Nucleon Three-Dimensional Spin Structures

第二十届全国中高能核物理大会 April 24th-28nd, 2025 @ Shanghai, Jiangsu

Tianbo Liu (刘天博)

Key Laboratory of Particle Physics and Particle Irradiation (MOE) Institute of Frontier and Interdisciplinary Science, Shandong University Southern Center for Nuclear-Science Theory, IMP, CAS

In collaboration with: Hongxin Dong, Bo-Qiang Ma, Peng Sun, Ke Yang, Chunhua Zeng, Yuxiang Zhao









How much do we understand our world?



Lepton-Hadron Deep Inelastic Scattering

Inclusive DIS at a large momentum transfer: $Q \gg \Lambda_{\text{OCD}}$

- dominated by the scattering of the lepton off an active quark/parton
- not sensitive to the dynamics at a hadronic scale ~ 1/fm
- collinear factorization:

 $\sigma \propto H(Q) \otimes f_{i/P}(x,\mu^2)$

- overall corrections suppressed by $1/Q^n$
- indirectly "see" quarks, gluons and their dynamics
- predictive power relies on
- precision of the probe
- universality of $f_{i/P}(x, \mu^2)$

Modern "Rutherford" experiment.





Lepton-Hadron Deep Inelastic Scattering



H. Abramowicz et al., EPJC 78, 580 (2015).





Lepton-Hadron Deep Inelastic Scattering



Nucleon Spin Structure

Proton spin puzzle

$$\Delta \Sigma = \Delta u + \Delta d + \Delta s \sim 0.3$$

Spin decomposition

$$J = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$



JAM17: $\Delta\Sigma=0.36\pm0.09$

JAM Collaboration, PRL 119, 132001 (2017).

Quark spin only contributes a small fraction to the nucleon spin.

J. Ashman et al., PLB 206, 364 (1988); NP B328, 1 (1989).



Gluon spin from LQCD: $S_g = 0.251(47)(16)$

50% of total proton spin Y.-B. Yang *et al.* (χQCD Collaboration), PRL 118, 102001 (2017).





Wigner Rotation Effect

Melosh-Wigner rotation

quark spin in a rest proton \neq quark spin in a moving proton

If applying a kinetic boost, one may relate the spin states in *proton rest frame* to the spin states in *infinite momentum frame*

$$\chi_T^{\uparrow} = w \left[\left(k^+ + m \right) \chi_F^{\uparrow} - \left(k^1 + ik^2 \right) \chi_F^{\downarrow} \right] \qquad \qquad k^+ = k^0 + k^3$$
$$\chi_T^{\downarrow} = w \left[\left(k^+ + m \right) \chi_F^{\downarrow} + \left(k^1 - ik^2 \right) \chi_F^{\uparrow} \right] \qquad \qquad w = \left[2k^+ \left(k^0 + m \right) \right]^{-1/2}$$

E.P. Wigner, Ann. Math 40 (1939) 149; H.J. Melosh, Phys. Rev. D 9 (1974) 1095.

The effect on quark polarization

$$\Delta q = \int \mathrm{d}^3 \mathbf{k} \mathscr{M} \left[q^{\uparrow}(k) - q^{\downarrow}(k) \right] \qquad \qquad \mathscr{M} = \frac{(k^+ + m)^2 - k_T^2}{2k^+(k^0 + m)}$$

B.-Q. Ma, J. Phys. G 17 (1991) L53-L58; B.-Q. Ma, Q.-R. Zhang, Z. Phys. C 58 (1993) 479.

It predicts decreasing polarization with k_T , which should be tested by data. This interpretation is based on a kinetic boost, but a complete boost including QCD dynamics is challenging.



The Sivers Function: Early Story

Transverse single spin asymmetry observed in experiments



Data: J. Antille et al., Phys. Lett B94 (1980) 523.

Data: 7th Symposium on High Energy Spin Physics (1986).

D. Sivers proposed to explain such SSA a new distribution function

Sivers function $\Delta^N G_{a/p(\uparrow)}(x, \mathbf{k}_T; \mu^2)$ D. Sivers, Phys. Rev. D 41 (1990) 83.

However it was soon shown this function was T-odd and prohibited by QCD

J. Collins, Nucl. Phys. B 396 (1993) 161.

For the next decade, the "Sivers effect" was thought to vanish.



