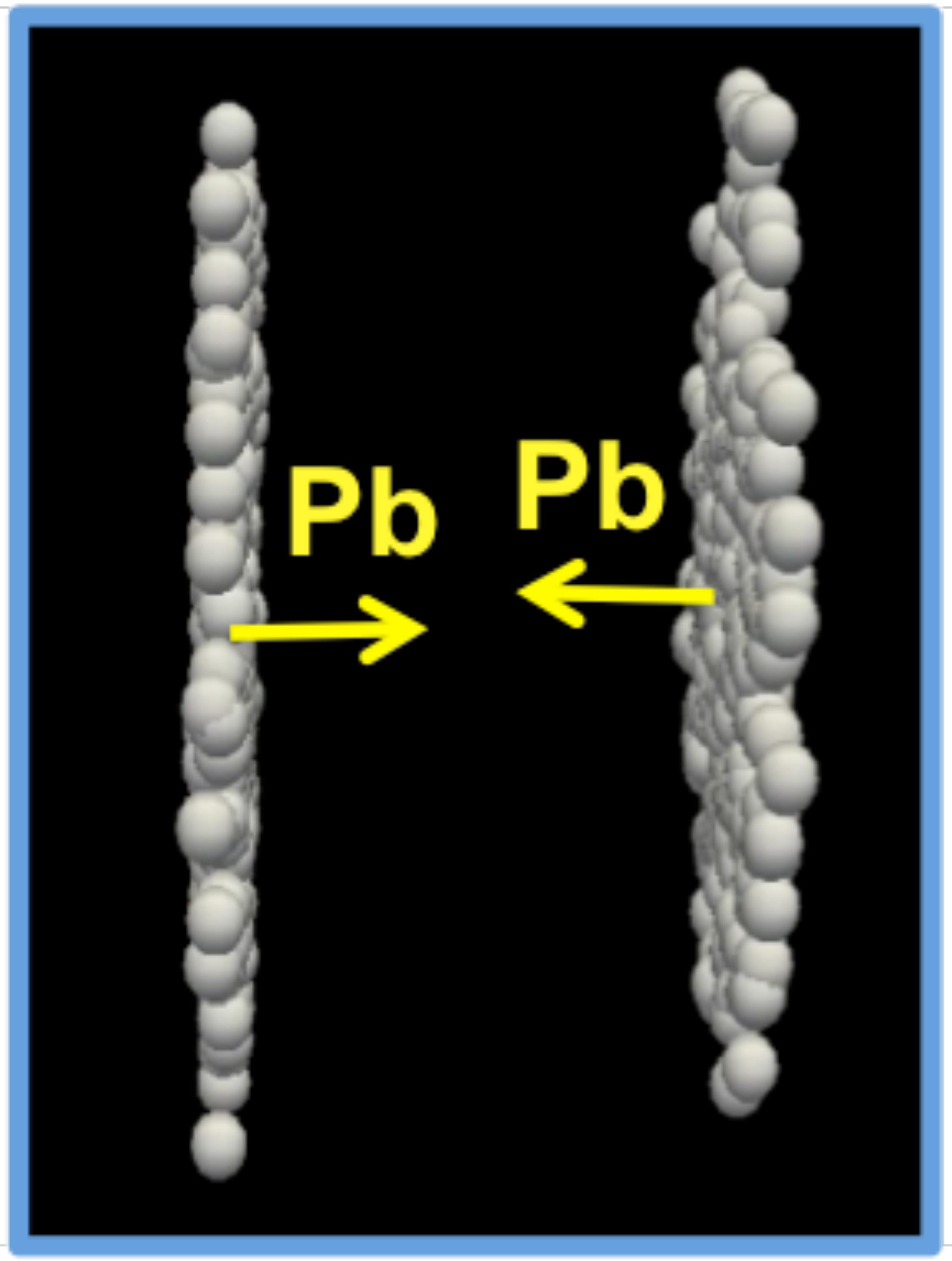
Quantum thermalization and *hydrodynamization* in high energy collisions

施舒哲，清华大学

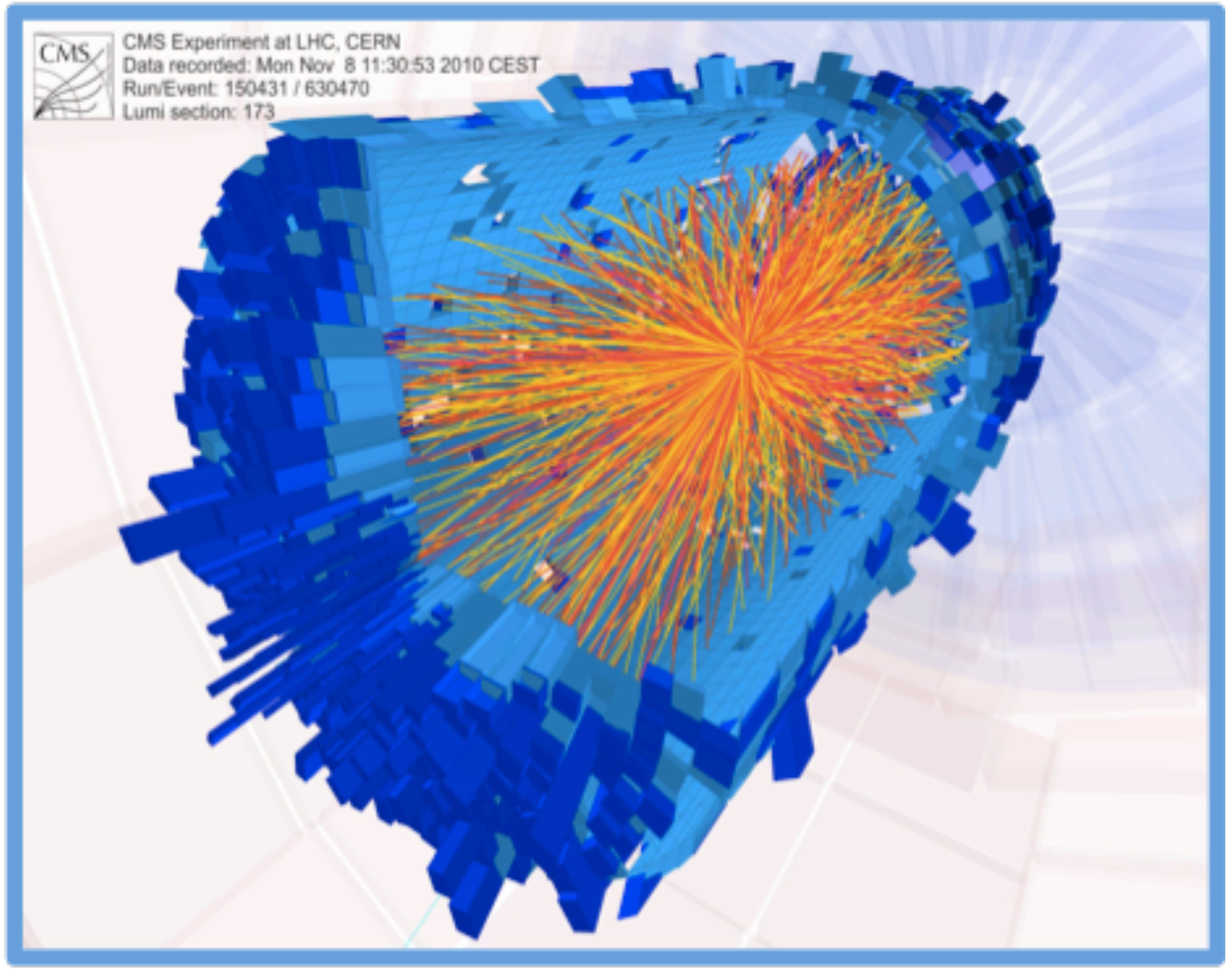
w/ Shile Chen (陈诗乐) & Li Yan (严力), 2412.00662.

w/ Haiyang Shao (邵海洋) & Shile Chen (陈诗乐), in preparation.

Pre-reaction



Detection

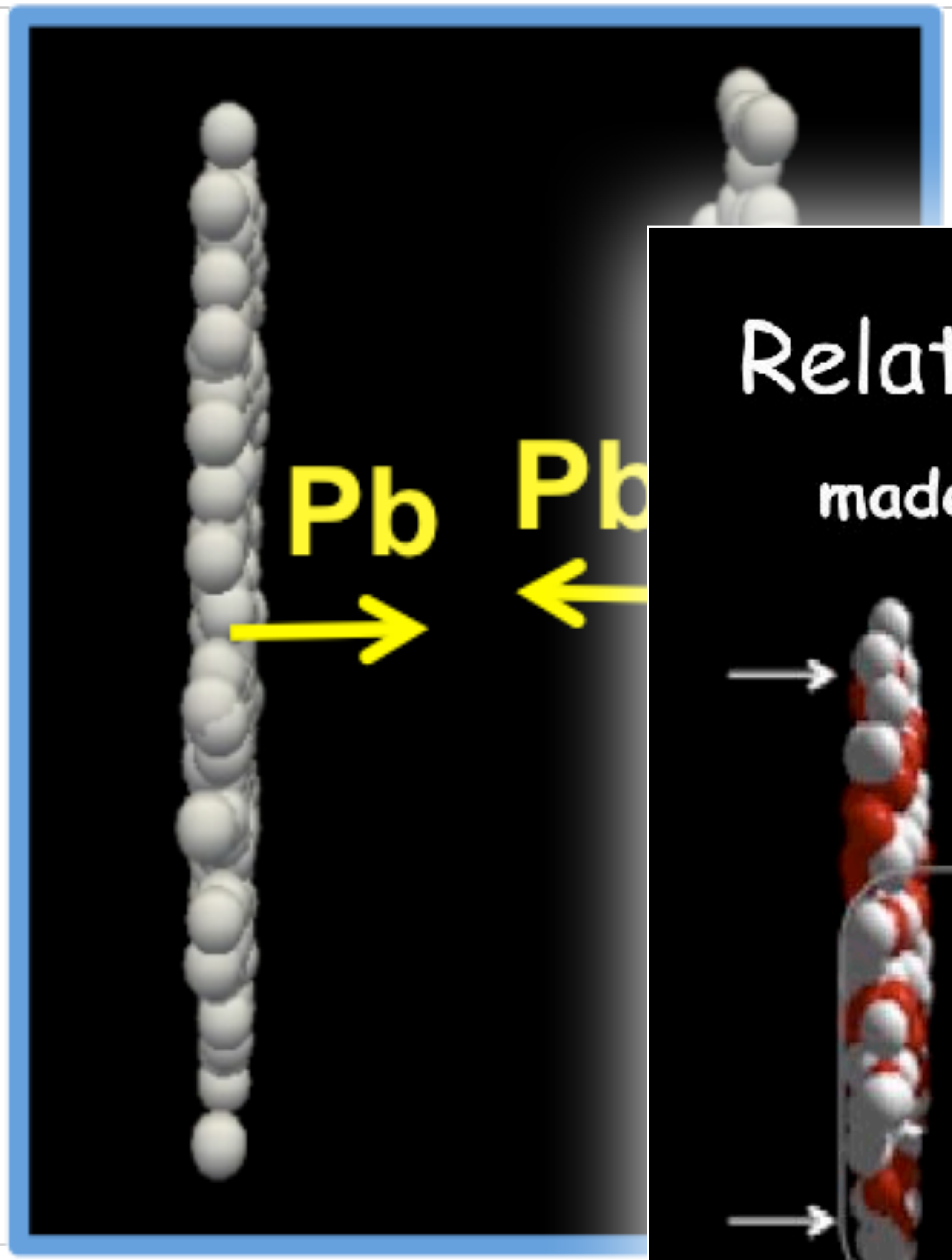


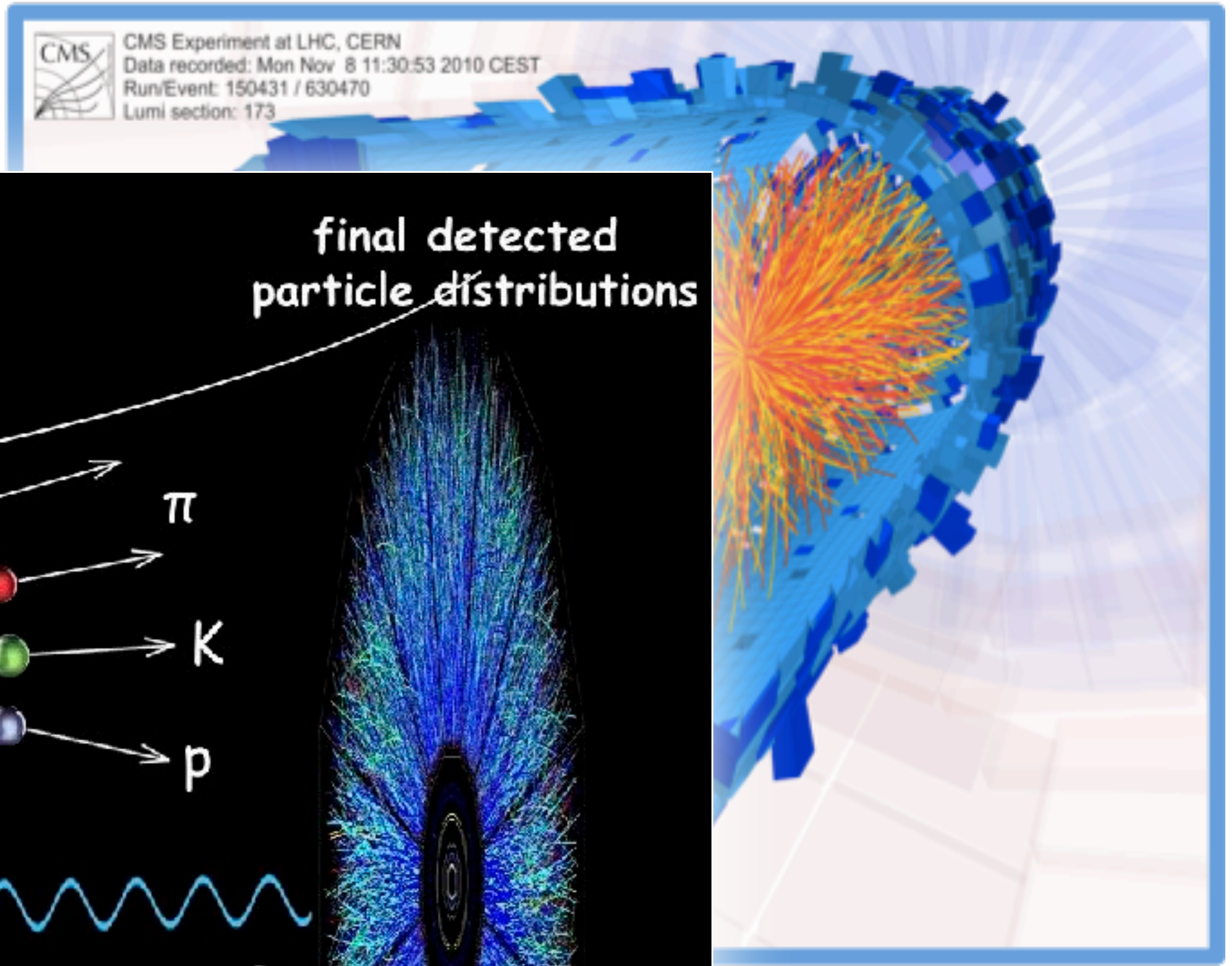
final state particles

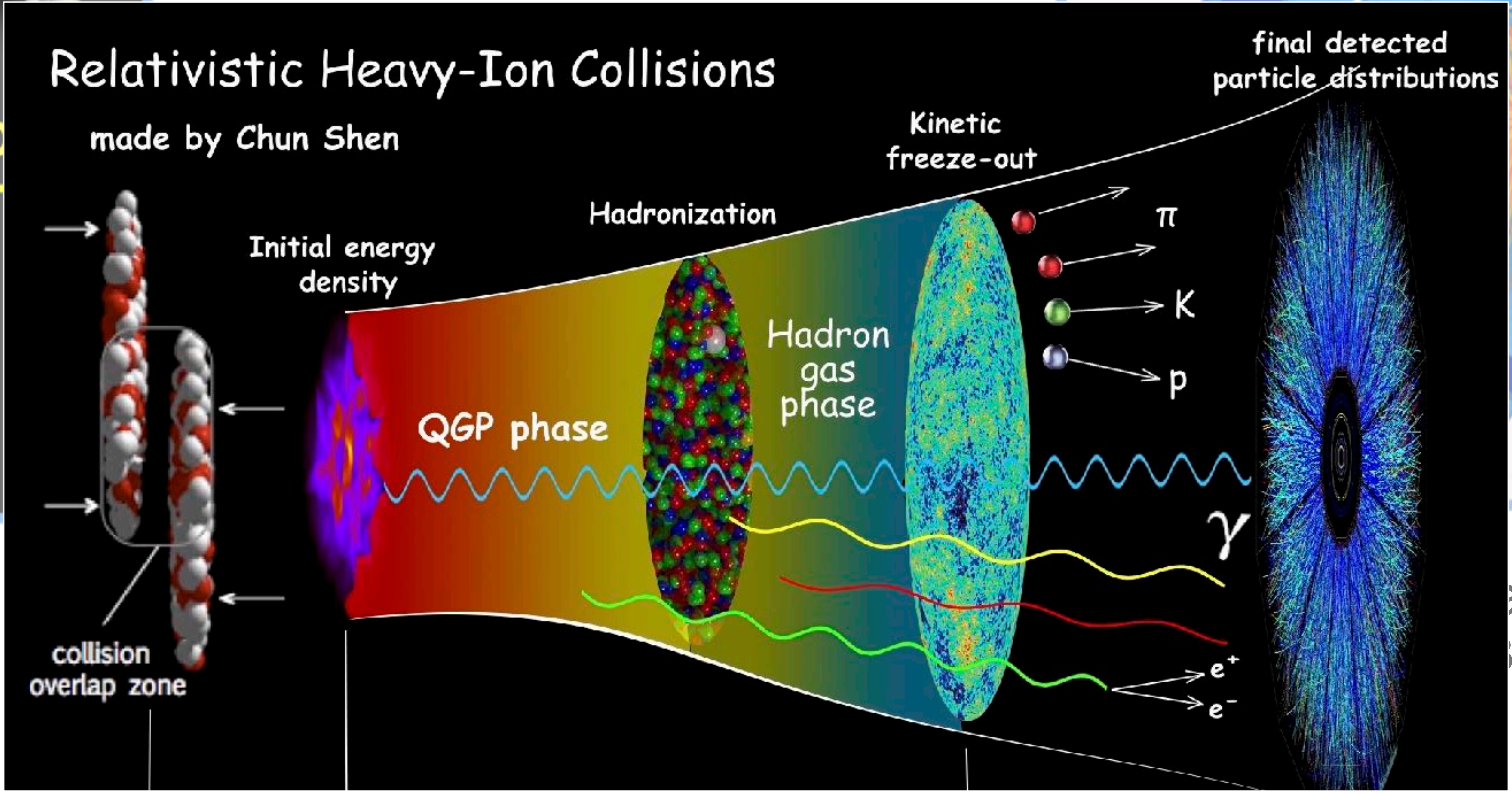
$$N \sim 10^{3-4}$$

Pre-reaction

Detection







articles

–4

Hydrodynamic attractors

- focus on properties of *hydrodynamic equations*
- *common behavior* developed very rapidly

rapidity distribution? *Shile Chen's talk tomorrow 09:00, Parallel I*

Hydrodynamic attractors

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Thermalization in phase space distribution

- solve Boltzmann equations, compared with hydro
- usually *linearized* Boltzmann, e.g., Relaxation Time Approximation

non-linear collision kernel? multiparticle correlations? *Shuai Lu's poster #223*

Quark-Gluon Plasma:

A strongly coupled, quantum, (quasi)many-body system!

Ideally, full quantum simulation of QCD in 3+1D

Practically, strong coupling QED in 1+1D

– confinement, chiral condensate

Hamiltonian

1+1D Schwinger model

$$H = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1 \partial_x - g\gamma^1 A - m)\psi \right) \mathrm{d}x \,.$$

$$\uparrow$$

$$L(t) = \int \left(-\frac{F^{\mu\nu}F_{\mu\nu}}{4} + \bar{\psi}(i\gamma^\mu \partial_\mu - g\gamma^\mu A_\mu - m)\psi \right) \mathrm{d}x \,.$$

E: electric field

A: electric potential

$\psi, \bar{\psi}$: fermion field

1+1D Schwinger model

$$H = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1 \partial_x - g\gamma^1 A - m)\psi \right) \mathrm{d}x \,.$$

$|0\rangle$:empty
$|1\rangle$:occupied

$$\{\psi_a(x), \psi_b^\dagger(y)\} = \delta_{a,b}\delta(x-y)$$

field operators represented by matrices

$$\{\chi_n^\dagger, \chi_m\} = \delta_{nm} \,, \quad \{\chi_n^\dagger, \chi_m^\dagger\} = \{\chi_n, \chi_m\} = 0 \,.$$

Jordan-Wigner

$$\chi_n = \frac{X_n - iY_n}{2} \prod_{m=1}^{n-1} (-iZ_m)$$

$$X_n \equiv I \otimes \cdots \otimes I \otimes \underset{n^{\mathrm{th}}}{\sigma_x} \otimes I \otimes \cdots \otimes I$$

1+1D Schwinger model

$$H = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1 \partial_x - g\gamma^1 A - m)\psi \right) \mathrm{d}x \, .$$

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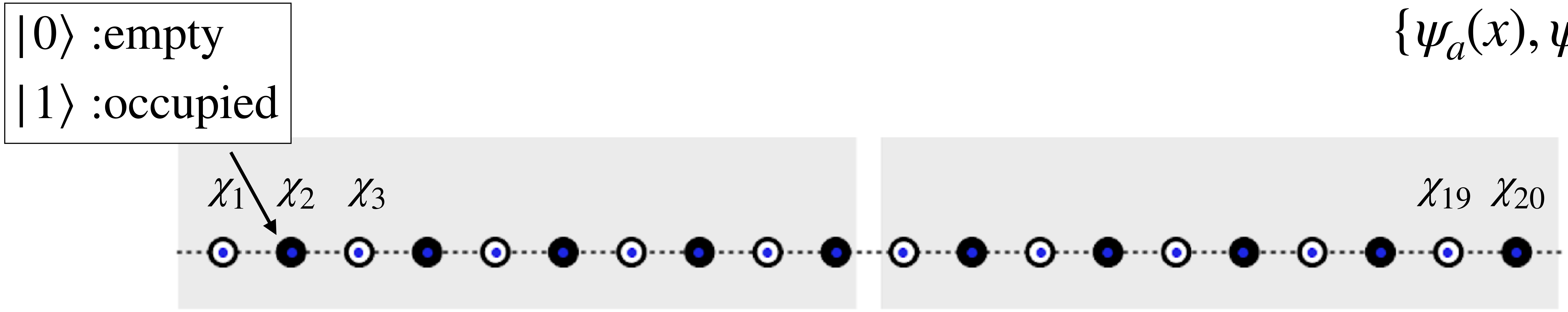


field operators represented by matrices

$$\{\chi_n^\dagger, \chi_m\} = \delta_{nm} \, , \quad \{\chi_n^\dagger, \chi_m^\dagger\} = \{\chi_n, \chi_m\} = 0 \, .$$

$$H = \frac{1}{4a} \sum_{n=1}^{N-1} (X_n X_{n+1} + Y_n Y_{n+1}) + \frac{m}{2} \sum_{n=1}^{N} (-1)^n Z_n + \frac{a\,g^2}{2} \sum_{n=1}^{N-1} L_n^2 \, .$$

