Shear and Bulk Viscosities of Gluon Plasma across the Transition Temperature from Lattice QCD

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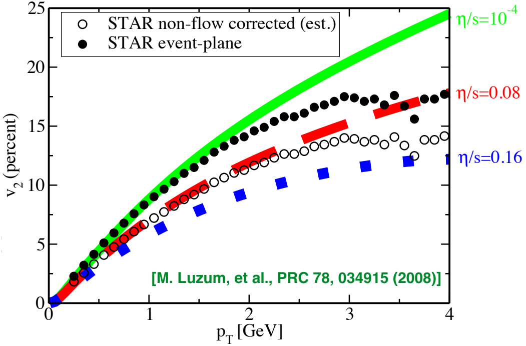
Introduction



• inputs for tranport/hydro models

- η/s quantifies the dissipation processes in the hydrodynamics.

  G. Denicol et al., PRC 80, 064901 (2008)

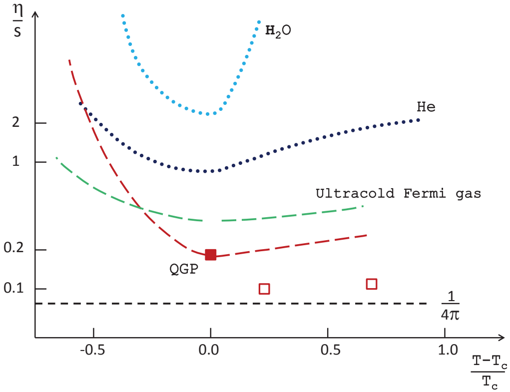
- Small η/s suggested by phenomenological interpretation of experimental data.

  K. H. Ackermann et al., (STAR), PRL 86, 402 (2001)

- Extracting η/s from experiments needs accurate inputs: initial condition, EoS ...

  U. Heinz et al., Annu. Rev. Nucl. Part. Sci. 63, 123 (2013)

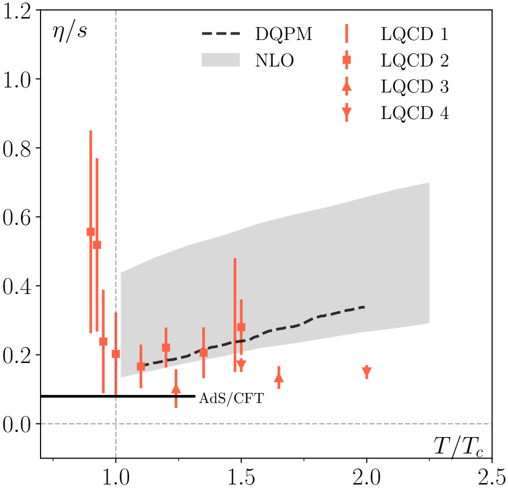
Introduction



S.Cremonini et al., JHEP 1208 (2012) 167

- η/s of QGP is sensitive to the phase transitions.
- Determinations of viscosities require theoretical inputs.

Determinations of viscosities from theory



- NLO weak-coupling calculation
  J. Ghiglieri et al., JHEP 03, 179 (2018)
- Dynamical Quasi-Particle Model (DQPM)
  R. Marty et al., PRC 88, 045204 (2013)
- ...
- Lattice QCD (all quenched):
  multi-level: 2: N. Astrakhantsev et al., JHEP 04, 101 (2017)
  3: H. B. Meyer, PRD 76, 101701 (2007)
  4: S. Borsanyi et al., PRD 98, 014512 (2018)
  gradient flow (extendable to full QCD):
  1: H. T. Shu et al., PRD 108, 014503 (2023)

1.5 $T_c \Rightarrow$ **this work:** $0.76 \leq T/T_c \leq 2.25$

Theoretical framework

$$G_{\text{shear}}(\tau,\tau_F) = \frac{1}{10}\int \mathrm{d}^3x \left\langle \pi_{ij}(0,\vec{0},\tau_F)\,\pi_{ij}(\tau,\vec{x},\tau_F)\right\rangle$$

Renormalization | $a \to 0, \tau_F \to 0$

$$G(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \frac{\cosh[\omega(1/2T-\tau)]}{\sinh(\omega/2T)} \rho(\omega,T)$$

Inverse problem

Spectra reconstruction $\rho(\omega,T)$

Kubo formula

$$\eta(T) = \lim_{\omega\to 0} \frac{\rho_{\text{shear}}(\omega,T)}{\omega}$$

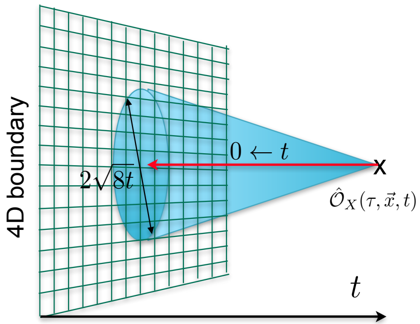
Key Points:

- Lattice computation of EMT correlators.
- Spectra reconstruction from the correlators.

