

# Heavy quark probes of the QCD matter: a theoretical perspective

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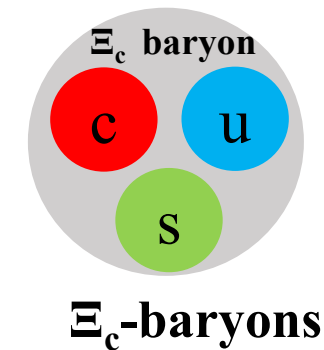
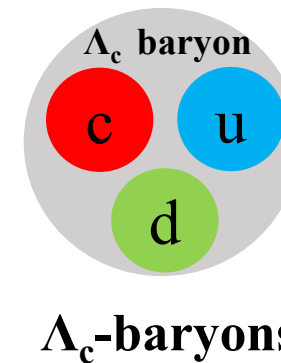
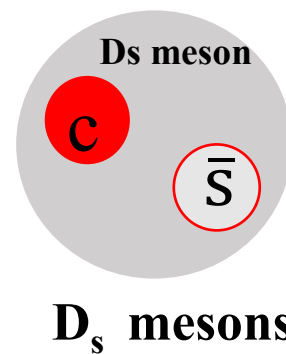
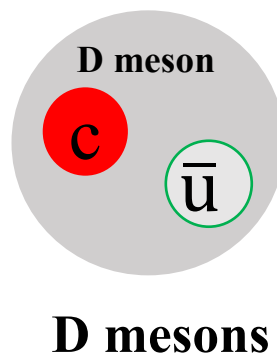
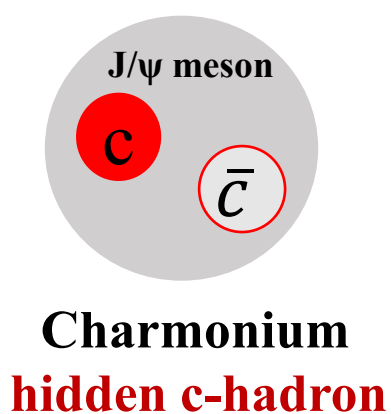


# Heavy quarks & heavy hadrons

- Heavy quark  $m_c \sim 1.5$ ,  $m_b \sim 4.5$  GeV  $\gg \Lambda_{\text{QCD}}$   
 $\rightarrow$  produced in early hard process  $\tau \sim 1/2m_Q$

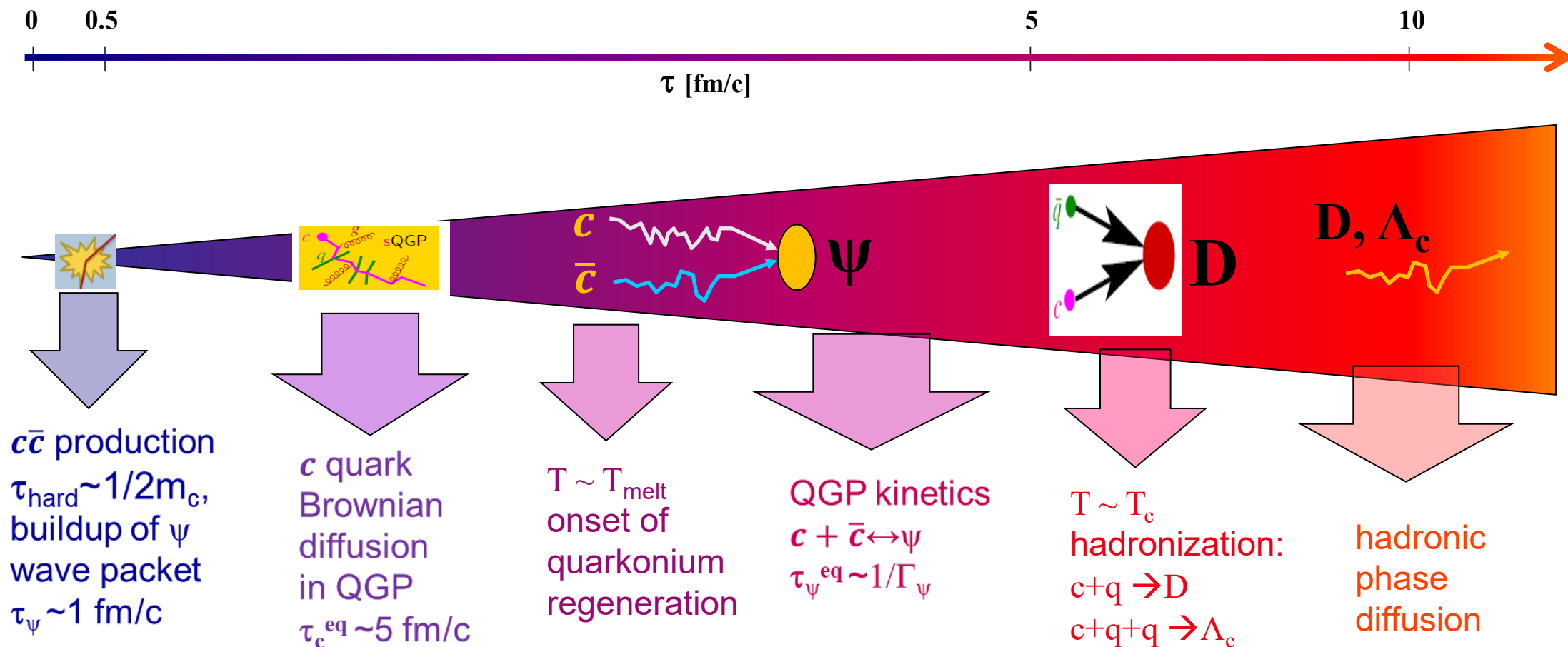
|                  |          |          |           |          |
|------------------|----------|----------|-----------|----------|
| 质量 $\rightarrow$ | 2.4 MeV  | 1.27 GeV | 171.2 GeV | 0        |
| 电荷 $\rightarrow$ | $2/3$    | $2/3$    | $2/3$     | 0        |
| 自旋 $\rightarrow$ | $1/2$    | $1/2$    | $1/2$     | 1        |
| 名字 $\rightarrow$ | 上夸克      | 粲夸克      | 顶夸克       | 光子       |
|                  | <b>u</b> | <b>c</b> | <b>t</b>  | $\gamma$ |
|                  |          |          |           |          |
|                  | 4.8 MeV  | 104 MeV  | 4.2 GeV   | 0        |
|                  | $-1/3$   | $-1/3$   | $-1/3$    | 0        |
|                  | $1/2$    | $1/2$    | $1/2$     | 1        |
| 夸克               | 下夸克      | 奇夸克      | 底夸克       | 胶子       |
|                  | <b>d</b> | <b>s</b> | <b>b</b>  | <b>g</b> |
|                  |          |          |           |          |

- Hadronization  $\rightarrow$  charmonium vs open charm hadrons



open c-hadrons

# Heavy flavor transport as probes of QGP



- $m_Q \gg T \rightarrow$  number conserved through diffusion/hadronization: **tagged & traceable probes**
- $\tau_Q^{\text{eq}} \geq \tau_{\text{QGP}} \rightarrow$  carrying a memory of interaction history: **quantitative gauge of coupling strength**

# Outline of the presentation

## Part I: Open heavy-hadron production in pp

- Heavy flavor hadro-chemistry: non-universal
- Statistical hadronization: grand-canonical  $\rightarrow$  canonical

## Part II: Open heavy-flavor transport in Pb-Pb

- Interaction of HF with medium: T-matrix approach
- Diffusion & hadronization
- Collective flow &  $p_T$ -dependent modifications of hadro-chemistry

## Part III: Heavy quarkonium transport in Pb-Pb

- Semi-classical transport of heavy quarkonia in QGP
- LO vs NLO reaction rate
- $J/\psi$  &  $\psi(2S)$  collective flow

*Disclaimer: selection of topics constrained by my knowledge  
focused on low & intermediate  $p_T$ , no high  $p_T$  HF jets*





## Part I: Open heavy-hadron production in pp

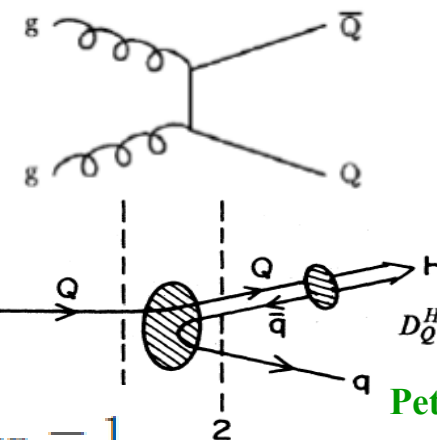
- Heavy flavor **hadro-chemistry**: non-universal
- **Statistical hadronization**: grand-canonical  $\rightarrow$  canonical
- Providing **baseline** heavy-hadron  $p_T$ -spectra for Pb-Pb



# Heavy quark hadronization in pp collisions

- Heavy hadron production cross section: **factorization**

$$\frac{d\sigma^{H_c}}{dp_T^{H_c}}(p_T; \mu_F, \mu_R) = \underbrace{\text{PDF}(x_1, \mu_F) \cdot \text{PDF}(x_2, \mu_F)}_{\text{Parton distribution functions (PDFs)}} \otimes \underbrace{\frac{d\sigma^c}{dp_T^c}(x_1, x_2, \mu_R, \mu_F)}_{\text{Hard scattering cross section (pQCD)}} \otimes \underbrace{D_{c \rightarrow H_c}(z = p_{H_c}/p_c, \mu_F)}_{\text{Fragmentation function (hadronization)}}$$

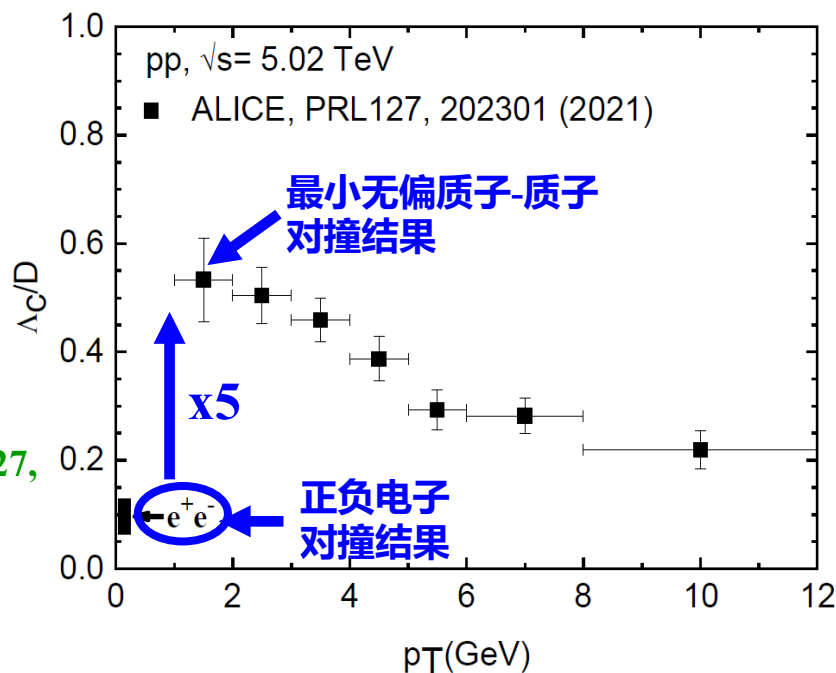


$$D_Q^H(z) = \frac{N}{z[1 - (1/z) - \epsilon_Q/(1-z)]^2}$$

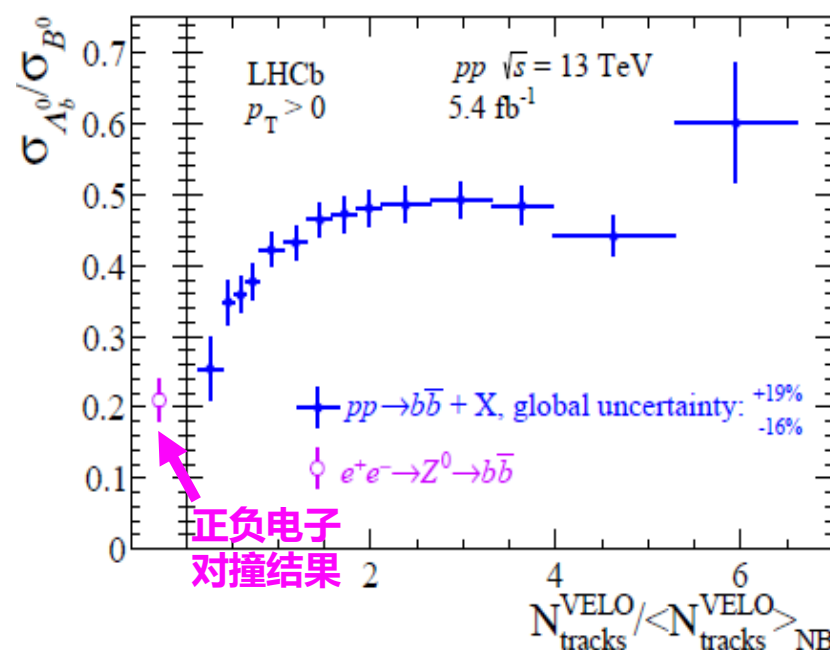
Peterson, PRD27, 105 (1983)

- Hadronization: heavy quark conservation  $f_u + f_d + f_s + f_{baryon} = 1$   
 - hadro-chemistry: universal?  $\rightarrow$  non-universal!

ALICE, PRL127, 202301 (2021)



$\Lambda_b/B$  vs 质子-质子对撞系统大小扫描

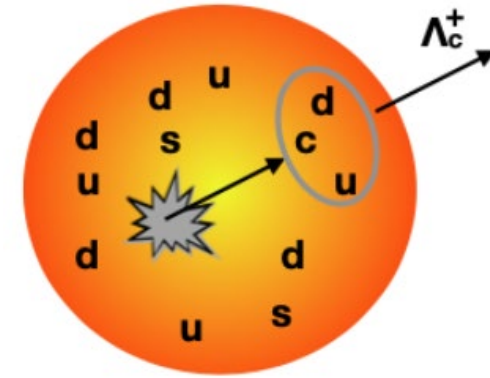


LHCb, PRL132, 081901 (2024)



# Statistical Hadronization Model (SHM)

- High-energy pp collisions = light-**quark-rich** environment  
→ **stochastic/statistical coalescence** of c/b with surrounding q
- Statistical hadronization model for heavy-hadrons



{ Hadronic population born into equilibrium = maximum entropy → ‘thermal’ production  
*Khazeev & Satz, EPJC 52,187 (2007)*  
**Relative chemical equilibrium** → primary production yields  $N_i \propto$  thermal densities  $n_i$

- Grand-canonical ensemble for mini.bias pp

$$n_i^{\text{primary}} = \frac{d_i}{2\pi^2} \gamma_s^{N_s^i} m_i^2 T_H K_2\left(\frac{m_i}{T_H}\right) \begin{cases} \gamma_s = 0.6 - \text{strangeness suppression factor} \\ T_H = 170 \text{ MeV} - \text{‘universal’ hadronization temperature} \end{cases}$$

# Heavy-hadron mass spectra as input of SHM

- Ground-state heavy-hadron total density = primary + feed-downs

$$n_{\alpha} = n_{\alpha}^{\text{primary}} + \sum_i n_i^{\text{primary}} \cdot BR(i \rightarrow \alpha)$$

Ebert et al., PRD 84 (2011) 014025

**PDG:** 5 B, 4 B<sub>s</sub>,  
5  $\Lambda_b$ , 2  $\Sigma_b$ , 4  $\Xi_b$ , 1  $\Omega_b$

**RQM:** 25 B, 20 B<sub>s</sub>,  
30  $\Lambda_b$ , 46  $\Sigma_b$ , 75  $\Xi_b$ , 42  $\Omega_b$

$\Lambda_b^0$

$$I(J^P) = 0(\frac{1}{2}^+)$$

$I(J^P)$  not yet measured;  $0(\frac{1}{2}^+)$  is the quark model prediction.

