



中国科学技术大学

交叉学科理论研究中心
Interdisciplinary Center for Theoretical Study

Baryon properties from Continuum Schwinger Function Methods (CSMs/DSEs)

Chen Chen

Interdisciplinary Center for Theoretical Study

&

Peng Huanwu Center for Fundamental Theory

April 27th, 2025

第二十届全国中高能核物理大会
暨 第十四届全国中高能核物理专题研讨会

- **Part A. Introduction**
- **Part B. The elastic and transition axial-vector form factors of the nucleon $N(940)$ and $\Delta(1232)$.**
- **Part C. Screening masses of positive- and negative-parity hadron ground-states, including those with strangeness.**

➤ **Part A. Introduction**

- **Part B. The elastic and transition axial-vector form factors of the nucleon $N(940)$ and $\Delta(1232)$.**
- **Part C. Screening masses of positive- and negative-parity hadron ground-states, including those with strangeness.**

Non-Perturbative QCD:

- **Hadrons, as bound states, are dominated by non-perturbative QCD dynamics – Two emergent phenomena**
 - **Confinement:** Colored particles have never been seen isolated
 - ✓ Explain how quarks and gluons bind together
 - **Dynamical Chiral Symmetry Breaking (DCSB):** Hadrons do not follow the chiral symmetry pattern
 - ✓ Explain the most important mass generating mechanism for visible matter in the Universe
- Neither of these phenomena is apparent in QCD's Lagrangian, HOWEVER, They play a dominant role in determining the characteristics of real-world QCD!

Non-Perturbative QCD:

- **From a quantum field theoretical point of view**, these emergent phenomena could be associated with dramatic, dynamically driven changes in the analytic structure of QCD's Schwinger functions (propagators and vertices). The Schwinger functions are solutions of the quantum equations of motion (**Dyson-Schwinger equations**).

Dyson-Schwinger 方程 (DSE)

Quark propagator:

$$\text{---}\bigcirc\text{---}^{-1} = \text{---}^{-1} + \text{---}\bigcirc\text{---}$$

Ghost propagator:

$$\text{---}\bigcirc\text{---}^{-1} = \text{---}^{-1} + \text{---}\bigcirc\text{---}$$

Ghost-gluon vertex:

$$\text{---}\bigcirc\text{---} = \text{---} + \text{---}\bigcirc\text{---}$$

Gluon propagator:

$$\text{---}\bigcirc\text{---}^{-1} = \text{---}^{-1} +$$

Quark-gluon vertex:

$$\text{---}\bigcirc\text{---} = \text{---} +$$

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- From a quantum field theoretical point of view, these emergent phenomena could be associated with dramatic, dynamically driven changes in the analytic structure of QCD's Schwinger functions (propagators and vertices). The Schwinger functions are solutions of the quantum equations of motion (**Dyson-Schwinger equations**).

- Dressed-quark propagator:

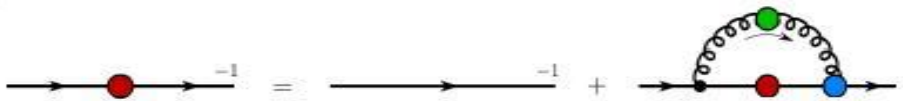


- Mass generated from the interaction of quarks with the gluon.
- Light quarks acquire a **HUGE** constituent mass.
- Responsible of the 98% of the mass of the proton and the large splitting between parity partners.

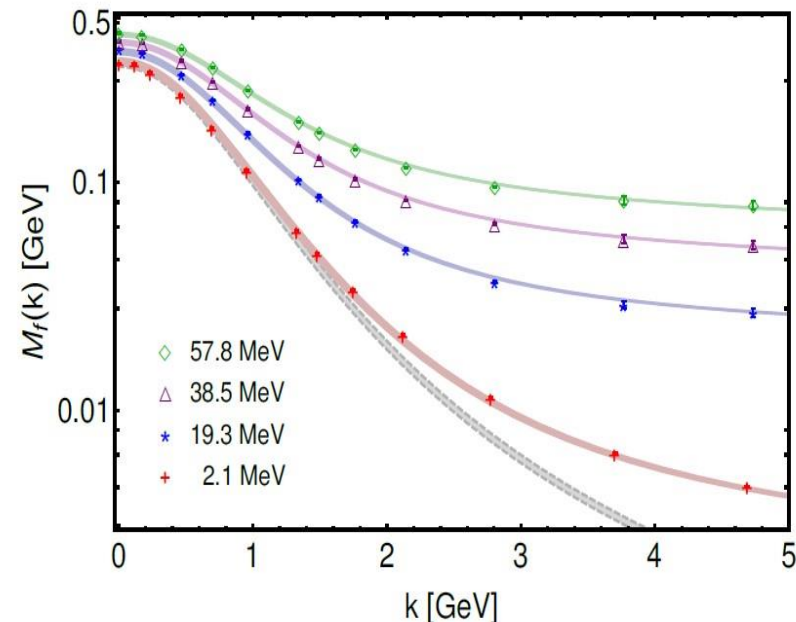
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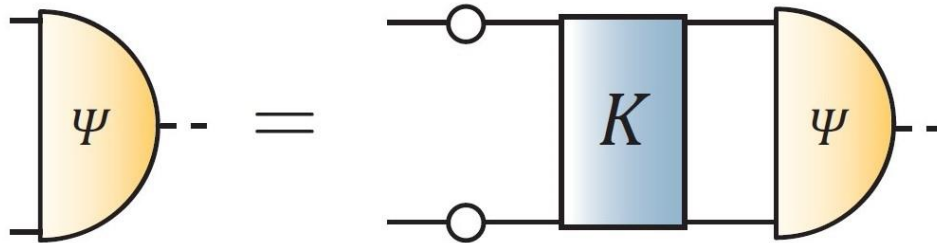
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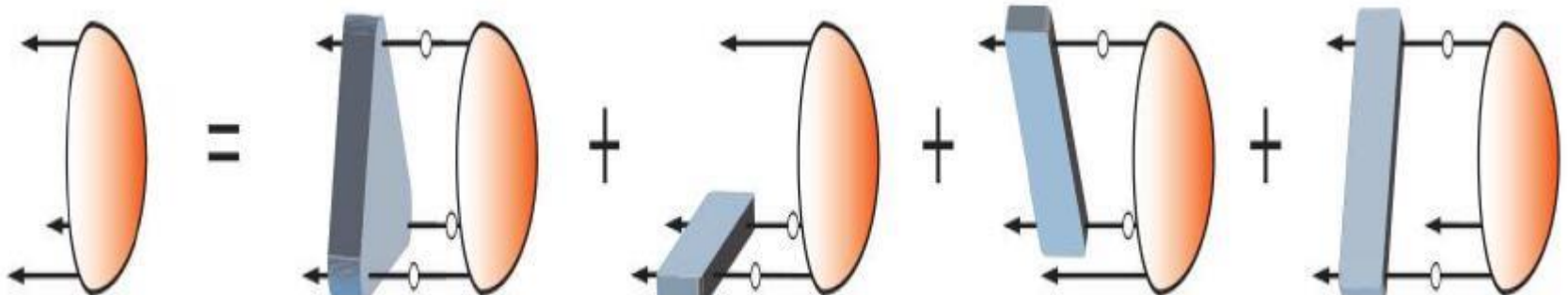
- Lei Chang, Yu-Bin Liu, Khépani Raya, J. Rodríguez-Quintero, and Yi-Bo Yang, Phys. Rev. D 104, no.9, 094509 (2021)

Hadrons: Bound-states in QFT

- **Mesons:** a 2-body bound state problem in QFT
 - Bethe-Salpeter Equation
 - **K - fully amputated, two-particle irreducible, quark-antiquark scattering kernel**



- **Baryons:** a 3-body bound state problem in QFT
- Faddeev equation: sums all possible quantum field theoretical exchanges and interactions that can take place between the three dressed-quarks that define its valence quark content.



2-body correlations

- Mesons: quark-antiquark correlations -- color-singlet
- Diquarks: quark-quark correlations within a color-singlet baryon.
- Diquark correlations:
 - In our approach: non-pointlike color-antitriplet and fully interacting.
 - Diquark correlations are soft, they possess an electromagnetic size.
 - Owing to properties of charge-conjugation, a diquark with spin-parity J^P may be viewed as a partner to the analogous J^{-P} meson.

$$\Gamma_{q\bar{q}}(p; P) = - \int \frac{d^4 q}{(2\pi)^4} g^2 D_{\mu\nu}(p - q) \frac{\lambda^a}{2} \gamma_\mu S(q + P) \Gamma_{q\bar{q}}(q; P) S(q) \frac{\lambda^a}{2} \gamma_\nu$$

$$\Gamma_{qq}(p; P) C^\dagger = -\frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} g^2 D_{\mu\nu}(p - q) \frac{\lambda^a}{2} \gamma_\mu S(q + P) \Gamma_{qq}(q; P) C^\dagger S(q) \frac{\lambda^a}{2} \gamma_\nu$$

2-body correlations

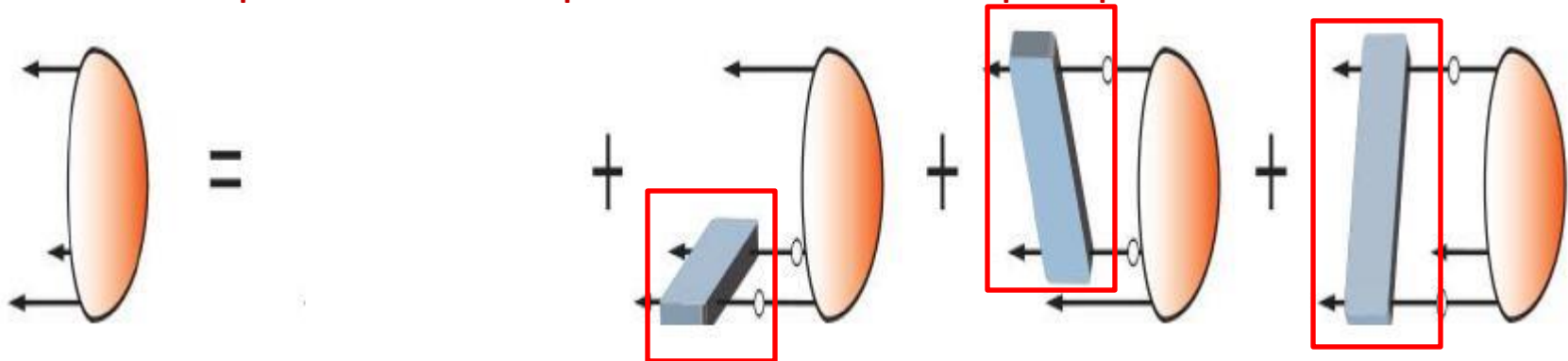
➤ Quantum numbers:

- $(I=0, J^P=0^+)$: isoscalar-scalar diquark
- $(I=1, J^P=1^+)$: isovector-pseudovector diquark
- $(I=0, J^P=0^-)$: isoscalar-pseudoscalar diquark
- $(I=0, J^P=1^-)$: isoscalar-vector diquark
- $(I=1, J^P=1^-)$: isovector-vector diquark

- ✓ G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer, Prog.Part.Nucl.Phys. 91 (2016) 1-100
- ✓ Chen Chen, B. El-Bennich, C. D. Roberts, S. M. Schmidt, J. Segovia, S-L. Wan, Phys.Rev. D97 (2018) no.3, 034016

➤ Three-body bound states

- The diquark *Ansatz* for the 4-point Green's function of the quark-quark correlations:



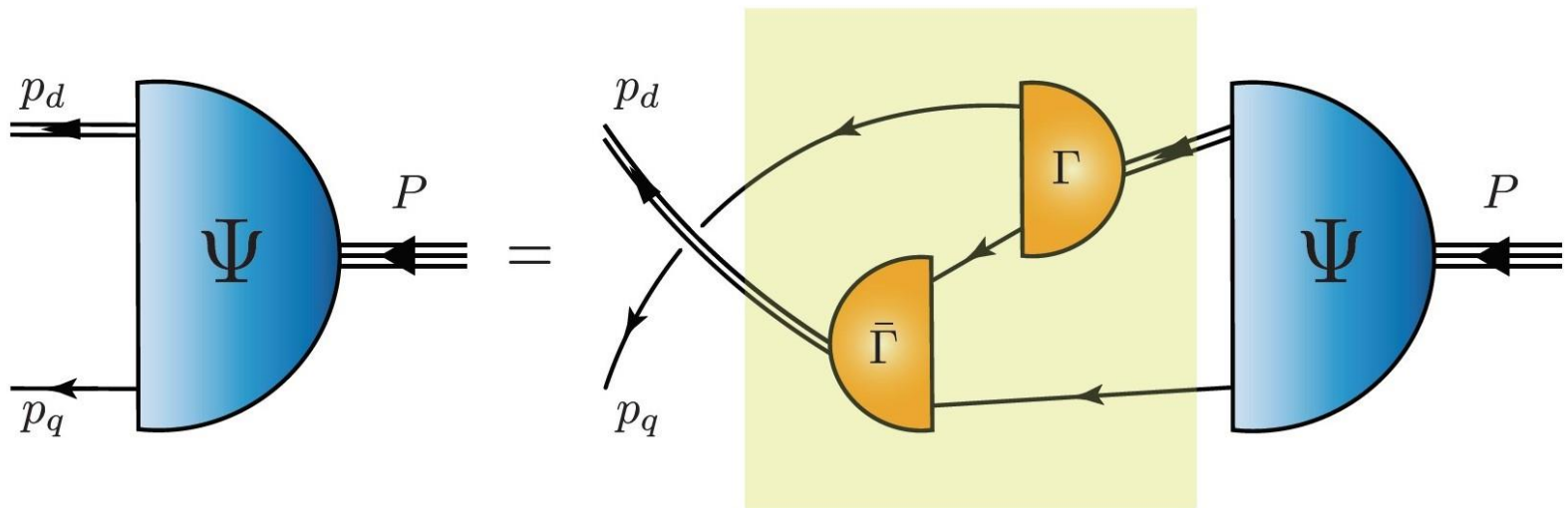
$$\begin{array}{c} k_1 \\ \leftarrow \\ \text{---} \\ \leftarrow \\ k_2 \end{array} \begin{array}{c} \text{---} \\ \leftarrow \\ \text{---} \\ \leftarrow \end{array} \begin{array}{c} -q_1 \\ \text{---} \\ \leftarrow \\ \text{---} \\ -q_2 \end{array} \begin{array}{c} \text{---} \\ \leftarrow \\ \text{---} \\ \leftarrow \end{array} G = \sum_{J^P} \begin{array}{c} \text{---} \\ \leftarrow \\ \text{---} \\ \leftarrow \end{array} \begin{array}{c} k \\ \updownarrow \\ \Lambda^{J^P} \end{array} \begin{array}{c} \text{---} \\ \leftarrow \\ \text{---} \\ \leftarrow \end{array} K \begin{array}{c} \text{---} \\ \leftarrow \\ \text{---} \\ \leftarrow \end{array} \begin{array}{c} \bar{\Lambda}^{J^P} \\ \updownarrow \\ q \end{array} \begin{array}{c} \text{---} \\ \leftarrow \\ \text{---} \\ \leftarrow \end{array}$$

Quark-diquark Faddeev equation

- Quantum numbers:
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 - $(I=1, J^P=1^-)$: isovector-vector diquark

- Three-body bound states  Quark-diquark two-body bound states

- ✓ R.T. Cahill, Craig D. Roberts, J. Praschifka, Phys. Rev. D 36 (1987) 2804
- ✓ R.T. Cahill, Craig D. Roberts, J. Praschifka, Austral.J.Phys. 42 (1989) 129-145





Progress in Particle and Nuclear Physics

Volume 116, January 2021, 103835



Review

Diquark correlations in hadron physics: Origin, impact and evidence

M.Yu. Barabanov¹, M.A. Bedolla², W.K. Brooks³, G.D. Cates⁴, C. Chen⁵, Y. Chen^{6, 7}, E. Cisbani⁸, M. Ding⁹, G. Eichmann^{10, 11}, R. Ent¹², J. Ferretti¹³
✉, R.W. Gothe¹⁴, T. Horn^{15, 12}, S. Liuti⁴, C. Mezrag¹⁶, A. Pilloni⁹, A.J.R. Puckett¹⁷, C.D. Roberts^{18, 19} ✉ ... B.B. Wojtsekhowski¹² ✉

Quark-diquark Faddeev equation

PRL **104**, 201601 (2010)

PHYSICAL REVIEW LETTERS

week ending
21 MAY 2010

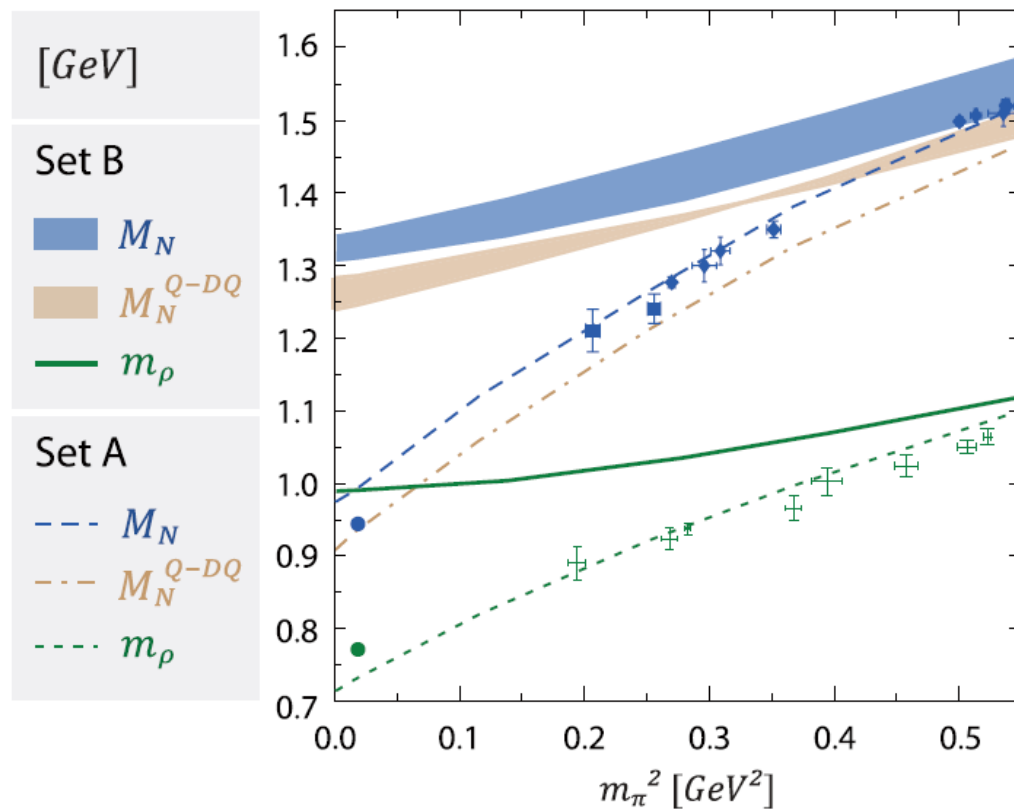
Nucleon Mass from a Covariant Three-Quark Faddeev Equation

G. Eichmann,^{1,2} R. Alkofer,² A. Krassnigg,² and D. Nicmorus²

¹*Institute for Nuclear Physics, Darmstadt University of Technology, 64289 Darmstadt, Germany*

²*Institut für Physik, Karl-Franzens-Universität Graz, 8010 Graz, Austria*

(Received 14 December 2009; published 21 May 2010)



Quark-diquark Faddeev equation

- Solution to the **60** year puzzle -- Roper resonance: Discovered in 1963, the Roper resonance appears to be an exact copy of the proton except that its mass is **50%** greater and it is unstable...