Challenges:

- Severe UV fluctuations in the correlators.
- Theoretical uncertainty in the spectra reconstruction.

Noise reduction technique: gradient flow



Flow equation :

$$\frac{\mathrm{d}B_\mu(x,\tau_\mathrm{F})}{\mathrm{d}\tau_\mathrm{F}} = D_\nu G_{\nu\mu}(x,\tau_\mathrm{F})$$

$$B_\nu(x,\tau_\mathrm{F}=0) = A_\nu(x)$$

LO solution:

$$B_\nu(x,\tau_F) \propto \exp\left(\frac{-(x-y)^2}{\sqrt{8\tau_F}^2/2}\right) B_\nu(y)$$

M. Lüscher, JHEP 08, 071 (2010)

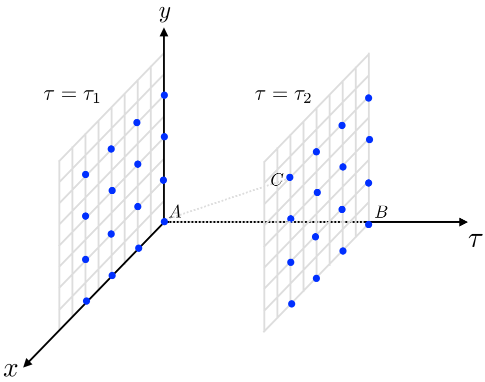
Smearing radius: $\sqrt{8\tau_\mathrm{F}}$.

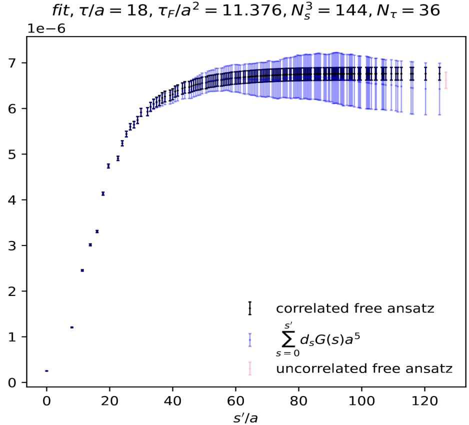
Advantage:

- The UV fluctuations strongly suppressed.
- Provide renormalization framework for EMT:

  $$T_{\mu\nu}(\tau_F,x) = c_1(\tau_F)U_{\mu\nu}(\tau_F,x) + 4c_2(\tau_F)\delta_{\mu\nu}E(\tau_F,x)$$
- Operator Product Expansion of $G(\tau,\tau_\mathrm{F})$ in τ_F/τ^2.

Noise reduction technique: blocking fit





$$G(\tau) = \frac{a^3}{V} \sum_{v_1} \Big[\sum_{\vec{m}\in v_1} \mathcal{O}(\tau_1, \vec{m}) \Big] \sum_{v_2} \Big[\sum_{\vec{n}\in v_2} \mathcal{O}(\tau_2, \vec{n}) \Big]$$

3-7 SNR improvement: save computation cost.