Mass  $m = 5619.60 \pm 0.17$  MeV

$m_{\Lambda_b^0} - m_{B^0} = 339.2 \pm 1.4$  MeV

$m_{\Lambda_b^0} - m_{B^+} = 339.72 \pm 0.28$  MeV

Mean life  $\tau = (1.471 \pm 0.009) \times 10^{-12}$  s  
 $c\tau = 441.0$   $\mu\text{m}$

$\Lambda_b(5912)^0$

$$J^P = \frac{1}{2}^-$$

Mass  $m = 5912.20 \pm 0.21$  MeV

Full width  $\Gamma < 0.66$  MeV, CL = 90%

$\Lambda_b(5920)^0$

$$J^P = \frac{3}{2}^-$$

Mass  $m = 5919.92 \pm 0.19$  MeV ( $S = 1.1$ )

Full width  $\Gamma < 0.63$  MeV, CL = 90%

$\Lambda_b(6146)^0$

$$J^P = \frac{3}{2}^+$$

Mass  $m = 6146.2 \pm 0.4$  MeV

Full width  $\Gamma = 2.9 \pm 1.3$  MeV

Full width  $\Gamma = 526.55 \pm 0.34$  MeV

$\Lambda_b(6152)^0$

$$J^P = \frac{5}{2}^+$$

Mass  $m = 6152.5 \pm 0.4$  MeV

Full width  $\Gamma = 2.1 \pm 0.9$  MeV

Full width  $\Gamma = 532.89 \pm 0.28$  MeV

Full width  $\Gamma = 6.34 \pm 0.32$  MeV

TABLE II. Masses of the  $\Lambda_Q$  ( $Q = c, b$ ) heavy baryons (in MeV).

| $I(J^P)$           | $Qd$ state | $Q = c$ |                            | $Q = b$ |                      |
|--------------------|------------|---------|----------------------------|---------|----------------------|
|                    |            | $M$     | $M^{\text{exp}} [1]$       | $M$     | $M^{\text{exp}} [1]$ |
| $0(\frac{1}{2}^+)$ | 1S         | 2286    | 2286.46(14)                | 5620    | 5620.2(1.6)          |
| $0(\frac{1}{2}^+)$ | 2S         | 2769    | 2766.6(2.4)?               | 6089    |                      |
| $0(\frac{1}{2}^+)$ | 3S         | 3130    | 6455                       |         |                      |
| $0(\frac{1}{2}^+)$ | 4S         | 3437    | 6756                       |         |                      |
| $0(\frac{1}{2}^+)$ | 5S         | 3715    | 7015                       |         |                      |
| $0(\frac{1}{2}^+)$ | 6S         | 3973    | 7256                       |         |                      |
| $0(\frac{1}{2}^-)$ | 1P         | 2598    | 2595.4(6)                  | 5930    |                      |
| $0(\frac{1}{2}^-)$ | 2P         | 2983    | 2939.3( $\frac{1}{1,3}$ )? | 6326    |                      |
| $0(\frac{1}{2}^-)$ | 3P         | 3303    | 6645                       |         |                      |
| $0(\frac{1}{2}^-)$ | 4P         | 3588    | 6917                       |         |                      |
| $0(\frac{1}{2}^-)$ | 5P         | 3852    | 7157                       |         |                      |
| $0(\frac{3}{2}^-)$ | 1P         | 2627    | 2628.1(6)                  | 5942    |                      |
| $0(\frac{3}{2}^-)$ | 2P         | 3005    | 6333                       |         |                      |
| $0(\frac{3}{2}^-)$ | 3P         | 3322    | 6651                       |         |                      |
| $0(\frac{3}{2}^-)$ | 4P         | 3606    | 6922                       |         |                      |
| $0(\frac{3}{2}^-)$ | 5P         | 3869    | 7171                       |         |                      |
| $0(\frac{3}{2}^+)$ | 1D         | 2874    | 6190                       |         |                      |
| $0(\frac{3}{2}^+)$ | 2D         | 3189    | 6526                       |         |                      |
| $0(\frac{3}{2}^+)$ | 3D         | 3480    | 6811                       |         |                      |
| $0(\frac{3}{2}^+)$ | 4D         | 3747    | 7060                       |         |                      |
| $0(\frac{5}{2}^+)$ | 1D         | 2880    | 2881.53(35)                | 6196    |                      |
| $0(\frac{5}{2}^+)$ | 2D         | 3209    | 6531                       |         |                      |
| $0(\frac{5}{2}^+)$ | 3D         | 3500    | 6814                       |         |                      |

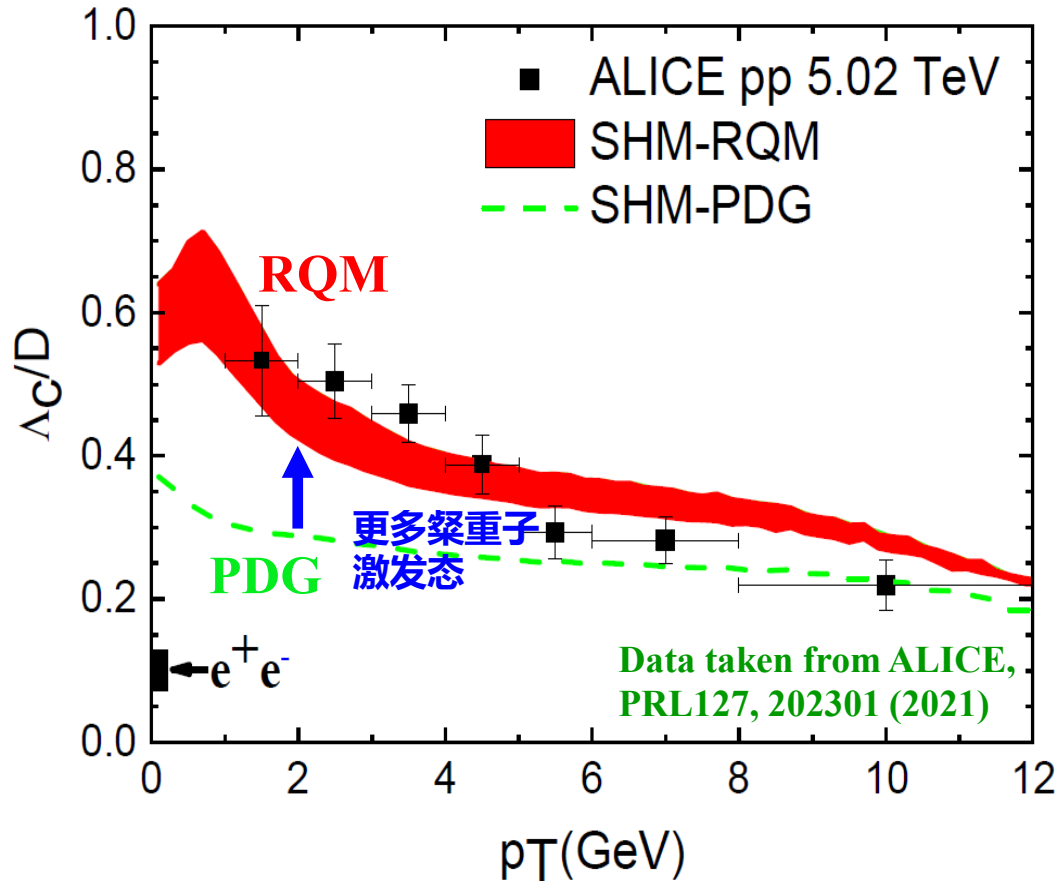
TABLE III. Masses of the  $\Sigma_Q$  ( $Q = c, b$ ) heavy baryons (in MeV).

| $I(J^P)$           | $Qd$ state | $Q = c$ |                            | $Q = b$ |                      |
|--------------------|------------|---------|----------------------------|---------|----------------------|
|                    |            | $M$     | $M^{\text{exp}} [1]$       | $M$     | $M^{\text{exp}} [1]$ |
| $1(\frac{1}{2}^+)$ | 1S         | 2443    | 2453.76(18)                | 5808    | 5807.8(2.7)          |
| $1(\frac{1}{2}^+)$ | 2S         | 2901    |                            | 6213    |                      |
| $1(\frac{1}{2}^+)$ | 3S         | 3271    |                            | 6575    |                      |
| $1(\frac{1}{2}^+)$ | 4S         | 3581    |                            | 6869    |                      |
| $1(\frac{1}{2}^+)$ | 5S         | 3861    |                            | 7124    |                      |
| $1(\frac{3}{2}^+)$ | 1S         | 2519    | 2518.0(5)                  | 5834    | 5829.0(3.4)          |
| $1(\frac{3}{2}^+)$ | 2S         | 2936    | 2939.3( $\frac{1}{1,3}$ )? | 6226    |                      |
| $1(\frac{3}{2}^+)$ | 3S         | 3293    |                            | 6583    |                      |
| $1(\frac{3}{2}^+)$ | 4S         | 3598    |                            | 6876    |                      |
| $1(\frac{3}{2}^+)$ | 5S         | 3873    |                            | 7129    |                      |
| $1(\frac{1}{2}^-)$ | 1P         | 2799    | 2802( $\frac{4}{1}$ )      | 6101    |                      |
| $1(\frac{1}{2}^-)$ | 2P         | 3172    |                            | 6440    |                      |
| $1(\frac{1}{2}^-)$ | 3P         | 3488    |                            | 6756    |                      |
| $1(\frac{1}{2}^-)$ | 4P         | 3770    |                            | 7024    |                      |
| $1(\frac{1}{2}^-)$ | 1P         | 2713    |                            | 6095    |                      |
| $1(\frac{1}{2}^-)$ | 2P         | 3125    |                            | 6430    |                      |
| $1(\frac{1}{2}^-)$ | 3P         | 3455    |                            | 6742    |                      |
| $1(\frac{1}{2}^-)$ | 4P         | 3743    |                            | 7008    |                      |
| $1(\frac{3}{2}^-)$ | 1P         | 2798    | 2802( $\frac{4}{1}$ )      | 6096    |                      |
| $1(\frac{3}{2}^-)$ | 2P         | 3172    |                            | 6430    |                      |
| $1(\frac{3}{2}^-)$ | 3P         | 3486    |                            | 6742    |                      |
| $1(\frac{3}{2}^-)$ | 4P         | 3768    |                            | 7009    |                      |
| $1(\frac{5}{2}^-)$ | 1P         | 2773    | 2766.6(2.4)?               | 6087    |                      |
| $1(\frac{5}{2}^-)$ | 2P         | 3151    |                            | 6423    |                      |
| $1(\frac{5}{2}^-)$ | 3P         | 3469    |                            | 6736    |                      |
| $1(\frac{5}{2}^-)$ | 4P         | 3753    |                            | 7003    |                      |
| $1(\frac{5}{2}^-)$ | 1P         | 2789    |                            | 6084    |                      |



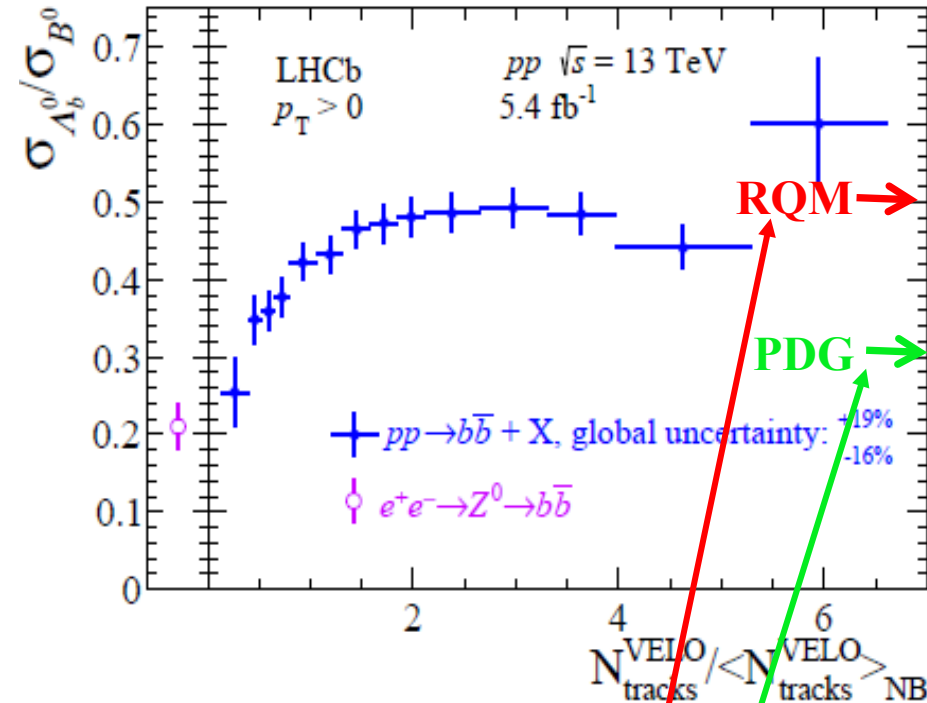
# Heavy baryon-to-meson ratio: mini.bias pp

$\Lambda_c/D$  vs  $p_T$



MH & Rapp, PLB 795, 117 (2019)  
 Chen & MH, PLB 815, 136144 (2021)

$\Lambda_b/B$  vs system-size



| $r_\alpha$ | $\bar{B}^0/B^-$ | $\bar{B}_s^0/B^-$ | $\Lambda_b^0/B^-$ | $\Xi_b^{0,-}/B^-$ |
|------------|-----------------|-------------------|-------------------|-------------------|
| PDG        | 0.9995          | 0.2904            | 0.3129            | 0.1000            |
| RQM        | 0.9994          | 0.2699            | 0.5122            | 0.1623            |

MH & Rapp, PRL 131, 012301 (2023)

# Canonical ensemble (CE) SHM

- Canonical ensemble partition function: **strict conservation** of quantum charges

$$Z(\vec{Q}) = \int_0^{2\pi} \frac{d^5\phi}{(2\pi)^5} e^{i\vec{Q}\cdot\vec{\phi}} \exp\left[\sum_j \gamma_s^{N_{sj}} \gamma_c^{N_{cj}} \gamma_b^{N_{bj}} e^{-i\vec{q}_j\cdot\vec{\phi}} z_j\right] \quad \vec{Q} = (Q, N, S, C, B)$$

$$z_j = (2J_j + 1) \frac{V T_H}{2\pi^2} m_j^2 K_2\left(\frac{m_j}{T_H}\right)$$

correlation volume ~ system size

- Primary hadron yield: CE vs GCE

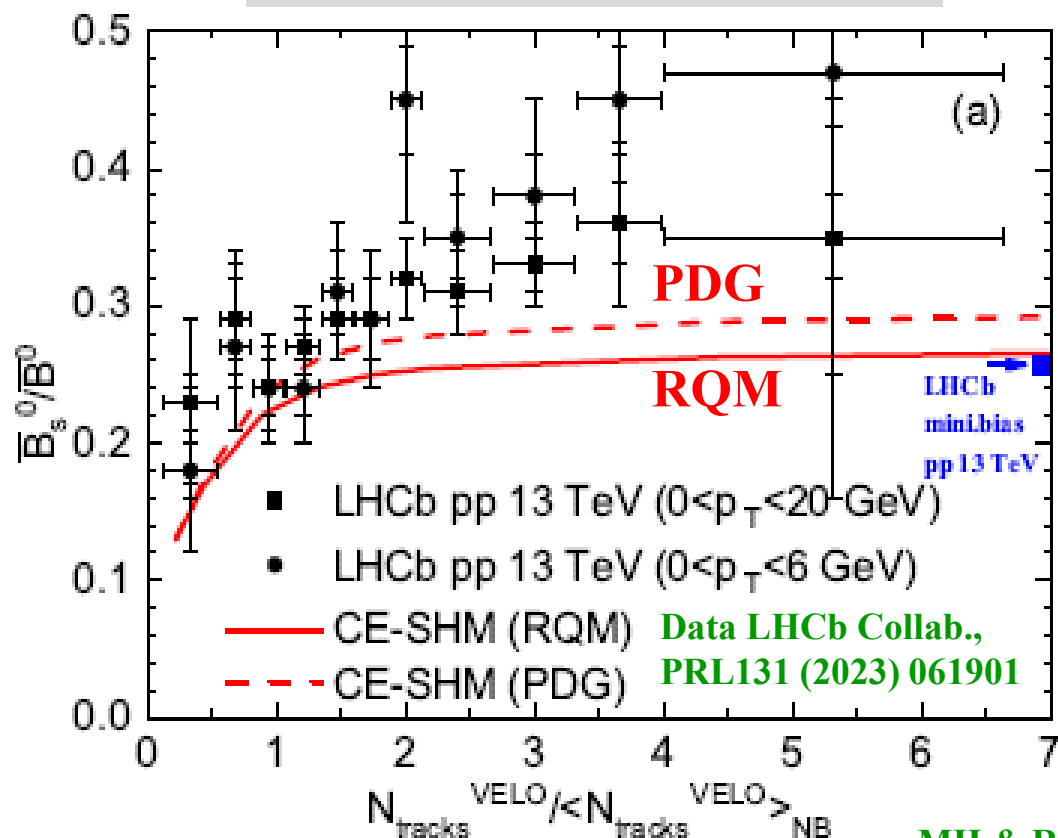
$$\begin{aligned} \langle N_j \rangle^{CE} &= \gamma_s^{N_{sj}} \gamma_c^{N_{cj}} \gamma_b^{N_{bj}} z_j \frac{Z(\vec{Q} - \vec{q}_j)}{Z(\vec{Q})} \\ &= \langle N_j \rangle^{GCE} \frac{Z(\vec{Q} - \vec{q}_j)}{Z(\vec{Q})} \end{aligned}$$

chemical factor <1:  
canonical suppression for  
charged hadron with  $\vec{q}_j \neq 0$

- E.g. exact baryon-number conservation requires: simultaneous creation of a pair of baryon and antibaryon → energy-expensive  $\exp(-2m_N/T_H)$   
→ **canonical suppression** for baryon production

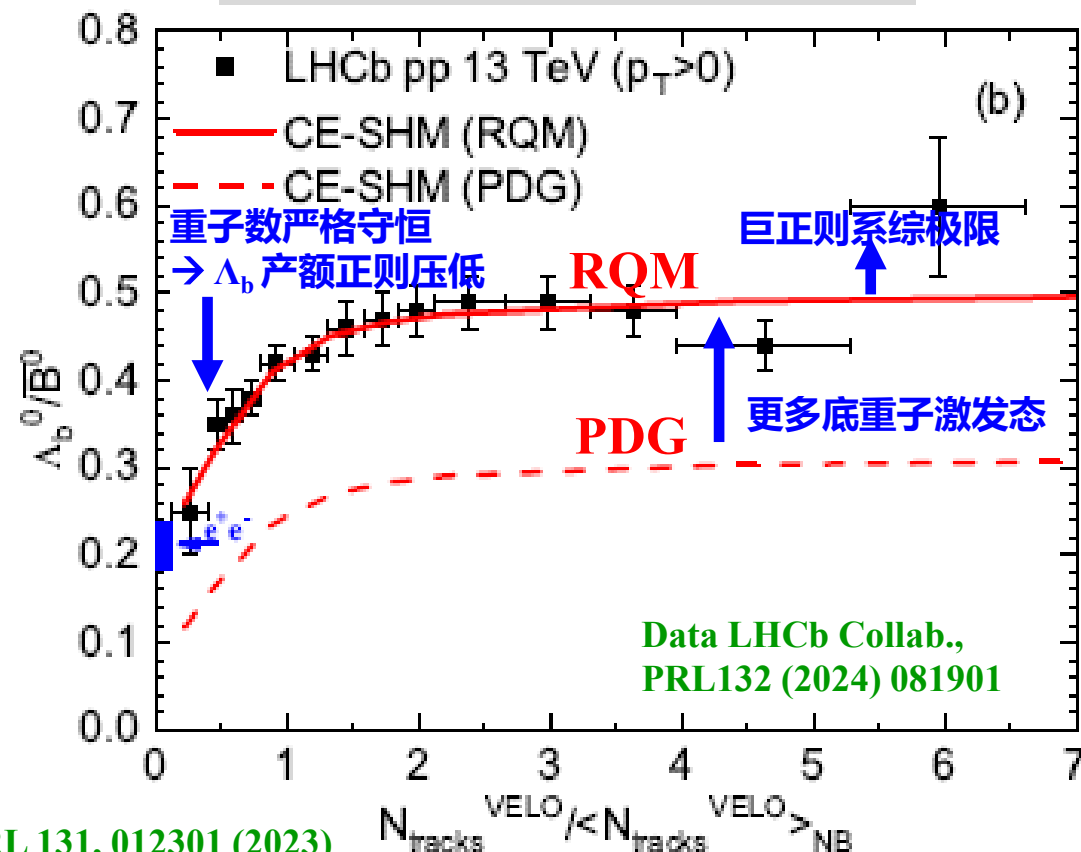
# Ground-state b-hadron production ratios

