

拥抱高精度强子谱学时代

刘翔

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兰州大学

“第八届强子谱和强子结构研讨会”

桂林 2025年7月11日-15日

Outline

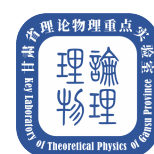
- 五夸克态发现十周年 **精度匹配**
- 典型新强子态的“低质量疑难”与
非淬火效应 **精度提升**
- 强子体系少体问题 **计算精度**
- 小结



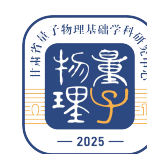
兰州理论物理中心
Lanzhou Center for Theoretical Physics



量子理论及应用基础
教育部重点实验室
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甘肃省量子物理
基础学科研究中心
Key Gansu Provincial Research Center for Basic
Disciplines of Quantum Physics

1. 五夸克态发现十周年—精度匹配



<https://physics.aps.org/articles/v8/126>

PRL **115**, 072001 (2015)

Selected for a **Viewpoint** in *Physics*
PHYSICAL REVIEW LETTERS

week ending
14 AUGUST 2015



Observation of $J/\psi p$ Resonances Consistent with Pentaquark States in $\Lambda_b^0 \rightarrow J/\psi K^- p$ Decays

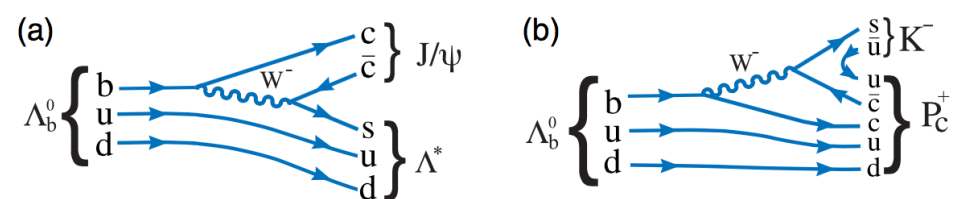
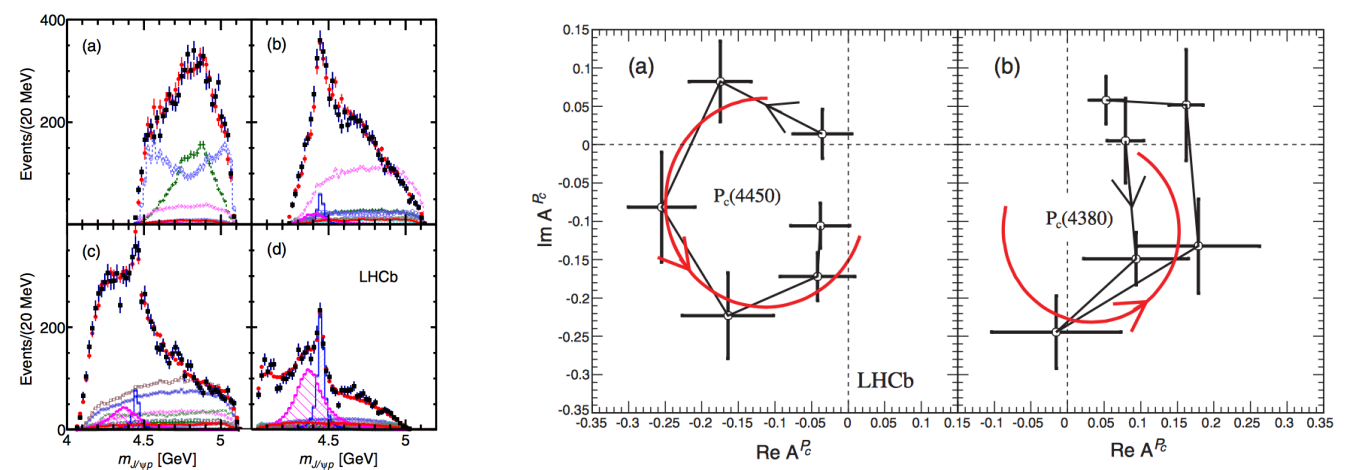


FIG. 1 (color online). Feynman diagrams for (a) $\Lambda_b^0 \rightarrow J/\psi \Lambda^*$ and (b) $\Lambda_b^0 \rightarrow P_c^+ K^-$ decay.



84. Pentaquarks

Revised March 2024 by M. Karliner (Tel Aviv U.) and T. Skwarnicki (Syracuse U.).

Experimental searches for pentaquark hadrons comprised of light flavors have a long and vivid history. No undisputed candidates had been found in 50 years. The first wave of claimed observations of pentaquark candidates containing a strange antiquark occurred in the early seventies, see e.g. a review in the 1976 edition of Particle Data Group listings for $Z_0(1780)$, $Z_0(1865)$ and $Z_1(1900)$ [1]. The last mention of these candidates can be found in the 1992 edition [2] with the perhaps prophetic comment “the results permit no definite conclusion - the same story for 20 years. [...] The skepticism about baryons not made of three quarks, and lack of any experimental activity in this area, make it likely that another 20 years will pass before the issue is decided.” A decade later, a second wave of observations occurred, possibly motivated by specific theoretical predictions for their existence [3–5]. The evidence for pentaquarks was based on observations of peaks in the invariant mass distributions of their decay products. More data and subsequent more sensitive experiments did not confirm these claims [6]. In the last mention of the best known candidate

from that period, conclusion that pelling.” which we of $\Lambda_b^0 \rightarrow J/\psi p K^-$ an analysis of 3 f a significant $J/\psi p$ with the minimal Ref. [9], where it v accounted for by from the earlier m angles describing together with the in the number of narrow $J/\psi p$ struc second horizontal

The 4450 MeV Performing a rigor ing and has not b it is not necessary of interfering Λ^* r have been charact tributions, with di masses (Fig. 84.1) analyzed by the L

The fit chosen $P_{cc}(4312)^+$ state statistical signific while the $P_{cc}(4440)$ one-peak hypothes of this region obs addition to the str obsolete since it u

While $\Sigma_c \bar{D}^{(*)}$ states had been predicted [14–17] before the first LHCb results [8], after these results became known, many theoretical groups interpreted the $P_{cc}(4450)^+$ and $P_{cc}(4380)^+$ states in terms of diquarks and triquarks as building blocks of a compact pentaquark [18–24]. In a different strategy, a tentative attempt has been made to treat the full 5-body dynamics, leading to states below the lowest threshold for spontaneous dissociation [25]. In the first implementation of the former approach [18], the pentaquark mass splitting was generated mostly by the change of angular momentum between the sub-components (L) from zero to one, which would also make

- [14] Z.-C. Yang *et al.*, *Chin. Phys.* **C36**, 6 (2012), [[arXiv:1105.2901](#)].
- [15] J.-J. Wu *et al.*, *Phys. Rev. Lett.* **105**, 232001 (2010), [[arXiv:1007.0573](#)].
- [16] J.-J. Wu, T. S. H. Lee and B. S. Zou, *Phys. Rev.* **C85**, 044002 (2012), [[arXiv:1202.1036](#)].
- [17] M. Karliner and J. L. Rosner, *Phys. Rev. Lett.* **115**, 12, 122001 (2015), [[arXiv:1506.06386](#)].

五夸克态的发现得益于 理论精度与实验精度相匹配

“ $\Sigma_c \bar{D}^{(*)}$ states had been predicted before the first LHCb result”



Identifying Exotic Hidden-Charm Pentaquarks

Rui Chen and Xiang Liu^{*}

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School of Physics, Nankai University, Tianjin 300071, China

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and Center of High Energy Physics, Peking University, Beijing 100871, China
(Received 14 July 2015; published 25 September 2015)*

The LHCb Collaboration at the Large Hadron Collider at CERN discovered two pentaquark states $P_c(4380)$ and $P_c(4450)$. These two hidden-charm states are interpreted as the loosely bound $\Sigma_c(2455)D^*$ and $\Sigma_c^*(2520)D^*$ molecular states in the boson exchange interaction model, which provides an explanation for why the experimental width of $P_c(4450)$ is much narrower than that of $P_c(4380)$. The discovery of the new resonances $P_c(4380)$ and $P_c(4450)$, indeed, opens a new page for hadron physics. The partners of $P_c(4380)$ and $P_c(4450)$ should be pursued in future experiments.



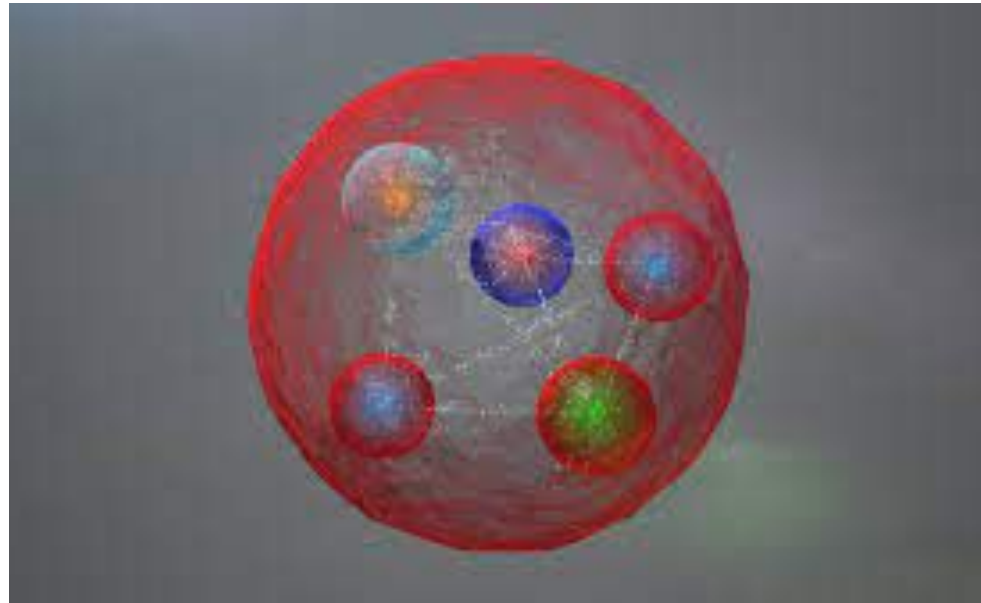
EDITORS' SUGGESTION

Identifying Exotic Hidden-Charm Pentaquarks

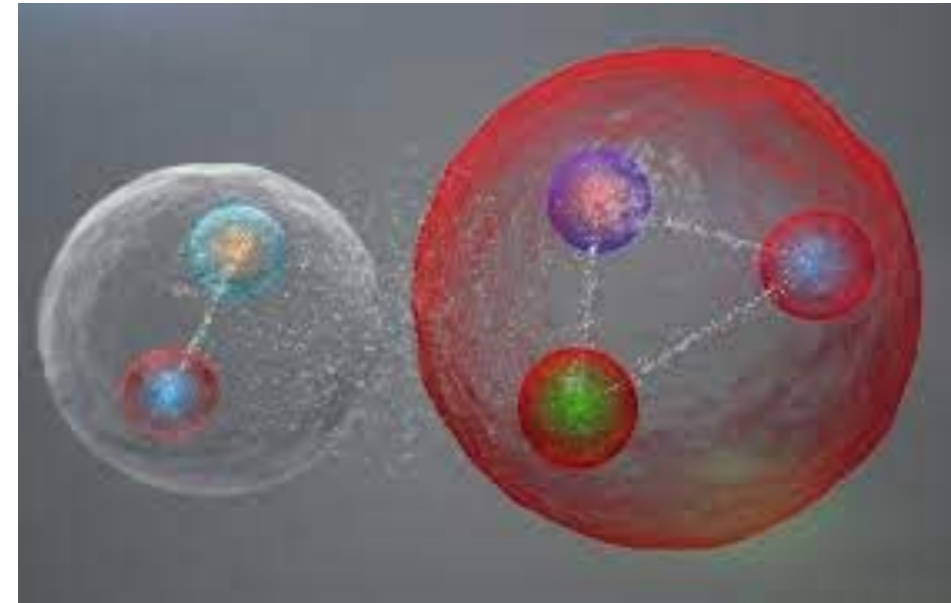
The pentaquarks discovered by the LHCb Collaboration could be molecular bound states of a charmed baryon and a meson. Observing the predicted isospin partners would allow for

**Decode the properties of two
observed P_c consistent with
hadronic molecular states**

LHCb实验2015的数据不足以区分紧致态和分子态构型



Compact pentaquark



Molecular state

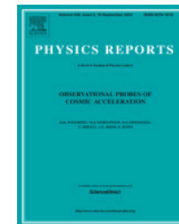
Physics Reports 639 (2016) 1–121



Contents lists available at [ScienceDirect](https://www.sciencedirect.com)

Physics Reports

journal homepage: www.elsevier.com/locate/physrep



需要更为精确的数据

The hidden-charm pentaquark and tetraquark states

Hua-Xing Chen^{a,b,1}, Wei Chen^{c,1}, Xiang Liu^{d,e,*}, Shi-Lin Zhu^{a,f,g,**}



猜测LHCb发现的 P_c 态还有子结构

As pointed out in Ref. [260], there may exist two or more resonant signals around 4380 MeV which are close to each other but may carry different parity. If the P-wave or higher excitation is very broad with a width around 500 MeV, such a state may easily be mistaken as the background. On the other hand, if an excitation lies several MeV within 4380 MeV but with a width as narrow as several MeV, then it may probably be buried by the $P_c(4380)$ resonance with a width around 205 MeV! The same situation may also occur around 4450 MeV.

