Nuclear equation of state at finite μ_B using deep learning assisted quasi-parton model

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arXiv: 2211.07994, 2501.10012
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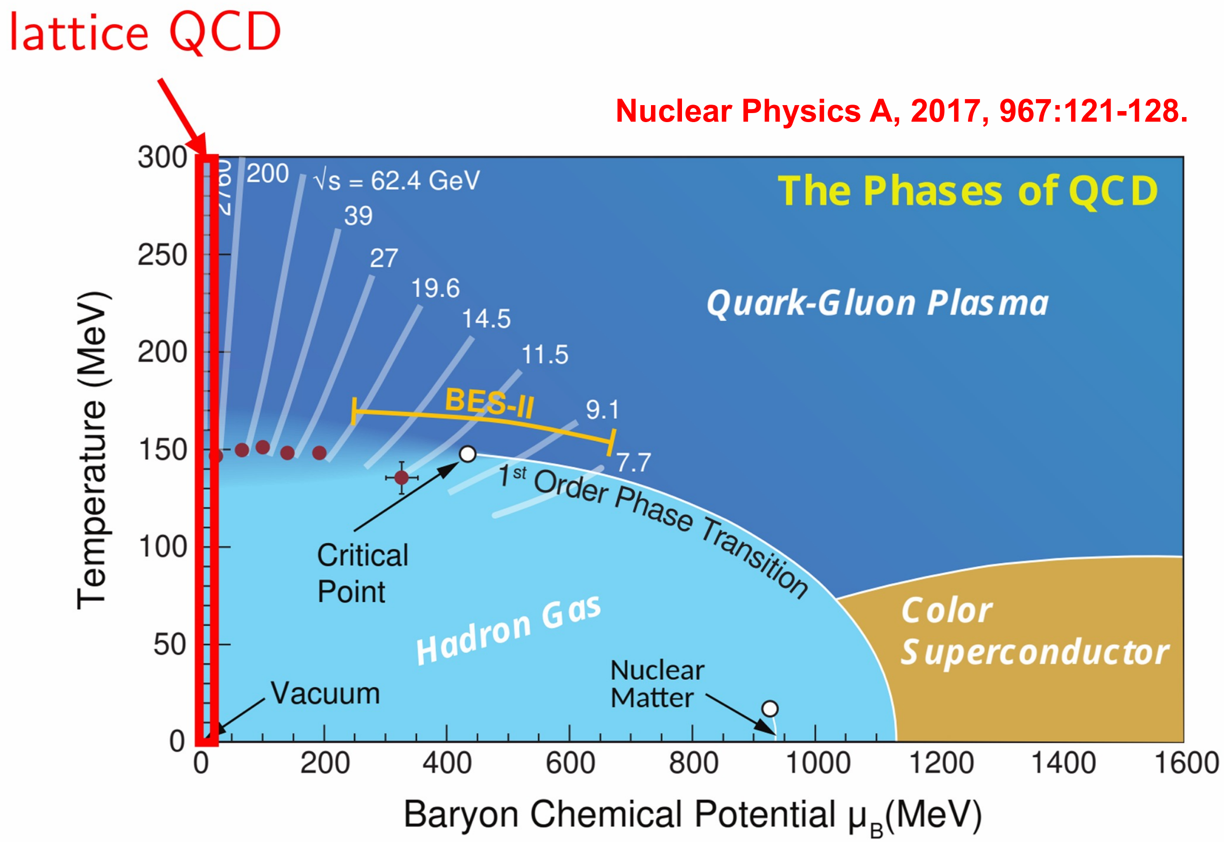
OUTLINE

- Motivation
  1. QCD equation of state
  2. Quasi-particles

- Deep-learning quasi-particles model
  1. QCD equation of state at $\mu_B = 0$
  2. QCD equation of state at $\mu_B > 0$

- Summary

Motivation: QCD equation of state



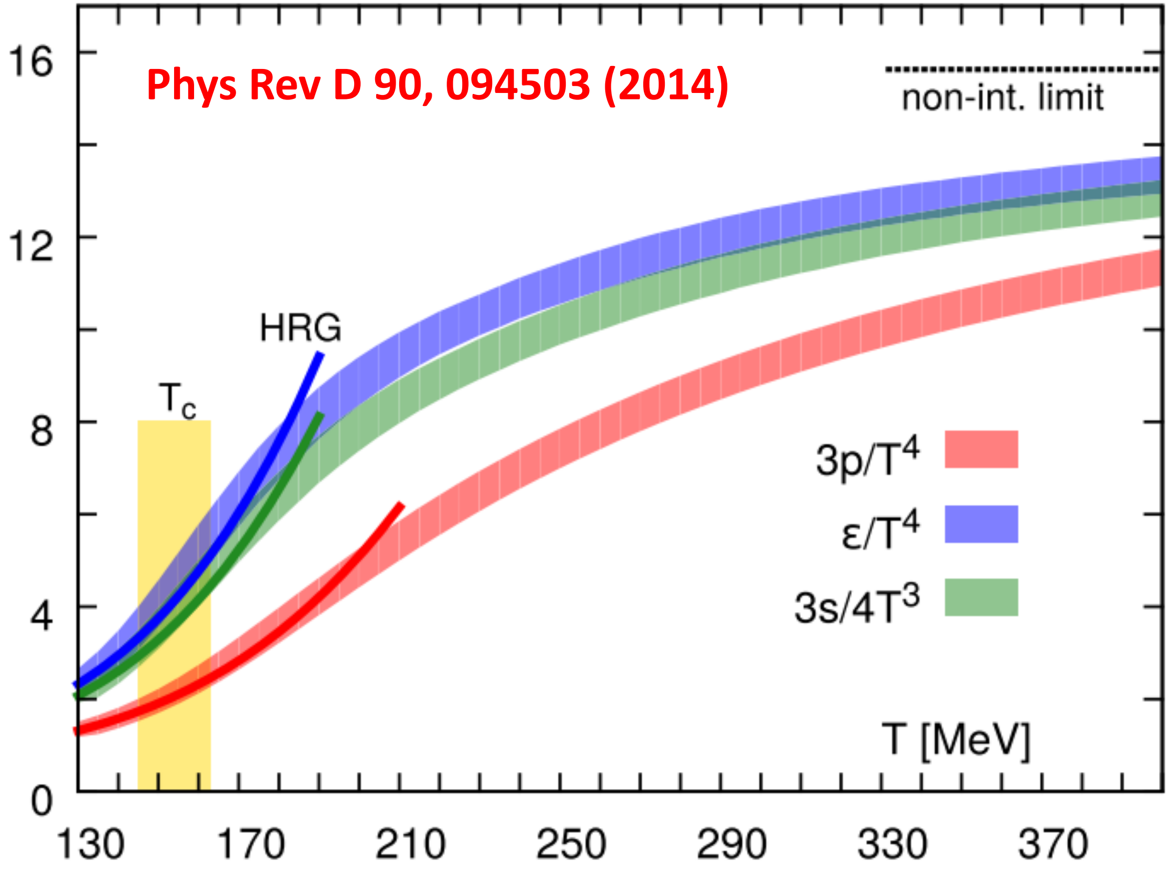
LQCD samples the quark field and gluon field from this partition function using the MCMC method

$$p(\phi) = e^{-S(\phi)}/Z, \quad \text{with} \quad Z = \int \prod_{j=1}^{D} d\phi_j e^{-S(\phi)},$$

It is vary expensive and time-consuming

1. The transition between QGP and the hadron gas is smooth crossover
2. At finite baryon chemical potential the transition is a 1st order phase transition