L. Altenkort et al. PRD 105, 094505 (2022)

Lattice setup

- **Pure SU(3) Yang-Mills gauge theory:**

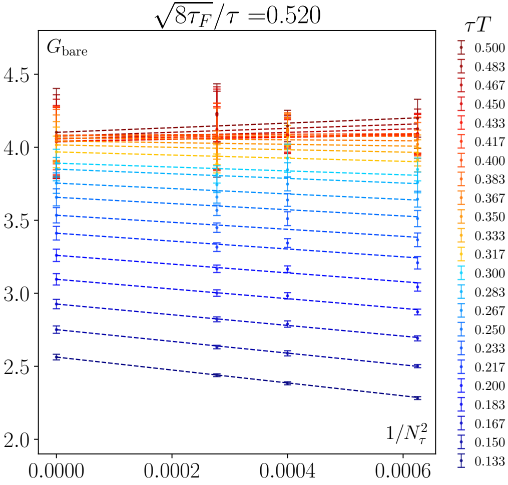
T/T_c	0.76			0.9			1.125			1.267			1.5			1.9			2.25		
N_σ	96	120	144	96	120	144	96	120	144	96	120	144	96	120	144	96	120	144	96	120	144
N_τ	40	50	60	40	50	60	32	40	48	24	30	36	24	30	36	16	20	24	16	20	24
#Conf.	5000			5000			5000			5000			5000			5000			5000		

- **Lattice spacing**:

β	7.0606	7.2005	7.2456	7.3874	7.3986	7.5416
a (fm)	0.02068	0.01746	0.01654	0.01397	0.01379	0.01164

H.-T. Ding, H.-T. Shu and CZ, work in progress

Continuum extrapolation



$a \to 0$ at $0.9\,T_c$ in the shear channel.

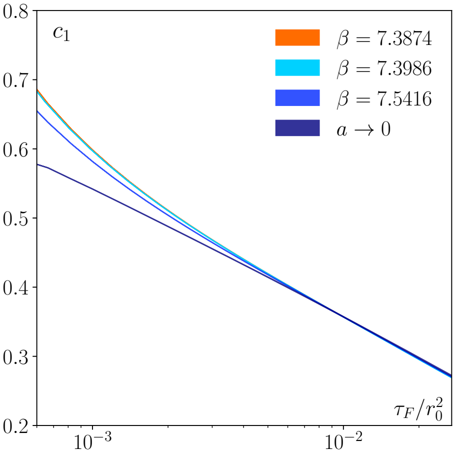
The joint fit Ansatz:

$$G_{\mathrm{bare}}(N_\tau) = G_{\mathrm{bare}}^{\tau T}(a=0) + \left(b + m_1 \cdot \tau T + \frac{m_2}{\tau T}\right)/N_\tau^2$$

$$G_{\mathrm{bare}} = \frac{G^{\mathrm{t.l.}}(\tau T, \tau_{\mathrm{F}})}{G^{\mathrm{norm}}(\tau T)}$$

The Ansatz describes the data well.

Renormalization



c_1 & c_2: renormalization constants matching Gradient Flow scheme to $\overline{\text{MS}}$ scheme.

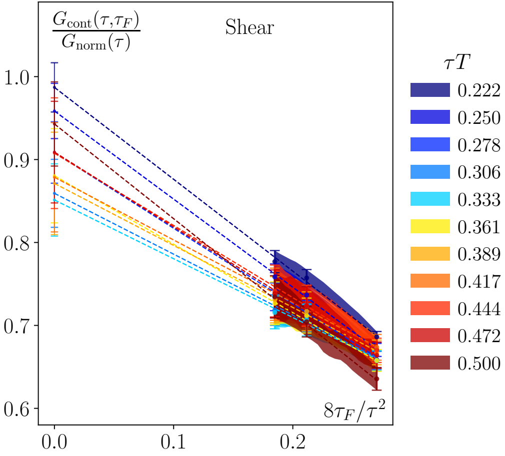
$$T_{\mu\nu}(\tau_F, x) = c_1(\tau_F)\, U_{\mu\nu}(\tau_F, x) + 4c_2(\tau_F)\, \delta_{\mu\nu} E(\tau_F, x)$$

$$c_1(\tau_F) = \frac{1}{g^2(\mu)} \sum_{n=0}^{2} k_1^{(n)}(L(\mu, \tau_F)) \left[\frac{g^2(\mu)}{(4\pi)^2}\right]^n$$

To control lattice spacing effects in c_1:

$$\tau_F/r_0^2 \geq 9 \times 10^{-4}$$

Flow time extrapolation



$\tau_F \to 0$ at $1.5\,T_c$.