PRL 115, 171801 (2015)

PHYSICAL REVIEW LETTERS

week ending
23 OCTOBER 2015

Completing the Picture of the Roper Resonance

Jorge Segovia,¹ Bruno El-Bennich,^{2,3} Eduardo Rojas,^{2,4} Ian C. Cloët,⁵ Craig D. Roberts,⁵
Shu-Sheng Xu,⁶ and Hong-Shi Zong⁶

¹*Grupo de Física Nuclear and Instituto Universitario de Física Fundamental y Matemáticas (IUFFyM),
Universidad de Salamanca, E-37008 Salamanca, Spain*

²*Laboratório de Física Teórica e Computacional, Universidade Cruzeiro do Sul, 01506-000 São Paulo, SP, Brazil*

³*Instituto de Física Teórica, Universidade Estadual Paulista, 01140-070 São Paulo, SP, Brazil*

⁴*Instituto de Física, Universidad de Antioquia, Calle 70 No. 52-21, Medellín, Colombia*

⁵*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

⁶*Department of Physics, Nanjing University, Nanjing 210093, China*

(Received 16 April 2015; revised manuscript received 29 July 2015; published 21 October 2015)

We employ a continuum approach to the three valence-quark bound-state problem in relativistic quantum field theory to predict a range of properties of the proton's radial excitation and thereby unify them with those of numerous other hadrons. Our analysis indicates that the nucleon's first radial excitation is the Roper resonance. It consists of a core of three dressed quarks, which expresses its valence-quark content and whose charge radius is 80% larger than the proton analogue. That core is complemented by a meson cloud, which reduces the observed Roper mass by roughly 20%. The meson cloud materially affects long-wavelength characteristics of the Roper electroproduction amplitudes but the quark core is revealed to probes with $Q^2 \gtrsim 3m_N^2$.

DOI: 10.1103/PhysRevLett.115.171801

PACS numbers: 13.40.Gp, 14.20.Dh, 14.20.Gk, 11.15.Tk

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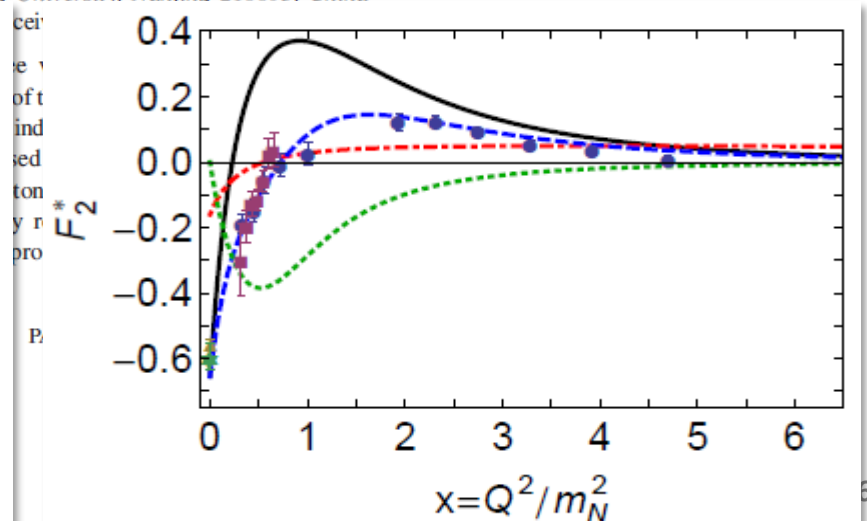
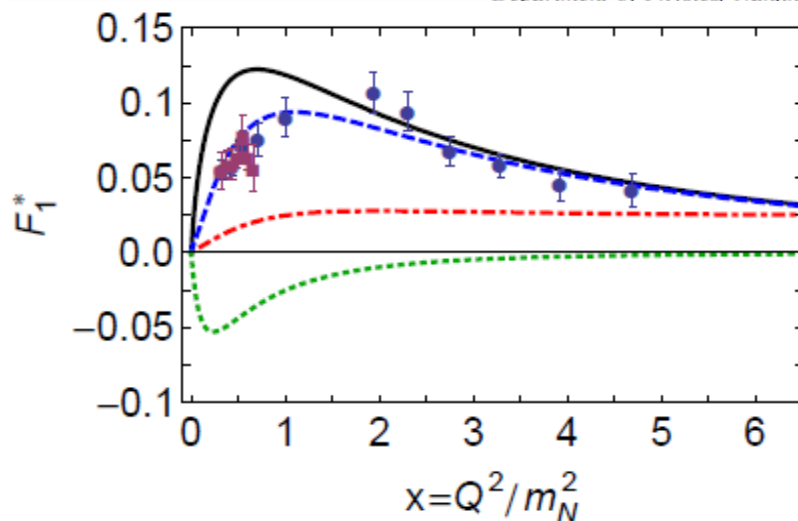
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Quark-diquark Faddeev equation

- Roper resonance -- solution to the 60 year puzzle

REVIEWS OF MODERN PHYSICS

REVIEWS OF MODERN PHYSICS, VOLUME 91, JANUARY–MARCH 2019

Colloquium: Roper resonance: Toward a solution to the fifty year puzzle

Volker D. Burkert^{*}

Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA

Craig D. Roberts[†]

Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA



(published 14 March 2019)

Quark-diquark Faddeev equation

- The electromagnetic nucleon-to- $\Delta(1600)$ transition form factors

PHYSICAL REVIEW D **100**, 034001 (2019)

Transition form factors: $\gamma^* + p \rightarrow \Delta(1232), \Delta(1600)$

Y. Lu,^{1,*} C. Chen,^{2,†} Z.-F. Cui,^{1,‡} C. D. Roberts,^{3,§} S. M. Schmidt,^{4,||} J. Segovia,^{5,¶} and H.-S. Zong^{1,6,**}

¹*Department of Physics, Nanjing University, Nanjing, Jiangsu 210093, China*

²*Instituto de Física Teórica, Universidade Estadual Paulista,*

Rua Dr. Bento Teobaldo Ferraz, 271, 01140-070 São Paulo, São Paulo, Brazil

³*Physics Division, Argonne National Laboratory, Lemont, Illinois 60439, USA*

⁴*Institute for Advanced Simulation, Forschungszentrum Jülich and JARA, D-52425 Jülich, Germany*

⁵*Departamento de Sistemas Físicos, Químicos y Naturales, Universidad Pablo de Olavide, E-41013 Sevilla, Spain*

⁶*Joint Center for Particle, Nuclear Physics and Cosmology, Nanjing, Jiangsu 210093, China*

- In 2023, those (parameter-free) predictions were confirmed in an analysis of data obtained using the CLAS detector at JLab

PHYSICAL REVIEW C **108**, 025204 (2023)

First results on nucleon resonance electroexcitation amplitudes from $ep \rightarrow e' \pi^+ \pi^- p'$ cross sections at $W = 1.4\text{--}1.7$ GeV and $Q^2 = 2.0\text{--}5.0$ GeV²

V. I. Mokeev^{1,*} P. Achenbach¹ V. D. Burkert¹ D. S. Carman¹ R. W. Gothe² A. N. Hiller Blin³
E. L. Isupov⁴ K. Joo⁵ K. Neupane² and A. Trivedi²

¹*Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA*

²*University of South Carolina, Columbia, South Carolina 29208, USA*

³*Institute for Theoretical Physics, Tübingen University, D-72076 Tübingen, Germany*

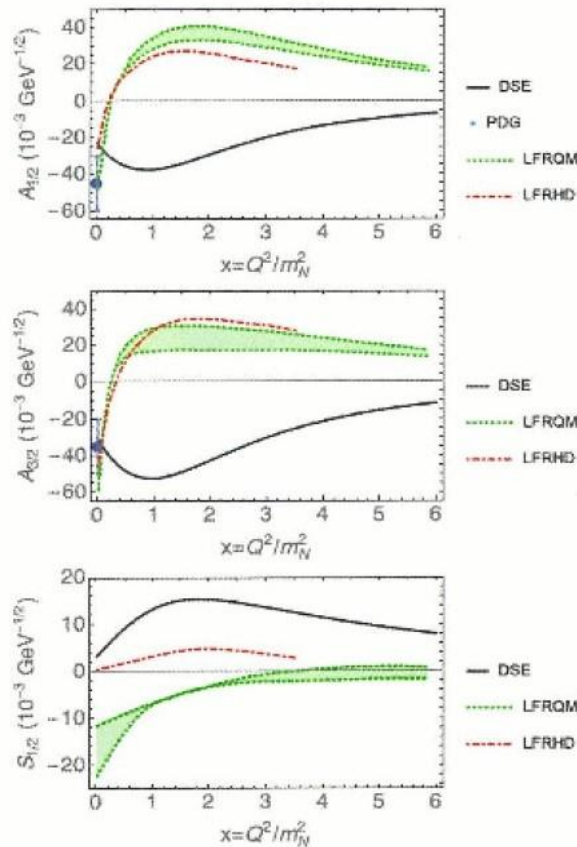
⁴*Skobeltsyn Institute of Nuclear Physics and Physics Department, Lomonosov Moscow State University, 119234 Moscow, Russia*

⁵*University of Connecticut, Storrs, Connecticut 06269, USA*

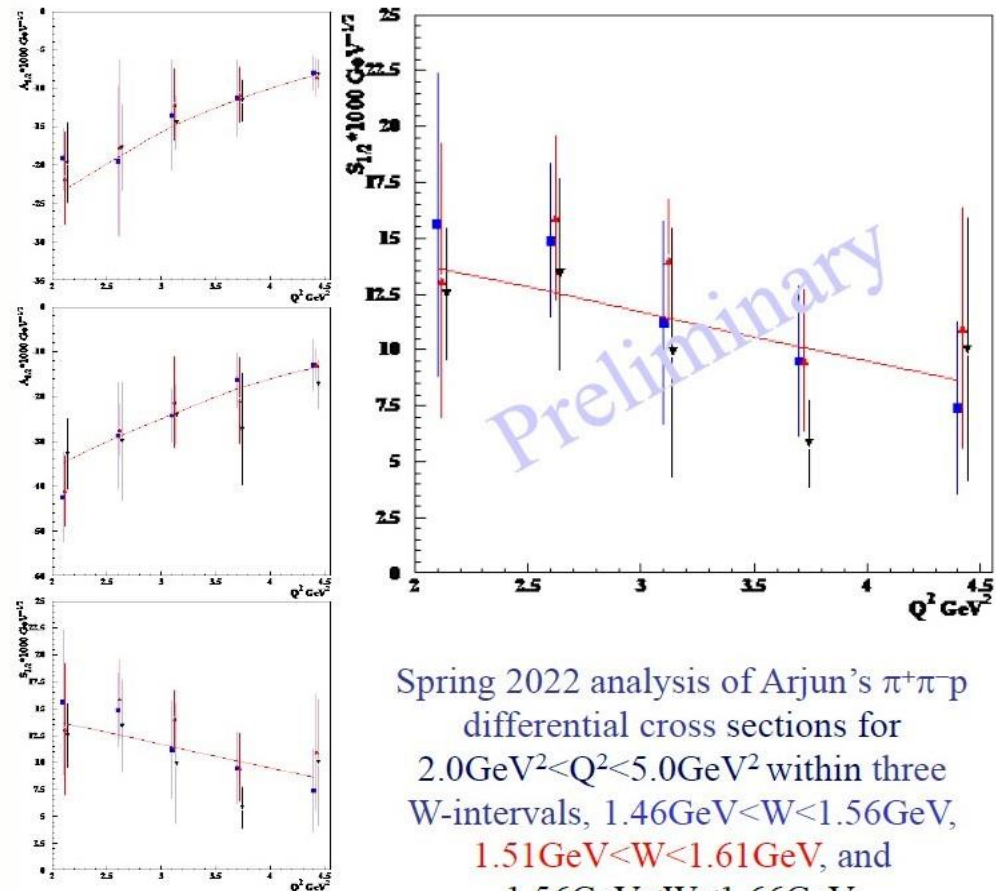
$\Delta(1600)3/2^+$ Form Factors in CSM Approach

Viktor Mokeev

CSM predictions of the $\Delta(1600)3/2^+$ electrocouplings



Ya Lu et al., PRD 100, 034001 (2019)



Spring 2022 analysis of Arjun's $\pi^+\pi^-\pi$ differential cross sections for $2.0\text{GeV}^2 < Q^2 < 5.0\text{GeV}^2$ within three W-intervals, $1.46\text{GeV} < W < 1.56\text{GeV}$, $1.51\text{GeV} < W < 1.61\text{GeV}$, and $1.56\text{GeV} < W < 1.66\text{GeV}$.

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Electromagnetic Form Factors

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Nucleon axial and pseudoscalar form factors from the covariant Faddeev equation

Gernot Eichmann and Christian S. Fischer

Institut für Theoretische Physik, Justus-Liebig-Universität Giessen, D-35392 Giessen, Germany
(Dated: November 2, 2018)

We compute the axial and pseudoscalar form factors of the nucleon in the Dyson-Schwinger approach. To this end, we solve a covariant three-body Faddeev equation for the nucleon wave function and determine the matrix elements of the axialvector and pseudoscalar isotriplet currents. Our only input is a well-established and phenomenologically successful ansatz for the nonperturbative quark-gluon interaction. As a consequence of the axial Ward-Takahashi identity that is respected at the quark level, the Goldberger-Treiman relation is reproduced for all current-quark masses. We discuss the timelike pole structure of the quark-antiquark vertices that enters the nucleon matrix elements and determines the momentum dependence of the form factors. Our result for the axial charge underestimates the experimental value by 20–25% which might be a signal of missing pion-cloud contributions. The axial and pseudoscalar form factors agree with phenomenological and lattice data in the momentum range above $Q^2 \sim 1 \dots 2 \text{ GeV}^2$.

PACS numbers: 11.80.Jy 12.38.Lg, 11.40.Ha 14.20.Dh

I. INTRODUCTION

The nucleon's axial and pseudoscalar form factors are of fundamental significance for the properties of the nucleon that are probed in weak interaction processes. Their momentum dependence can be experimentally tested by (anti)neutrino scattering off nucleons or nuclei, charged pion electroproduction and muon capture processes; see [1–3] for reviews. Both form factors are experimentally hard to extract and therefore considerably less well known than their electromagnetic counterparts. Precisely measured is only the low-momentum limit g_A of the axial form factor which is determined from neutron β -decay. Planned experiments at major facilities are expected to change this situation in the near future.

The theoretical calculation of the nucleon's axial and pseudoscalar form factors requires genuinely non-perturbative methods. Chiral perturbation theory has been successful in this respect [1, 4, 5] although it is generally limited to the region of low momentum transfer. Recent studies in lattice gauge theory are getting closer to the physical pion mass region [6–8] but finite-volume effects become increasingly important. Another non-perturbative approach is the one via functional meth-

The study of axial and pseudoscalar form factors in the functional approach has so far been limited to an approximation where the nucleon is treated as a bound object of a quark and a diquark that interact via quark exchange [12, 13]. The entire gluonic substructure appears here only implicitly within the dressing of quark and diquark propagators as well as diquark vertex functions. There are several conceptual issues that complicate the treatment of form factors in the quark-diquark model. First, the requirement of current conservation induces the appearance of intricate 'seagull' diagrams [14]. Such terms have been taken into account for electromagnetic form factors, but their implementation in the case of axial form factors has not yet been possible for technical reasons [13]. Second, to comply with chiral Ward identities, a current-conserving quark-diquark model requires vector diquarks in addition to the usual scalar and axialvector diquark degrees of freedom [15]. Such an elaborate treatment of the quark-diquark model has not yet been performed.

The situation is somewhat different when the nucleon is treated as a genuine three-body problem. The resulting Faddeev equation in rainbow-ladder truncation has been solved only recently for the nucleon and Δ masses [16–17] and the corresponding nucleon states

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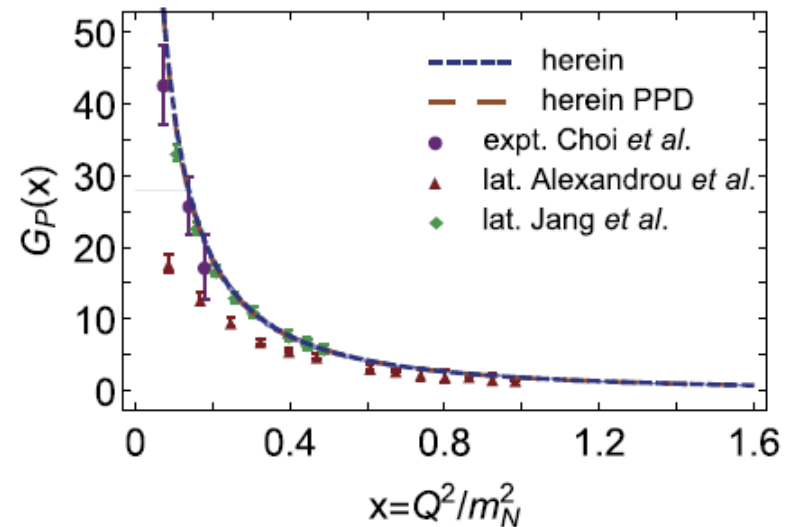
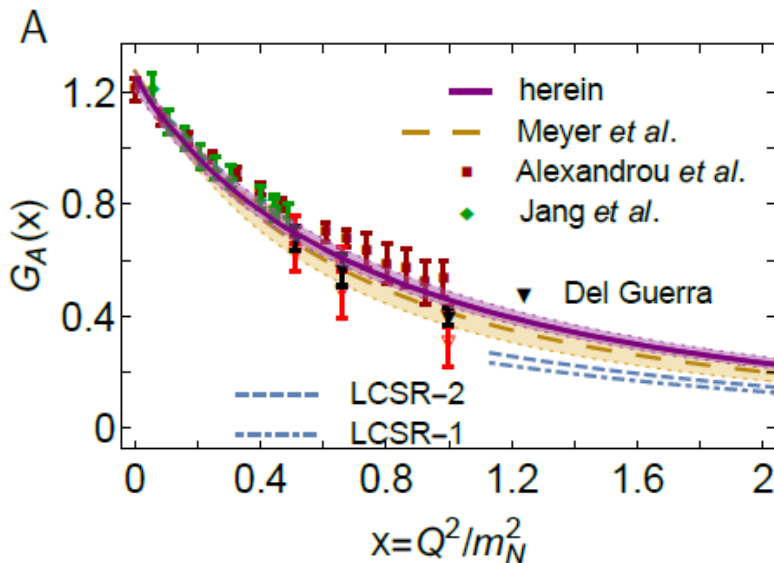
Goldberger-Treiman relation and $g_{\pi NN}$ from the three quark BS / Faddeev approach in the NJL model

Noriyoshi Ishii (Erlangen - Nuremberg U.) (Apr 28, 2000)

Published in: *Nucl.Phys.A* 689 (2001) 793-845 • e-Print: nucl-th/0004063 [nucl-th]

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Axial Form Factors

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PHYSICAL REVIEW D **105**, 094022 (2022)

Nucleon axial-vector and pseudoscalar form factors and PCAC relations

Chen Chen (陈晨) ^{1,2,3,4,*} Christian S. Fischer ^{3,4,†} Craig D. Roberts ^{5,6,‡} and Jorge Segovia ^{7,6,§}

¹*Interdisciplinary Center for Theoretical Study, University of Science and Technology of China,
Hefei, Anhui 230026, China*

²*Peng Huanwu Center for Fundamental Theory, Hefei, Anhui 230026, China*

³*Institut für Theoretische Physik, Justus-Liebig-Universität Gießen, D-35392 Gießen, Germany*

⁴*Helmholtz Forschungsakademie Hessen für FAIR (HFHF), GSI Helmholtzzentrum
für Schwerionenforschung, Campus Gießen, D-35392 Gießen, Germany*

⁵*School of Physics, Nanjing University, Nanjing, Jiangsu 210093, China*

⁶*Institute for Nonperturbative Physics, Nanjing University, Nanjing, Jiangsu 210093, China*

