CLQCD

利用国产格点组态计算高精度ρ介子质量和宽度

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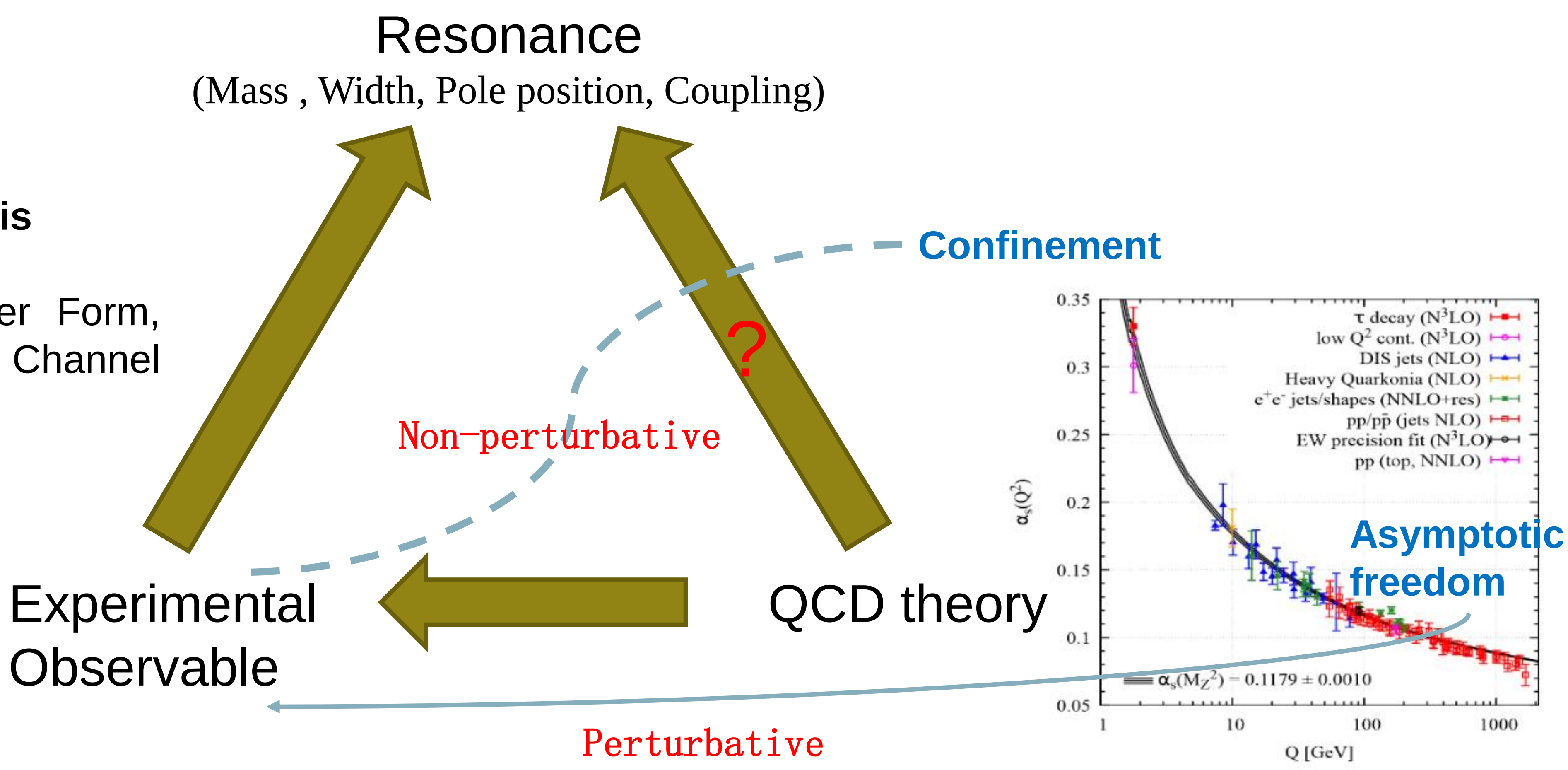
Outline

- Motivation
- Lattice QCD introduction
- ρ meson spectra on LQCD
- Summary

Motivation

Partial Wave Analysis
With various Model:
Such as Breit-Wigner Form, Flatte Form, Coupled Channel Form, and so on



Motivation: Spectrum & Scattering

- Spectrum: Table of the energy of the physical states, can reflect the inner structure information.
- Scattering: The line shapes of Cross section or T-matrix show the properties of the states and interaction.

- These observables can constrain the models.

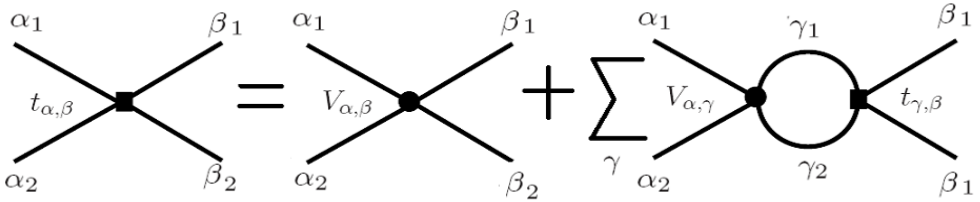
Powerful Parameters, Powerful Model

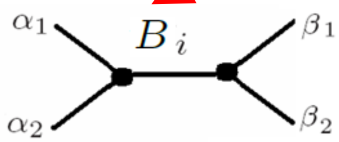
Motivation: Scattering

Lattice Spectrum !!!

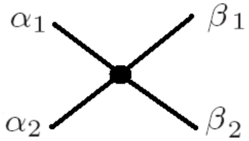
The dimension of m_π

$$t_{\alpha,\beta}(k_\alpha,k_\beta,E)=V_{\alpha,\beta}(k_\alpha,k_\beta)+\sum_\gamma\int k_\gamma{}^2dk_\gamma\frac{V_{\alpha,\gamma}(k_\alpha,k_\gamma)t_{\gamma,\beta}(k_\gamma,k_\beta,E)}{E-\sqrt{m_{\gamma1}^2+k_\gamma^2}-\sqrt{m_{\gamma2}^2+k_\gamma^2}+i\varepsilon}$$





$$g_{i,\alpha}^*\frac{1}{E-m_i}g_{i,\beta}$$



$$v_{\alpha,\beta}$$

- $T^{-1}(E)\sim E-m_i-\Sigma(\mathrm{E})$
  $=E-m_i-\mathrm{Re}\big(\Sigma(\mathrm{E})\big)-\mathrm{iIm}\big(\Sigma(\mathrm{E})\big)$
- $T^{-1}(E)\sim E-m_i-\mathrm{Re}\big(\Sigma(\mathrm{E})\big)+i\Gamma(E)/2$
- $m_{phys}=m_i+\mathrm{Re}\big(\Sigma(\mathrm{E})\big)$
  $=m_0+am_\pi^2+\cdots+\mathrm{Re}\big(\Sigma(\mathrm{E})\big)$

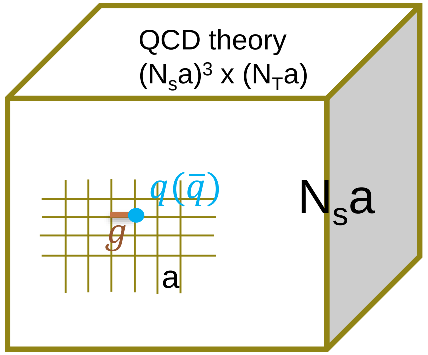
$$\Sigma_{ij}(\mathrm{E})=\sum_\alpha\int k^2dk\frac{g_{i,\alpha}^*G_{i,\alpha}}{E-\sqrt{m_{\alpha1}^2+k^2}-\sqrt{m_{\alpha2}^2+k^2}+i\epsilon}$$

Λ

$$\mathrm{G}_{i,\alpha}=g_{i,\alpha}+\sum_\beta\int\frac{k^2dk\tilde{t}_{\alpha\beta}g_{j,\beta}}{E-\sqrt{m_{\beta1}^2+k^2}-\sqrt{m_{\beta2}^2+k^2}+i\epsilon}\qquad \tilde{t}_{\alpha\beta}=v_{\alpha\beta}+\sum_\gamma\int\frac{k_\gamma^2dk_\gamma\, v_{\alpha\gamma}\tilde{t}_{\gamma\beta}}{E-\sqrt{m_{\gamma1}^2+k_\gamma{}^2}-\sqrt{m_{\gamma2}^2+k_\gamma{}^2}+i\epsilon}$$

Lattice QCD

1. QCD theory: on a box in the Euclid four space



2. a -> UV cutoff, $N_s a$ -> Infrared truncation
3. Lattice QCD -> a model of statistical physics.

$$\langle O \rangle = \int D\phi O[\phi] P[\phi] \qquad P[\phi] = \frac{1}{Z} e^{-S[\phi]} \qquad Z = \int D\phi e^{-S[\phi]}$$

Φ: field quantity, *S*[*ϕ*]: Action, *O*[*ϕ*]:physical quantity

4. Monte Carlo method
5. Three steps for Lattice QCD to real world

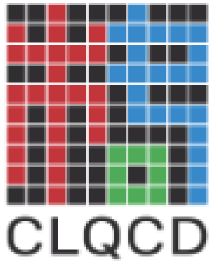
a, Configuration

b, Measurement $\sum_{(\vec{y}-\vec{x})\in Z^3} e^{i\vec{p}.(\vec{y}-\vec{x})} \left\langle T\left(\psi(t;\vec{y}), \psi^{\dagger}(t;\vec{y})\right)\right\rangle \sim \sum_{\Gamma,i} Z_i^{\Gamma} e^{-E_i^{\Gamma} t}$

c, Transformation

Build configuration for different pion mass

Cost!