$\tau_{\mathrm{F}} \to 0$ extrapolation Ansatz:

$$G(\tau_{\mathrm{F}}/\tau^2, \tau T) = G^{\tau T}_{\tau_{\mathrm{F}}=0} + \left(b + m_1 \cdot \tau T + \frac{m_2}{\tau T}\right) \cdot \tau_{\mathrm{F}}/\tau^2$$

Flow time window:

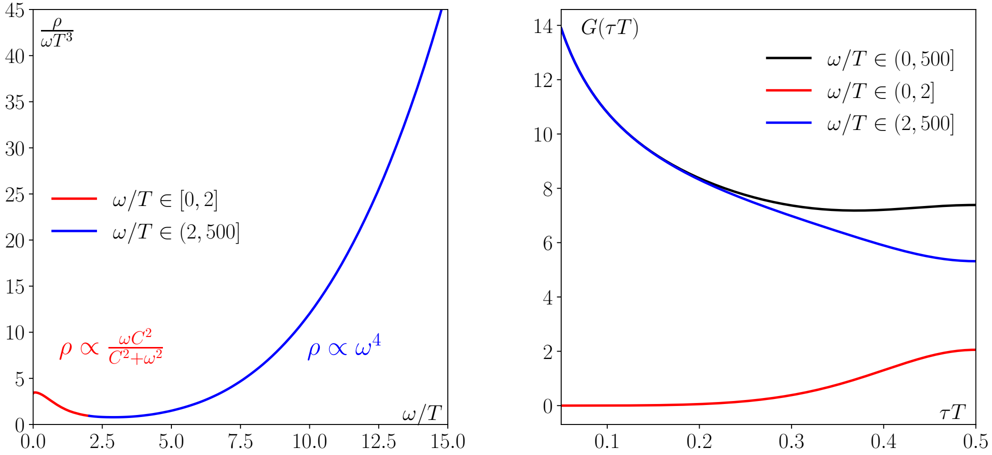
$$\sqrt{8\tau_{\mathrm{F}}}/\tau \in [0.43,\ 0.52]$$

L. Altenkort et al. PRD 103, 114513 (2021)

Avoid over smearing:

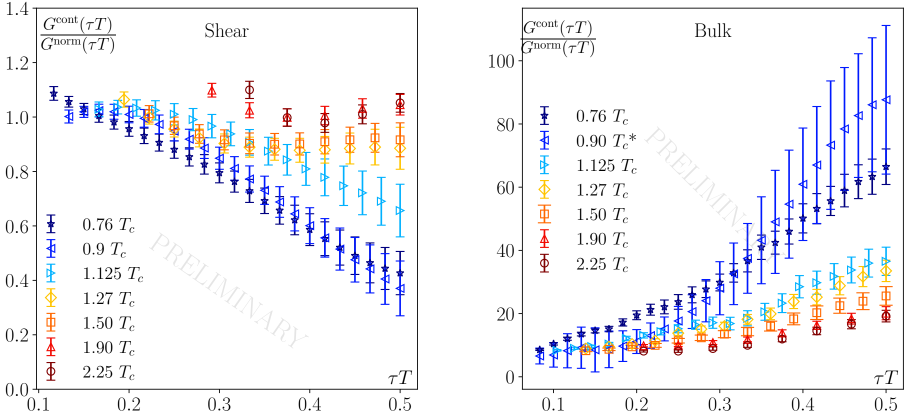
$$\sqrt{8\tau_{\mathrm{F}}} \geq \sqrt{2}a$$

Illustration of sensitivity of correlators to the transport peak



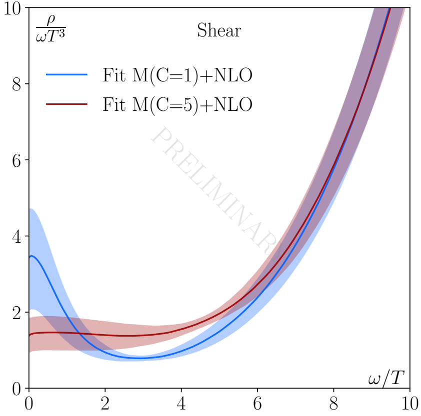
$G(\tau T)$ at $\tau T \sim 0.5$ are more sensitive to the transport peak.

Normalized correlators in the continuum limit



Clear temperature dependencies for both channels.
Negative slope for low temperatures in the shear channel.