⁷*Dpto. Sistemas Físicos, Químicos y Naturales, Universidad Pablo de Olavide, E-41013 Sevilla, Spain*

Axial Form Factors

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Axial **Form Factors**

- Nucleon Axial Form Factor at large momentum transfers
- $\Delta(1232)$ Axial Form Factors
- $N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Transition Form Factors

Eur. Phys. J. A (2022) 58:206
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THE EUROPEAN
PHYSICAL JOURNAL A



Regular Article - Theoretical Physics

Nucleon axial form factor at large momentum transfers

Chen Chen^{1,2,a} , Craig D. Roberts^{3,4,b} 

¹ Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei 230026, Anhui, China

² Peng Huanwu Center for Fundamental Theory, Hefei 230026, Anhui, China

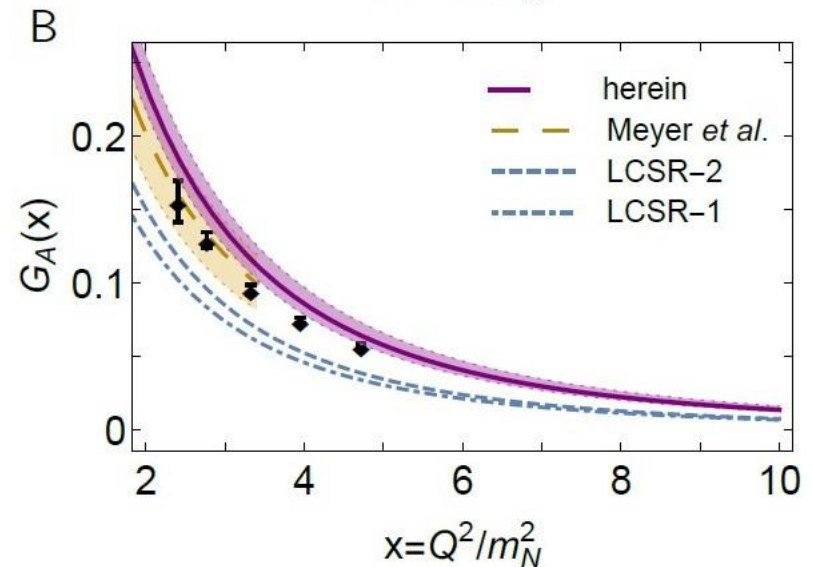
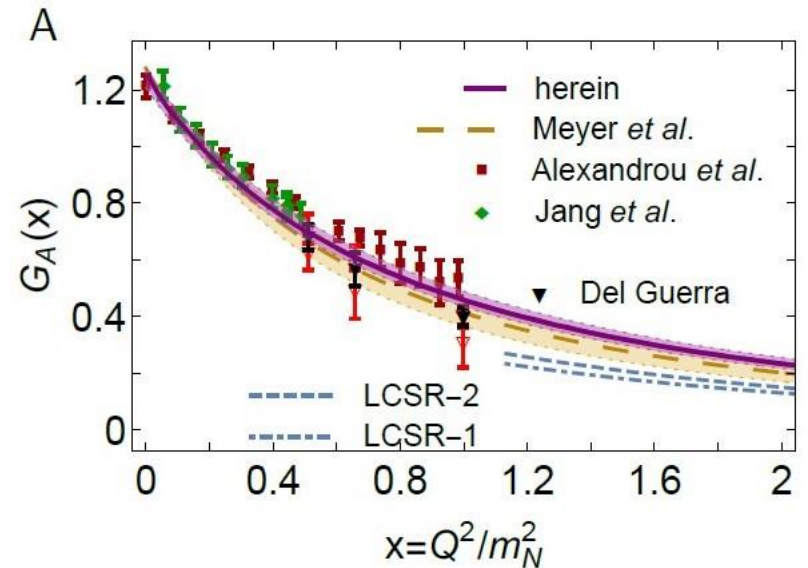
³ School of Physics, Nanjing University, Nanjing 210093, Jiangsu, China

⁴ Institute for Nonperturbative Physics, Nanjing University, Nanjing 210093, Jiangsu, China

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Large Q^2 Nucleon Axial Form Factor

- Parameter-free CSM predictions to $Q^2 = 10 \cdot m_N^2$
- CSM prediction agrees with available data: small & large Q^2





Δ -Baryon axialvector and pseudoscalar form factors, and associated PCAC relations

Pei-Lin Yin¹ , Chen Chen^{2,3,a} , Christian S. Fischer^{4,5} , Craig D. Roberts^{6,7,b} 

¹ College of Science, Nanjing University of Posts and Telecommunications, Nanjing 210023, China

² Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei, Anhui 230026, China

³ Peng Huanwu Center for Fundamental Theory, Hefei 230026, Anhui, China

⁴ Institut für Theoretische Physik, Justus-Liebig-Universität Gießen, 35392 Gießen, Germany

⁵ Helmholtz Forschungsakademie Hessen für FAIR (HFHF), GSI Helmholtzzentrum für Schwerionenforschung, Campus Gießen, 35392 Gießen, Germany

⁶ School of Physics, Nanjing University, Nanjing 210093, Jiangsu, China

⁷ Institute for Nonperturbative Physics, Nanjing University, Nanjing 210093, Jiangsu, China

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Δ -Baryon axialvector and pseudoscalar form factors, and associated PCAC relations

Pei-Lin Yin¹ , Chen Chen^{2,3,a} , Christian S. Fischer^{4,5} , Craig D. Roberts

¹ College of Science, Nanjing University of Posts and Telecommunications, Nanjing 2

² Interdisciplinary Center for Theoretical Study, University of Science and Technology

³ Peng Huanwu Center for Fundamental Theory, Hefei 230026, Anhui, China

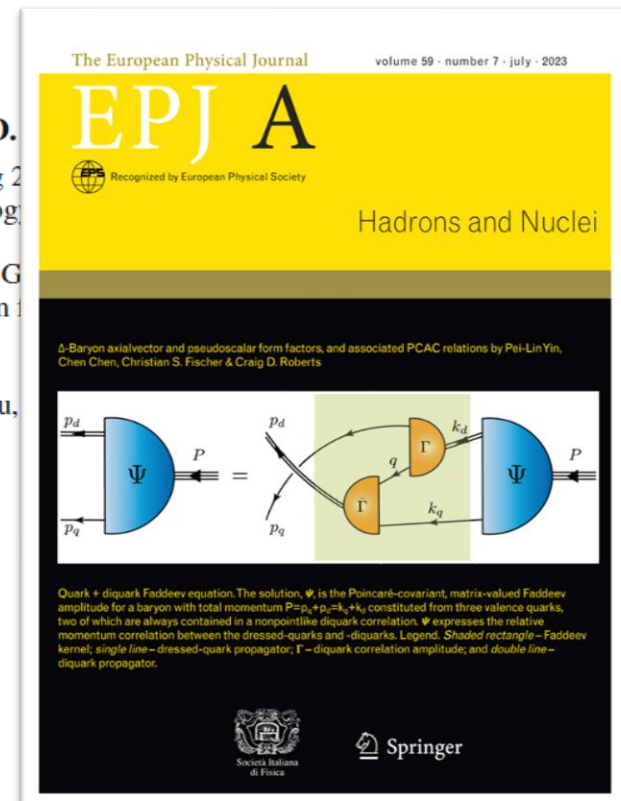
⁴ Institut für Theoretische Physik, Justus-Liebig-Universität Gießen, 35392 Gießen, G

⁵ Helmholtz Forschungsakademie Hessen für FAIR (HFHF), GSI Helmholtzzentrum für Schwerionenforschung, 65924 Frankfurt am Main, Germany

⁶ School of Physics, Nanjing University, Nanjing 210093, Jiangsu, China

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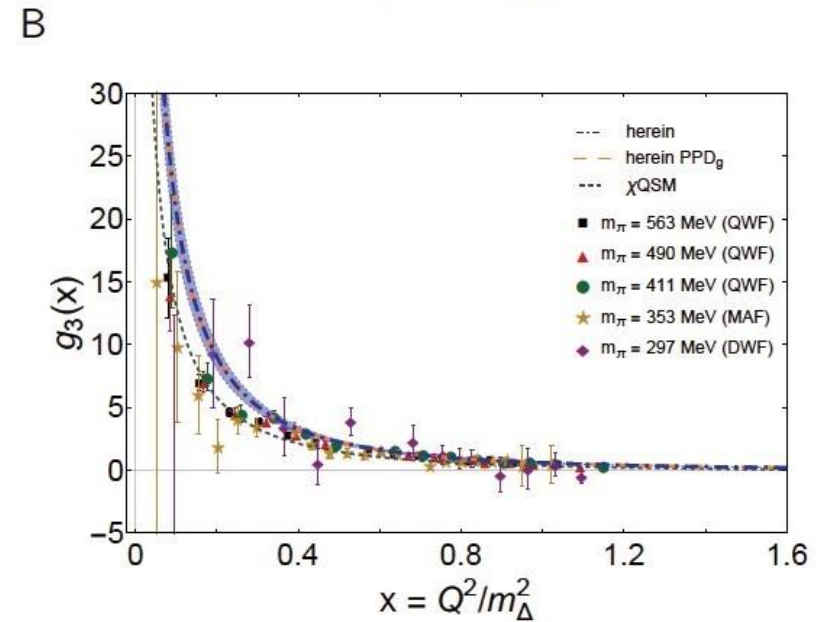
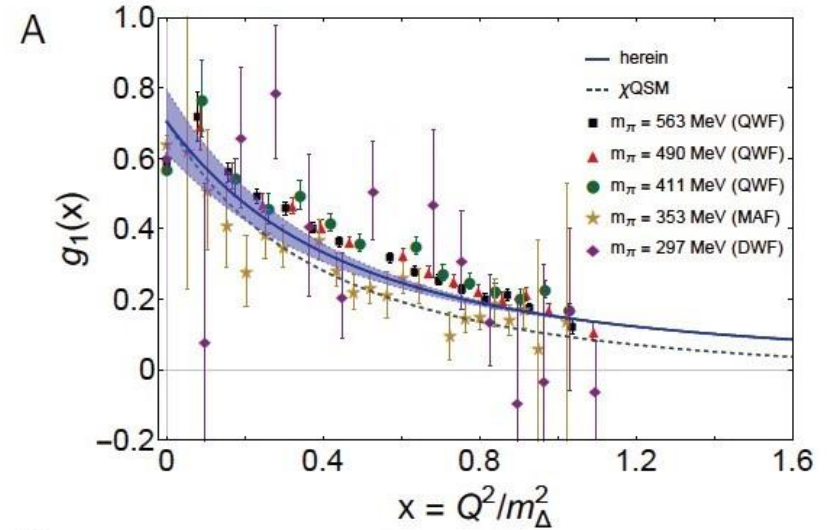
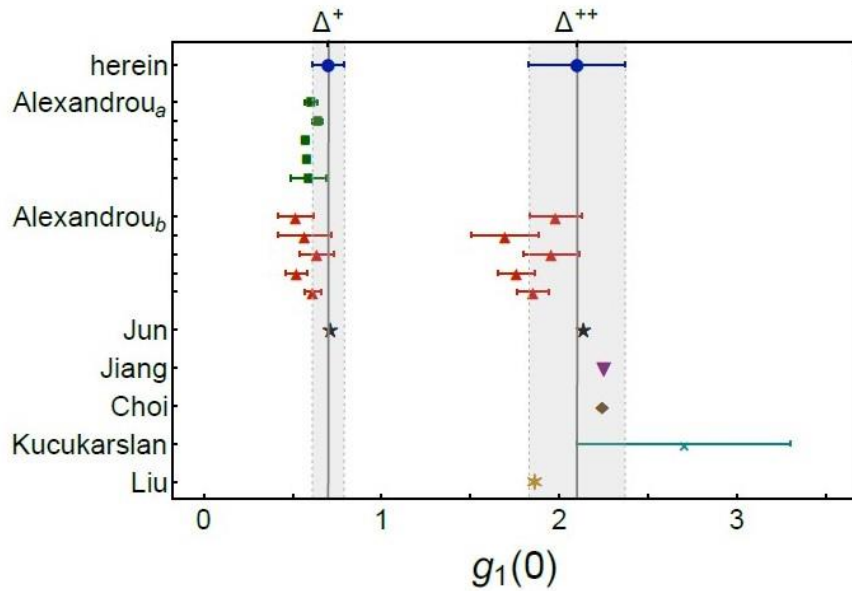
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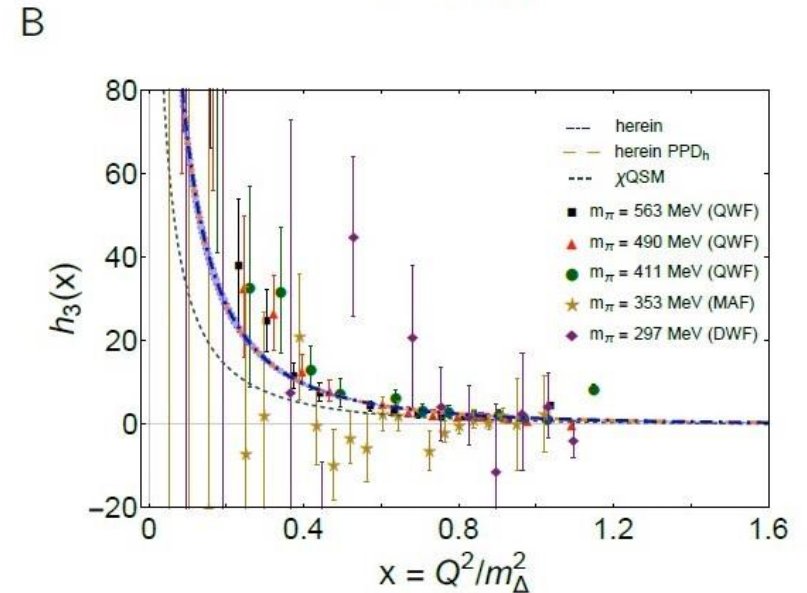
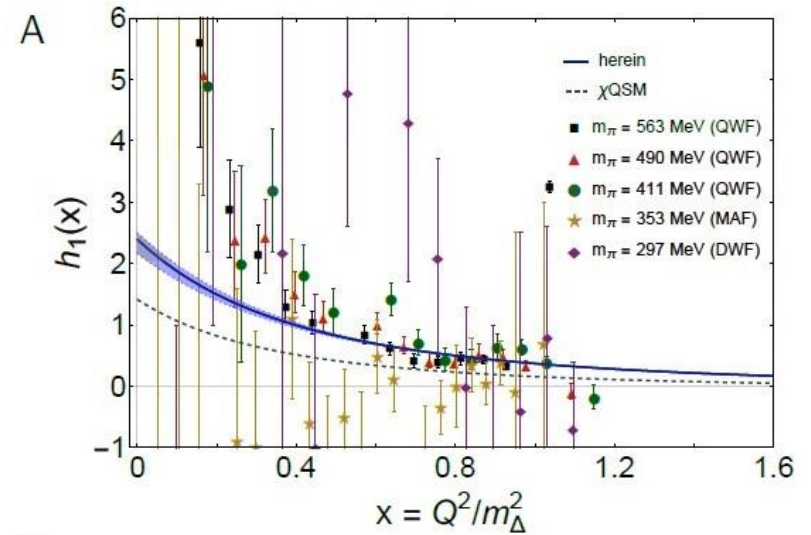
$\Delta(1232)$ Axial Form Factors

➤ Axial charge: $g_1(0) = 0.71(9)$



$\Delta(1232)$ Axial Form Factors

- $h_1(x)$: regular
- $h_1(0) = 2.25(17)$



PHYSICAL REVIEW LETTERS **133**, 131901 (2024)

Nucleon-to- Δ Axial and Pseudoscalar Transition Form Factors

Chen Chen (陈晨)^{1,2,*} Christian S. Fischer^{3,4,†} and Craig D. Roberts^{5,6,‡}

¹*Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei, Anhui 230026, China*

²*Peng Huanwu Center for Fundamental Theory, Hefei, Anhui 230026, China*

³*Institut für Theoretische Physik, Justus-Liebig-Universität Gießen, D-35392 Gießen, Germany*

⁴*Helmholtz Forschungsakademie Hessen für FAIR (HFHF), GSI Helmholtzzentrum für Schwerionenforschung, Campus Gießen, 35392 Gießen, Germany*

⁵*School of Physics, Nanjing University, Nanjing, Jiangsu 210093, China*

⁶*Institute for Nonperturbative Physics, Nanjing University, Nanjing, Jiangsu 210093, China*



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$$J_{\mu}^{\Delta N}(K, Q) = \bar{u}_{\alpha}^{\Delta}(P_f) \Gamma_{(5)(\mu), \alpha}(K, Q) u^N(P_i),$$

$$\Gamma_{\mu, \alpha}^{\text{EM}}(K, Q) = b \left[\frac{i\omega}{2\lambda_+} (G_M^* - G_E^*) \gamma_5 \varepsilon_{\alpha\mu\gamma\delta} K_{\gamma} \hat{Q}_{\delta} - G_E^* T_{\alpha\gamma}^Q T_{\gamma\mu}^K - \frac{i\tau}{\omega} G_C^* \hat{Q}_{\alpha} K_{\mu} \right],$$

$$\Gamma_{5\mu, \alpha}^{\text{AX}}(Q) = \sqrt{\frac{2}{3}} \left[i(\gamma_{\mu} Q_{\lambda} - \delta_{\mu\lambda} \not{Q}) \frac{C_3^A}{m_N} - (\delta_{\mu\lambda} (P_f \cdot Q) - P_{f\mu} Q_{\lambda}) \frac{C_4^A}{m_N^2} + \delta_{\mu\lambda} C_5^A - Q_{\mu} Q_{\lambda} \frac{C_6^A}{m_N^2} \right],$$

$$\Gamma_{5, \alpha}^{\text{PS}}(Q) = \sqrt{\frac{2}{3}} \left[i \frac{Q_{\lambda}}{4m_N} \frac{m_{\pi}^2}{Q^2 + m_{\pi}^2} \frac{f_{\pi}}{m_q} G_{\pi N \Delta}(Q^2) \right].$$

$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors

PHYSICAL REVIEW LETTERS **133**, 131901 (2024)

Nucleon-to- Δ Axial and Pseudoscalar Transition Form Factors

Chen Chen (陈晨)^{1,2,*} Christian S. Fischer^{3,4,†} and Craig D. Roberts^{5,6,‡}

¹*Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei, Anhui 230026, China*

²*Peng Huanwu Center for Fundamental Theory, Hefei, Anhui 230026, China*

³*Institut für Theoretische Physik, Justus-Liebig-Universität Gießen, D-35392 Gießen, Germany*

⁴*Helmholtz Forschungsakademie Hessen für FAIR (HFHF), GSI Helmholtzzentrum für Schwerionenforschung,
Campus Gießen, 35392 Gießen, Germany*