The Sivers Function: Early Story



Tianbo Liu

Semi-inclusive Deep Inelastic Scattering

Semi-inclusive DIS: a final state hadron (P_h) is identified

- enable us to explore the emergence of color neutral hadrons from colored quarks/gluons
- flavor dependence by selecting different types of observed hadrons: pions, kaons, ...
- a large momentum transfer *Q* provides a shortdistance probe
- an additional and adjustable momentum scale P_{hT}
- multidimensional imaging of the nucleon







SIDIS Kinematic Regions

Sketch of kinematic regions of the produced hadron



Structure Functions of SIDIS

SIDIS differential cross section in terms of 18 structure functions $F_{ABC}(x_{B}, z, P_{hT}^{2}, Q^{2})$ A: lepton polarization P_h B: nucleon polarization C: virtual photon polarization ϕ_S $\frac{\mathrm{d}\sigma}{\mathrm{d}x_B \,\mathrm{d}y \,\mathrm{d}z \,\mathrm{d}P_{hT}^2 \,\mathrm{d}\phi_h \,\mathrm{d}\phi_S}$ $= \frac{\alpha^2}{x_B y Q^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x_B}\right)$ $\times \left\{ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)} F_{UU}^{\cos\phi_h} \cos\phi_h + \epsilon F_{UU}^{\cos2\phi_h} \cos2\phi_h + \lambda_e \sqrt{2\epsilon(1-\epsilon)} F_{LU}^{\sin\phi_h} \sin\phi_h \right\}$ $x_B = \frac{Q^2}{2P \cdot a}$ $+S_{L}\left[\sqrt{2\epsilon(1+\epsilon)}F_{UL}^{\sin\phi_{h}}\sin\phi_{h}+\epsilon F_{UL}^{\sin2\phi_{h}}\sin2\phi_{h}\right]+\lambda_{e}S_{L}\left[\sqrt{1-\epsilon^{2}}F_{LL}+\sqrt{2\epsilon(1-\epsilon)}F_{LL}^{\cos\phi_{h}}\cos\phi_{h}\right]$ $y = \frac{P \cdot q}{P \cdot l}$ $+S_T \left[\left(F_{UT,T}^{\sin(\phi_h - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \sin(\phi_h - \phi_S) + \epsilon F_{UT}^{\sin(\phi_h + \phi_S)} \sin(\phi_h + \phi_S) \right]$ $z = \frac{P \cdot P_h}{P \cdot a}$ $+\epsilon F_{UT}^{\sin(3\phi_h-\phi_S)}\sin\left(3\phi_h-\phi_S\right) + \sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin\phi_S}\sin\phi_S + \sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin(2\phi_h-\phi_S)}\sin\left(2\phi_h-\phi_S\right)$ $\gamma = \frac{2x_BM}{Q}$ $+ \lambda_e S_T \left[\sqrt{1 - \epsilon^2} F_{LT}^{\cos(\phi_h - \phi_S)} \cos\left(\phi_h - \phi_S\right) \right]$ $+\sqrt{2\epsilon(1-\epsilon)}F_{LT}^{\cos\phi_{S}}\cos\phi_{S}+\sqrt{2\epsilon(1-\epsilon)}F_{LT}^{\cos(2\phi_{h}-\phi_{S})}\cos\left(2\phi_{h}-\phi_{S}\right)\right\}$



Leading Twist TMDs



See also Poster 214 马凌泉 Lattice calculation of BM function

レイズス (青岛)

Longitudinal Double Spin Asymmetry

Longitudinal DSA in SIDIS

$$A_{LL} \equiv \frac{\sigma_{++} - \sigma_{+-} + \sigma_{--} - \sigma_{-+}}{\sigma_{++} + \sigma_{+-} + \sigma_{--} + \sigma_{-+}} = \frac{\sqrt{1 - \varepsilon^2} F_{LL} \left(x, z, P_{hT}^2, Q^2 \right)}{F_{UU} \left(x, z, P_{hT}^2, Q^2 \right)}$$

In TMD region:
$$F_{LL}\left(x, z, P_{hT}^2, Q^2\right) \sim g_{1L}(x, k_T^2) \otimes D_1(z, p_T^2)$$
$$F_{UU}\left(x, z, P_{hT}^2, Q^2\right) \sim f_1(x, k_T^2) \otimes D_1(z, p_T^2)$$



Several global analyses of collinear helicity but no extraction of TMD helicity before!

 P_{hT} dependent DSA measurements

HERMES: proton (H_2) and deuteron (D_2) targets

HERMES Collaboration, Phys. Rev. D 99 (2019) 112001.

JLab CLAS: proton (NH₃) target

CLAS Collaboration, Phys. Lett. B 782 (2018) 662.



