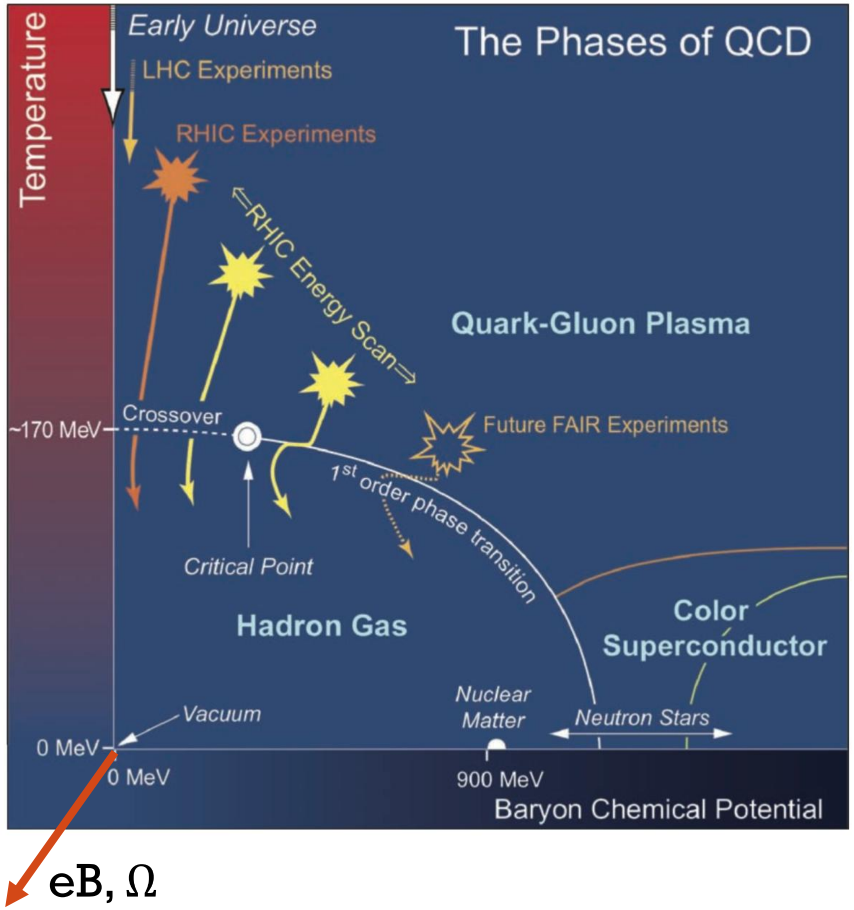



Non-monotonic behavior

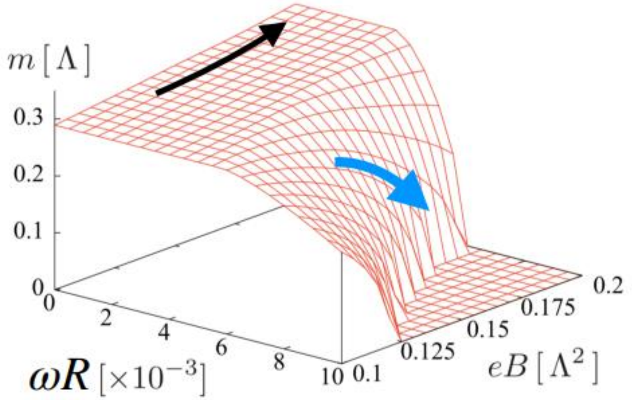
$\rho_{00} \neq 1/3$ indicates spin alignment

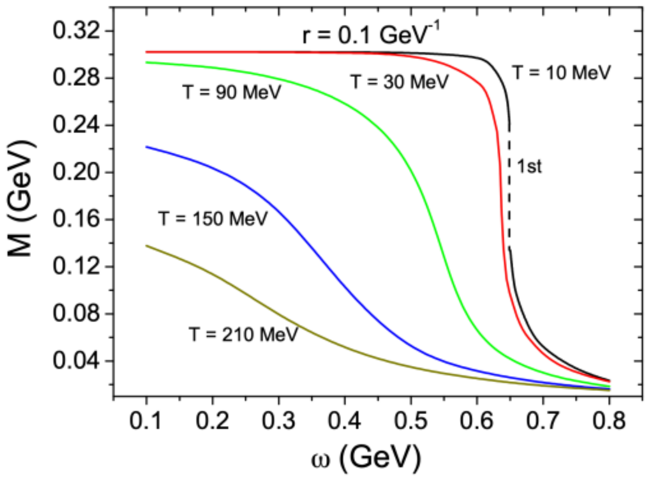
Other vector mesons

# QCD PHASE DIAGRAM UNDER ROTATION



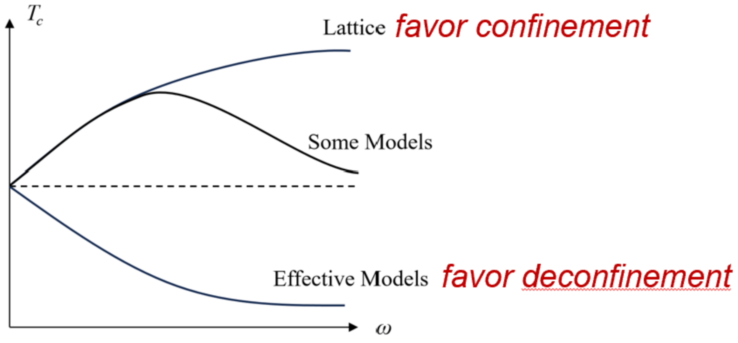

HLC, K. Fukushima, X-G. Huang, K. Mameda, Phys. Rev. D 93, 104052 (2016), 1512.08974




Y. Jiang and J. Liao, Phys. Rev. Lett. 117, 192302 (2016), 1606.03808

- First studied by NJL model (order parameter: quark mass)
- Rotation behaves like chemical potential
- However, lattice studies give opposite result!