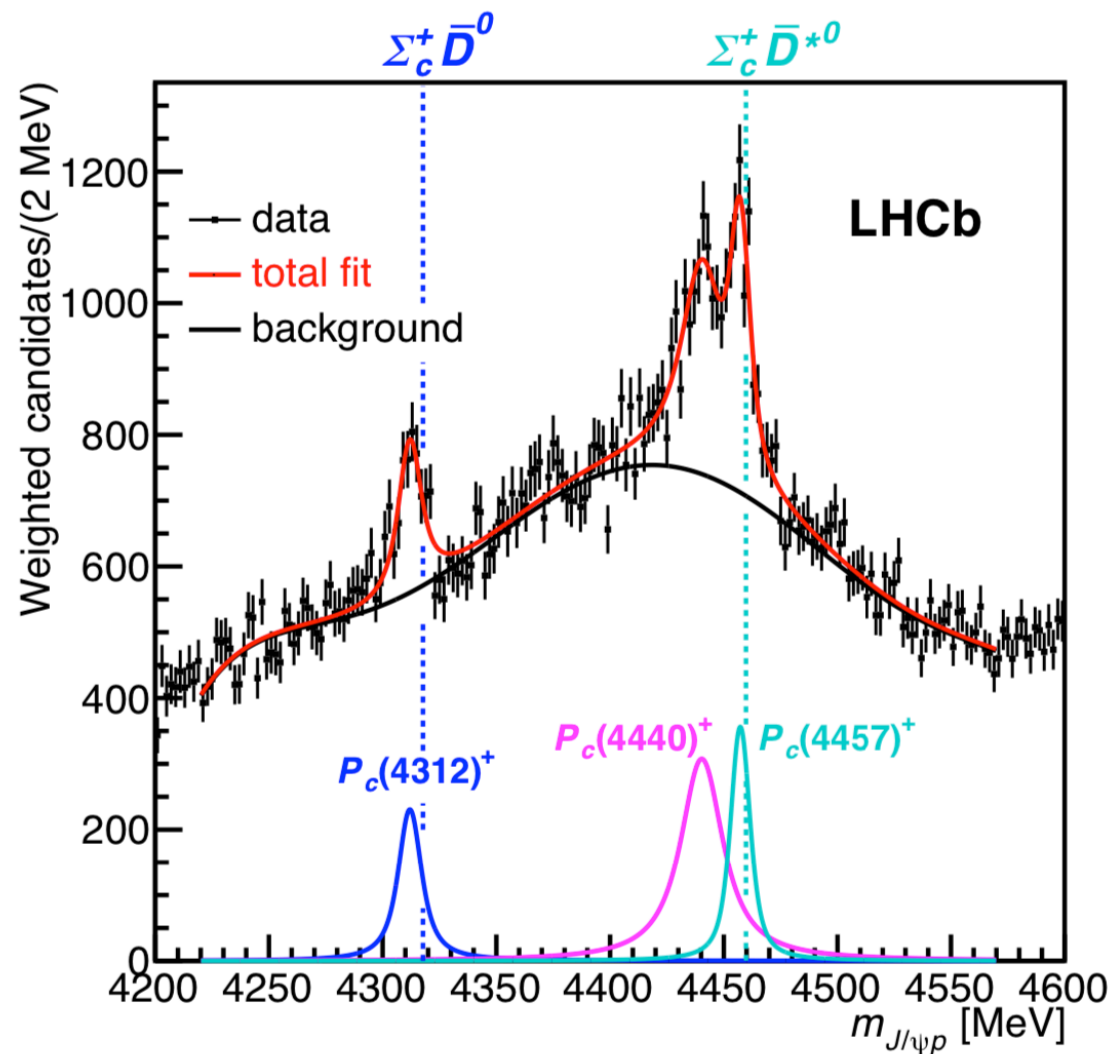
精度的重要性再次显现

PHYSICAL REVIEW LETTERS **122**, 222001 (2019)

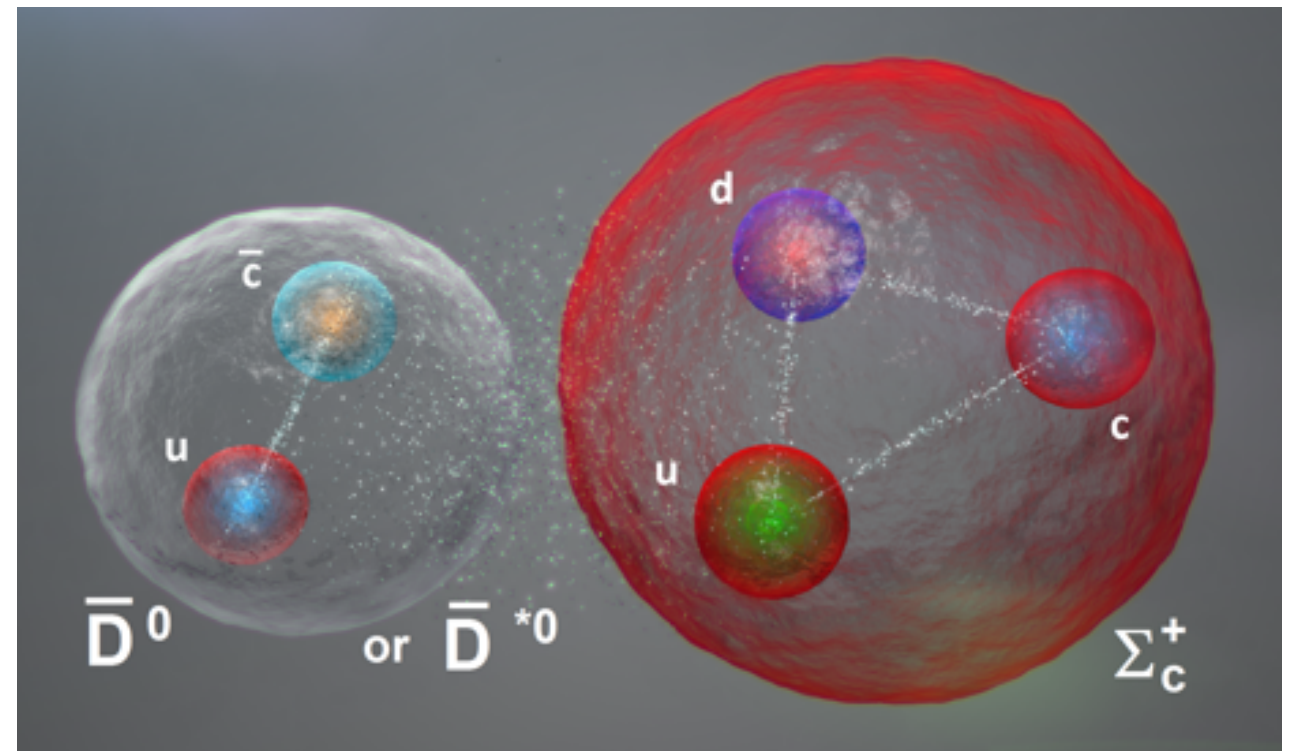
Editors' Suggestion

Featured in Physics

Observation of a Narrow Pentaquark State, $P_c(4312)^+$,
and of the Two-Peak Structure of the $P_c(4450)^+$

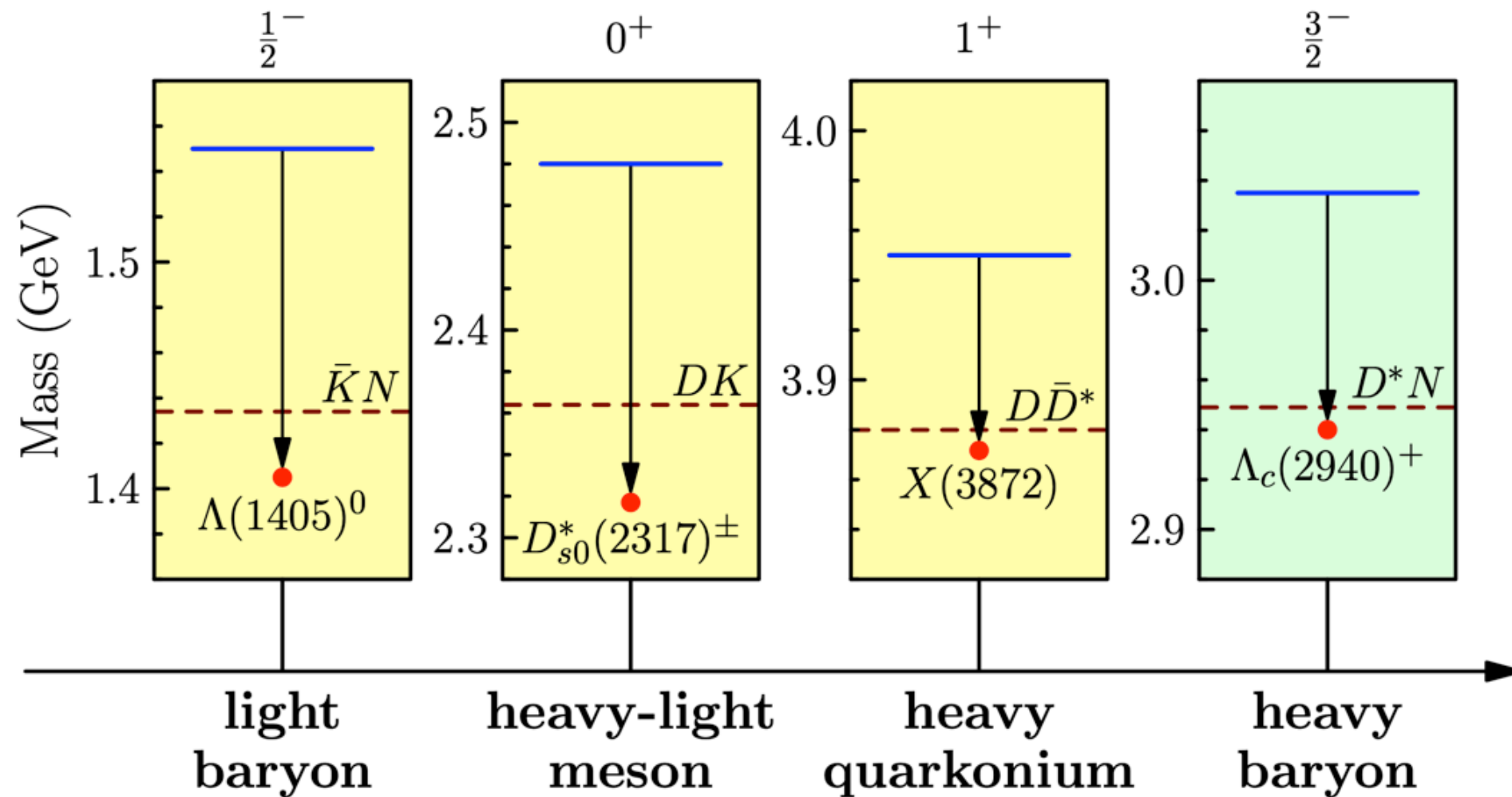


2019年高精度的实验数据支持分子态构型



2. 典型新强子态的“低质量疑难”与非淬火效应

“该粒子作为常规强子态时，其质量显著低于淬火夸克势模型的预期值”