Eigenstates of H: vacuum and (quasi)-particles

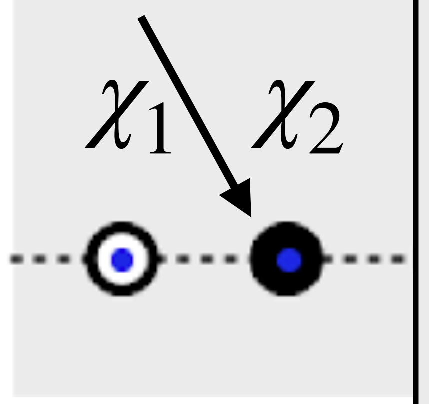
Time evolution:

$$\frac{\partial}{\partial t}|\psi(t)\rangle = -\,i\,H\,|\psi(t)\rangle$$

$$O(t) = \langle \psi(t)|\,\hat{O}\,|\psi(t)\rangle$$

$|0\rangle$:empty

$|1\rangle$:occupied



$\ldots_{a,b}\delta(x-y)$

d operators
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trices

$\{\chi_n, \chi_m\} = 0\,.$

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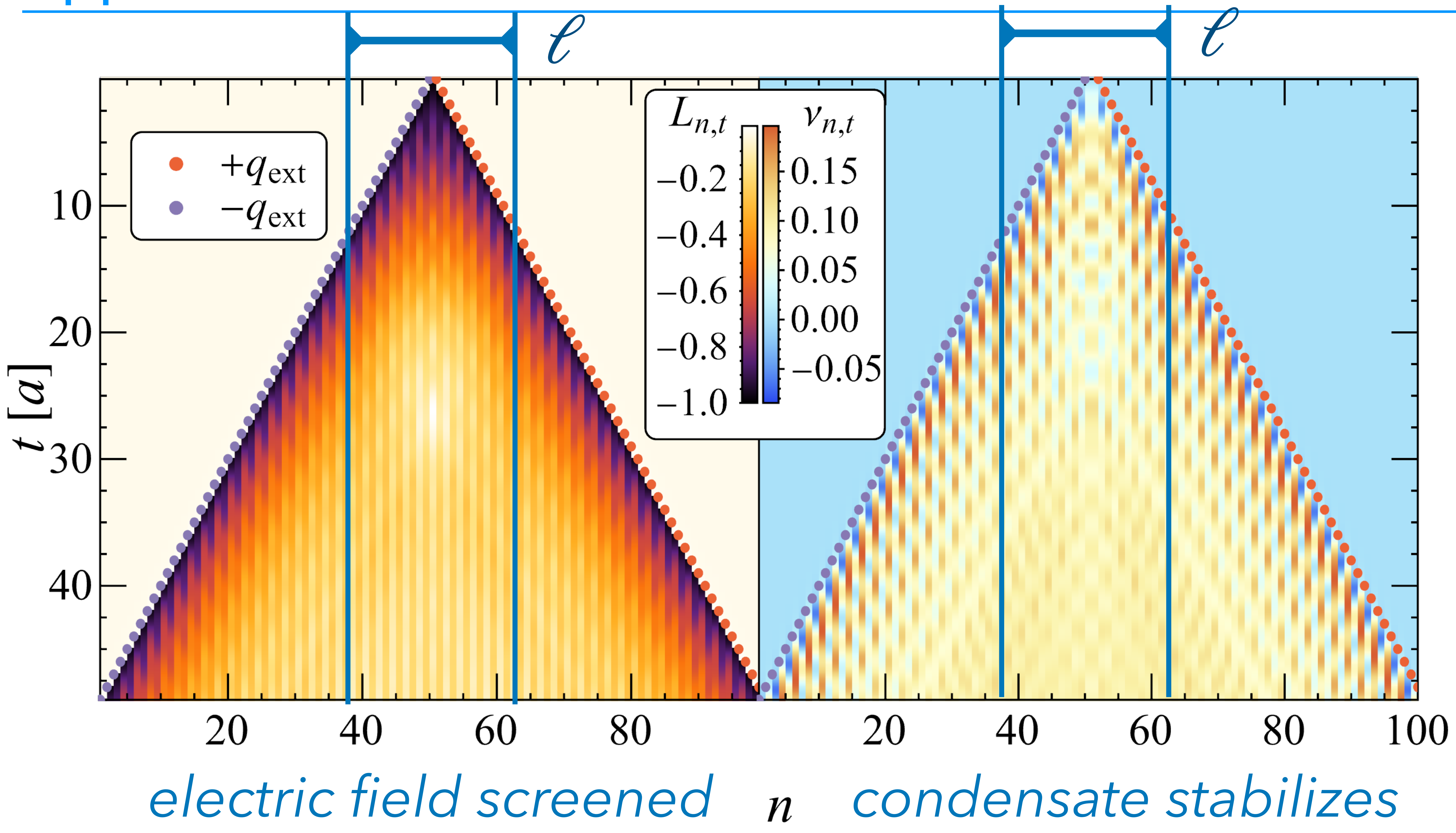
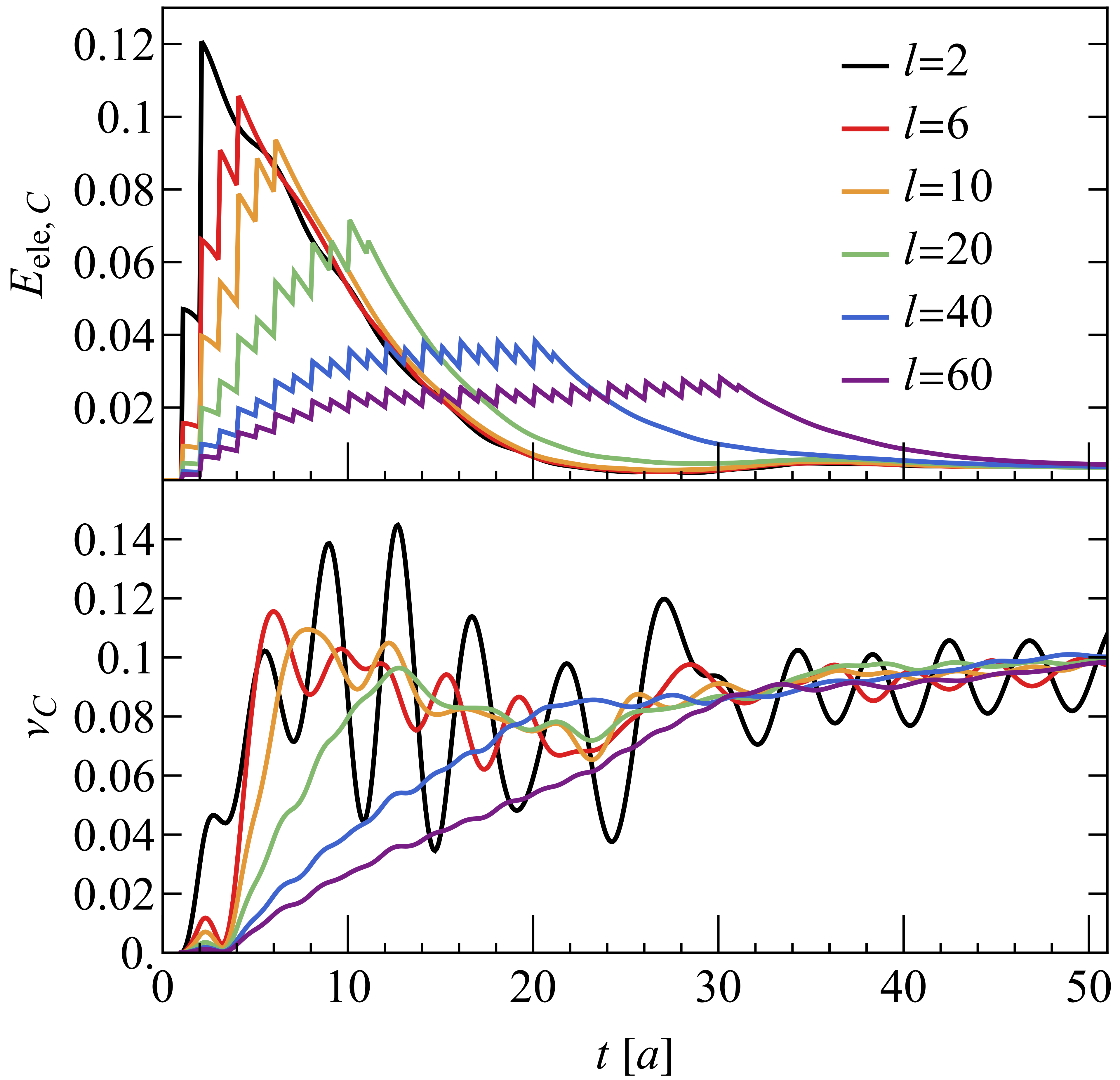
approach to the continuum limit



observables at the center

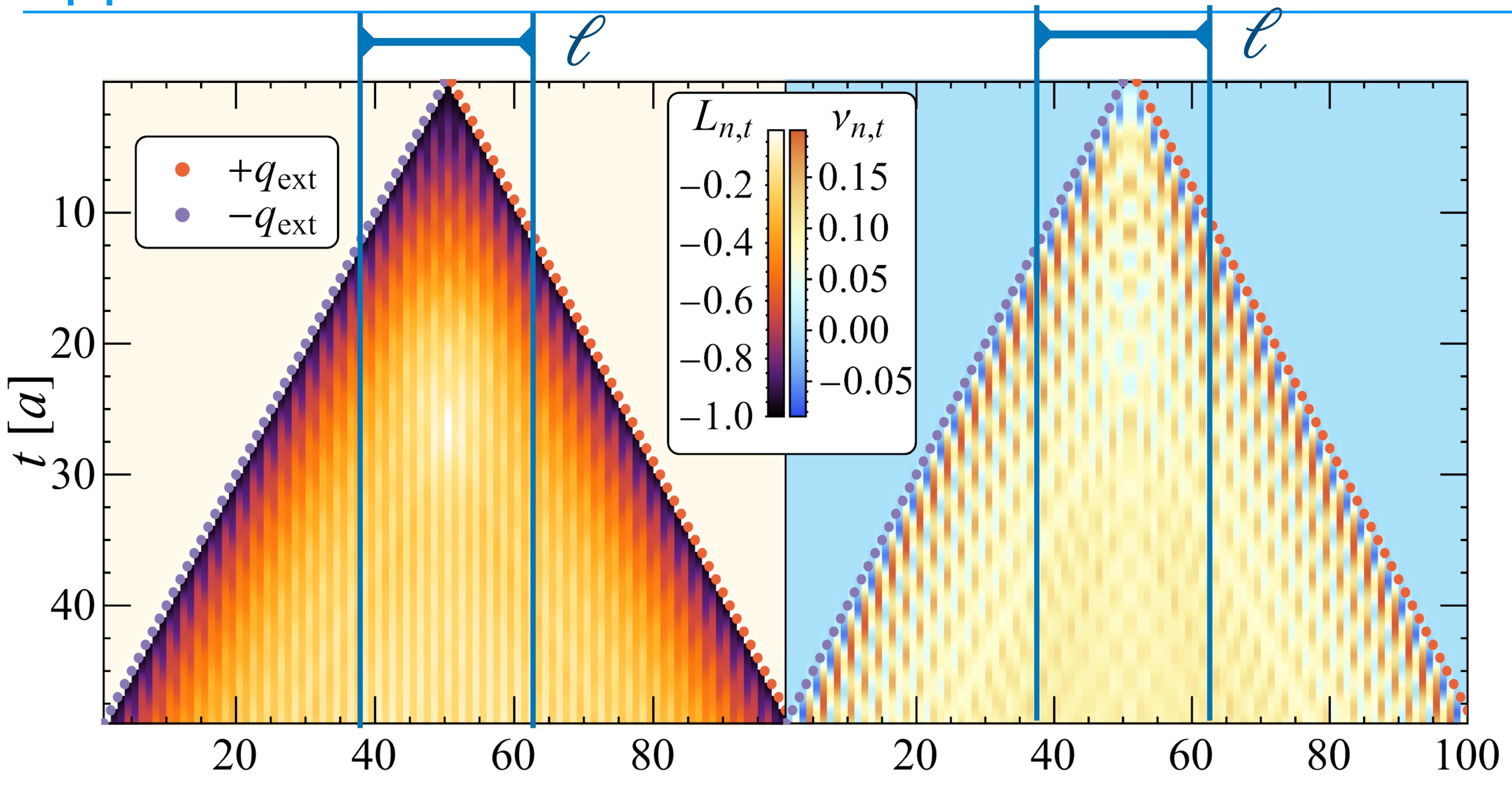


$$H_{\rm Schwinger}\,|\psi(t=0)\rangle = E_0\,|\psi(t=0)\rangle$$

$$\frac{\partial}{\partial t}|\psi(t)\rangle = -\,i\left(H_{\rm Schwinger} + H_{\rm source}(t)\right)|\psi(t)\rangle$$

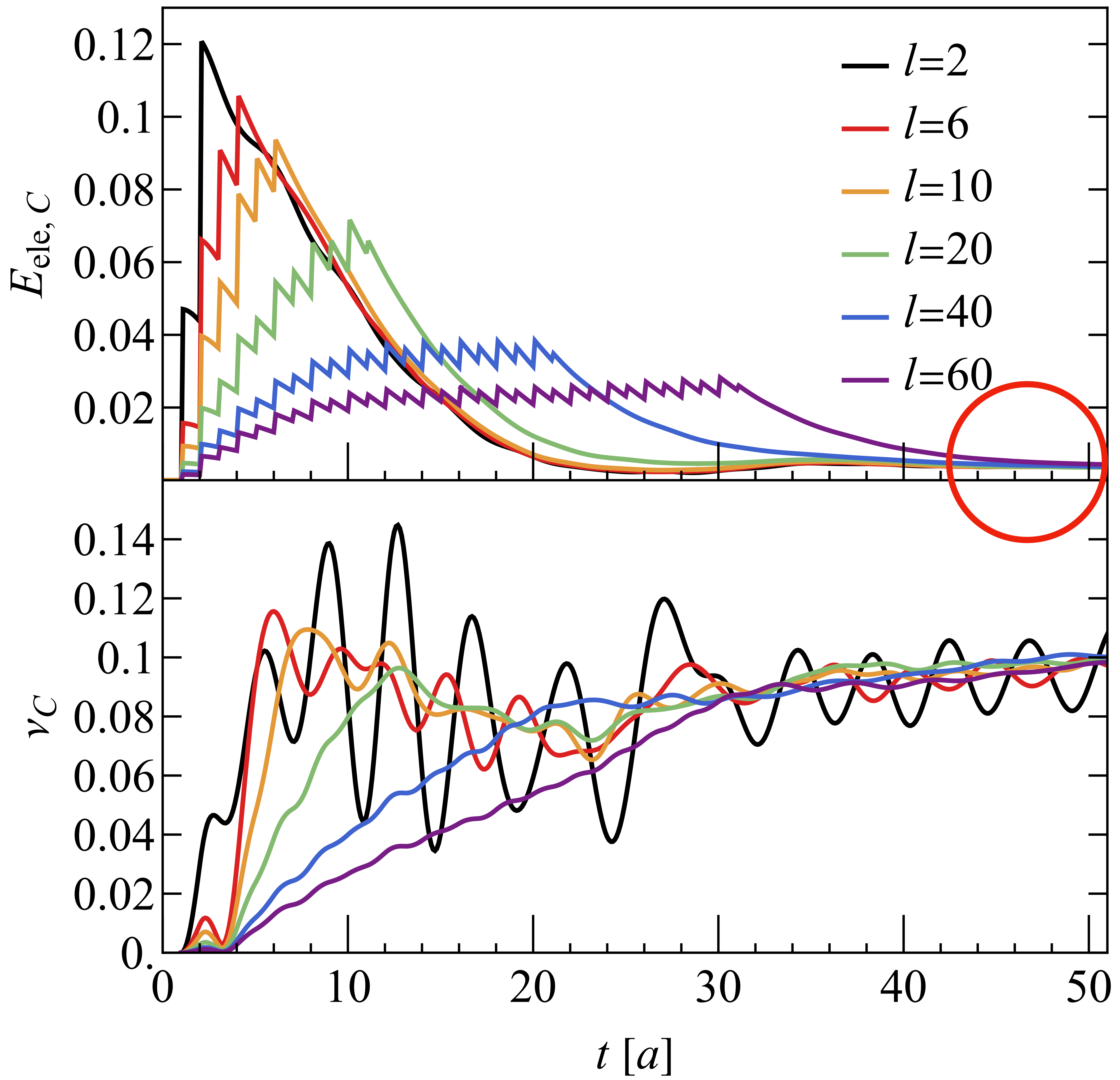
w/ Florio, Frenklakh, Ikeda, Kharzeev, Korepin, Yu,
PhysRevLett.131.021902; PhysRevD.110. 094029; in progress

approach to the continuum limit



electric field screened   *condensate stabilizes*

observables at the center



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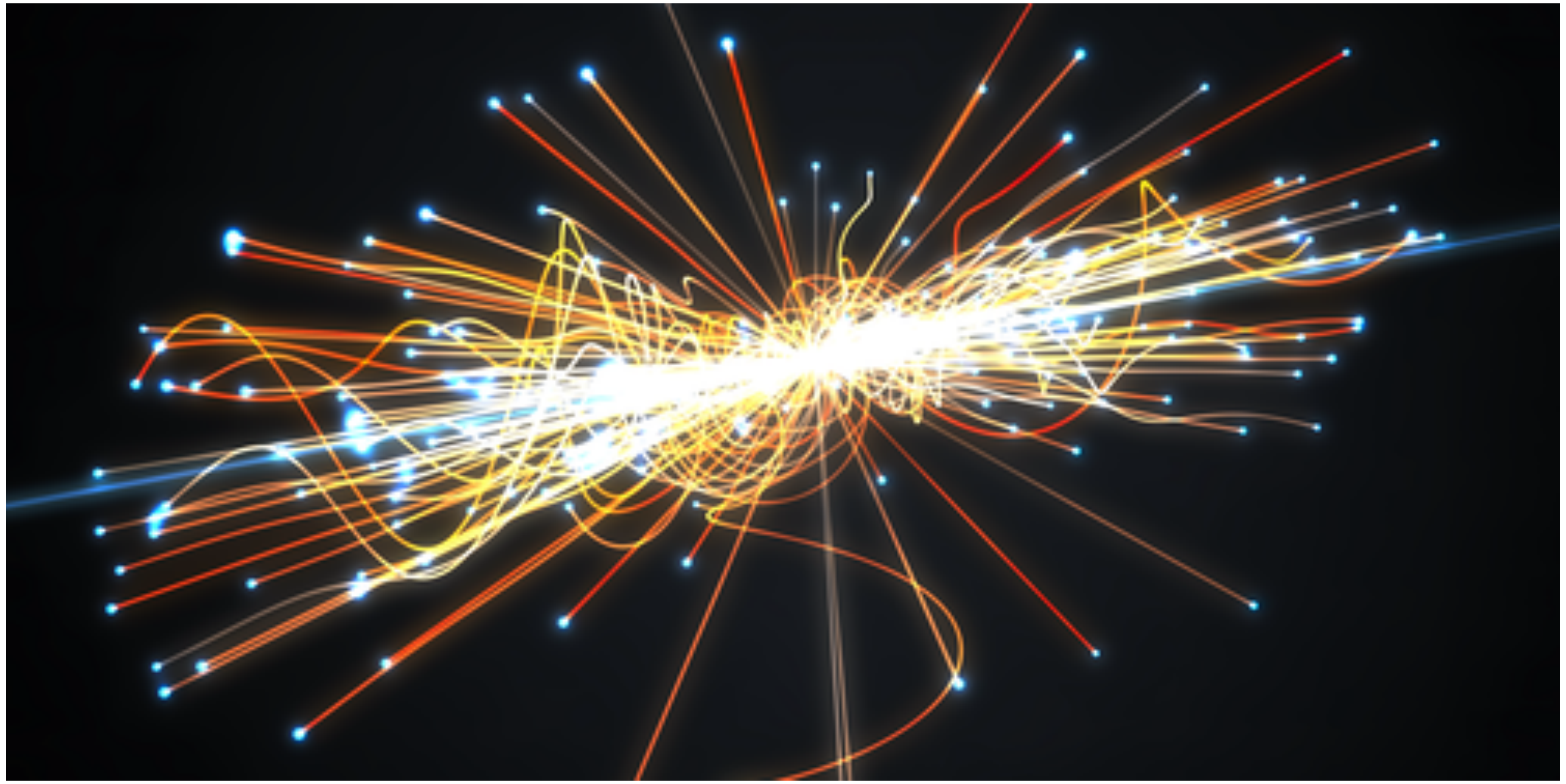


$$H = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1\partial_x - g\gamma^1 A - m)\psi \right) \mathrm{d}x \,.$$

$$|\psi(t)\rangle = \exp(-i\,H\,t)\,|\psi(0)\rangle$$

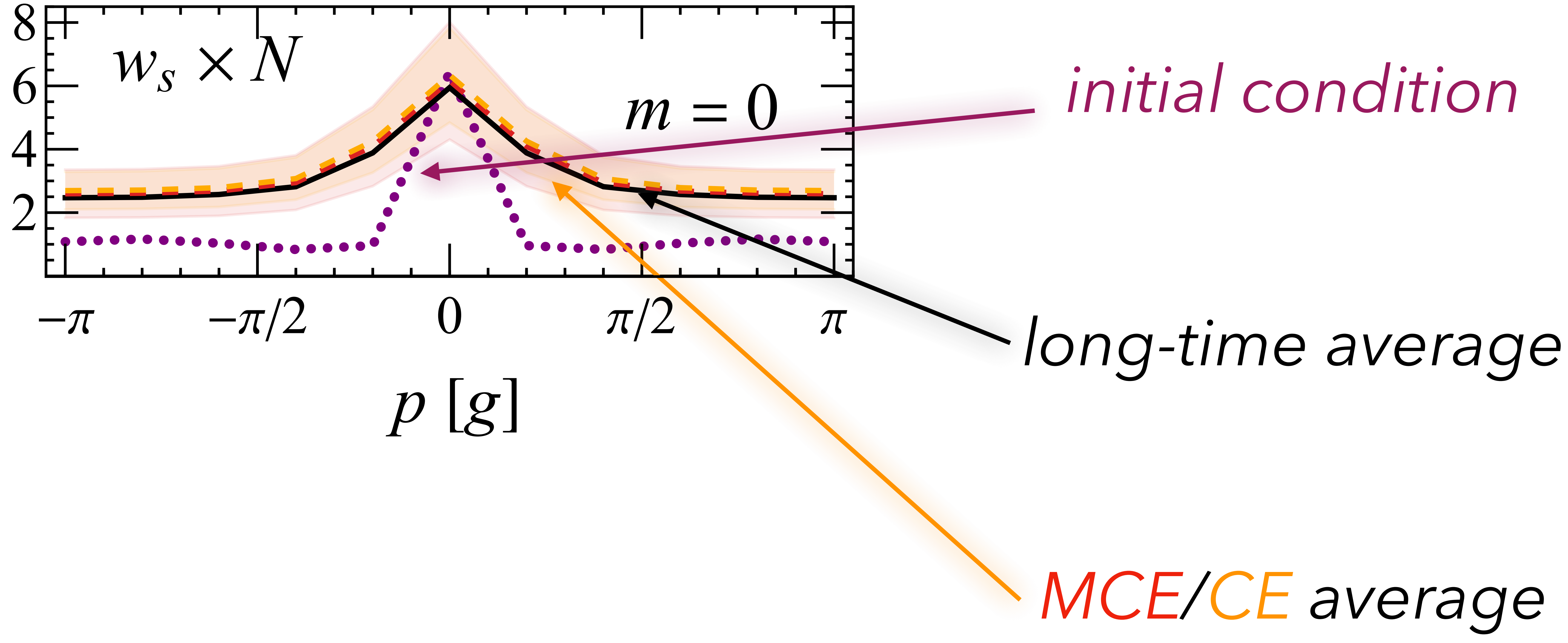
$$W(t) = \langle \psi(t)\,|\,\hat{W}\,|\,\psi(t)\rangle$$

Thermalization in an isolated quantum system?