⁵*School of Physics, Nanjing University, Nanjing, Jiangsu 210093, China*

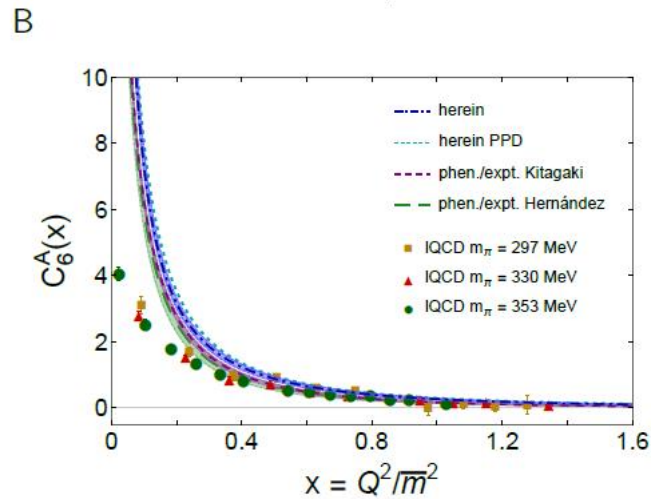
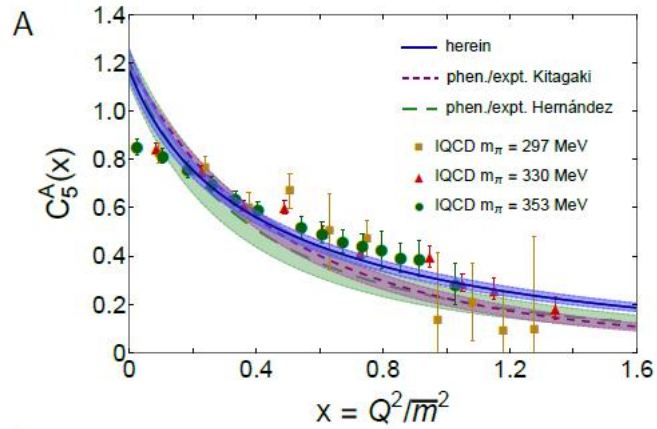
⁶*Institute for Nonperturbative Physics, Nanjing University, Nanjing, Jiangsu 210093, China*



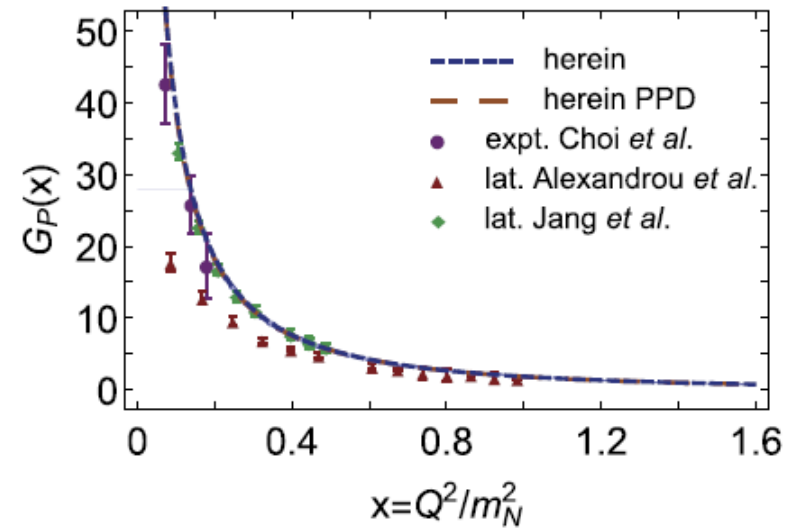
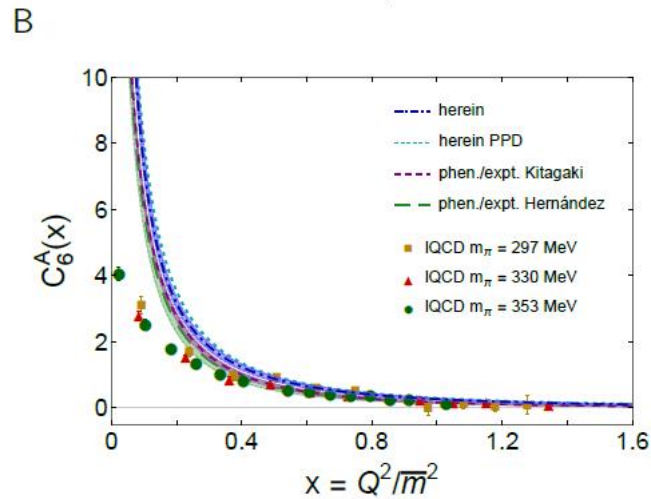
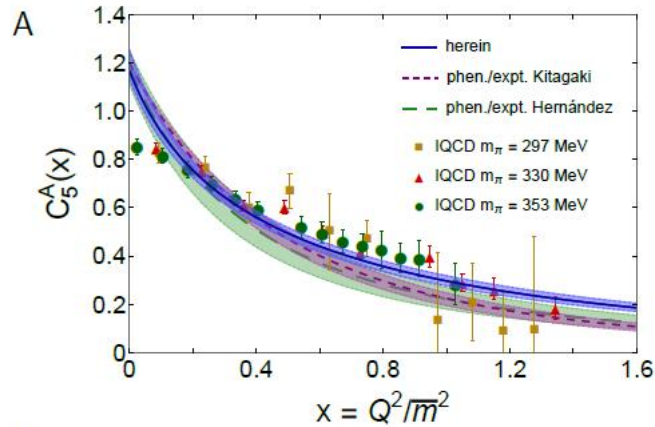
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2. Ian C. Cloet, Craig D. Roberts, and Anthony W. Thomas, *Phys.Rev.Lett.* 111 (**2013**) 101803
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4. Chen Chen, Christian S. Fischer, and Craig D. Roberts, *Phys.Rev.Lett.* 133 (**2024**) 13, 131901

$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors



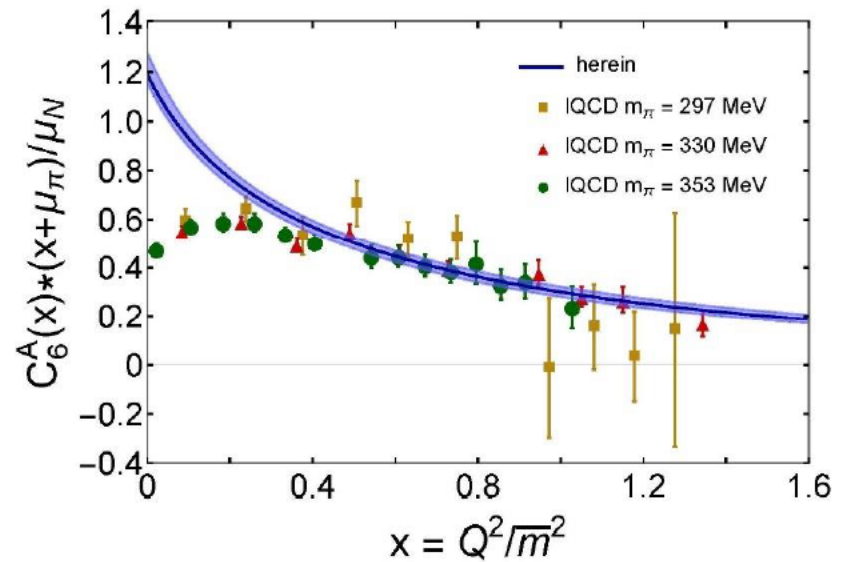
$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors



$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors

Thus, the discrepancy between our prediction and the IQCD results is not explained by the larger than physical pion masses used in their simulation. Instead, the existing simulations are affected by some as yet unknown and uncontrolled systematic lattice artifact.

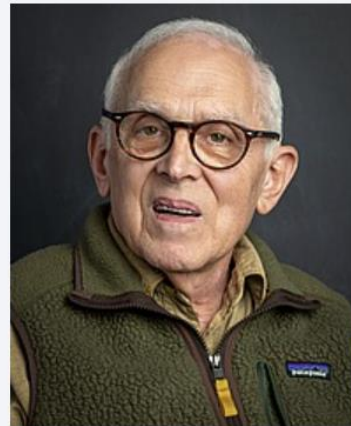
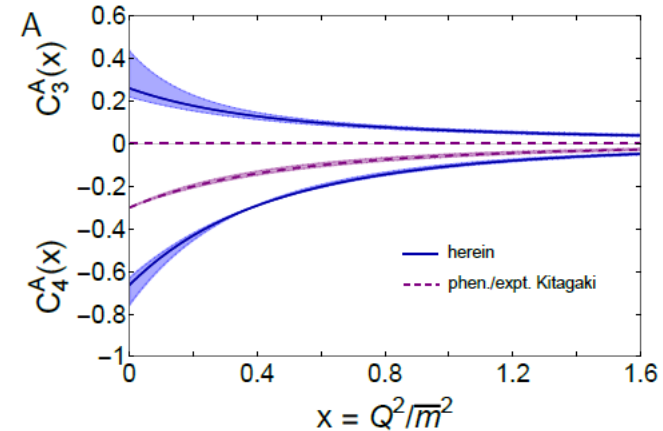
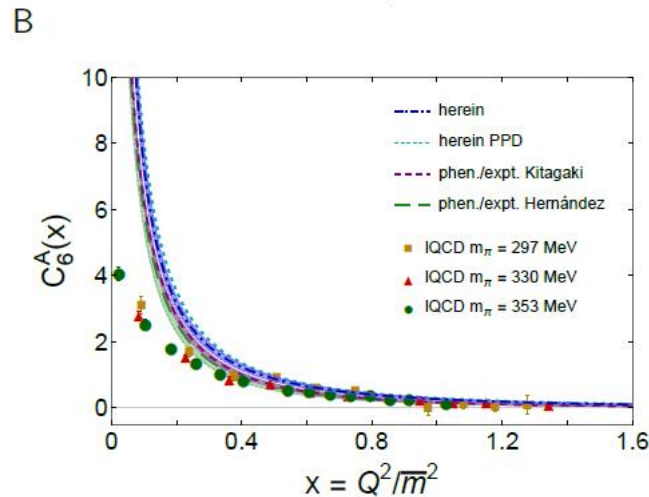
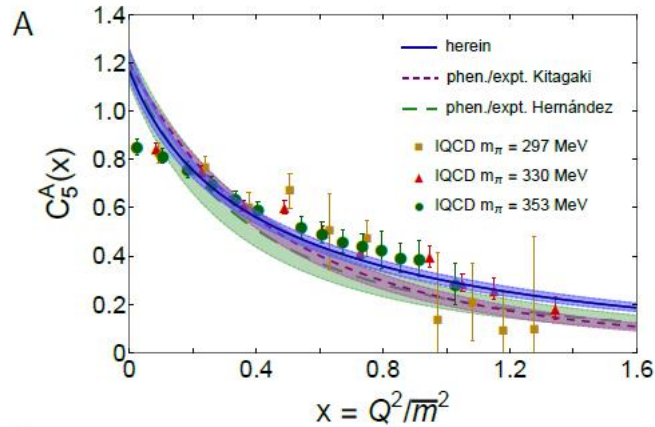
The figure at right augments Figs. 6, 7 in the IQCD paper. It highlights the issue and shows that factorizing a pion pole component consistent with PPD accentuates the visual power of the discrepancy between our prediction and the IQCD results.



We elect to keep the original version of our figure because we are not in a position to explain systematic issues with lattice simulations, which have, furthermore, not been explained by the IQCD practitioners themselves. Notwithstanding that, we have highlighted the challenge in the revised manuscript, adding a closing sentence to the penultimate paragraph of Sec. 4:

It seems fair to judge that these $C_{5,6}^A$ discrepancies owe partly to the larger than physical pion masses which characterise the lattice configurations. However, there are other systematic issues with the IQCD results – see, *e.g.*, Ref. [37, Figs. 6, 7].

$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors



Stephen L. Adler

PROFESSOR EMERITUS

School of Natural Sciences
Particle Physics

Photoproduction, electroproduction and weak single pion production in the (3,3) resonance region

Stephen L. Adler (Princeton, Inst. Advanced Study) (1968)

Published in: *Annals Phys.* 50 (1968) 189-311

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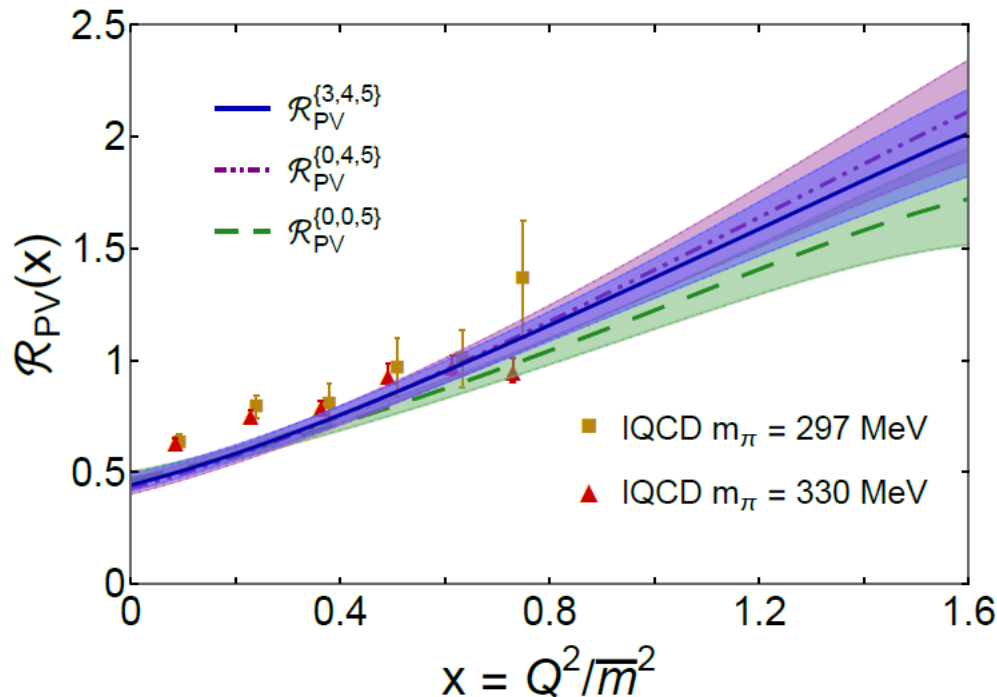
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#1

$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors

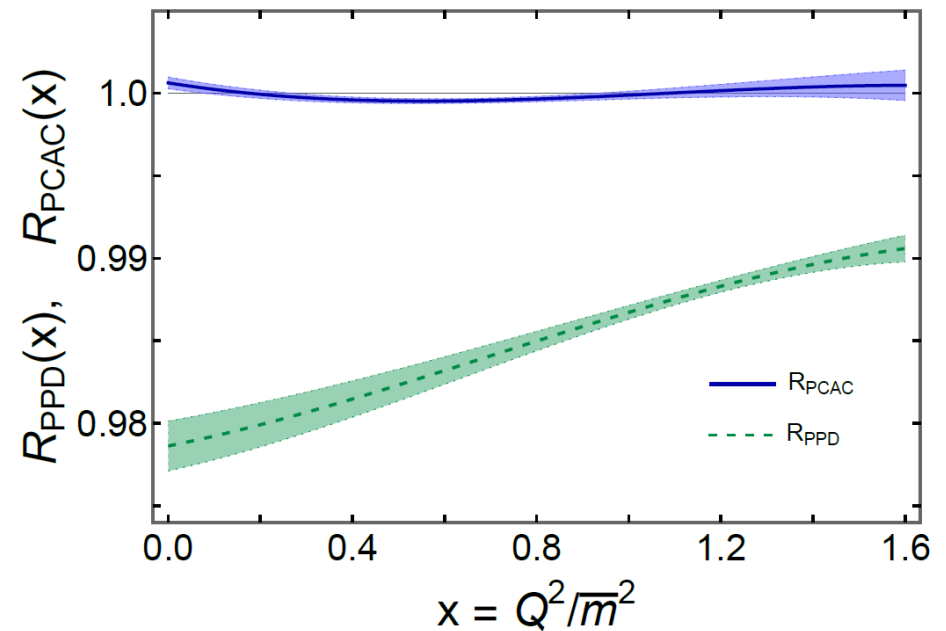
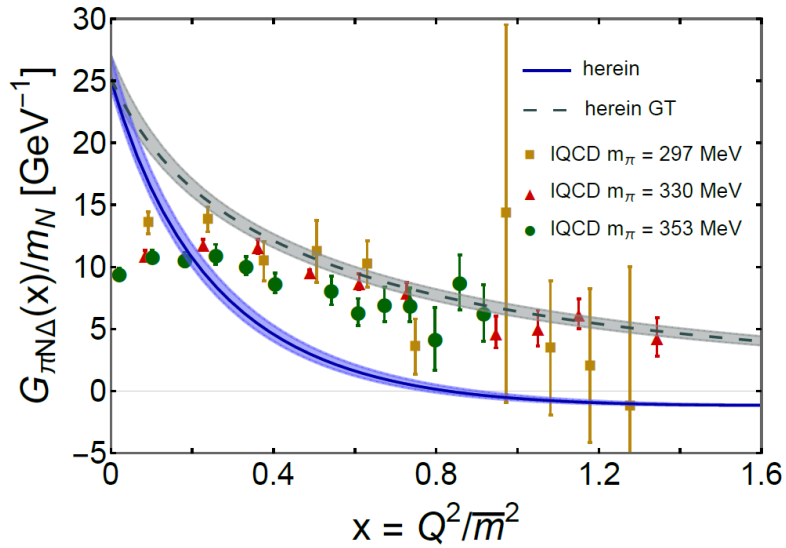
➤ The parity violation asymmetry

$$\mathcal{R}_{PV}^{\{3,4,5\}}(Q^2) := \frac{C_5^A}{C_3^V} \left[1 + \frac{m_\Delta^2 - Q^2 - m_N^2}{2m_N^2} \frac{C_4^A}{C_5^A} - \frac{m_N^2 + Q^2 + 2m_N m_\Delta - 3m_\Delta^2}{4m_N m_\Delta} \frac{C_3^A}{C_5^A} \right]$$



$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors

➤ Pseudoscalar form factor



- **Part A. Introduction**
- **Part B. The elastic and transition axial-vector form factors of the nucleon $N(940)$ and $\Delta(1232)$.**
- **Part C. Screening masses of positive- and negative-parity hadron ground-states, including those with strangeness.**

DSEs at finite temperature & density



PERGAMON

Progress in Particle and Nuclear Physics 45 (2000) S1–S103

**Progress in
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Dyson-Schwinger Equations: Density, Temperature and Continuum Strong QCD

C. D. ROBERTS and S. M. SCHMIDT

Physics Division, Building 203, Argonne National Laboratory, Argonne, IL 60439-4843, USA

e-mail: cdroberts@anl.gov basti@theory.phy.anl.gov

Progress in Particle and Nuclear Physics 105 (2019) 1–60



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Review

QCD at finite temperature and chemical potential from Dyson–Schwinger equations

Christian S. Fischer*

*Institut für Theoretische Physik, Justus-Liebig-Universität Giessen, 35392 Giessen, Germany
HIC for FAIR Giessen, 35392 Giessen, Germany*



DSEs at finite temperature

- Currently, it is widely accepted that strongly interacting matter transitions from hadronic excitations to quark-gluon plasma (QGP) at high temperatures, where quarks and gluons become deconfined and chiral symmetry is restored.
- For strongly interacting matter, one key aspect of studying is the behavior of **screening masses**, which are defined by the exponential decay of the spatial correlation functions.
- The **screening mass** is physically connected to the response of the medium inserted into the probe hadron. On the chiral symmetry restoration domain, for any hadronic parity-partner-pair, i.e., a hadron and its parity partner, the associated screening masses are expected to degenerate.