TMD Evolution

Evolution equations

$$\mu^{2} \frac{dF(x,b;\mu^{2},\zeta)}{d\mu^{2}} = \frac{\gamma_{F}(\mu,\zeta)}{2} F(x,b;\mu^{2},\zeta) \qquad -\zeta \frac{d\gamma_{F}(\mu,\zeta)}{d\zeta} = \mu \frac{d\mathscr{D}(\mu,b)}{d\mu} = \Gamma_{\text{cusp}}(\mu)$$

$$\zeta \frac{dF(x,b;\mu^{2},\zeta)}{d\zeta} = -\mathscr{D}(\mu,b)F(x,b;\mu^{2},\zeta) \qquad \gamma_{F}(\mu,\zeta) = \Gamma_{\text{cusp}}(\mu) \ln \frac{\mu^{2}}{\zeta} - \gamma_{V}(\mu)$$

$$F\left(x,b;\mu_{f},\zeta_{f}\right) = \exp\left[\int_{P}\left(\gamma_{F}(\mu,\zeta)\frac{d\mu}{\mu} - \mathscr{D}(\mu,b)\frac{d\zeta}{\zeta}\right)\right] F\left(x,b;\mu_{i},\zeta_{i}\right)$$

 ζ -prescription

equipotential lines: $\frac{d \ln \zeta_{\mu}(\mu, b)}{d \ln \mu^{2}} = \frac{\gamma_{F}\left(\mu, \zeta_{\mu}(\mu, b)\right)}{2\mathscr{D}(\mu, b)}$ $\mathscr{D}\left(\mu_{0}, b\right) = 0, \quad \gamma_{F}\left(\mu_{0}, \zeta_{\mu}\left(\mu_{0}, b\right)\right) = 0$ $F\left(x, b; Q, Q^{2}\right) = \left(\frac{Q^{2}}{\zeta_{Q}(b)}\right)^{-\mathscr{D}(Q,b)} F(x, b), \quad \mu_{f}^{2} = \zeta_{f} = Q^{2}$



Tianbo Liu

First Extraction of TMD Helicity

NLO+NNLL analysis results



Nonzero signals for *u* and *d* quarks, while sea quarks and gluons are loosely constrained.

K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

ふまたる(青岛)

Compare with HERMES data



K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

ふまたる(青岛)

Compare with HERMES data



K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

レネズス(青岛)

Compare with HERMES data



K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

 $e^{\pm}p \rightarrow e^{\pm}\pi^{+}X$

0.14 < x < 0.2

 $e^{\pm}p \rightarrow e^{\pm}\pi^{-}X$

 $e^{\pm}d \to e^{\pm}\pi^+X$

 $e^{\pm}d \rightarrow e^{\pm}\pi^{-}X$

 $e^{\pm}d \to e^{\pm}K^+X$

 $e^{\pm}d \rightarrow e^{\pm}K^{-}X$

₹

ふまたる(青岛)

δ

Ō

₫

Compare with HERMES data





K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

レネスス(青岛)

Compare with HERMES data



K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

レネスス(青岛)

Compare with CLAS data



K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

しまたる(青岛)

Transverse Momentum Dependent Polarization



 $g_{1L}(x, k_T^2)$ gives the absolute number density difference between spin-parallel and spin-antiparallel quarks.

The ratio $g_{1L}(x, k_T^2)/f_1(x, k_T^2)$ measures the polarization rate of quarks.

• At large x, where valence components dominate, the polarization decreases with increasing k_T

Qualitatively consistent with kinetic Wigner rotation effects

• At low *x*, where the valence component is no longer adequate, distributions are highly driven by complex QCD dynamics The polarization is found increasing with *k*_T

> K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

See also:Poster 179 杨科 TMD helicity; 70 陈毅 axial radius



Transversity Distribution

Transversity distribution

$$h_1$$
 (Collinear & TMD)

A transverse counter part to the longitudinal spin structure: helicity g_{1L} , but NOT the same.

Phenomenological extractions



Z.-B. Kang, A. Prokudin, P. Sun, F. Yuan, PRD 93, 014009 (2016).

Chiral-odd:

No mixing with gluons Valence dominant Couple to another chiral-odd function.

Effect in SIDIS:

transverse single spin asymmetry (Collins asymmetry)



JAM Collaboration, PRD 104, 034014 (2022).

Assuming vanishing transverse polarization of sea quarks!