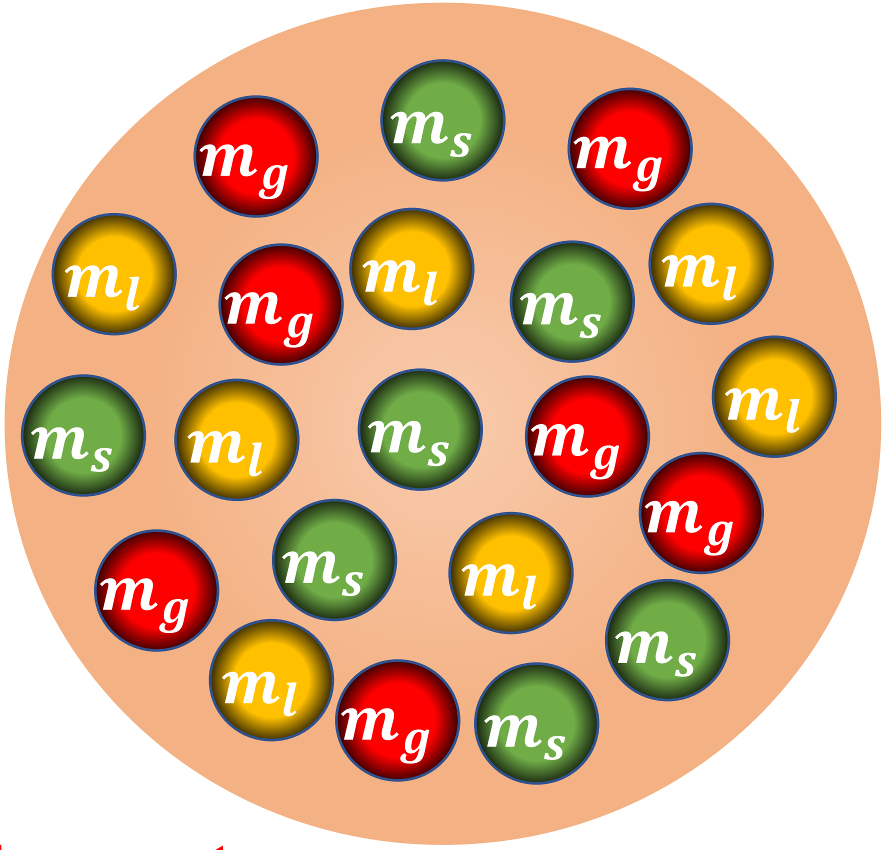
Motivation: QCD equation of state

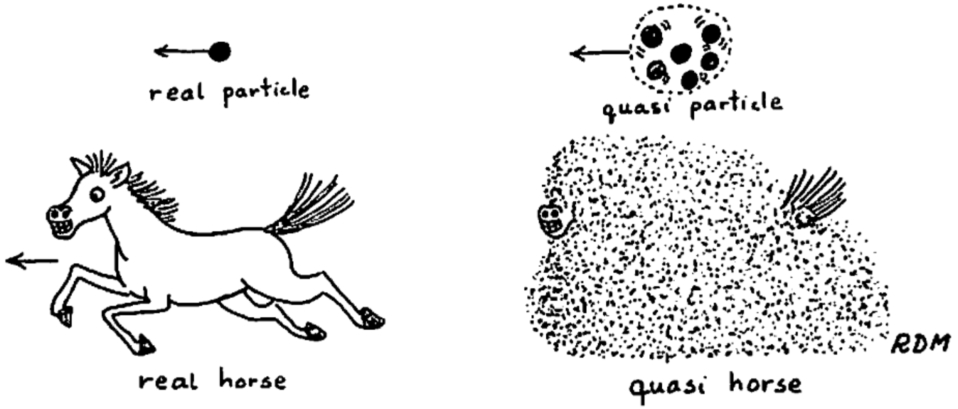


HRG model: the hot and dense QCD matter is considered as non-interacting hadrons.

- Tc < 200 MeV, QCD equation of state well described by HRG
- Tc > 200 MeV, nuclear matter transitions into the Quark-Gluon Plasma (QGP) phase.

Motivation: quasi-particle method





A Guide to Feynman Diagrams in the Many-Body Problem(1992), R.D.Mattuck

quasi particles of this particular system. Many different types of systems of interacting particles may be described in this manner, and in general we have

real particle + 'coat' or 'cloud' of other particles = quasi particle. (0.1)

Sometimes this same equation is stated in a more powerful terminology coming from quantum field theory:

'bare' particle + 'clothing' or 'cloud' = 'dressed' or 'clothed' or 'physical' or 'renormalized' particle. (0.2)

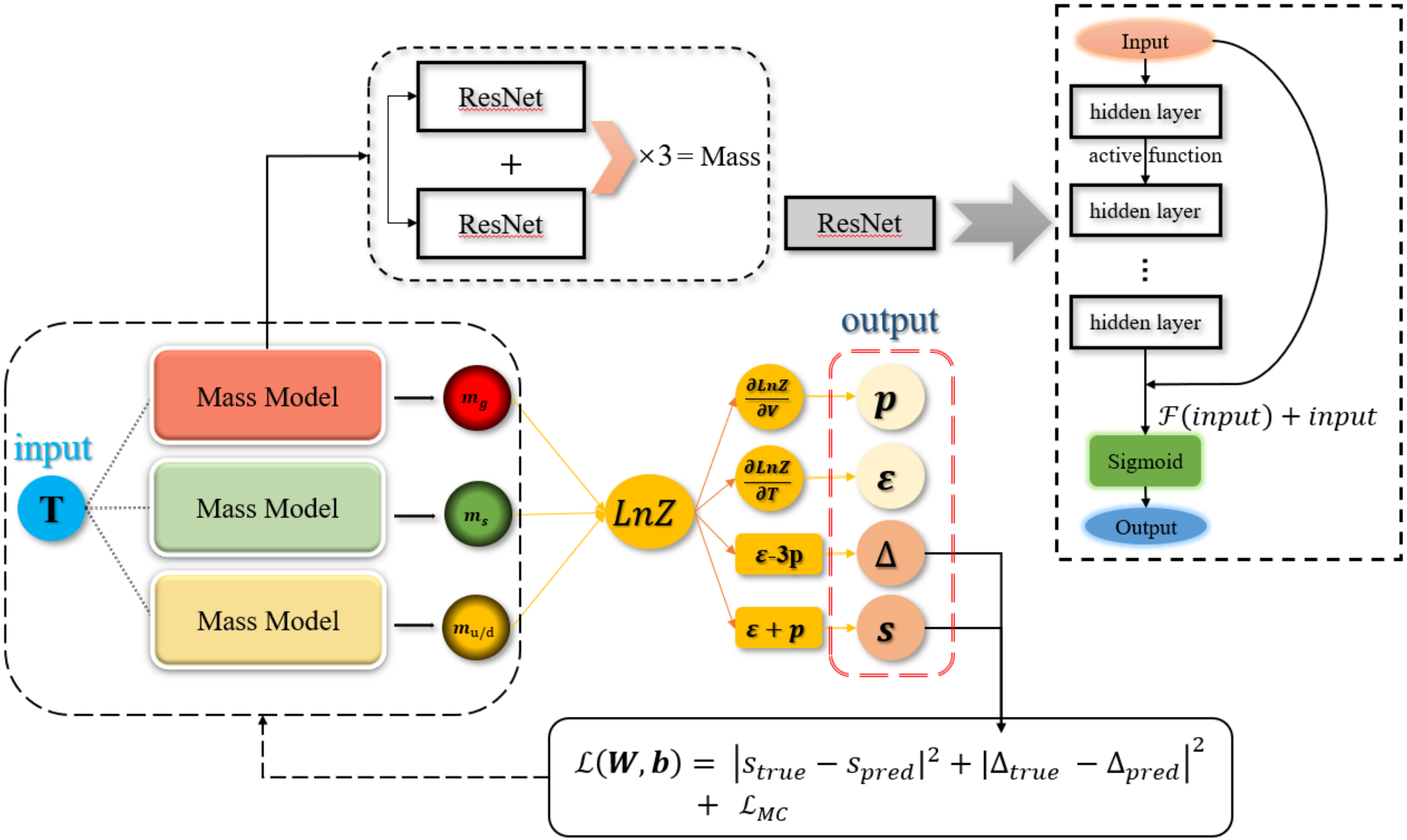
$$H = T + V_{eff} = \frac{P^2}{2M} + V_{eff}$$