Quantum distribution function: Wigner function

$$\hat{W}_{\alpha\beta}(t, z, p) = \int \bar{\psi}_\alpha(z_+) U(z_+, z_-) \psi_\beta(z_-)\, e^{i\frac{py}{\hbar}}\, \mathrm{d}y$$

thermalization of quantum distribution function



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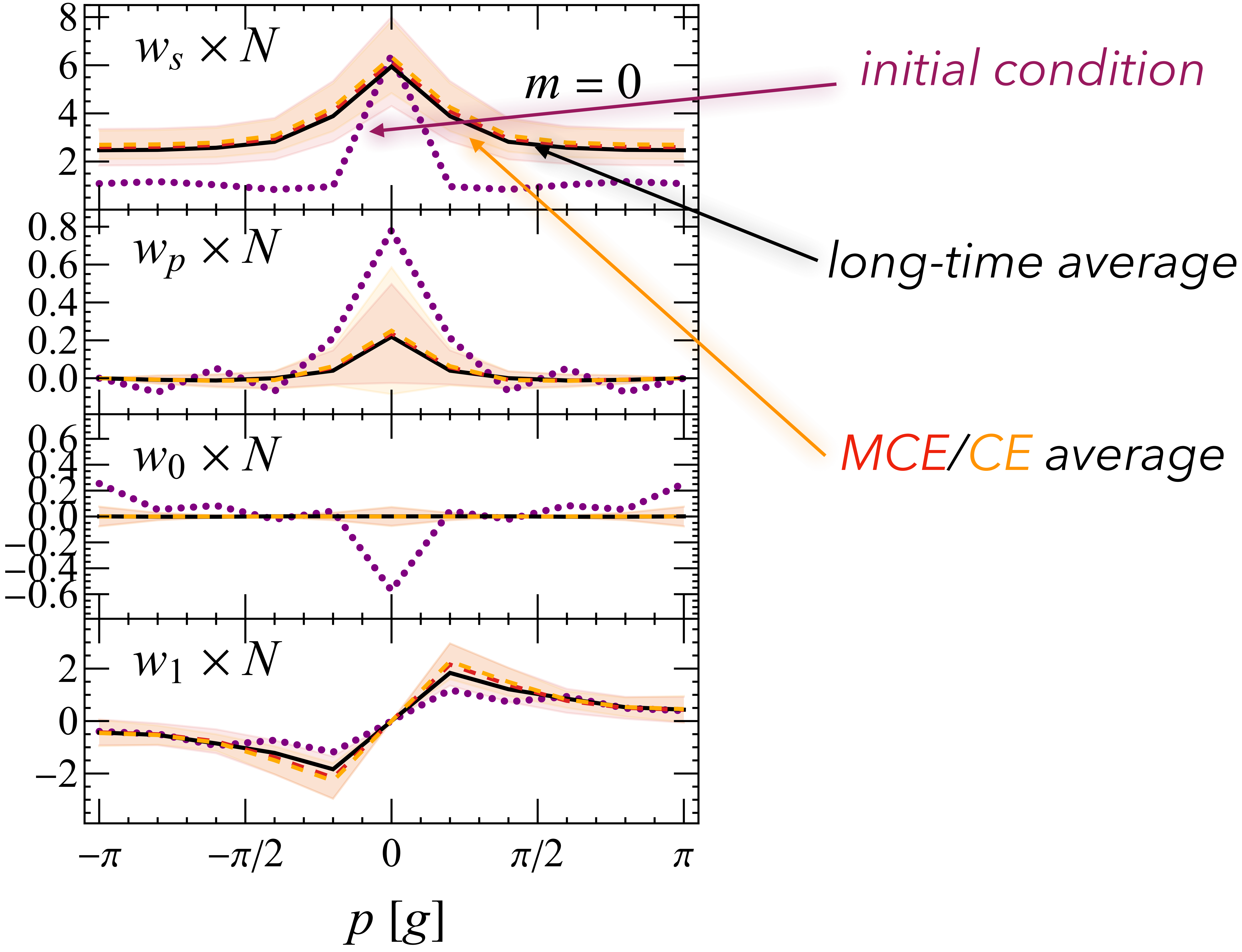
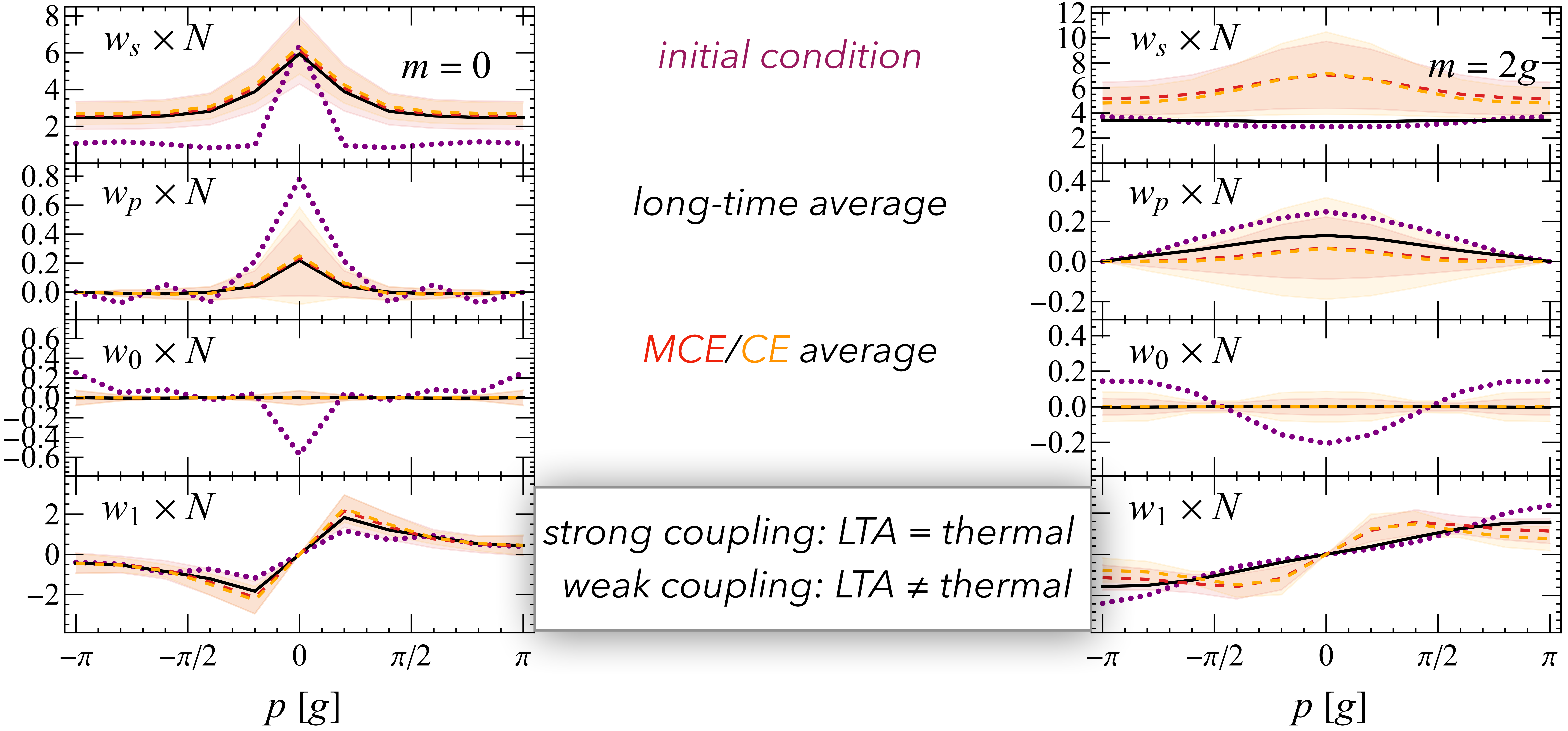


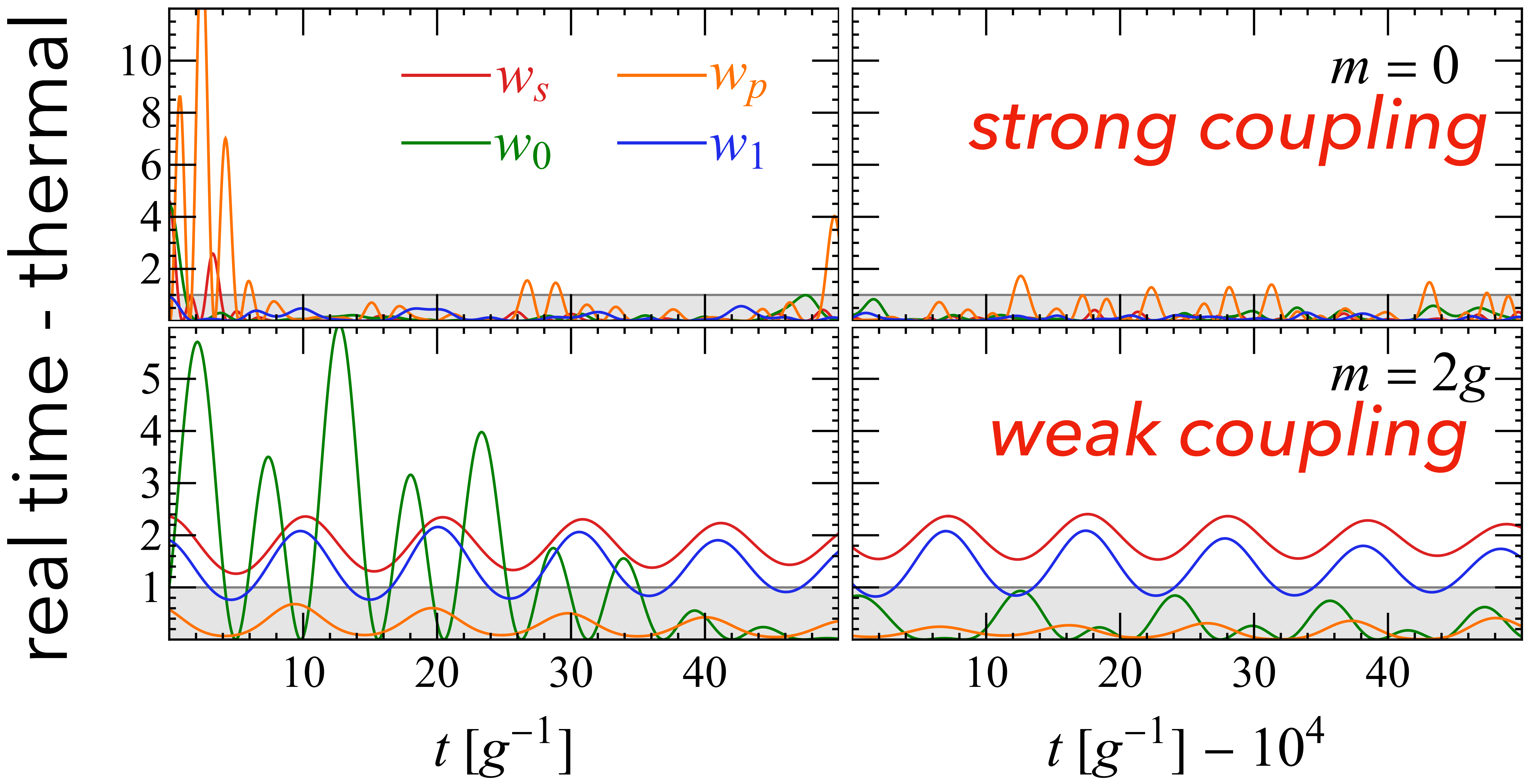
thermalization of quantum distribution function



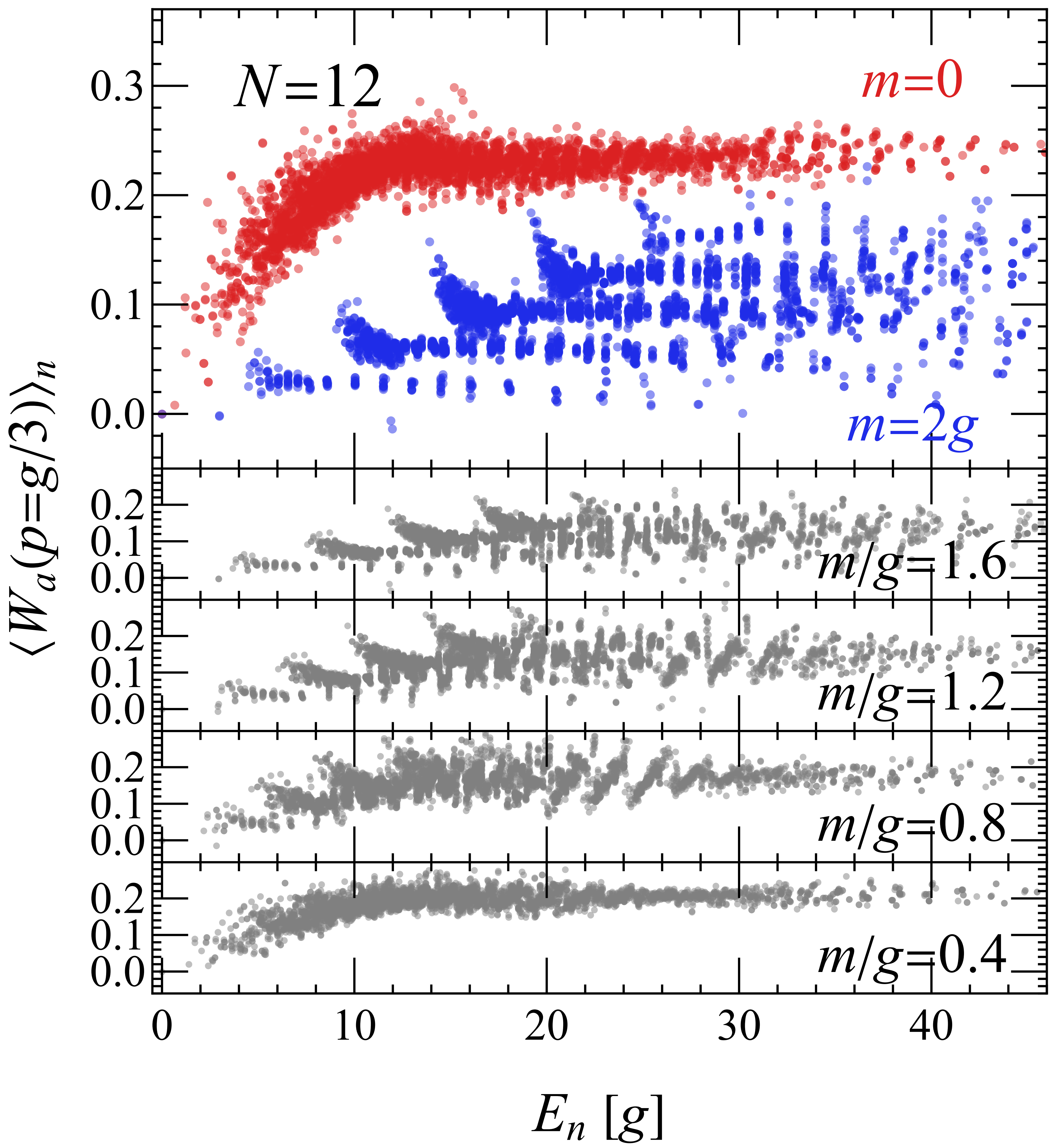
strong coupling

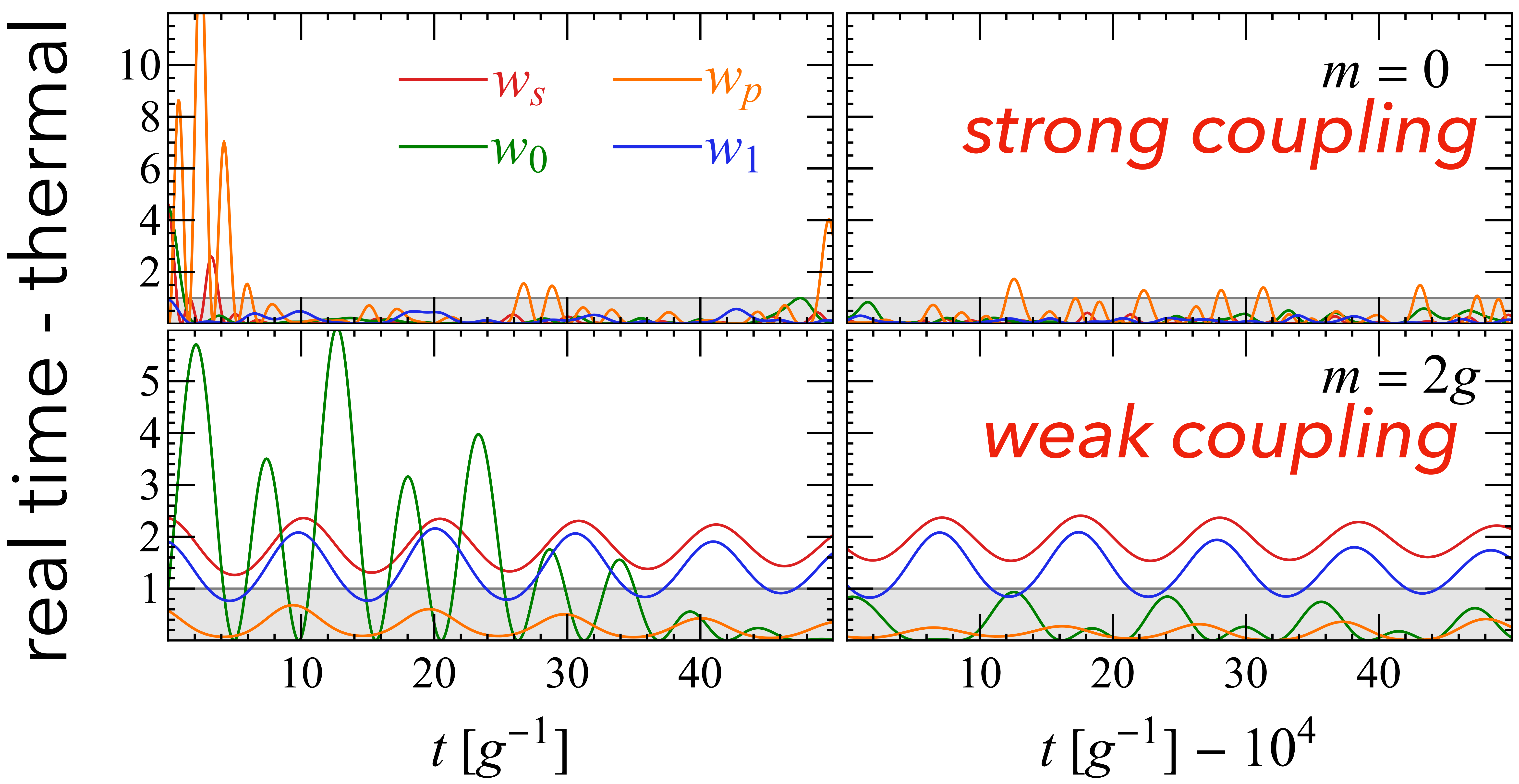
weak coupling

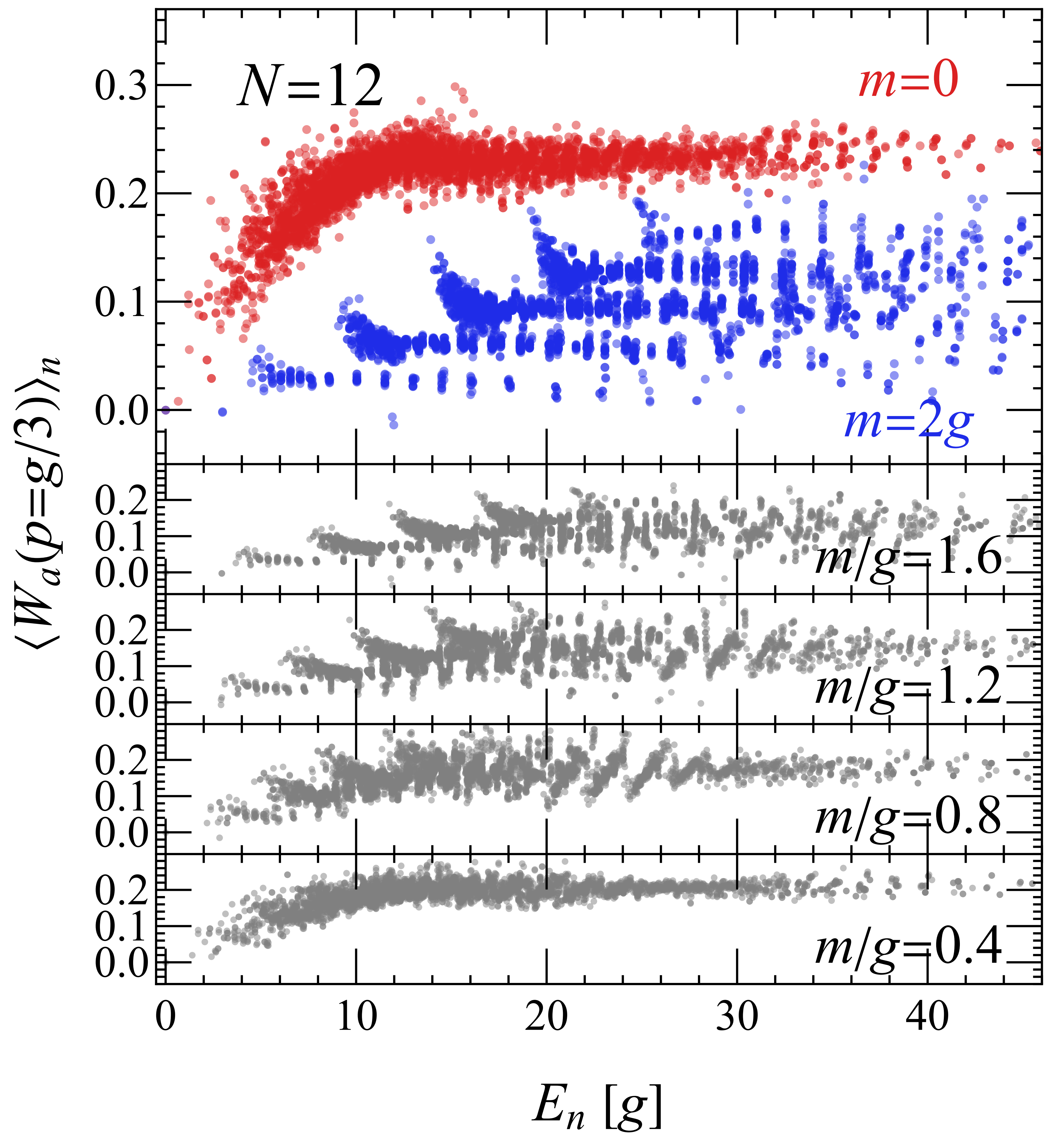
thermalization of quantum distribution function



thermalization of quantum distribution function



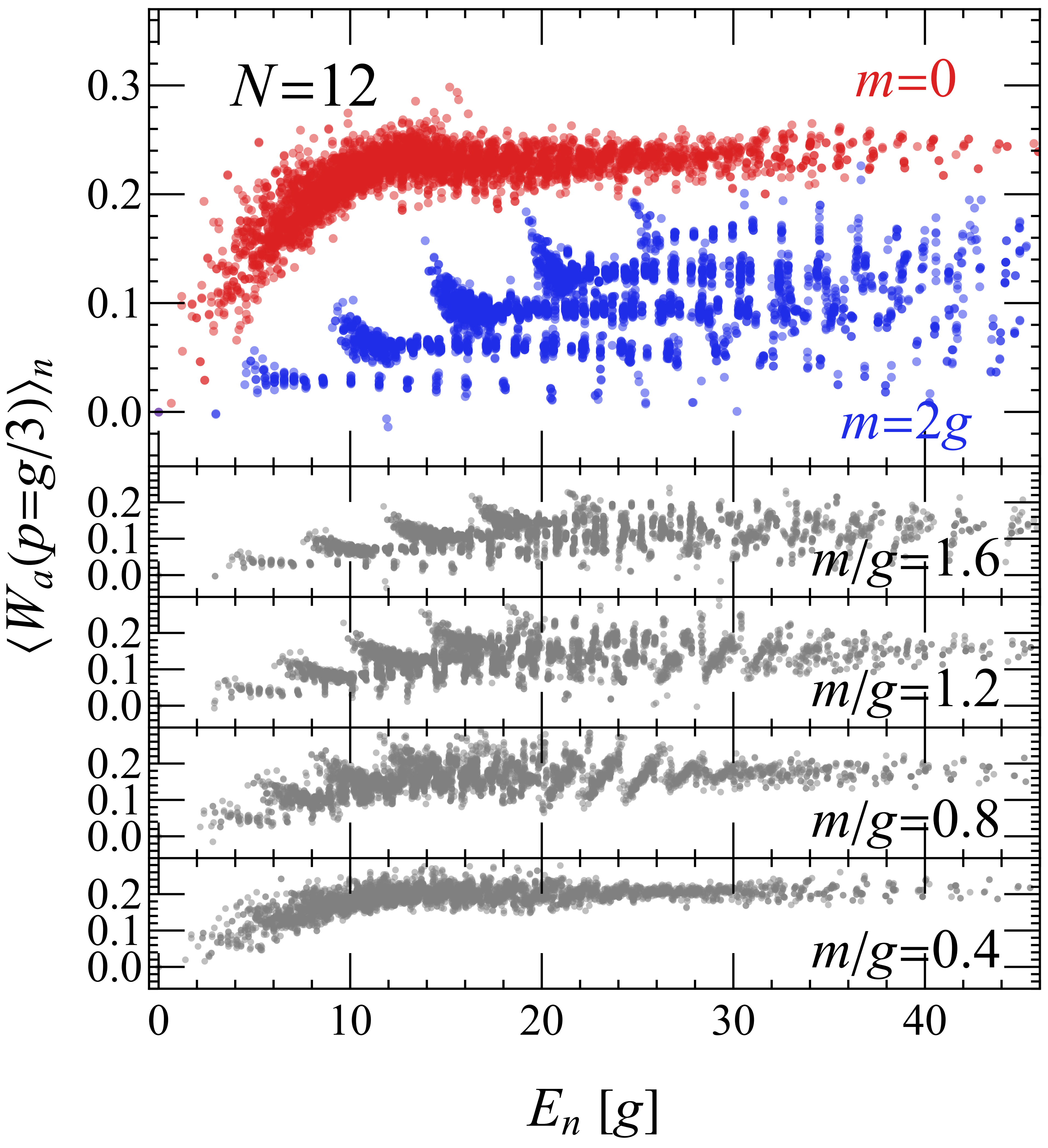




eigenstate thermalization hypothesis

$$\langle n | \hat{O} | n \rangle \approx f_O(E_n)$$

thermalization of quantum distribution function



eigenstate thermalization hypothesis

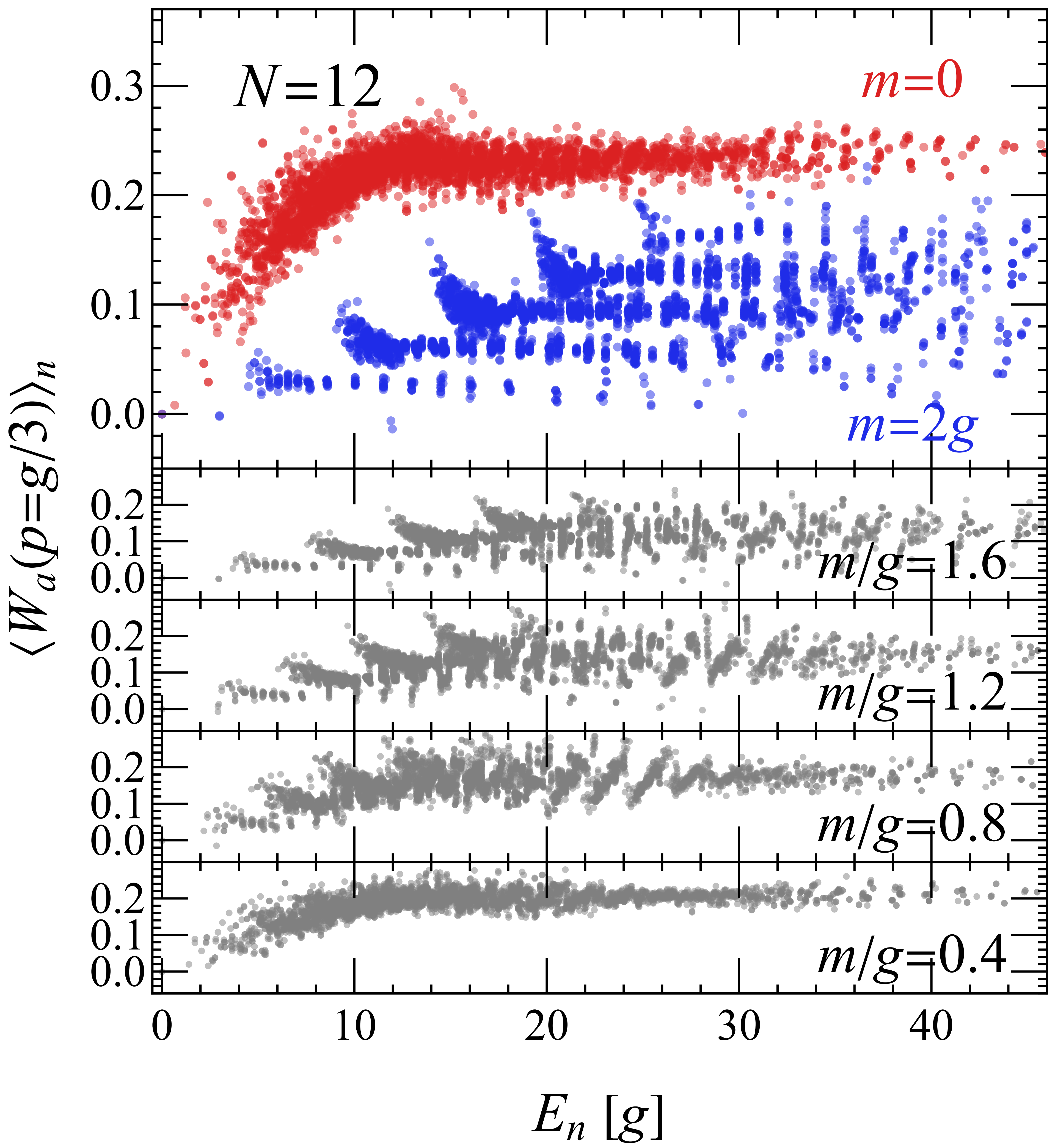
$$\langle n|\hat{O}|n\rangle \approx f_O(E_n)$$