Operator in quark and gluon level

Calculate Correlation function

Extract energy levels

Luescher

Translate energy levels to observable

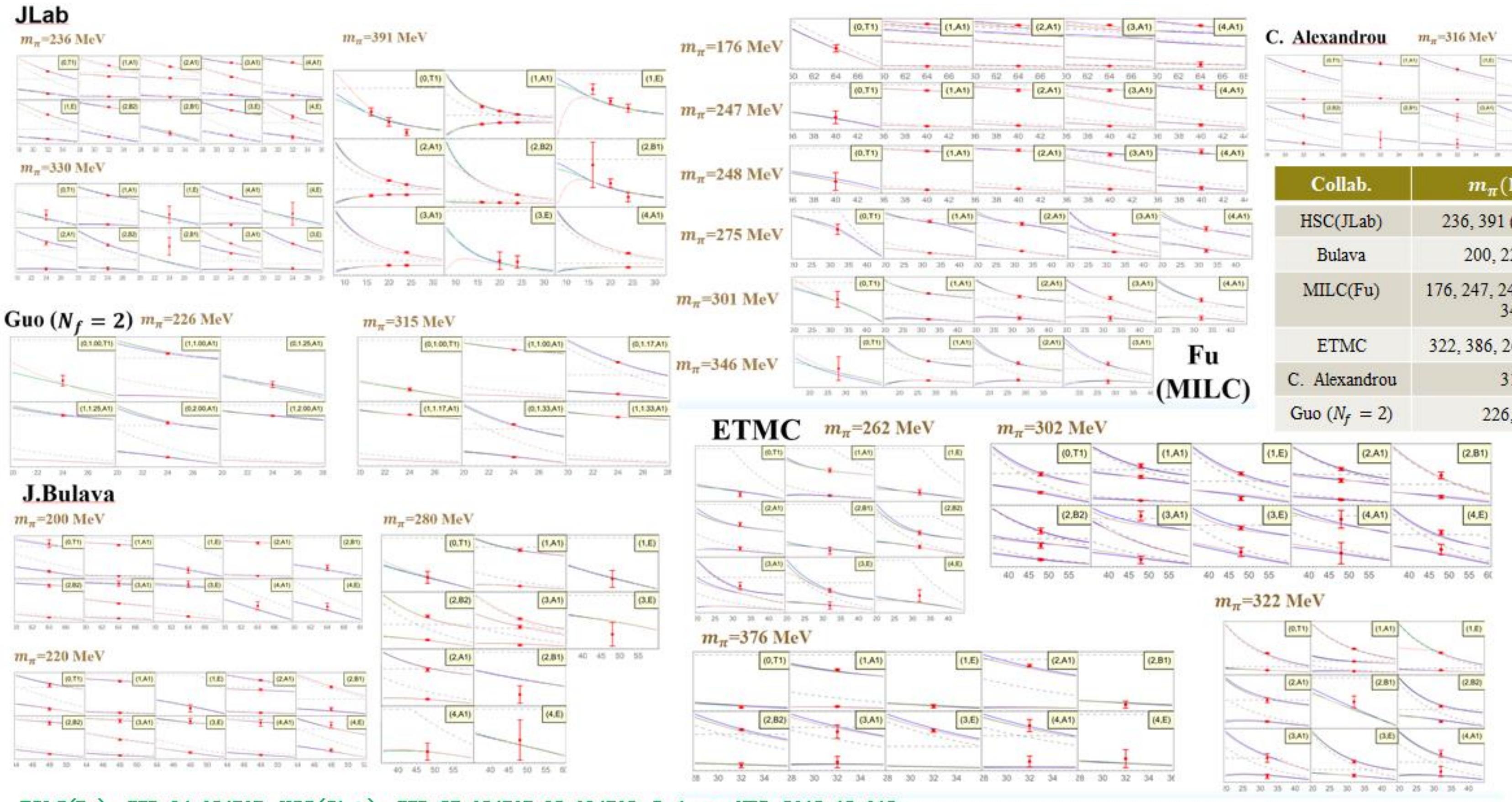
HEFT

Model/EFT

Extract physics parameters from observables

Motivation: π mass dependence

- **The ρ is the best one!**
- **The largest lattice data of ρ.**
- **The lowest state can decay through strong interaction.**
- **We almost clearly understand ρ.**



Collab.	m_π(MeV)
HSC(JLab)	236, 391 (330, 283)
Bulava	200, 220, 280
MILC(Fu)	176, 247, 248, 275, 301, 346
ETMC	322, 386, 262, 302, 376
C. Alexandrou	316
Guo (N_f = 2)	226, 315

MILC(Fu): PRD. 94. 034505; HSC(Jlab): PRD. 87. 034505, 92. 094502; Bulava: NPB. 2018. 12. 018;
ETMC: EPJA56, 61; Guo: PRD. 94. 034501; C. Alex:PRD. 96. 034525

Kang Yu, D. Leinweber,
A. Thomas, J.j. Wu
Phys.Rev.D 109 (2024) 3, 034505

Motivation: π mass dependence

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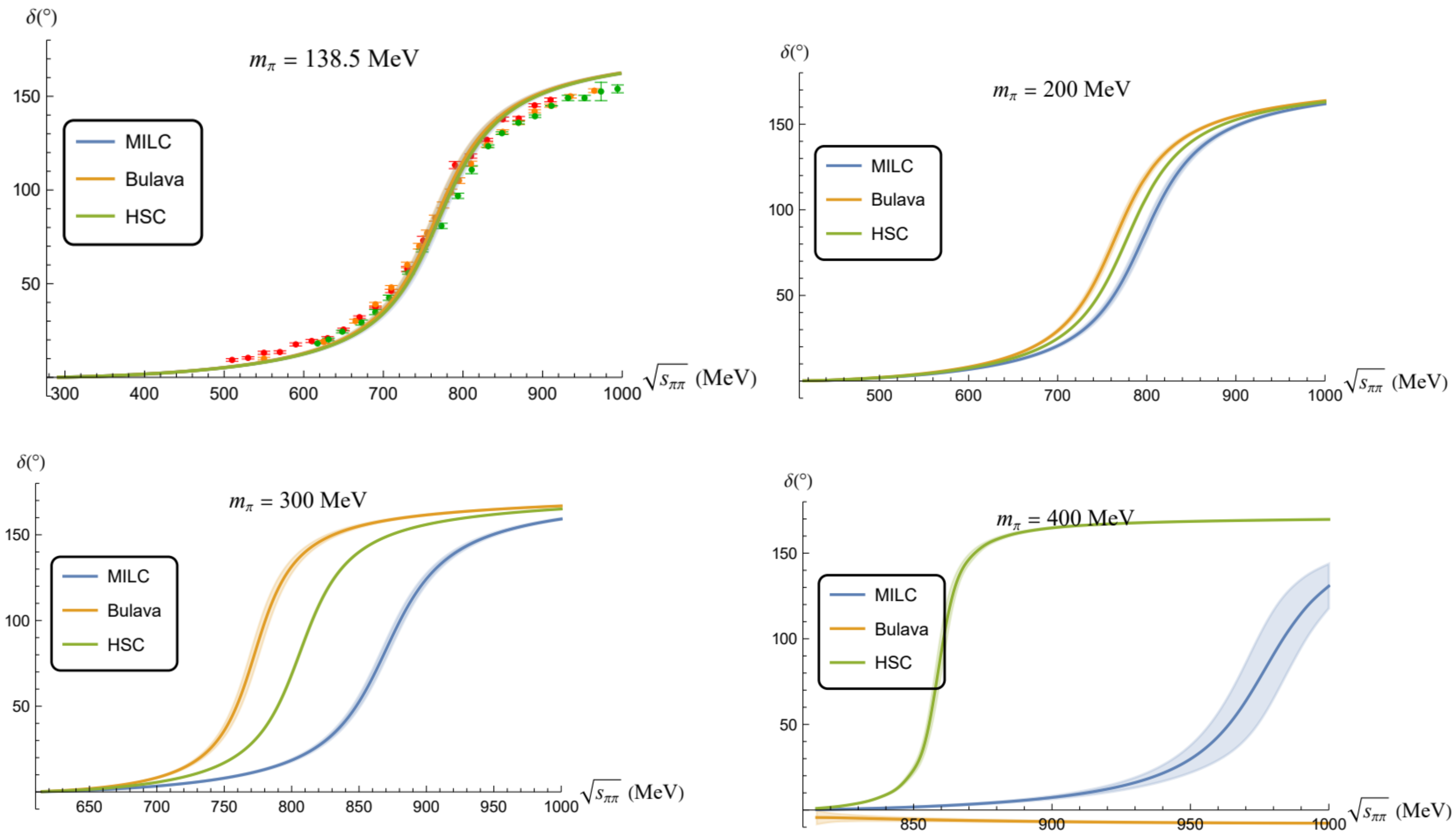