Luo, Chen, Liu, XL, EPJC 80 (2020) 301

康奈尔势模型是典型的淬火势模型之一

PHYSICAL REVIEW D

VOLUME 17, NUMBER 11

1 JUNE 1978

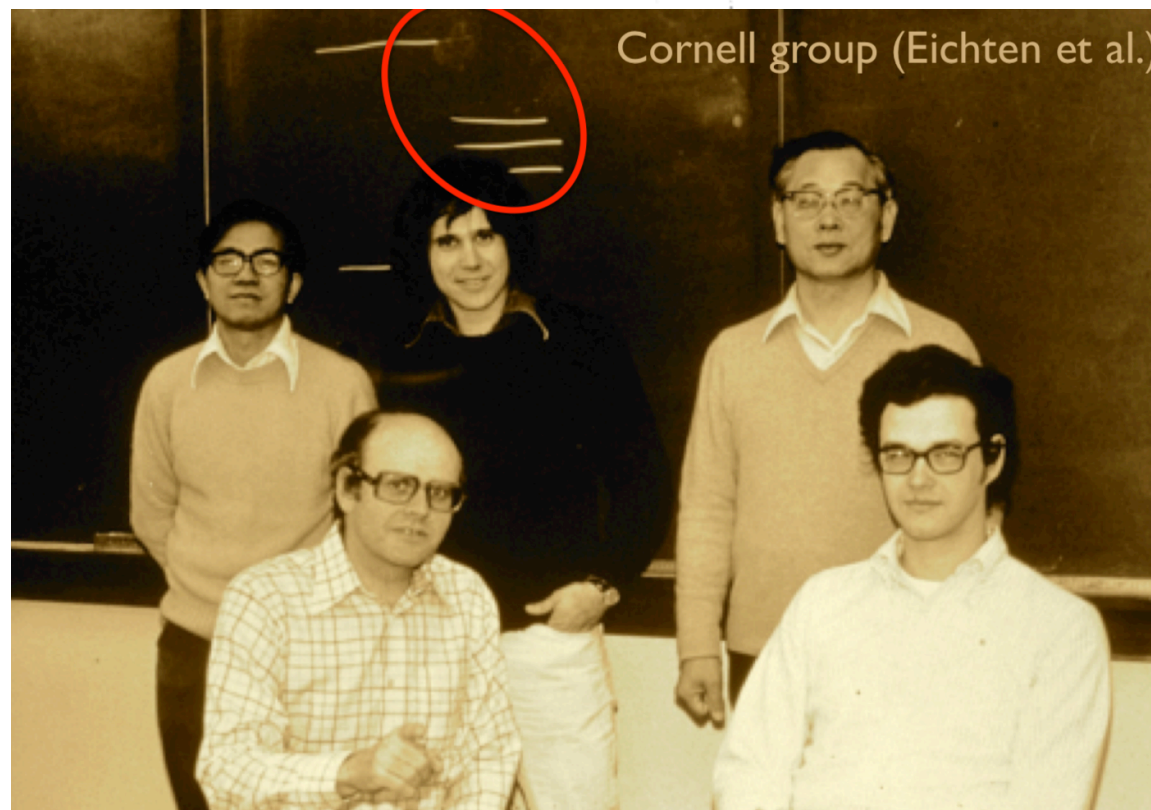
Charmonium: The model

E. Eichten,* K. Gottfried, T. Kinoshita, K. D. Lane,* and T.-M. Yan†

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

(Received 9 February 1978)

A comprehensive treatment of the charmonium model of the ψ family is presented. The model's basic assumption is a flavor-symmetric instantaneous effective interaction between quark color densities. This interaction describes both quark-antiquark binding and pair creation, and thereby provides a unified approach for energies below and above the threshold for charmed-meson production. If coupling to decay channels is ignored, one obtains the "naive" model wherein the dynamics is completely described by a single charmed-quark pair. A detailed description of this "naive" model is presented for the case where the instantaneous potential is a superposition of a linear and Coulombic term. A far more realistic picture is attained by incorporating those terms in the interaction that couple charmed quarks to light quarks. The coupled-channel formalism needed for this purpose is fully described. Formulas are given for the inclusive e^+e^- cross section and for e^+e^- annihilation into specific charmed-meson pairs. The influence of closed decay channels on ψ states below charm threshold is investigated, with particular attention to leptonic and radiative widths.



Cornell group (Eichten et al.)

color gauge interaction leads to forces that are so strong at large distances that quarks are permanently confined in color-neutral bound states—the mesons and baryons. We also adopt this assumption.

Secondly, the large masses of the ψ resonances and charmed mesons lead to the assumption that the charmed quarks are so heavy that they may be treated nonrelativistically.⁴ No one has yet succeeded in calculating the effective form of the interquark forces from quantum chromodynamics,¹⁶ even in the nonrelativistic limit. To fill this gap we postulate that in this limit many of the gross features of the potential between the charmed quarks can be simulated by the potential

$$V(r) = -\frac{\kappa}{r} + \frac{\gamma}{a^2} \cdot \quad (1.1)$$

Cornell potential

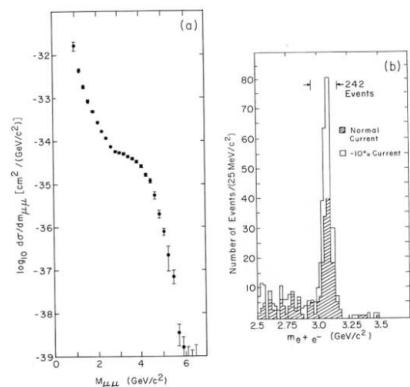
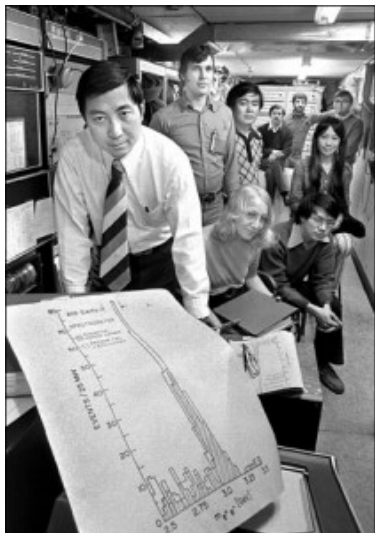
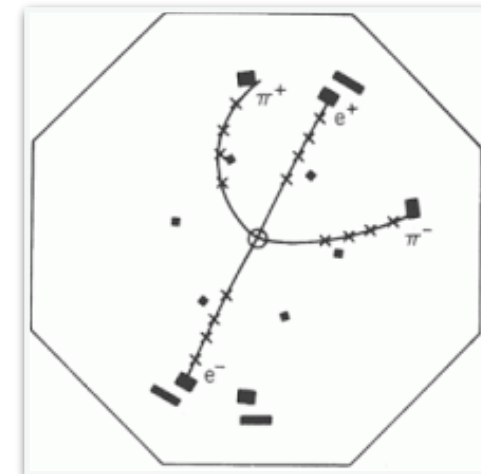
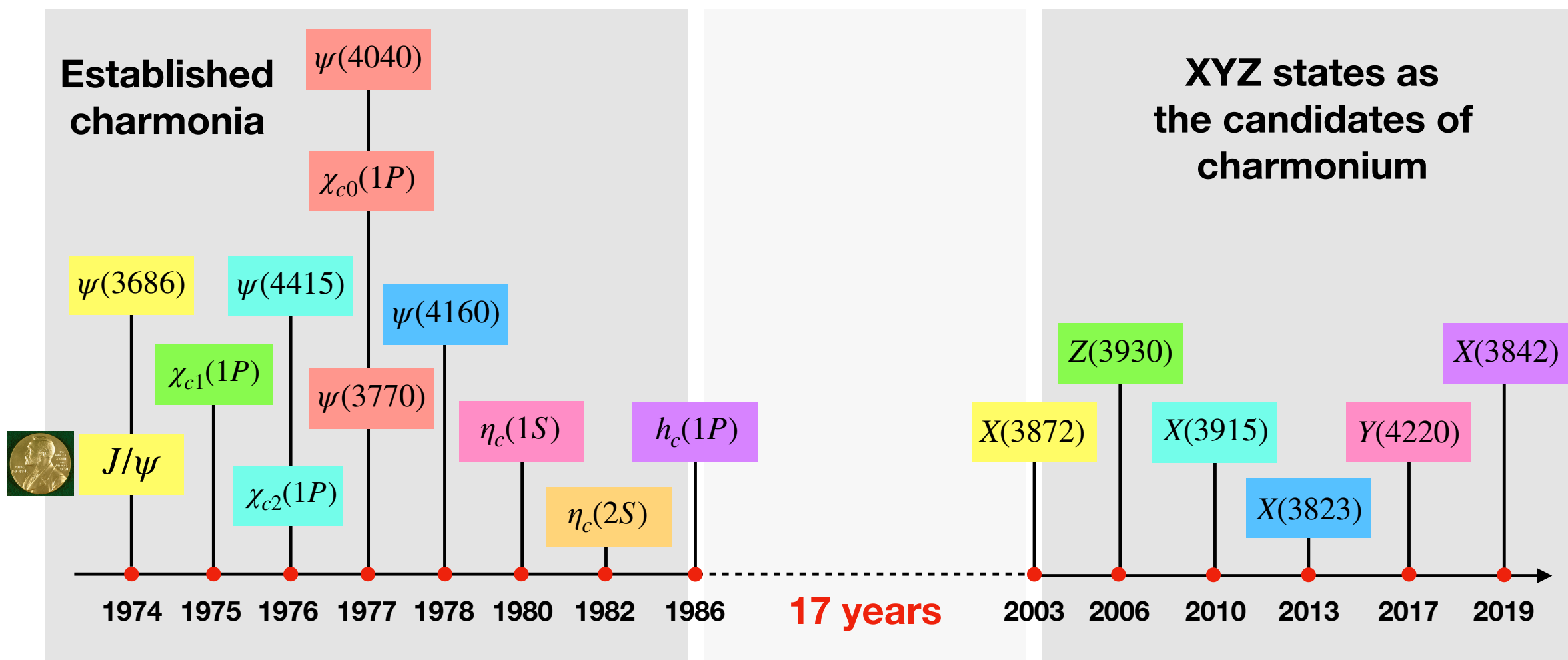


Fig. 1 - (left) Yield of the dimuons from the 1968 experiments by Lederman and his group. (left) Dielectron yield from Ting's 1974 experiment. Source: Lederman L. (1997) The discovery of the Upsilon, Bottom Quark, and B Mesons. In Hoddeson, L. Brown, L. M., Riordan, M. & Dresden, M., (1997) The Rise of the Standard Model: Particle Physics in the 1960s and the 1970s. Cambridge University Press, Cambridge. p. 101-113, on p. 104.