$$H = \frac{P^2}{2M_{eff}}$$

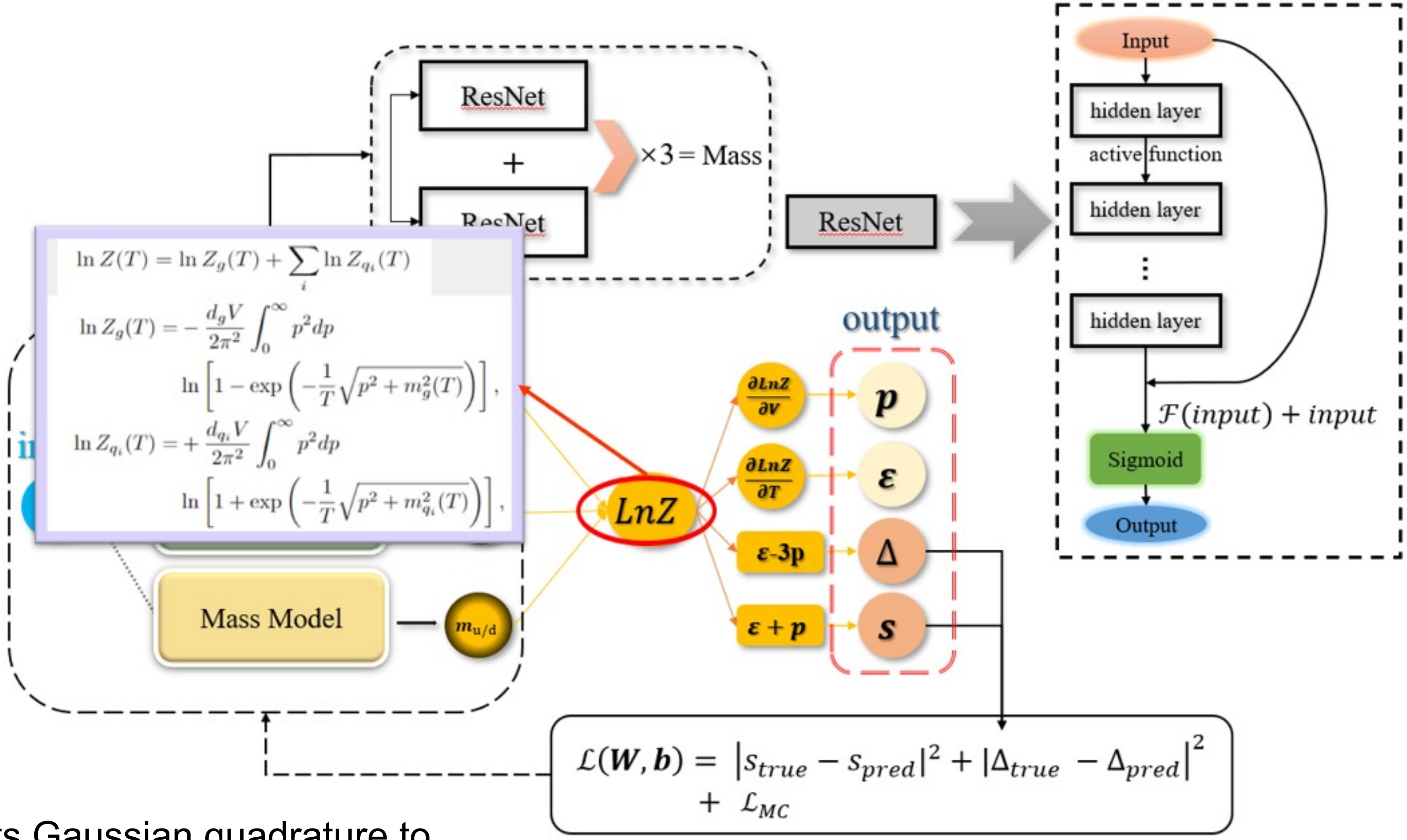
◆ **basic concept**

1. Absorbing the interaction potential into the mass
2. The non-interacting quasi-particles whose mass depends on the temperature of the medium
3. We construct a weakly interacting quasi-partons gas model which is an effective theory for strongly coupled QGPs

QCD equation of state at $\mu_B = 0$

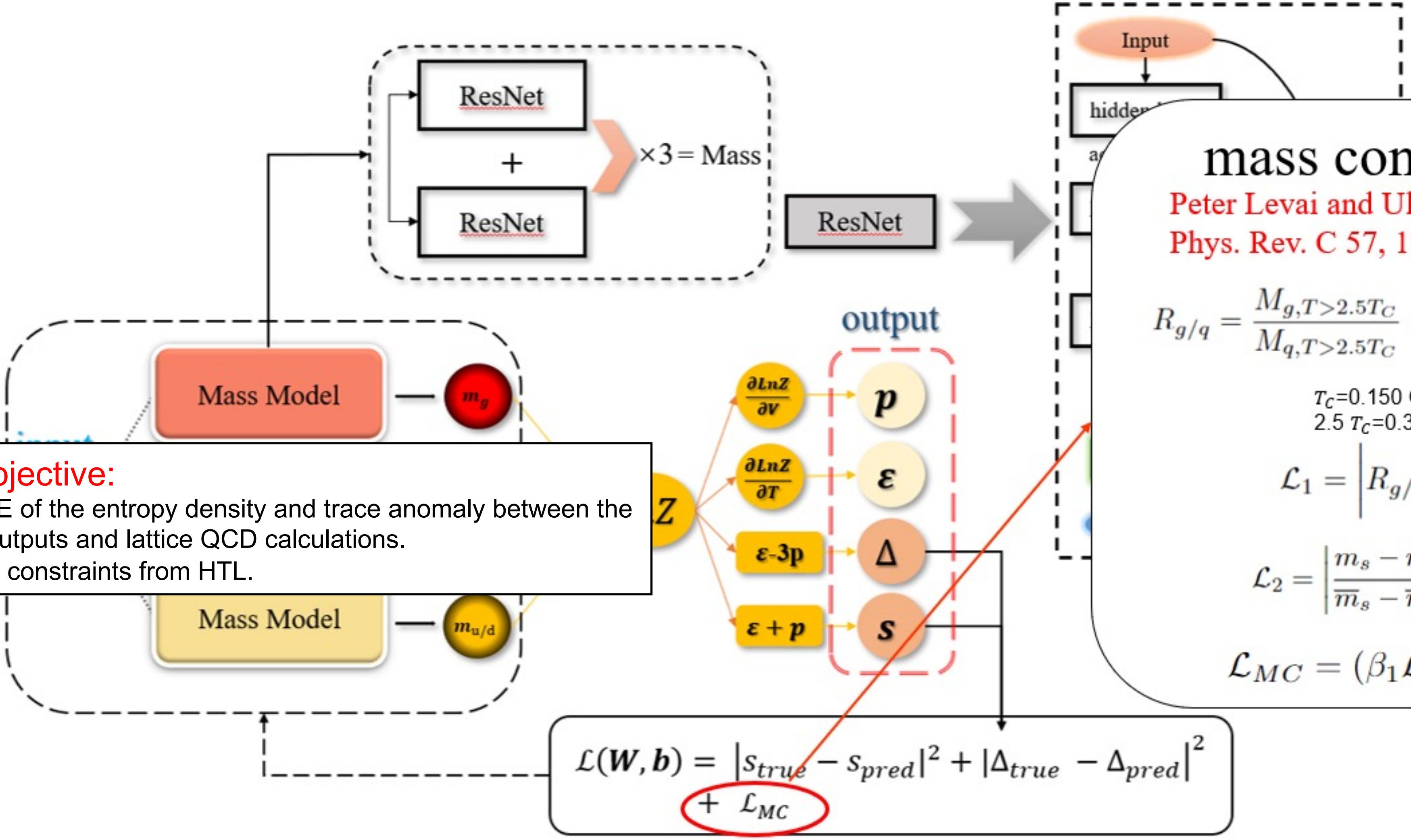


QCD equation of state at $\mu_B = 0$



✓ Using 25 points Gaussian quadrature to compute the partition function

QCD equation of state at $\mu_B = 0$



Training objective:

1. The RMSE of the entropy density and trace anomaly between the network outputs and lattice QCD calculations.
2. The mass constraints from HTL.