However

CLQCD

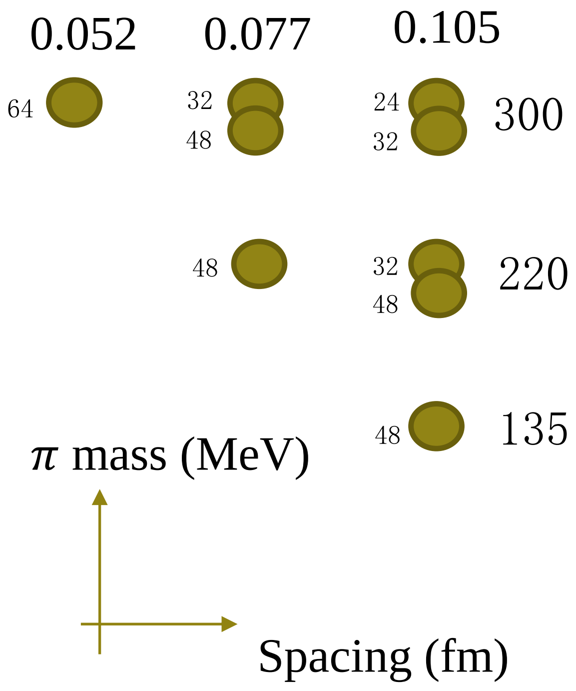
Lattice spacing effect,
Fermion action effect…

We need a systematic Lattice data group to study ρ !

Kang Yu, D. Leinweber, A. Thomas, J.j. Wu Phys.Rev.D 109 (2024) 3, 034505

ρ meson spectra on LQCD -- Configuration

Name	Volume	Spacing	β	m_π/MeV	$m_\pi L$
C24P34	$24^3 \times 64$	$0.10530 fm$	6.20	340	4.38
C24P29	$24^3 \times 72$			292	3.75
C32P29	$32^3 \times 64$			292	5.01
C32P23	$32^3 \times 64$			228	3.91
C48P23	$48^3 \times 96$			225	5.79
C48P14	$48^3 \times 96$			135	3.56
F32P30	$32^3 \times 96$	$0.07746 fm$	6.41	303	3.81
F48P30	$48^3 \times 96$			303	5.72
F32P21	$32^3 \times 64$			210	2.67
F48P21	$48^3 \times 96$			207	3.91
H48P32	$48^3 \times 144$	$0.05187 fm$	6.72	321	4.06
H64P32	$64^3 \times 128$			321	5.41



CLQCD, PRD109 (2024) 5, 054507

ρ meson spectra on LQCD -- Operator

$[000]T_1^-$	$[001]A_1$	$[001]E_2$	$[011]A_1$
$\rho[000]$	$\rho[001]$	$\rho[001]$	$\rho[011]$
$\pi[001]\pi[00{-}1]$	$\pi[000]\pi[001]$	$\pi[0{-}10]\pi[011]$	$\pi[000]\pi[011]$
$\pi[011]\pi[0{-}1{-}1]$	$\pi[0{-}10]\pi[011]$	$\pi[0{-}1{-}1]\pi[111]$	$\pi[{-}100]\pi[111]$
$\pi[111]\pi[{-}1{-}1{-}1]$	$\pi[{-}1{-}10]\pi[111]$		$\pi[01{-}1]\pi[002]$
	$\pi[00{-}1]\pi[002]$		

$[011]B_1$	$[011]B_2$	$[111]A_1$	$[002]A_1$
$\rho[011]$	$\rho[011]$	$\rho[111]$	$\rho[002]$
$\pi[010]\pi[001]$	$\pi[{-}100]\pi[111]$	$\pi[000]\pi[111]$	$\pi[000]\pi[002]$
$\pi[110]\pi[{-}101]$	$\pi[110]\pi[{-}101]$	$\pi[100]\pi[011]$	
$\pi[0{-}11]\pi[002]$		$\pi[200]\pi[{-}111]$	

$$\rho^0 = \frac{1}{\sqrt{2}}(\bar{u}\Gamma^P u - \bar{d}\Gamma^P d)$$

$$\rho^{0\dagger} = \frac{1}{\sqrt{2}}(\bar{u}\bar{\Gamma}^P u - \bar{d}\bar{\Gamma}^P d)$$

$$\pi^+ = \bar{d}\Gamma^k u, \quad \pi^- = \bar{u}\Gamma^k d$$

$$\pi\pi = \frac{1}{\sqrt{2}}(\pi^+(k_1)\pi^-(k_2) - \pi^-(k_1)\pi^+(k_2))$$

$$= \frac{1}{\sqrt{2}}(\bar{d}\Gamma^{k_1} u\bar{u}\Gamma^{k_2} d - \bar{u}\Gamma^{k_1} d\bar{d}\Gamma^{k_2} u)$$

$$(\pi\pi)^\dagger = \frac{1}{\sqrt{2}}(\bar{d}\bar{\Gamma}^{k_2} u\bar{u}\bar{\Gamma}^{k_1} d - \bar{u}\bar{\Gamma}^{k_2} d\bar{d}\bar{\Gamma}^{k_1} u)$$

ρ meson spectra on LQCD -- correlation function

$$C_{ij} = \left\langle 0 \left| \mathcal{O}_i \mathcal{O}_j^\dagger \right| 0 \right\rangle = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

Computed based on wick contraction and quark propagator calculated based on the configuration.

To parameterize the correlation function !

Energy level in the finite volume

- **Solve the generalized eigenvalue problem(GEVP)**

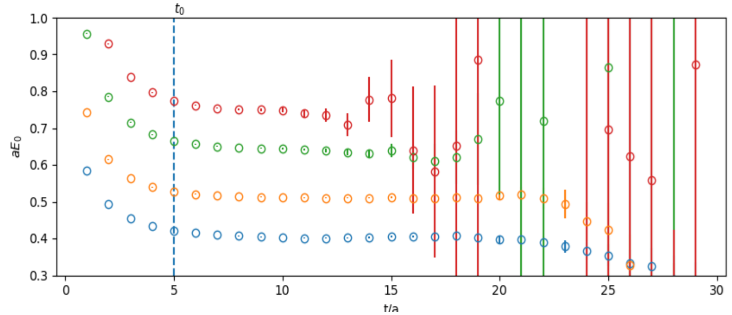
$$\mathcal{G}_{ij}(t_0 + dt) u_j^\alpha = \mathrm{e}^{-m_\alpha dt} \mathcal{G}_{ij}(t_0) u_j^\alpha,$$

$$[\mathcal{G}^{-1}(t_0)\mathcal{G}(t_0 + dt)]_{ij} u_j^\alpha = c^\alpha u_i^\alpha$$

$$v_i^\alpha [\mathcal{G}(t_0 + dt)\mathcal{G}^{-1}(t_0)]_{ij} = c^\alpha v_j^\alpha,$$

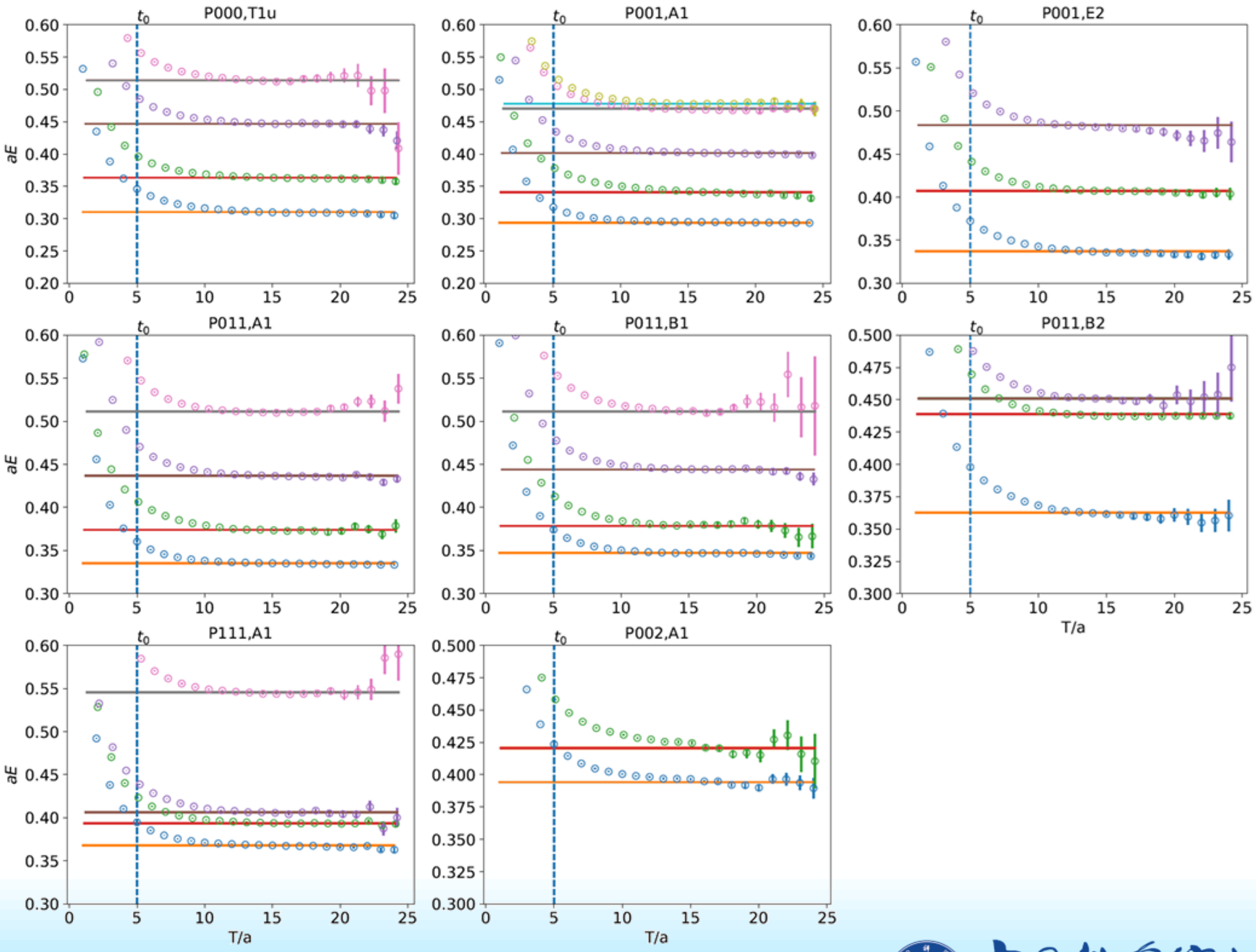
$$\mathcal{G}^\alpha(t) = v_i^\alpha \mathcal{G}_{ij}(t) u_j^\alpha,$$