$$\sum_n p_n \langle n|\hat{O}|n\rangle \approx f_O\left(\sum_n p_n E_n\right)$$

general pure state: $|c_n|^2$

thermal: $e^{-\beta E_n}/Z$

thermalization of quantum distribution function



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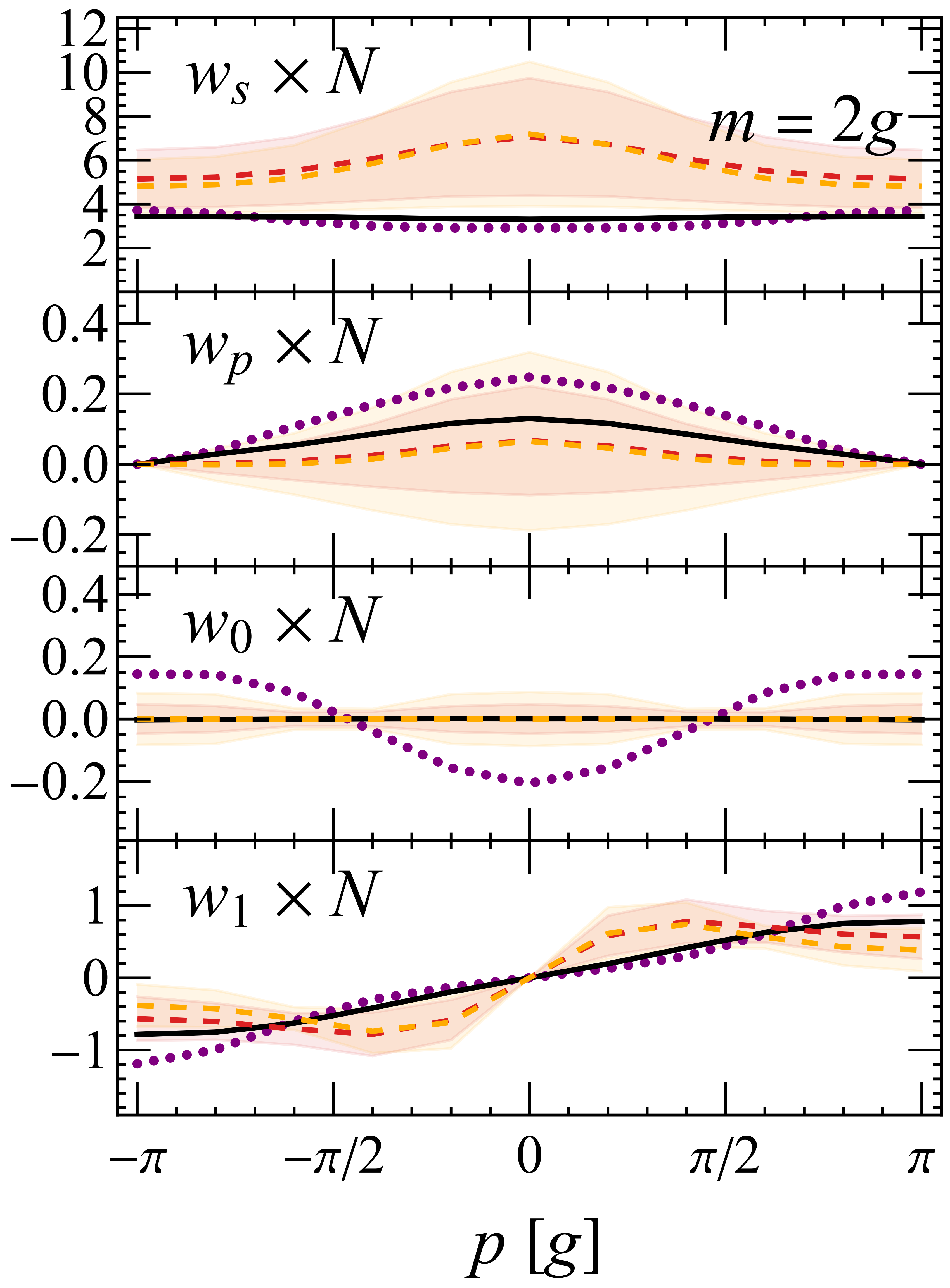
general pure state: $p_n = |c_n|^2$

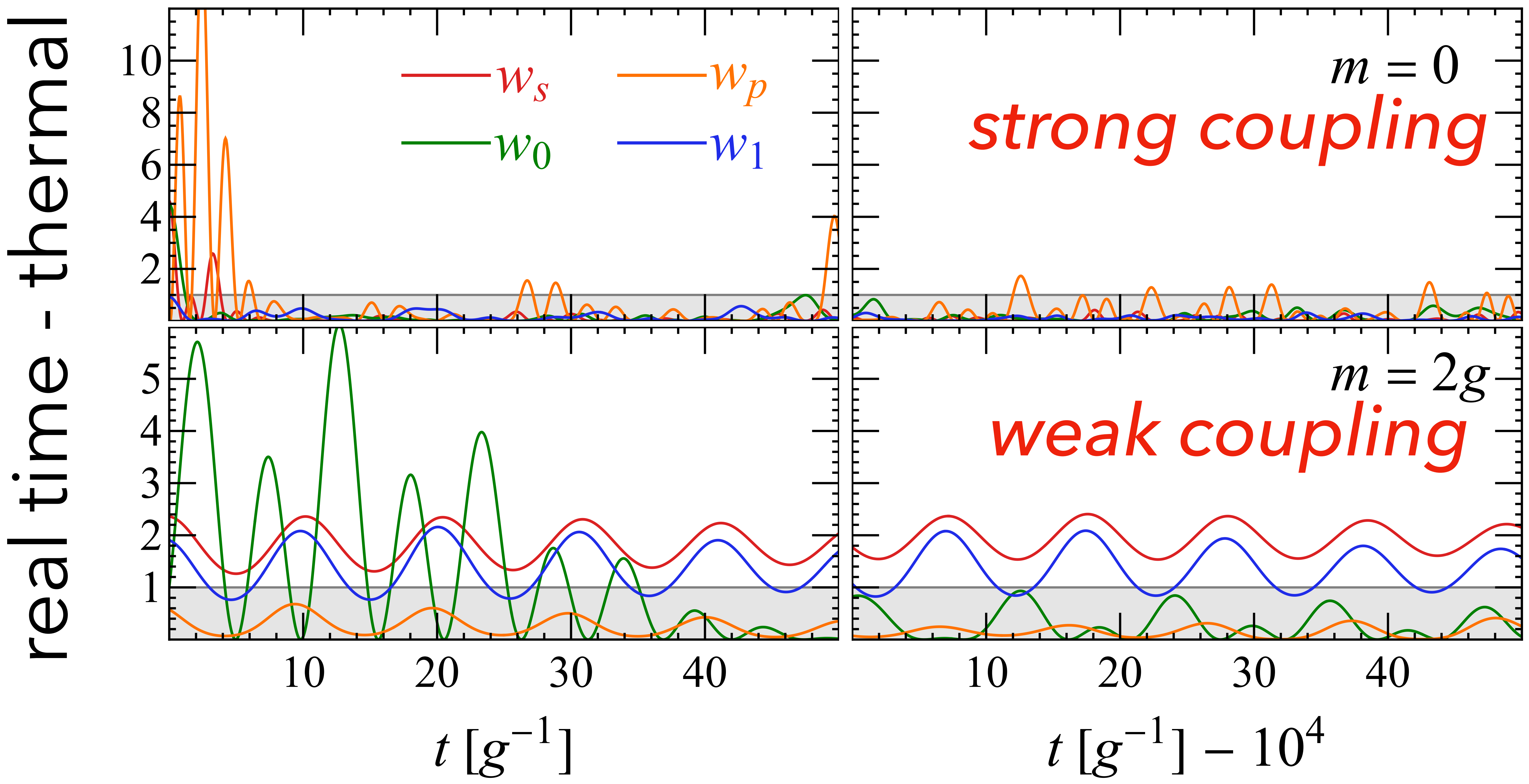
thermal: $p_n = e^{-\beta E_n}/Z$

$$\langle O\rangle_{PS} \approx \langle O\rangle_{th}$$

if $\langle E\rangle_{PS} = \langle E\rangle_{th}$

Partial thermalization?





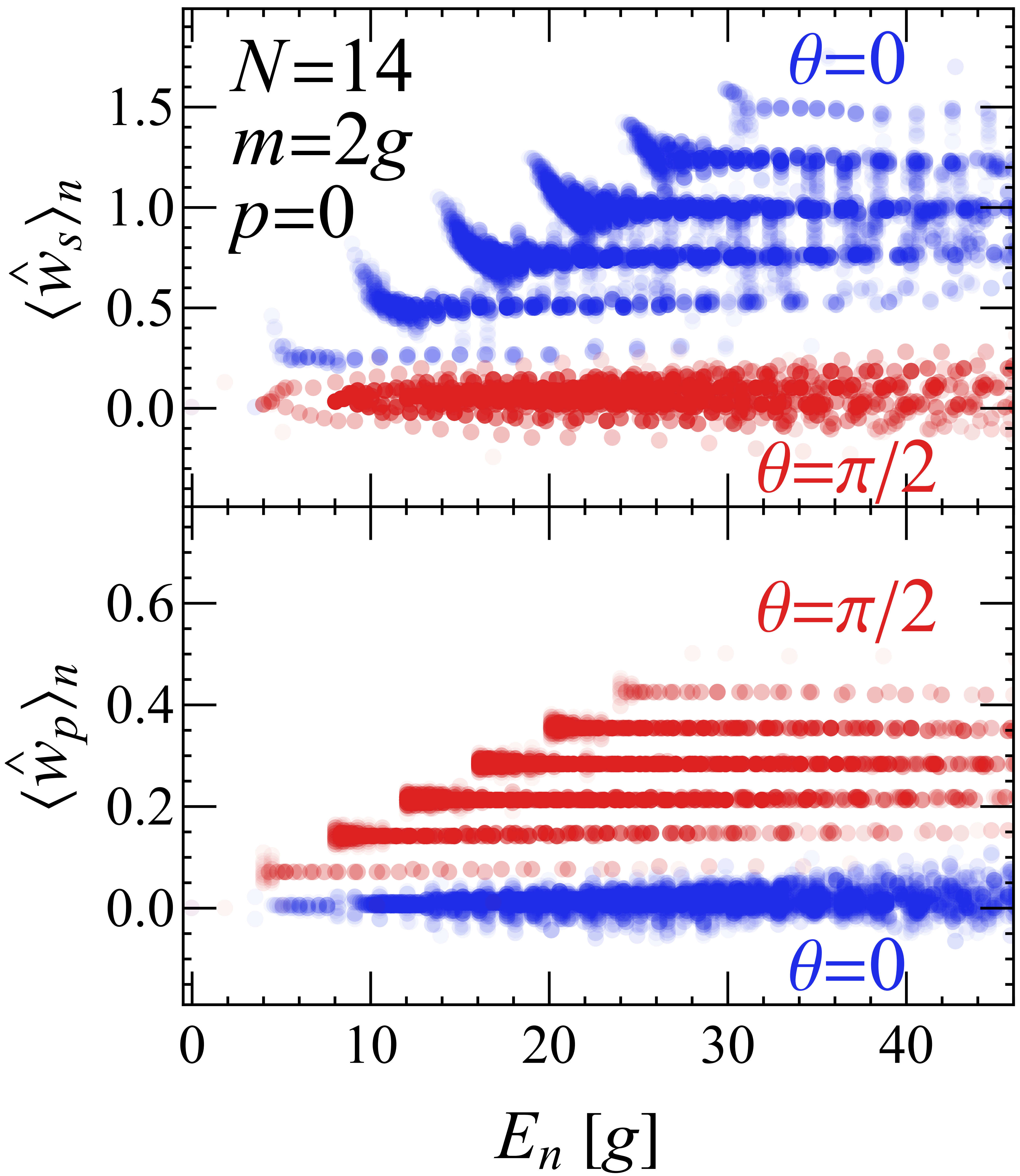
$$H = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1\partial_x - g\gamma^1 A - \underline{m e^{i\theta\gamma^5}})\psi \right) \mathrm{d}x$$

$\theta = \frac{\pi}{2}$: $m\,\bar{\psi}\gamma^5\psi$

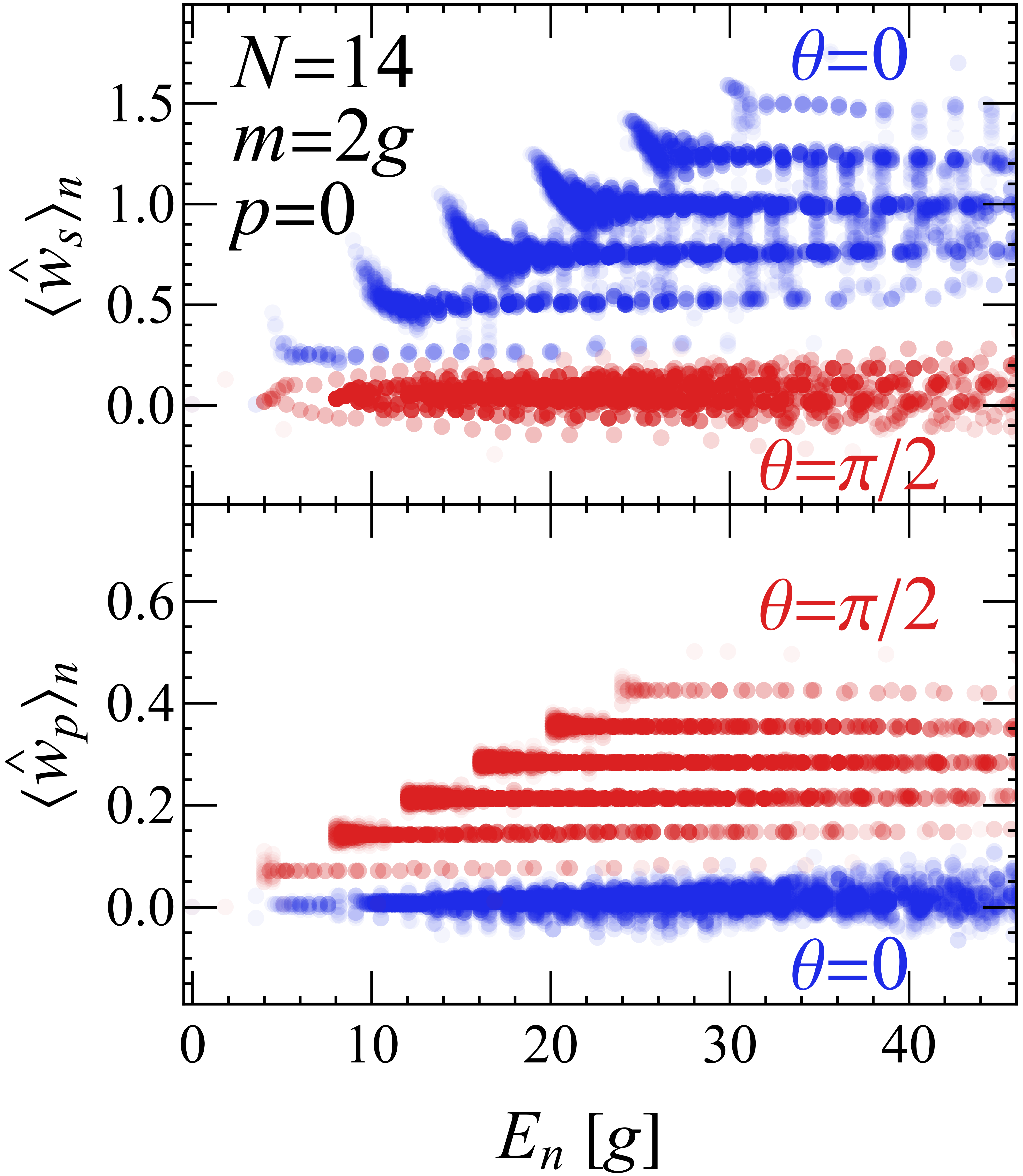
$\theta = 0$: $m\,\bar{\psi}\psi$

$$w_s(x,p) \sim \int_{x-y} e^{ip(x-y)}\bar{\psi}(x)\psi(y)$$

$$w_p(x,p) \sim \int_{x-y} e^{ip(x-y)}\bar{\psi}(x)\gamma^5\psi(y)$$



Shile Chen (陈诗乐)



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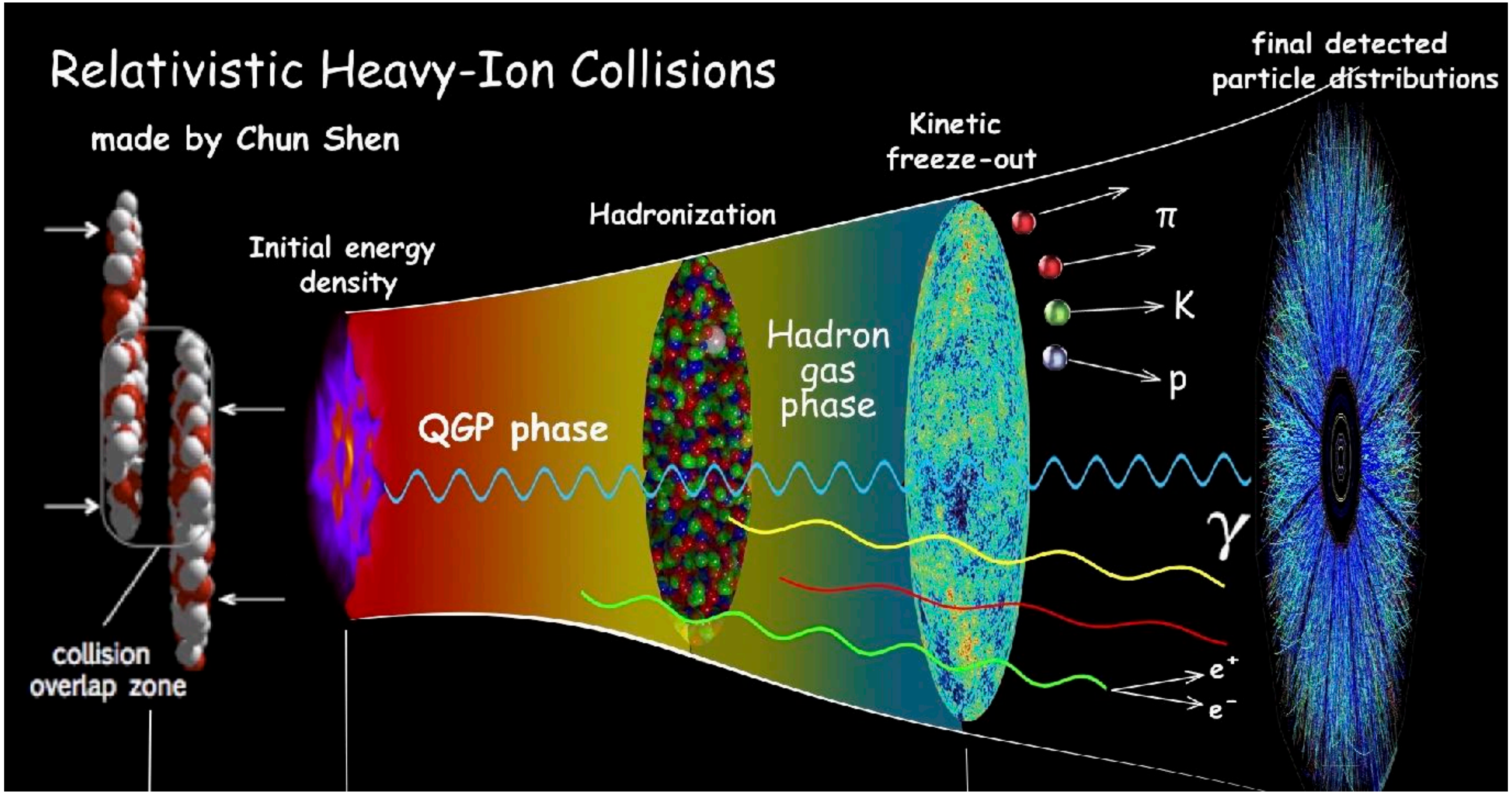
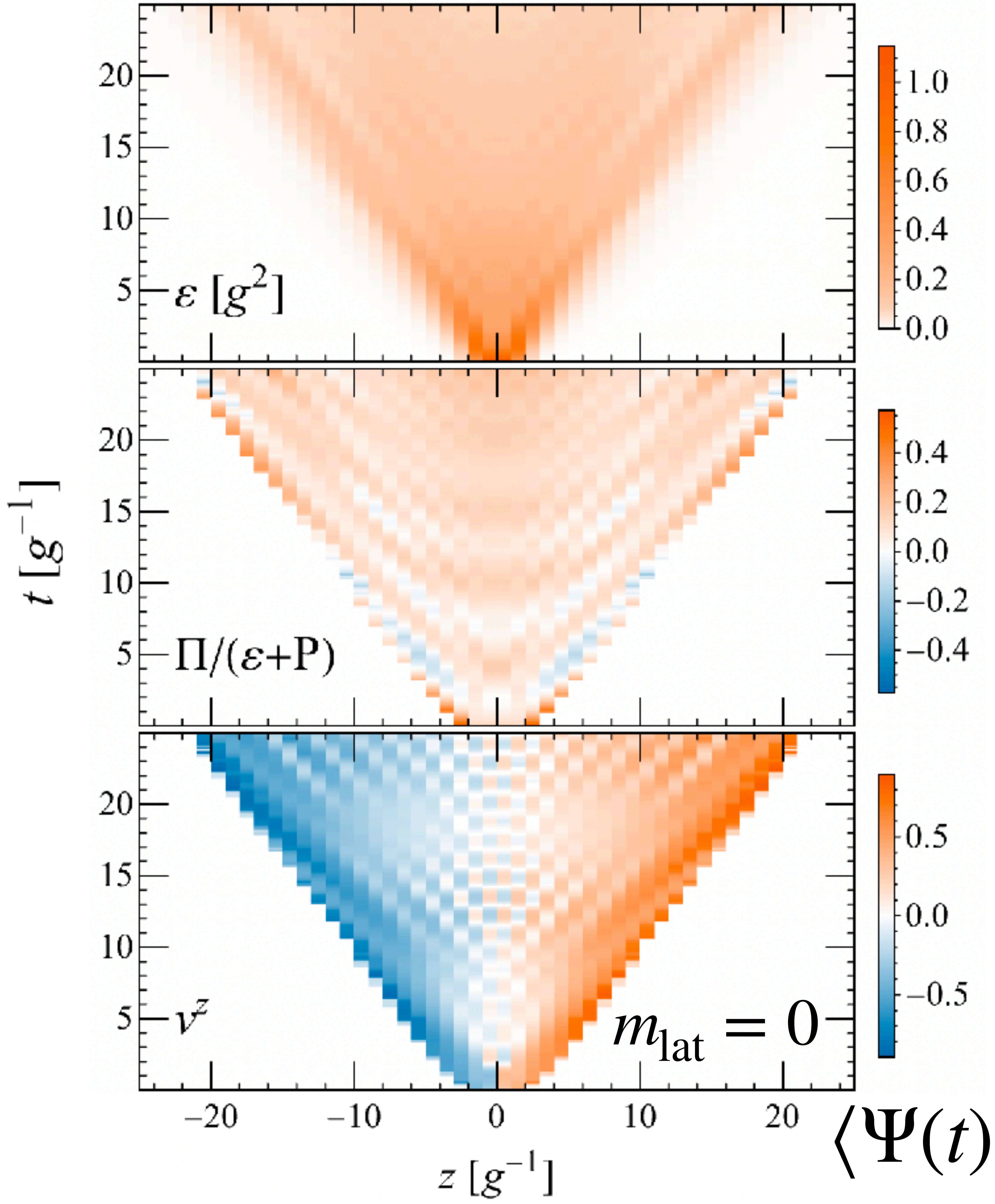