Complementary Process

Semi-inclusive e^+e^- annihilation: $e^+e^- \rightarrow h_1h_2X$



$$\frac{d^5\sigma}{dz_1 dz_2 d^2 \boldsymbol{P}_{h\perp} d\cos\theta}$$

= $\frac{3\pi\alpha^2}{2Q^2} z_1^2 z_2^2 \Big[(1 + \cos^2\theta) F_{UU}^{h_1 h_2} + \sin^2\theta \cos(2\phi_0) F_{Collins}^{h_1 h_2} \Big]$

In TMD region: h_1 and h_2 are near back-to-back, $P_{hT} \ll Q$ $F_{\text{Collins}}^{h_1h_2} \sim H_1^{\perp h_1} \otimes H_1^{\perp h_2}$

Experimental measurements:

Belle: $\sqrt{s} = 10.58 \text{ GeV}$ BaBar: $\sqrt{s} = 10.6 \text{ GeV}$ BESIII: $\sqrt{s} = 3.68 \text{ GeV}$

Phys. Rev. D 78 (2008) 032011; 86 (2012) 039905(E). Phys. Rev. D 90 (2014) 052003; Phys. Rev. D 92 (2015) 111101. Phys. Rev. Lett. 116 (2016) 042001.

> See also: 4月26日10:30 邵鼎煜 and Poster 195 张宜新 transverse SSA and hadron in jets





Sea Quark Transversity

First determination of sea quark transversity, including TMD evolution



ふまたる(青岛)

New COMPASS Data

SIDIS on transversely polarized deuteron target



COMPASS Collaboration, Phys. Rev. Lett. 133 (2024) 101903.



Transversity Distributions



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

ふまたる(青岛)

Transversity Distributions



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

レネスズ(青岛)

Collins Fragmentation Functions



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

ふまたる(青岛)

Collins Fragmentation Functions



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

レネスズ(青岛)

Tensor Charge

Tensor charge

$$\langle P, S | \bar{\psi}^q i \sigma^{\mu\nu} \gamma_5 \psi^q | P, S \rangle = g_T^q \bar{u}(P, S) i \sigma^{\mu\nu} \gamma_5 u(P, S)$$

$$g_T^q = \int_0^1 [h_1^q(x) - h_1^{\bar{q}}(x)] \, dx$$

- A fundamental QCD quantity: matrix element of local operators.
- Moment of the transversity distribution: valence quark dominant.
- Calculable in lattice QCD.



Larger uncertainties when including anti-quarks (less biased) Compatible with lattice QCD calculations C. Zeng, H. Don

C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324



The Sivers Function

Sivers TMD distribution function

$$\frac{\epsilon_{ij}k_T^i S_T^j}{M} f_{1T}^{\perp}(x, k_T^2) \quad \textcircled{\bullet} - \bigodot_{\bullet}$$

A naive T-odd distribution function

Transverse momentum distribution distorted by nucleon transverse spin



Effect in SIDIS:

transverse single spin asymmetry (Sivers asymmetry)

$$A_{UT}^{\sin(\phi_h - \phi_S)} \sim f_{1T}^{\perp} \otimes D_1$$

sizable Sivers asymmetry observed by HERMES, COMPASS, JLab

Sign change prediction:

 $f_{1T}^{\perp}(x, k_T^2) |_{\text{SIDIS}} = -f_{1T}^{\perp}(x, k_T^2) |_{\text{DY}}$



COMPASS Collaboration, PRL 119, 112002 (2017).



Sivers Functions

Global analysis of SIDIS, Drell-Yan, W^{\pm}/Z^{0} production data



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

ふまたる(青岛)

Sivers Functions

Global analysis of SIDIS, Drell-Yan, W^{\pm}/Z^0 production data



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

ふまたる(青岛)

Double Spin Asymmetry and Worm-gear

Trans-helicity worm-gear distribution



Effect in SIDIS: A longitudinal-transverse double spin asymmetry

- Longitudinally polarized quark density in a transversely polarized nucleon
- Overlap between wave functions differing by one unit of orbital angular momentum

Phenomenological extraction





K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. D 110 (2024) 034036.



Summary

- Spin always surprises since its discovery nearly 100 years ago
- Nucleon spin structure is still not well understood
- Rich information is contained in TMDs
 - helicity: quark polarization has nontrivial dependence on transverse momentum;
 - transversity: sea quarks may have nonzero transverse polarization, suggest intrinsic sea;
 - Sivers: quark transverse momentum is distorted by the nucleon transverse spin;

- ...

- SIDIS with polarized beam and target is a main process to study polarized TMDs
- Electron-positron annihilation is an important complementary reaction to constrain TMDs and to understand the role of spin in hadronization process
- There are still challenges on the theoretical side
 - power correction, radiative correction, target fragmentation, ...
- Opportunities from existing experiments at JLab12, BESIII, BelleII, and future facilities, EIC, EicC, STCF, to understand nucleon spin structures and fragmentation functions.

Thank you!