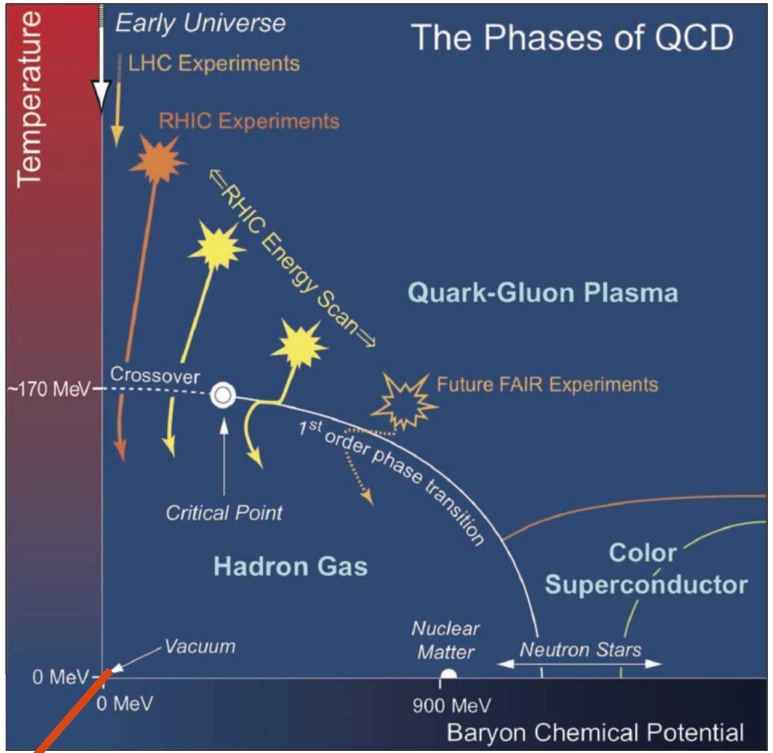

P. Zhuang's talk @PHD2024

# MOTIVATION

- Almost studied separately

Spin polarization/alignment


Phase transition



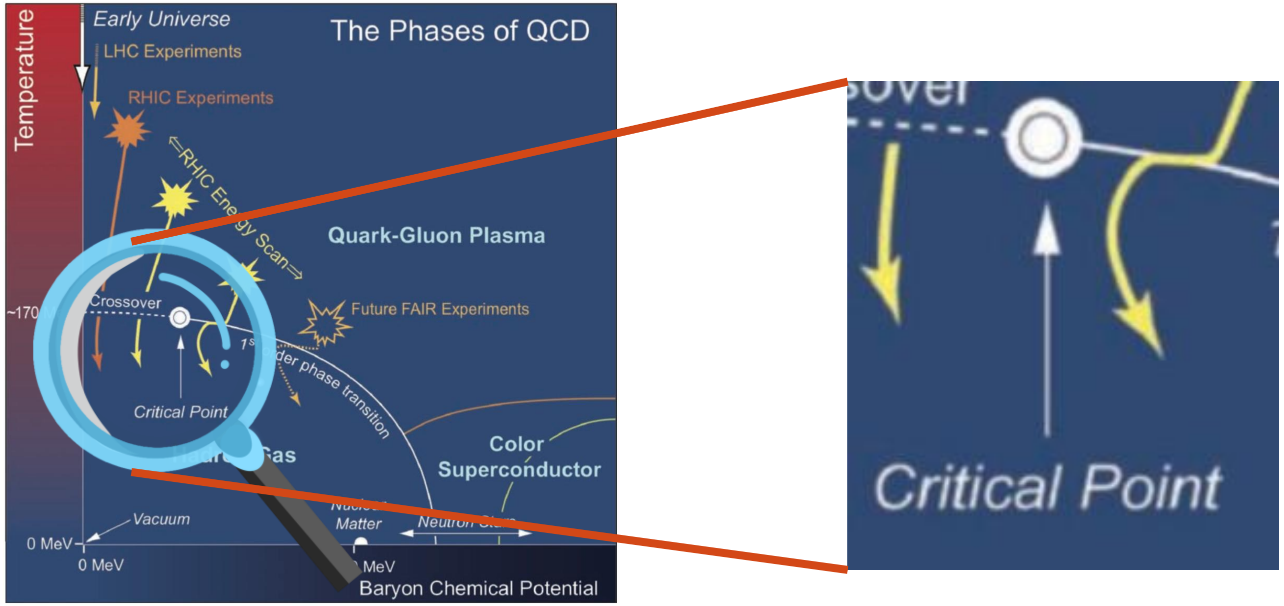

$\Omega$

Q: Are these two aspects related?