## $B_s^0/B$ vs system-size



MH & Rapp, PRL 131, 012301 (2023)  
Dai & MH, PRC 110, 034905 (2024)

## $\Lambda_b^0/B$ vs system-size

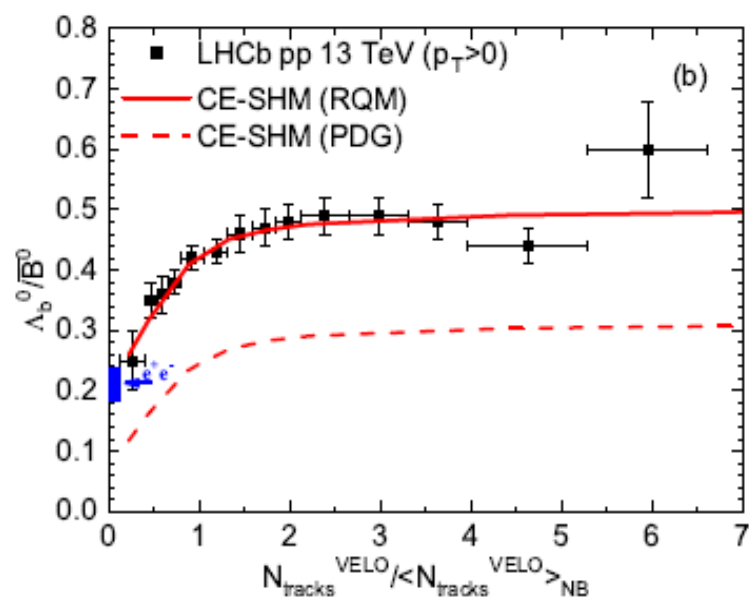
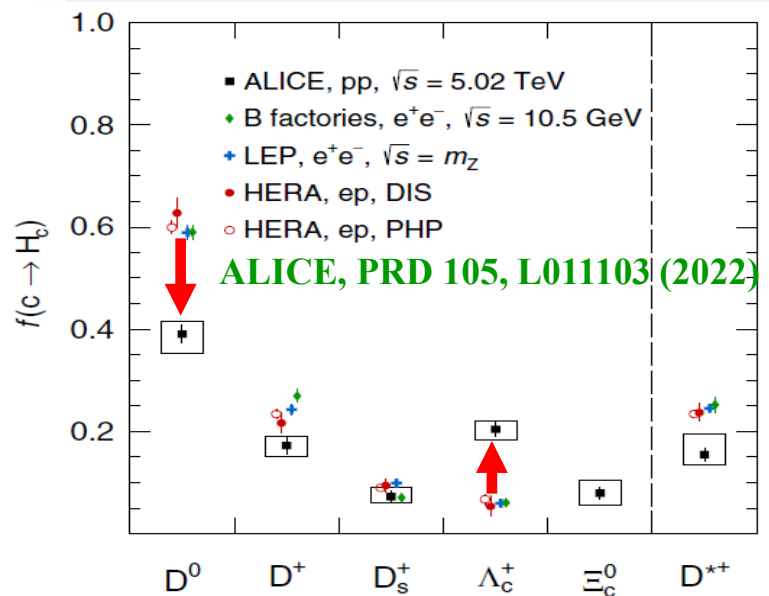


-  $\Lambda_b^0/B$  GCE saturation limit  $\rightarrow$   $e^+e^-$  vacuum fragmentation limit

- RQM strongly favored by data



# Take-aways from Part I



- Heavy flavor **hadro-chemistry: non-universal**
  - $c/b \rightarrow \Lambda_{c/b}$  enhanced &  $c/b \rightarrow D/B$  reduced in pp vs  $e^+e^-$
- Grand-canonical  $\rightarrow$  canonical **statistical hadronization**
  - exact conservation of quantum charges  $\rightarrow$  canonical suppr.
  - large volume saturation limit  $\rightarrow$  vacuum fragmentation  $e^+e^-$  limit
- “Missing” heavy-baryon excited states highlighted  $\rightarrow$  awaiting measurement & confirmation



# Outline of Part II

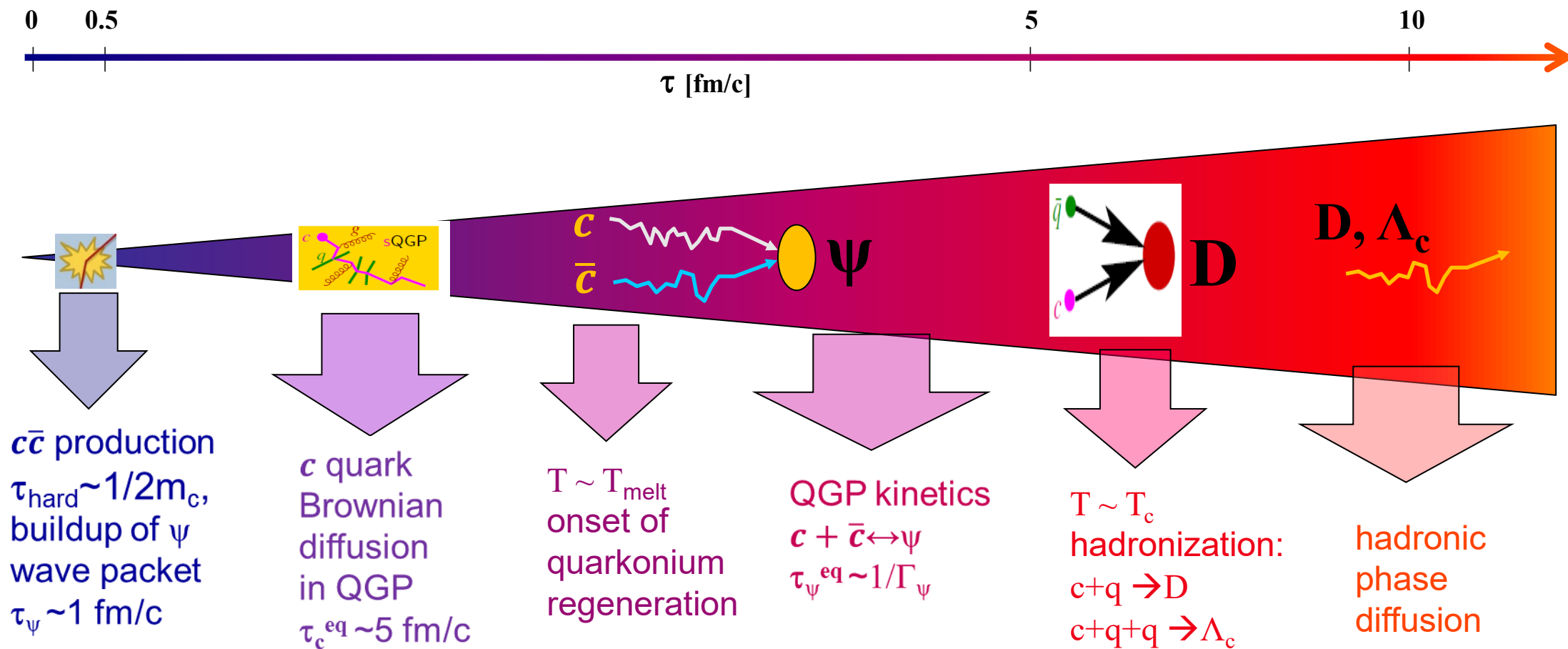
Hadro-chemistry &  $p_T$  spectra computed above in minimum bias pp collisions  
= a controlled reference for studying modifications in heavy-ion collisions →

## Part II: Open heavy-flavor transport in Pb-Pb

- Interaction of HF with medium: T-matrix approach
- Heavy quark diffusion & hadronization in QGP
- Collective flow &  $p_T$ -dependent modifications of hadro-chemistry



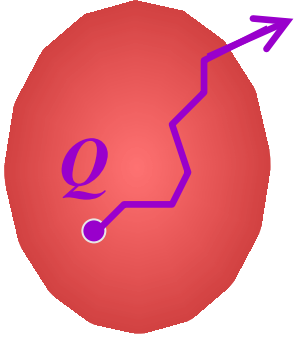
# Heavy flavor transport as probes of QGP



- $m_Q \gg T \rightarrow$  number conserved through diffusion/hadronization: **tagged & traceable probes**
- $\tau_Q^{\text{eq}} \geq \tau_{\text{QGP}} \rightarrow$  carrying a memory of interaction history: **quantitative gauge of coupling strength**

# HQ interaction & diffusion in QGP

- Heavy quark Brownian motion: Fokker-Planck/Langevin equation



$$q_0 \sim q^2/2m_Q \ll q \sim T \ll m_Q$$

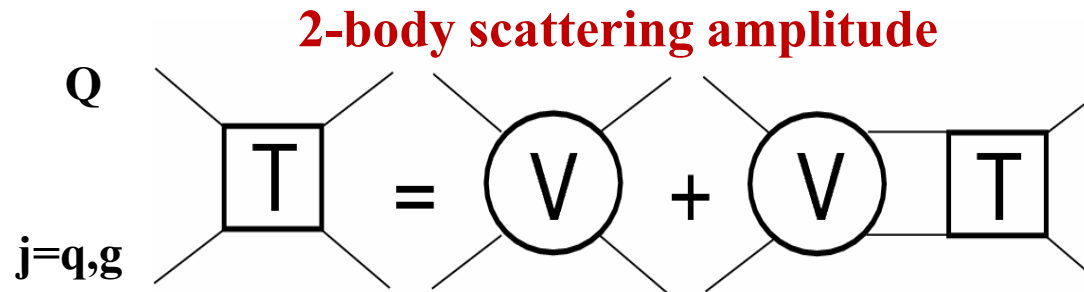
$$\frac{\partial}{\partial t} f_Q(t, p) = \underbrace{\gamma}_{\text{thermal relaxation rate}} \frac{\partial}{\partial p_i} [p_i f_Q(t, p)] + \underbrace{D_p}_{\text{momentum diffusion coeff.}} \Delta_{\vec{p}} f_Q(t, p)$$

thermal relaxation rate

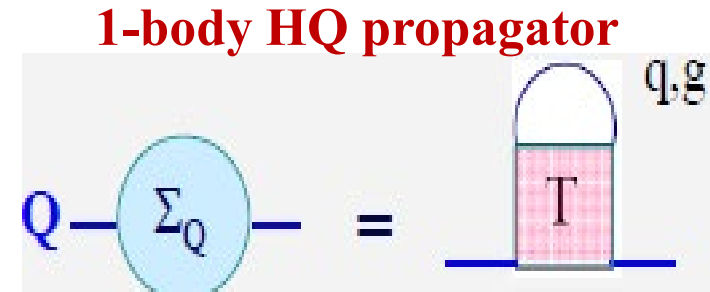
$$\gamma \sim \int |\mathbf{T}_{Qj}|^2 (1 - \cos\theta) f^j$$

momentum diffusion coeff.  
Einstein relation  $D_p = \gamma m_Q T$

- Q-q/g soft scatterings: T-matrix resummation of lattice-constrained in-medium HQ potential



$$T_{Qj} = V_{Qj} + \int V_{Qj} D_Q D_j T_{Qj}$$



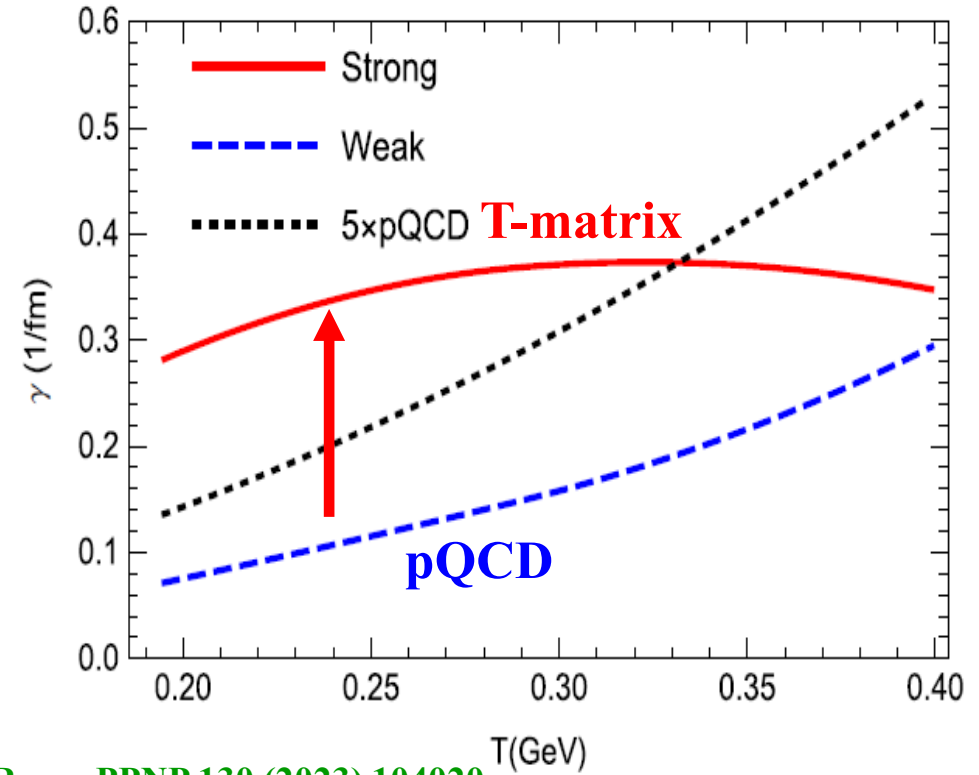
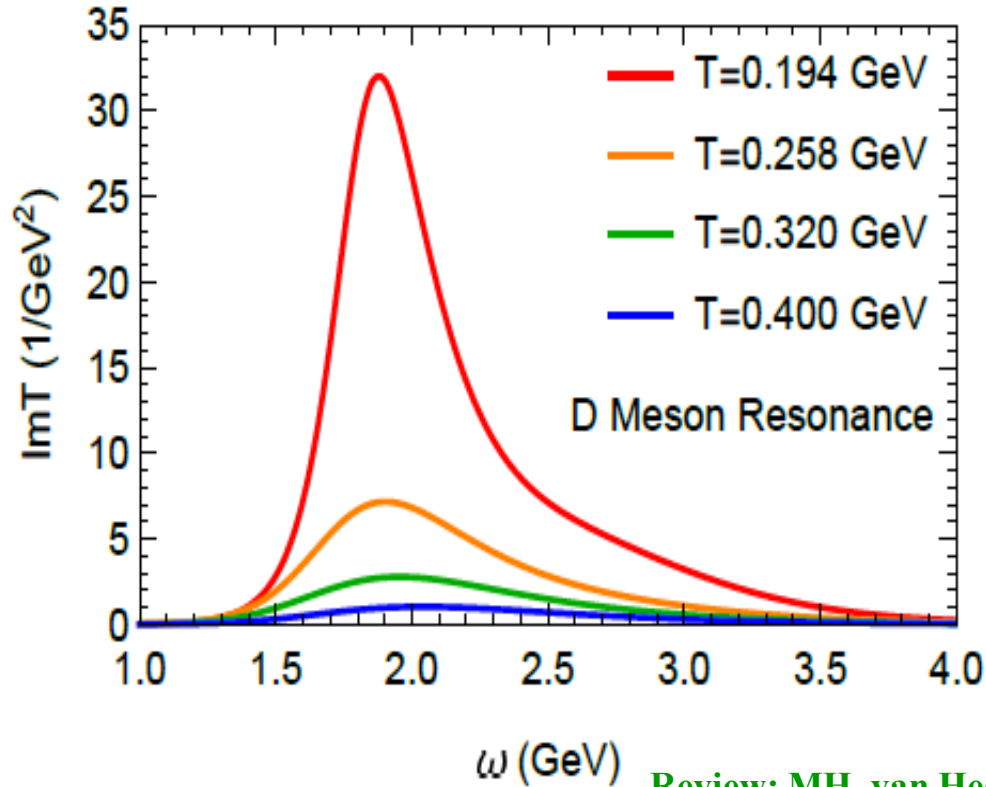
$$D = 1 / [\omega - \omega_k - \Sigma(\omega, k)]$$

Review: MH, van Hees & Rapp, PPNP 130 (2023) 104020

# Charm quark thermal relaxation rate in QGP

$$\gamma = A \sim \int |T_{Qj}|^2 (1 - \cos\theta) f^j$$

**T-matrix w/ strong potential  $\longrightarrow$  Resonance formation near  $T_c \longrightarrow$  Accelerating charm thermalization**



Review: MH, van Hees & Rapp, PPNP 130 (2023) 104020

- Non-perturbative enhancement at low  $p$  &  $T$ ; approaching pQCD at high  $p$  &  $T$
- x K-factor=1.6 for mimicking spin-dependent force/radiative contributions

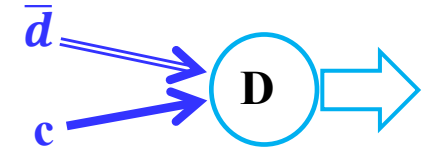
# HQ hadronization: resonance recombination