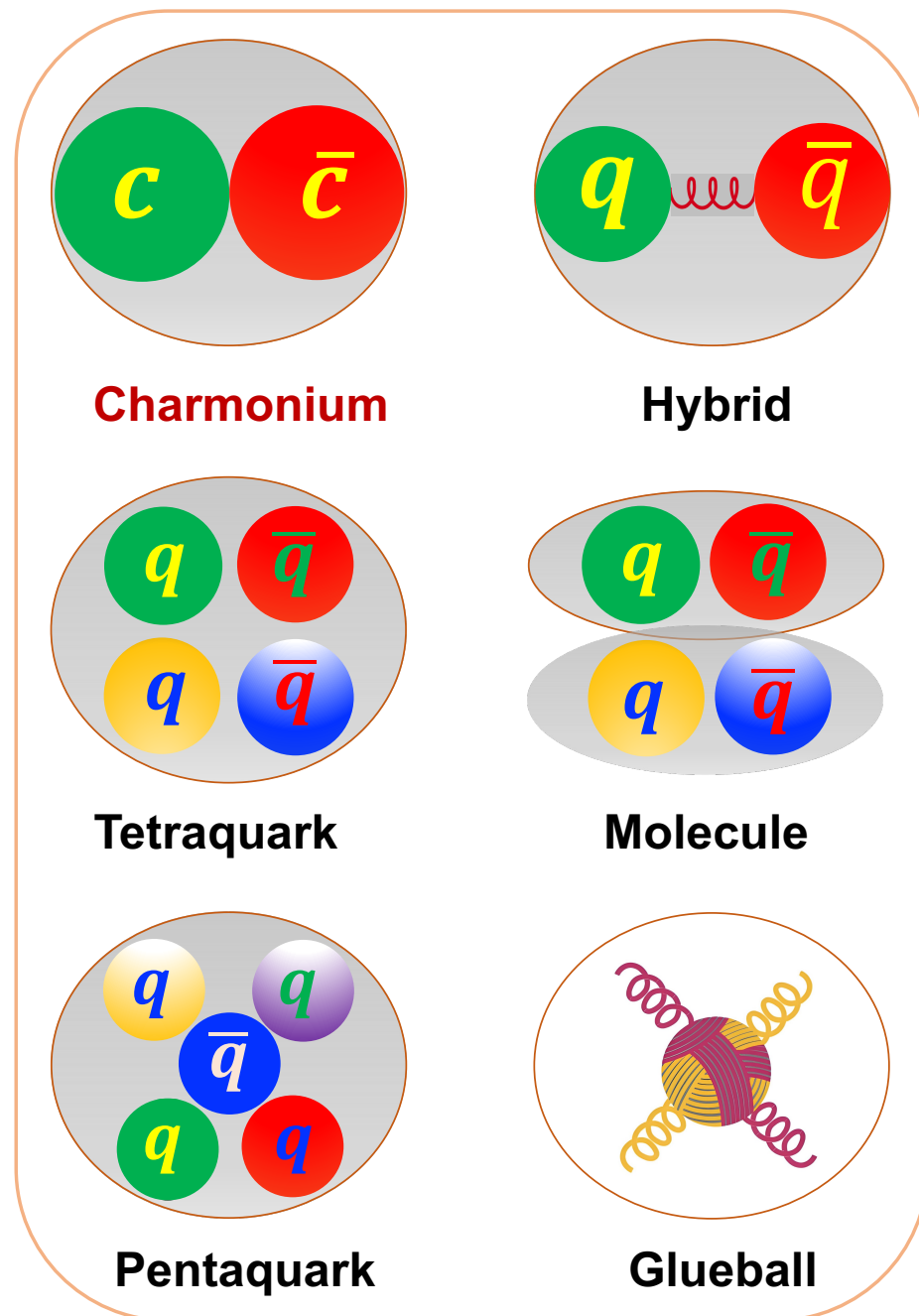
粲偶素的发现催生了淬火势模型

淬火势模型落伍于新强子态的发现



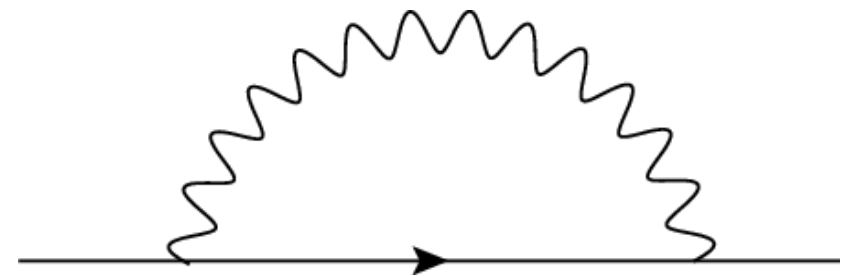
解决低质量疑难的两条路径

① 引入奇特态

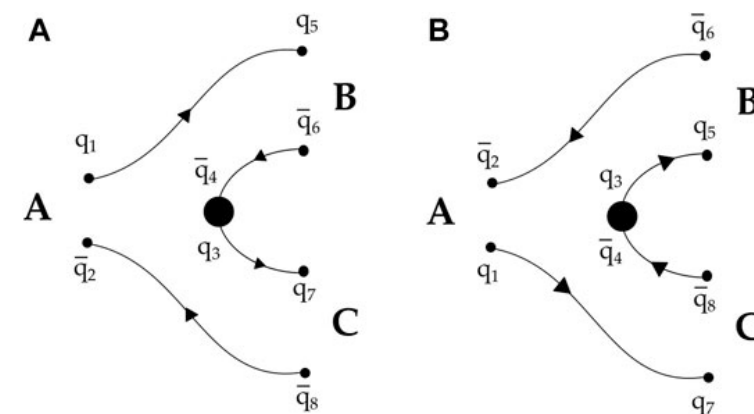


② 审视淬火热模型

- 电子自能修正



- 耦合道效应



淬火模型向非淬火模型转变

用非淬火势模型来解决典型新强子态低质量疑难是可行的

Eur. Phys. J. C (2020) 80:301
https://doi.org/10.1140/epjc/s10052-020-7874-1

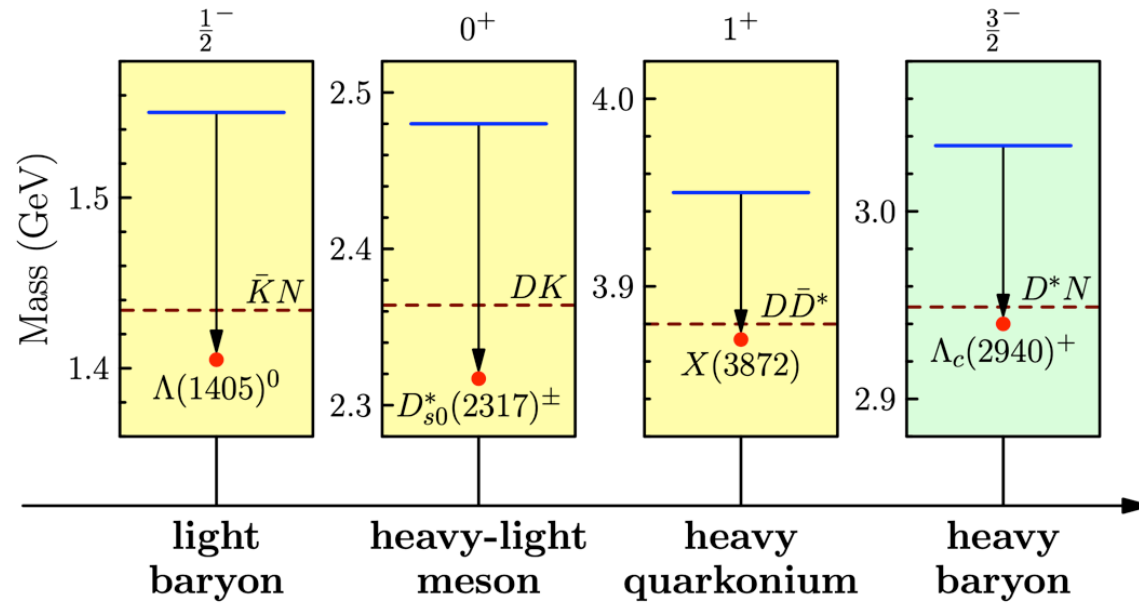
THE EUROPEAN
PHYSICAL JOURNAL C



Regular Article - Theoretical Physics

Resolving the low mass puzzle of $\Lambda_c(2940)^+$

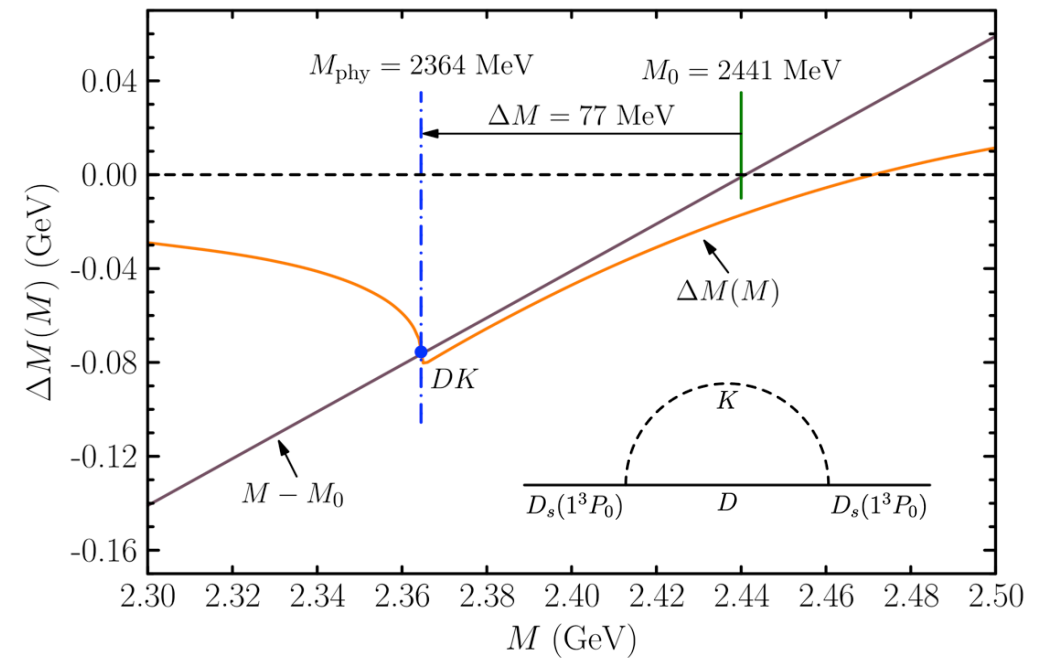
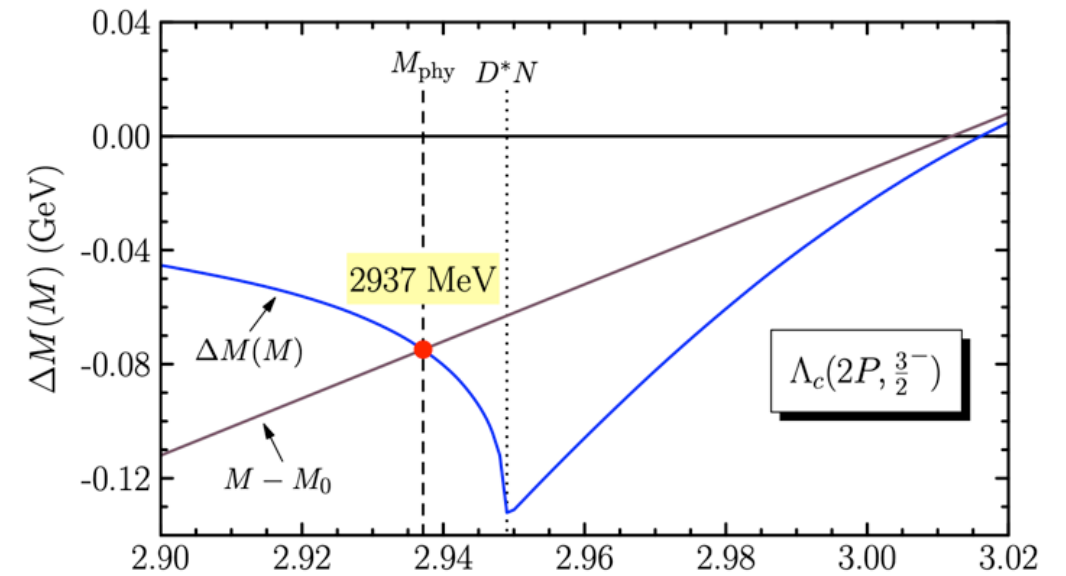
Si-Qiang Luo^{1,3,a}, Bing Chen^{2,3,b}, Zhan-Wei Liu^{1,3,c}, Xiang Liu^{1,3,d}