mass constraints

Peter Levai and Ulrich W. Heinz
Phys. Rev. C 57, 1879–1890 (1998

$$R_{g/q} = \frac{M_{g,T>2.5T_C}}{M_{q,T>2.5T_C}} = \sqrt{\frac{3}{2}\left(\frac{N_C}{3} + \frac{N_f}{6}\right)}$$

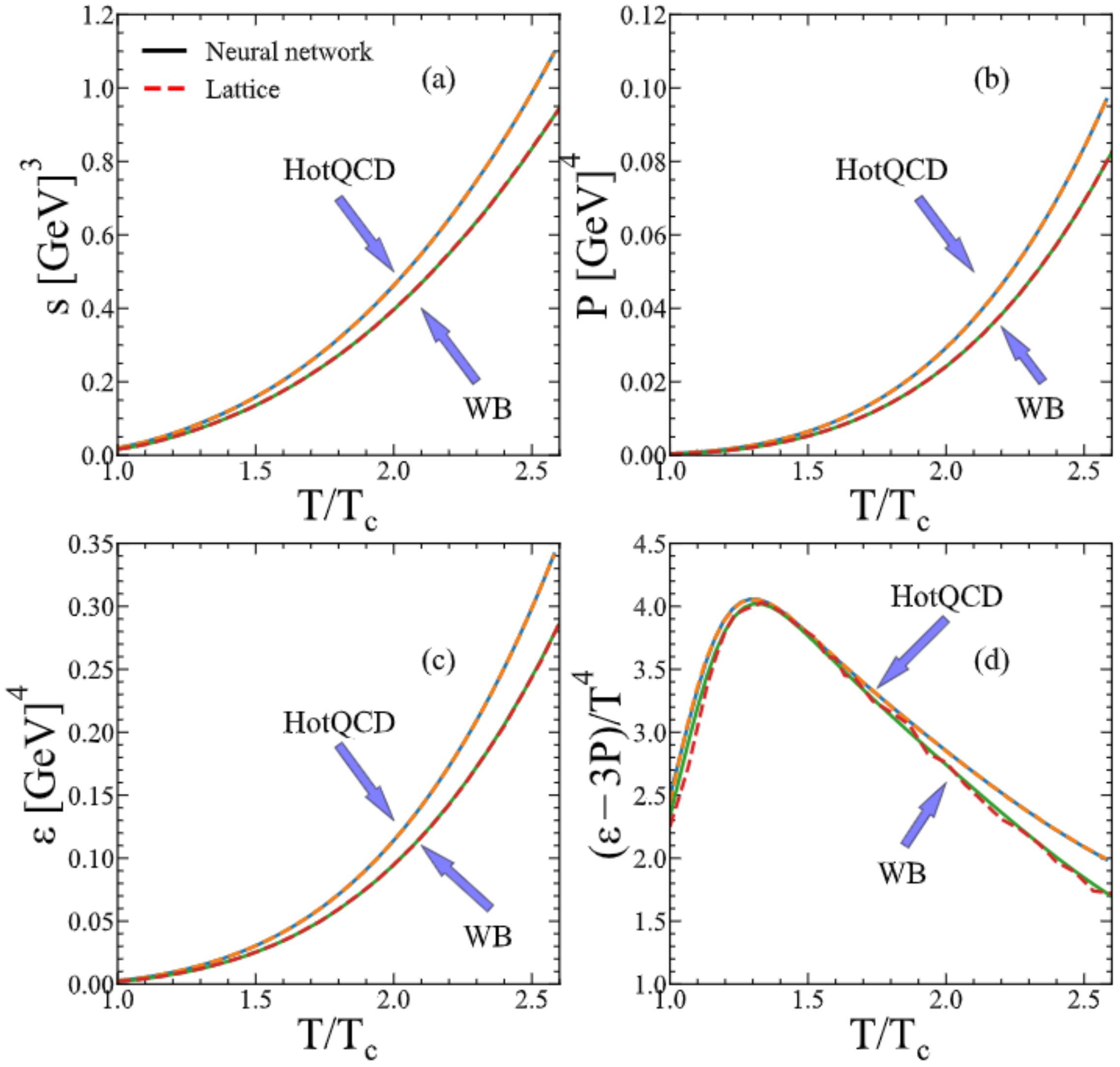
T_C=0.150 GeV
2.5 T_C=0.375GeV

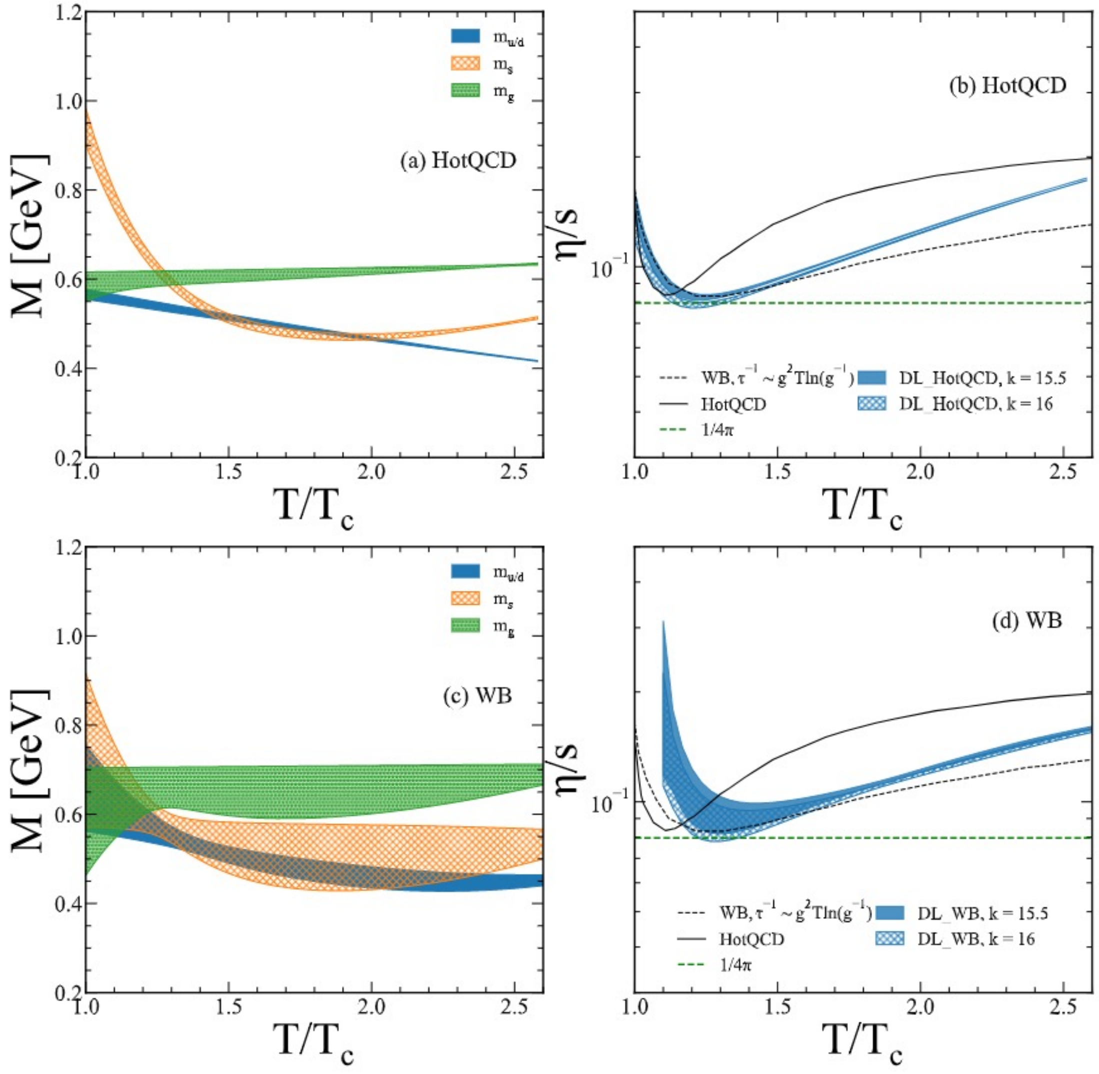
$$\mathcal{L}_1 = \left| R_{g/q} - \frac{3}{2} \right|.$$