$$E^\alpha(t) = \frac{1}{n} \log \frac{\mathcal{G}^\alpha(t)}{\mathcal{G}^\alpha(t+n)},$$



Kiratidis, et al., PRD 91, 094509 (2015)

ρ meson spectra on LQCD -- Spectra



Configuration: F48P30

There are 8 different spectra with different total momentum and irreps, including following irreducible representations.:

$p^2 = 0 \ : \ \ T_1^-,$

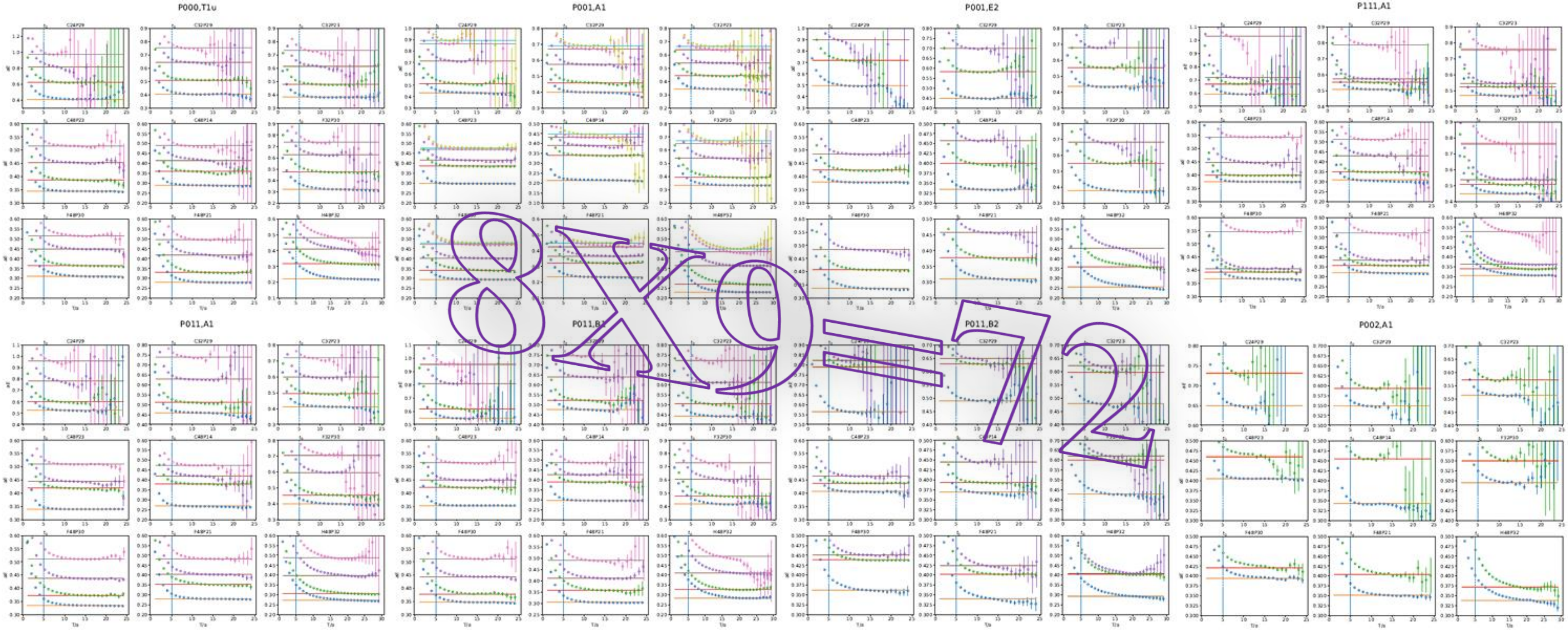
$p^2 = 1 \ : \ \ A_1$ and $E_2,$

$p^2 = 2 \ : \ \ A_1, B_1$ and $B_2,$

$p^2 = 3 \ : \ \ A_1,$

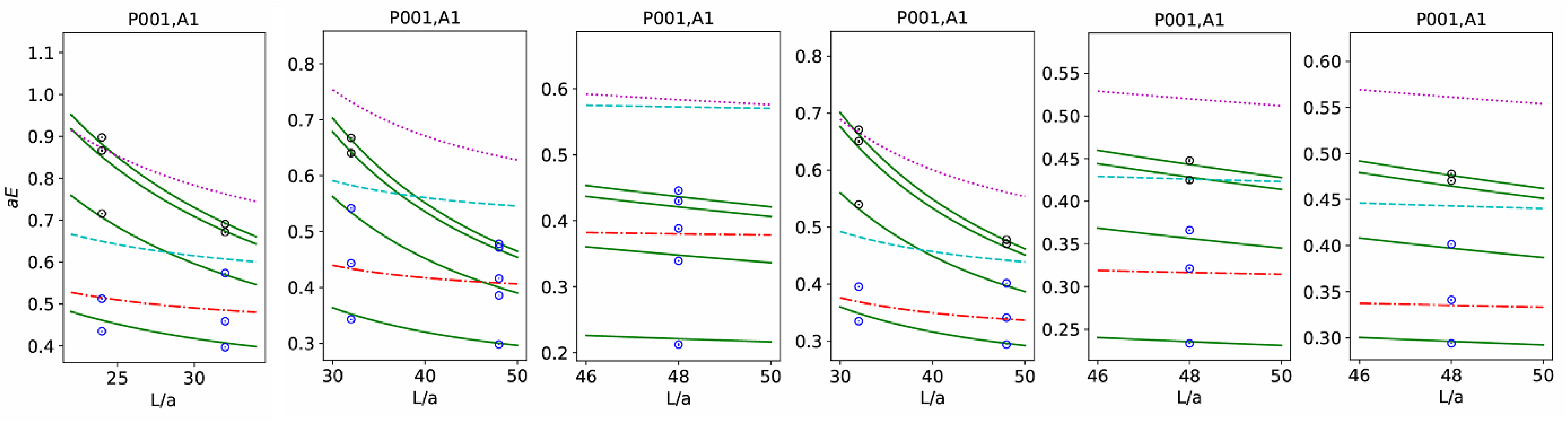
$p^2 = 4 \ : \ \ A_1.$

ρ meson spectra on LQCD -- Spectra



ρ meson spectra on LQCD -- Spectra

C32P29/C48P29	C48P23/C32P23	C48P14	F48P30/F32P30	F48P21	H48P32
m_π ~ 300 MeV	~ 230 MeV	~ 135 MeV	~ 300 MeV	~ 220 MeV	~ 300 MeV