- 2→1 Resonance Recombination Model (RRM) via Boltzmann equation

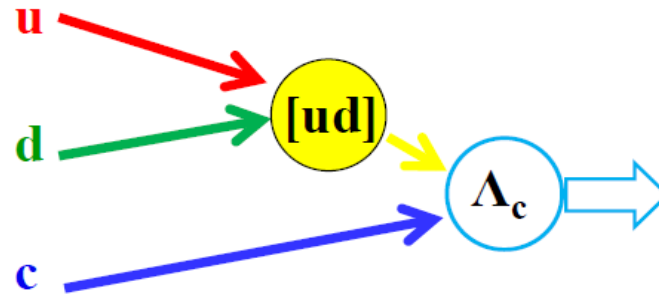
$$f_M(\vec{x}, \vec{p}) = \frac{\gamma_M(p)}{\Gamma_M} \int \frac{d^3\vec{p}_1 d^3\vec{p}_2}{(2\pi)^3} f_q(\vec{x}, \vec{p}_1) f_{\bar{q}}(\vec{x}, \vec{p}_2) \sigma_M(s) v_{\text{rel}}(\vec{p}_1, \vec{p}_2) \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2)$$

- Resonant cross section  $\sigma_M(s)$  ← T-matrix resonance amplitude

- Encoding **energy conservation** & **correct equilibrium limit**

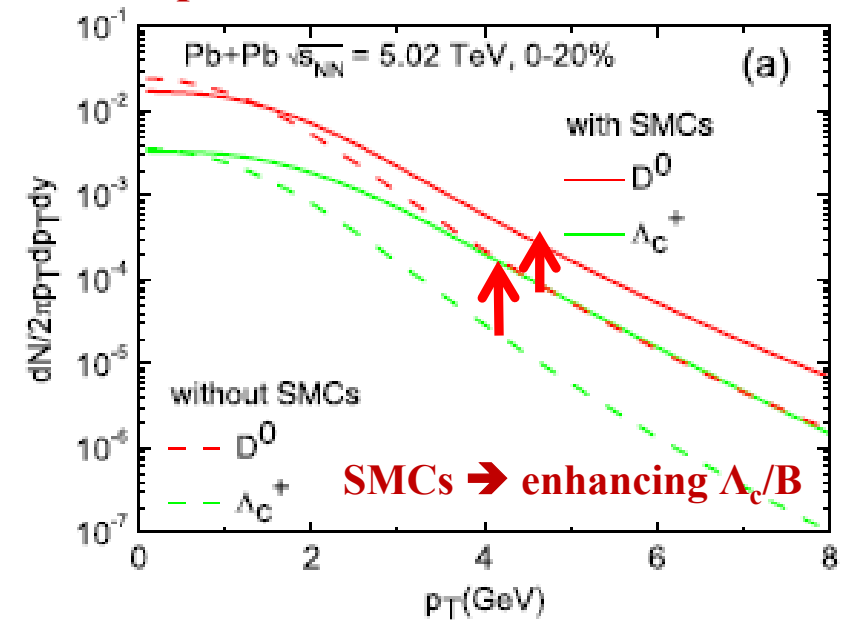


- 3→1 RRM: **diquark correlations** in heavy-baryons



$$f_B(\vec{x}, \vec{p}) = \frac{E_B(\vec{p})}{\Gamma_B m_B} \int \frac{d^3p_1 d^3p_2 d^3p_3}{(2\pi)^6} \frac{E_d(\vec{p}_{12})}{\Gamma_d m_d} f_1(\vec{x}, \vec{p}_1) f_2(\vec{x}, \vec{p}_2) f_3(\vec{x}, \vec{p}_3) \\ \times \sigma_{12}(s_{12}) v_{\text{rel}}^{12}(\vec{p}_1, \vec{p}_2) \sigma_B(s_{d3}) v_{\text{rel}}^{d3}(\vec{p}_{12}, \vec{p}_3) \big|_{\vec{p}_{12}=\vec{p}_1+\vec{p}_2} \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$

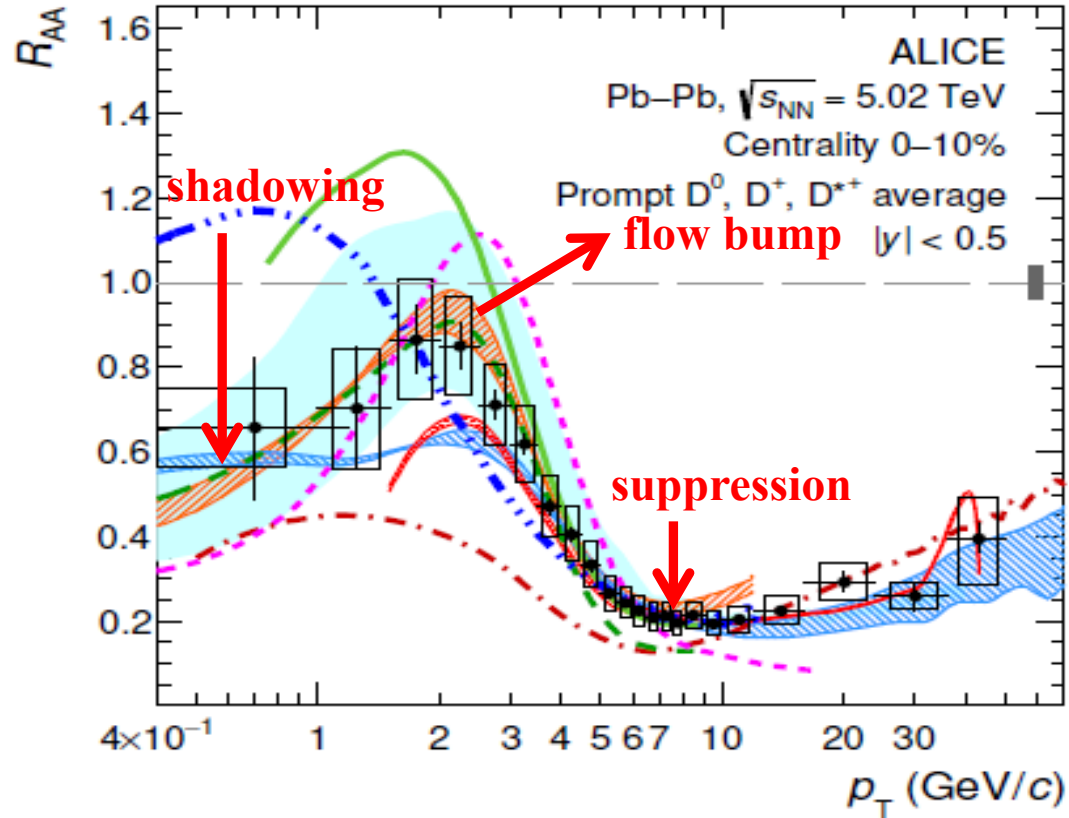
## Space-Momentum Correlations



MH & Rapp PRL 124, 042301 (2020); Lu & MH, PRC 106, 064903 (2022)

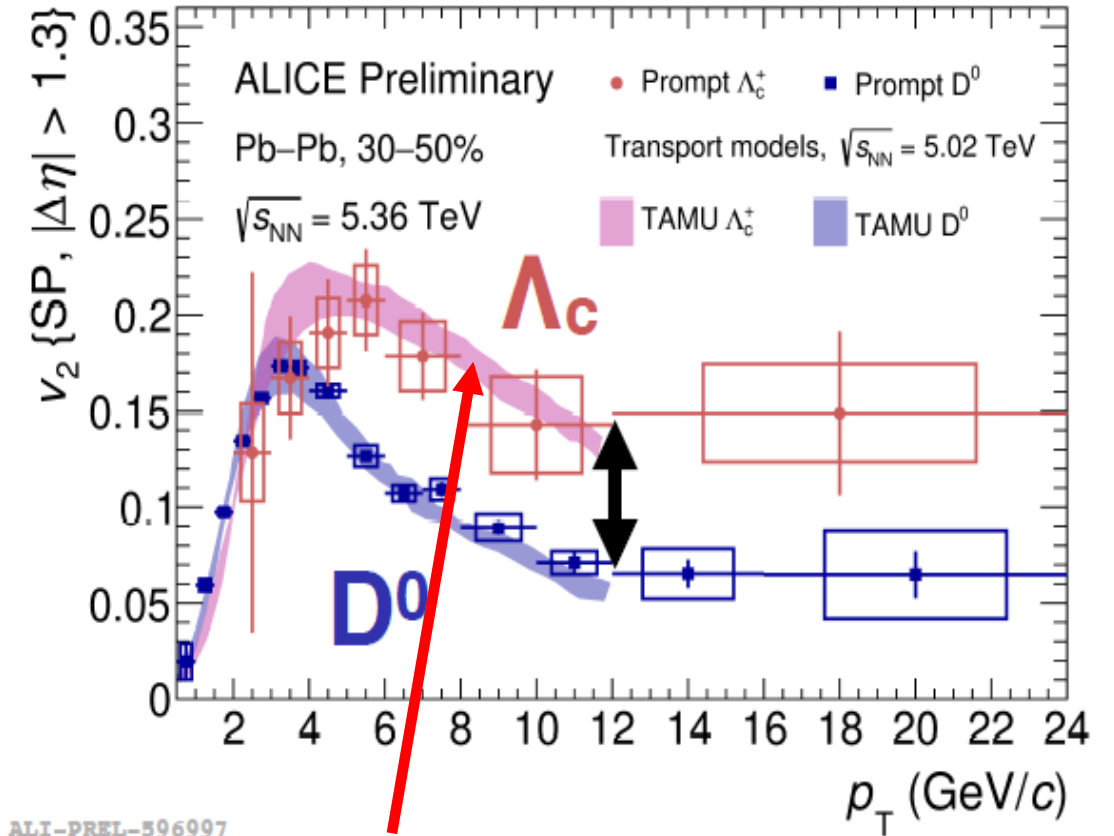
# D-meson & $\Lambda_c$ -baryon's flow @ 5 TeV Pb-Pb

ALICE, JHEP 01 (2022) 174



- Brown curve taken from our model calculation in PRL 124, 042301 (2020)
- Simultaneous description of D's  $R_{AA}$  &  $v_2$

ALICE highlights @ Quark Matter 2025

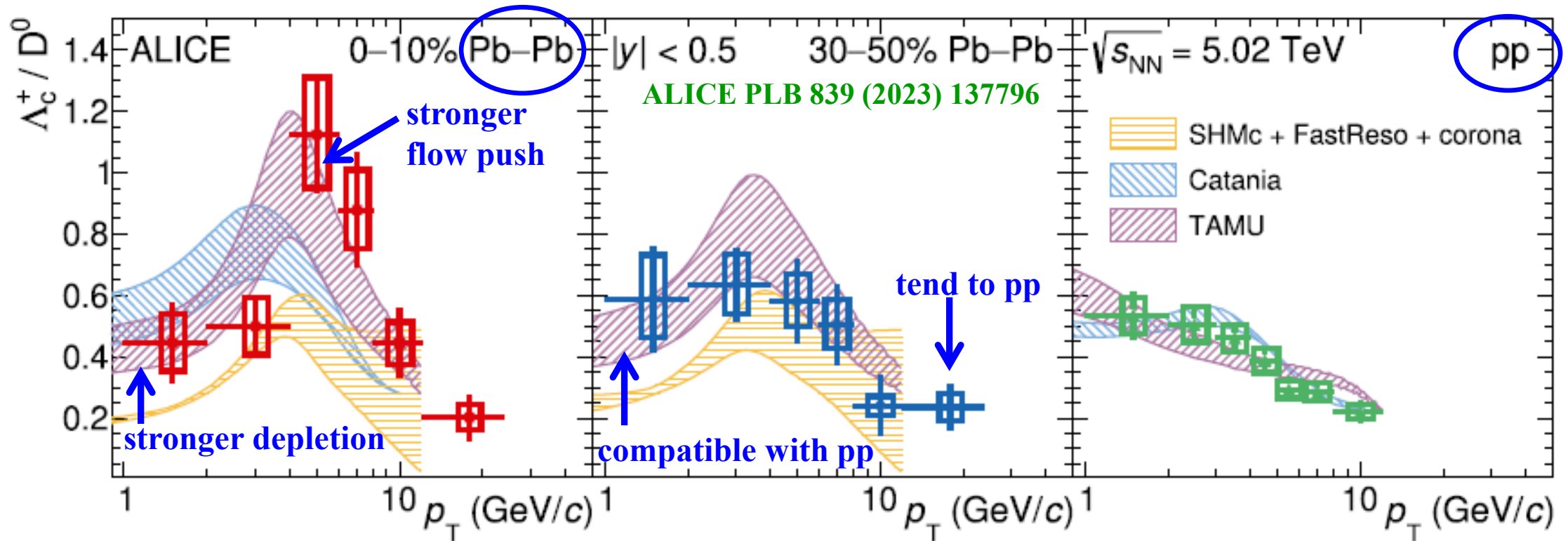


ALI-PREL-596997

- In particular,  $\Lambda_c$ 's  $v_2$  curve taken from our predictions in PRL 124, 042301 (2020)
- $v_2$  at low  $p_T$ : gauge of coupling strength

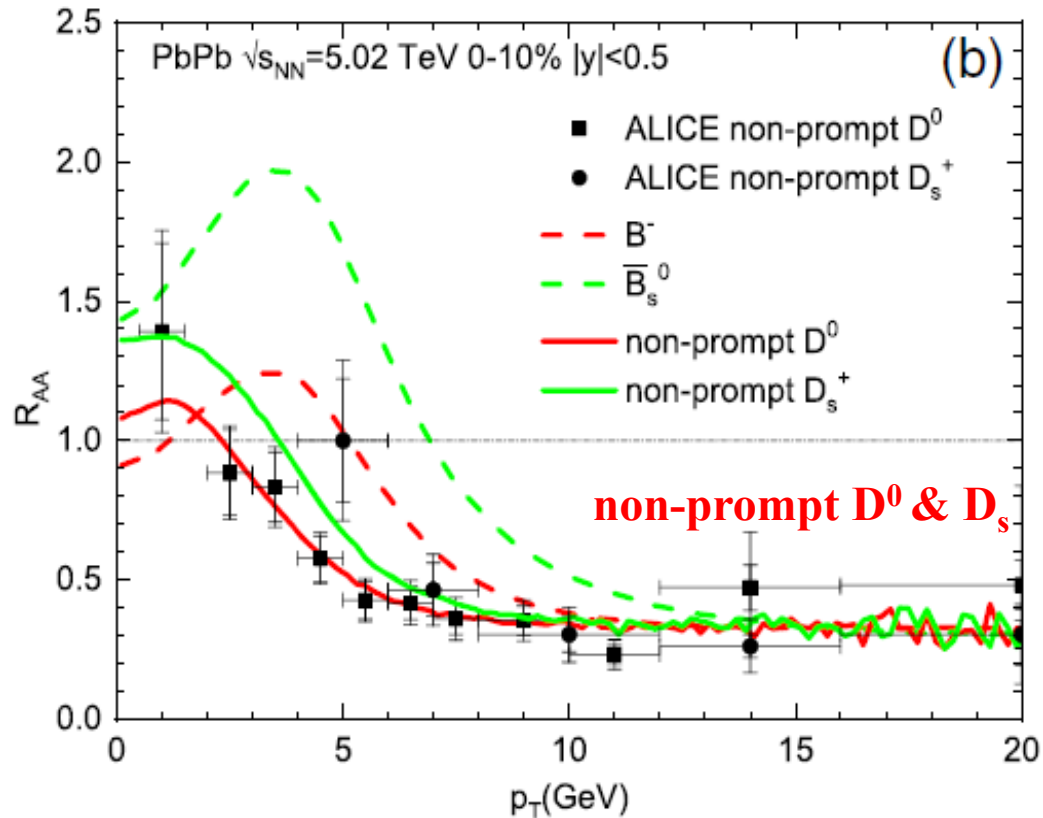
# Charm hadro-chemistry $\Lambda_c/D$ : $pp \rightarrow Pb-Pb$

purple curves taken from our model predictions in [PRL 124, 042301 \(2020\)](#) & [PLB 795, 117 \(2019\)](#)



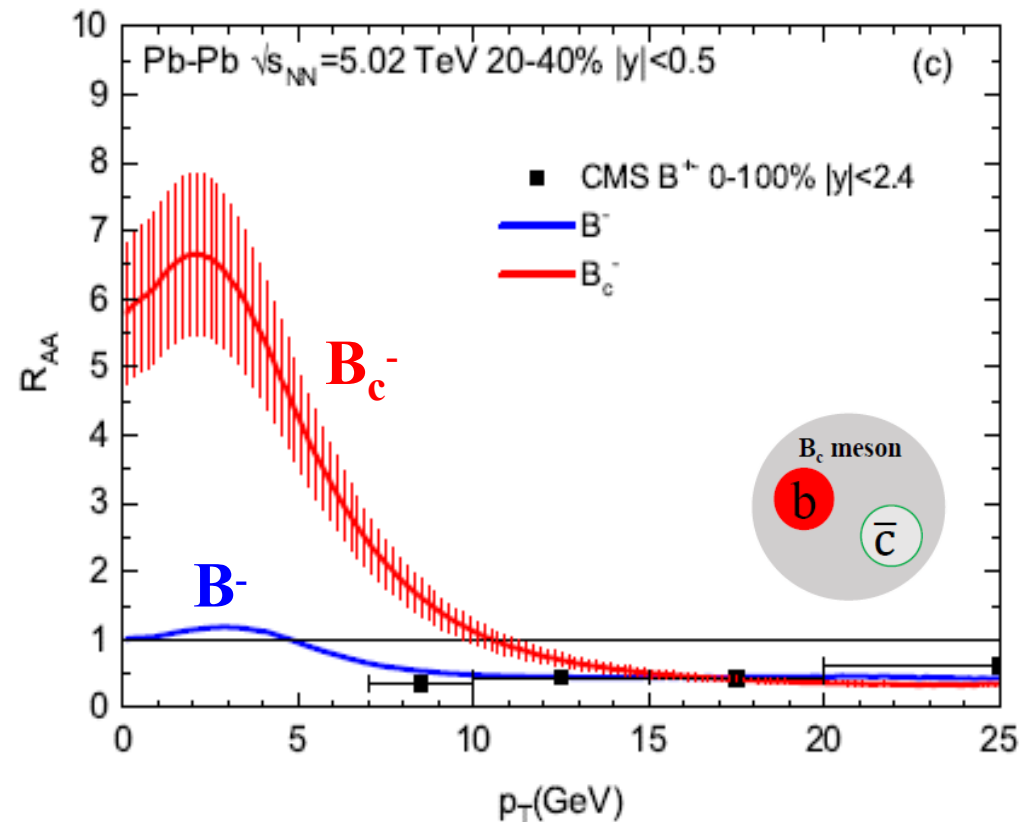
- Same RQM charm-hadron spectra (in particular baryons) in Pb-Pb & pp
- **RRM** satisfying correct **relative chemical equilibrium limit**  $\rightarrow$  same integrated  $\Lambda_c/D$  as in pp
- **RRM** with **SMCs** capturing full flow effects  $\rightarrow$  enhancement of  $\Lambda_c/D$  at intermediate  $p_T$