$$\mathcal{L}_2 = \left| \frac{m_s - m_{u/d}}{\overline{m}_s - \overline{m}_{u/d}} - 1 \right|.$$

$$\mathcal{L}_{MC} = (\beta_1 \mathcal{L}_1 + \beta_2 \mathcal{L}_2)^2$$

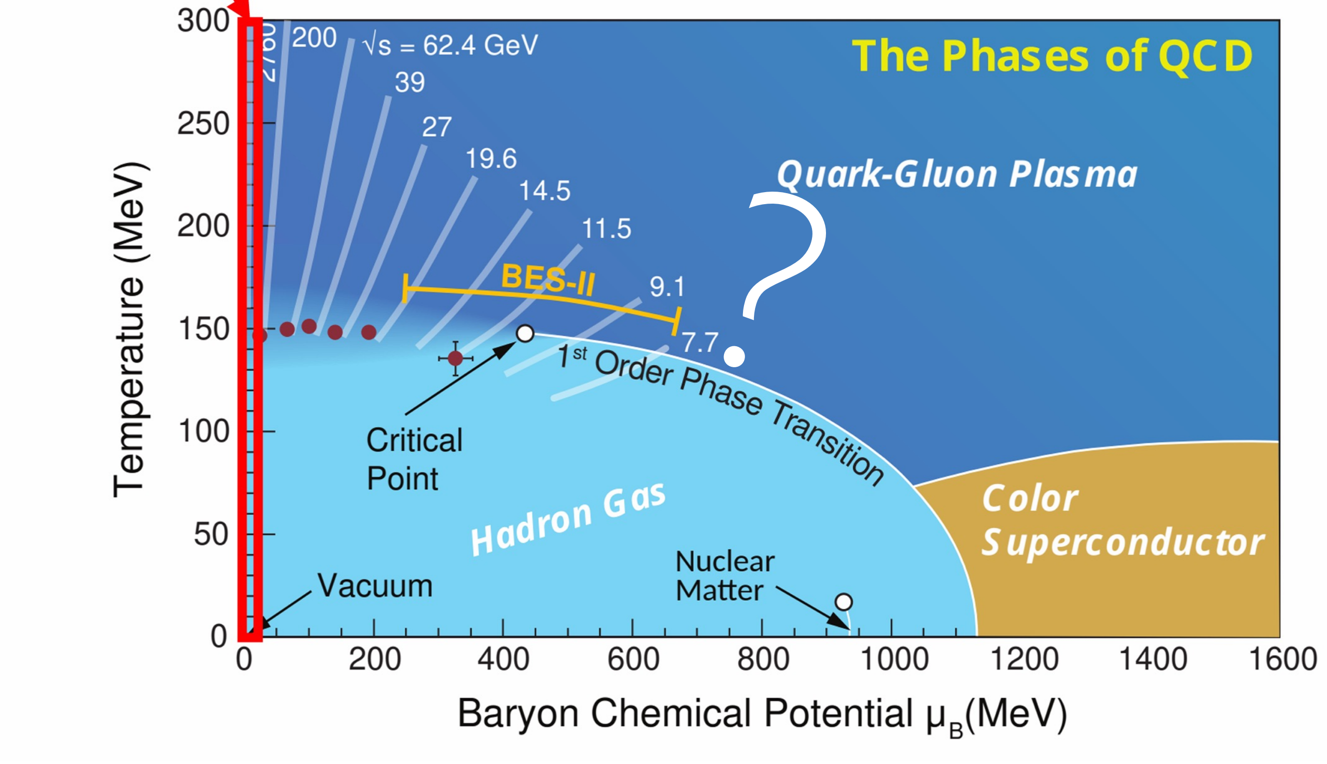
Result : $\mu_B = 0$


Neural network
Lattice
(a)
(b)
(c)
(d)
HotQCD
WB
s [GeV]^3
P [GeV]^4
ε [GeV]^4
(ε − 3P)/T^4
T/T_c

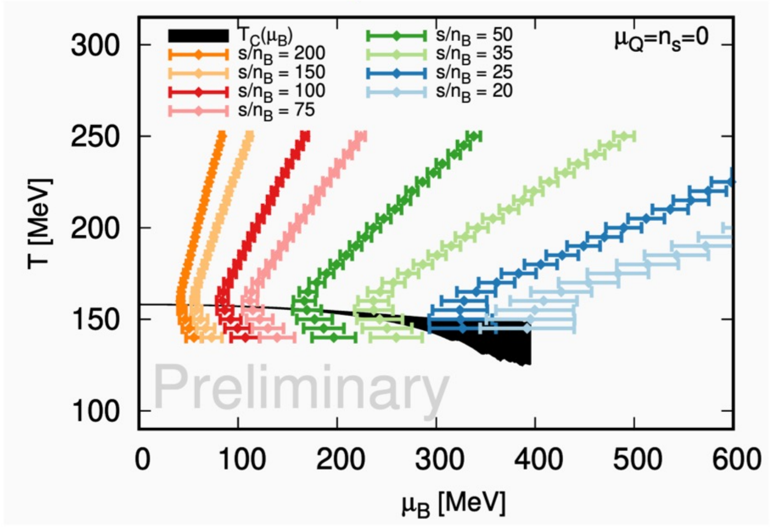

m_u/d
m_s
m_g
(a) HotQCD
(c) WB
(b) HotQCD
(d) WB
M [GeV]
η/s
T/T_c
WB, τ^-1 ~ g^2Tln(g^-1)
HotQCD
1/4π
DL_HotQCD, k = 15.5
DL_HotQCD, k = 16
DL_WB, k = 15.5
DL_WB, k = 16