Emergence of Hydrodynamics?

$$\mathscr{L} \Rightarrow \hat{T}^{\mu\nu} \qquad \langle \Psi(t) \,|\, \hat{T}^{\mu\nu}(x) \,|\, \Psi(t) \rangle = (\varepsilon + P + \Pi) u^\mu u^\nu - (P + \Pi) g^{\mu\nu}$$

emergence of hydrodynamics

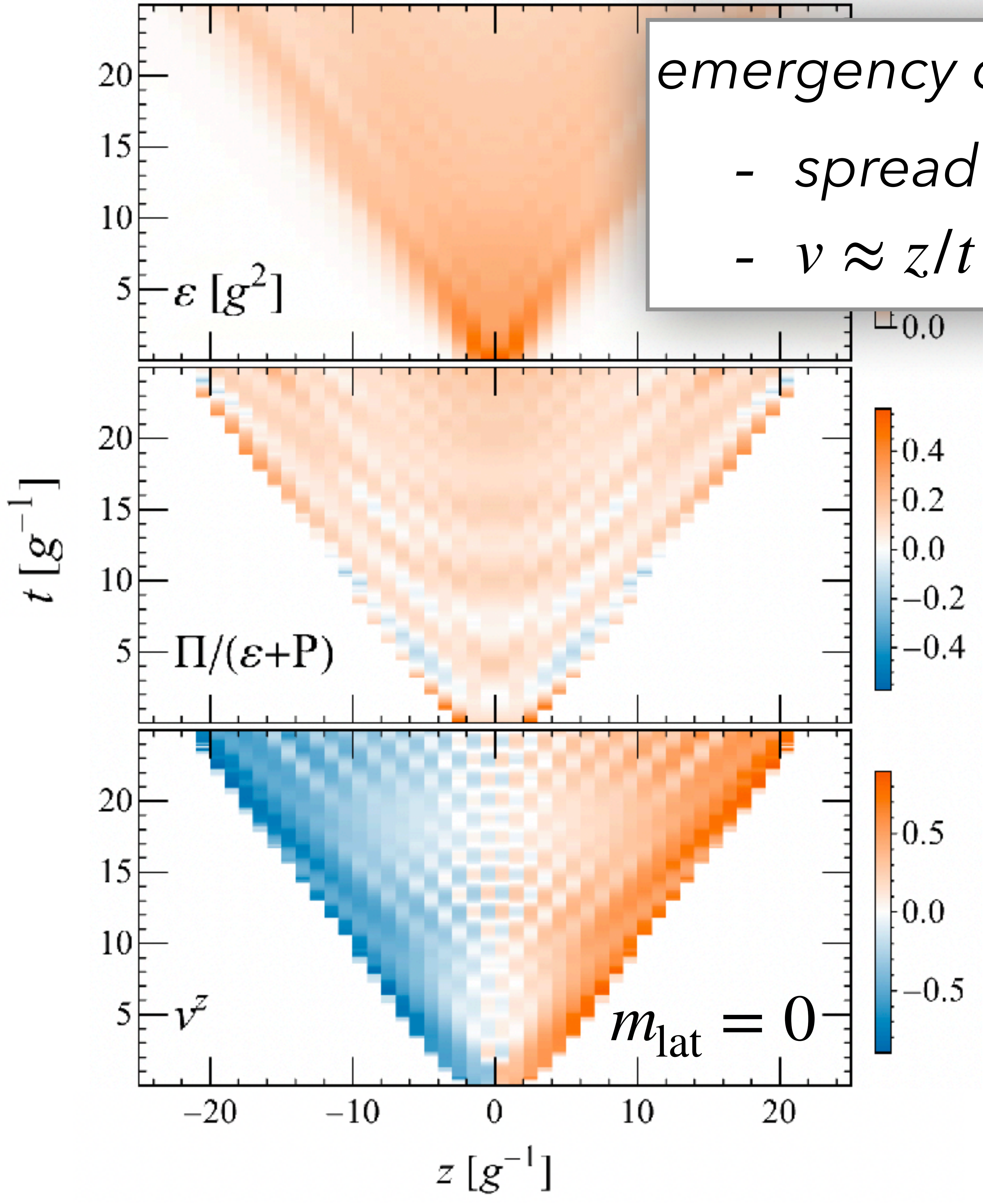


initial state:

vacuum + [excitation @ center]

$$\langle \Psi(t) \,|\, \hat{T}^{\mu\nu}(x) \,|\, \Psi(t) \rangle = (\varepsilon + P + \Pi) u^\mu u^\nu - (P + \Pi) g^{\mu\nu}$$

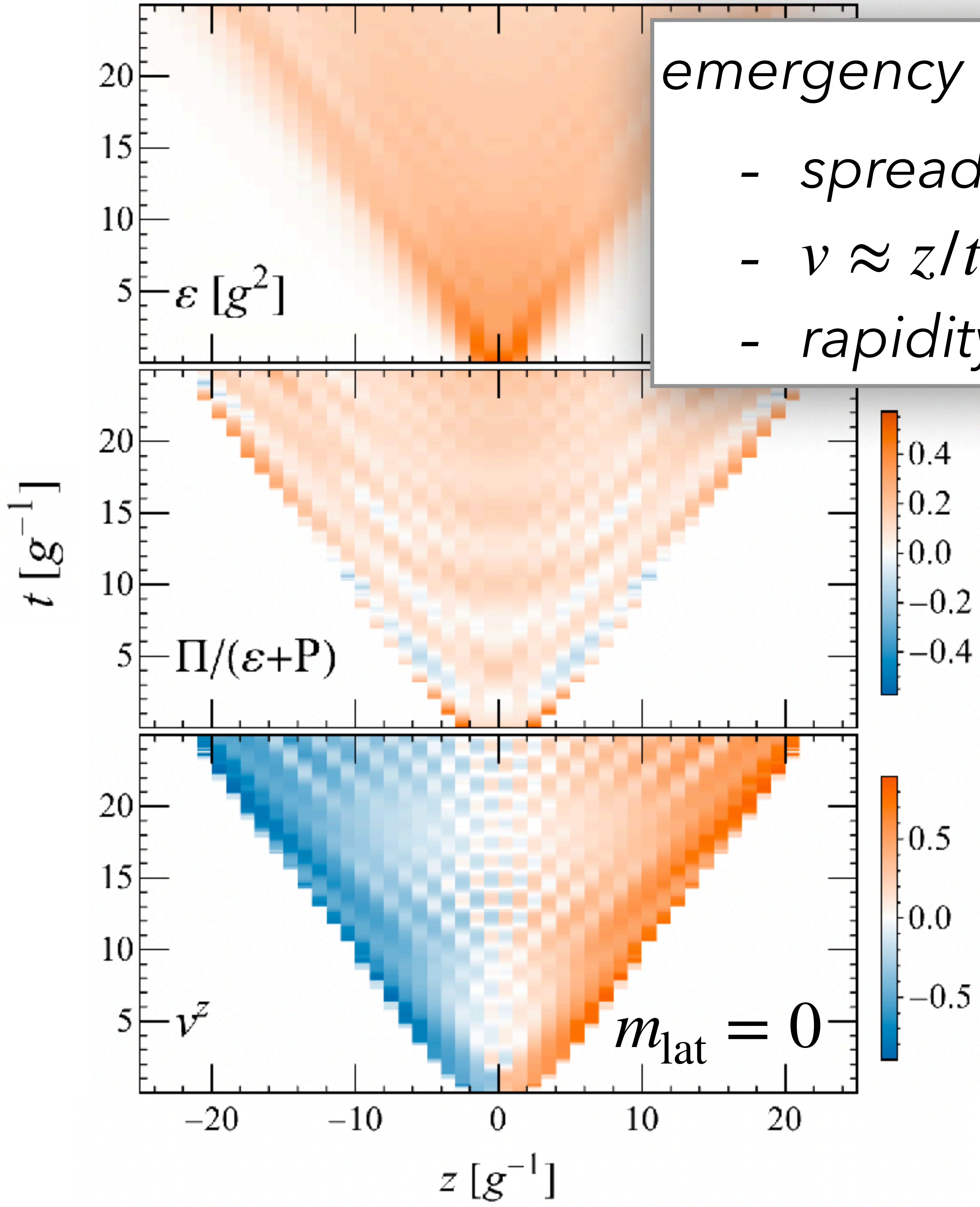
emergence of hydrodynamics



emergency of hydrodynamics

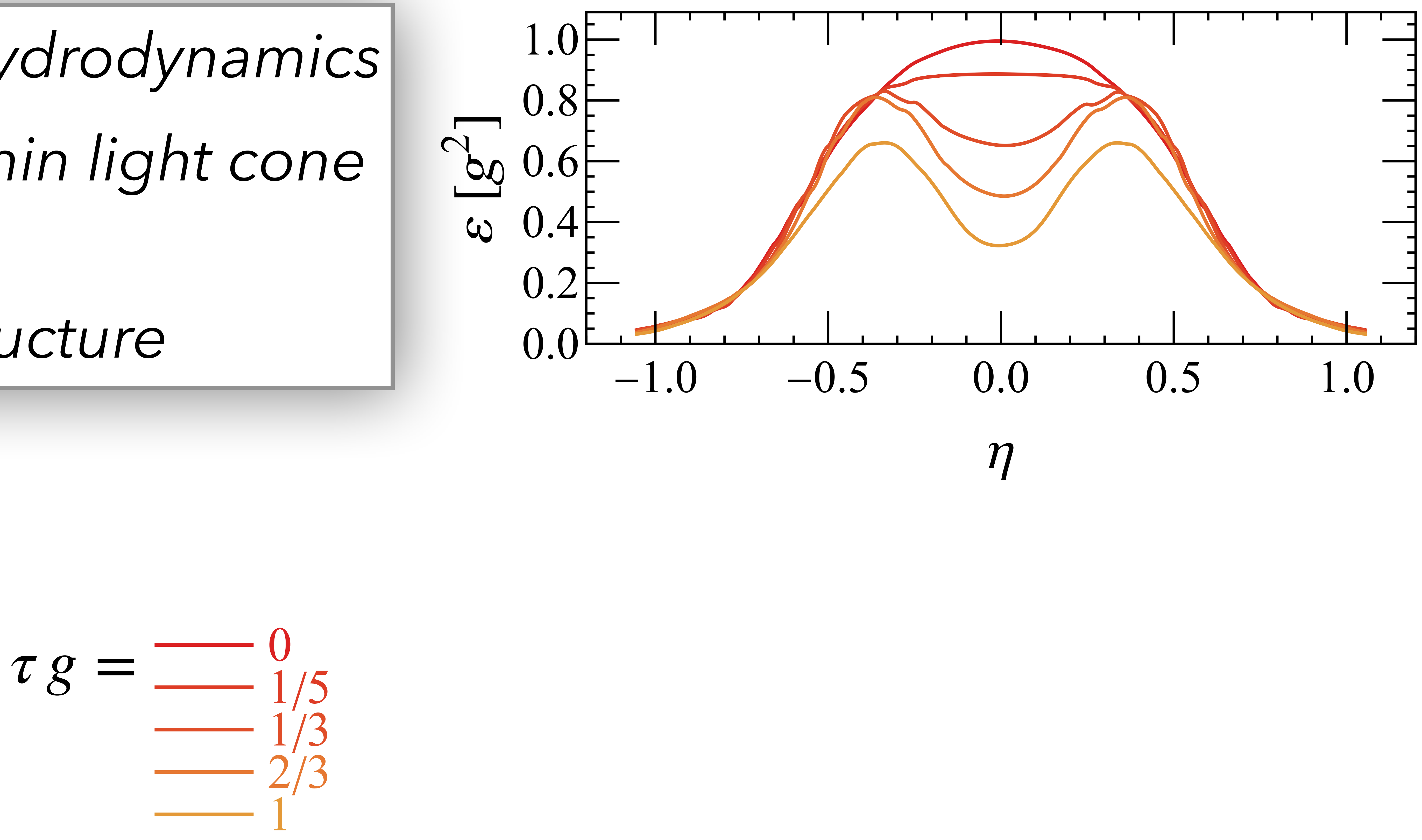
- *spread within light cone*
- $v \approx z/t$

emergence of hydrodynamics



emergency of hydrodynamics

- *spread within light cone*
- $v \approx z/t$
- *rapidity structure*



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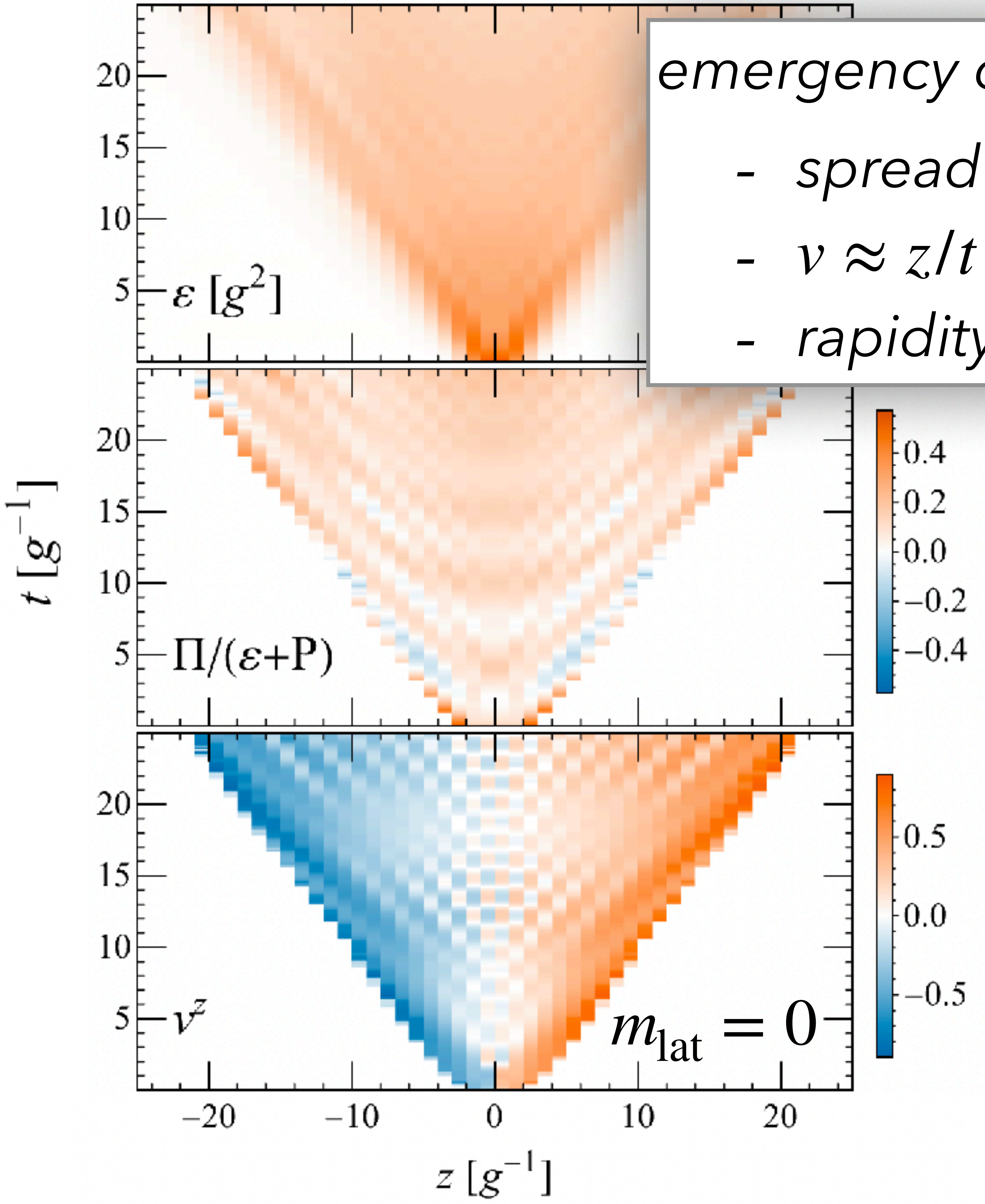

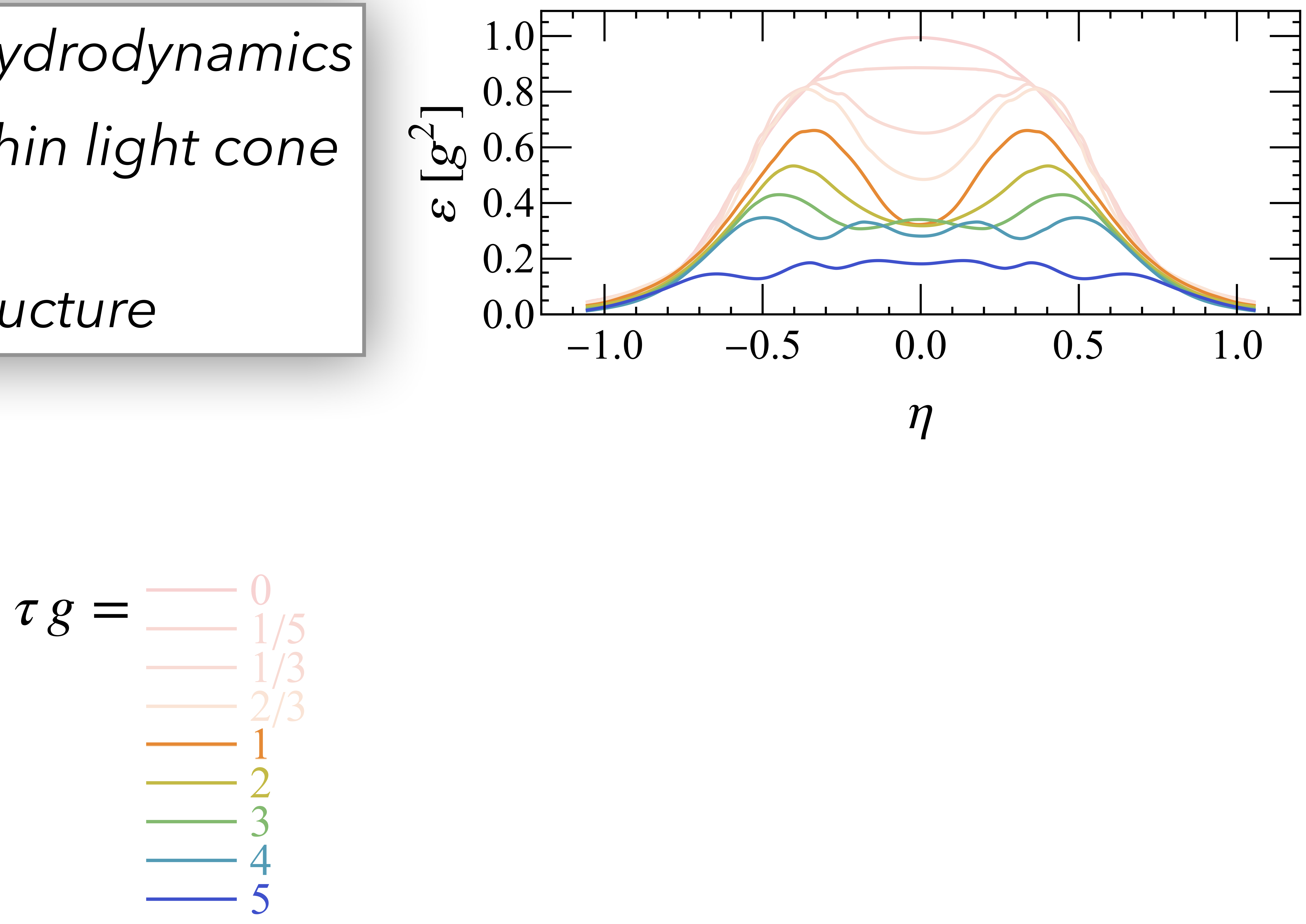

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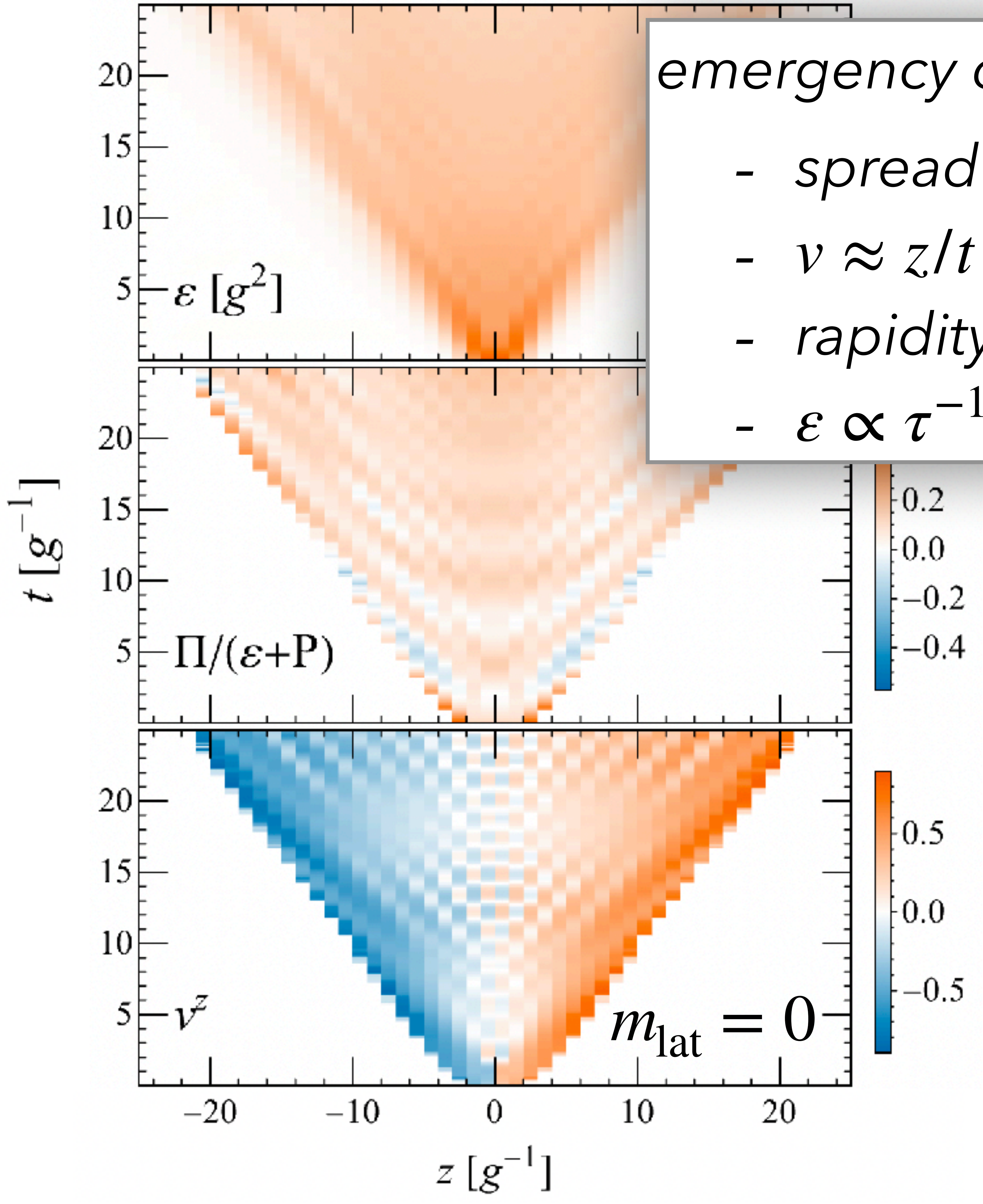

emergency of hydrodynamics

- *spread within light cone*
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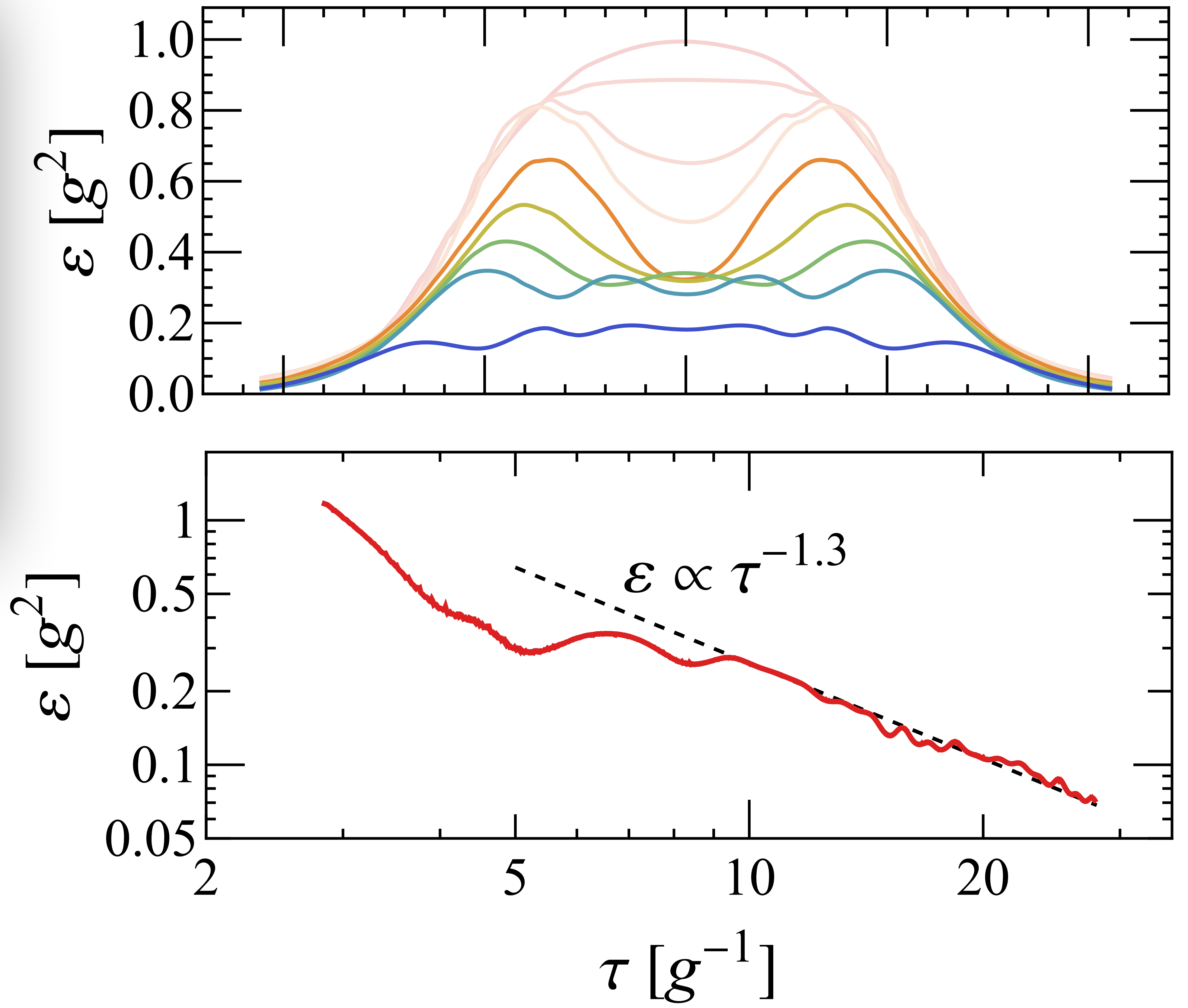
- $\varepsilon \propto \tau^{-1-c_s^2}$