a ~ 0.105 fm

a ~ 0.077 fm

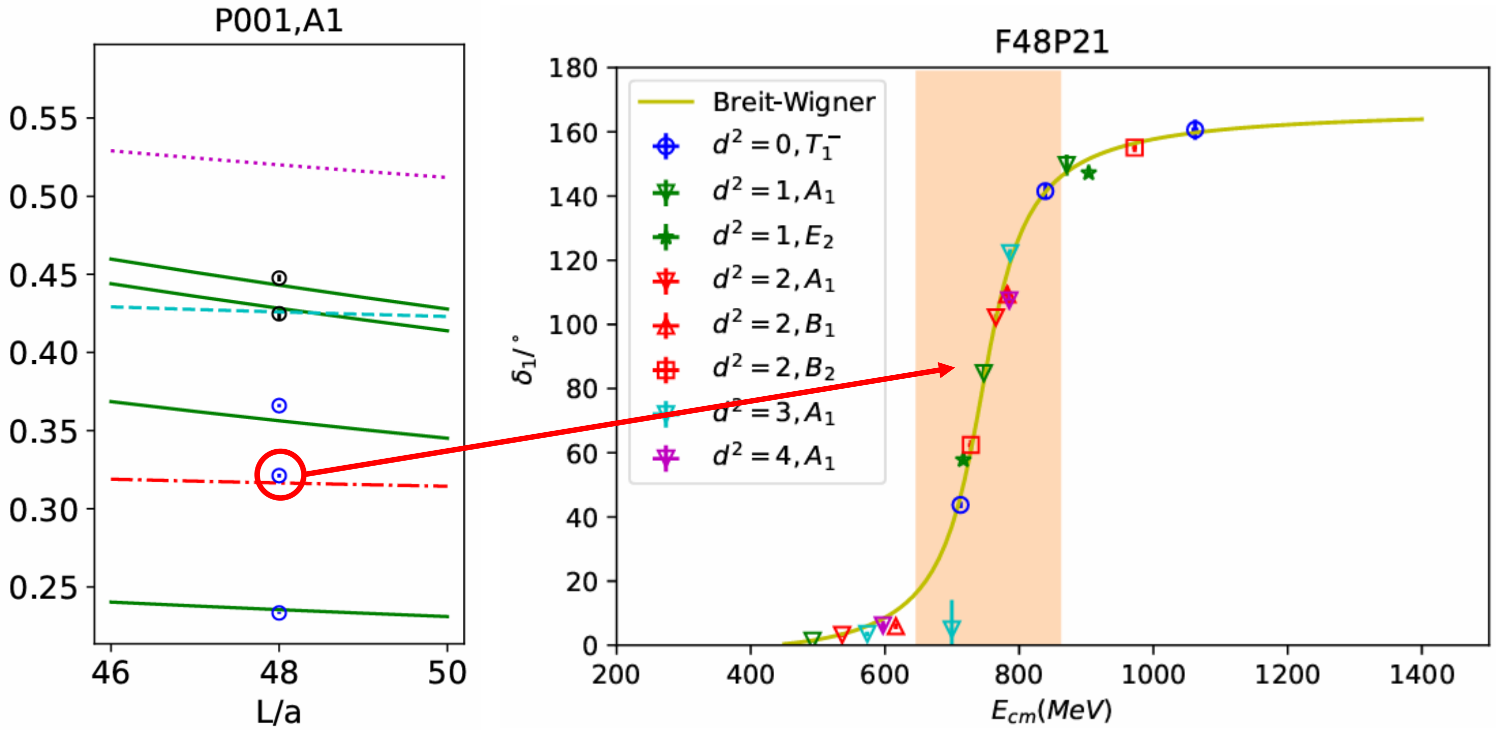
a ~ 0.052 fm

ρ meson spectra on LQCD -- Phase shift of $\pi\pi$

d^2	Λ, μ	$M_{11,11}^{(\vec{P},\Lambda,\mu)}$
0	$T_1^-, 2$	$w_{0,0}$
1	$A_1, 1$	$w_{0,0} + 2w_{2,0}$
1	$E_2, 1$	$w_{0,0} - w_{2,0}$
1	$E_2, 2$	$w_{0,0} - w_{2,0}$
2	$A_1, 1$	$w_{0,0} + \frac{1}{2}w_{2,0} + i\sqrt{6}w_{2,1} - \frac{\sqrt{6}}{2}w_{2,2}$
2	$B_1, 1$	$w_{0,0} + \frac{1}{2}w_{2,0} - i\sqrt{6}w_{2,1} - \frac{\sqrt{6}}{2}w_{2,2}$
2	$B_2, 1$	$w_{0,0} - w_{2,0} + \sqrt{6}w_{2,2}$
3	$A_1, 1$	$w_{0,0} - i2\sqrt{6}w_{2,2}$

$$\det\left(M_{ln,l'n'}(k) - \delta_{ll'}\delta_{nn'}\cot(\delta_l)\right) = 0, \quad \delta_1 = \text{arccot}\, M_{11,1}^{(\vec{P},}$$

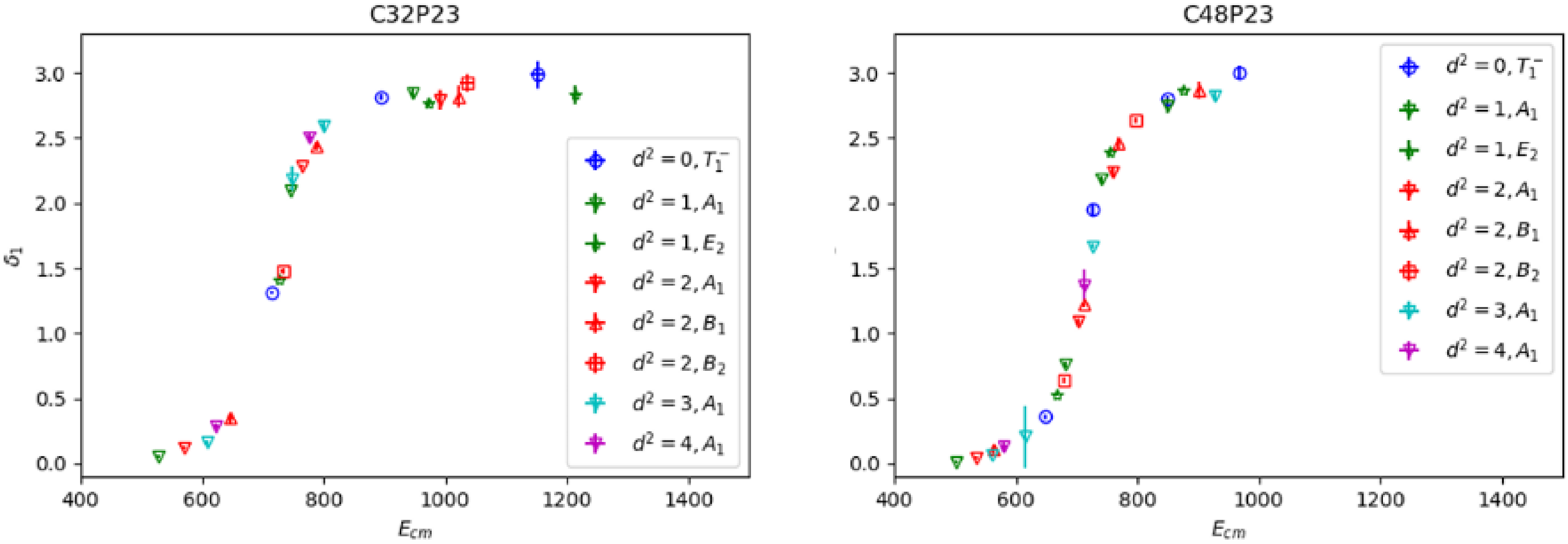
$$w_{j,s} = \frac{\mathcal{Z}_{js}(1, q^2)}{\pi^{3/2}\sqrt{2j+1}\gamma q^{j+1}}, \quad q = \frac{kL}{2\pi}, \quad \delta_{l\geq 3} \text{ are negligible}$$



$$E_{cm} = \sqrt{E_{001}^2 - 4\pi^2/L^2} = \sqrt{(0.34/a)^2 - 4\pi^2/(48a)^2} = 771 \text{ MeV}$$

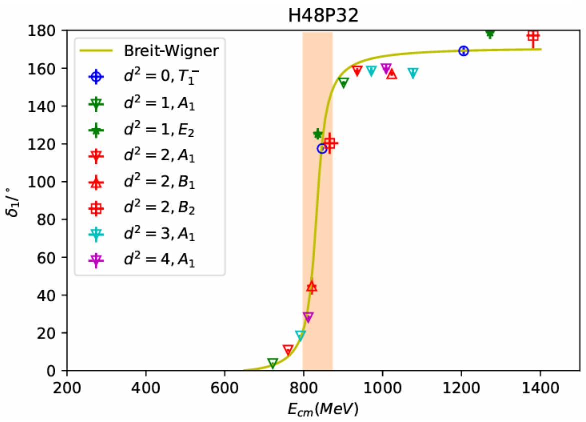
ρ meson spectra on LQCD -- Phase shift of $\pi\pi$

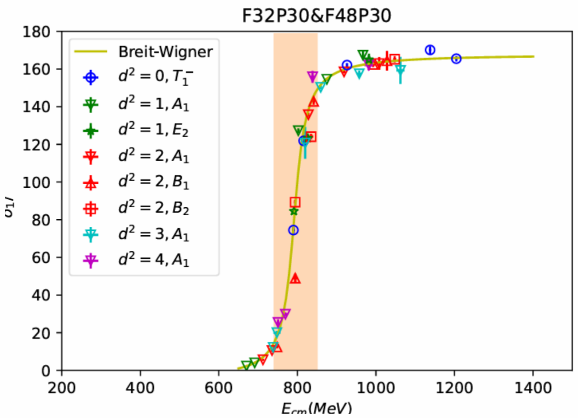
Name	Volume	Spacing	β	m_π/MeV	$m_\pi L$
C32P23	$32^3 \times 64$	$0.10530fm$	6.20	228	3.91
C48P23	$48^3 \times 96$			225	5.79