QCD equation of state at $\mu_B > 0$

lattice QCD

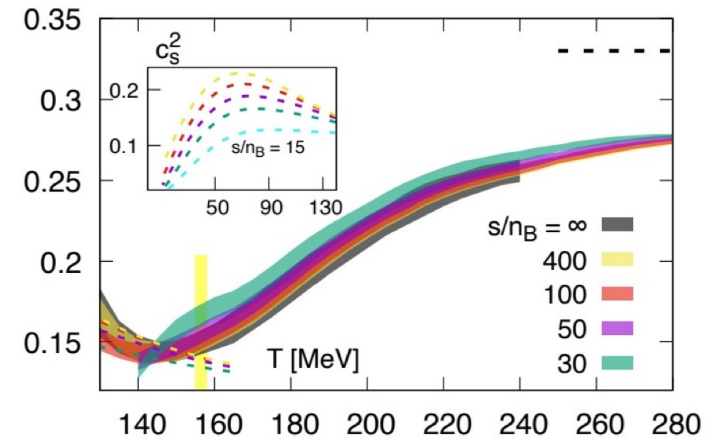


T' expansion scheme



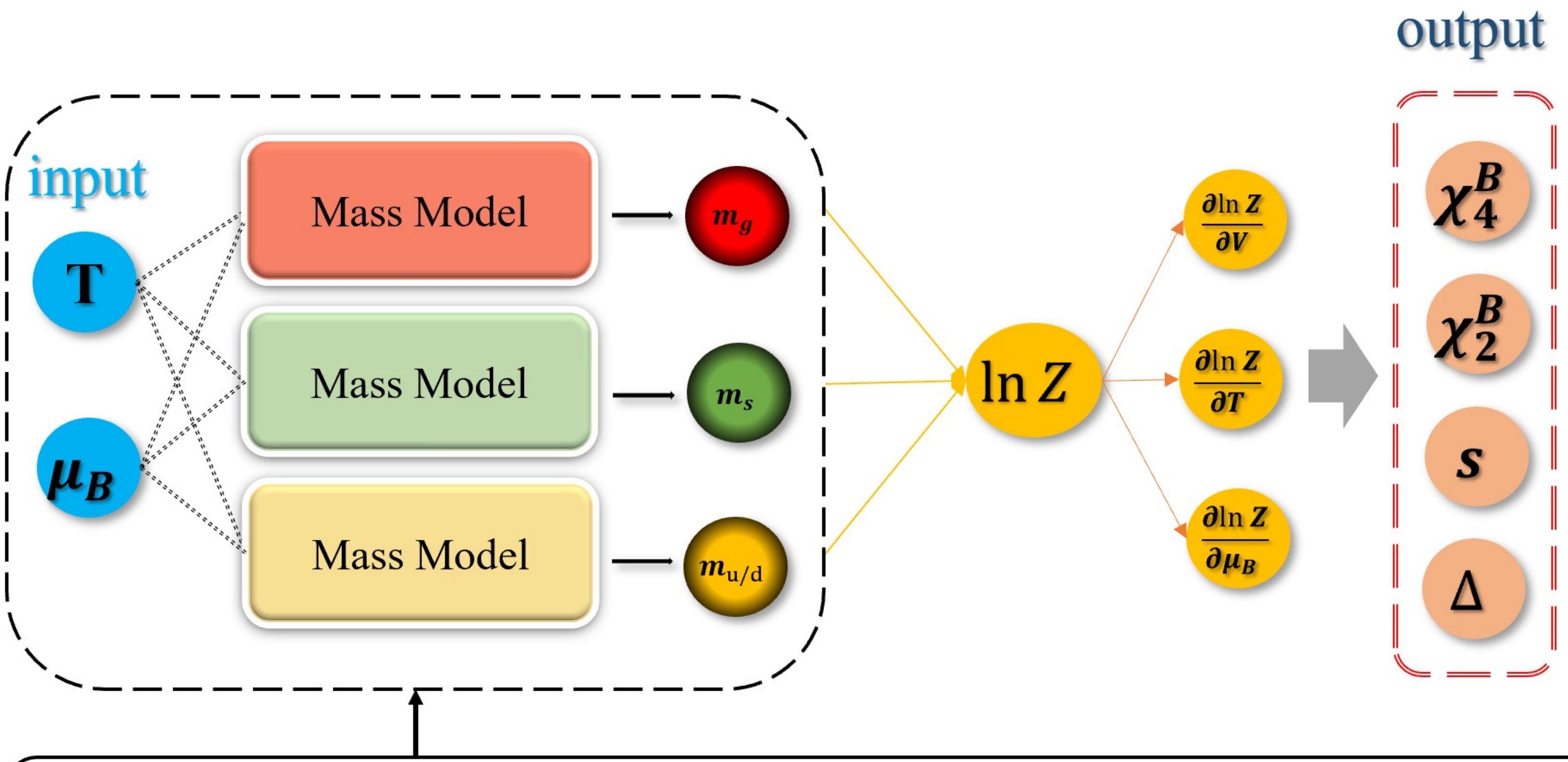
[Borsanyi et al. (WB), Phys. Rev. D 105, 114504 (2022)]

Padé approximants



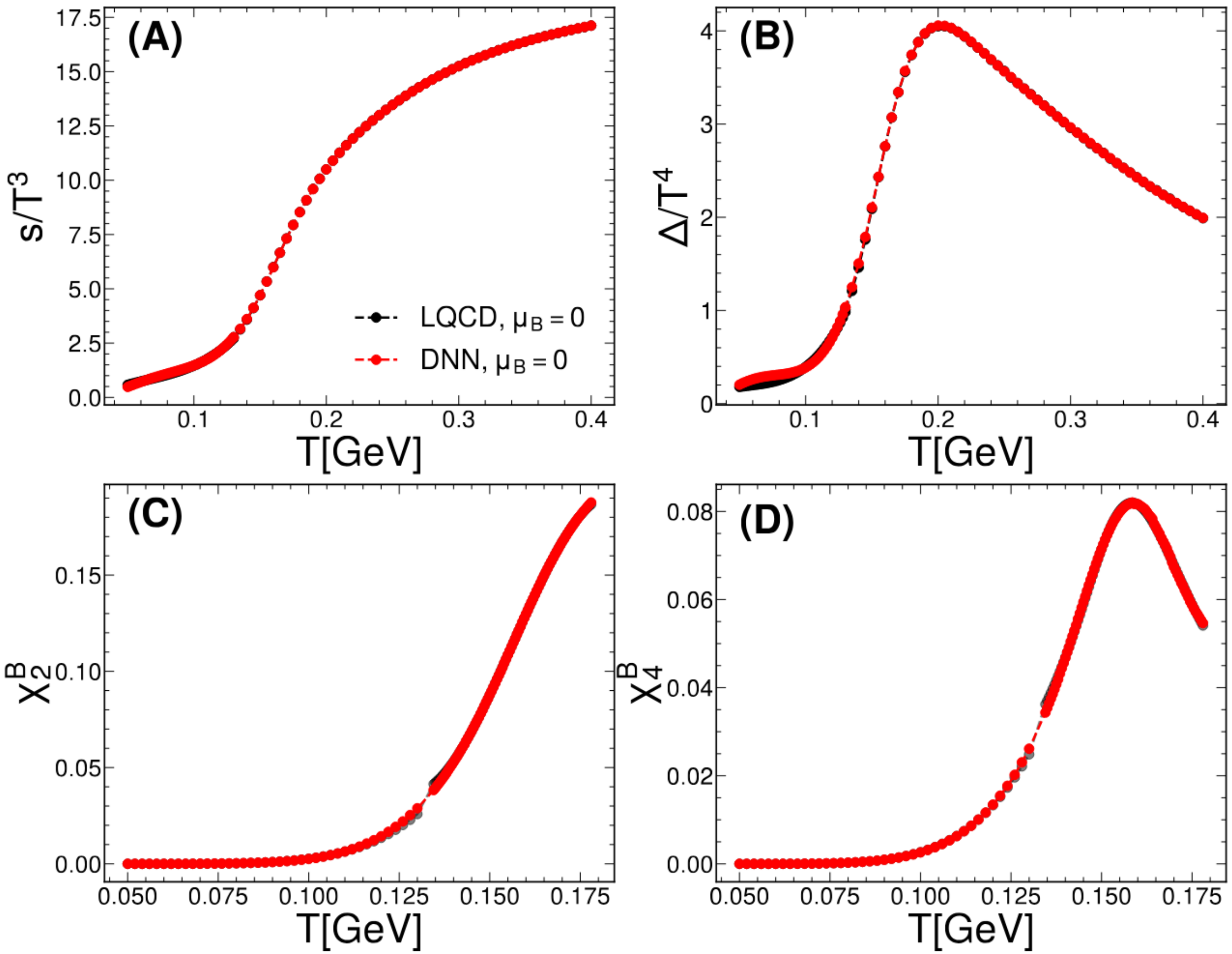
[Bollweg et al. (HotQCD), Phys. Rev. D 108, 014510 (2023)]

QCD equation of state at $\mu_B > 0$

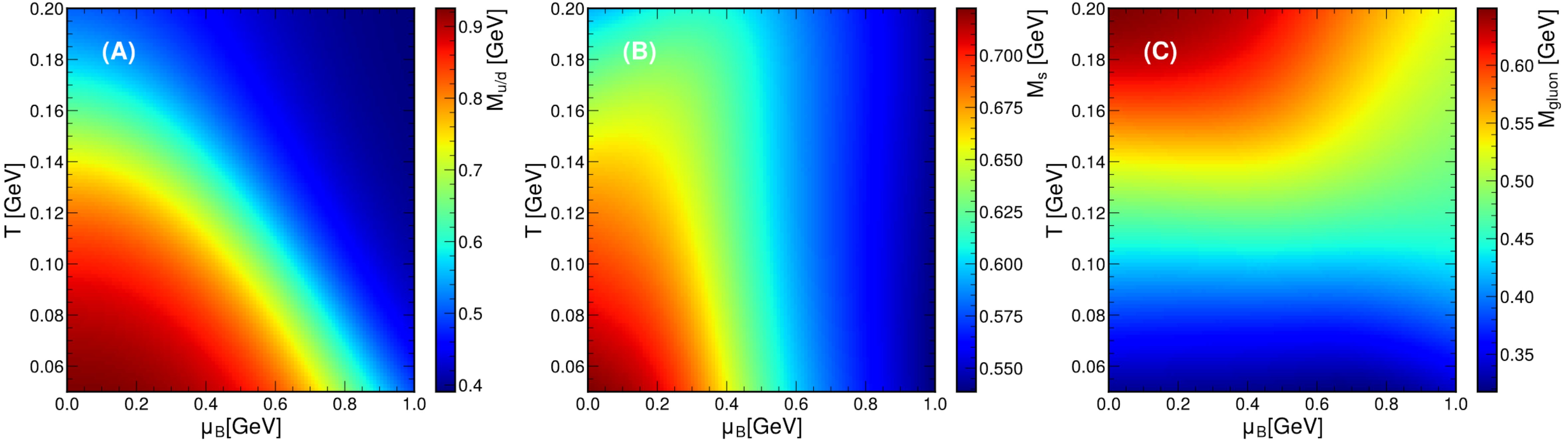


$$\mathcal{L}(\boldsymbol{W},\boldsymbol{b}) = |s_{NN} - s_{input}| + \left|\frac{\Delta_{NN} - \Delta_{input}}{T}\right| + |\chi^B_{2,NN} - \chi^B_{2,input}| + |\chi^B_{4,NN} - \chi^B_{4,input}| + \mathcal{L}_{MC}$$