➤ arXiv:2412.15045 [hep-ph]

Preprint no. USTC-ICTS/PCFT-24-49

Screening masses of positive- and negative-parity hadron ground-states, including those with strangeness

Chen Chen,^{1,2,*} Fei Gao,^{3,†} and Si-xue Qin^{4,‡}

¹*Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei, Anhui 230026, China*

²*Peng Huanwu Center for Fundamental Theory, Hefei, Anhui 230026, China*

³*School of Physics, Beijing Institute of Technology, Beijing 100081, China*

⁴*Department of Physics and Chongqing Key Laboratory for Strongly Coupled Physics, Chongqing University, Chongqing 401331, China*

(Dated: December 24, 2024)

Using a symmetry-preserving treatment of a vector $\times$ vector contact interaction (SCI) at nonzero temperature, we compute the screening masses of flavour-SU(3) ground-state $J^P = 0^\pm, 1^\pm$ mesons, and $J^P = 1/2^\pm, 3/2^\pm$ baryons. We find that all correlation channels allowed at $T = 0$ persist when the temperature increases, even above the QCD phase transition. The results for mesons qualitatively agree with those obtained from the contemporary lattice-regularised quantum chromodynamics (lQCD) simulations. One of the most remarkable features is that each parity-partner-pair degenerates when $T > T_c$, with T_c being the critical temperature. For each pair, the screening mass of the negative parity meson increases monotonously with temperature. In contrast, the screening mass of the meson with positive parity is almost invariant on the domain $T \lesssim T_c/2$; when T gets close to T_c , it decreases but soon increases again and finally degenerates with its parity partner, which signals the restoration of chiral symmetry. We also find that the T -dependent behaviours of baryon screening masses are quite similar to those of the mesons. For baryons, the dynamical, nonpointlike diquark correlations play a crucial role in the screening mass evolution. We further calculate the evolution of the fraction of each kind of diquark within baryons respective to temperature. We observe that, at high temperatures, only $J = 0$ scalar and pseudoscalar diquark correlations can survive within $J^P = 1/2^\pm$ baryons.

➤ Phys.Rev.D 87 (2013) 7, 074038

PHYSICAL REVIEW D **87**, 074038 (2013)

Baryon and meson screening masses

Kun-lun Wang,¹ Yu-xin Liu,^{1,*} Lei Chang,² Craig D. Roberts,^{3,4,†} and Sebastian M. Schmidt⁵

¹*Department of Physics, Center for High Energy Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*

²*Institut für Kernphysik, Forschungszentrum Jülich, D-52425 Jülich, Germany*

³*Argonne National Laboratory, Physics Division, Argonne, Illinois 60439, USA*

⁴*Department of Physics, Illinois Institute of Technology, Chicago, Illinois 60616-3793, USA*

⁵*Institute for Advanced Simulation, Forschungszentrum Jülich and JARA, D-52425 Jülich, Germany*
(Received 28 January 2013; published 30 April 2013)

In a strongly coupled quark-gluon plasma, collective excitations of gluons and quarks should dominate over the excitation of individual quasifree gluon and quark modes. To explore this possibility, we computed screening masses for ground-state light-quark mesons and baryons at leading order in a symmetry-preserving truncation scheme for the Dyson-Schwinger equations using a confining formulation of a contact interaction at nonzero temperature. Meson screening masses are obtained from Bethe-Salpeter equations, and baryon analogues from a novel construction of the Faddeev equation, which employs an improved quark-exchange approximation in the kernel. Our treatment implements a deconfinement transition that is coincident with chiral symmetry restoration in the chiral limit, when both transitions are second order. Despite deconfinement, in all $T = 0$ bound-state channels, strong correlations persist above the critical temperature, $T > T_c$; and, in the spectrum defined by the associated screening masses, degeneracy between parity-partner correlations is apparent for $T \gtrsim 1.3T_c$. Notwithstanding these results, there are reasons (including Golberger-Treiman relations) to suppose that the inertial masses of light-quark bound states, when they may be defined, vanish at the deconfinement temperature, and that this is a signal of bound-state dissolution. Where a sensible comparison is possible, our predictions are consistent with results from contemporary numerical simulations of lattice-regularized QCD.

DSEs at finite temperature

- A symmetry-preserving treatment of a vector $\times$ vector contact interaction (SCI) at nonzero temperature:

$$\mathcal{K}_{\alpha\beta,\gamma\delta} = g^2 D_{\mu\nu}(k) [i\gamma_\mu]_{\alpha\beta} [i\gamma_\nu]_{\gamma\delta}, \quad (1)$$

where $D_{\mu\nu}$ is the gluon propagator. Furthermore, in this work, we use the following SCI *Ansatz*

$$g^2 D_{\mu\nu}(k) = \delta_{\mu\nu} \frac{4\pi\alpha_{\text{IR}}}{m_G^2}, \quad (2)$$

- Dressed-quark propagator

$$\begin{aligned} S_f^{-1}(p; T) &= i\vec{\gamma} \cdot \vec{p} + i\gamma_4 \omega_n + m_f \\ &+ \frac{16\pi\alpha_{\text{IR}}}{3m_G^2} \int_{l,dq} \gamma_\mu S_f(q; T) \gamma_\mu \end{aligned}$$

- Dressed-quark mass

$$M_f(T) = m_f + \frac{16\pi\alpha_{\text{IR}}}{3m_G^2} \int_{l,dq} \frac{4M_f(T)}{s_l + M_f(T)^2}$$

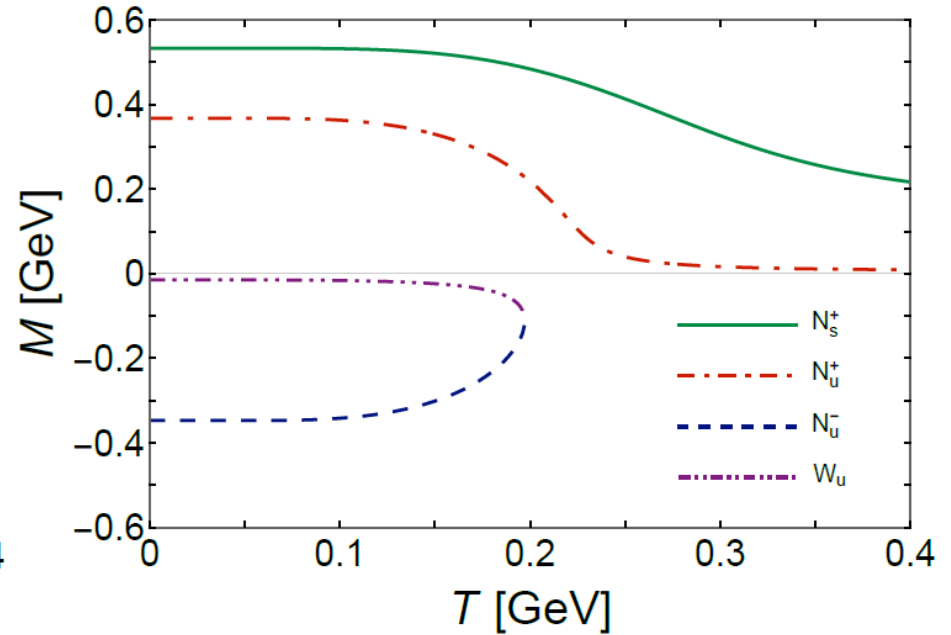
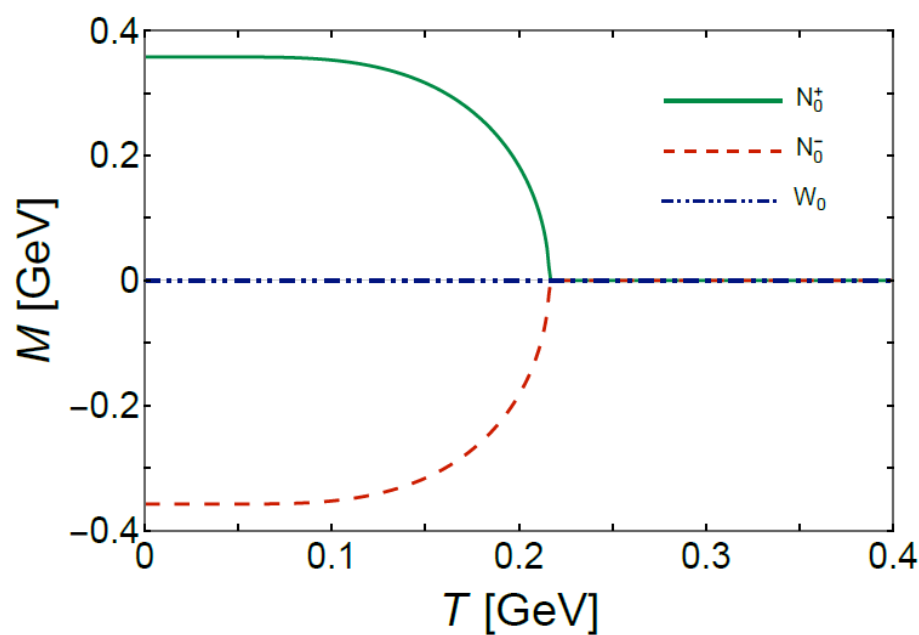
DSEs at finite temperature

- A symmetry-preserving treatment of a vector $\times$ vector contact interaction (SCI) at nonzero temperature:

$$\mathcal{K}_{\alpha\beta,\gamma\delta} = g^2 D_{\mu\nu}(k) [i\gamma_\mu]_{\alpha\beta} [i\gamma_\nu]_{\gamma\delta}, \quad (1)$$

where $D_{\mu\nu}$ is the gluon propagator. Furthermore, in this work, we use the following SCI *Ansatz*

$$g^2 D_{\mu\nu}(k) = \delta_{\mu\nu} \frac{4\pi\alpha_{\text{IR}}}{m_G^2}, \quad (2)$$



Screening masses: mesons

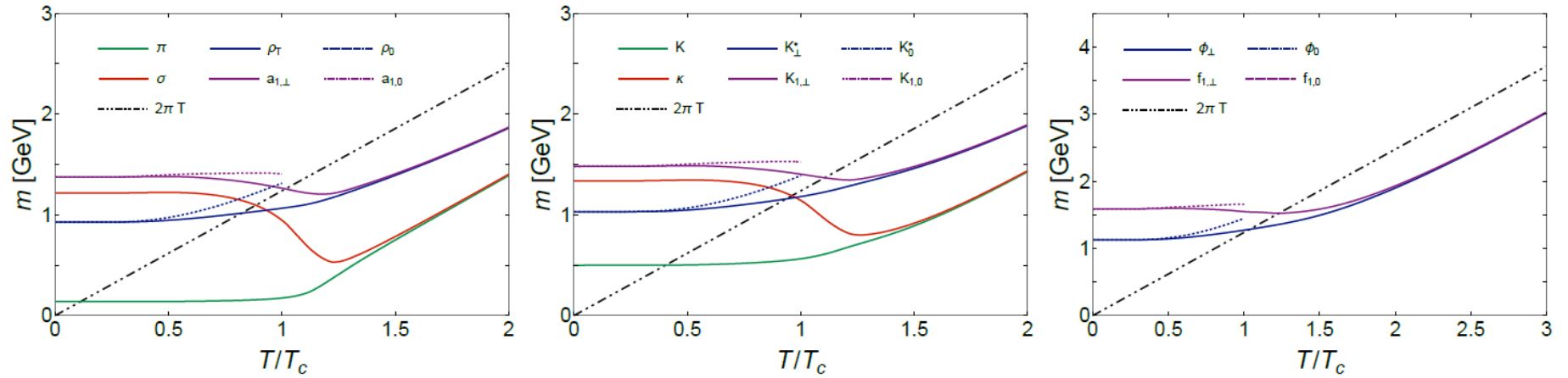


FIG. 3. (Left to right) Screening masses of mesons with strangeness $S = 0, 1, 2$. In each figure: *green curve*: pseudoscalar meson; *red curve*: scalar meson; *blue curves*: vector meson; *purple curves*: axial-vector meson; and *black dot-dot-dashed curve*: free theory limit of $m = 2\pi T$. For the $J = 1$ mesons, *i.e.*, vector and axial-vector mesons, transverse modes are traced with solid lines and longitudinal modes with dotted lines.

Screening masses: diquarks

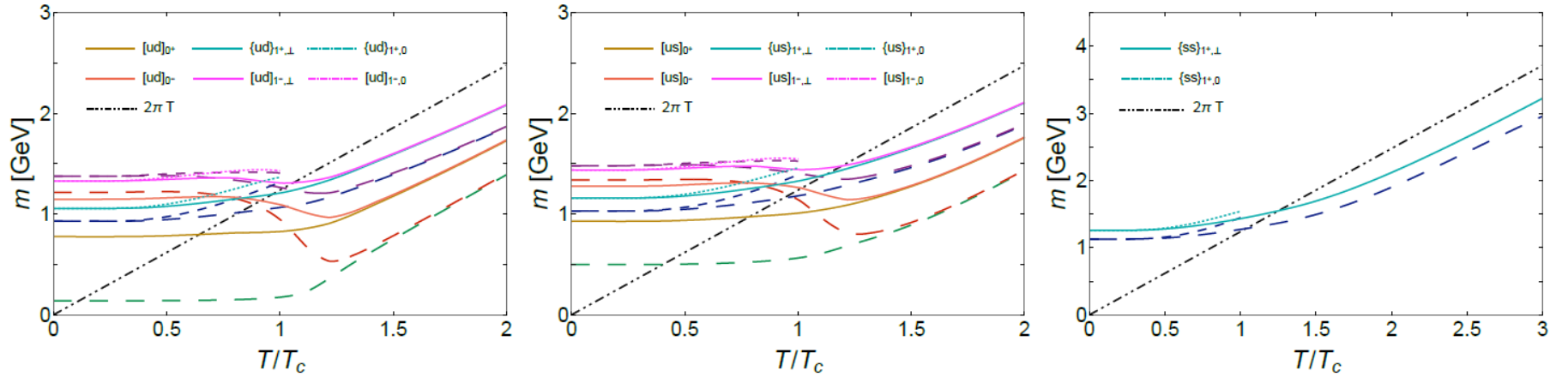


FIG. 5. (Left to right) Screening masses of diquarks with strangeness $S = 0, 1, 2$, in comparison with the results of their meson partners from Fig. 3. In each figure: *gold curve*: scalar diquark; *coral curve*: pseudoscalar diquark; *cyan curves*: axial-vector diquark; *magenta curves*: vector diquark; and *black dot-dot-dashed curve*: free theory limit of $m = 2\pi T$. For the $J = 1$ diquark correlations, *i.e.*, axial-vector and vector diquarks, transverse modes are traced with solid lines and longitudinal modes with dotted lines. For the legends of mesons: the colours are the same as those in Fig. 3, except that the *long dashed curves* represent the $J = 0$ and the transverse modes of the $J = 1$ mesons, and the *short dashed curves* are for the longitudinal modes of the $J = 1$ mesons.

Screening masses: $J^P = 1/2^\pm$ baryons

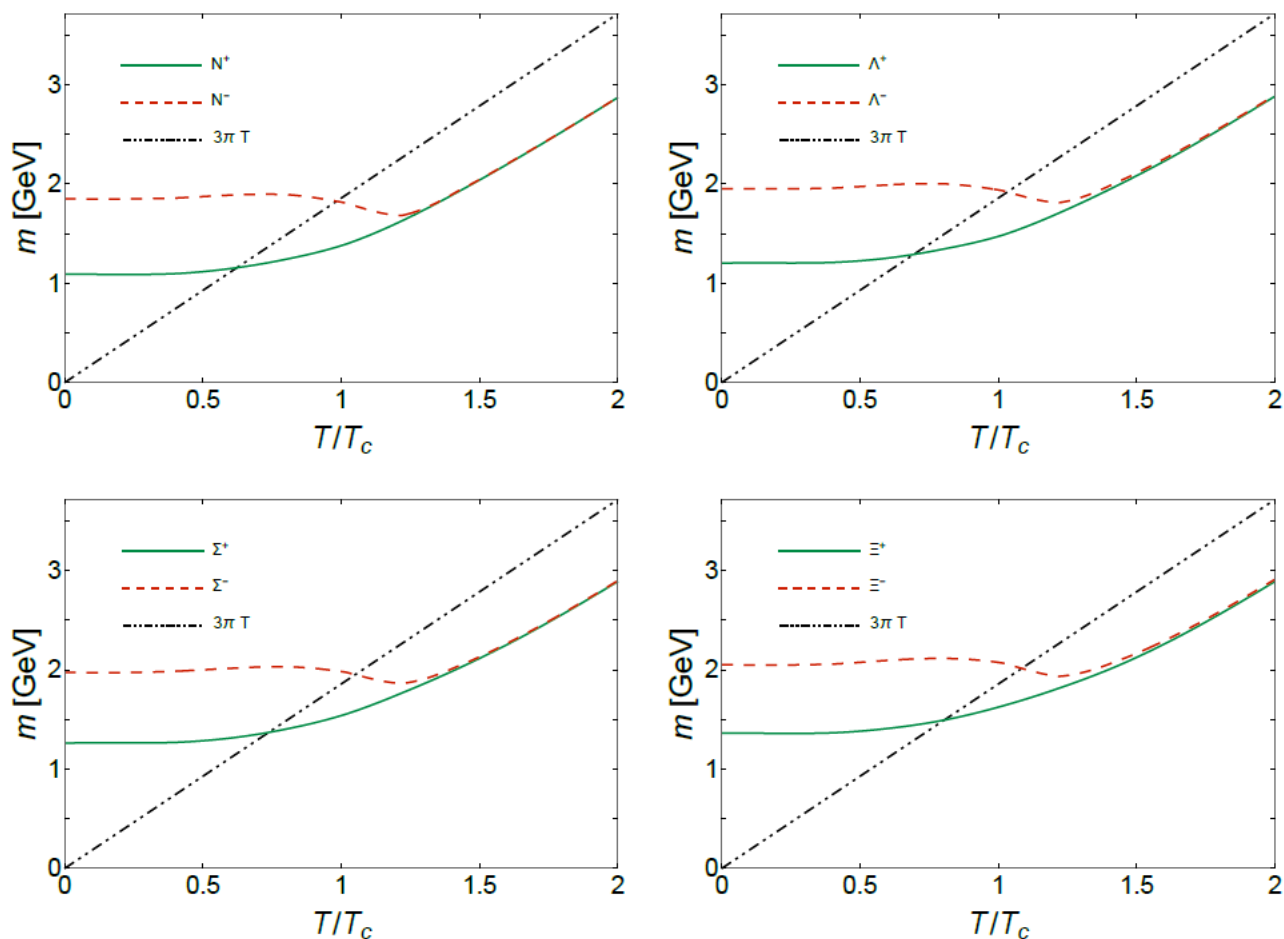
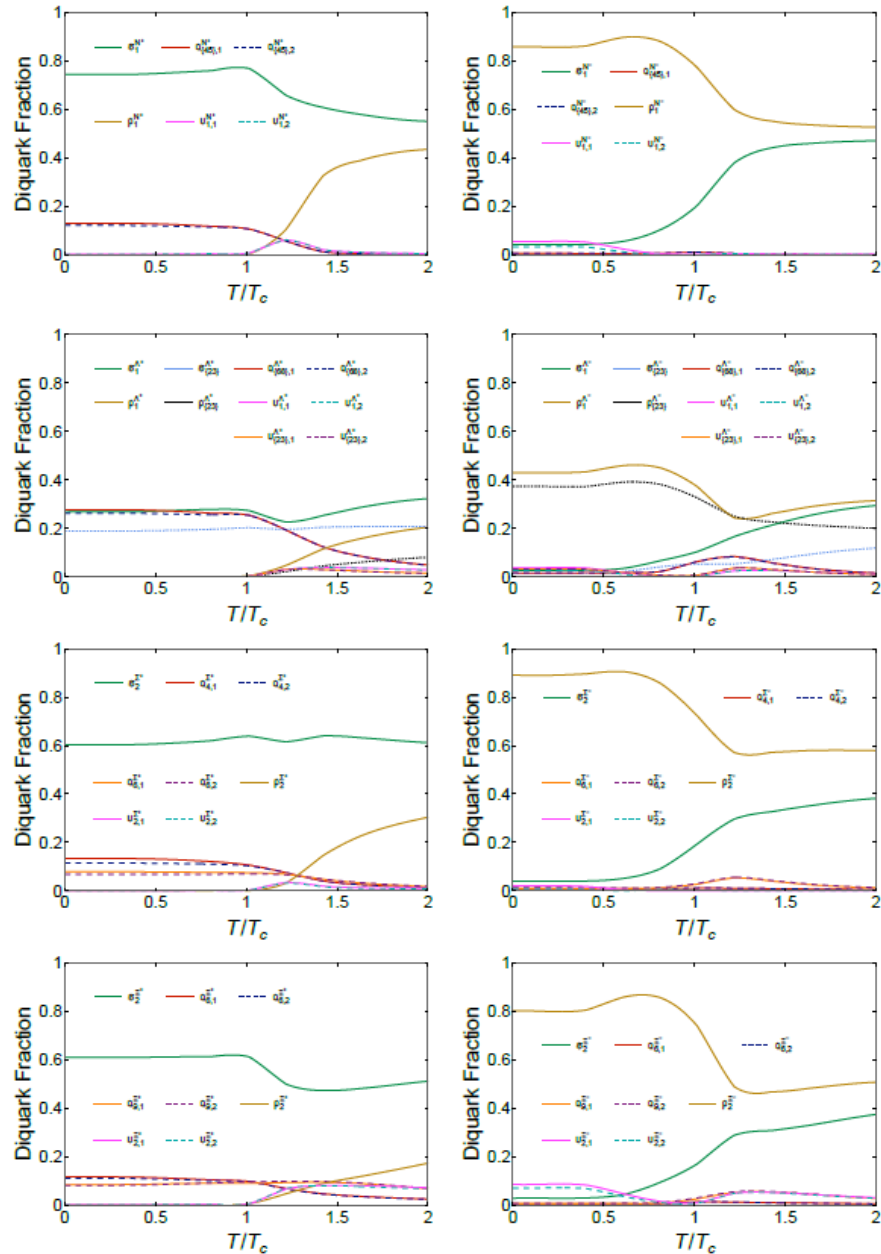


FIG. 6. Screening masses of $J^P = 1/2^\pm$ baryon ground states. In each figure: *solid green curve*: positive-parity baryon; *dashed red curve*: negative-parity baryon; and *black dot-dot-dashed curve*: free theory limit of $m = 3\pi T$.