Reconstructed spectral functions in the shear channel at 1.5 T_c



Spectral function at 1.5 T_c.

$$G(\tau) = \int_0^\infty \frac{d\omega}{\pi} \frac{\cosh[\omega(1/2T - \tau)]}{\sinh(\omega/2T)} \rho(\omega, T)$$

$$\frac{\rho(\omega)}{\omega T^3} = \frac{A}{T^3} \frac{C^2}{C^2 + (\omega/T)^2} + B \frac{\rho_{\text{pert}}(\omega)}{\omega T^3}$$

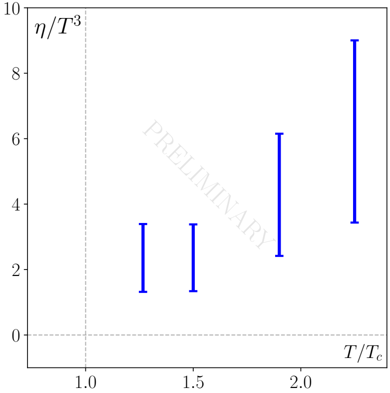
$$\rho_{\text{pert}}(\omega) \propto (\omega/T)^4$$

Y. Zhu et al. JHEP 03, 002 (2013) (shear)

M. Laine et al., JHEP 09, 084(2011 (bulk)

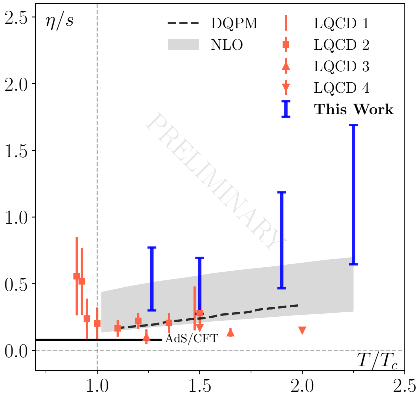
$C = 1$, sharp peak, long-lived excitation.
$C = 5$, broad peak, short-lived excitation.

Temperature dependencies of shear viscosity



DQPM: PRC 88, 045204 (2013).
NLO: JHEP 03, 179 (2018).
Gradient Flow:
LQCD1: PRD 108, 014503 (2023).

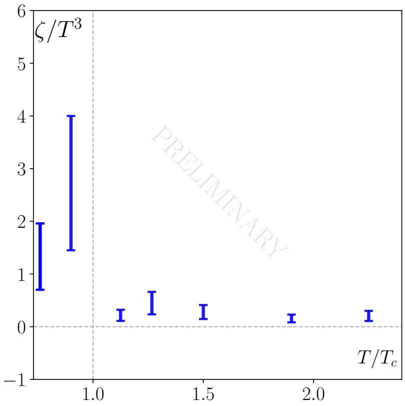
Multi-Level:
LQCD2: JHEP 04, 101 (2017).
LQCD3: PRD 76, 101701 (2007).
LQCD4: PRD 98, 014512 (2018).

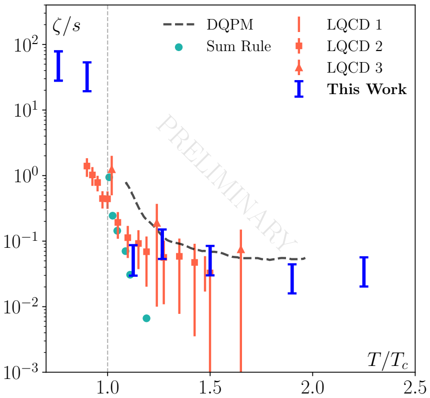


Mild increase with temperature in η/s at $T \geq 1.27\,T_c$.

η/s agrees with LQCD1 & NLO at $T \geq 1.27\,T_c$.

Temperature dependencies of bulk viscosity





DQPM: PRC 88, 045204 (2013)
Sum Rule: JHEP 09, 093 (2008)
LQCD1 (GF): PRD 108, 014503 (2023). $a = 0.0117$ fm
LQCD2 (ML): PRD 98, 054515 (2018). $a \geq 0.0253$ fm
LQCD3 (ML): PRL 100, 162001 (2008). $a \geq 0.0475$ fm

Smaller values of ζ/s at $T > T_c$.

ζ/s agrees with LQCD1 & LQCD2 at $T > T_c$.

Results

- Large and fine lattices are generated to extract the viscosities.
- High-precision EMT correlators are obtained via gradient flow and blocking methods.
- Temperature dependencies of η/s and ζ/s are investigated in SU(3) across the phase transition region.
- η/s increases mildly with temperature at $T \geq 1.27T_c$.
- $\zeta/s(T < T_c) \gg \zeta/s(T > T_c)$, and ζ/s is most flat at $T \geq 1.13T_c$.
- The full QCD investigation (including dynamical quarks) is progressing.