Minghua Wei , Mei Huang, Chin.Phys.C 47 (2023) 10, 104105
Fei Sun, Jingdong Shao, Rui, Kun Xu, Mei Huang, PhysRevD.109.116017
Sushant K. Singh, Jan-e Alam, Eur. Phys. J. C 83, 585 (2023)

# NAÏVE ESTIMATIONS

- Spin polarization $\sim\Omega$
- Spin alignment $\sim\Omega^2$
- Chiral condensate does not change much at small $\Omega$
- Seems not large enough for measurement
- However

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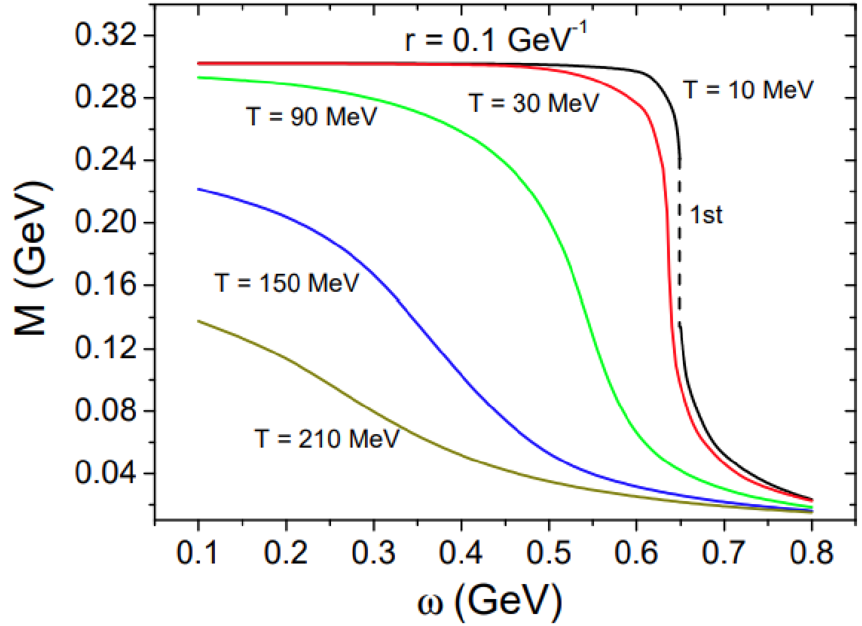
Y. Jiang and J. Liao, Phys. Rev. Lett. 117, 192302 (2016),




Spin fluctuation? Just like baryon number?

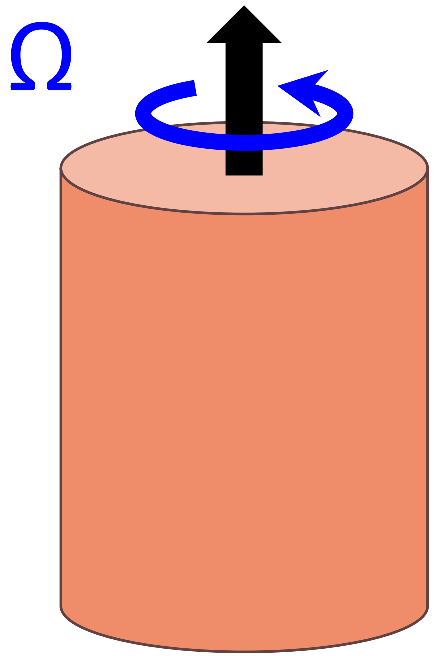
# QUALITATIVE STUDY: NJL MODEL

$$g_{\mu\nu} = \begin{pmatrix} 1-(x^2+y^2)\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Vierbein formulism (QFT in curved spacetime)

$$\Gamma_\mu = \frac{1}{4}\times\frac{1}{2}[\gamma^a,\gamma^b]\Gamma_{ab\mu} \qquad \Gamma_{ab\mu} = \eta_{ac}\left(e^c_\sigma G^\sigma_{\mu\nu} e^\nu_b - e^\nu_b \partial_\mu e^c_\nu\right) \qquad g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu$$

$$e^a_\mu = \delta^a_\mu + \delta^a_i \delta^0_\mu v_i \qquad e^\mu_a = \delta^\mu_a - \delta^0_a \delta^\mu_i v_i$$




- NJL model under rotation

$$\mathcal{L} = \bar{\psi}[ie^\mu_a\gamma^a\nabla_\mu - m_0 + \mu\gamma^0]\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\vec{\tau}\psi)^2]$$

$$\nabla_\mu = \partial_\mu + \boxed{\Gamma_\mu}$$

spinor connection

# QUALITATIVE STUDY: NJL MODEL

- General thermodynamic potential under rotation

$$V_{eff}(r) = \frac{(m-m_0)^2}{4G} - N_c N_f \sum_l \int_0^\Lambda \frac{p_t \mathrm{d}p_t \mathrm{d}p_z}{(2\pi)^2} [\varepsilon_p + T\ln(1+\mathrm{e}^{-\beta(\varepsilon_p-\mu-\Omega j)}) + T\ln(1+\mathrm{e}^{-\beta(\varepsilon_p+\mu+\Omega j)})](J_l^2(p_t r) + J_{l+1}^2(p_t r)).$$

- Away from the center, contribution from orbital angular momentum is dominant
- Since we are interested in spin, we first focus on the physics near the center (r=0)

$$\begin{aligned} V^0_{eff}(\Omega,\mu) =& \frac{(m-m_0)^2}{4G} - N_c N_f \int_0^\Lambda \frac{\mathrm{d}^3p}{(2\pi)^2} 2\varepsilon_p \\ &+ N_c N_f \int_0^\infty \frac{\mathrm{d}^3p}{(2\pi)^2} [\overset{q,\uparrow}{T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu-\Omega/2)/T})} + \overset{q,\downarrow}{T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu+\Omega/2)/T})} \\ &+ \underset{\bar{q},\uparrow}{T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu-\Omega/2)/T})} + \underset{\bar{q},\downarrow}{T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu+\Omega/2)/T})}]. \end{aligned}$$

- However, we can only get information about average total spin from this expression
- What we want is the correlation between quark and antiquark