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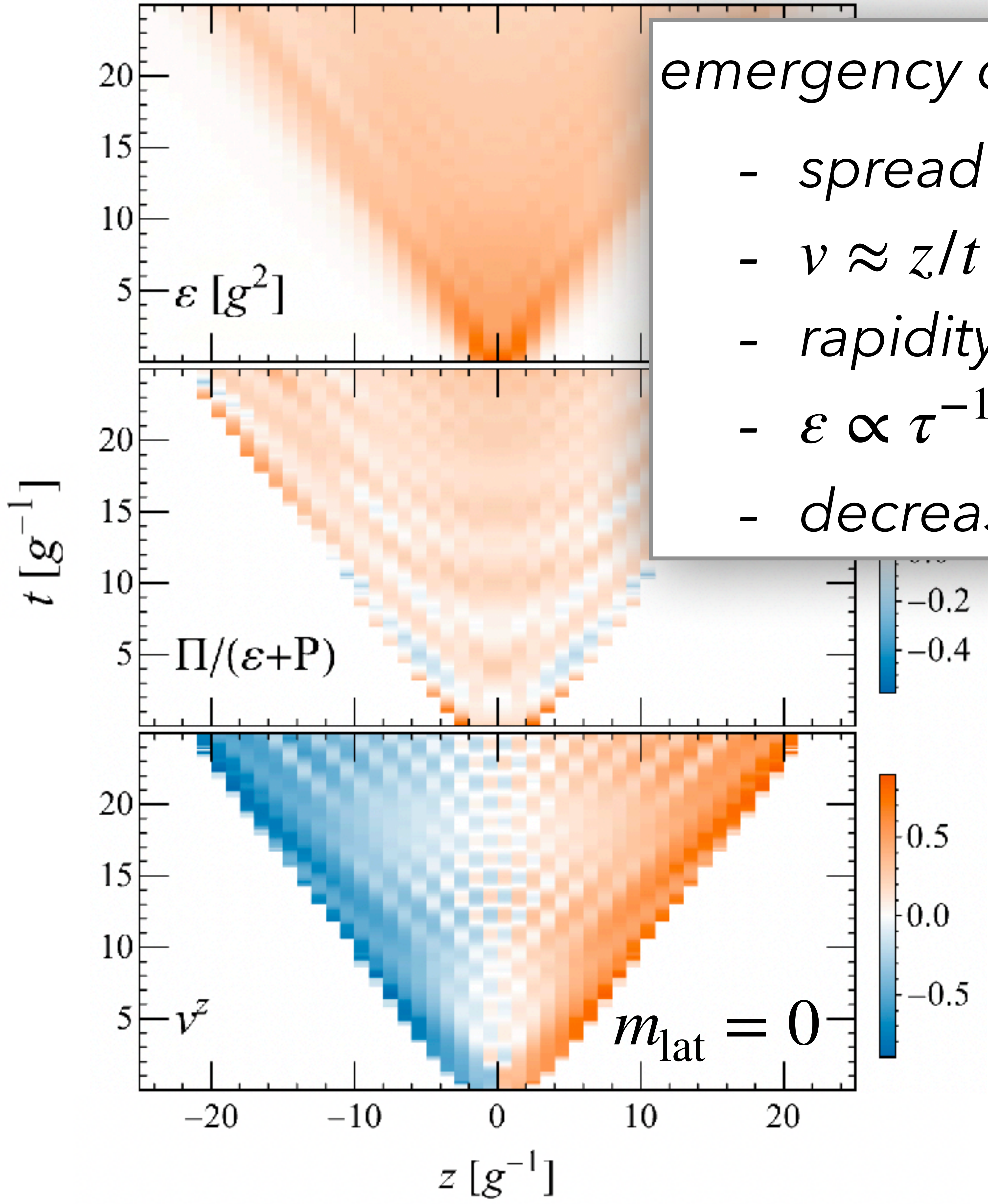

emergency of hydrodynamics

- *spread within light cone*
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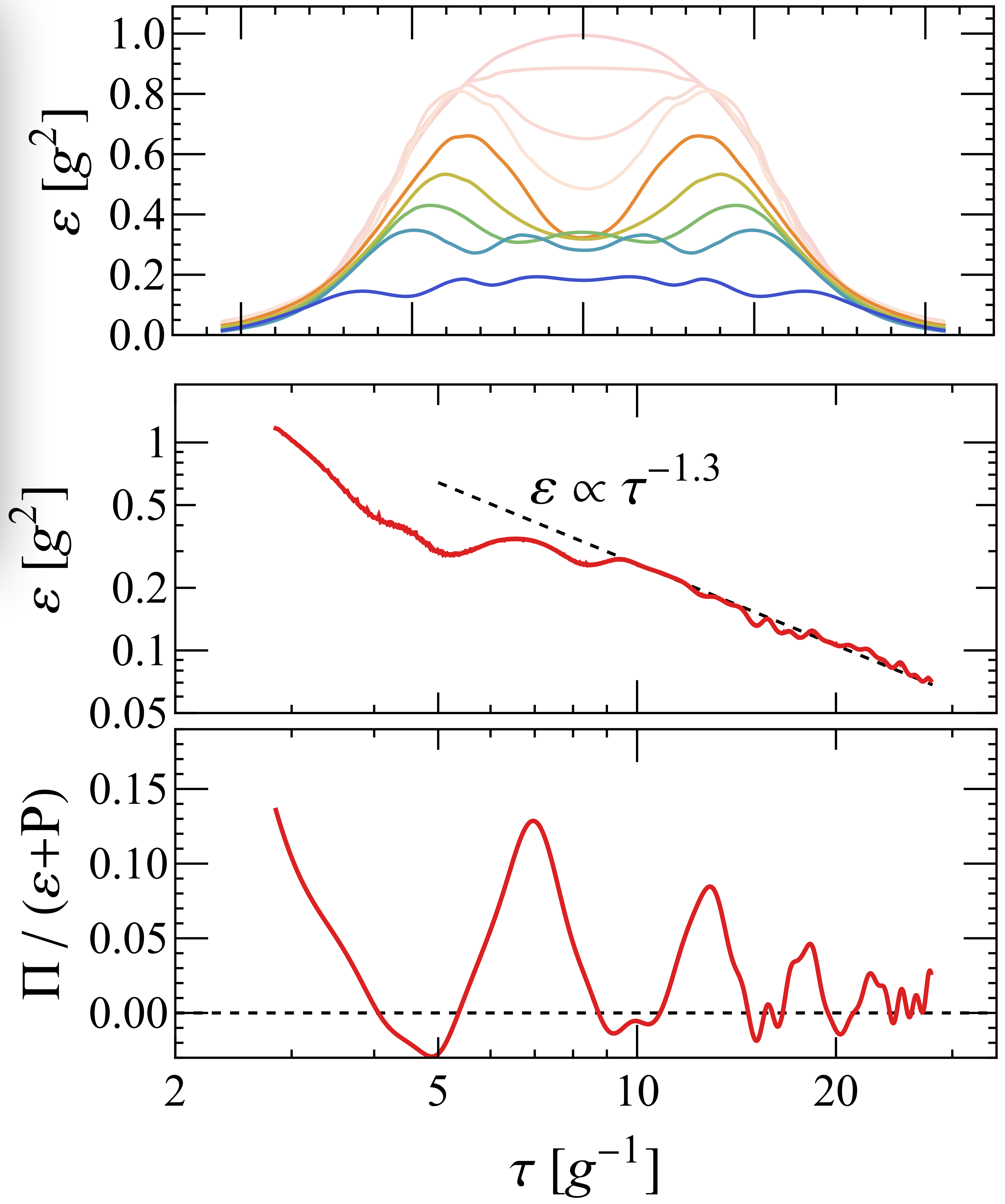
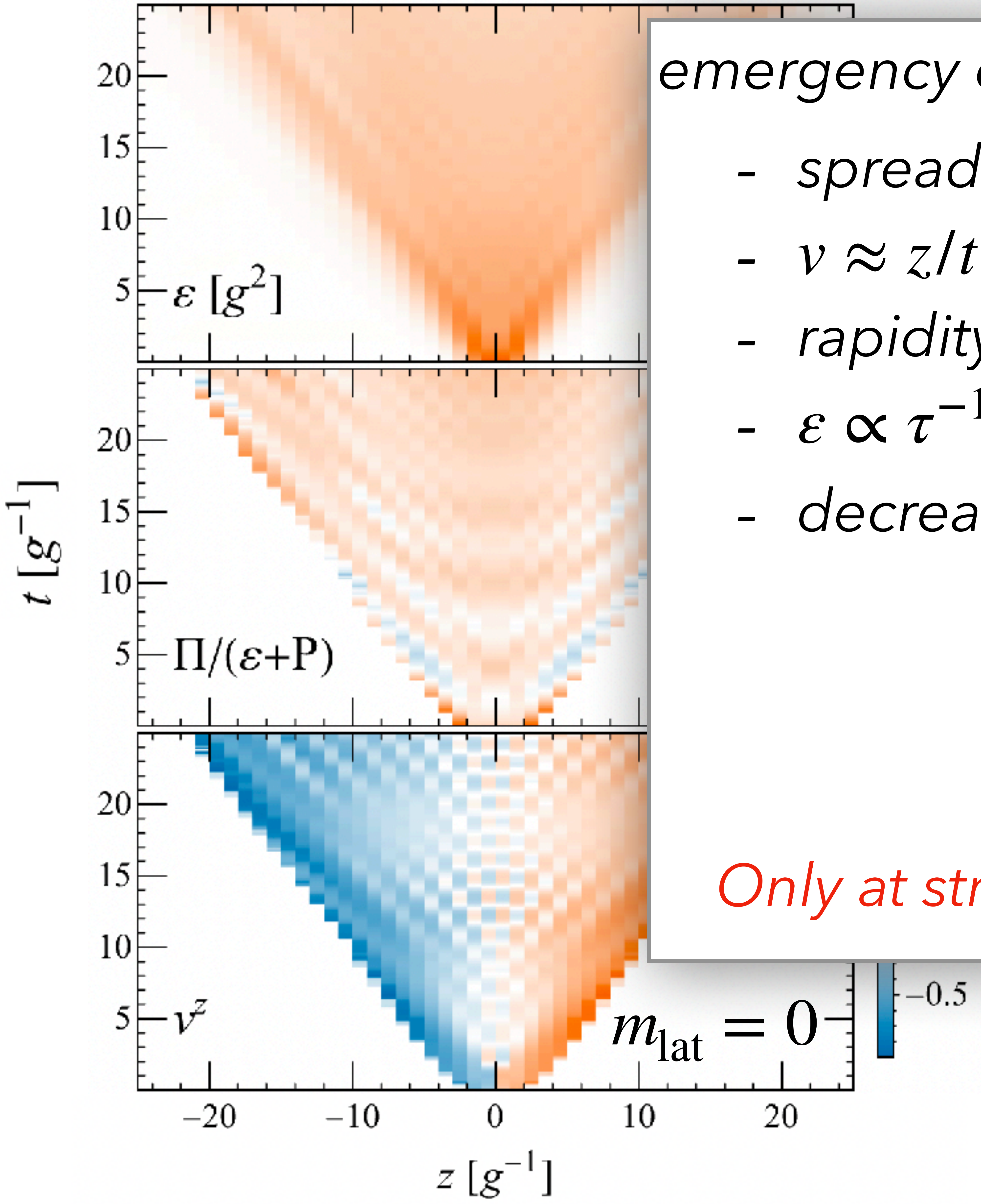
- *decreasing of* Π



emergence of hydrodynamics



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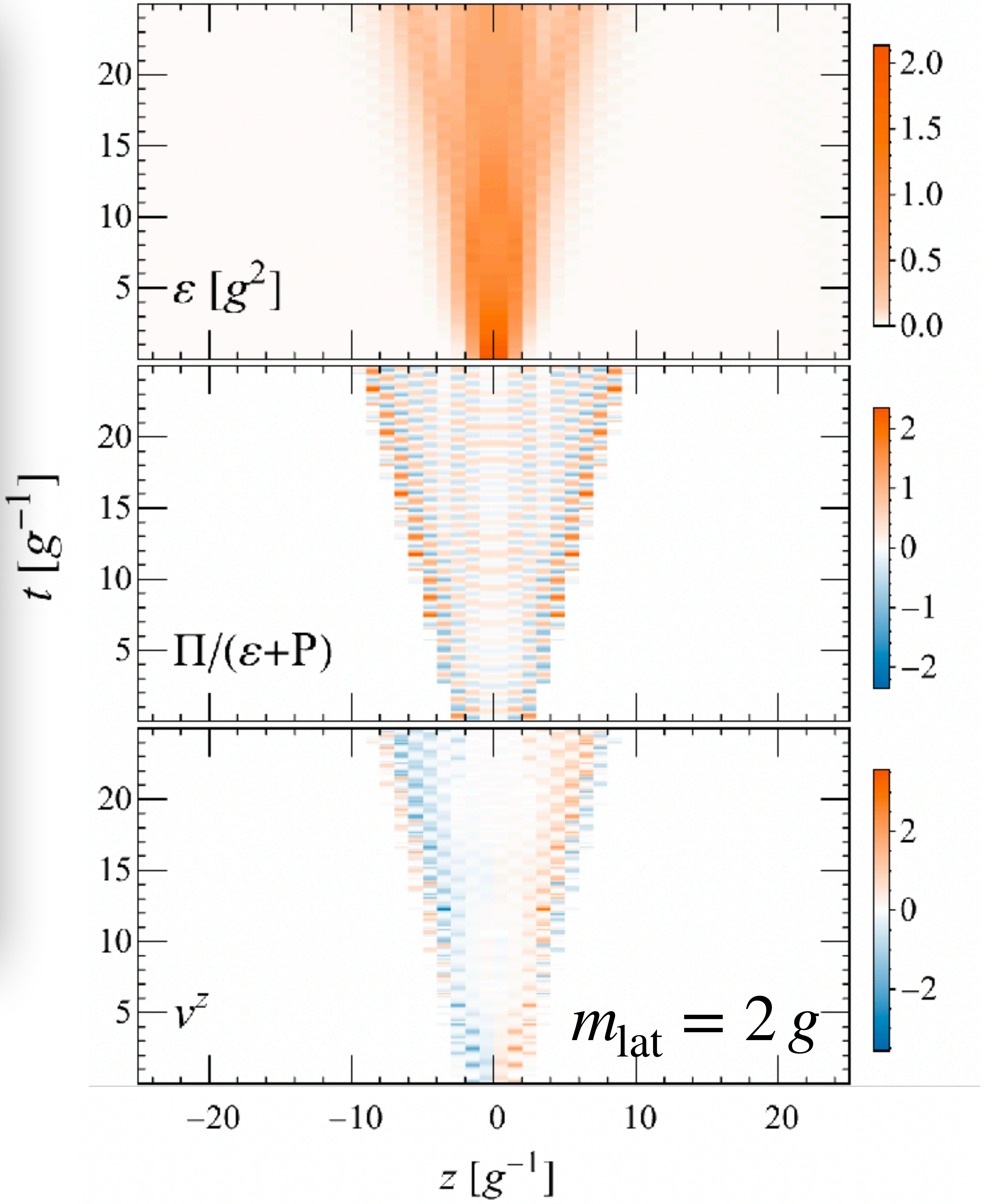
Only at strong coupling!!!



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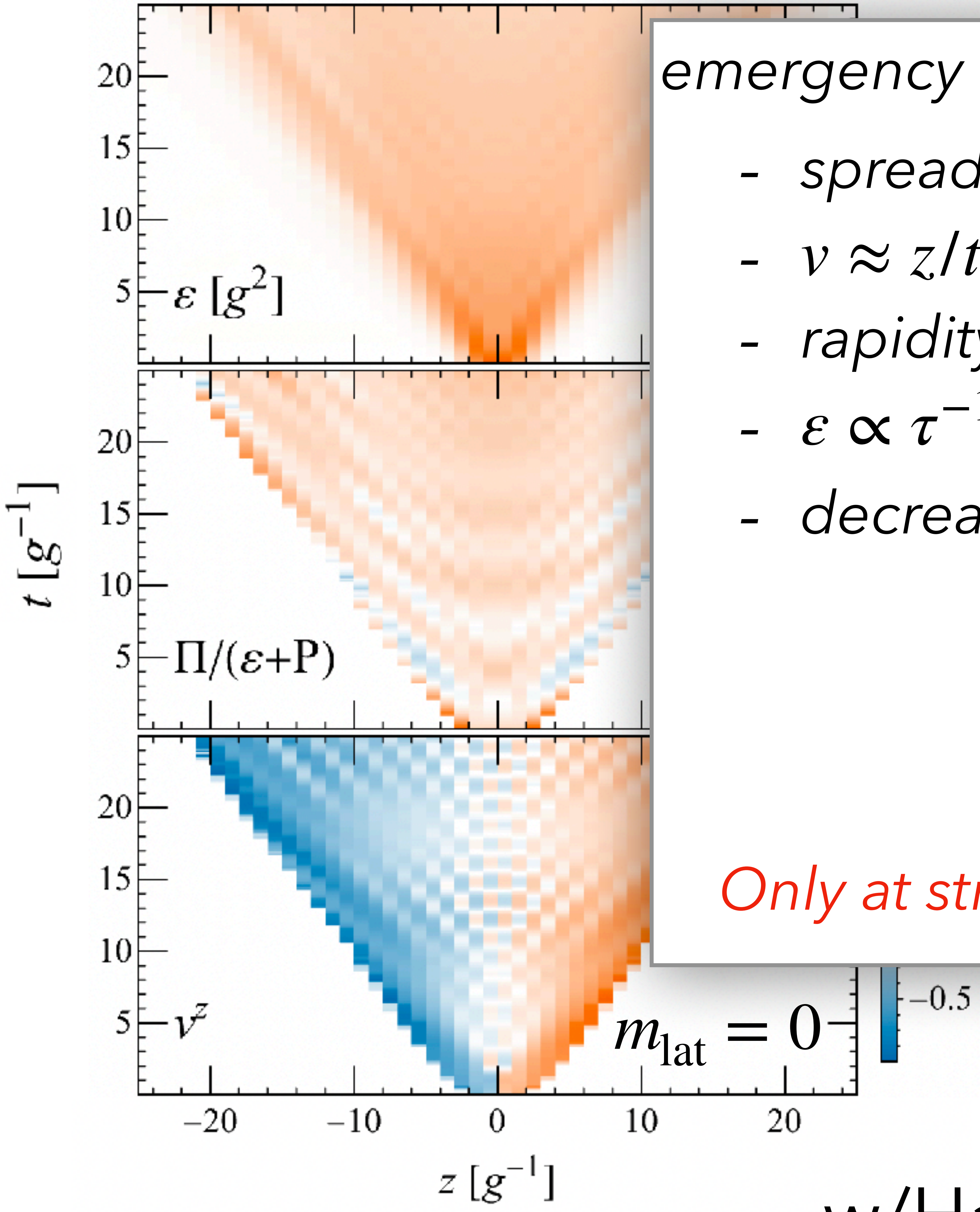

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Only at strong coupling!!!