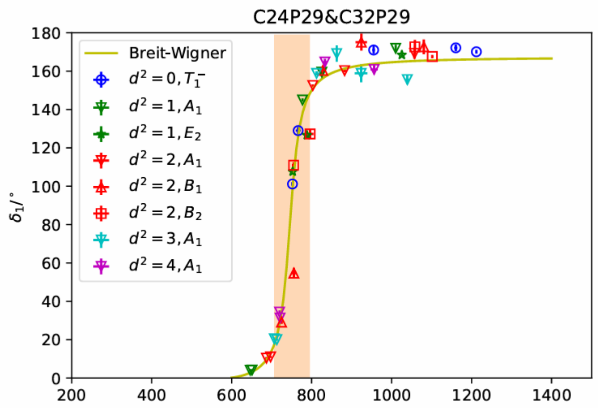


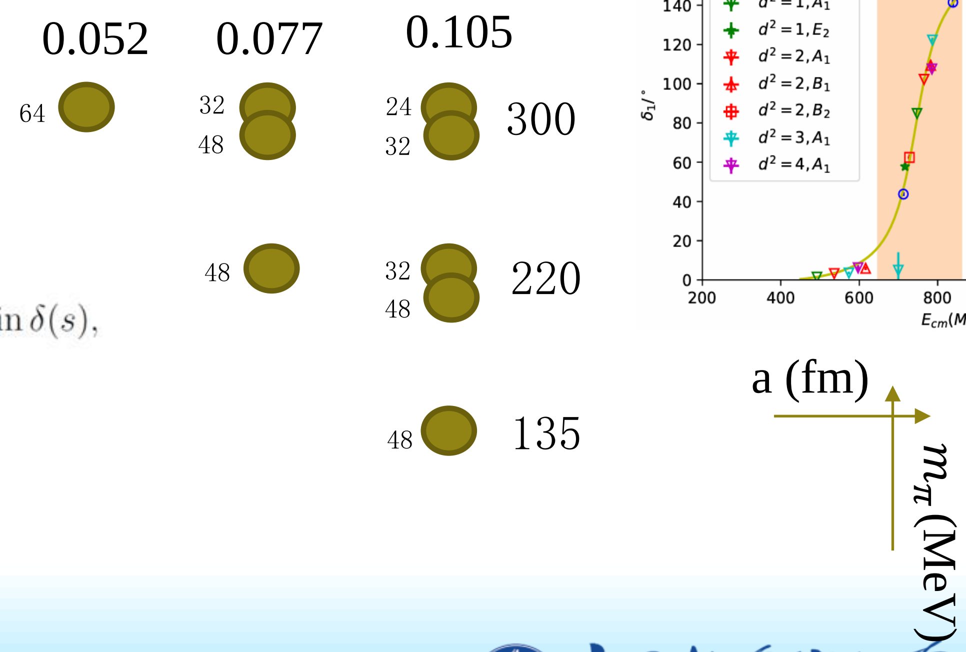
ρ meson spectra on LQCD -- Phase shift of $\pi\pi$

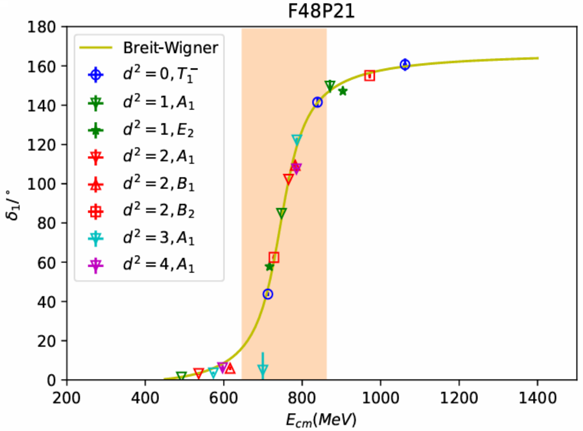
	m_ρ (MeV)	$g_{\rho\pi\pi}$
C24P29/C32P29	749.0(6.0)	5.859(79)
C32P23/C48P23	723.9(5.6)	5.827(14)
C48P14	714.0(34.5)	6.300(440)
F32P30/F48P30	795.5(4.2)	5.775(58)
F48P21	754.2(3.0)	5.954(38)
H48P32	834.8(7.4)	4.916(45)

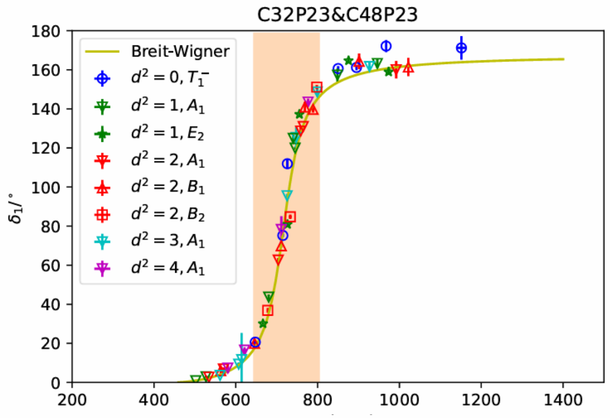






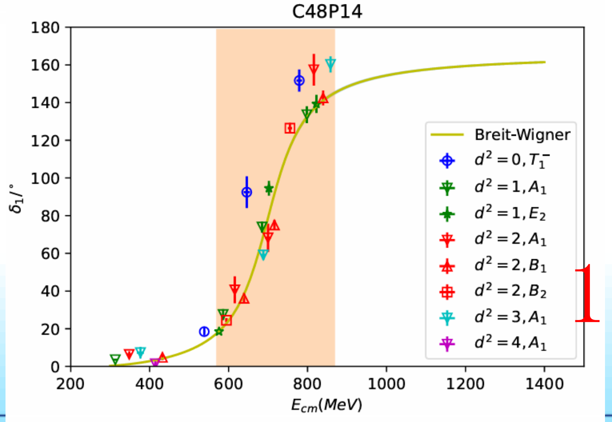






$$a_1 = \frac{-\sqrt{s}\Gamma(s)}{s - m_\rho^2 + i\sqrt{s}\Gamma(s)} = e^{i\delta(s)} \sin\delta(s),$$

$$\Gamma(s) = \frac{p^{*3}}{s}\frac{g_{\rho\pi\pi}^2}{6\pi}.$$



University of Chinese Academy of Sciences

ρ meson spectra on LQCD -- M & Γ of ρ meson

	m_ρ (MeV)	$g_{\rho\pi\pi}$
C24P29/C32P29	749.0(6.0)	5.859(79)
C32P23/C48P23	723.9(5.6)	5.827(14)
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H48P32	834.8(7.4)	4.916(45)

$m_\rho = c_0 + c_1 m_\pi^2 + c_2 a^2 \qquad \chi^2/d.o.f = 0.23$

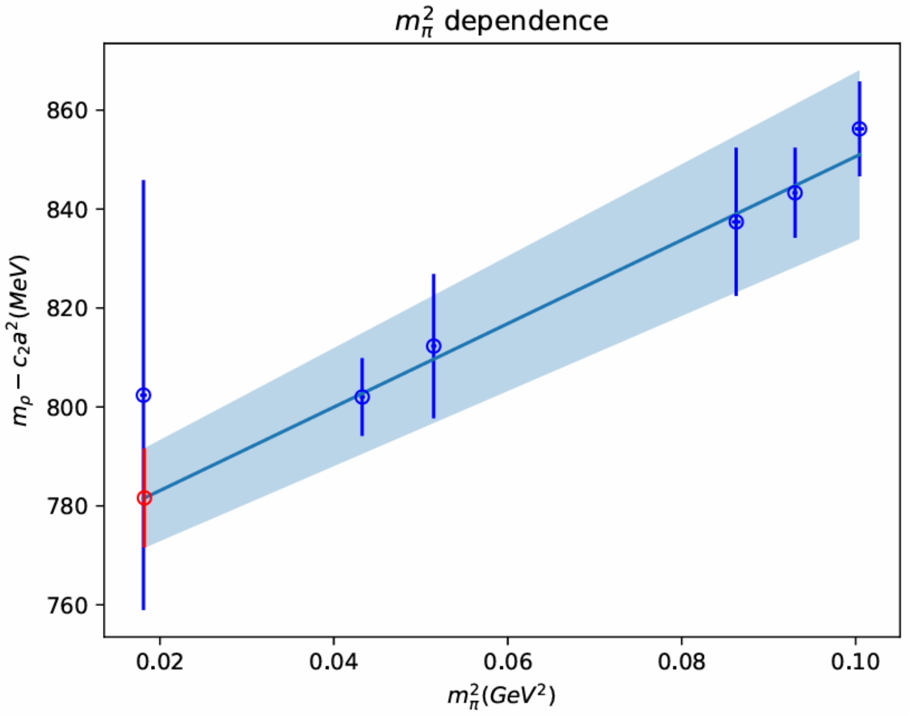
$(c_0, c_1, c_2)^{\rm BW} = (\,766.2(8.5),\ 0.84(9),\ -7.97(81)\,)$

(MeV, GeV^{-1}, $\mathrm{GeV}\cdot\mathrm{fm}^{-2}$)

$g = \tilde{c}_0 + \tilde{c}_1 m_\pi^2 + \tilde{c}_2 a^2 \qquad \chi^2/d.o.f = 0.25$

$(\tilde{c}_0, \tilde{c}_1, \tilde{c}_2)^{\rm BW} = (\,5.74(67),\ -5.3(6.4),\ 46(48)\,).$