# Bottom-hadron nuclear modification factors



MH & Rapp, PRL 131, 012301 (2023)

- Enhanced strangeness coupled to b-quark via RRM  $\rightarrow$  larger  $R_{AA}$  for non-prompt  $D_s$

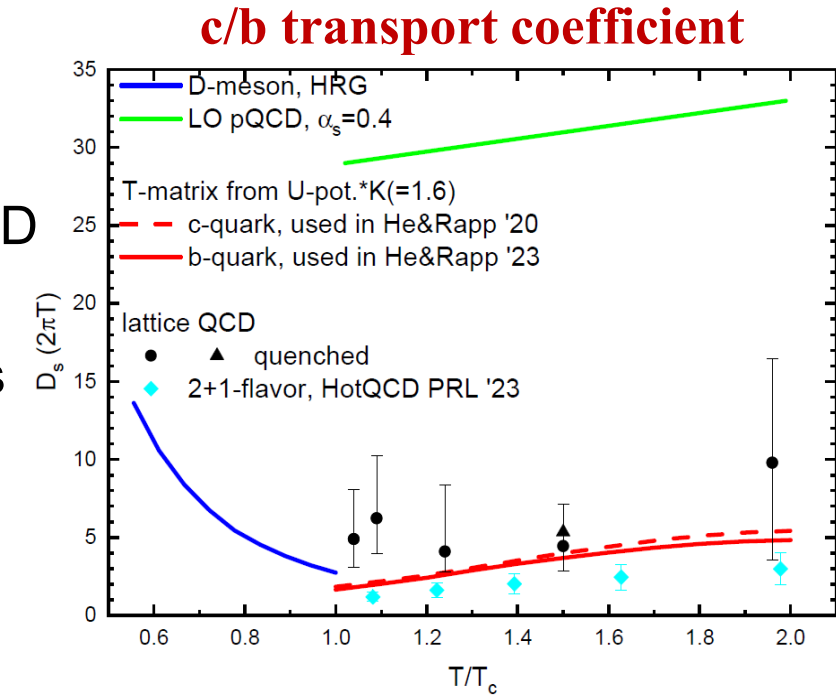


Zhao & MH, PLB 861, 139283 (2025)

- Statistical recombination of b-quark with plenty of near-thermalized c-quarks  $\rightarrow$  x5-6 enhancement of  $B_c$  at low  $p_T$

# Take-aways from Part II

- Heavy quark diffusion
  - spatial diffusion coefficient:  $\mathcal{D}_s = T/m_Q \gamma \rightarrow \langle x^2 \rangle \sim \mathcal{D}_s t$   
model & lattice  $\mathcal{D}_s(2\pi T) \sim 1-3$  near  $T_c$ , x10 smaller than pQCD
  - collisional width  $\Gamma_{\text{coll}} \sim 3/\mathcal{D}_s \sim 1 \text{ GeV} > M_{q,g} \rightarrow$  thermal partons melted, while HQs as Brownian markers survive
  - maximum coupling strength near  $T_c \rightarrow$  **strongly coupled QGP liquid**
- Heavy-quark hadronization
  - recombination on top of diffusion  $\rightarrow$  characteristic **flow features** (c vs b, meson vs baryon)
  - recombination  $\rightarrow$   **$p_T$ -dependent modifications of hadro-chemistry**, but only kinematic redistribution in  $p_T$  with integrated  $\Lambda_c/D$  unchanged





## Part III: Heavy quarkonium transport in Pb-Pb

- Semi-classical transport of heavy quarkonia in QGP
- Reaction rate: LO vs NLO
- $J/\psi$  &  $\psi(2S)$  collective flow



# Semi-classical transport of charmonium in QGP

- Boltzmann  $\rightarrow$  rate equation for charmonium yields

$$\frac{dN_\psi}{dt} = -\Gamma_\psi (N_\psi - N_\psi^{eq})$$

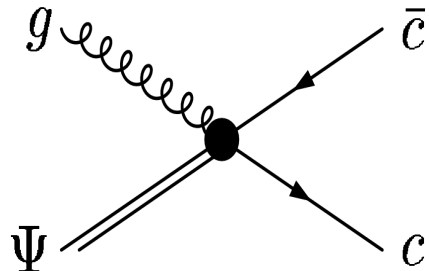
dissociation/  
reaction rate  $\Gamma_\psi$

regeneration  
toward equilibrium

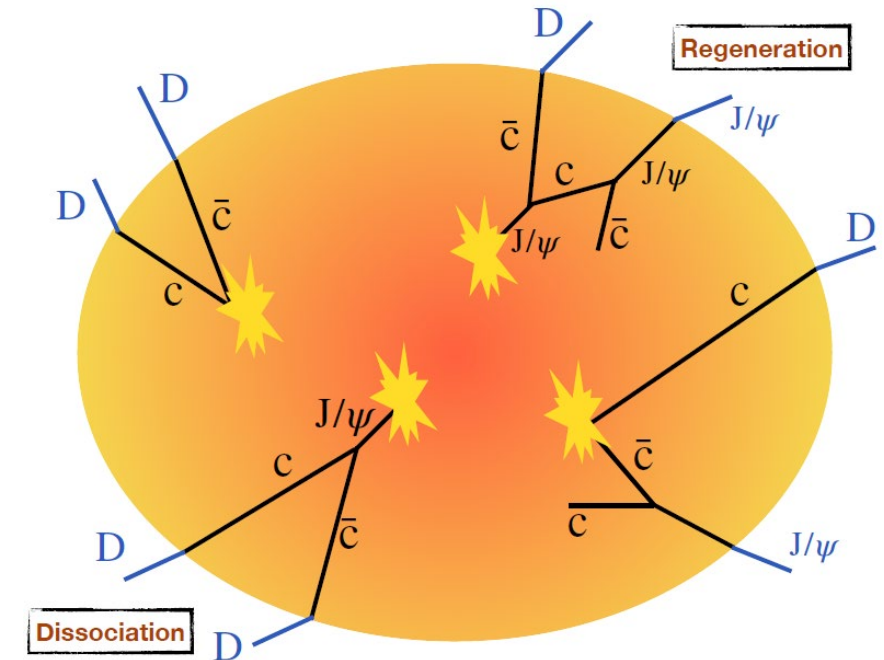
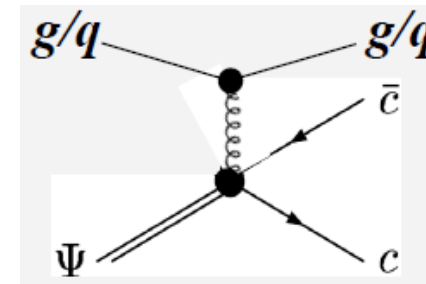
Review: MH, van Hees & Rapp, PNP 130 (2023) 104020

- Transport coefficients
  - equilibrium limit  $N_\psi^{eq}(T) = \gamma_c^2 n_\psi(T) V_{FB}$ , with  $\gamma_c \propto \sigma_{c\bar{c}}$
  - dissociation/reaction rate  $\Gamma_\psi [E_B(T)]$ :

LO: gluo-dissociation  $g + \psi \rightarrow c + \bar{c}$



NLO: parton-inelastic  $g/q + \psi \rightarrow g/q + c + \bar{c}$



# Charmonium dissociation rate in QGP

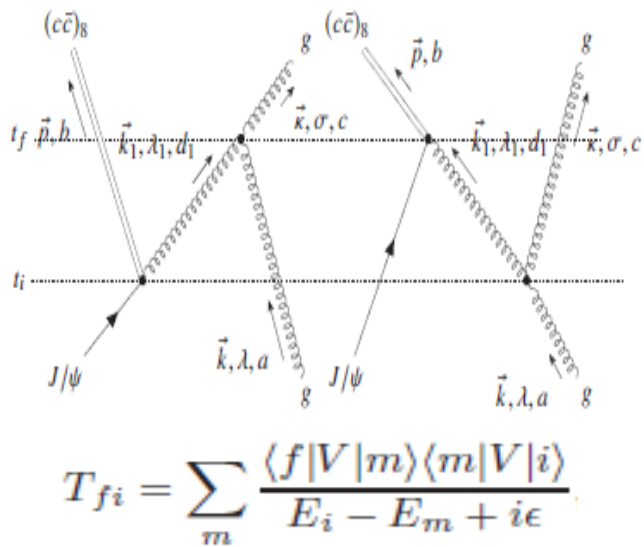
- Heavy quarkonium-gluon coupling: pNRQCD color-electric dipole

Singlet J/ψ  $S$  Octet (c $\bar{c}$ )<sub>8</sub>  $O^a$

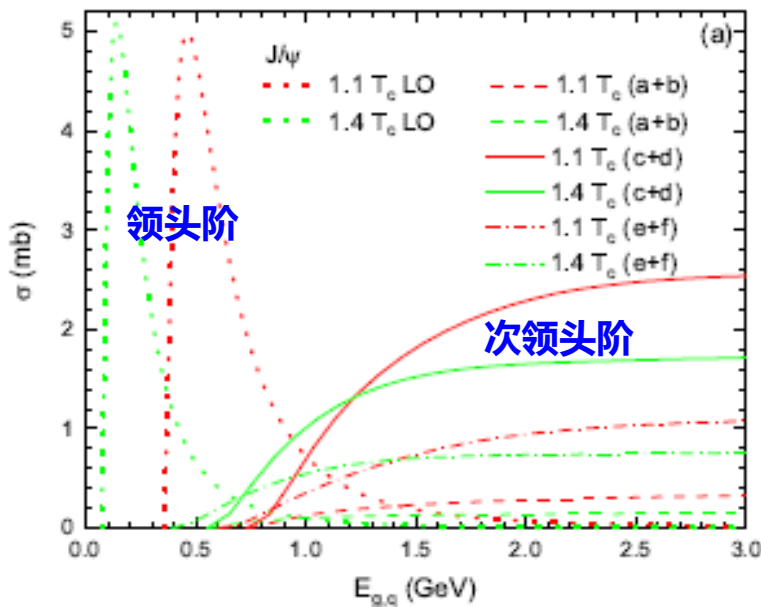
$$\langle O, a | V_{SO} | S \rangle = \langle O, a | \frac{1}{2} g_s \vec{r} \left( \frac{\lambda^b}{2} - \frac{\bar{\lambda}^b}{2} \right) \cdot \vec{E}^b | S \rangle$$

- Second-order quantum-mechanical perturbation

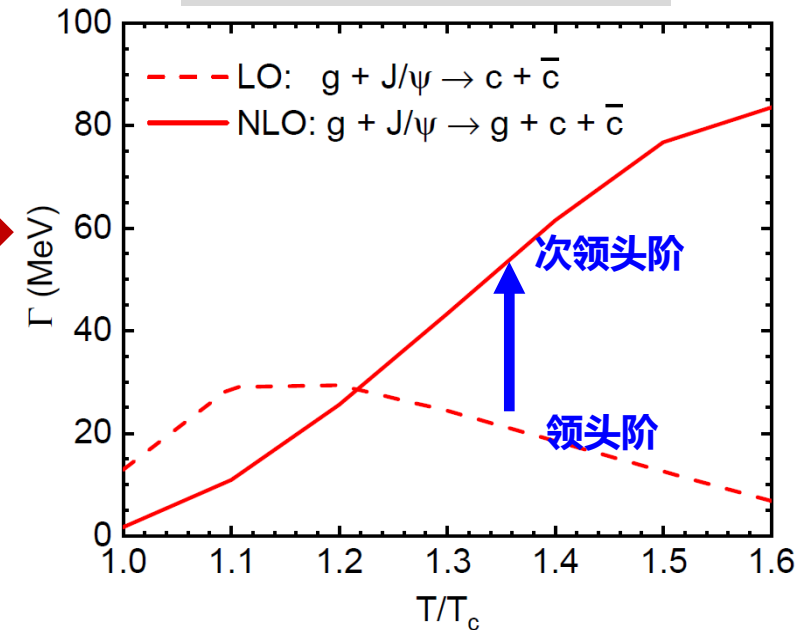
## Feynman diagrams



## Cross sections



## Dissociation rates



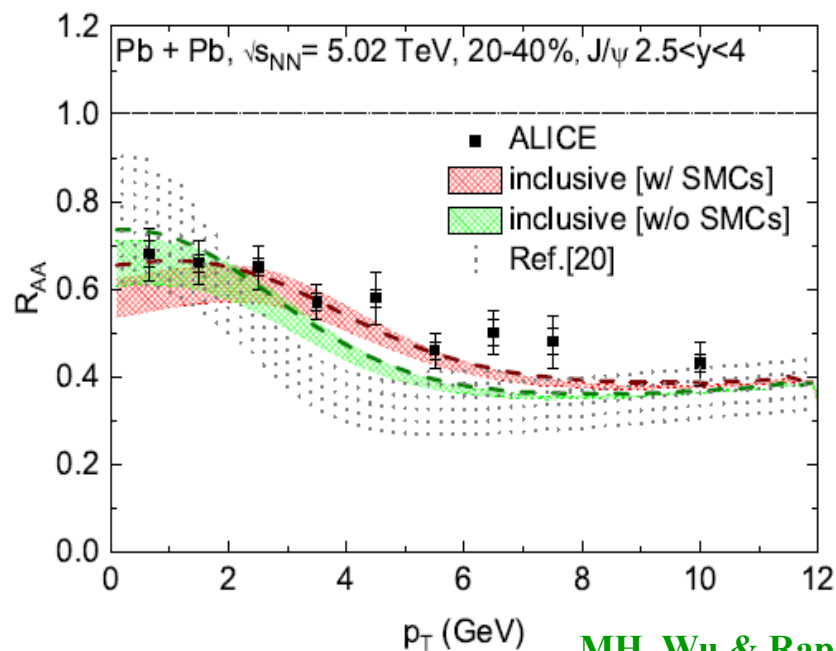
Chen & MH, PLB 786 (2018) 260–267; Zhao & MH, PRD 110, 074040 (2024)

# Resolving the 'J/ψ v<sub>2</sub> puzzle'

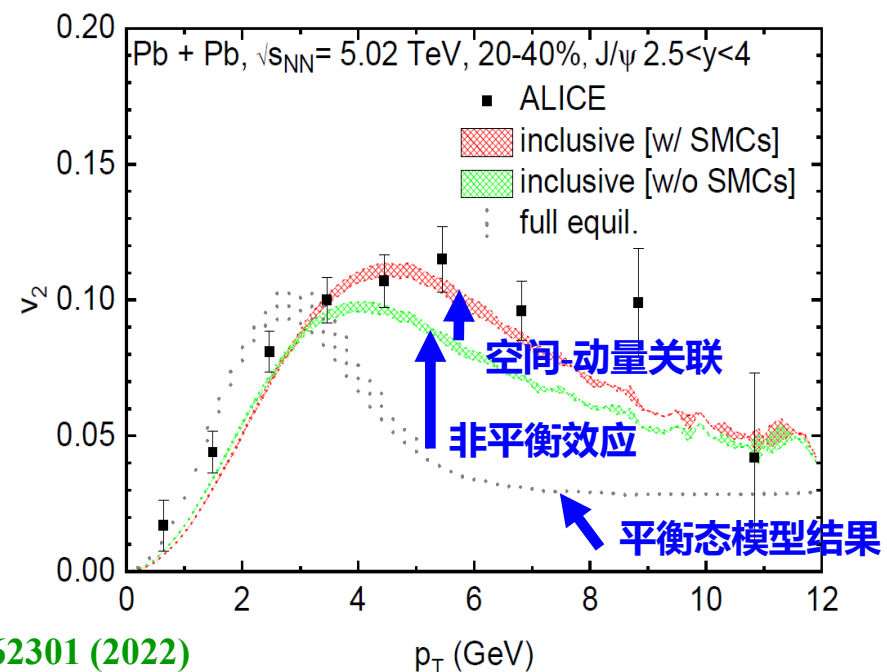
- RRM for J/ψ p<sub>T</sub>-shape: normalized to N<sup>regeneration</sup>

$$f_{\Psi}(\vec{x}, \vec{p}) = C_{\Psi} \frac{E_{\Psi}(\vec{p})}{m_{\Psi} \Gamma_{\Psi}} \int \frac{d^3 \vec{p}_1 d^3 \vec{p}_2}{(2\pi)^3} f_c(\vec{x}, \vec{p}_1) f_{\bar{c}}(\vec{x}, \vec{p}_2) \times \sigma_{\Psi}(s) v_{\text{rel}}(\vec{p}_1, \vec{p}_2) \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2)$$

Off-equilibrium Langevin c/cbar distributions constrained by D-meson observables



MH, Wu & Rapp, PRL 128, 162301 (2022)