# QUALITATIVE STUDY: NJL MODEL

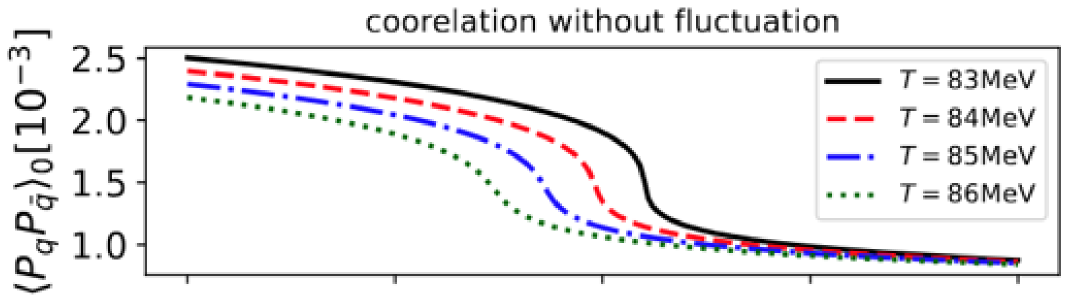
- Thermodynamic contribution

$$
\begin{aligned}
V_{eff}^{0}(\Omega,\mu) =& \frac{(m-m_0)^2}{4G} - N_c N_f \int_0^{\Lambda} \frac{\mathrm{d}^3 p}{(2\pi)^2} 2\varepsilon_p \\
&+ N_c N_f \int_0^{\infty} \frac{\mathrm{d}^3 p}{(2\pi)^2} [\overbrace{T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu-\Omega/2)/T})}^{q,\uparrow} + \overbrace{T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu+\Omega/2)/T})}^{q,\downarrow} \\
&+ \underbrace{T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu-\Omega/2)/T})}_{\bar{q},\uparrow} + \underbrace{T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu+\Omega/2)/T})}_{\bar{q},\downarrow}].
\end{aligned}
$$

$$
\langle P_q P_{\bar{q}} \rangle_0 = \frac{\int \mathrm{d}^3 p (f_q^{\uparrow} - f_q^{\downarrow})(f_{\bar{q}}^{\uparrow} - f_{\bar{q}}^{\downarrow})}{\int \mathrm{d}^3 p (f_q^{\uparrow} + f_q^{\downarrow})(f_{\bar{q}}^{\uparrow} + f_{\bar{q}}^{\downarrow})}.
$$

$$
\langle S \rangle = -\frac{V}{T}\frac{\mathrm{d}V_{eff}^{0}}{\mathrm{d}(\frac{\Omega}{T})} = -\frac{V}{T}\frac{\partial V_{eff}^{0}}{\partial(\frac{\Omega}{T})} - \frac{V}{T}\underbrace{\boxed{\frac{\partial V_{eff}^{0}}{\partial m}}}_{0 \text{ (Gap Eq)}}\frac{\partial m}{\partial(\frac{\Omega}{T})}.
$$

$$
f_q^{\uparrow/\downarrow} = \frac{1}{\mathrm{e}^{\beta(\epsilon_p - \mu \mp \frac{\Omega}{2})}+1}, \quad f_{\bar{q}}^{\uparrow/\downarrow} = \frac{1}{\mathrm{e}^{\beta(\epsilon_p + \mu \mp \frac{\Omega}{2})}+1}.
$$



# QUALITATIVE STUDY: NJL MODEL

- To get correlation, further techniques are needed
- Introducing rotation and chemical potential only act on quark or antiquark

$$
\begin{aligned}
&V_{\text{eff}}(\Omega_q^s, \Omega_{\bar{q}}^s, \Omega, \mu_q, \mu_{\bar{q}}, \mu; r) \\
=&\frac{[m(r)-m_0]^2}{4G} - N_c N_f \int_0^\Lambda \frac{\mathrm{d}^3 p}{(2\pi)^3} 2\varepsilon_p - \sum_{l=-\infty}^{\infty} N_c N_f \int_0^\infty \frac{\mathrm{d}^3 p}{(2\pi)^3} J_l^2(p_t r) \Big[ T \ln(1+\mathrm{e}^{-(\varepsilon_p - \mu - \Omega_q^s/2 - \Omega l - \mu_q)/T}) \\
&+ T \ln(1+\mathrm{e}^{-(\varepsilon_p - \mu + \Omega_q^s/2 - \Omega l - \mu_q)/T}) + T \ln(1+\mathrm{e}^{-(\varepsilon_p + \mu - \Omega_{\bar{q}}^s/2 + \Omega l - \mu_{\bar{q}})/T}) + T \ln(1+\mathrm{e}^{-(\varepsilon_p + \mu + \Omega_{\bar{q}}^s/2 + \Omega l - \mu_{\bar{q}})/T}) \Big]
\end{aligned}
$$

  - Then by taking derivative, we can get correlation of quark/antiquark spin and particle number

$$
\langle S_q S_{\bar{q}} \rangle - \langle S_q \rangle \langle S_{\bar{q}} \rangle = \frac{\partial^2 V_{eff}}{\partial \Omega_q^s \partial \Omega_{\bar{q}}^s} \Big|_{\Omega_q^s = \Omega_{\bar{q}}^s = \Omega} \qquad \langle N_q N_{\bar{q}} \rangle - \langle N_q \rangle \langle N_{\bar{q}} \rangle = \frac{\partial^2 V_{eff}}{\partial \mu_q \partial \mu_{\bar{q}}} \Big|_{\mu_q = \mu_{\bar{q}} = 0}
$$