(None, GeV^{-2}, fm^{-2})



$$T^{l=1}_{\pi\pi\to\pi\pi}(z) = \frac{\left|V_{\rho\pi\pi}(\bar{k}(z))\right|^2}{z - m_\rho^B - \Sigma(z)},$$

$$\Sigma(z) = \int q^2 dq \frac{\left|V_{\rho\pi\pi}(q)\right|^2}{z - 2\sqrt{q^2 + m_\pi^2} + i\epsilon}.$$

HEFT

↑

BW

$$\frac{-\sqrt{s}\Gamma(s)}{s - m_\rho^2 + \mathrm{i}\sqrt{s}\Gamma(s)} = e^{\mathrm{i}\delta(s)} \sin\delta(s)$$

$$\Gamma(s) = \frac{g^2_{\rho\pi\pi}}{6\pi s}\left(\frac{s}{4} - m_\pi^2\right)^{3/2}$$

Here we take $m_{\pi-phy} = m_{\pi^0} \sim 135$ MeV

$m_\rho = 781.6(10.0)\,\mathrm{MeV}, \quad \Gamma_\rho(m_\rho) = 146.5(9.9)\,\mathrm{MeV}$

$Z_{\rm pole} = 768.1\,(10.0) - i\,70.5\,(4.9)\,\mathrm{MeV}$

Introduction of HEFT

J. M. M. Hall etc. PRD 87(2013), 094510
J.-j. Wu etc. PRC90 (2014), 055206
Y. Li etc. PRD 101(2020), 114501
PRD 103(2021), 094518

$$H = H_0 + H_I$$

$$H_0 = \sum_{i=1,n} |B_i\rangle m_i \langle B_i| + \sum_{\alpha} |\alpha(k_\alpha)\rangle \left[\sqrt{m_{\alpha 1}^2 + k_\alpha^2} + \sqrt{m_{\alpha 2}^2 + k_\alpha^2}\right] \langle \alpha(k_\alpha)|$$

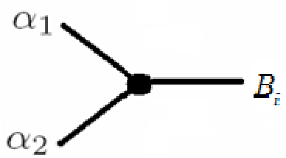
$|B_i>$ bare state， bare mass m_i

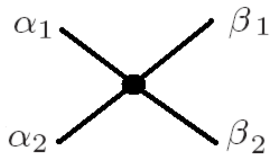
$|\alpha(k_\alpha)>$ non-interaction channels

$$H_I = \hat{g} + \hat{v}$$

$$\hat{g} = \sum_{\alpha} \sum_{i=1,n} \left[|\alpha(k_\alpha)\rangle g_{i,\alpha}^{+} \langle B_i| + |B_i\rangle g_{i,\alpha} \langle \alpha(k_\alpha)| \right]$$

$$\hat{v} = \sum_{\alpha,\beta} |\alpha(k_\alpha)\rangle v_{\alpha,\beta} \langle \beta(k_\beta)|$$





Resonance
(Mass , Width, Pole position, Coupling)

HEFT

T matrix
(Phase Shifts, inelasticity)

Lattice Spectrum

ρ meson spectra on LQCD -- Pole of T

$$T^{l=1}_{\pi\pi\to\pi\pi}(z) = \frac{\left|V_{\rho\pi\pi}(\bar{k}(z))\right|^2}{z - m_\rho^B - \Sigma(z)}$$

$$e^{i\delta(s)}\sin\delta(s) = -\rho(s)\, T^{l=1}_{\pi\pi\to\pi\pi}(\sqrt{s})$$

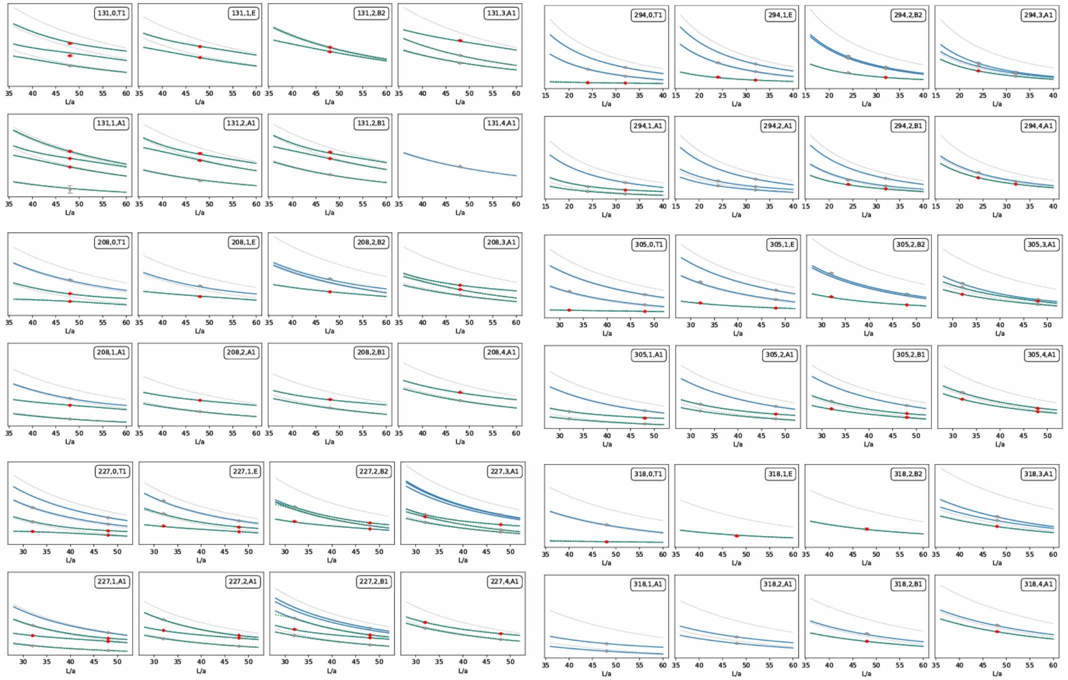
$$\rho(s) = \frac{\pi}{4}\left(s\left(\frac{s}{4} - m_\pi^2\right)\right)^{1/2}$$

$$\Sigma(z) = \int q^2 dq \frac{\left|V_{\rho\pi\pi}(q)\right|^2}{z - 2\sqrt{q^2 + m_\pi^2} + i\epsilon}.$$

$$V_{\rho\pi\pi}(k) = \frac{g_{\rho\pi\pi} k}{\sqrt{m_\rho^B}\sqrt{k^2 + m_\pi^2}}\left(\frac{\Lambda_{\rho\pi\pi}^2}{k^2 + \Lambda_{\rho\pi\pi}^2}\right)^2$$

$$\Sigma_{\omega\pi}(z) = \int q^2 dq \frac{\left|V_{\rho\omega\pi}(q)\right|^2}{z - \sqrt{q^2 + m_\pi^2} - \sqrt{q^2 + m_\omega^2} + i\epsilon}$$

$$V_{\rho\omega\pi}(k) = \frac{g_{\rho\omega\pi} k \sqrt{m_\rho^B}}{\sqrt{2E_\pi(k)\,E_\omega(k)}}\left(\frac{\Lambda_{\rho\omega\pi}^2 - \mu_\pi^2}{k^2 + \Lambda_{\rho\omega\pi}^2}\right)^2$$



- Five free parameters, m_ρ^B, $g_{\rho\pi\pi}$, $\Lambda_{\rho\pi\pi}$, $g_{\rho\omega\pi}$, $\Lambda_{\rho\omega\pi}$
- Here we take $g_{\rho\pi\pi}$, $\Lambda_{\rho\pi\pi}$ as constant for different m_π and a.
- $g_{\rho\omega\pi}$=18 GeV^{-1}, $\Lambda_{\rho\omega\pi}$=1 GeV by $\omega \to 3\pi$
- But m_ρ^B is pion mass dependence.

Scheme A, $g_{\rho\pi\pi} = 7.0 \pm 0.1$, $\Lambda_{\rho\pi\pi} = 0.9 \pm 0.1$ GeV without $\omega\pi$ channel.
Scheme B, $g_{\rho\pi\pi} = 7.4 \pm 0.1$, $\Lambda_{\rho\pi\pi} = 1.0 \pm 0.1$ GeV with $\omega\pi$ channel.