Diquark fractions: $J^P = 1/2^\pm$ baryons



Di-quark fractions: $J^P = 1/2^\pm$ baryons

➤ Phys.Rev.D 87 (2013) 7, 074038

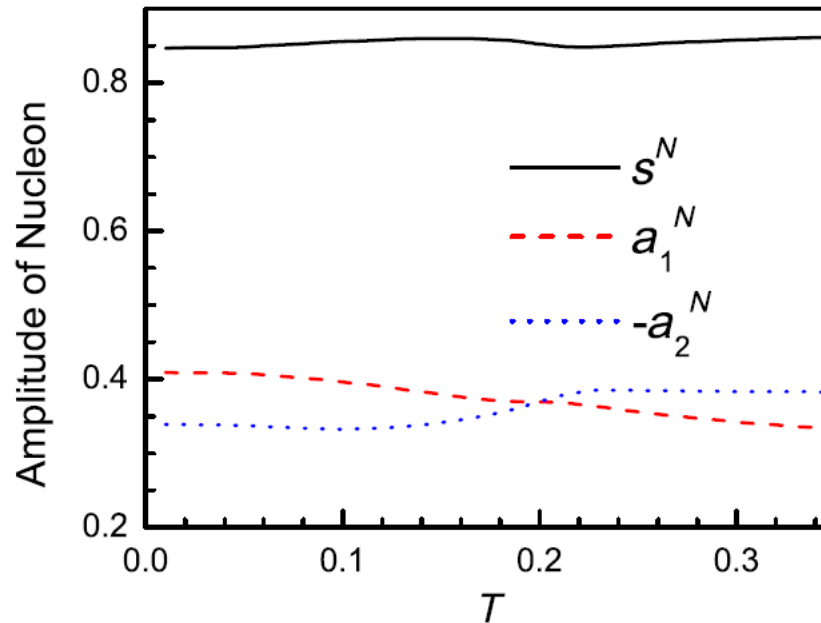


FIG. 11 (color online). Evolution of the nucleon's Faddeev amplitude with temperature: *solid curve*: scalar diquark component; and *dashed, dotted curves*: the two distinct axial-vector diquark structures. [See Eqs. (C1) and (C2).]

Screening masses: $J^P = 3/2^\pm$ baryons

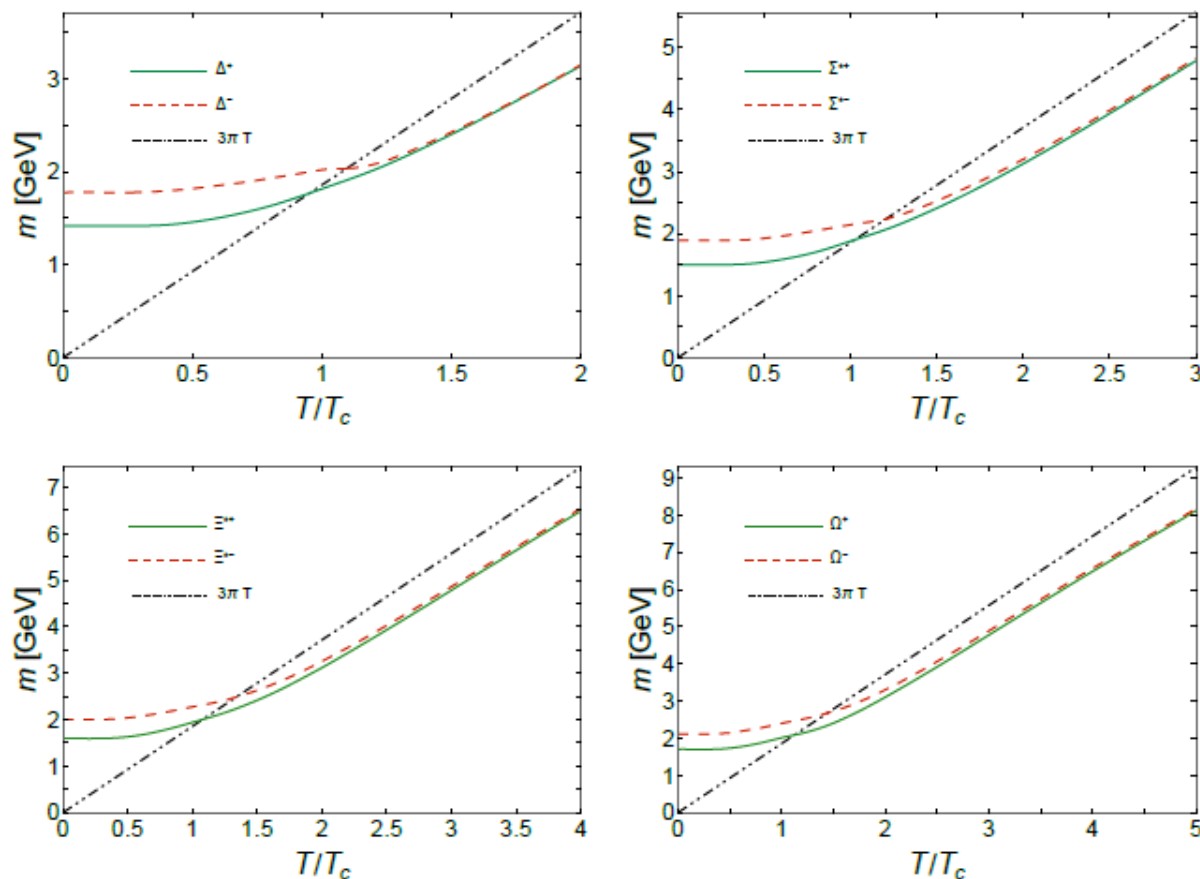
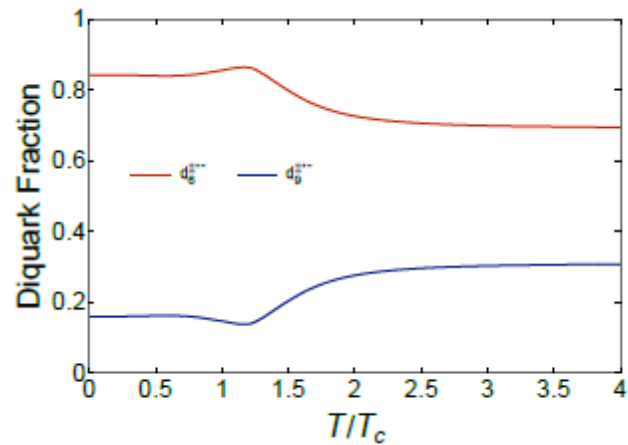
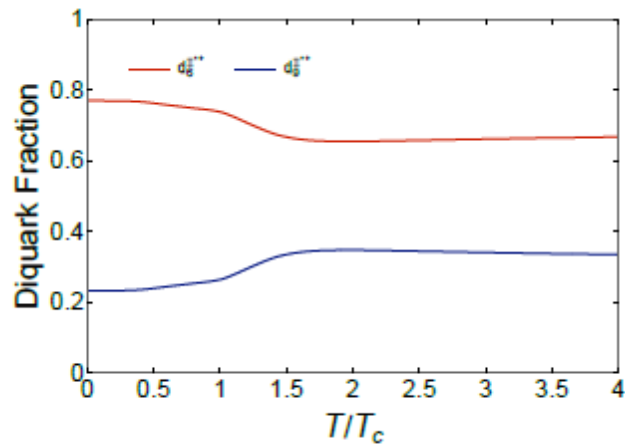
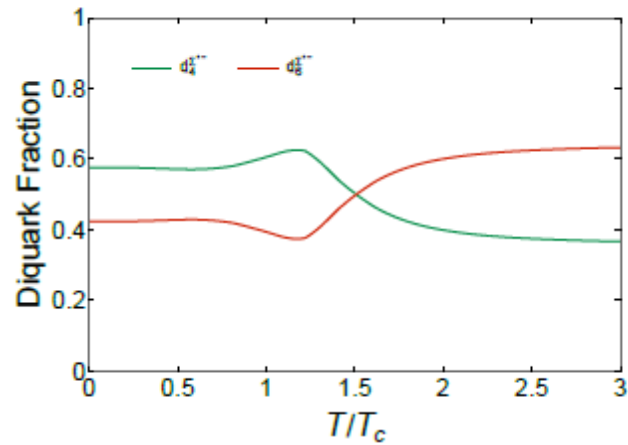
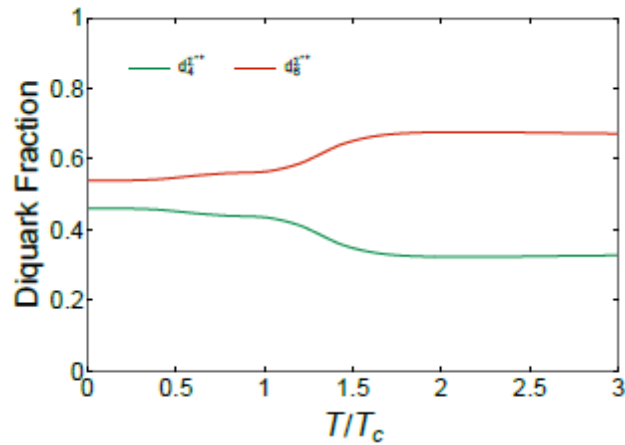


FIG. 8. Screening masses of $J^P = 3/2^\pm$ baryon ground states. In each figure: *solid green curve*: positive-parity baryon; *dashed red curve*: negative-parity baryon; and *black dot-dot-dashed curve*: free theory limit of $m = 3\pi T$.

Diquark fractions: $J^P = 3/2^\pm$ baryons



Summary & Perspective

- Nucleon Axial Form Factor at large momentum transfers
- $\Delta(1232)$ Axial Form Factors
- $N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Transition Form Factors
- Longer plan:

	质量谱		电磁形状因子		轴矢形状因子		引力形状因子	
	双夸克	三体	双夸克	三体	双夸克	三体	双夸克	三体
$N(940)1/2^+$	★★★★	★★★★	★★★★	★★★★	★★★	★		★
$\Delta(1232)3/2^+$	★★★★	★★★★	★★★	★★★	★			
$N(1440)1/2^+$	★★★★		★★★					
$N(1535)1/2^-$	★★★★							
$N(1520)3/2^-$	★★★★							
超子（正宇称）	★★★★	★★★★		★★★				
超子（负宇称）								

- Screening masses of positive- and negative-parity hadron ground-states, including those with strangeness

Summary & Perspective

- Nucleon Axial Form Factor at large momentum transfers
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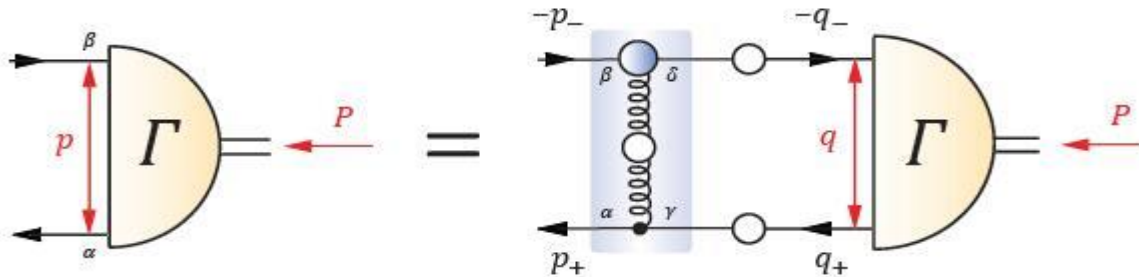
	质量谱		电磁形状因子		轴矢形状因子		引力形状因子	
	双夸克	三体	双夸克	三体	双夸克	三体	双夸克	三体
$N(940)1/2^+$	★★★★	★★★★	★★★★	★★★★	★★★	★		★
$\Delta(1232)3/2^+$	★★★★	★★★★	★★★	★★★	★			
$N(1440)1/2^+$	★★★★		★★★					
$N(1535)1/2^-$	★★★★							
$N(1520)3/2^-$	★★★★							
超子（正宇称）	★★★★	★★★★		★★★				
超子（负宇称）								

- Screening masses of positive- and negative-parity hadron ground-states, including those with strangeness

Thank you!

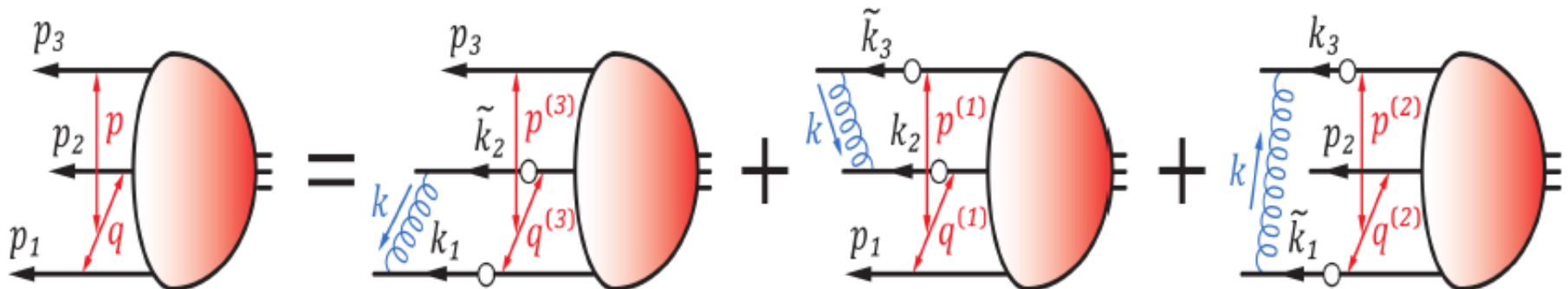
Hadrons: Bound-states in QFT

- **Mesons:** a 2-body bound state problem in QFT
 - Bethe-Salpeter Equation
 - **K - fully amputated, two-particle irreducible, quark-antiquark scattering kernel**

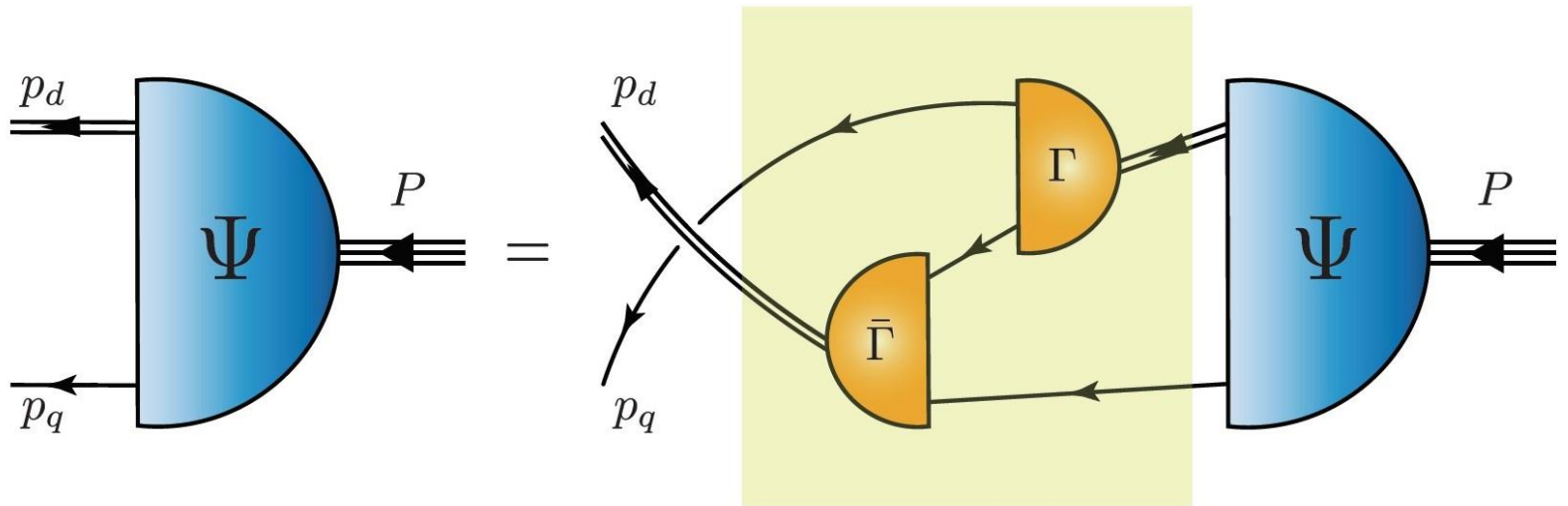


- **Baryons:** a 3-body bound state problem in QFT
- Faddeev equation: sums all possible quantum field theoretical exchanges and interactions that can take place between the three dressed-quarks that define its valence quark content.

Faddeev equation in rainbow-ladder truncation

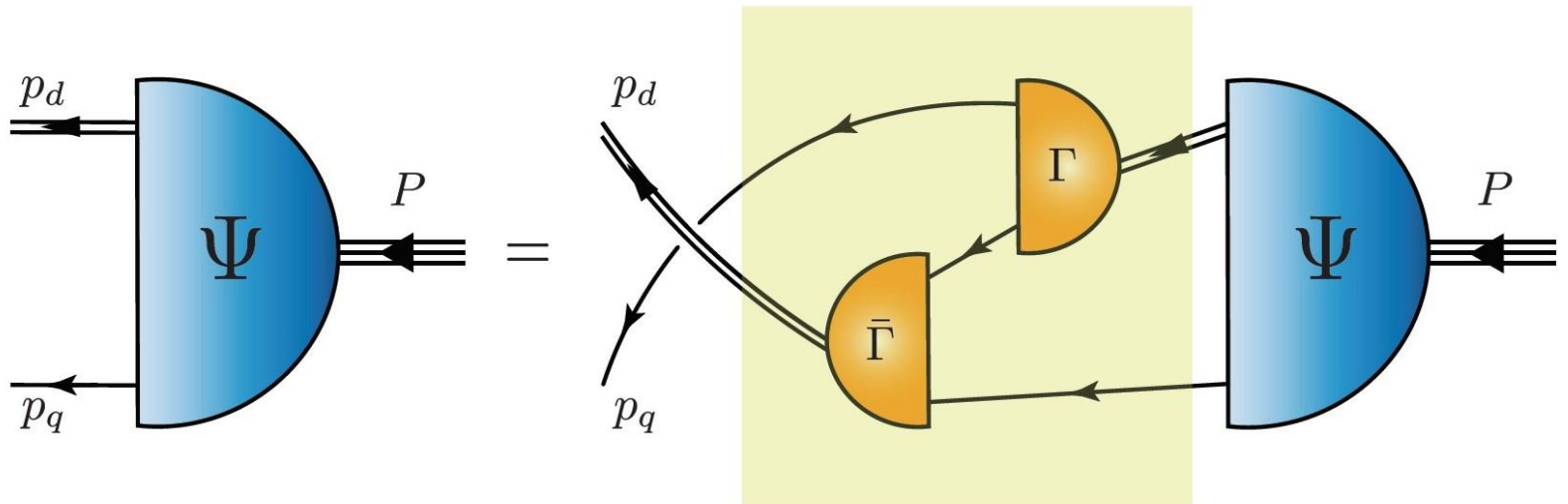


How to solve?