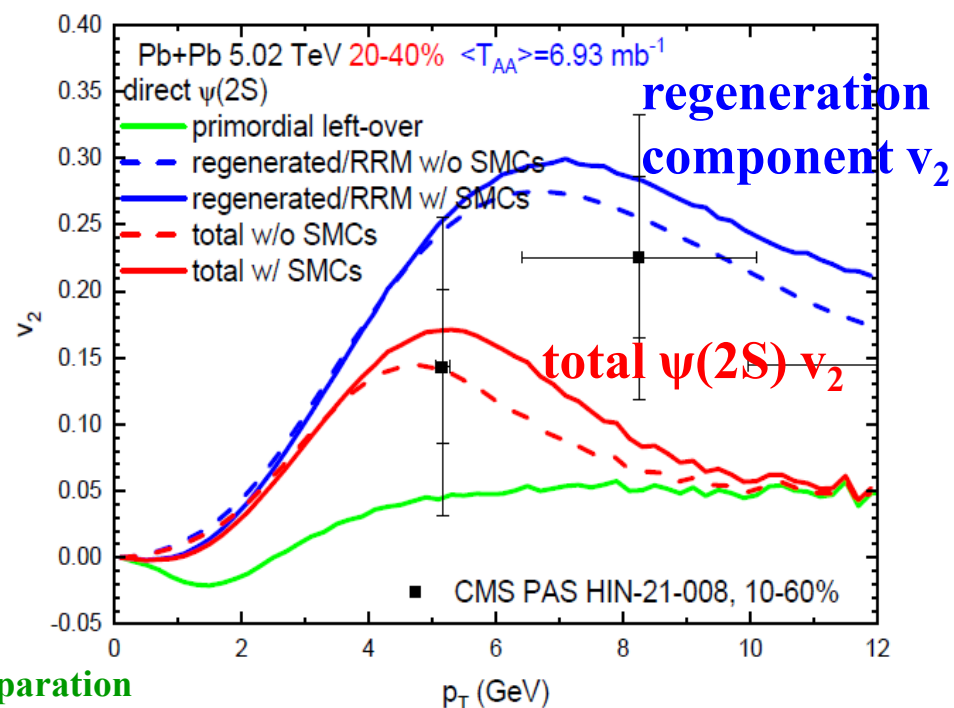
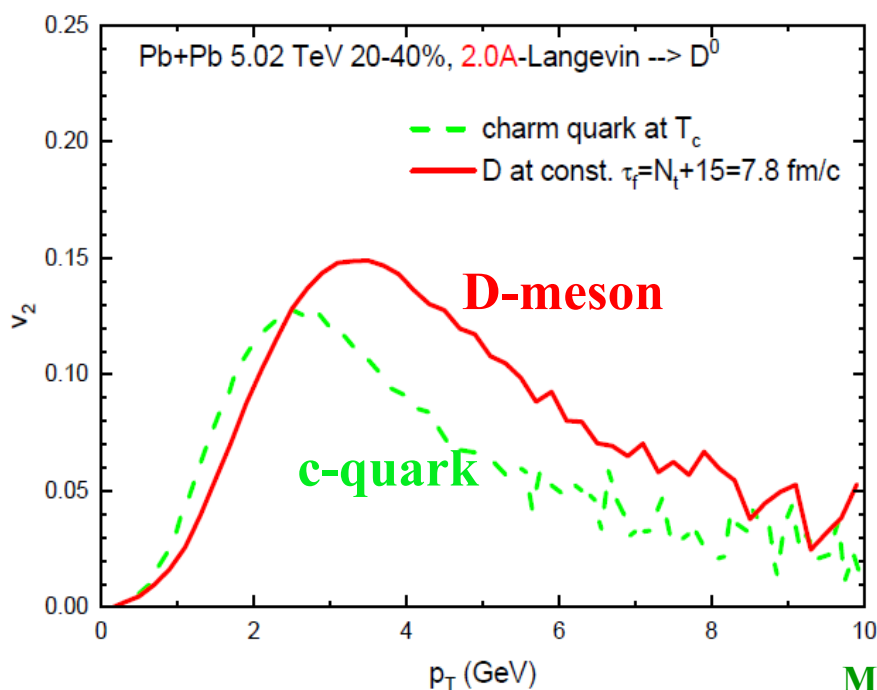


- Intermediate p<sub>T</sub>: **harder off-equi. c/cbar spectra + SMCs** → p<sub>T</sub> reach of **regeneration** much extended
- High p<sub>T</sub>: hydro simulation of suppression of primordial ψ → path-length dependent v<sub>2</sub> ~ 0.05

# Sequential regeneration: $v_2$ of $\psi(2S)$

- Binding energy  $J/\psi > \psi(2S) \rightarrow$  dissociation temperature  $T_d J/\psi > \psi(2S)$   
 $\rightarrow$  regeneration of  $\psi(2S)$  at  $T_d \sim T_c$  in hadronic phase, significantly later than  $J/\psi$

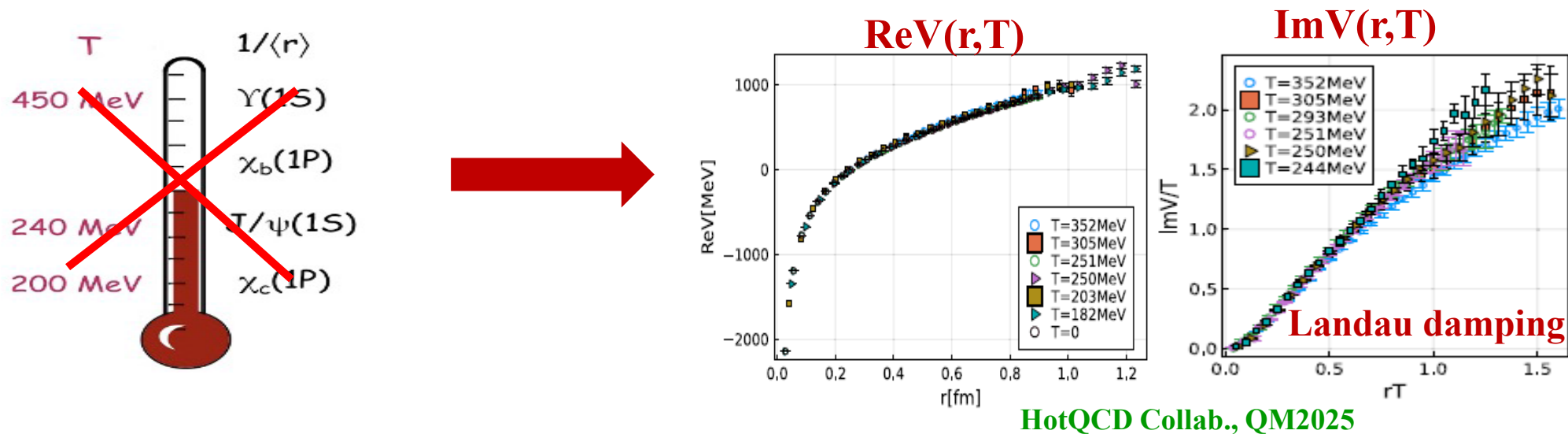
$$f_{\psi'}(\vec{x}, \vec{p}) = C_{\psi'} \frac{\gamma_{p, \psi'}}{\Gamma_{\psi'}} \int \frac{d^3 \vec{p}_1 d^3 \vec{p}_2}{(2\pi)^3} f_D(\vec{x}, \vec{p}_1) f_{\bar{D}}(\vec{x}, \vec{p}_2) \sigma_{\psi'}(s) v_{\text{rel}}(\vec{p}_1, \vec{p}_2) \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2)$$



- Regeneration  $c + c\bar{c} \rightarrow J/\psi$   
but  $D + D\bar{c} \rightarrow \psi(2S)$
- $\psi(2S) v_2 \sim 17\%$  vs.  $J/\psi \sim 11\%$ , a strong indication of sequential regene. if confirmed

# Take-aways from Part III

- Heavy quarkonia melt through large reaction rates ( $\leftrightarrow$  small HQ  $\mathcal{D}_s$ ), rather than static screening of color force
  - lattice QCD: strong HQ potential ReV with little screening near  $T_c$ , but large  $\text{Im}V \sim \Gamma_{\text{diss}}$
  - probe of in-medium force via in-medium “spectroscopy”, not “thermometer” via static screening

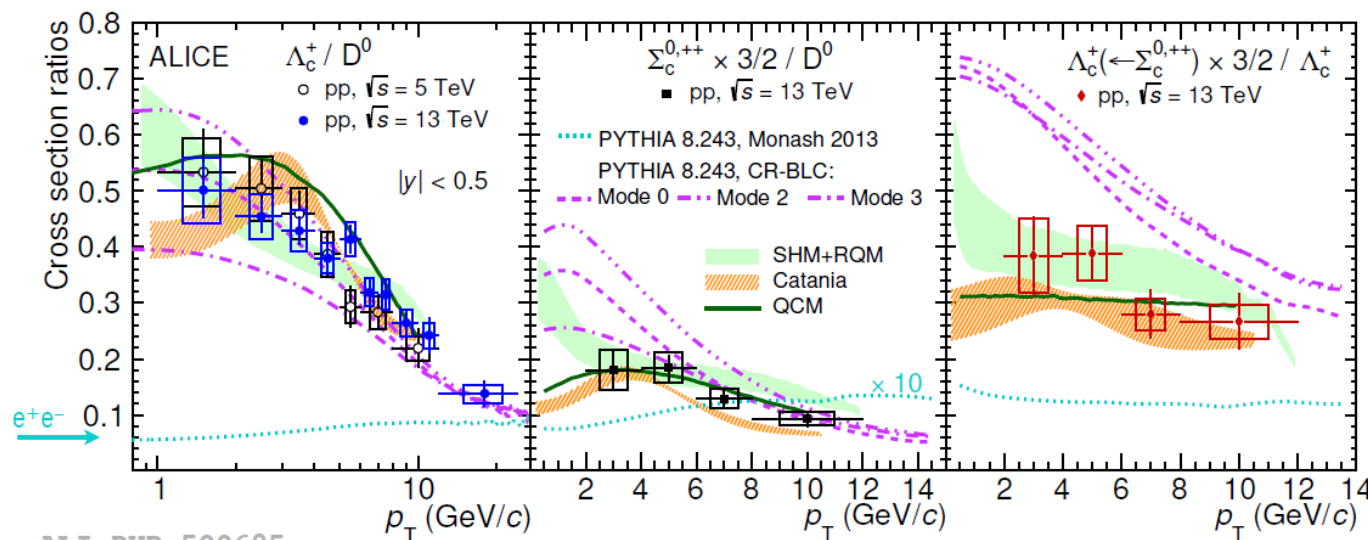


- Quantitative connections between open-  $\leftrightarrow$  hidden-charm transport
  - transported c/cbar distributions &  $d\sigma^{cc}/dy$  via regeneration

# Summary & outlook

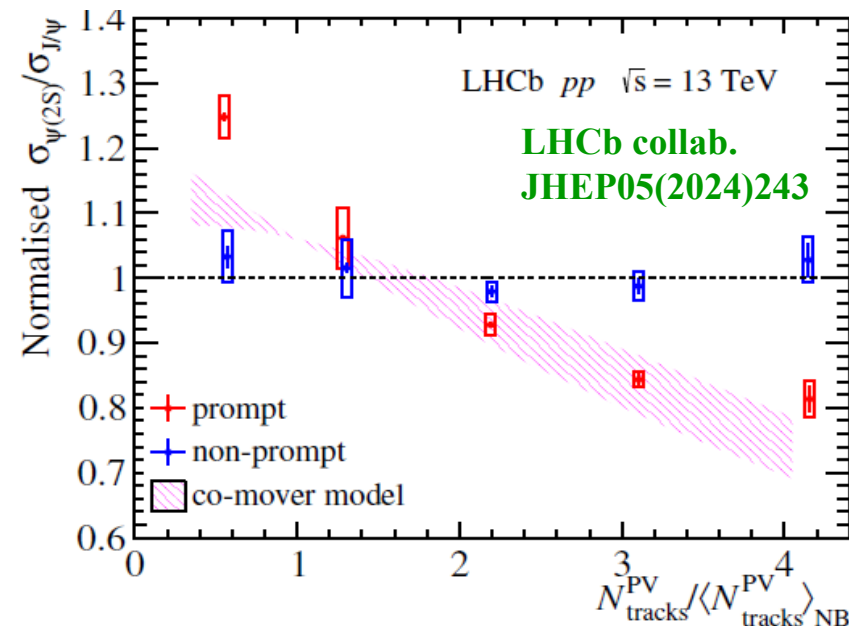
- HFs: excellent probes of sQGP transport properties, hadronization mechanisms, in-medium force & and inner structure/working
    - a small open HF diffusion coefficient  $\mathcal{D}_s$
    - recombination/color neutralization important
    - quarkonia melting by large reaction rates
    - close connection between open- & hidden-HF
- strongly coupled QGP likely supported by significant remnant confining force well above  $T_c$
- Outlook: LHC Run 3/4, sPHENIX, SHINE/NA61, ALICE3 ...
    - pp:  $\Xi_c$  production puzzle, rapidity dependence of  $\Lambda_c/D$ ?
    - p-dependence of  $\mathcal{D}_s$ : nonperturbative diffusion  $\rightarrow$  perturbative radiative e-loss
    - quantum effects in open & hidden HF transport

# Back-up: $\Sigma_c/D$ & $\psi(2S)/J/\psi$ in pp collisions



- Ground and excited charm baryons enhanced compared to  $e^+e^-$  collisions

ALICE, PRL 128 (2022) 012001



# Ground-state b-hadron densities/ratios

- total density & production fractions of ground state b-hadrons @  $T_H=170$  MeV

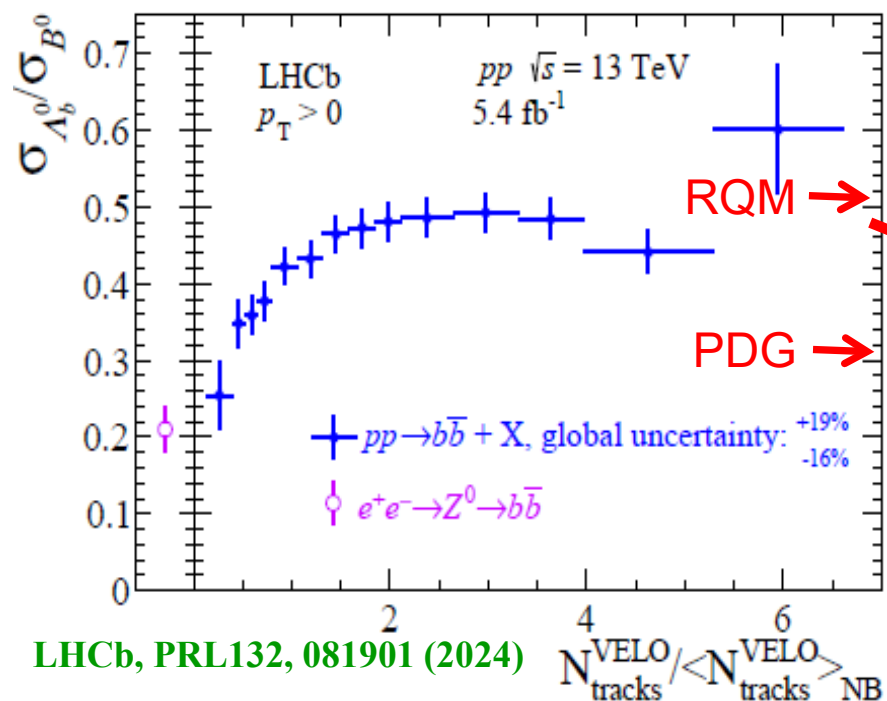
$$n_\alpha = n_\alpha^{\text{primary}} + \sum_i n_i^{\text{primary}} \cdot BR(i \rightarrow \alpha)$$

| $n_\alpha$ ( $\cdot 10^{-12} \text{ fm}^{-3}$ ) | $B^-$         | $\bar{B}^0$ | $\bar{B}_s^0$ | $\Lambda_b^0$  | $\Xi_b^{0,-}$ | $\Omega_b^-$ |
|-------------------------------------------------|---------------|-------------|---------------|----------------|---------------|--------------|
| PDG                                             | <u>1.0094</u> | 1.0089      | 0.29308       | <u>0.31591</u> | 0.10097       | 0.002341     |
| RQM                                             | <u>1.2045</u> | 1.2041      | 0.32513       | <u>0.61702</u> | 0.19548       | 0.0063204    |

$$f_u + f_d + f_s + f_{\Lambda_b^0} + f_{\Xi_b^{0,-}} + f_{\Omega_b^-} = 1$$

| $f_\alpha$ | $B^-$  | $\bar{B}^0$   | $\bar{B}_s^0$ | $\Lambda_b^0$ | $\Xi_b^{0,-}$ |
|------------|--------|---------------|---------------|---------------|---------------|
| PDG        | 0.3697 | 0.3695        | 0.1073        | 0.1157        | 0.03698       |
| RQM        | 0.3391 | <u>0.3389</u> | 0.09152       | <u>0.1737</u> | 0.05503       |