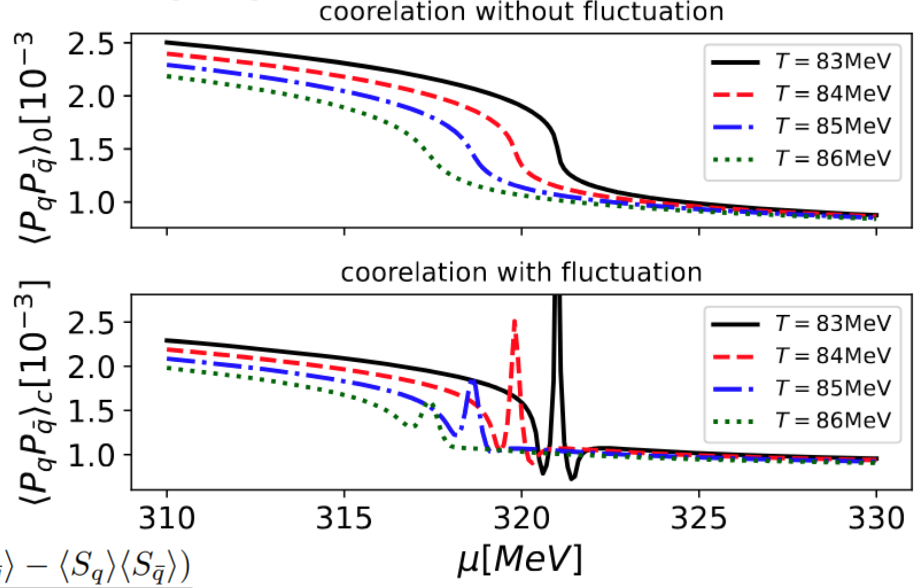
  - Then we can define the spin correlation of quark-antiquark as

$$
\langle P_q P_{\bar{q}} \rangle_c = \frac{4(\langle S_q S_{\bar{q}} \rangle - \langle S_q \rangle \langle S_{\bar{q}} \rangle)}{\langle N_q N_{\bar{q}} \rangle - \langle N_q \rangle \langle N_{\bar{q}} \rangle}
$$

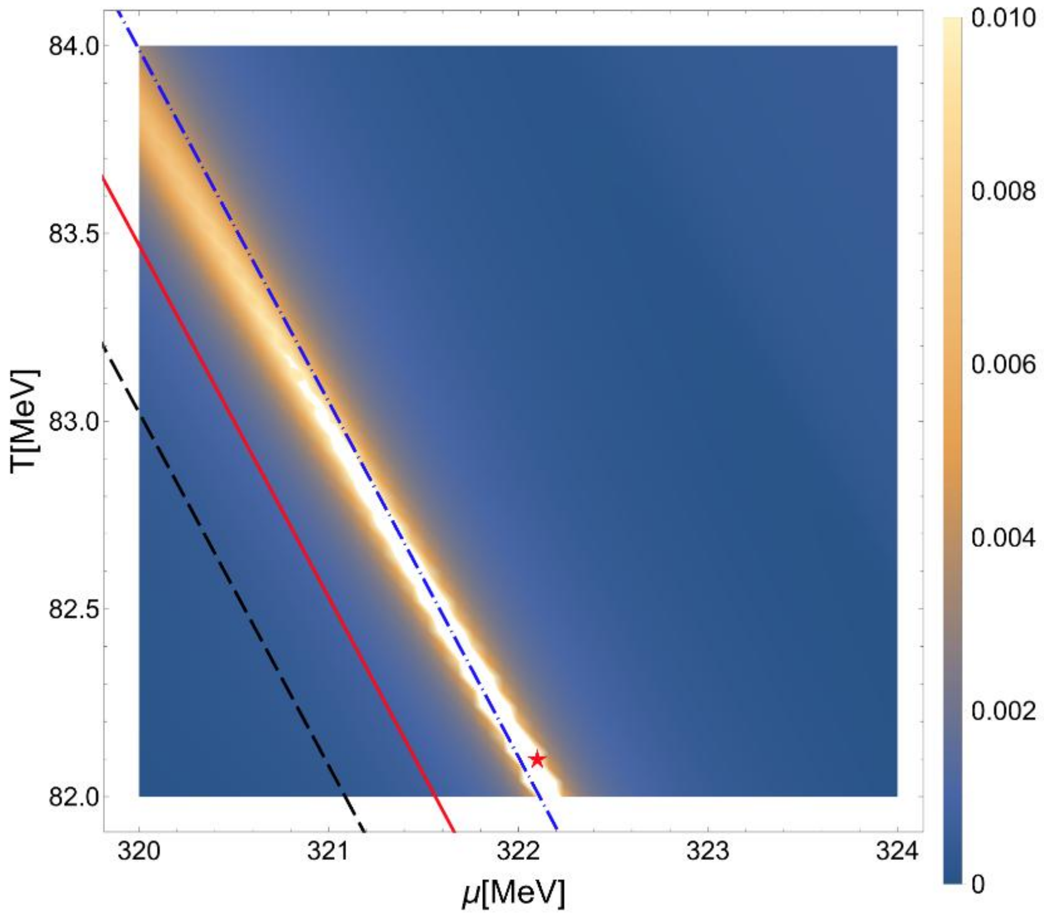
# SPIN CORRELATION ENHANCED BY CEP!