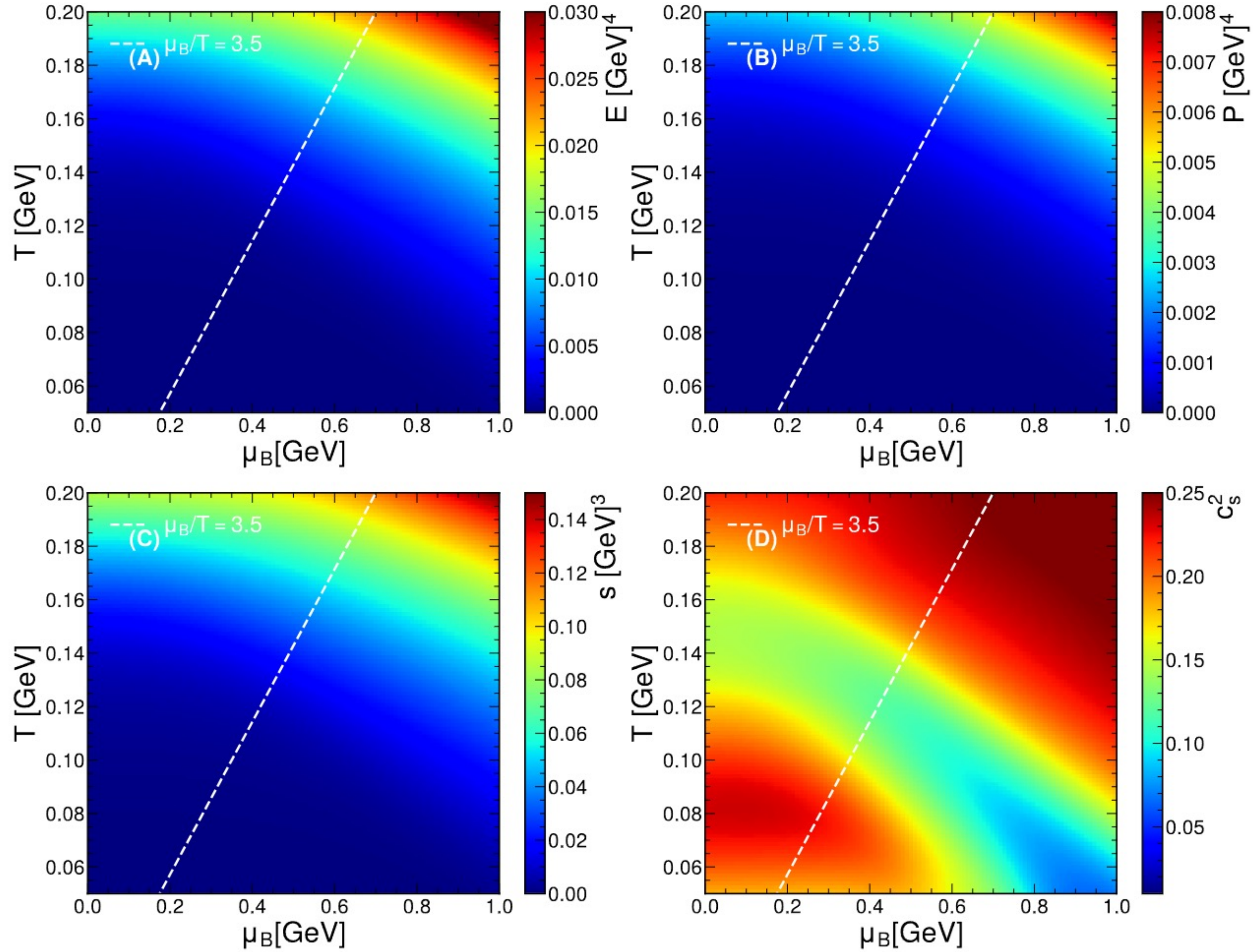
Result : $\mu_B > 0$



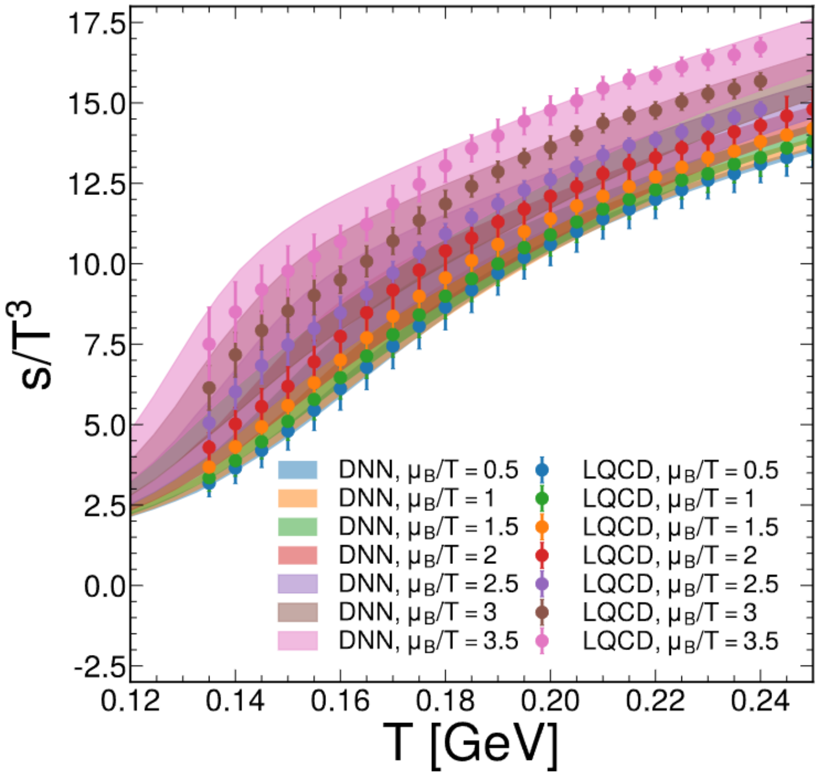
Result : $\mu_B > 0$


(A)
(B)
(C)
T [GeV]
μ_B[GeV]
$M_{u/d}$ [GeV]
M_s [GeV]
M_{gluon} [GeV]
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.0
0.2
0.4
0.6
0.8
1.0