考虑耦合道效应解决低质量疑难

$$M - M_0 - \Delta M(M) = 0,$$

$$\Delta M(M) = \text{Re} \int_0^\infty dq q^2 \frac{|\mathcal{M}^{\Lambda_c^{\text{bare}}(2P, 3/2^-) \rightarrow D^* N}(q)|^2}{M - E_{D^* N}(\mathbf{q})}$$



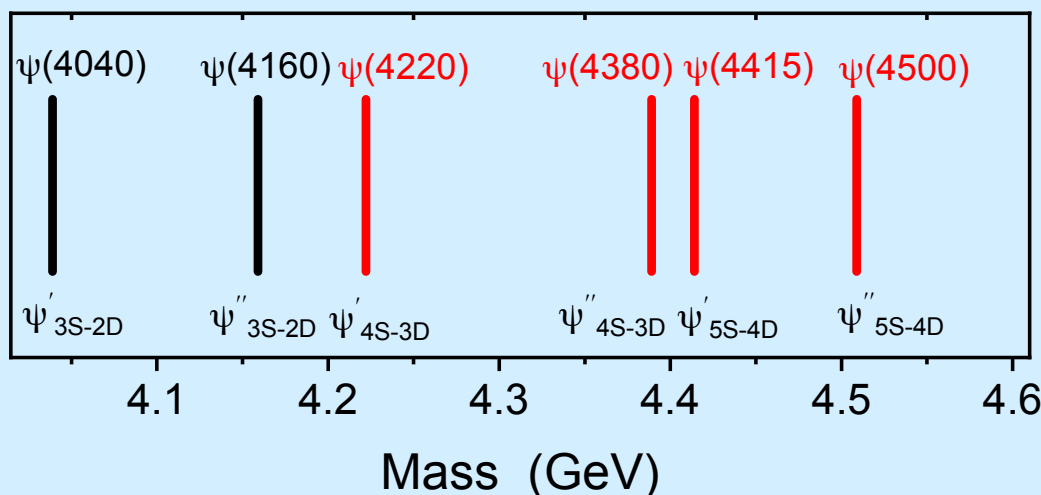
Luo, Chen, X. Liu, Matsuki, PRD 103 (2021) 074027

非淬火势模型给出的4-4.5 GeV能区粲偶素特征质量谱

屏幕势

$$S^{scr}(r) = \frac{b(1 - e^{-\mu r})}{\mu} + c$$

Wang, Chen, X. Liu, Matsuki, PRD 99 (2019) 114003



康奈尔势模型（淬火）的粲偶素质量谱

PHYSICAL REVIEW D

VOLUME 21, NUMBER 1

1 JANUARY 1980

Charmonium: Comparison with experiment

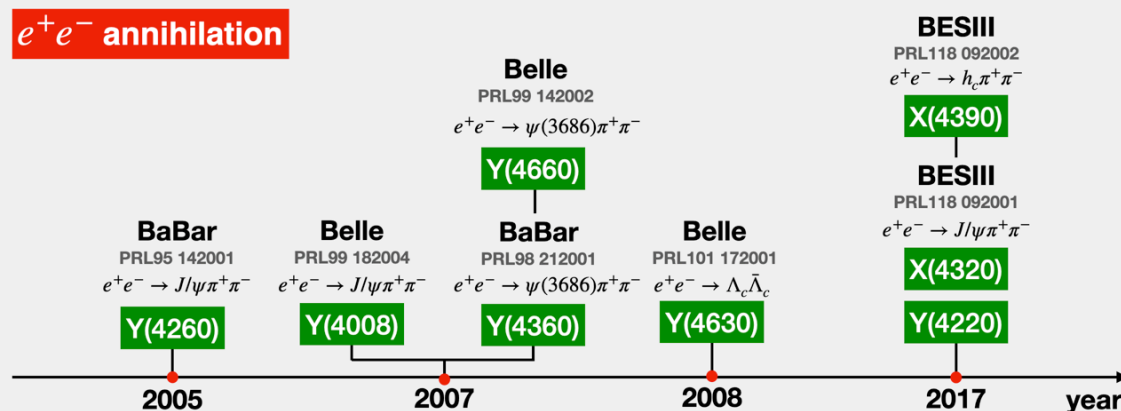
E. Eichten,* K. Gottfried, T. Kinoshita, K. D. Lane,* and T. M. Yan
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853
(Received 25 June 1979)

TABLE II. $c\bar{c}$ bound states in naive model, and their properties. Parameters used are $m_c = 1.84$ GeV, $a = 2.34$ GeV⁻¹, and $\kappa = 0.52$.

State	Mass (GeV)	Γ_{ee} (keV) ^b	$\langle \frac{v^2}{c^2} \rangle$	$\langle r^2 \rangle^{1/2}$ (fm)	Candidate
1S	3.095 ^a	4.8	0.20	0.47	$\psi(3095)$
1P	3.522 ^a		0.20	0.74	$\chi_{0,1,2}(3522 \pm 5)$
2S	3.684 ^a	2.1	0.24	0.96	$\psi'(3684)$
1D	3.81		0.23	1.0	$\psi'(3772)$ ^c
3S	4.11	1.5	0.30	1.3	$\psi(4028)$
2D	4.19		0.29	1.35	$\psi(4160)$ ^d
4S	4.46	1.1	0.35	1.7	$\psi(4414)$
5S	4.79	0.8	0.40	2.0	

非淬火势模型可为“Y problem”给出一览子解决方案

e^+e^- annihilation

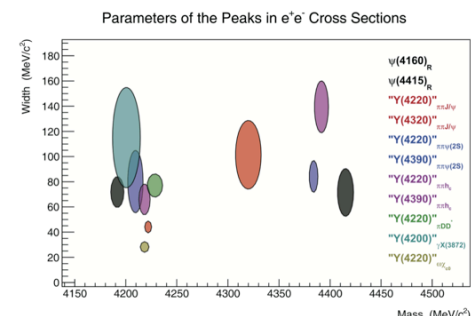


Chinese Physics C Vol. 44, No. 4 (2020)

Future Physics Programme of BESIII*

(1) The Y problem

Exclusive e^+e^- cross-sections have shown surprisingly complex behavior as a function of cms energy. The $Y(4260)$ is more complex than a single ordinary resonance, as shown by the complicated line shape of the $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ cross-section in Fig. 3.10(e); the $Y(4360)$ and $Y(4660)$ are seen in $e^+e^- \rightarrow \pi^+\pi^- \psi(3686)$; two other peaks are seen in $e^+e^- \rightarrow \pi^+\pi^- h_c$ in Fig. 3.10(f); the $Y(4220)$ is seen in $e^+e^- \rightarrow \omega \chi_{c0}$ in Fig. 3.10(g), etc. A summary of the masses and widths of resonances extracted from recent BESIII results is shown in Fig. 3.11.



PRD 83 (2011) 054021
PRD 83 (2011) 074012
EPJC 74 (2014), 3208
PRD 91 (2015) 094023
PRD 93 (2016) 014011
PRD 93 (2016) 034028
PRD 96 (2017) 094004
EPJC 78 (2018) 136
EPJC 79 (2019) 613
PRD 99 (2019) 114003

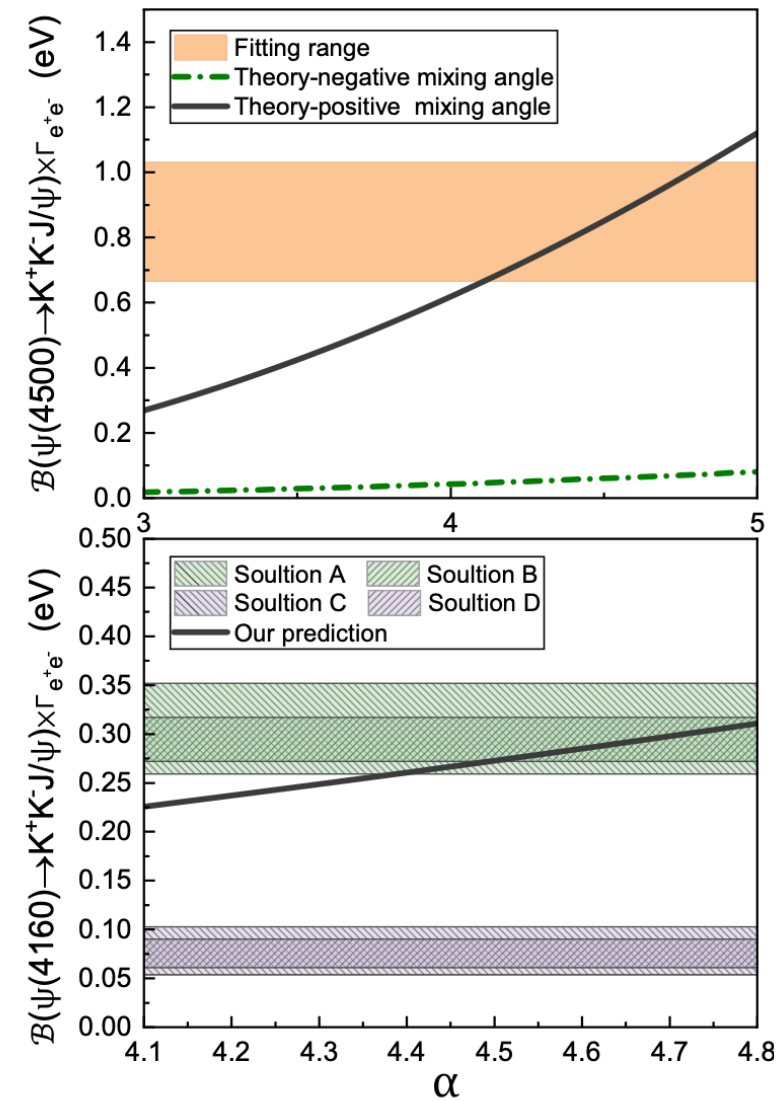
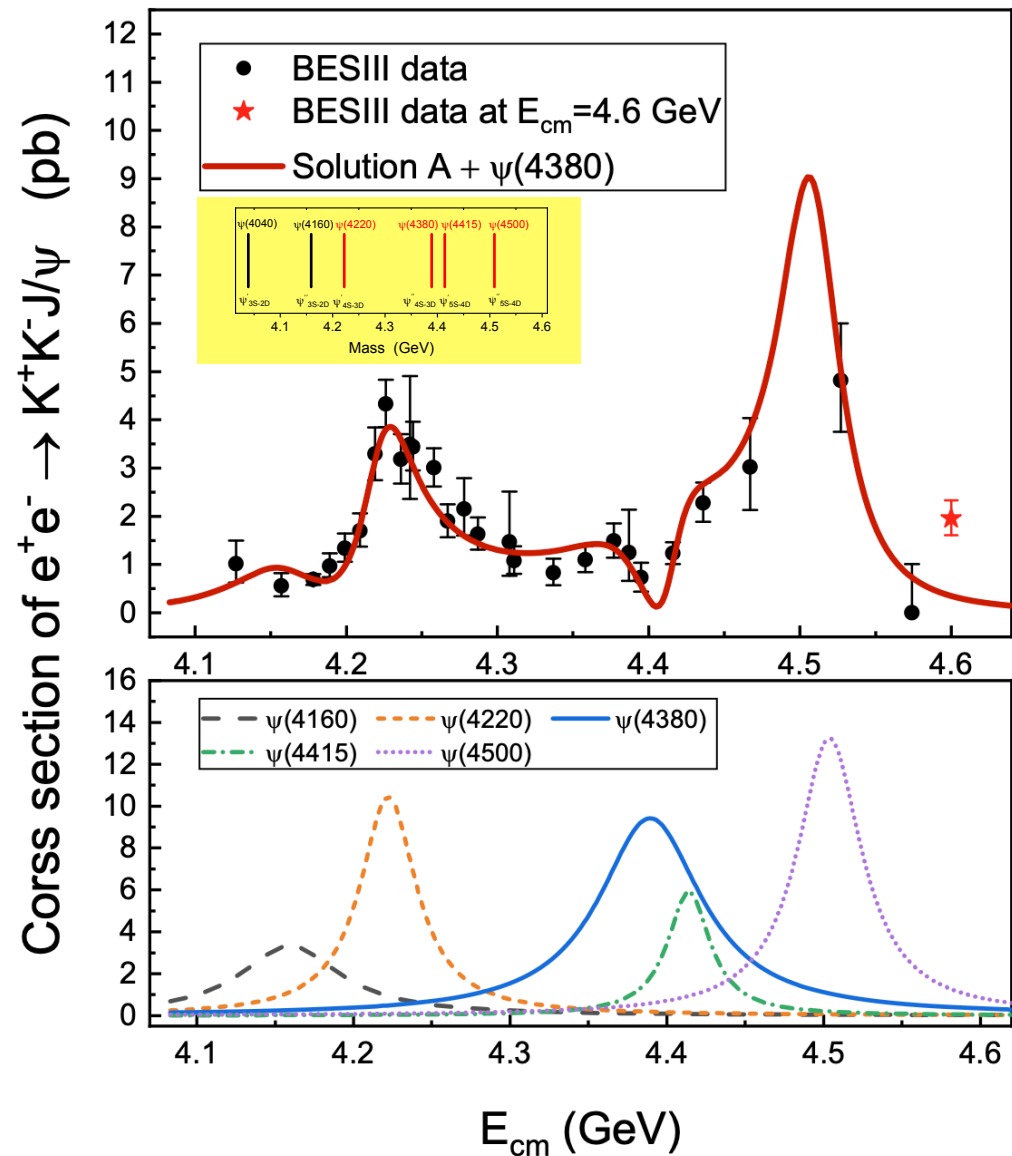
PRD 101 (2020) 034001
PRD 104 (2021) 094001
PLB 833 (2022) 137292
PRD 107 (2023) 054016
PLB 849 (2024) 138456
PRD 109 (2024) 094048
PRD 111 (2025) 054021
PRD 111 (2025) 054023
PLB 868 (2025) 139644