- First we consider r=0
- Comparison with the case w/o fluctuation

$$\langle P_q P_{\bar{q}} \rangle_0 = \frac{\int \mathrm{d}^3 p (f_q^{\uparrow} - f_q^{\downarrow})(f_{\bar{q}}^{\uparrow} - f_{\bar{q}}^{\downarrow})}{\int \mathrm{d}^3 p (f_q^{\uparrow} + f_q^{\downarrow})(f_{\bar{q}}^{\uparrow} + f_{\bar{q}}^{\downarrow})}.$$




$$\langle P_q P_{\bar{q}} \rangle_c = \frac{4(\langle S_q S_{\bar{q}} \rangle - \langle S_q \rangle \langle S_{\bar{q}} \rangle)}{\langle N_q N_{\bar{q}} \rangle - \langle N_q \rangle \langle N_{\bar{q}} \rangle}$$



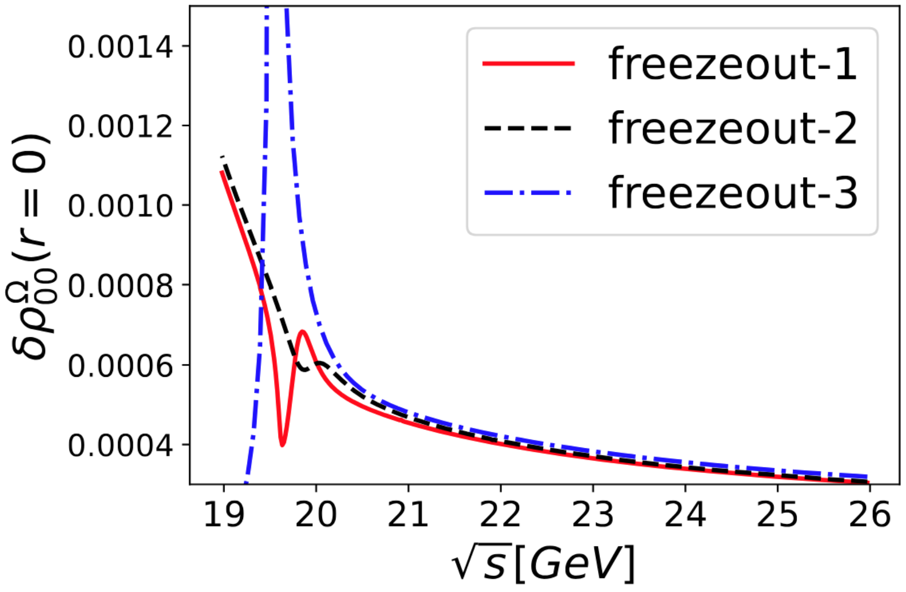
# VECTOR MESON SPIN ALIGNMENT

- Along imaginary freezeout lines

$$\rho_{00} = \frac{1 - \langle P_q P_{\bar{q}} \rangle}{3 + \langle P_q P_{\bar{q}} \rangle} \approx \bar{\rho}_{00} \boxed{- \delta\rho_{00}^{\Omega}}.$$

Contribution from critical fluctuation

At r=0

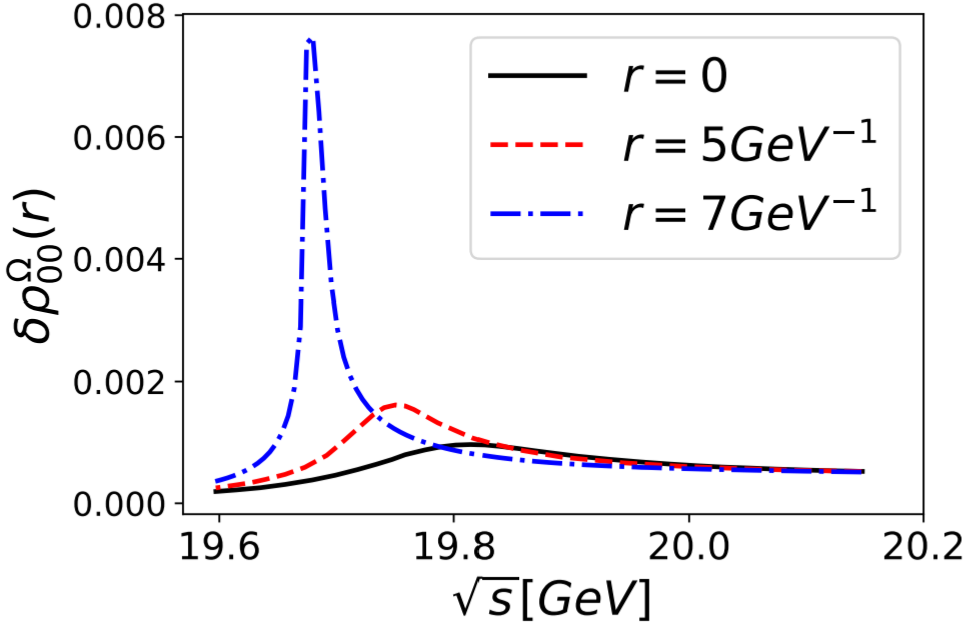



$$V_{\rm eff}(\Omega_q^s, \Omega_{\bar{q}}^s, \Omega, \mu_q, \mu_{\bar{q}}, \mu; r)$$

$$= \frac{[m(r) - m_0]^2}{4G} - N_c N_f \int_0^{\Lambda} \frac{d^3p}{(2\pi)^3} 2\varepsilon_p - \sum_{l=-\infty}^{\infty} N_c N_f \int_0^{\infty} \frac{d^3p}{(2\pi)^3} J_l^2(p_t r) \Big[ T \ln(1 + e^{-(\varepsilon_p - \mu - \Omega_q^s/2 - \Omega l - \mu_q)/T})$$

$$+ T \ln(1 + e^{-(\varepsilon_p - \mu + \Omega_q^s/2 - \Omega l - \mu_q)/T}) + T \ln(1 + e^{-(\varepsilon_p + \mu - \Omega_{\bar{q}}^s/2 + \Omega l - \mu_{\bar{q}})/T}) + T \ln(1 + e^{-(\varepsilon_p + \mu + \Omega_{\bar{q}}^s/2 + \Omega l - \mu_{\bar{q}})/T}) \Big]$$