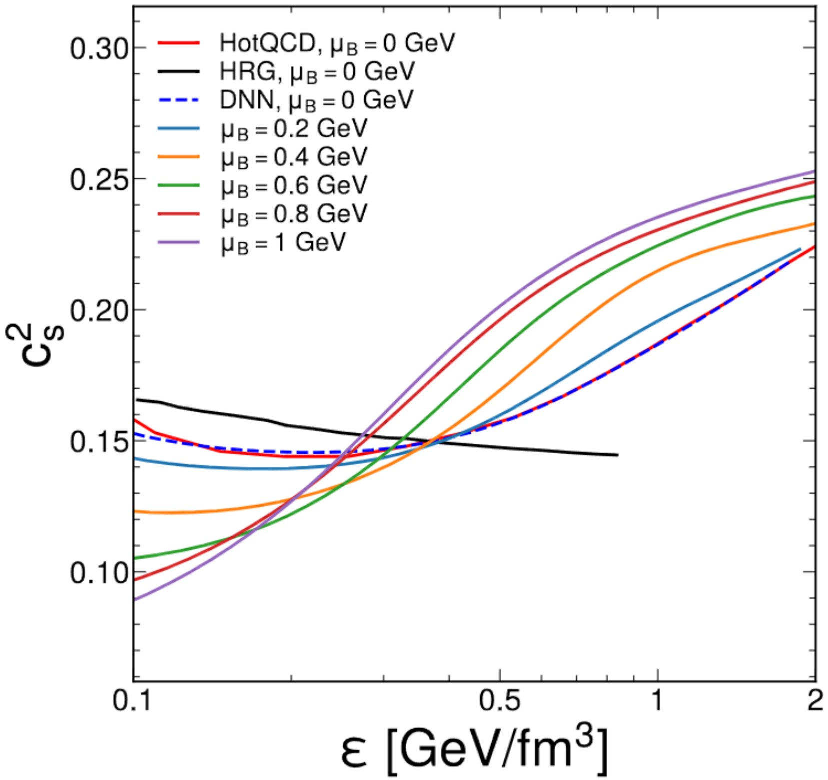
Result : $\mu_B > 0$


(A)
μB/T = 3.5
E [GeV]4
T [GeV]
μB[GeV]
(B)
μB/T = 3.5
P [GeV]4
T [GeV]
μB[GeV]
(C)
μB/T = 3.5
s [GeV]3
T [GeV]
μB[GeV]
(D)
μB/T = 3.5
c2s
T [GeV]
μB[GeV]

Result : $\mu_B > 0$

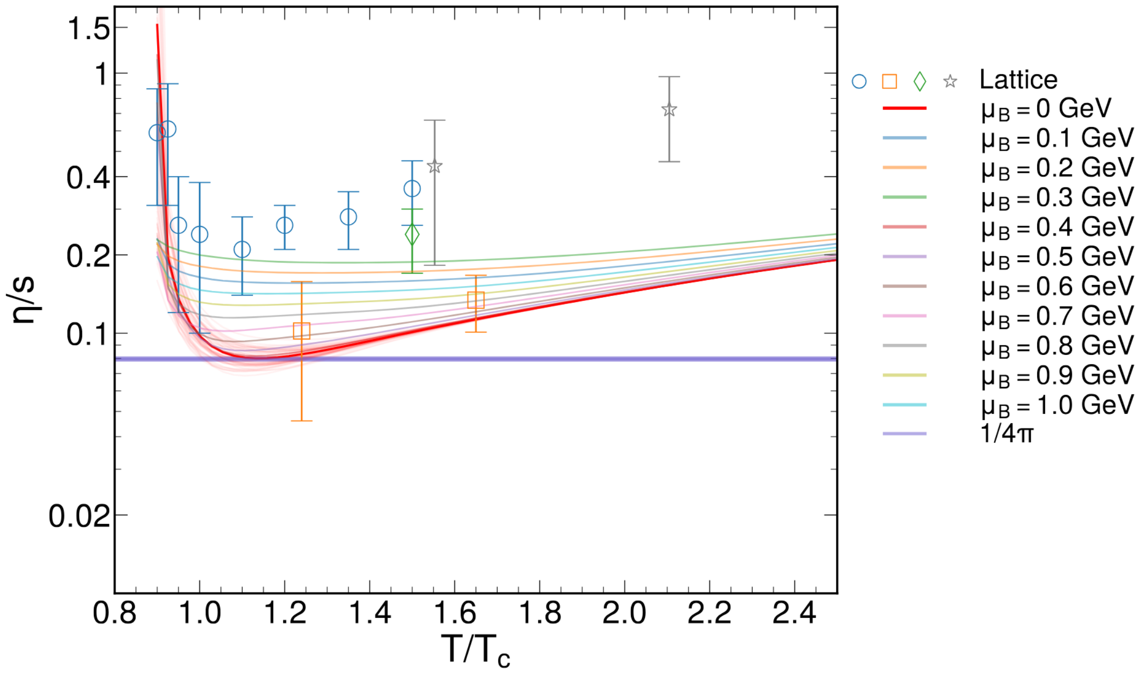


s/T3
T [GeV]
DNN, μB/T = 0.5
DNN, μB/T = 1
DNN, μB/T = 1.5
DNN, μB/T = 2
DNN, μB/T = 2.5
DNN, μB/T = 3
DNN, μB/T = 3.5
LQCD, μB/T = 0.5
LQCD, μB/T = 1
LQCD, μB/T = 1.5
LQCD, μB/T = 2
LQCD, μB/T = 2.5
LQCD, μB/T = 3
LQCD, μB/T = 3.5

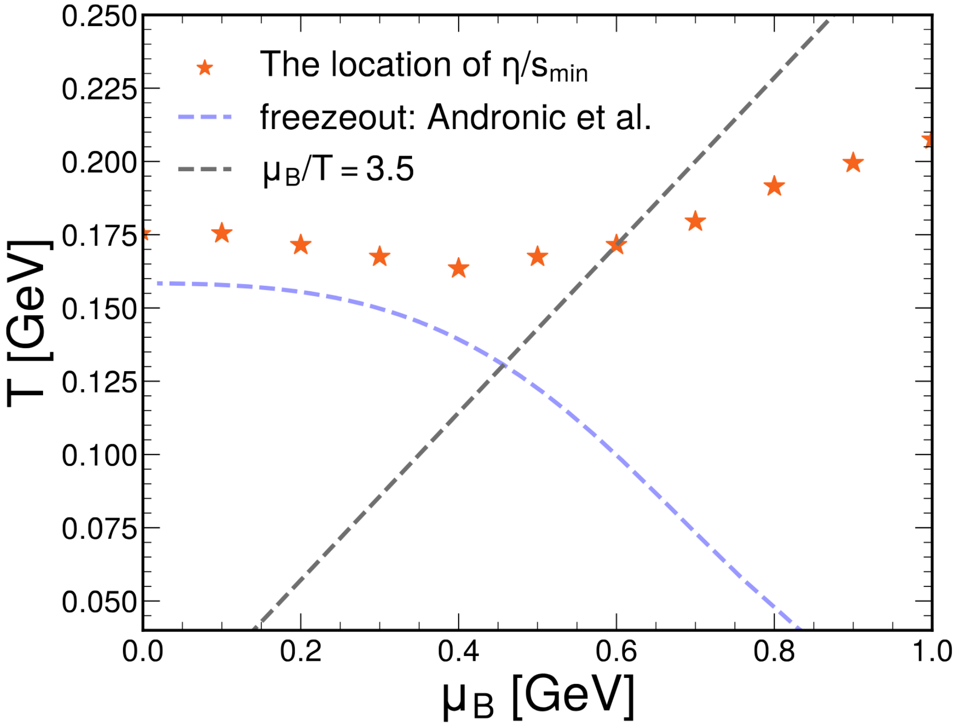


c2s
ε [GeV/fm3]
HotQCD, μB = 0 GeV
HRG, μB = 0 GeV
DNN, μB = 0 GeV
μB = 0.2 GeV
μB = 0.4 GeV
μB = 0.6 GeV
μB = 0.8 GeV
μB = 1 GeV

Result : $\mu_B > 0$



η/s
1.5
1
0.4
0.2
0.1
0.02
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
T/Tc
Lattice
μB = 0 GeV
μB = 0.1 GeV
μB = 0.2 GeV
μB = 0.3 GeV
μB = 0.4 GeV
μB = 0.5 GeV
μB = 0.6 GeV
μB = 0.7 GeV
μB = 0.8 GeV
μB = 0.9 GeV
μB = 1.0 GeV
1/4π



The location of η/smin
freezeout: Andronic et al.
μB/T = 3.5
T [GeV]
0.250
0.225
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.0
0.2
0.4
0.6
0.8
1.0
μB [GeV]

• Summary

1. We use three neural networks to represent the quasi-particles masses can well reproduce the lattice QCD EoS at zero chemical potential.

2. We can calculate the entropy density at finite baryon chemical potentials, which is consisted with Lattice QCD result using Taylor expansions.

3. The QCD equation of state at finite chemical potential can be used in relativistic hydrodynamics simulations to study the QCD matter produced in the BESII region.