高精度实验数据使得验证非淬火势模型在4-4.5 GeV能区预测的粲偶素特征质量谱成为可能

PHYSICAL REVIEW D **107**, 054016 (2023)

Confirming the existence of a new higher charmonium $\psi(4500)$ by the newly released data of $e^+e^- \rightarrow K^+K^-J/\psi$

Jun-Zhang Wang^{1,2,*} and Xiang Liu^{2,3,4,†}



BESIII: CPC 46 (2022) 111002

高精度实验数据使得验证非淬火势模型在4-4.5 GeV能区预测的粲偶素特征质量谱成为可能

Phys. Lett. B 849 (2024) 138456



Contents lists available at ScienceDirect

Physics Letters B

journal homepage: www.elsevier.com/locate/physletb



Letter

Identifying a characterized energy level structure of higher charmonium well matched to the peak structures in $e^+e^- \rightarrow \pi^+ D^0 D^{*-}$

Jun-Zhang Wang^{a,b,} , Xiang Liu^{b,c,d,e,*}



ABSTRACT

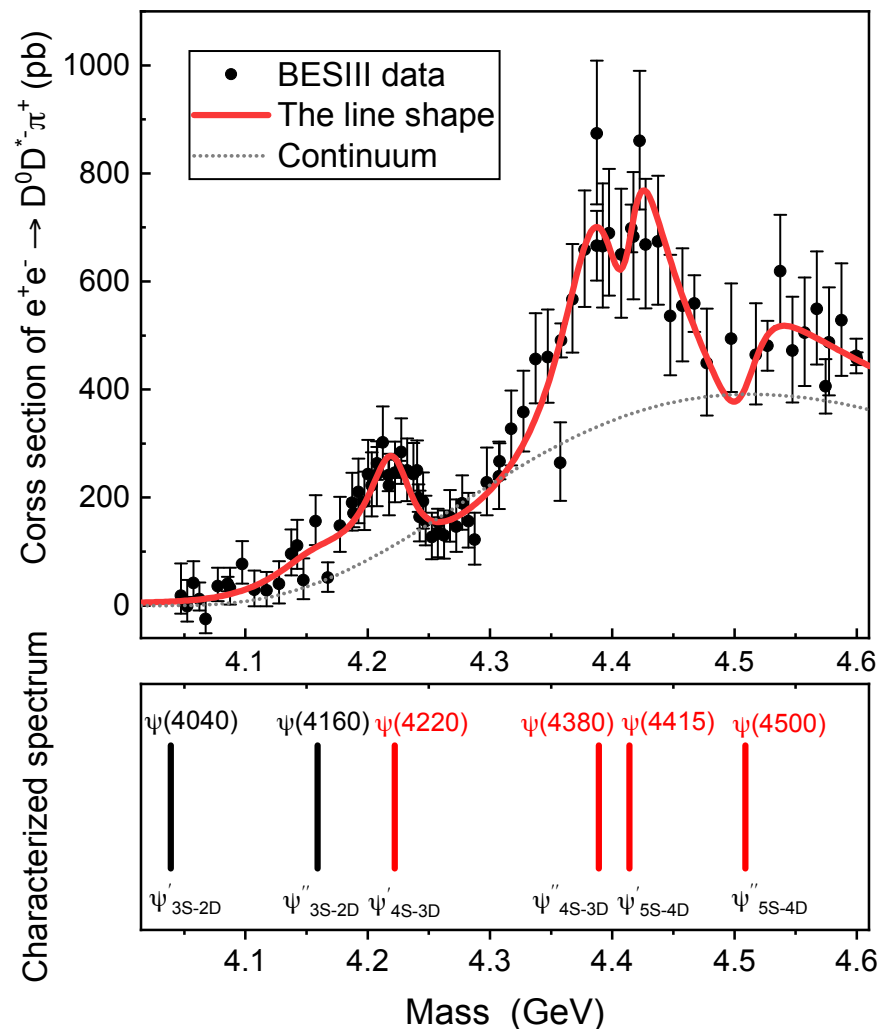
Recent progresses on charmoniumlike state have significantly enriched the discovery of new hadronic states, providing exciting opportunities for further investigations into the fascinating realm of charmonium physics. In this letter, we focus on the vector charmonium family and perform a detailed analysis of the recently observed $e^+e^- \rightarrow \pi^+ D^0 D^{*-}$ process. Our findings demonstrate a agreement between the observed peak structures and the predicted characterized energy level structure of higher vector charmonia including the $\psi(4220)$, $\psi(4380)$, $\psi(4415)$, and $\psi(4500)$, which are derived from an unquenched potential model. This discovery challenges conventional understanding of higher charmonia above 4 GeV and offers fresh insights into the dynamics of charm and anti-charm quarks in the formation of these states. Furthermore, the identification of these higher charmonia in the precisely measured $\pi^+ D^0 D^{*-}$ open-charm decay channel would serve as compelling evidence supporting the unquenched scenario and contribute to a deeper understanding of the nonperturbative aspects of the strong interaction.

Table 1

The fitted parameters of describing the cross section distribution of $e^+e^- \rightarrow \pi^+ D^0 D^{*-}$ based on the scheme involving the characterized energy level structures of higher charmonia, where the $\chi^2/d.o.f. = 0.74$.

Parameters	a_0 (GeV ⁻²)	g (GeV ⁻³)	$R_{\psi(4160)}$ (eV)	$R_{\psi(4220)}$ (eV)	$R_{\psi(4380)}$ (eV)	$R_{\psi(4415)}$ (eV)
Values	0.445 ± 0.025	34.5 ± 18.1	0.726 ± 0.527	2.70 ± 0.63	19.0 ± 4.6	2.34 ± 1.23
Parameters	$R_{\psi(4500)}$ (eV)	$\phi_{\psi(4160)}$ (rad)	$\phi_{\psi(4220)}$ (rad)	$\phi_{\psi(4380)}$ (rad)	$\phi_{\psi(4415)}$ (rad)	$\phi_{\psi(4500)}$ (rad)
Values	1.60 ± 0.42	1.97 ± 0.65	2.07 ± 0.15	1.44 ± 0.16	6.04 ± 0.24	5.76 ± 0.31

BESIII: Phys. Rev. Lett. 130, 121901 (2023)



PHYSICAL REVIEW D **109**, 094048 (2024)

How higher charmonia shape the puzzling data of the $e^+e^- \rightarrow \eta J/\psi$ cross section

Tian-Cai Peng^{1,2,3,4,*}, Zi-Yue Bai^{1,2,3,4,†}, Jun-Zhang Wang^{1,5,‡} and Xiang Liu^{1,2,3,4,6,§}

¹School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China

²Lanzhou Center for Theoretical Physics, Key Laboratory of Theoretical Physics of Gansu Province, Lanzhou University, Lanzhou 730000, China

³Key Laboratory of Quantum Theory and Applications of MoE, Lanzhou University, Lanzhou 730000, China

⁴Research Center for Hadron and CSR Physics, Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China

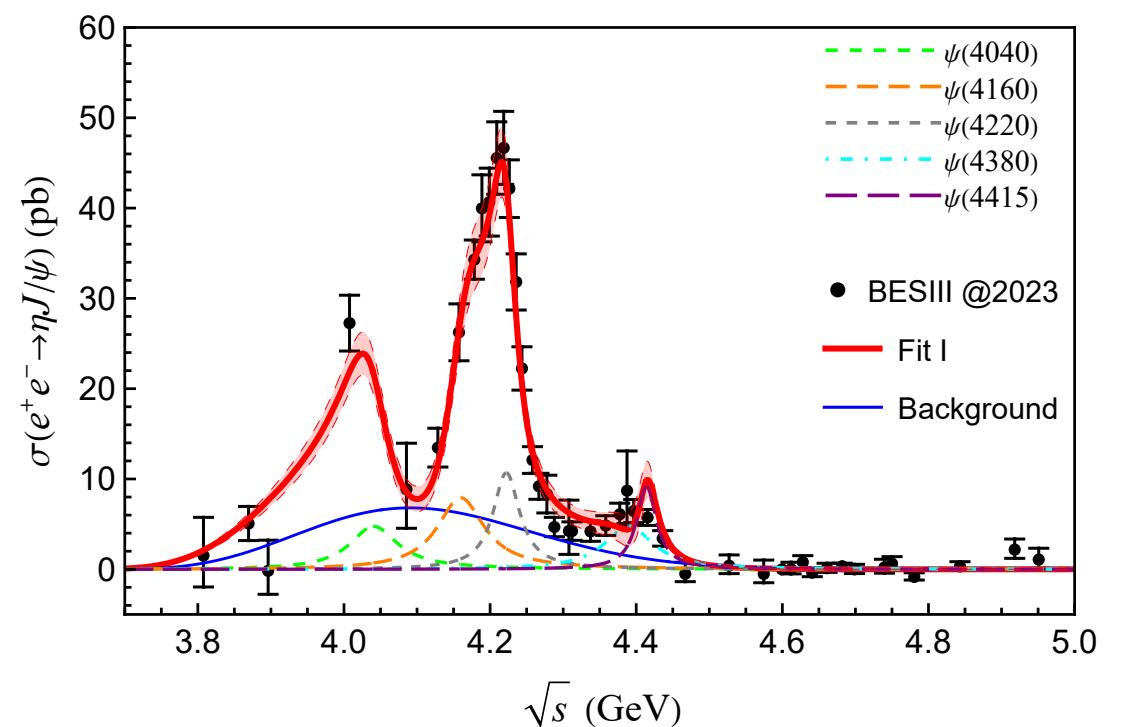
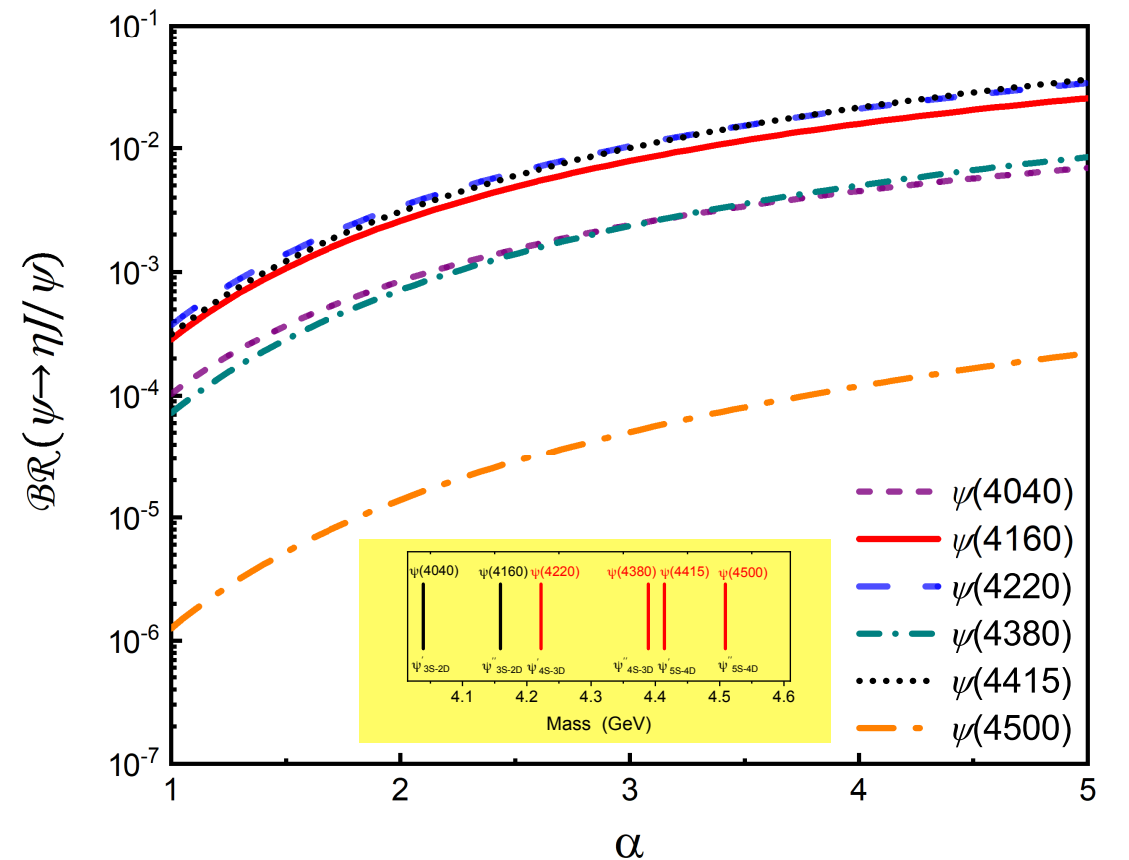
⁵School of Physics and Center of High Energy Physics, Peking University, Beijing 100871, China