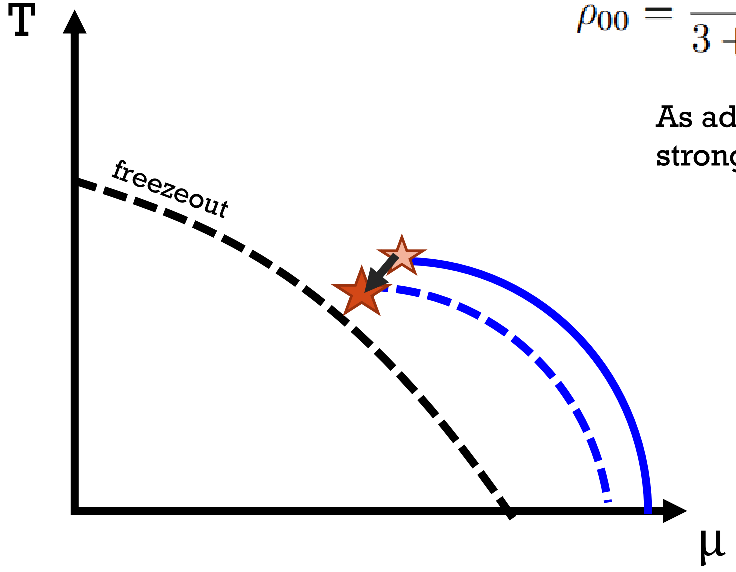
Orbit angular momentum contribution to Freezeout – 2



# PHI MESON SPIN ALIGNMENT

- The CEP is shifted closer to freezeout line by orbit angular momentum contribution
- The peak structure is enhanced

## A SCHEMATIC FIGURE

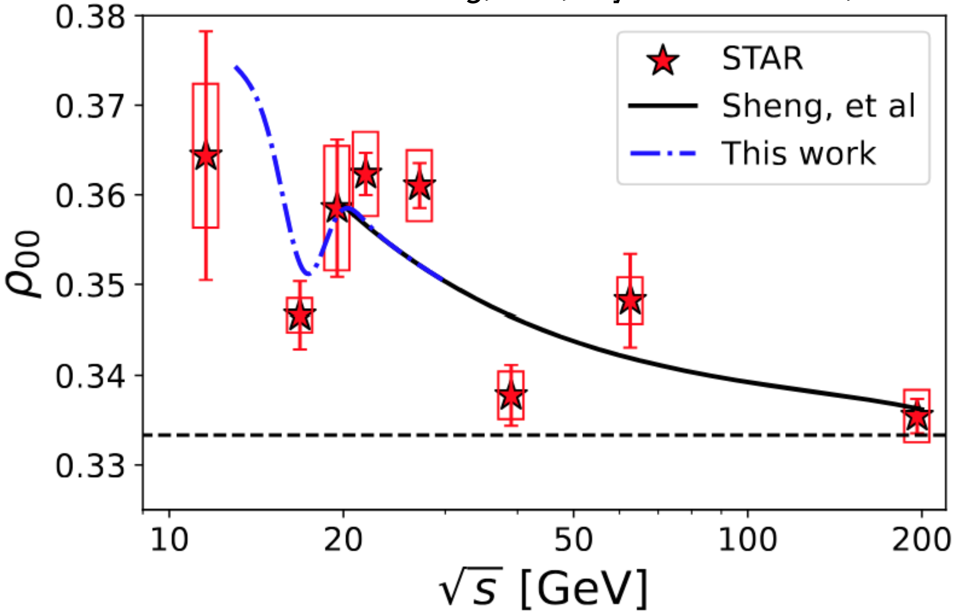



$$\rho_{00} = \frac{1 - \langle P_q P_{\bar{q}} \rangle}{3 + \langle P_q P_{\bar{q}} \rangle} \approx \bar{\rho}_{00} - \delta\rho_{00}^{\Omega}.$$

As additional contribution (to strong force field contribution)

G. Wilks' talk@SQM2024

X.-L. Sheng, et al, Phys. Rev. Lett. 131, 042304 (2023)



# SUMMARY

- Quark-antiquark spin correlation has a peak near the CEP
- Critical fluctuation near CEP can lead to non-monotonic behavior of spin alignment & Hyperon-anti-Hyperon correlation
- Spin alignment & Hyperon-anti-Hyperon correlation can serve as signatures for CEP

$$\frac{N_{H\bar{H}}^{\uparrow\uparrow} + N_{H\bar{H}}^{\downarrow\downarrow} - N_{H\bar{H}}^{\uparrow\downarrow} - N_{H\bar{H}}^{\downarrow\uparrow}}{N_{H\bar{H}}^{\uparrow\uparrow} + N_{H\bar{H}}^{\downarrow\downarrow} + N_{H\bar{H}}^{\uparrow\downarrow} + N_{H\bar{H}}^{\downarrow\uparrow}}$$

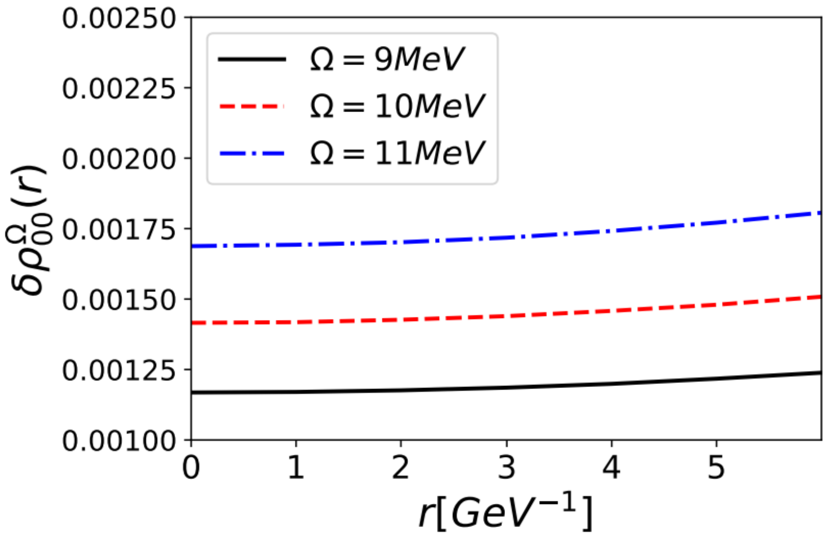
- Connection between spin and phase transition is an interesting direction
- More realistic and detailed studies in future