→ Agree with Tevatron p-pbar



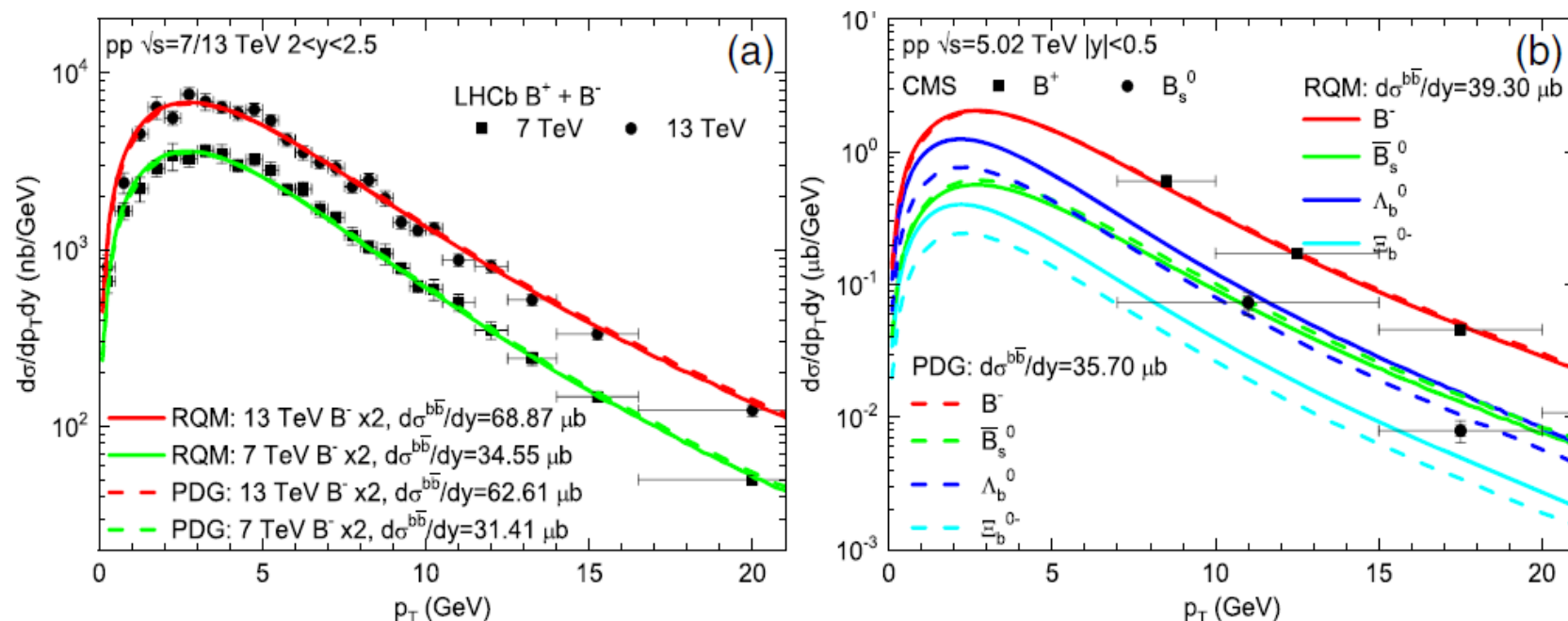
- production ratios at large  $dN_{\text{ch}}/d\eta$ :  
RQM favored!

| $r_\alpha$ | $\bar{B}^0/B^-$ | $\bar{B}_s^0/B^-$ | <u><math>\Lambda_b^0/B^-</math></u> | $\Xi_b^{0,-}/B^-$ |
|------------|-----------------|-------------------|-------------------------------------|-------------------|
| PDG        | 0.9995          | 0.2904            | <u>0.3129</u>                       | 0.1000            |
| RQM        | 0.9994          | 0.2699            | <u>0.5122</u>                       | 0.1623            |

# Fragmentation & $p_T$ -spectra

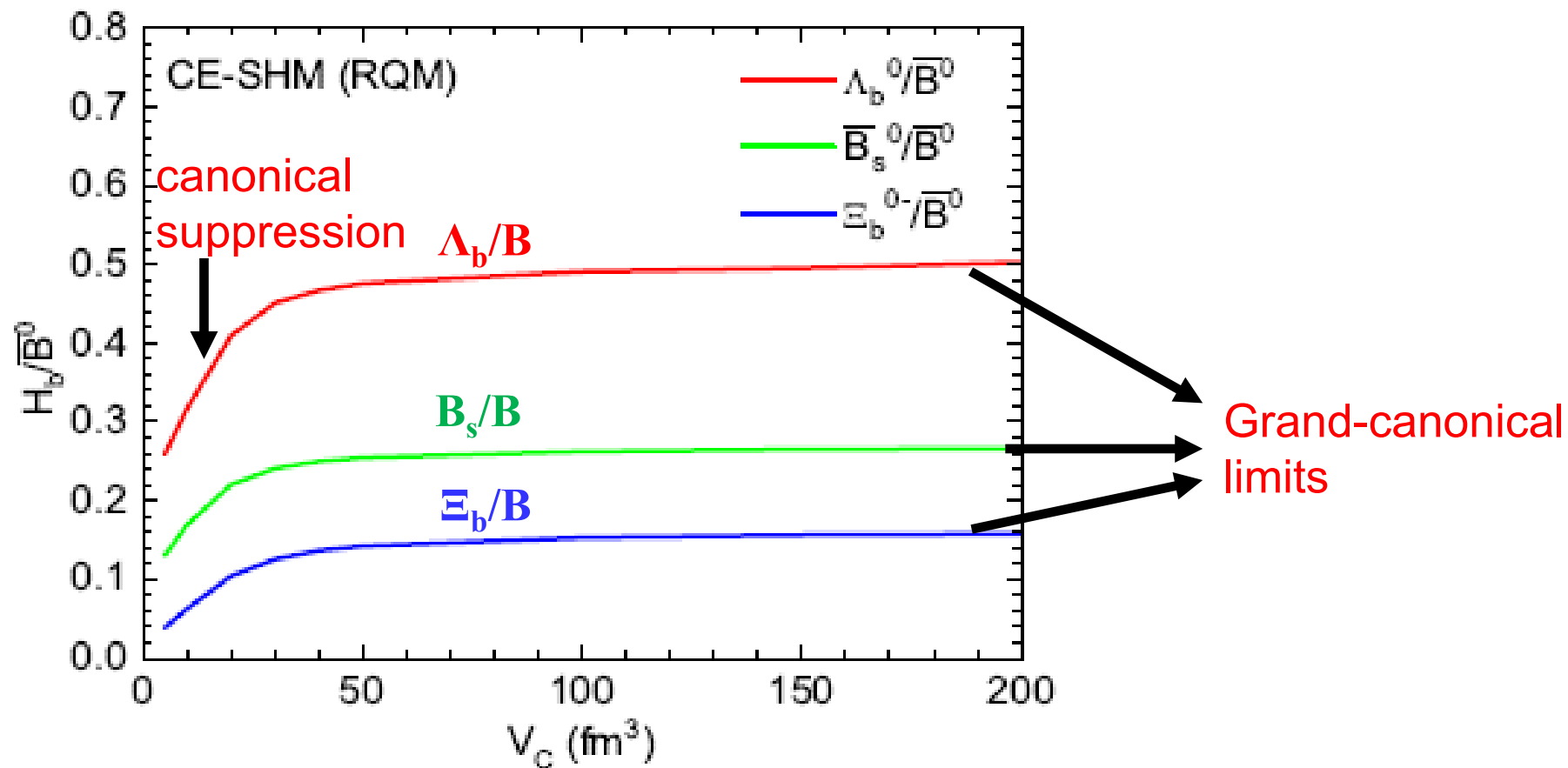
- FONLL b-quark  $p_t$ -spectrum + fragmentation into **all primary** states + decay simulations  
 $\rightarrow$  ground-state b-hadrons  $p_T$ -spectra:  $z = p_T/p_t$

$$D_{b \rightarrow H_b}(z) \propto z^\alpha (1-z), \quad \left\{ \begin{array}{l} \text{weight} \propto \text{primary density (relative chemical equilibrium)} \\ \alpha_B = 45, \alpha_{B_s} = 25, \alpha_{\text{baryon}} = 8 \text{ to tune the slope of spectra} \end{array} \right.$$



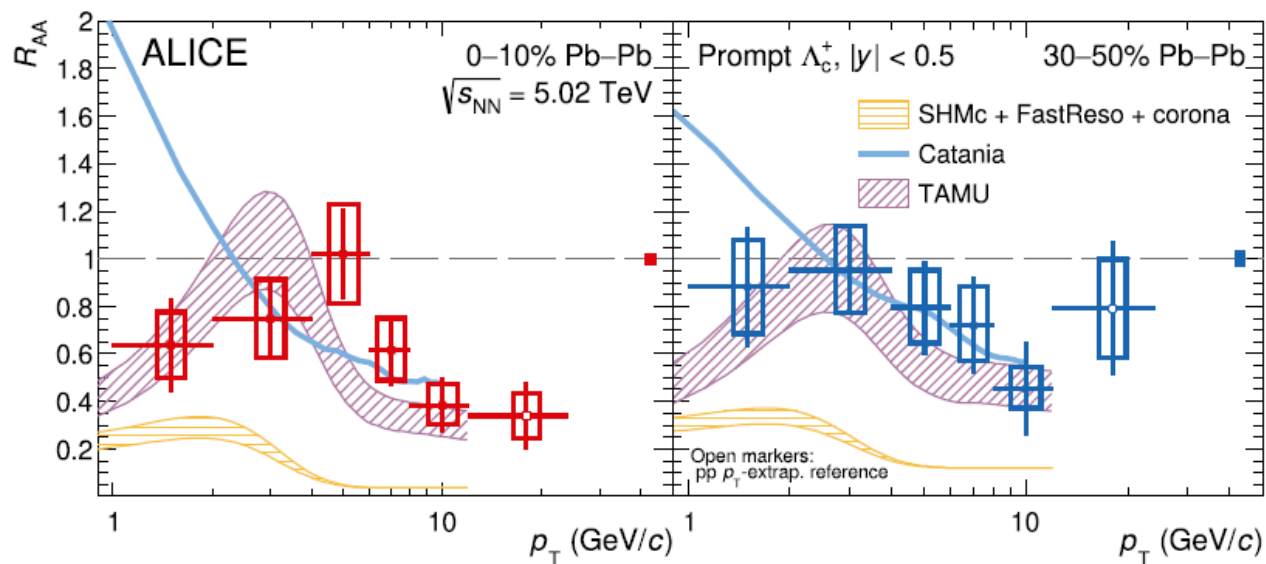
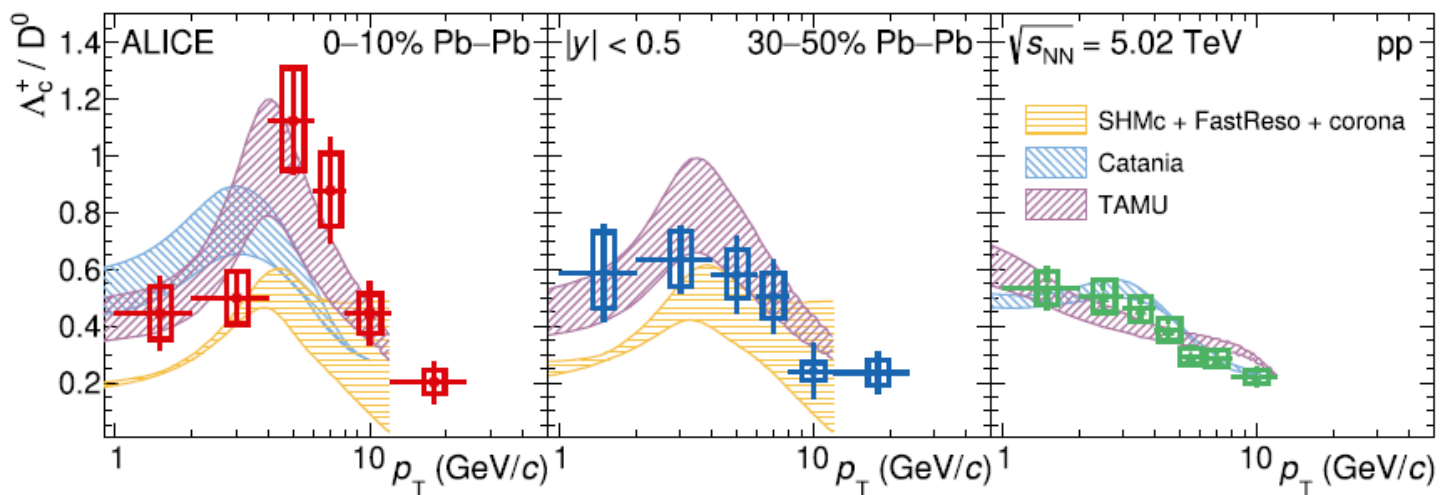
- Fitting meson spectra  $\rightarrow$  **predicting** baryon & total  $d\sigma^{b\bar{b}\text{bar}}/dy = 39.3 \mu\text{b}$  for 5.02 TeV mid-y based on SHM chemistry  $\rightarrow$  **baseline for b-hadron production in Pb-Pb collisions**

# Ground-state b-hadron ratios vs Volume

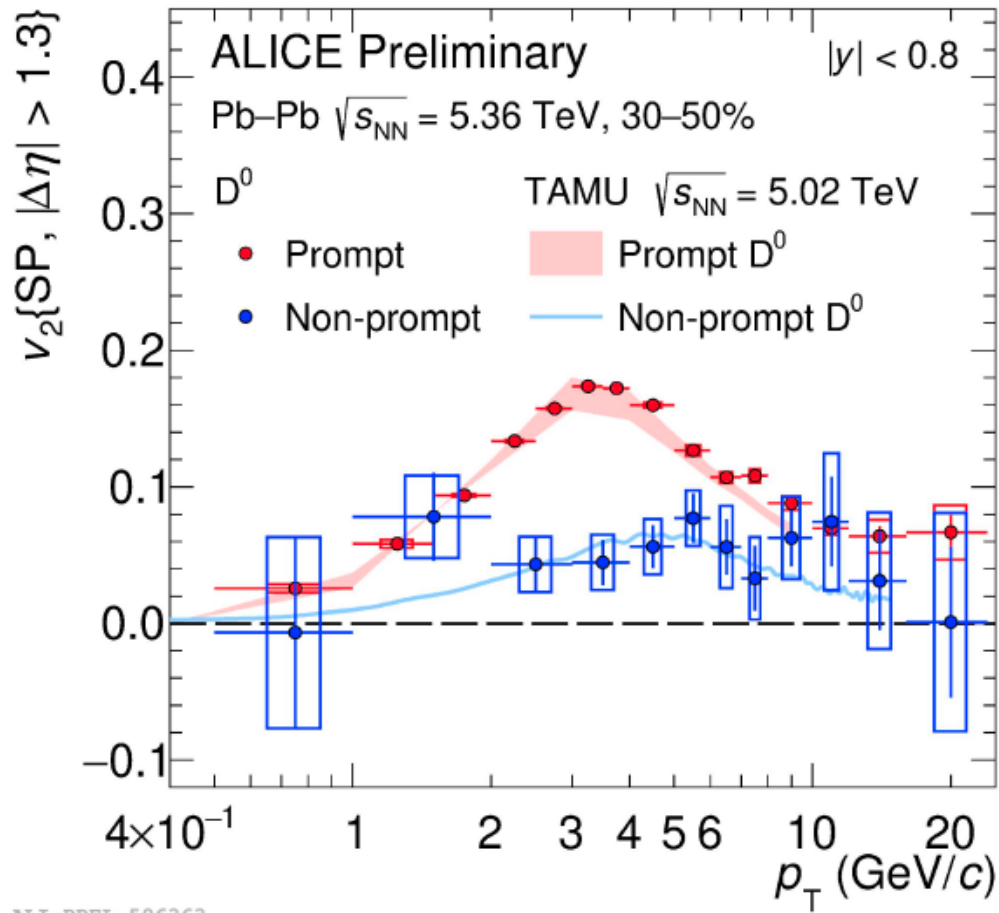


- As volume/system size reduces,  $B_s/B$ ,  $\Lambda_b/B$  suppressed by a factor 2;  $\Xi_b/B$  suppression stronger, two-fold role of baryon + strangeness
- All ratios tend to the corresponding GCE-SHM values at large system size

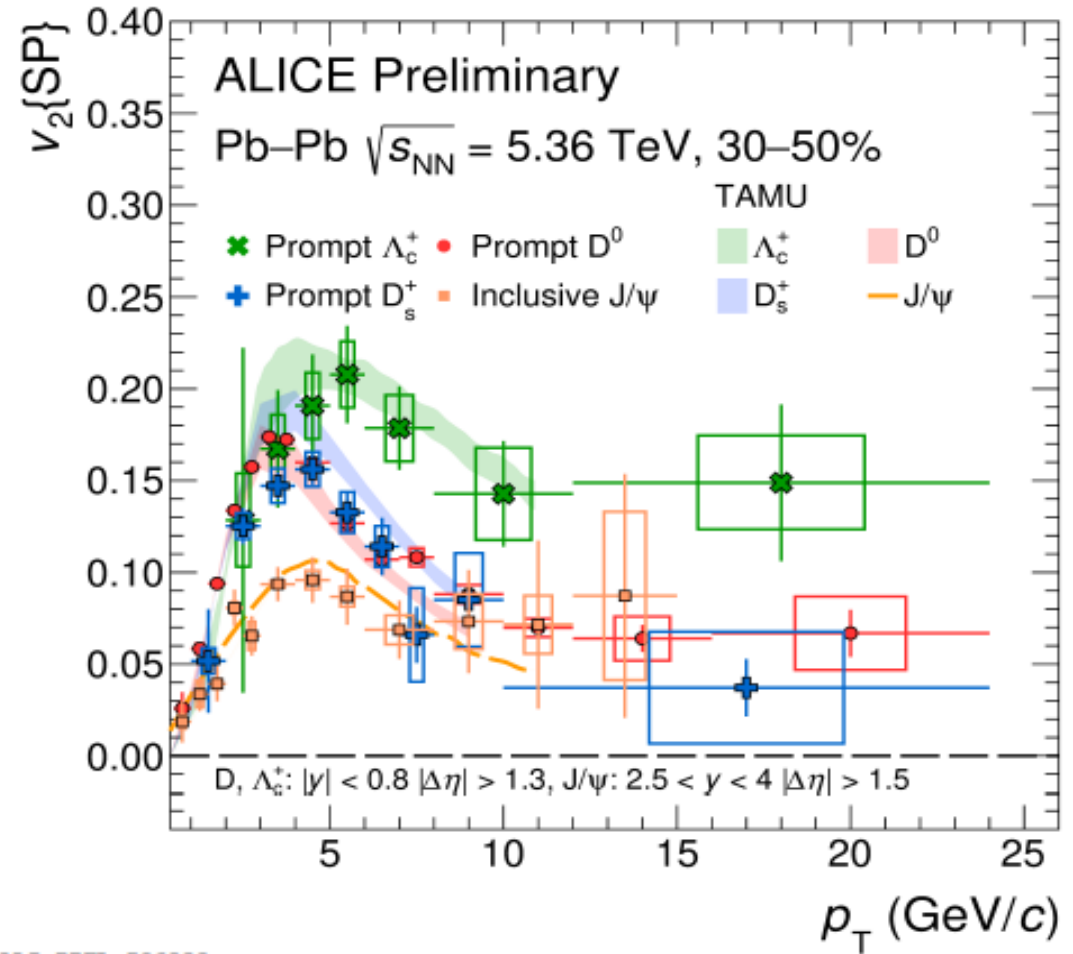
# $\Lambda_c/D$ & $\Lambda_c R_{AA}$ , ALICE PLB 839 (2023) 137796