⁶MoE Frontiers Science Center for Rare Isotopes, Lanzhou University, Lanzhou 730000, China

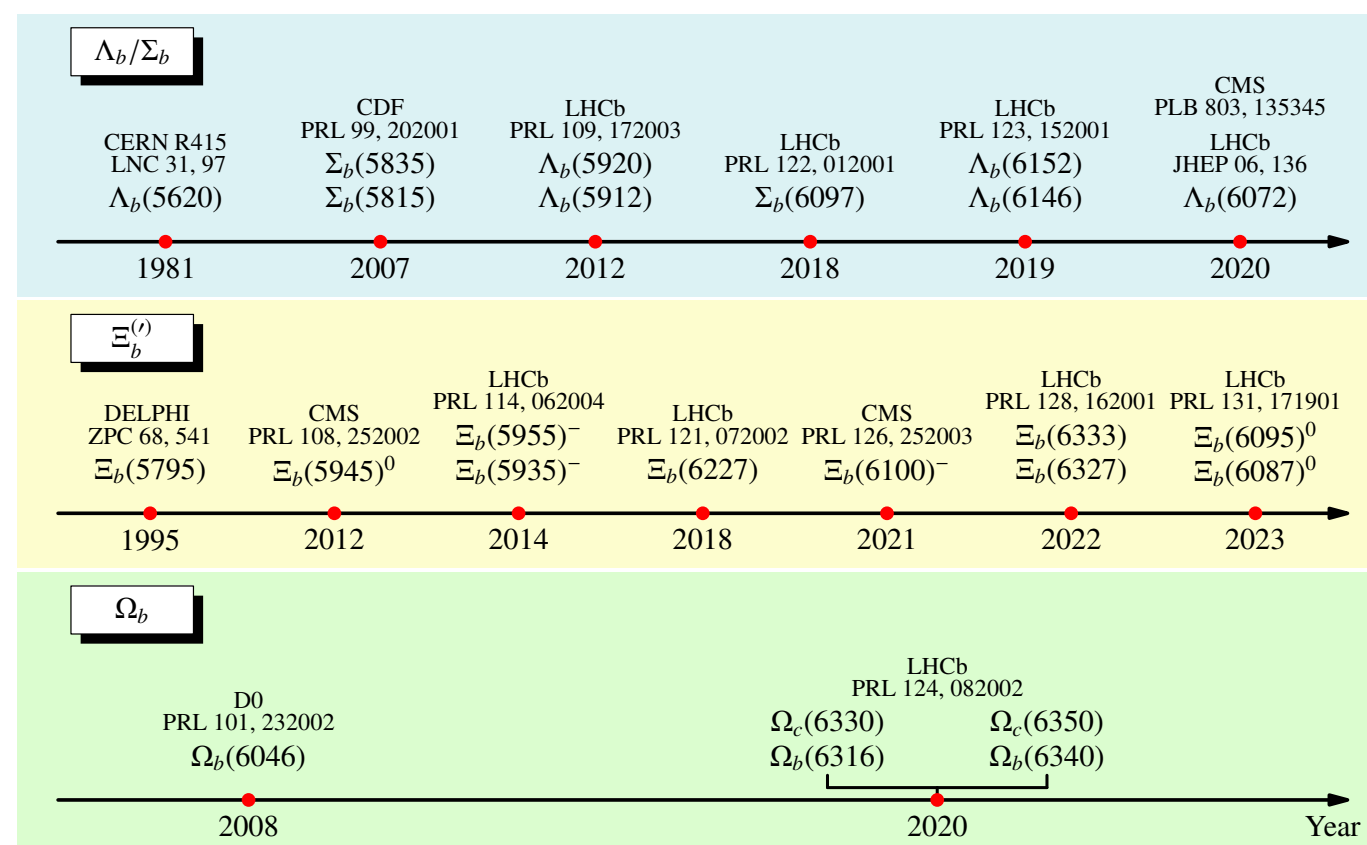
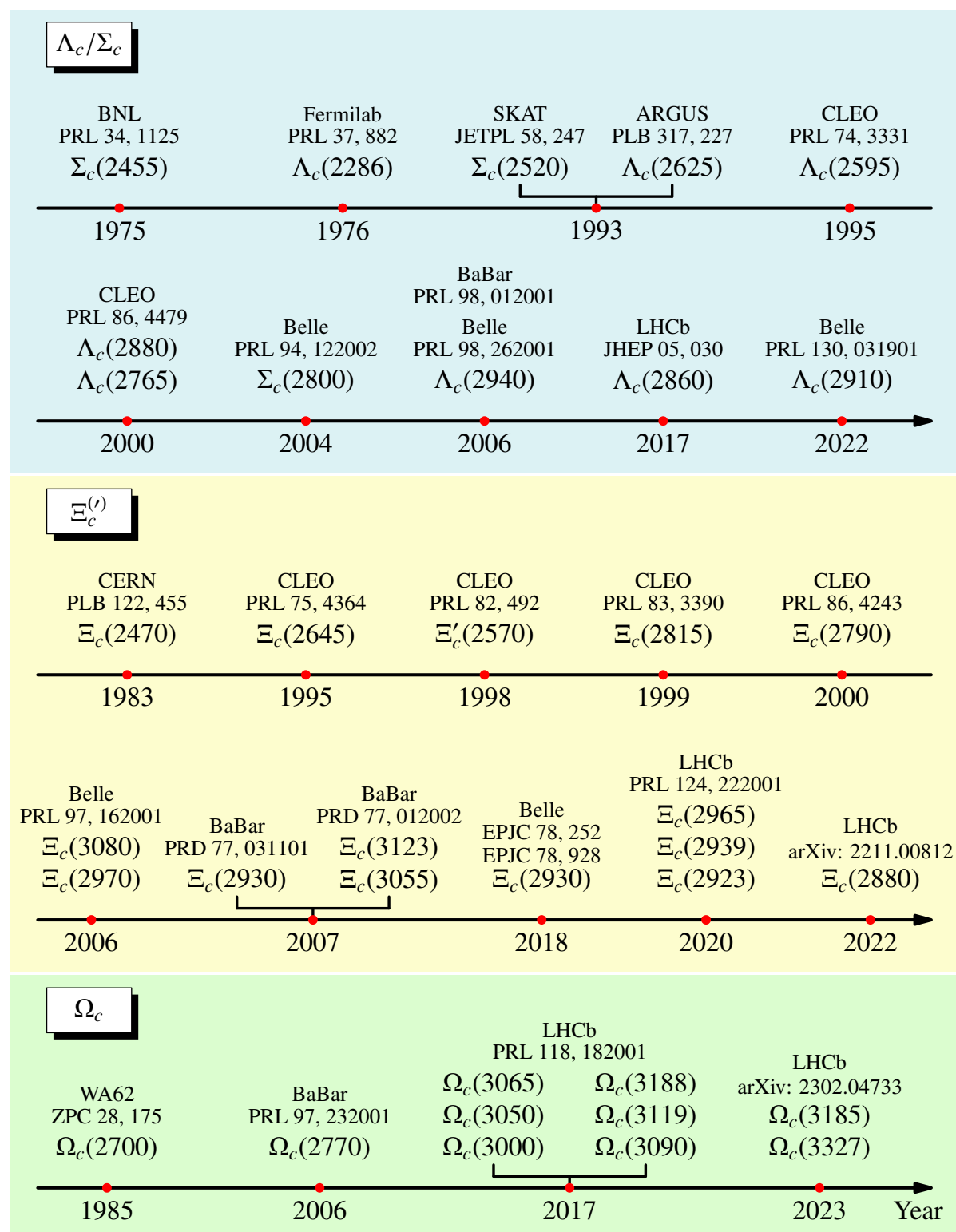
(Received 8 March 2024; accepted 6 May 2024; published 31 May 2024)

Recently, the BESIII collaboration performed a precise measurement of the $e^+e^- \rightarrow \eta J/\psi$ cross section. It is puzzling that the resonance parameters of the reported $Y(4230)$ show a substantial divergence from the previously measured results in both the open-charmed and hidden-charmed decay channels, and the line shape asymmetry of the data approaching 4.2 GeV also suggests that it might be difficult to characterize the details of the structure around 4.2 GeV by a single resonance. This has motivated our great curiosity about how the charmonium states are distributed in the measured energy range and how they shape the puzzling data of the $e^+e^- \rightarrow \eta J/\psi$ cross section. In this work, we use five theoretically constructed charmonia in the range of 4.0–4.5 GeV, i.e., $\psi(4040)$, $\psi(4160)$, $\psi(4220)$, $\psi(4380)$, and $\psi(4415)$, to apply a combined fit to the data, in which their calculated decay ratios into $\eta J/\psi$ via hadronic loop mechanism are taken as input. The fit results can reproduce the measured cross section data well, especially for the subtle line shape around 4.2 GeV, showing that the structure around 4.2 GeV is possible from the contribution of both $\psi(4160)$ and $\psi(4220)$.