Haiyang Shao(邵海洋)

w/Haiyang Shao & Shile Chen, final preparation

summary

Real-time non-perturbative quantum evolution

- thermalization of quantum distribution function
    - strong coupling limit: ETH satisfied
    - weak coupling: ETH violated depending on symmetry
- emergence of hydrodynamics

back up slides

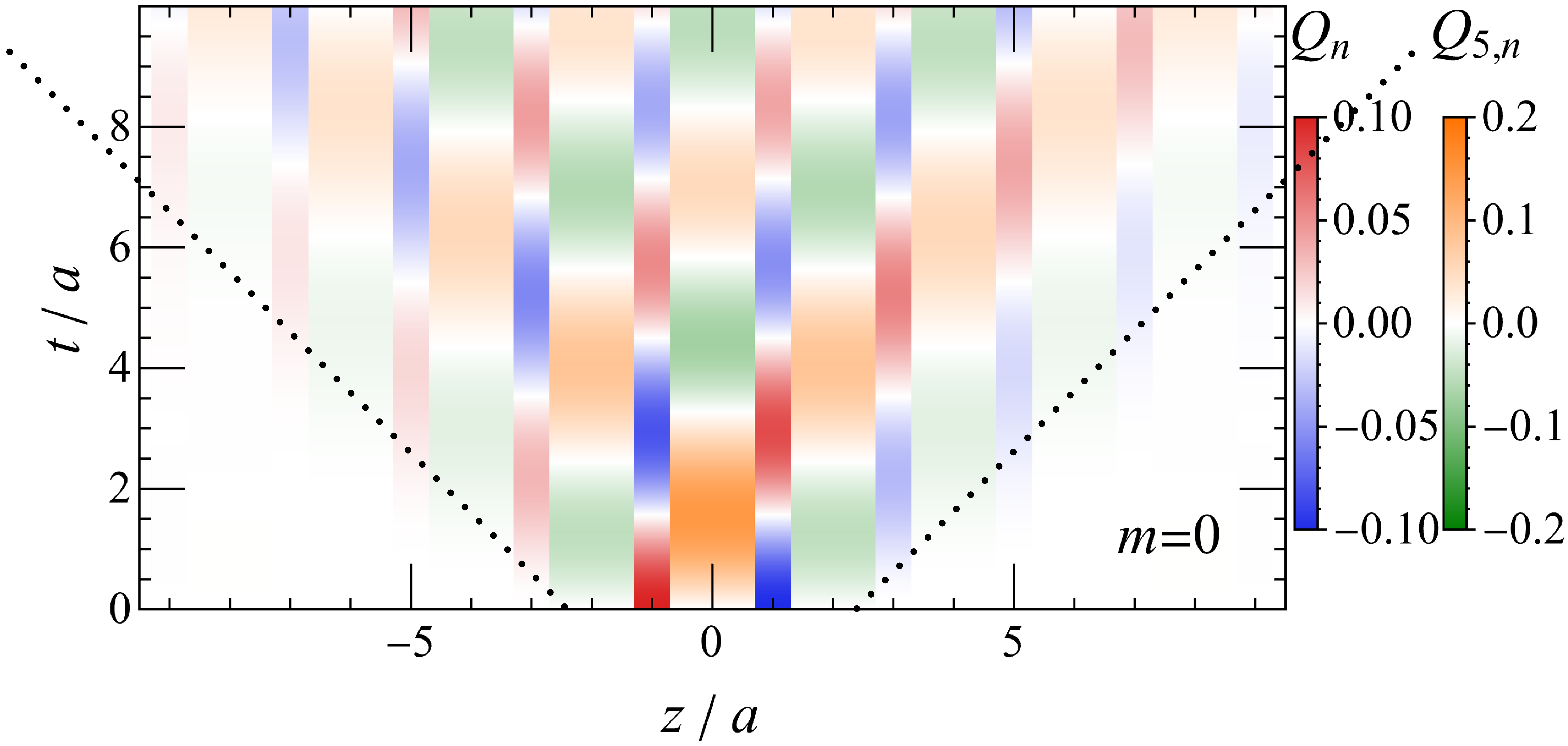
thermodynamic properties and phase structure

- finite temperature, finite chemical potential:

$$\langle O \rangle_{\rm th} \equiv {\rm Tr}(\rho_{\rm th}\, O) \qquad \rho_{\rm th} \equiv \frac{e^{-(H-\mu Q)/T}}{{\rm Tr}(e^{-(H-\mu Q)/T})}$$

K. Ikeda, D. Kharzeev, R. Meyer, *SS*, PhysRevD.108.L091501

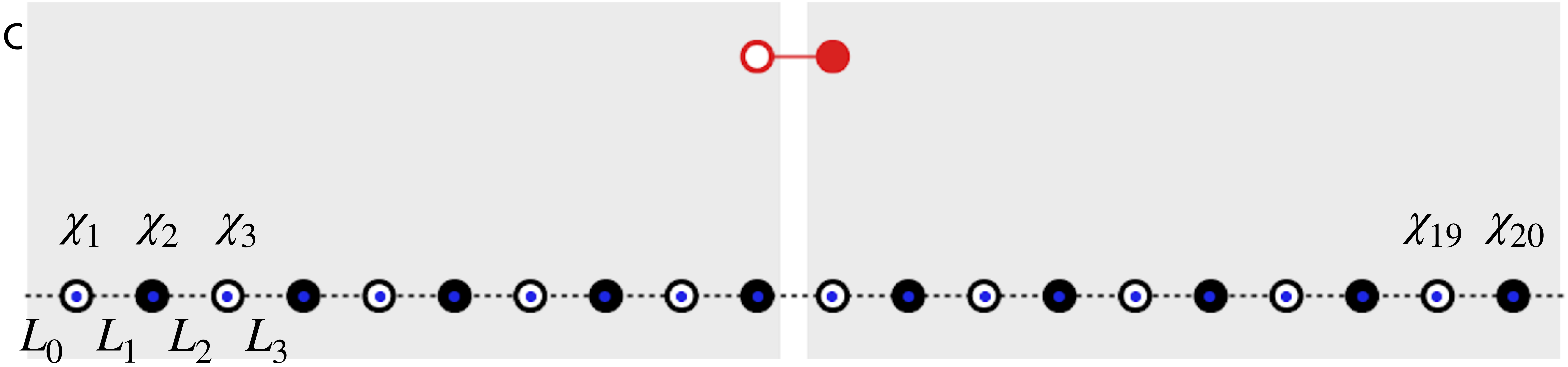
vector and axial charge propagation



K. Ikeda, D. Kharzeev, *SS*, PhysRevD.108.074001

Hamiltonian

1+1D Sc



discretize and matrix(gate) representation:

staggered fermion that satisfied anti-commutation: $\{\psi_a(x), \psi_b^\dagger(y)\} = \delta_{a,b}\delta(x-y)$

gauge field fixed by Gauss' law: $\partial_1 E - g\,\bar{\psi}\gamma^0\psi = j^0_{\text{ext}}$

$E(x = an) \quad \leftrightarrow \quad L_n$

$$L_n - L_{n-1} - \frac{Z_n + (-1)^n}{2} = \frac{1}{g}\int_{(n-1/2)a}^{(n+1/2)a} \mathrm{d}x\, j^0_{\text{ext}}(x,t) \ ,$$

thermal equilibrium property

$$\hat{\rho}_{\rm th} \equiv Z^{-1}\, e^{-(\hat{H}-\mu\hat{Q})/T}$$

$$\langle \hat{O} \rangle_{\rm th} \equiv {\rm Tr}(\hat{\rho}_{\rm th}\, \hat{O})$$

real time evolution

$$\partial_t \hat{\rho} = -\, i[\hat{H}, \hat{\rho}]$$

$$\frac{\partial}{\partial t} |\psi(t)\rangle = -\, i\, \hat{H} |\psi(t)\rangle$$

$$q_{n,t} \equiv \langle \psi^\dagger(a\,n)\psi(a\,n)\rangle_t = \frac{\langle Z_n \rangle_t + (-1)^n}{2a},$$

$$\nu_{n,t} \equiv \langle \bar{\psi}(a\,n)\psi(a\,n)\rangle_t = \frac{(-1)^n \langle Z_n \rangle_t}{2a},$$

$$\Pi_{n,t} \equiv \langle E(a\,n)\rangle_t = g\,\langle L_n \rangle_t,$$

$$\vdots$$

why quantum computer?

dimension of state vector $= 2^N$

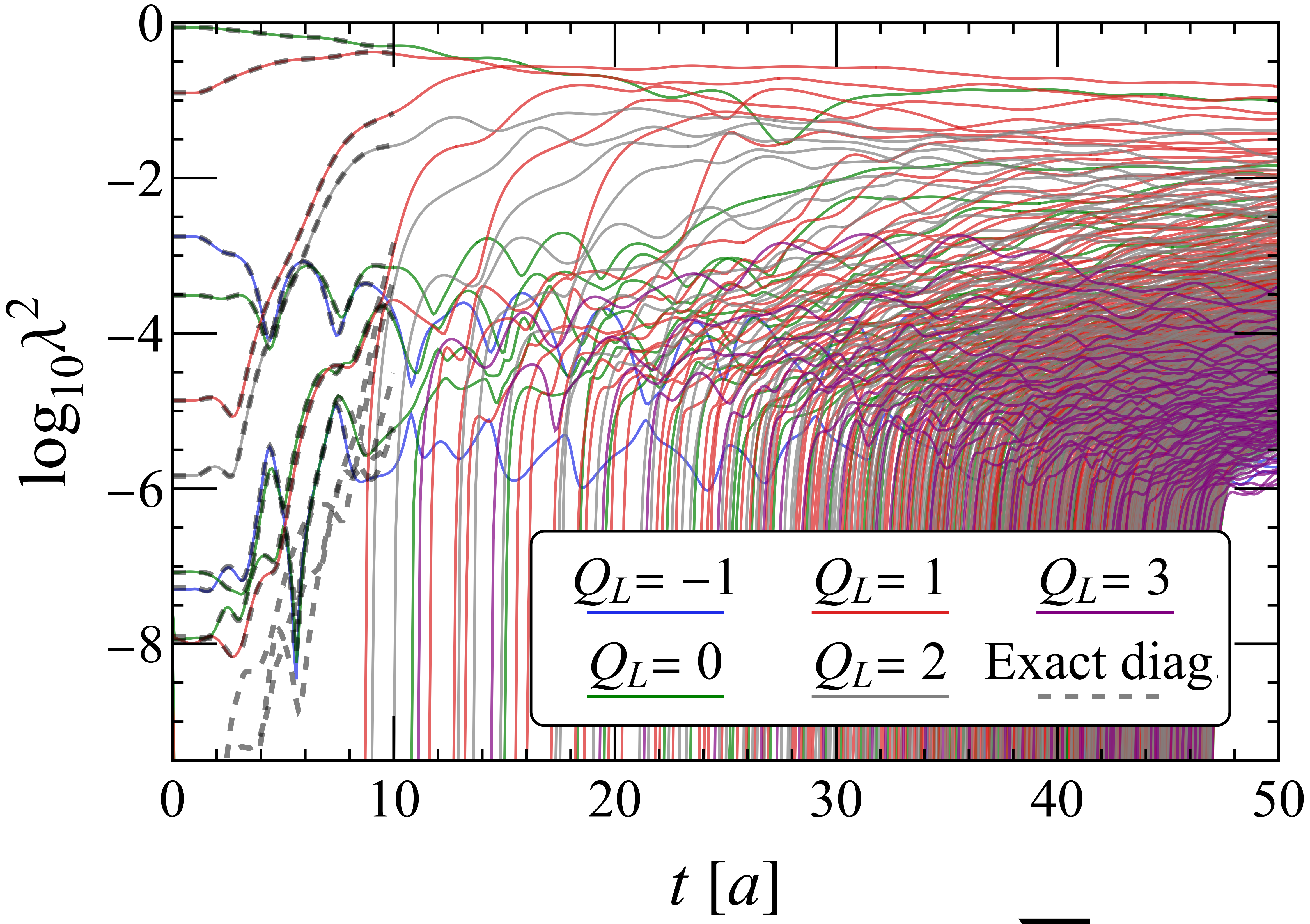
N : number of lattice sides

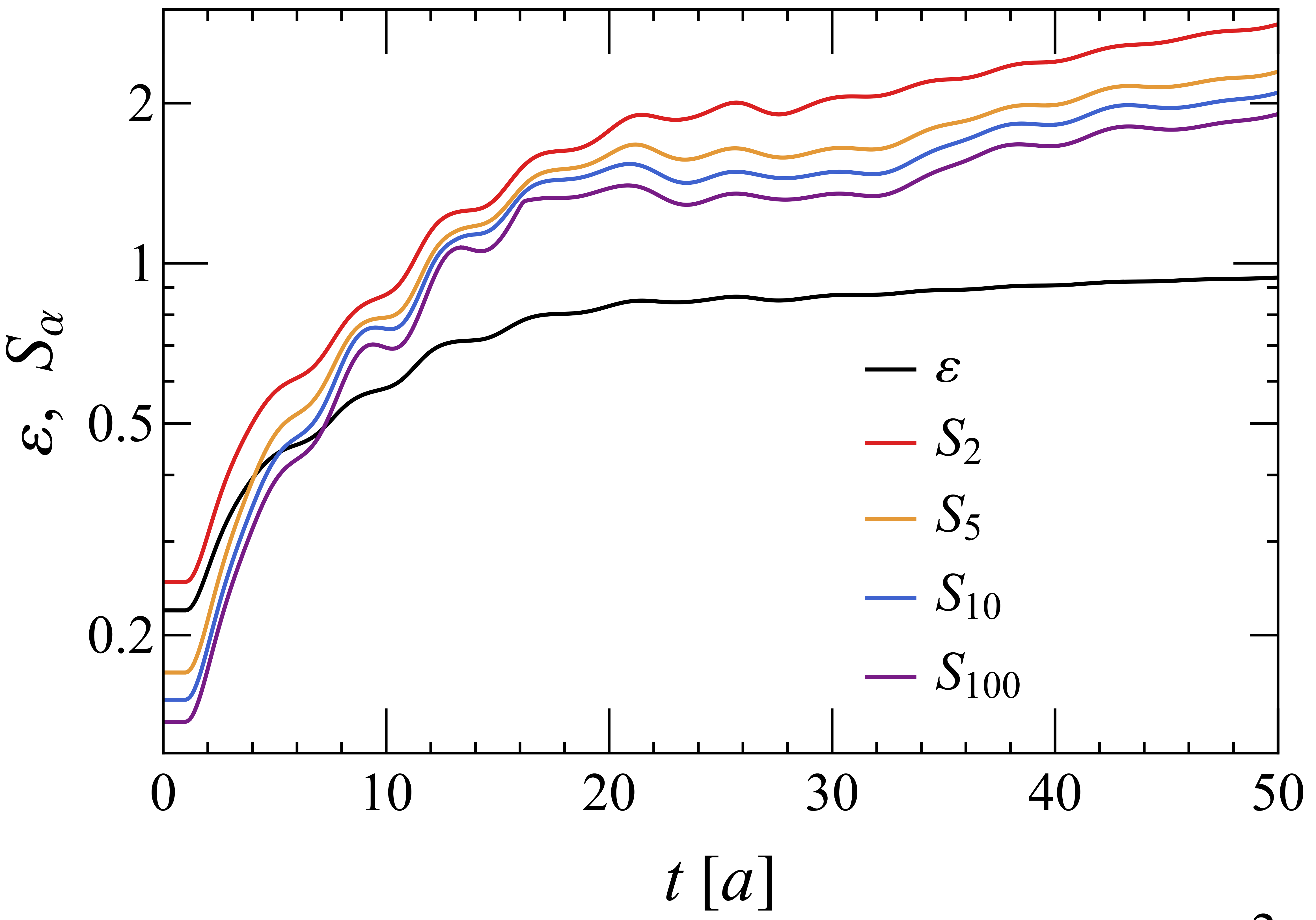
dimension of Hamiltonian $=$ ~~$2^N \times 2^N$~~ sparse $\sim 2N \times 2^N$

N	dimension	memory of Hamiltonian	# of qubit (N)
8	256	~ 131 kB	8
12	4,096	~ 3.1 MB	12
16	65,536	~ 67 MB	16
20	1,048,576	~ 1.3 GB	20
24	16,777,216	~ 26	
~~28~~	~~268,435,456~~	~~~ 481 GB~~	~~28~~

classical hardware in this work

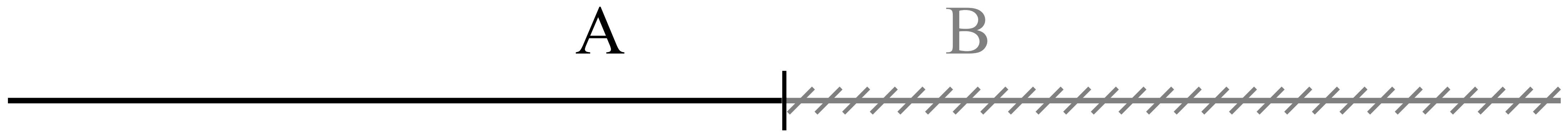
performance not satisfying…





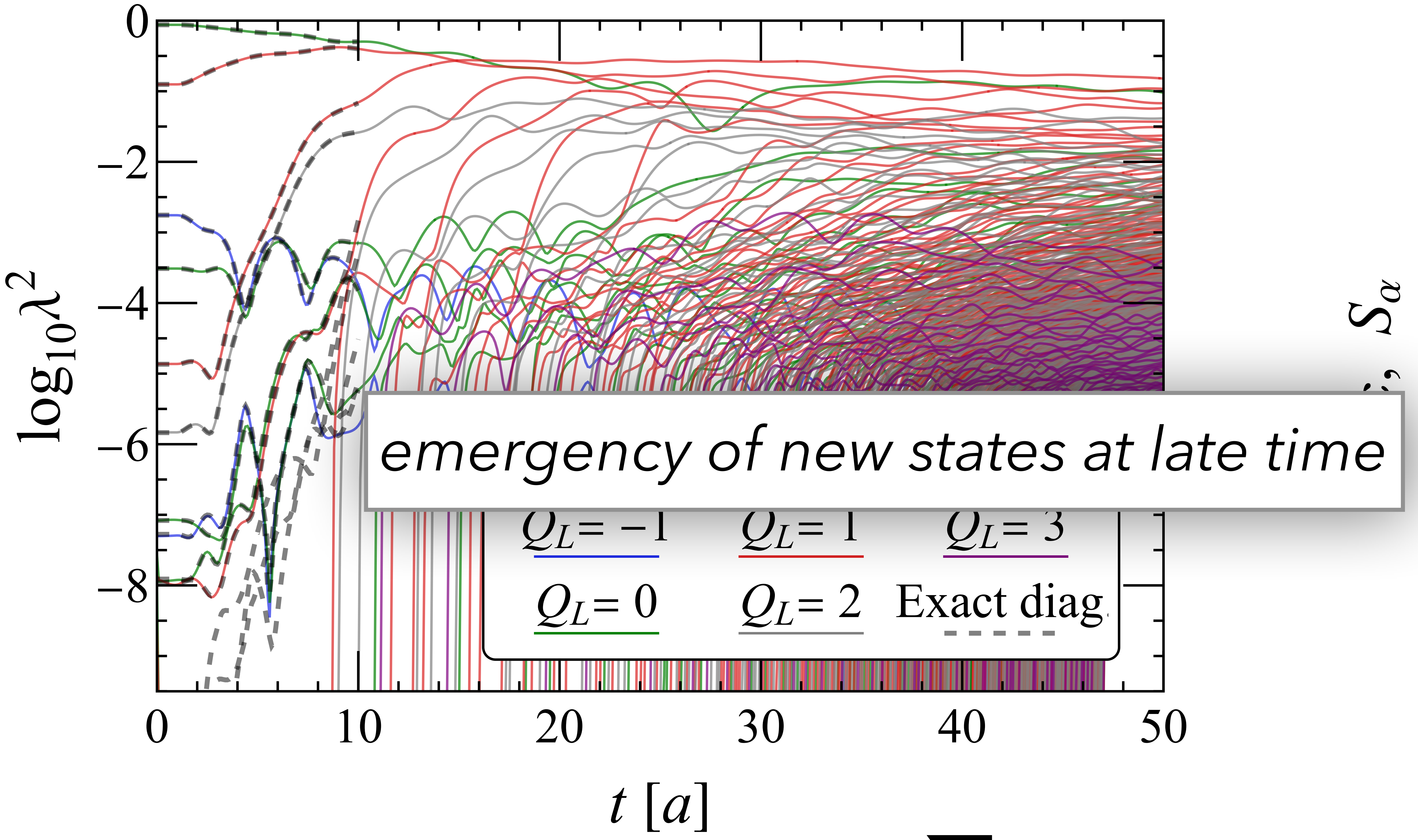
$$\rho_A = \sum_i \lambda_i^2 |\Psi_i\rangle\langle\Psi_i|$$

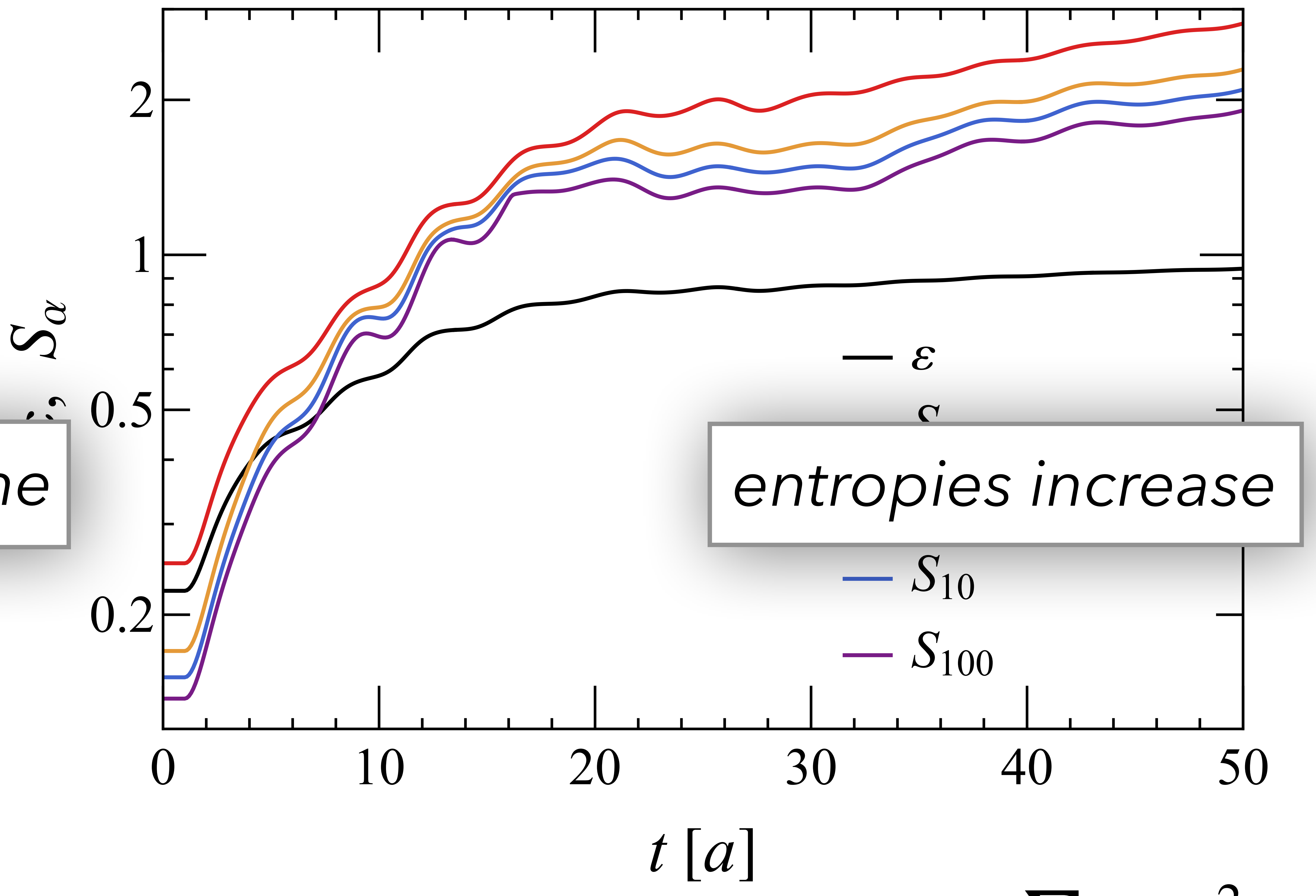
$$S_\alpha = -\frac{\ln \sum_i \ln \lambda_i^{2\alpha}}{1-\alpha}$$



w/ Florio, Frenklakh, Ikeda, Kharzeev, Korepin, Yu, 2301.11991(PRD)

entanglement entropy





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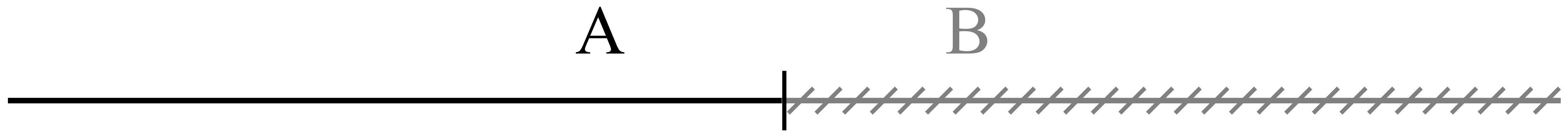