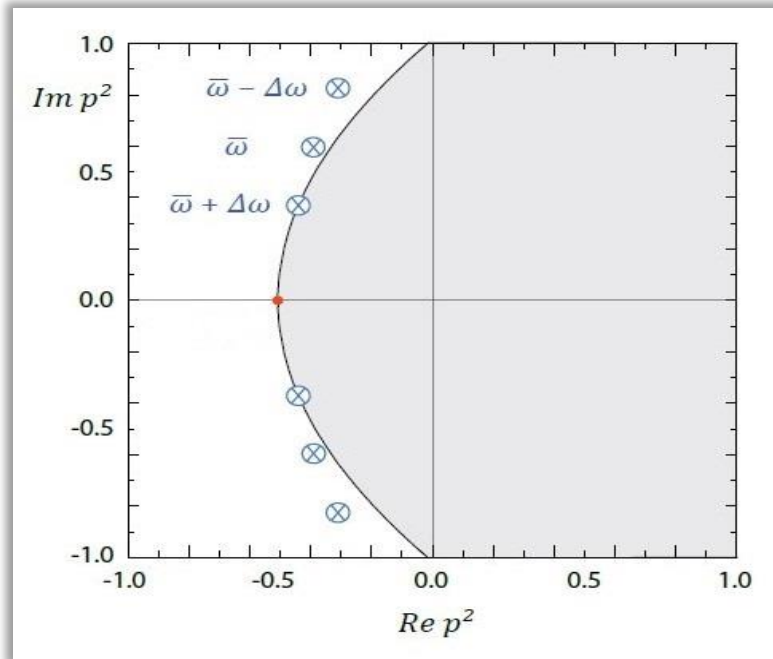
How to solve?

- ◆ The dressed-quark propagator
- ◆ Diquark amplitudes
- ◆ Diquark propagators
- ◆ **Faddeev amplitudes**

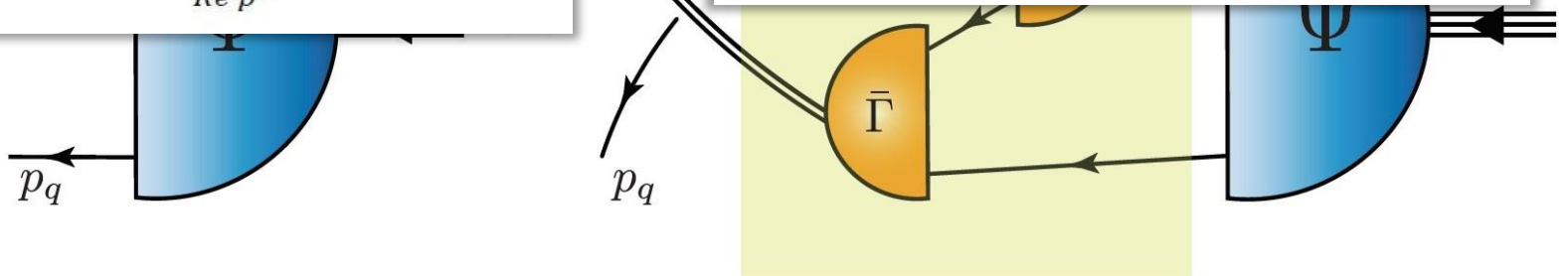
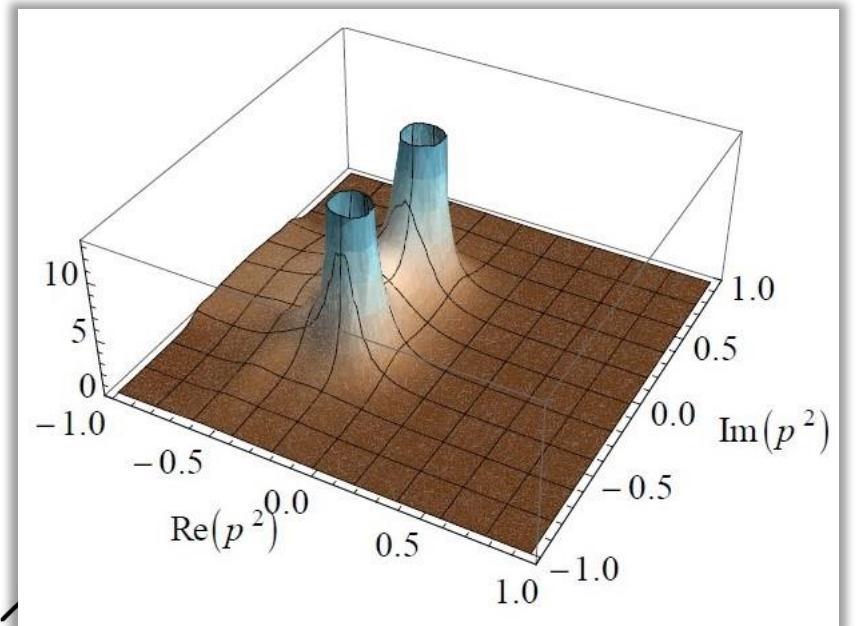


How to solve?

◆ The dressed-quark propagator

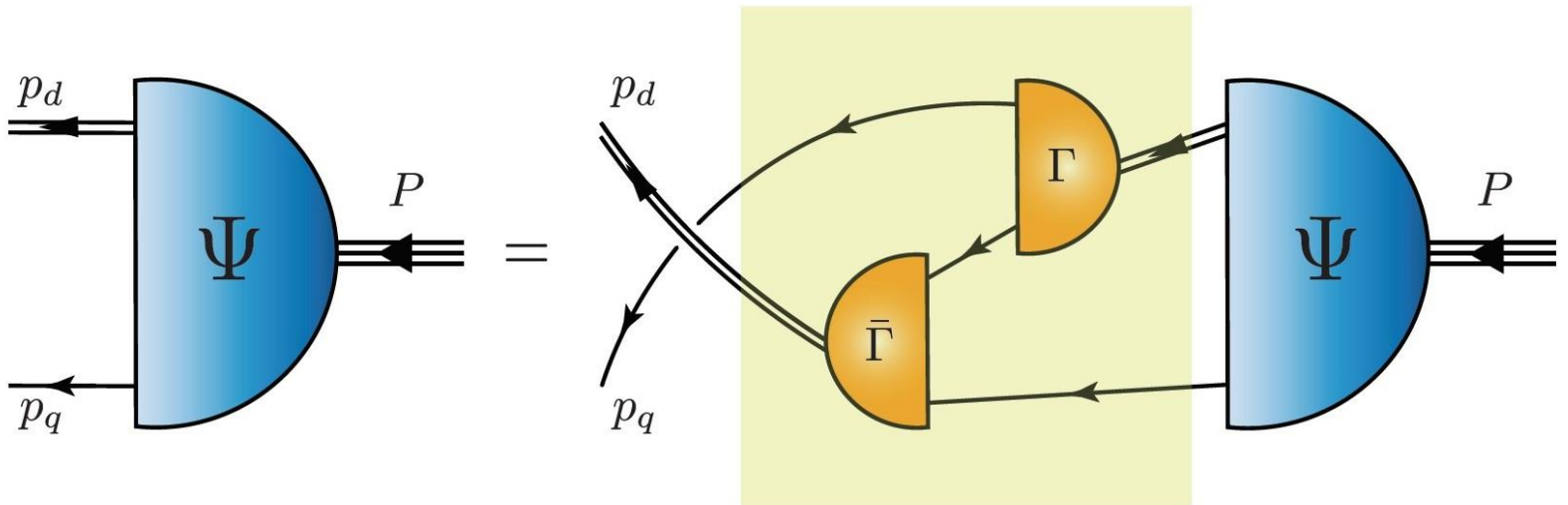


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QCD-kindred model

- ◆ The dressed-quark propagator
- ◆ Diquark amplitudes
- ◆ Diquark propagators
- ◆ **Faddeev amplitudes**



QCD-kindred model

➤ **Diquark masses (in GeV):**

$$m_{[ud]_{0+}} = 0.80 \text{ GeV}$$

$$m_{\{uu\}_{1+}} = m_{\{ud\}_{1+}} = m_{\{dd\}_{1+}} = 0.89 \text{ GeV}$$

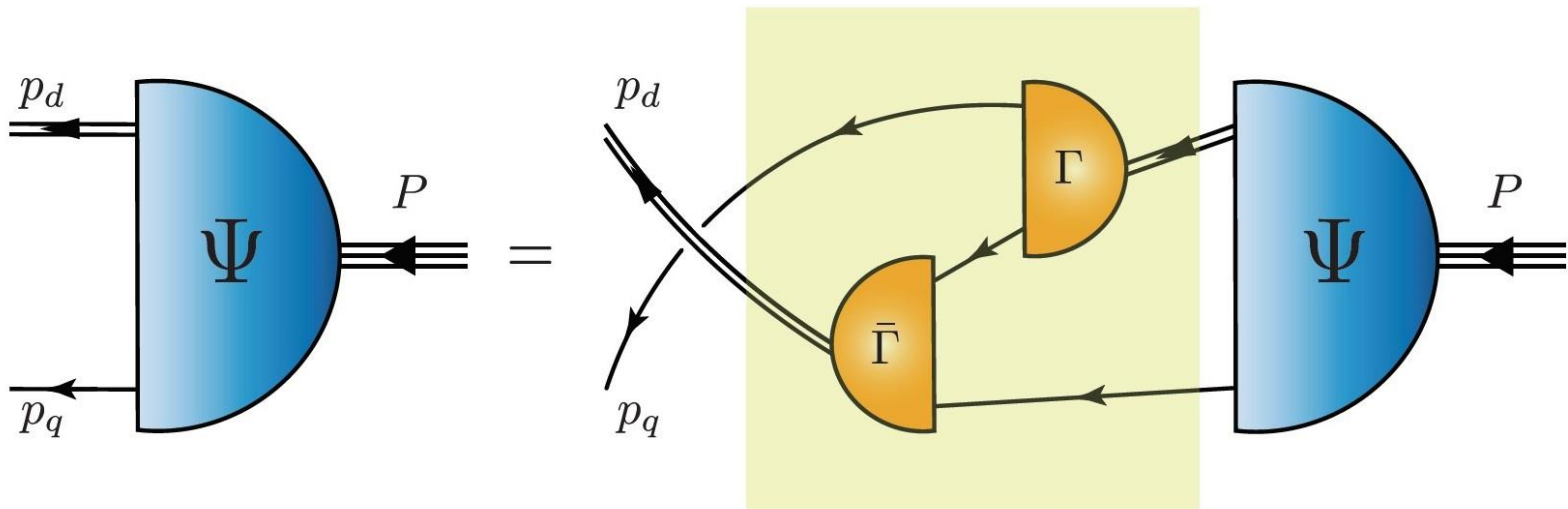
$$m_N = 1.18 \text{ GeV}$$

$$m_R = 1.72 \text{ GeV}$$

$$m_\Delta = 1.35 \text{ GeV}$$

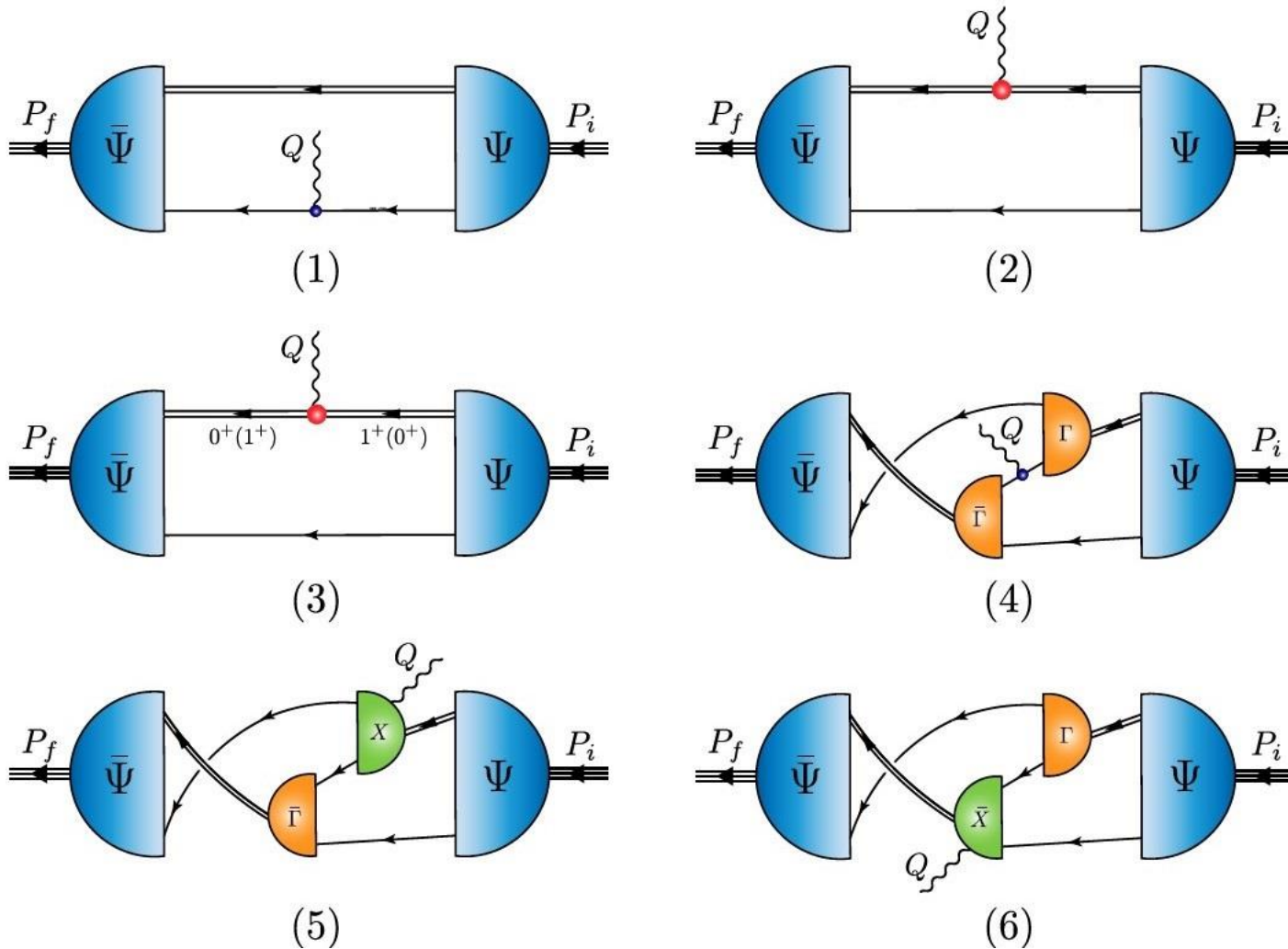


- **These two values provide for a good description of numerous dynamical properties of the nucleon, Δ -baryon and low-lying excitations, e.g., N(1440), N(1535), Δ (1600).**



How to compute Form Factors?

- In the quark-diquark framework, the associated symmetry-preserving current:



Proton Spin Structure

- Flavour separation of proton axial charge
- **d**-quark receives large contribution from probe+quark in presence of axialvector diquark

$$\frac{g_A^d}{g_A^u} = 0^+ \& 1^+ -0.32(2)$$

$$\frac{g_A^d}{g_A^u} = 0^+ \text{ only } -0.054(13)$$

- Experiment: **0.27(4)**
- Hadron scale: $g_A^u + g_A^d (+g_A^s = 0) = 0.65(2) \Rightarrow$ quarks carry **65%** of the proton spin
- Poincaré-covariant proton wave function: remaining **35%** lodged with quark+diquark orbital angular momentum



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Perspective on polarised parton distribution functions and proton spin

P. Cheng (程鹏)^{a,b}, Y. Yu (俞杨)^{a,b}, H.-Y. Xing (邢惠瑜)^{a,b}, C. Chen (陈晨)^{c,d,*},
Z.-F. Cui (崔著钊)^{a,b,*}, C.D. Roberts^{a,b,*}



Large Q^2 Nucleon Axial Form Factor

➤ Light-front transverse density profiles

➤ Scalar diquark only:

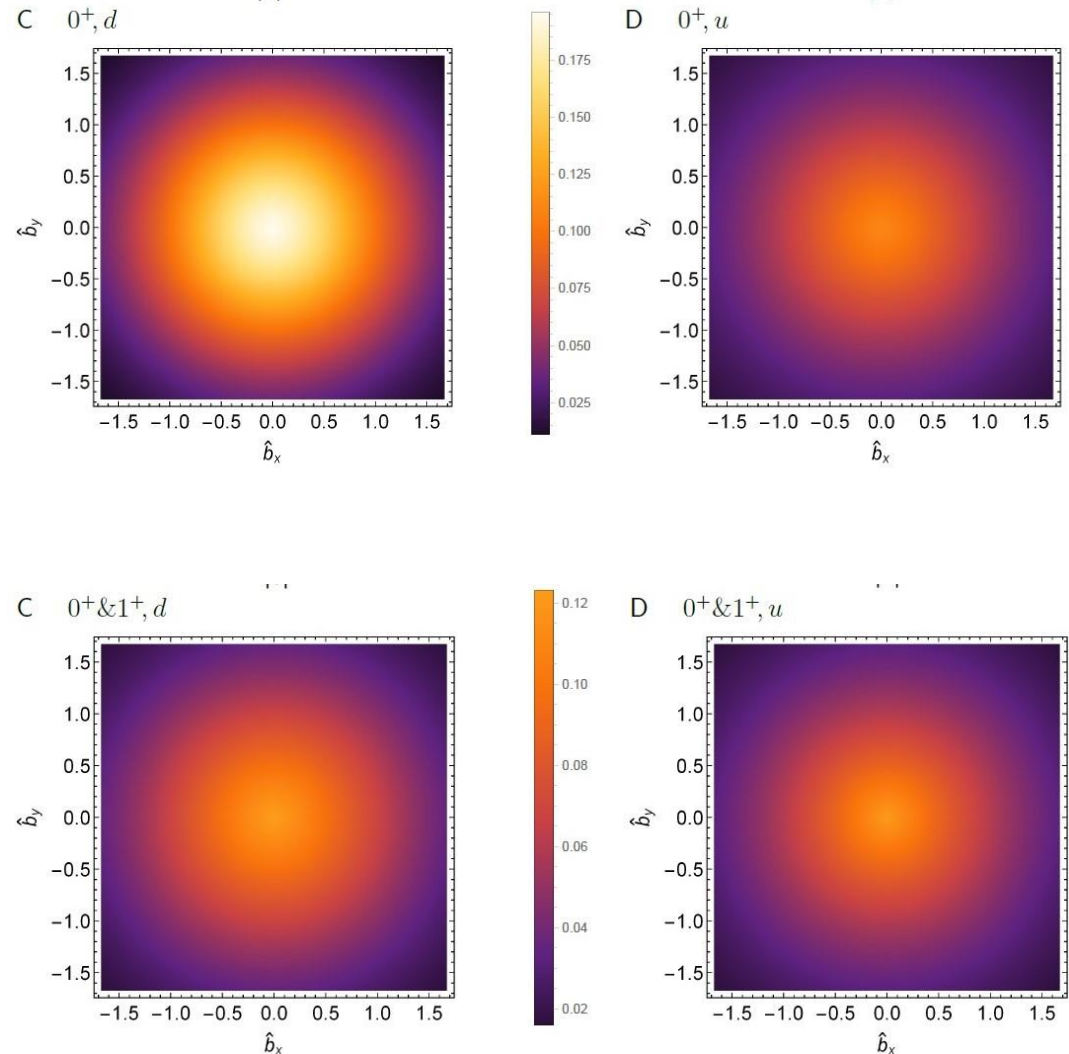
- magnitude of the d quark contribution to GA is just 10% of that from the u quark
- d quark is also much more localized

$$r_{A_d}^\perp \approx 0.5 r_{A_u}^\perp$$

➤ Scalar + axial-vector diquarks:

- d and u quark transverse profiles are quite similar

$$r_{A_d}^\perp \approx 0.9 r_{A_u}^\perp$$



The axial current – G_A & G_P

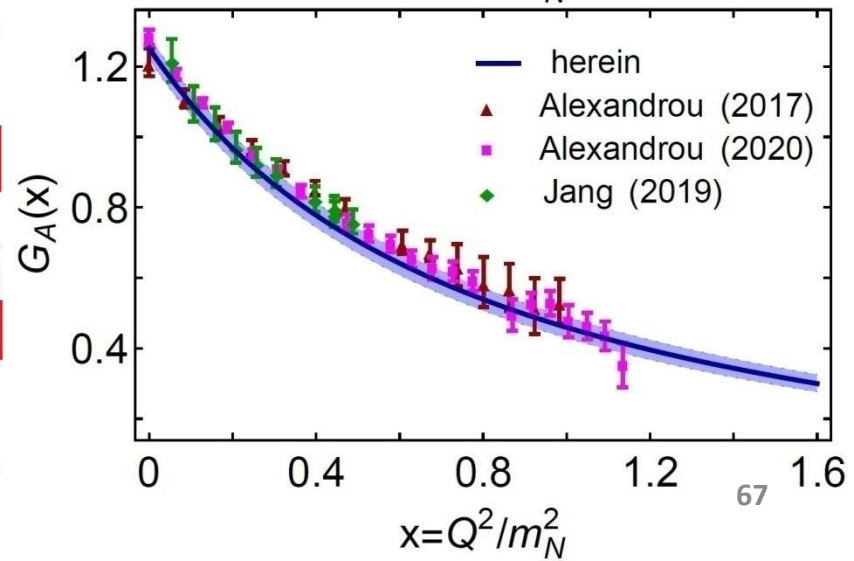
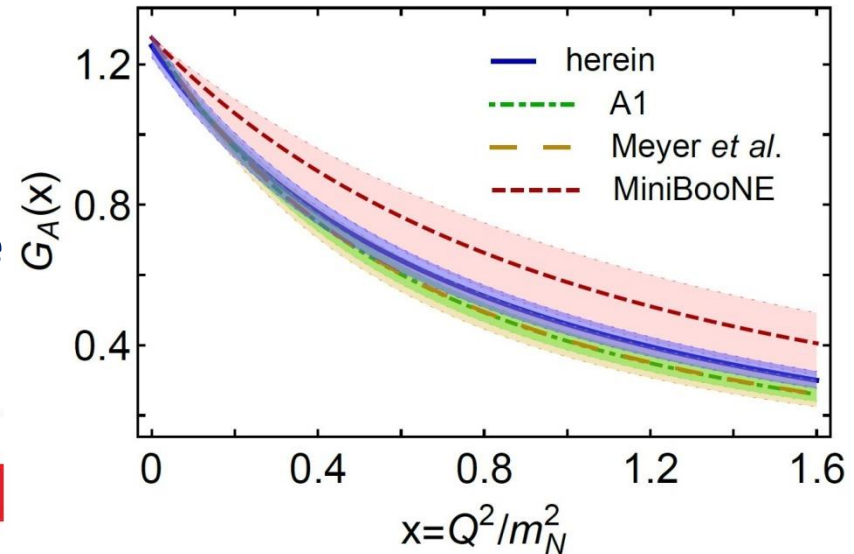
$$J_{5\mu}^j(K, Q) = \bar{u}(P_f) \frac{\tau^j}{2} \gamma_5 \left[\gamma_\mu G_A(Q^2) + i \frac{Q_\mu}{2m_N} G_P(Q^2) \right] u(P_i)$$

➤ Two form factors:

- G_A – axial form factor
- G_P – induced pseudoscalar form factor

➤ G_A can reliably be represented by dipole characterised by mass-scale m_A

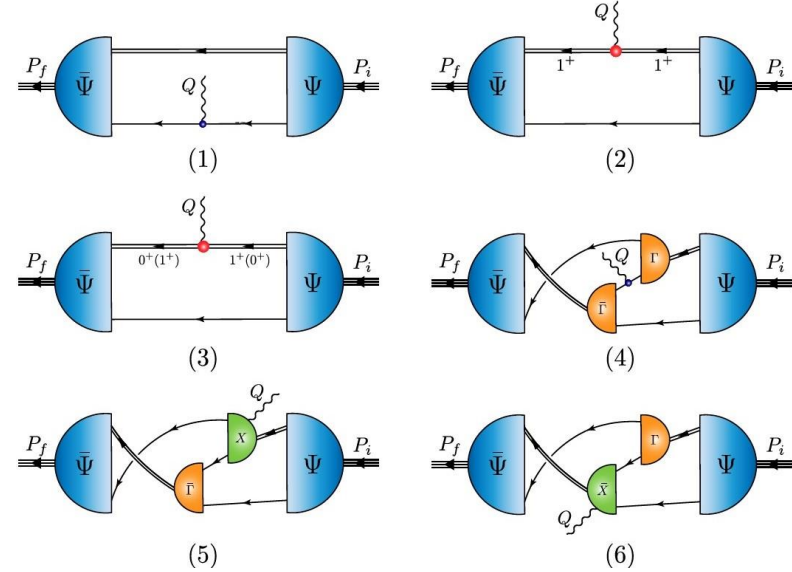
	g_A	$m_N \langle r_A^2 \rangle^{1/2}$	m_A/m_N
Herein	1.25(03)	3.25(04)	1.23(03)
Faddeev ₃ [31]	0.99(02)	2.63(06)	1.32(03)
Exp [4]	1.2756(13)	–	–
Exp [13]	–	3.02(11)	1.15(04)
Exp [14]	–	3.23(72)	1.15(08)
Exp [17]	–	2.41(31)	1.44(18)
IQCD [57]	1.21(3)(2)	2.45(08)(03)	1.41(04)(02)
IQCD [58]	1.30(6)	3.57(30)	0.97(16)
IQCD _d [59]	1.23(3)	2.48(15)	1.39(09)
IQCD _z [59]	1.30(9)	3.19(30)	1.09(11)



Fractions of $G_A(0)$, $G_P(0)$ and $G_5(0)$

TABLE I. Referring to Fig. 3, separation of $G_A(0)$, $G_P(0)$ and $G_5(0)$ into contributions from various diagrams, listed as a fraction of the total $Q^2 = 0$ value. Diagram (1): $\langle J \rangle_q^S$ – weak-boson strikes dressed-quark with scalar diquark spectator; and $\langle J \rangle_q^A$ – weak-boson strikes dressed-quark with axial-vector diquark spectator. Diagram (2): $\langle J \rangle_{qq}^{AA}$ – weak-boson interacts strikes axial-vector diquark with dressed-quark spectator. Diagram (3): $\langle J \rangle_{dq}^{SA+AS}$ – weak-boson mediates transition between scalar and axial-vector diquarks, with dressed-quark spectator. Diagram (4): $\langle J \rangle_{\text{ex}}$ – weak-boson strikes dressed-quark “in-flight” between one diquark correlation and another. Diagrams (5) and (6): $\langle J \rangle_{\text{sg}}$ – weak-boson couples inside the diquark correlation amplitude. The listed uncertainty in these results reflects the impact of $\pm 5\%$ variations in the diquark masses in Eq. (16), *e.g.* $0.71_{1\mp} \Rightarrow 0.71 \mp 0.01$.

	$\langle J \rangle_q^S$	$\langle J \rangle_q^A$	$\langle J \rangle_{qq}^{AA}$	$\langle J \rangle_{dq}^{SA+AS}$	$\langle J \rangle_{\text{ex}}$	$\langle J \rangle_{\text{sg}}$
$G_A(0)$	$0.71_{4\mp}$	$0.064_{2\pm}$	$0.025_{5\pm}$	$0.13_{0\mp}$	$0.072_{32\pm}$	0
$G_P(0)$	$0.74_{4\mp}$	$0.070_{5\pm}$	$0.025_{5\pm}$	$0.13_{0\mp}$	$0.22_{4\pm}$	$-0.19_{1\mp}$
$G_5(0)$	$0.74_{4\mp}$	$0.069_{5\pm}$	$0.025_{5\pm}$	$0.13_{0\mp}$	$0.22_{4\pm}$	$-0.19_{1\mp}$



➤ Projections:

$$G_A = -\frac{1}{4(1+\tau)} \text{tr}_D [J_{5\mu} \gamma_5 \gamma_\mu^T],$$

$$G_P = \frac{1}{\tau} \left(G_A - \frac{Q_\mu}{4im_N \tau} \text{tr}_D [J_{5\mu} \gamma_5] \right),$$

$$G_5 = \frac{1}{2\tau} \text{tr}_D [J_5 \gamma_5],$$

➤ $G_P(0) \sim G_5(0)$

$$G_P \sim \frac{Q_\mu}{\tau^2} \text{tr}_D [J_{5\mu} \gamma_5] \sim \frac{1}{\tau} \text{tr}_D [J_5 \gamma_5] \sim G_5,$$

when $Q^2 \sim 0 \text{ GeV}^2$.

QCD-kindred model

➤ The dressed-quark propagator

$$S(p) = -i\gamma \cdot p \sigma_V(p^2) + \sigma_S(p^2)$$

➤ algebraic form:

$$\begin{aligned} \bar{\sigma}_S(x) &= 2\bar{m}\mathcal{F}(2(x + \bar{m}^2)) \\ &\quad + \mathcal{F}(b_1x)\mathcal{F}(b_3x)[b_0 + b_2\mathcal{F}(\epsilon x)], \end{aligned} \quad (\text{A3a})$$

$$\bar{\sigma}_V(x) = \frac{1}{x + \bar{m}^2} [1 - \mathcal{F}(2(x + \bar{m}^2))], \quad (\text{A3b})$$

with $x = p^2/\lambda^2$, $\bar{m} = m/\lambda$,

$$\mathcal{F}(x) = \frac{1 - e^{-x}}{x}, \quad (\text{A4})$$

$\bar{\sigma}_S(x) = \lambda\sigma_S(p^2)$ and $\bar{\sigma}_V(x) = \lambda^2\sigma_V(p^2)$. The mass scale, $\lambda = 0.566$ GeV, and parameter values,

$$\begin{array}{ccccc} \bar{m} & b_0 & b_1 & b_2 & b_3 \\ \hline 0.00897 & 0.131 & 2.90 & 0.603 & 0.185 \end{array}, \quad (\text{A5})$$

associated with Eq. (A3) were fixed in a least-squares fit to light-meson observables [79,80]. [$\epsilon = 10^{-4}$ in Eq. (A3a) acts only to decouple the large- and intermediate- p^2 domains.]

QCD-kindred model

➤ The dressed-quark propagator

$$S(p) = -i\gamma \cdot p \sigma_V(p^2) + \sigma_S(p^2)$$

- Based on solutions to the gap equation that were obtained with a dressed gluon-quark vertex.
- Mass function has a real-world value at $p^2 = 0$, NOT the highly inflated value typical of **RL** truncation.
- Propagators are entire functions, consistent with sufficient condition for confinement and completely unlike known results from **RL** truncation.
- Parameters in quark propagators were fitted to a diverse array of meson observables. **ZERO** parameters changed in study of baryons.
- Compare with that computed using the DCSB-improved gap equation kernel (DB).
The parametrization is a sound representation of numerical results, although simple and introduced long beforehand.

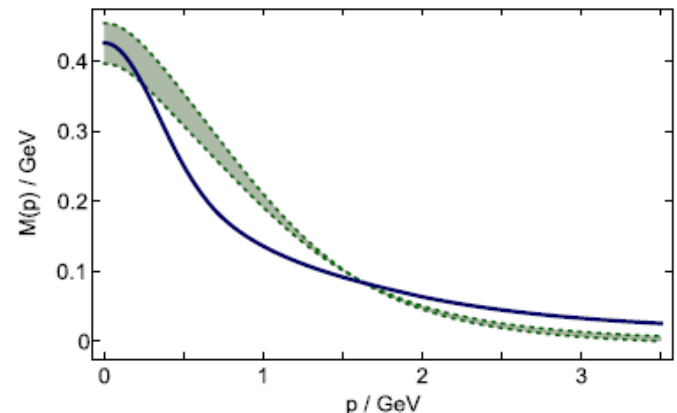


FIG. 6. Solid curve (blue)—quark mass function generated by the parametrization of the dressed-quark propagator specified by Eqs. (A3) and (A4) (A5); and band (green)—exemplary range of numerical results obtained by solving the gap equation with the modern DCSB-improved kernels described and⁷⁰ used in Refs. [16,81–83].

QCD-kindred model

- **Diquark amplitudes:** five types of correlation are possible in a $J=1/2$ bound state: isoscalar scalar($I=0, J^P=0^+$), isovector pseudovector, isoscalar pseudoscalar, isoscalar vector, and isovector vector.
- The **LEADING** structures in the correlation amplitudes for each case are, respectively (Dirac-flavor-color),

$$\Gamma^{0+}(k; K) = g_{0+} \gamma_5 C \tau^2 \vec{H} \mathcal{F}(k^2 / \omega_{0+}^2),$$

$$\vec{\Gamma}_{\mu}^{1+}(k; K) = i g_{1+} \gamma_{\mu} C \vec{\tau} \vec{H} \mathcal{F}(k^2 / \omega_{1+}^2),$$

$$\Gamma^{0-}(k; K) = i g_{0-} C \tau^2 \vec{H} \mathcal{F}(k^2 / \omega_{0-}^2),$$

$$\Gamma_{\mu}^{1-}(k; K) = g_{1-} \gamma_{\mu} \gamma_5 C \tau^2 \vec{H} \mathcal{F}(k^2 / \omega_{1-}^2),$$

$$\vec{\Gamma}_{\mu}^{1-}(k; K) = i g_{1-} [\gamma_{\mu}, \gamma \cdot K] \gamma_5 C \vec{\tau} \vec{H} \mathcal{F}(k^2 / \omega_{1-}^2),$$

- **Simple form. Just one parameter: diquark masses.**
- **Match expectations based on solutions of meson and diquark Bethe-Salpeter amplitudes.**

➤ The diquark propagators

$$\Delta^{0\pm}(K) = \frac{1}{m_{0\pm}^2} \mathcal{F}(k^2/\omega_{0\pm}^2),$$

$$\Delta_{\mu\nu}^{1\pm}(K) = \left[\delta_{\mu\nu} + \frac{K_\mu K_\nu}{m_{1\pm}^2} \right] \frac{1}{m_{1\pm}^2} \mathcal{F}(k^2/\omega_{1\pm}^2).$$

- The ***F-functions***: Simplest possible form that is consistent with infrared and ultraviolet constraints of confinement (IR) and **1/q²** evolution (UV) of meson propagators.
- Diquarks are **confined**.
- free-particle-like at spacelike momenta
 - pole-free on the timelike axis
 - This is **NOT** true of **RL** studies. It enables us to reach arbitrarily high values of momentum transfer.

➤ The Faddeev amplitudes:

$$\begin{aligned}
 \psi^\pm(p_i, \alpha_i, \sigma_i) = & [\Gamma^{0+}(k; K)]_{\sigma_1 \sigma_2}^{\alpha_1 \alpha_2} \Delta^{0+}(K) [\varphi_{0+}^\pm(\ell; P) u(P)]_{\sigma_3}^{\alpha_3} \\
 & + [\Gamma_\mu^{1+j}] \Delta_{\mu\nu}^{1+} [\varphi_{1+\nu}^{j\pm}(\ell; P) u(P)] \\
 & + [\Gamma^{0-}] \Delta^{0-} [\varphi_{0-}^\pm(\ell; P) u(P)] \\
 & + [\Gamma_\mu^{1-}] \Delta_{\mu\nu}^{1-} [\varphi_{1-\nu}^\pm(\ell; P) u(P)], \quad (9)
 \end{aligned}$$

➤ Quark-diquark vertices:

$$\varphi_{0+}^\pm(\ell; P) = \sum_{i=1}^2 \mathcal{S}_i^\pm(\ell^2, \ell \cdot P) S^i(\ell; P) \mathcal{G}^\pm,$$

$$\varphi_{1+\nu}^{j\pm}(\ell; P) = \sum_{i=1}^6 \omega_i^{j\pm}(\ell^2, \ell \cdot P) \gamma_5 \mathcal{A}_\nu^i(\ell; P) \mathcal{G}^\pm,$$

$$\varphi_{0-}^\pm(\ell; P) = \sum_{i=1}^2 \mathcal{P}_i^\pm(\ell^2, \ell \cdot P) S^i(\ell; P) \mathcal{G}^\mp,$$

$$\varphi_{1-\nu}^\pm(\ell; P) = \sum_{i=1}^6 v_i^\pm(\ell^2, \ell \cdot P) \gamma_5 \mathcal{A}_\nu^i(\ell; P) \mathcal{G}^\mp,$$

where $\mathcal{G}^{+(-)} = \mathbf{I}_D(\gamma_5)$ and

$$S^1 = \mathbf{I}_D, \quad S^2 = i\gamma \cdot \hat{\ell} - \hat{\ell} \cdot \hat{P} \mathbf{I}_D$$

$$\mathcal{A}_\nu^1 = \gamma \cdot \ell^\perp \hat{P}_\nu, \quad \mathcal{A}_\nu^2 = -i\hat{P}_\nu \mathbf{I}_D, \quad \mathcal{A}_\nu^3 = \gamma \cdot \hat{\ell}^\perp \hat{\ell}_\nu^\perp$$

$$\mathcal{A}_\nu^4 = i\hat{\ell}_\nu^\perp \mathbf{I}_D, \quad \mathcal{A}_\nu^5 = \gamma_\nu^\perp - \mathcal{A}_\nu^3, \quad \mathcal{A}_\nu^6 = i\gamma_\nu^\perp \gamma \cdot \hat{\ell}^\perp - \mathcal{A}_\nu^4,$$

QCD-kindred model

- Both the Faddeev amplitude and wave function are Poincare covariant, i.e. they are qualitatively identical in all reference frames.
- Each of the scalar functions that appears is frame independent, but the frame chosen determines just how the elements should be combined.
- In consequence, the manner by which the dressed quarks' spin, S , and orbital angular momentum, L , add to form the total momentum J , is **frame dependent**: L , S are not independently Poincare invariant.
- The set of baryon **rest-frame** quark-diquark angular momentum identifications:

$$^2S: S^1, \mathcal{A}_v^2, (\mathcal{A}_v^3 + \mathcal{A}_v^5),$$

$$^2P: S^2, \mathcal{A}_v^1, (\mathcal{A}_v^4 + \mathcal{A}_v^6),$$

$$^4P: (2\mathcal{A}_v^4 - \mathcal{A}_v^6)/3,$$

$$^4D: (2\mathcal{A}_v^3 - \mathcal{A}_v^5)/3,$$

- The scalar functions associated with these combinations of Dirac matrices in a Faddeev wave function possess the identified angular momentum correlation between the quark and diquark.