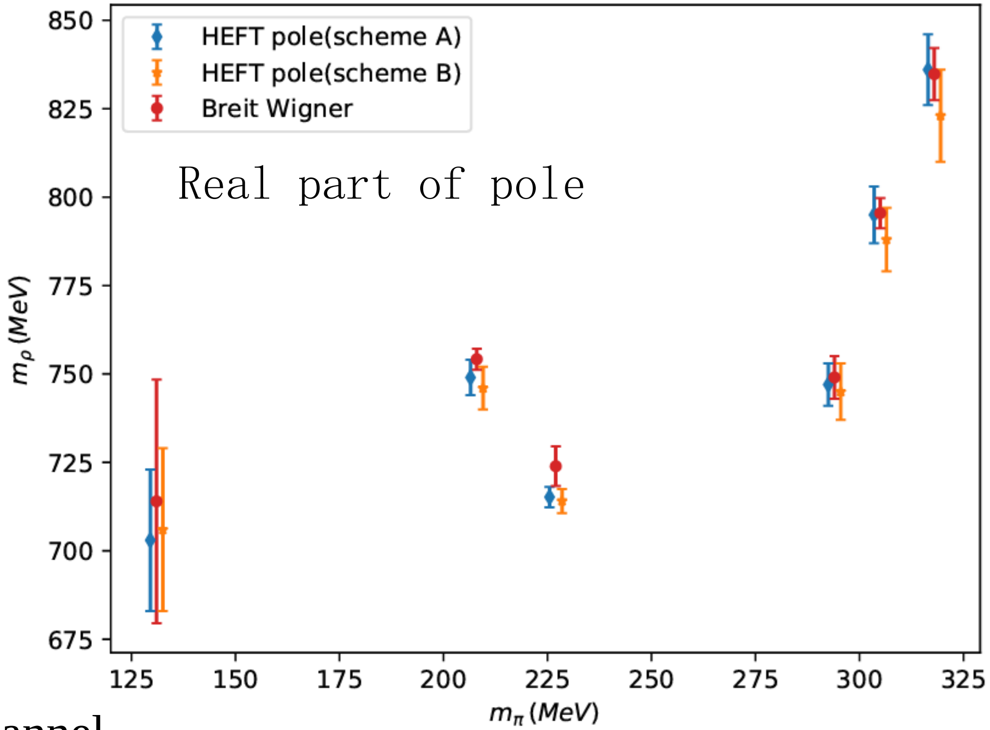
ρ meson spectra on LQCD -- Pole of T

$$T^{l=1}_{\pi\pi\to\pi\pi}(z) = \frac{\left|V_{\rho\pi\pi}(\bar{k}(z))\right|^2}{z - m_\rho^B - \Sigma(z)} \qquad e^{i\delta(s)} \sin\delta(s) = -\rho(s)\, T^{l=1}_{\pi\pi\to\pi\pi}(\sqrt{s}), \qquad \rho(s) = \frac{\pi}{4}\left(s\left(\frac{s}{4} - m_\pi^2\right)\right)^{1/2}$$

$$\Sigma(z) = \int q^2 dq \frac{\left|V_{\rho\pi\pi}(q)\right|^2}{z - 2\sqrt{q^2 + m_\pi^2} + i\epsilon}. \qquad V_{\rho\pi\pi}(k) = \frac{g_{\rho\pi\pi} k}{\sqrt{m_\rho^B}\sqrt{k^2 + m_\pi^2}}\left(\frac{\Lambda^2_{\rho\pi\pi}}{k^2 + \Lambda^2_{\rho\pi\pi}}\right)^2$$

$$\Sigma_{\omega\pi}(z) = \int q^2 dq \frac{\left|V_{\rho\omega\pi}(q)\right|^2}{z - \sqrt{q^2 + m_\pi^2} - \sqrt{q^2 + m_\omega^2} + i\epsilon} \qquad V_{\rho\omega\pi}(k) = \frac{g_{\rho\omega\pi} k \sqrt{m_\rho^B}}{\sqrt{2E_\pi(k)\, E_\omega(k)}}\left(\frac{\Lambda^2_{\rho\omega\pi} - \mu_\pi^2}{k^2 + \Lambda^2_{\rho\omega\pi}}\right)^2$$

- Five free parameters, m_ρ^B, $g_{\rho\pi\pi}$, $\Lambda_{\rho\pi\pi}$, $g_{\rho\omega\pi}$, $\Lambda_{\rho\omega\pi}$
- Here we take $g_{\rho\pi\pi}$, $\Lambda_{\rho\pi\pi}$ as constant for different m_π and a.
- $g_{\rho\omega\pi}$=18 GeV^{-1}, $\Lambda_{\rho\omega\pi}$=1 GeV by $\omega \to 3\pi$
- But m_ρ^B is pion mass dependence.



Scheme A, $g_{\rho\pi\pi} = 7.0 \pm 0.1$, $\Lambda_{\rho\pi\pi} = 0.9 \pm 0.1$ GeV without $\omega\pi$ channel.
Scheme B, $g_{\rho\pi\pi} = 7.4 \pm 0.1$, $\Lambda_{\rho\pi\pi} = 1.0 \pm 0.1$ GeV with $\omega\pi$ channel.

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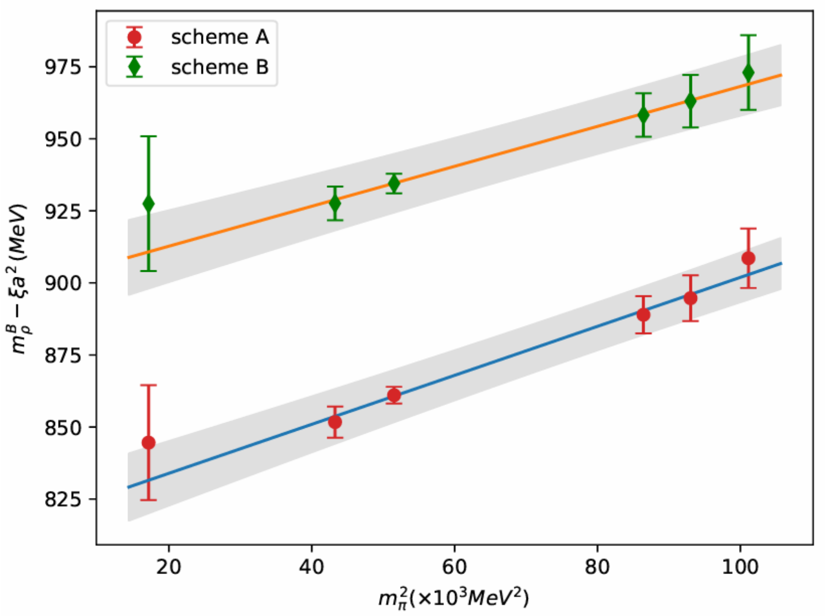
ρ meson spectra on LQCD -- Phase shift of ππ

Only m_ρ^B is pion mass dependence.

$$m_\rho^B(m_\pi, a) = c_0 + c_1 m_\pi^2 + c_2 a^2$$

$$(c_0, c_1, c_2)^{\mathrm{A}} = (817.0\,(13.0),\ 0.85\,(12),\ -7.61\,(89)).$$

$$(c_0, c_1, c_2)^{\mathrm{B}} = (840.0\,(11.2),\ 0.72\,(11),\ -7.91\,(86)).$$



$$m_\rho^{\text{pole,ext(A)}} = 787.0\,(15.0) - i\,59.0\,(6.0)\,\text{MeV}$$

$$m_\rho^{\text{pole,ext(B)}} = 777.0\,(15.0) - i\,60.0\,(6.0)\,\text{MeV}$$

$\rho(770)$ T-MATRIX POLE $\sqrt{s}$ $(761 - 765) - i(71 - 74)$ MeV

at $\delta = 90$ degree

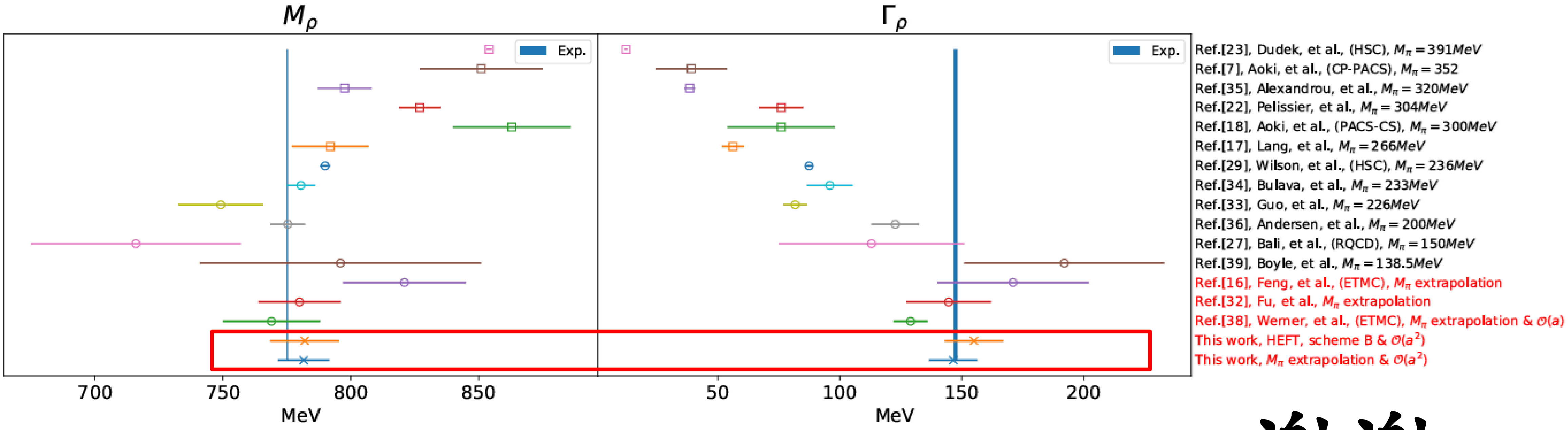
$$m_\rho = 782.0(13.5)\,\text{MeV}, \quad \Gamma_\rho(m_\rho) = 155.0(12.0)\,\text{MeV}$$

$$m_\rho = 781.6(10.0)\,\text{MeV}, \quad \Gamma_\rho(m_\rho) = 146.5(9.9)\,\text{MeV}$$

$$Z_{\text{pole}} = 768.1\,(10.0) - i\,70.5\,(4.9)\,\text{MeV}$$

Summary

We using CLQCD configuration measure the rho meson mass and width at the physical pion mass and continuum limit with the highest precision up to now!



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