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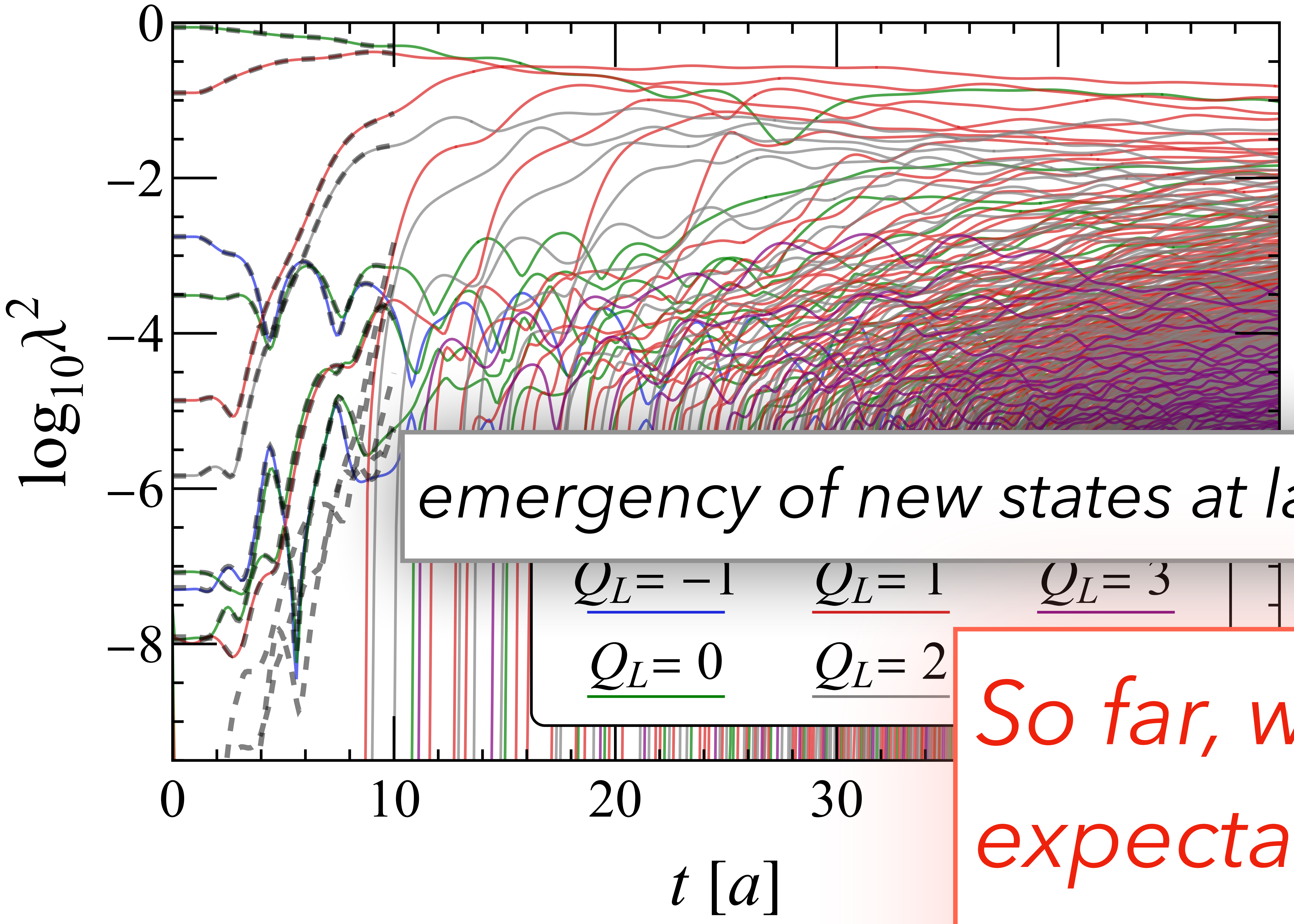
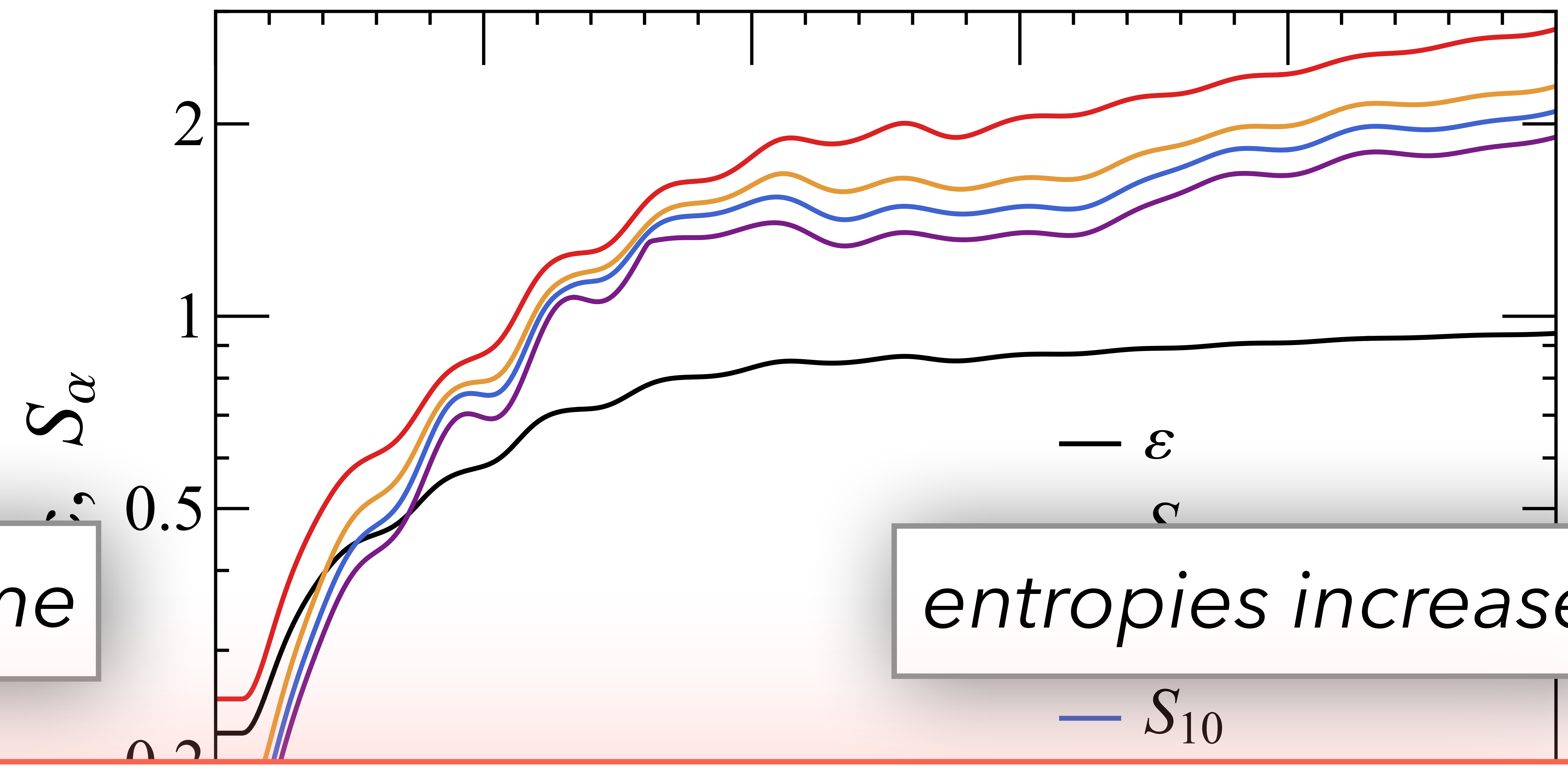




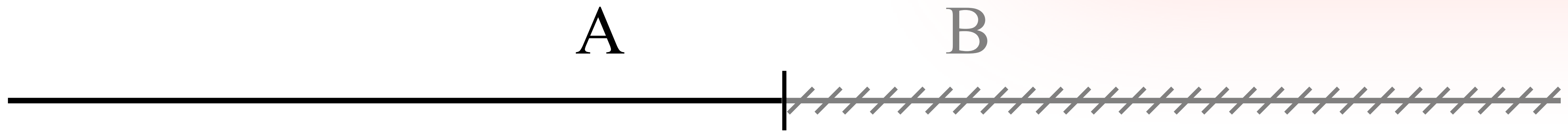
emergency of new states at late time

entropies increase

ρ_A =

So far, what have been observed fit the expectation of thermalization.

What are there in the "thermalized" state?



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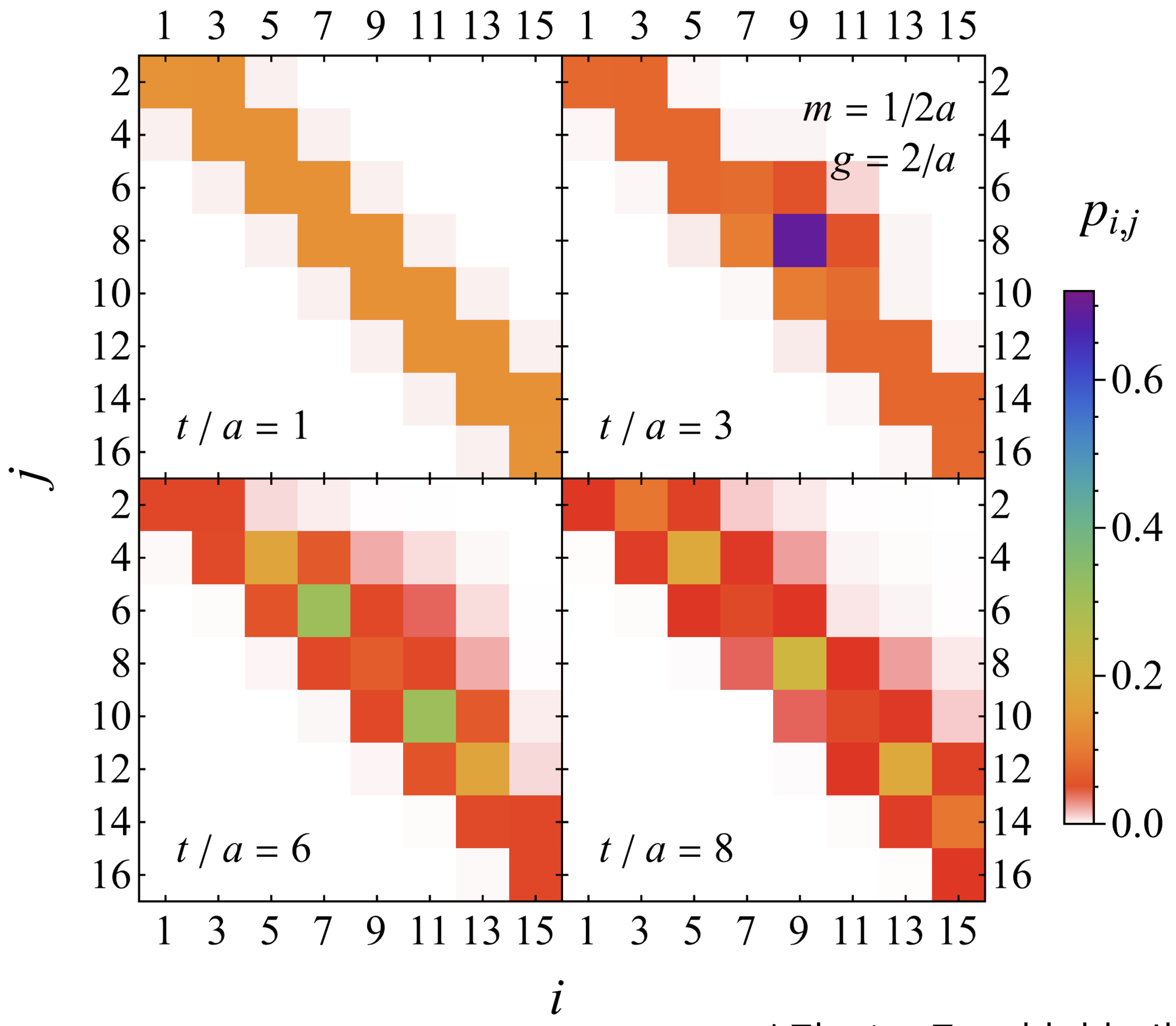
overlap with one-pair state



$$p_{i,j} = |\langle \Psi_t | \chi_i \chi_j^\dagger | \text{empty} \rangle|^2$$

probability of exciting the i^{th} antiquark and j^{th} quark

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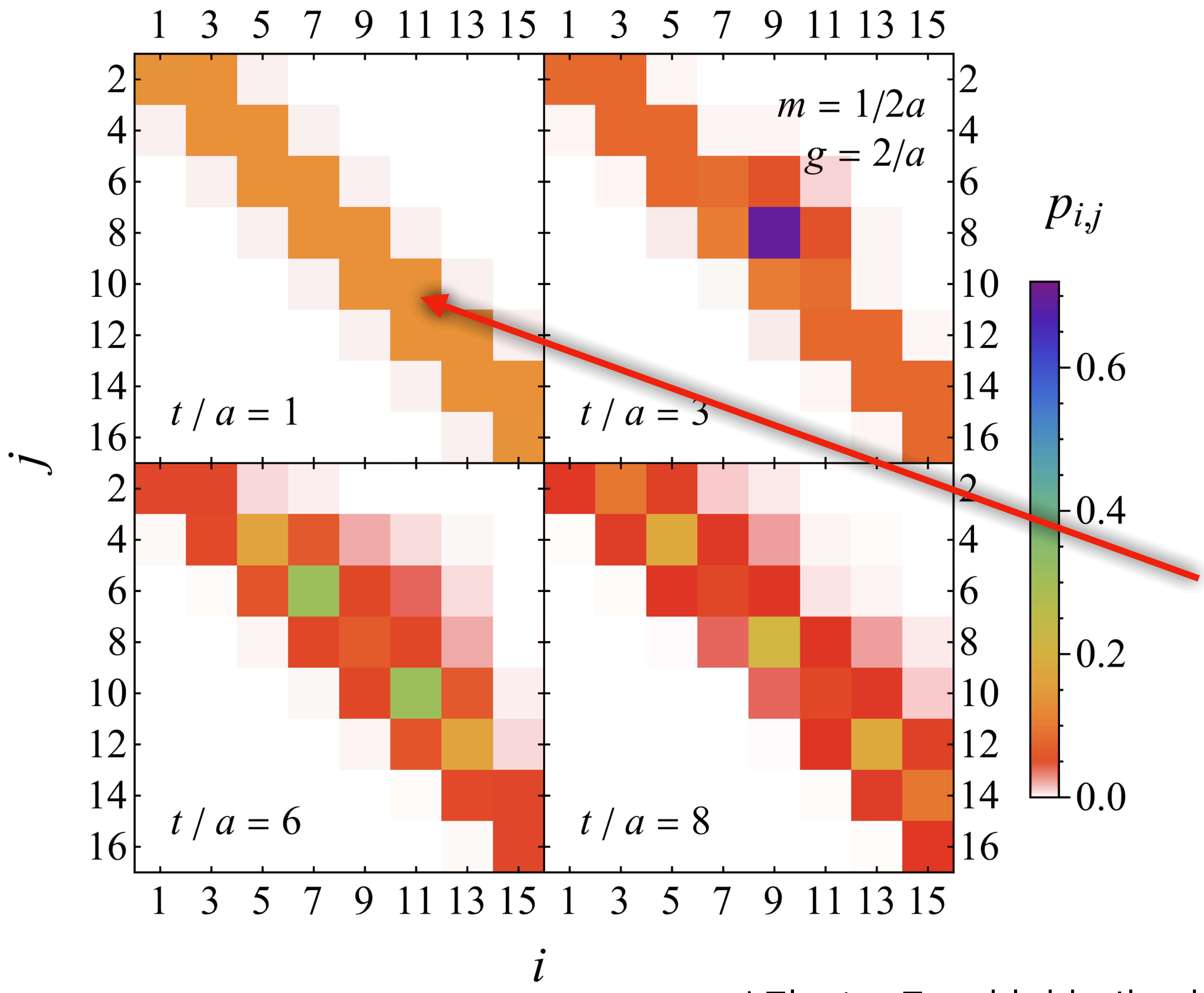
overlap with one-pair state



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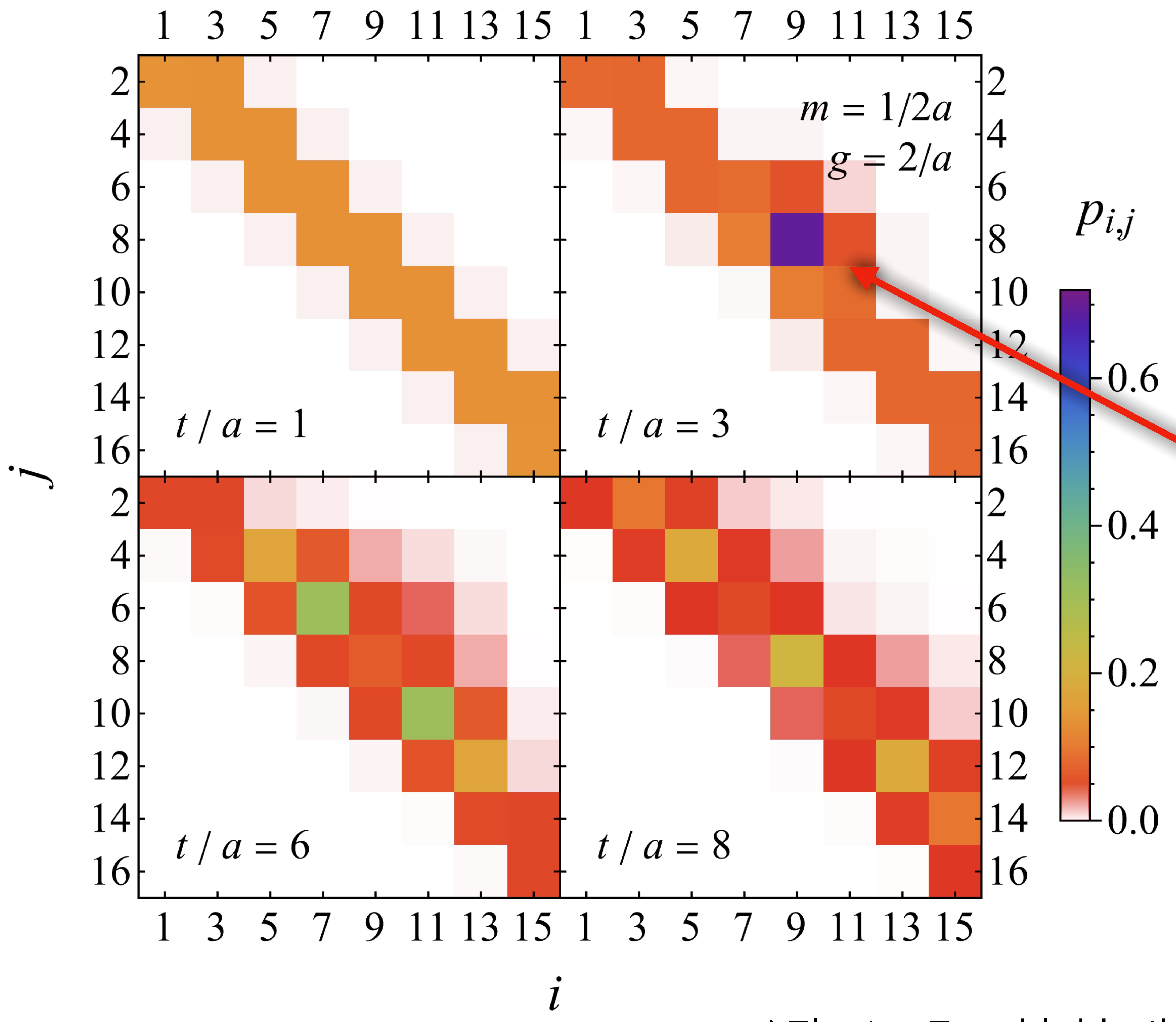
probability of exciting the i^{th} antiquark and j^{th} quark

homogeneous vacuum structure

w/ Florio, Frenklakh, Ikeda, Kharzeev, Korepin, Yu, 2301.11991(PRD)

overlap with one-pair state



$$p_{i,j} = |\langle \Psi_t | \chi_i \chi_j^\dagger | \text{empty} \rangle|^2$$

probability of exciting the i^{th} antiquark and j^{th} quark

creation of meson

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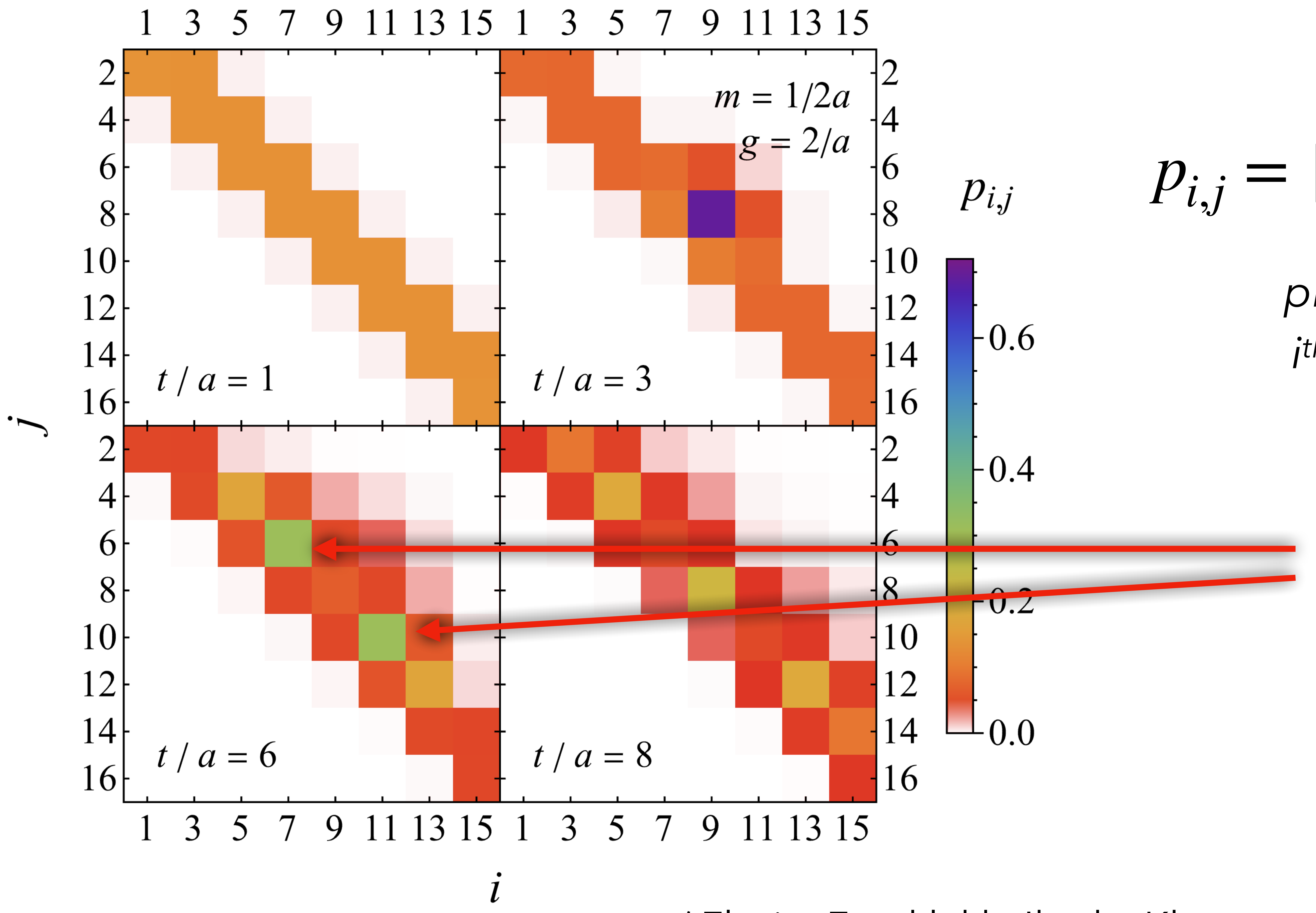
overlap with one-pair state



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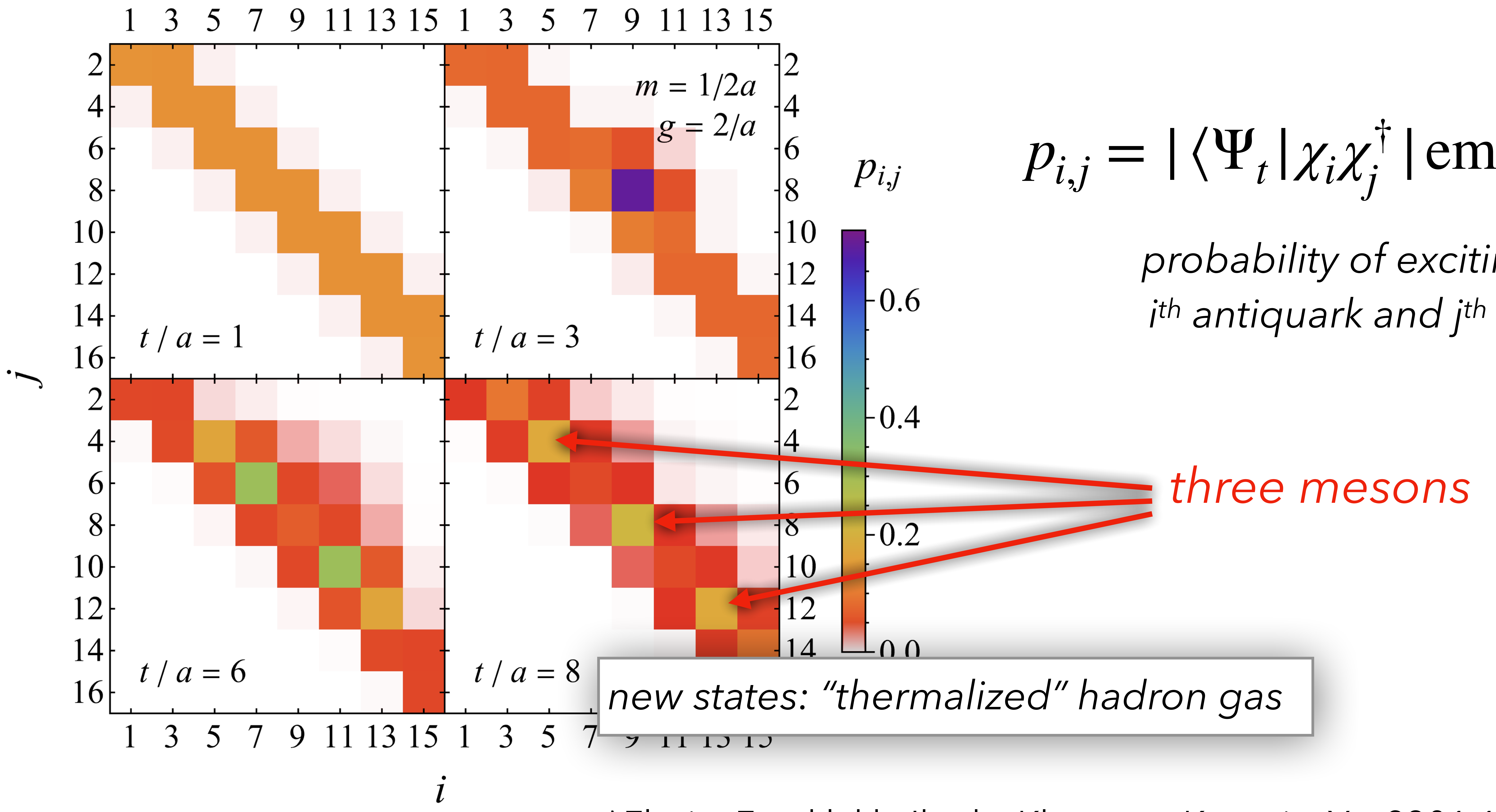
probability of exciting the i^{th} antiquark and j^{th} quark

w/ Florio, Frenklakh, Ikeda, Kharzeev, Korepin, Yu, 2301.11991(PRD)

overlap with one-pair state



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probability of exciting the i^{th} antiquark and j^{th} quark

new states: "thermalized" hadron gas

w/ Florio, Frenklakh, Ikeda, Kharzeev, Korepin, Yu, 2301.11991(PRD)