# THANKS！

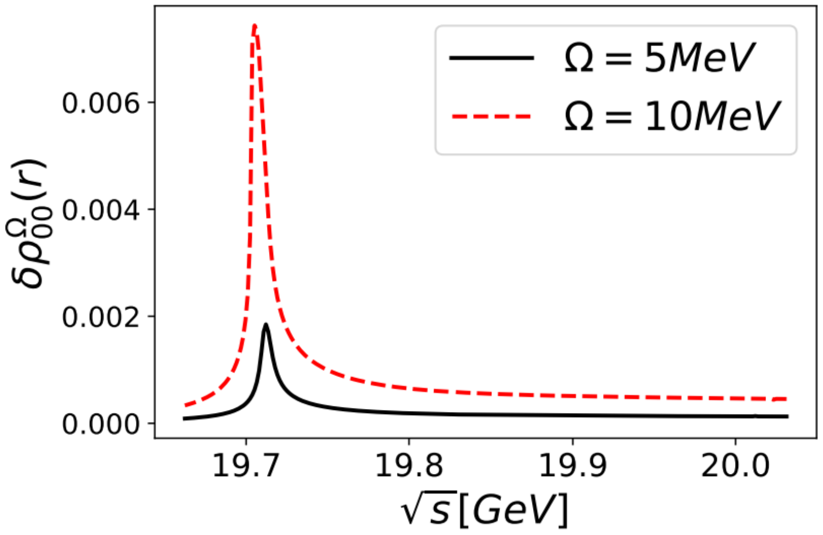
# BACK UP

# ROTATION DEPENDENCE

Far from CEP




Freezeout-2 at different rotation



# PNJL MODEL

| $\Lambda$ [MeV] | $m_0$ [MeV] | $G_{\mathrm{PNJL}}\Lambda^2$ | $N_f$ | $a_0$ | $a_1$ | $a_3$ | $b_3$ | $T_0$[MeV] |
|---|---|---|---|---|---|---|---|---|
| 651 | 5.5 | 2.135 | 2 | 3.51 | −2.47 | 15.2 | −1.75 | 210 |

$$
\begin{aligned}
&V_{\mathrm{PNJL}}(\Omega_q^s, \Omega_{\bar{q}}^s, \mu_q, \mu_{\bar{q}}; r=0)\\
=&\frac{[m-m_0]^2}{4G_{\mathrm{PNJL}}} - 2N_f \int_0^{\Lambda} \frac{\mathrm{d}^3 p}{(2\pi)^3} 3\varepsilon_p\\
&- N_f \int_0^{\infty} \frac{\mathrm{d}^3 p}{(2\pi)^3} \Big[ T \ln(1 + 3\Phi \mathrm{e}^{-(\varepsilon_p-\mu-\Omega_q^s/2-\mu_q)/T} + 3\bar{\Phi}\mathrm{e}^{-2(\varepsilon_p-\mu-\Omega_q^s/2-\mu_q)/T} + \mathrm{e}^{-3(\varepsilon_p-\mu-\Omega_q^s/2-\mu_q)/T})\\
&+ T \ln(1 + 3\Phi \mathrm{e}^{-(\varepsilon_p-\mu+\Omega_q^s/2-\mu_q)/T} + 3\bar{\Phi}\mathrm{e}^{-2(\varepsilon_p-\mu+\Omega_q^s/2-\mu_q)/T} + \mathrm{e}^{-3(\varepsilon_p-\mu+\Omega_q^s/2-\mu_q)/T})\\
&+ T \ln(1 + 3\Phi \mathrm{e}^{-(\varepsilon_p+\mu-\Omega_{\bar{q}}^s/2-\mu_{\bar{q}})/T} + 3\bar{\Phi}\mathrm{e}^{-2(\varepsilon_p+\mu-\Omega_{\bar{q}}^s/2-\mu_{\bar{q}})/T} + \mathrm{e}^{-3(\varepsilon_p+\mu-\Omega_{\bar{q}}^s/2-\mu_{\bar{q}})/T})\\
&+ T \ln(1 + 3\Phi \mathrm{e}^{-(\varepsilon_p+\mu+\Omega_{\bar{q}}^s/2-\mu_{\bar{q}})/T} + 3\bar{\Phi}\mathrm{e}^{-2(\varepsilon_p+\mu+\Omega_{\bar{q}}^s/2-\mu_{\bar{q}})/T} + \mathrm{e}^{-3(\varepsilon_p+\mu+\Omega_{\bar{q}}^s/2-\mu_{\bar{q}})/T})\Big]\\
&+ T^4 \Big\{ -\frac{1}{2}\Big[a_0 + a_1(\frac{T_0}{T}) + a_2(\frac{T_0}{T})^2\Big]\bar{\Phi}\Phi + b_3(\frac{T_0}{T})^3 \ln\Big[1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3+\Phi^3) - 3(\bar{\Phi}\Phi)^2\Big]\Big\},
\end{aligned}
$$

- Quantitatively agree with NJL model results