# HF elliptic flows, ALICE highlights @ QM2025



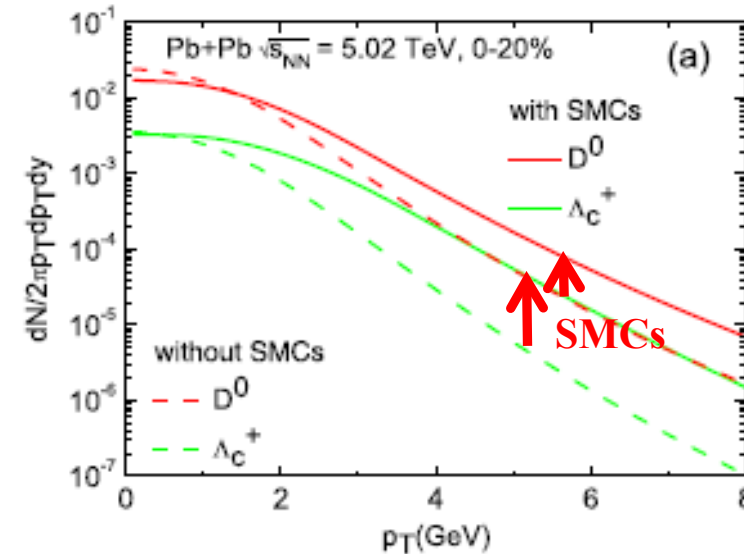
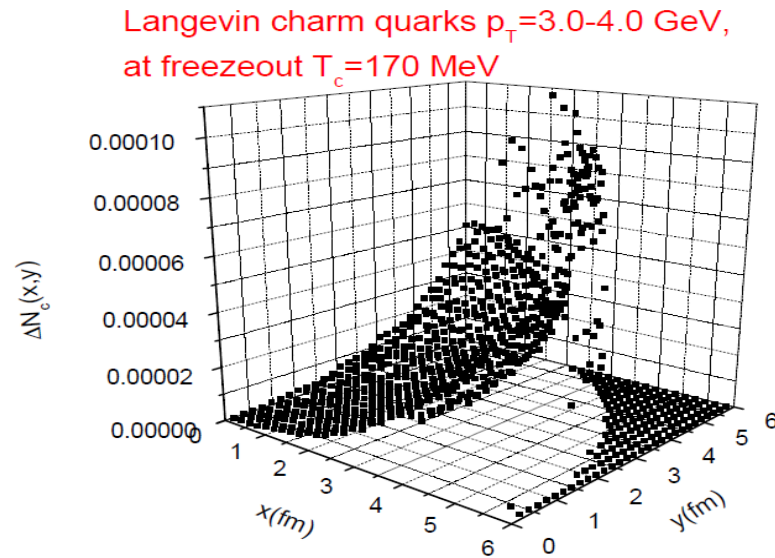
ALI-PREL-596363



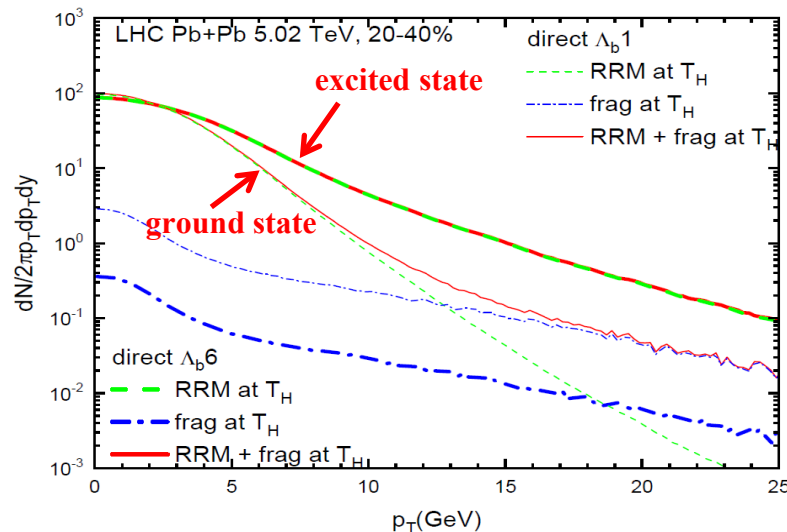
ALI-PREL-596333

# Recombination: space-momentum correlations

- Inhomogeneous distribution: SMCs → recombination beyond momentum space

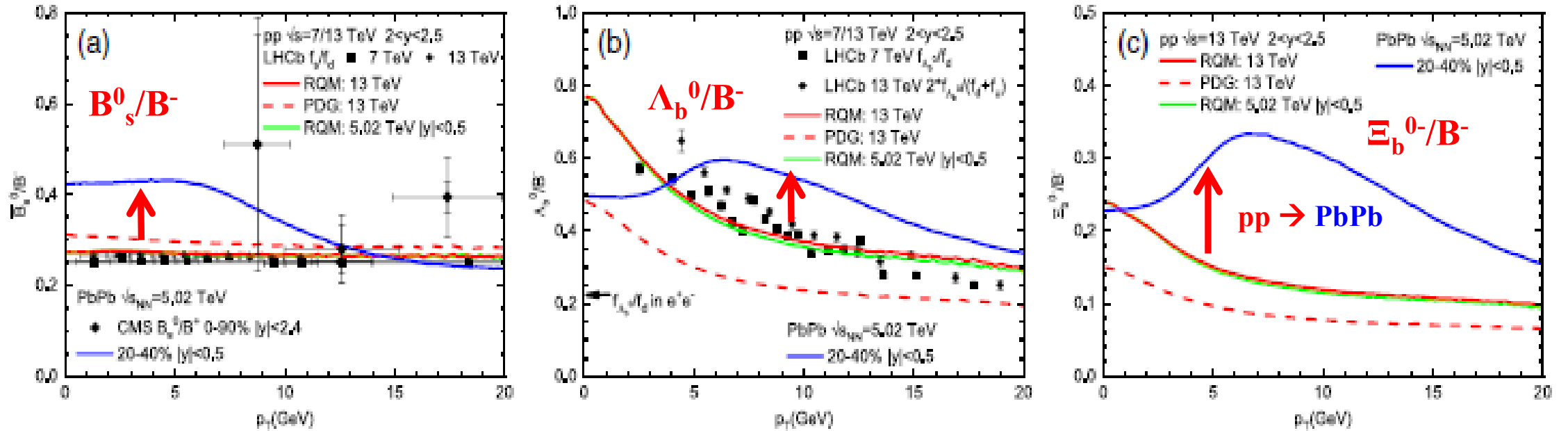


- Left-over b-quarks: fragmentation in the same manner as in pp



- Excited state more massive: **recombination spectrum harder** than ground state (SMCs/flow)
- RRM vs frag. crossing at  $p_T \sim 14$  GeV for  $\Lambda_b 1$  vs  $\Lambda_b 6$  at  $p_T > 25$  GeV

# Modifications of bottom hadro-chemistry



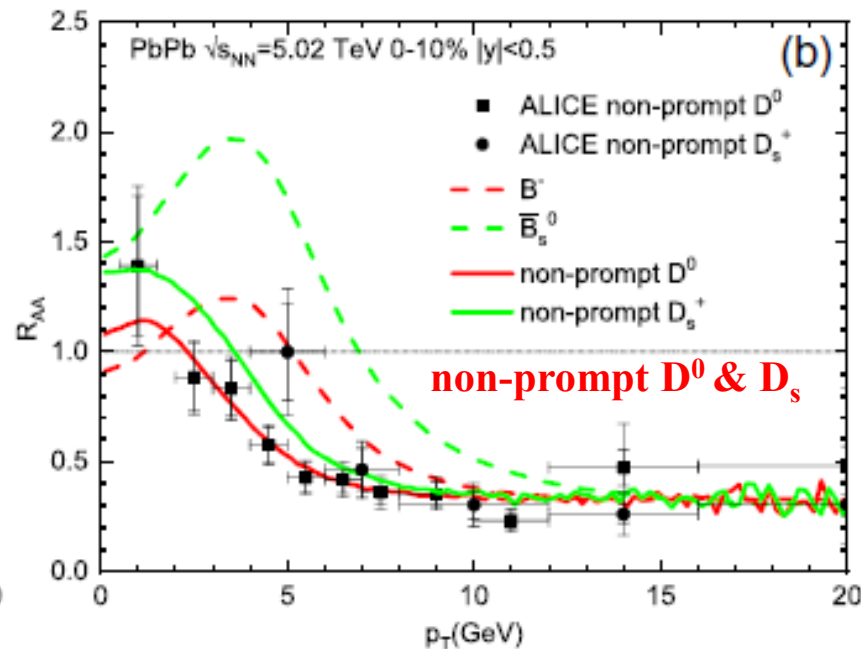
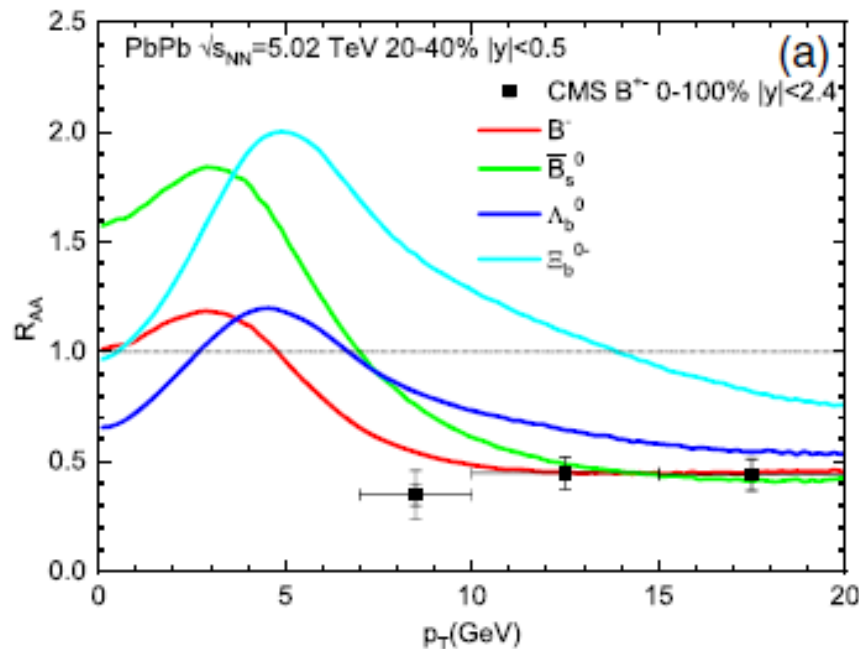
- $pp \rightarrow PbPb$

- $B_s/B^-$  – enhancement at low  $p_T$ : b coupled to equilibrated strangeness via recombination
- $\Lambda_b/B^-$  – flow-bump at intermediate  $p_T \sim 5-15$  GeV [significantly higher than c-sector]: stronger flow push on baryons, captured by 3-body RRM with SMCs
- $\Xi_b/B^-$  – enhancement more pronounced: combining two-fold role of containing a s-quark & being a 3-body baryon

# b-hadron nuclear modification factors

- $R_{AA}$ : hierarchy of **flow effects** and suppression driven by their quark content

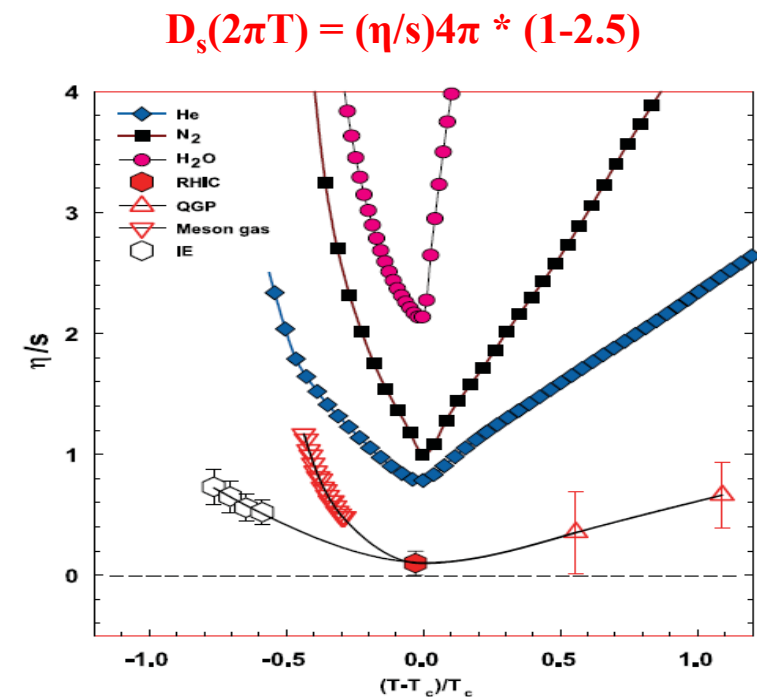
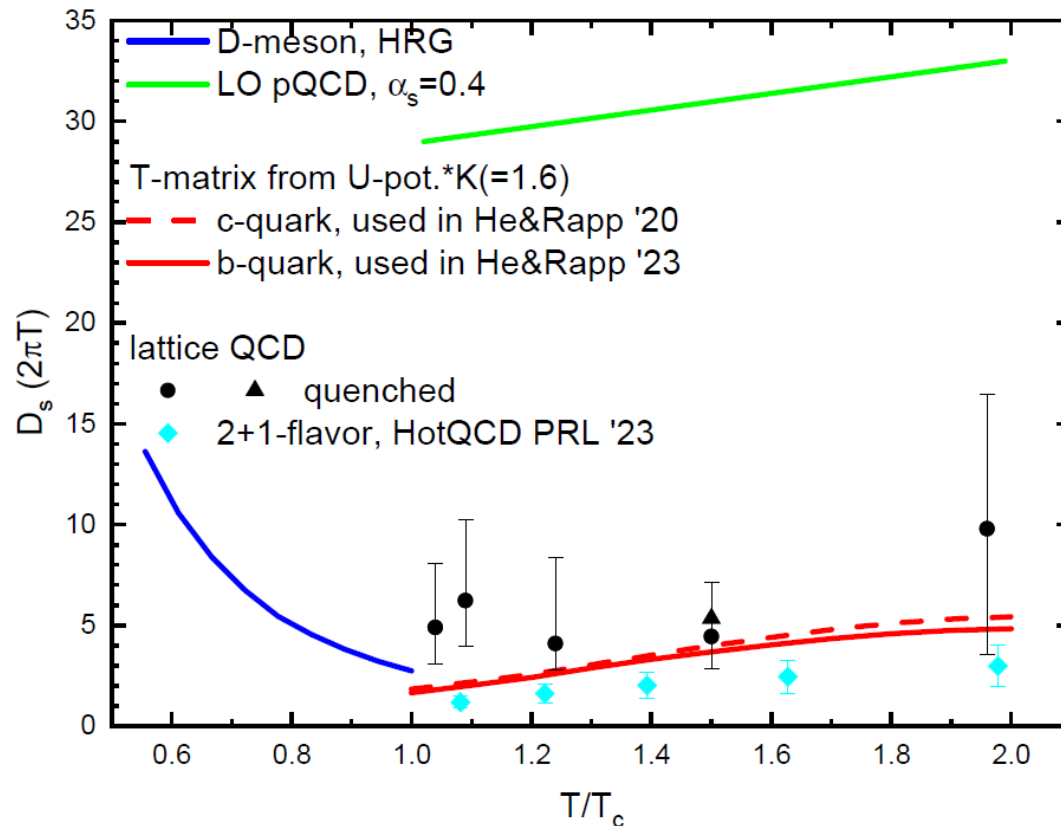
$$R_{AA}(p_T) = \frac{dN^{AA}/dp_T dy}{\langle T_{AA} \rangle d\sigma^{pp}/dp_T dy} \longrightarrow \begin{array}{l} dN^{b\bar{b}}/dy \sim 0.9 \text{ for } 0-10\% \\ \text{pp reference from SHM} \end{array}$$



- $B_s$ : b-quark coupled to equilibrated strangeness via recombination
- $\Lambda_b$ : 3-body baryon recombination, RRM with SMCs
- $\Xi_b$ : combining two-fold role of being baryon + containing a s-quark
- Non-prompt  $D^0$  &  $D_s$ : weak decays of **all b-hadrons** via PYTHIA8

# Transport coefficient $\mathcal{D}_s(2\pi T)$

- HQ spatial diffusion coefficient:  $\mathcal{D}_s = T/m_Q A(p=0) = T/m_Q \gamma \rightarrow \langle x^2 \rangle \sim \mathcal{D}_s t$



- models & lattice  $\mathcal{D}_s(2\pi T) \sim 1-3$  near  $T_c$ , x10 smaller than pQCD  $\rightarrow$  collisional rate  $\Gamma_{\text{coll}} \sim 3/\mathcal{D}_s \sim 1 \text{ GeV} > M_{q,g} \rightarrow$  thermal partons melted, Brownian markers/HQs survive
- maximum coupling strength near  $T_c \rightarrow$  small  $\mathcal{D}_s$  &  $\eta/s \rightarrow$  strongly coupled QGP