BESIII: PRD 109 (2024) 092012



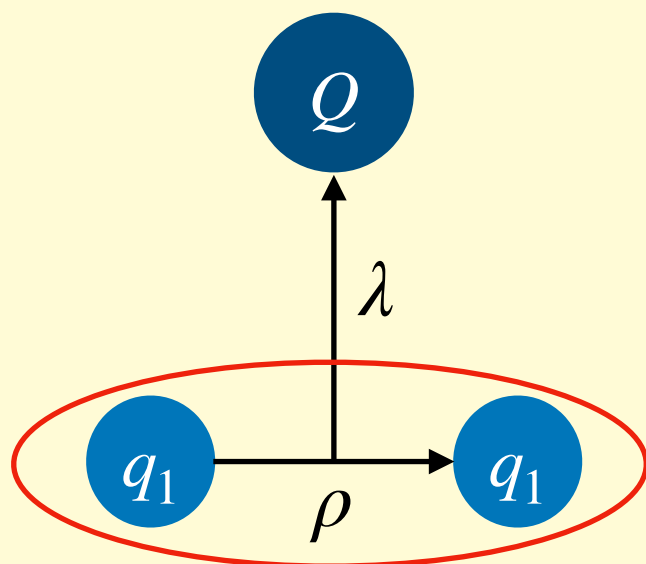
3. 强子体系少体问题—计算精度



随着实验精度的提升，过去20年间发现了丰富的粲重子和底重子

Rep.Prog.Phys. 86 (2023) 026201
Rep.Prog.Phys. 80 (2017) 076201
Chin.J.Phys. 78 (2022) 324-362

典型的少体系统



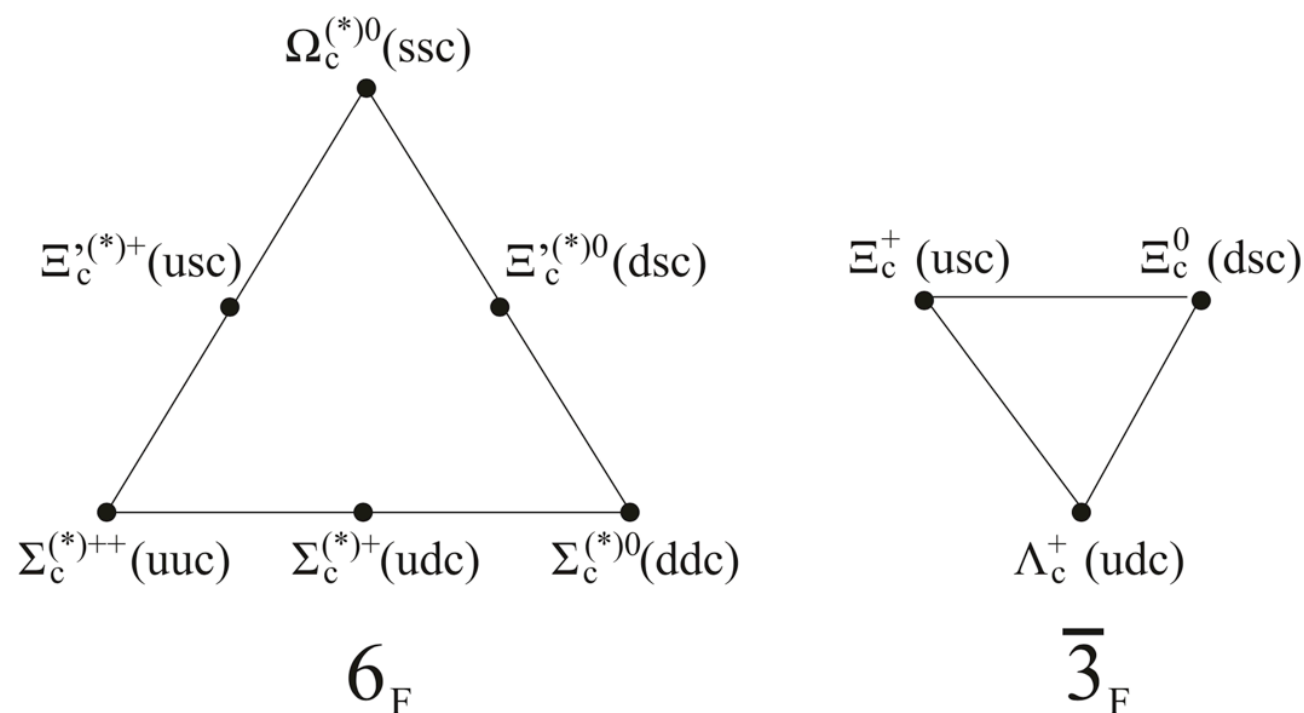
Jacobi coordinates for the three-body system

SU(3)对称性

$$3_f \otimes_f = \bar{3}_f \oplus 6_f$$

Rep.Prog.Phys. 80 (2017) 076201

粲重子的SU(3)味道多重态



$$\phi_{\Lambda_Q}^{\text{flavor}} = \frac{1}{\sqrt{2}}(ud - du)Q$$

$$\phi_{\Xi_Q}^{\text{flavor}} = \begin{cases} \frac{1}{\sqrt{2}}(us - su)Q \\ \frac{1}{\sqrt{2}}(ds - sd)Q \end{cases}$$

$$\phi_{\Sigma_Q}^{\text{flavor}} = \begin{cases} uuQ \\ \frac{1}{\sqrt{2}}(ud + du)Q \\ ddQ \end{cases}$$

$$\phi_{\Xi'_Q}^{\text{flavor}} = \begin{cases} \frac{1}{\sqrt{2}}(us + su)Q \\ \frac{1}{\sqrt{2}}(ds + sd)Q \end{cases},$$

$$\phi_{\Omega_Q}^{\text{flavor}} = ssQ.$$

Mass spectrum

Potential model

Diquark

早些年

$$\hat{H} = m_{\text{di}} + m_Q + \frac{p_{\text{di}}^2}{2m_{\text{di}}} + \frac{p_Q^2}{2m_Q} + V_{\text{di}-Q},$$

$$V_{\text{di}-Q} = H^{\text{conf}} + H^{\text{hyp}} + H^{\text{so(cm)}} + H^{\text{so(tp)}}.$$

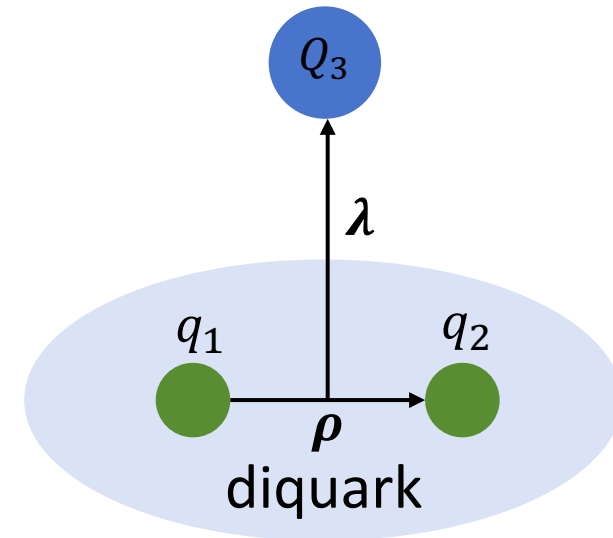
$$H^{\text{conf}} = -\frac{4\alpha_s}{3r} + br + C,$$

$$H^{\text{hyp}} = \frac{4\alpha_s}{3m_{\text{di}}m_Q} \left(\frac{8\pi}{3} \mathbf{s}_{\text{di}} \cdot \mathbf{s}_Q \tilde{\delta}(r) + \frac{1}{r^3} S(\mathbf{r}, \mathbf{s}_{\text{di}}, \mathbf{s}_Q) \right),$$

$$\tilde{\delta}(r) = \frac{\sigma^3}{\pi^{3/2}} e^{-\sigma^2 r^2}, \quad S(\mathbf{r}, \mathbf{s}_{\text{di}}, \mathbf{s}_Q) = \frac{3\mathbf{s}_{\text{di}} \cdot \mathbf{r} \mathbf{s}_Q \cdot \mathbf{r}}{r^2} - \mathbf{s}_{\text{di}} \cdot \mathbf{s}_Q,$$

$$H^{\text{so(cm)}} = \frac{4\alpha_s}{3r^3} \left(\frac{1}{m_{\text{di}}} + \frac{1}{m_Q} \right) \left(\frac{\mathbf{s}_{\text{di}}}{m_{\text{di}}} + \frac{\mathbf{s}_Q}{m_Q} \right) \cdot \mathbf{L},$$

$$H^{\text{so(tp)}} = -\frac{1}{2r} \frac{\partial H^{\text{conf}}}{\partial r} \left(\frac{\mathbf{s}_{\text{di}}}{m_{\text{di}}^2} + \frac{\mathbf{s}_Q}{m_Q^2} \right) \cdot \mathbf{L}.$$



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Q. F. Lu, K. L. Wang, L. Y. Xiao and X. H. Zhong, Phys. Rev. D 96, no.11, 114006 (2017)

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Mass spectrum

Potential model

Three-body potential

$$\hat{H} = \sum_i \left(m_i + \frac{p_i^2}{2m_i} \right) + \sum_{i<j} \left(V_{ij}^{\text{conf}} + V_{ij}^{\text{hyp}} + V_{ij}^{\text{so(cm)}} + V_{ij}^{\text{so(tp)}} \right).$$

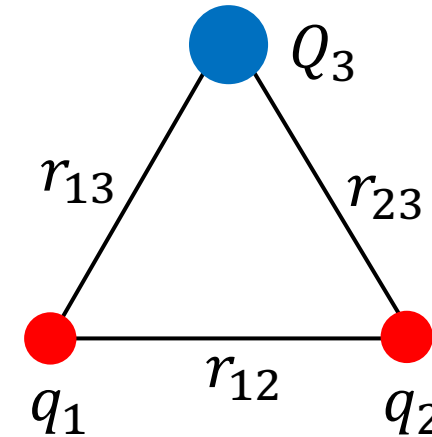
$$V_{ij}^{\text{conf}} = -\frac{2}{3} \frac{\alpha_s}{r_{ij}} + \frac{b}{2} r_{ij} + \frac{1}{2} C,$$

$$V_{ij}^{\text{hyp}} = \frac{2\alpha_s}{3m_i m_j} \left[\frac{8\pi}{3} \tilde{\delta}(r_{ij}) \mathbf{s}_i \cdot \mathbf{s}_j + \frac{1}{r_{ij}^3} S(\mathbf{r}, \mathbf{s}_i, \mathbf{s}_j) \right],$$

$$\tilde{\delta}(r) = \frac{\sigma^3}{\pi^{3/2}} e^{-\sigma^2 r^2}, \quad S(\mathbf{r}, \mathbf{s}_i, \mathbf{s}_j) = \frac{3\mathbf{s}_i \cdot \mathbf{r}_{ij} \mathbf{s}_j \cdot \mathbf{r}_{ij}}{r_{ij}^2} - \mathbf{s}_i \cdot \mathbf{s}_j,$$

$$V_{ij}^{\text{so(cm)}} = \frac{2\alpha_s}{3r_{ij}^3} \left(\frac{\mathbf{r}_{ij} \times \mathbf{p}_i \cdot \mathbf{s}_i}{m_i^2} - \frac{\mathbf{r}_{ij} \times \mathbf{p}_j \cdot \mathbf{s}_j}{m_j^2} - \frac{\mathbf{r}_{ij} \times \mathbf{p}_j \cdot \mathbf{s}_i - \mathbf{r}_{ij} \times \mathbf{p}_i \cdot \mathbf{s}_j}{m_i m_j} \right),$$

$$V_{ij}^{\text{so(tp)}} = -\frac{1}{2r_{ij}} \frac{\partial H_{ij}^{\text{conf}}}{\partial r_{ij}} \left(\frac{\mathbf{r}_{ij} \times \mathbf{p}_i \cdot \mathbf{s}_i}{m_i^2} - \frac{\mathbf{r}_{ij} \times \mathbf{p}_j \cdot \mathbf{s}_j}{m_j^2} \right).$$



近年来

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G. L. Yu, Z. Y. Li, Z. G. Wang, J. Lu and M. Yan, Nucl. Phys. B 990, 116183 (2023)

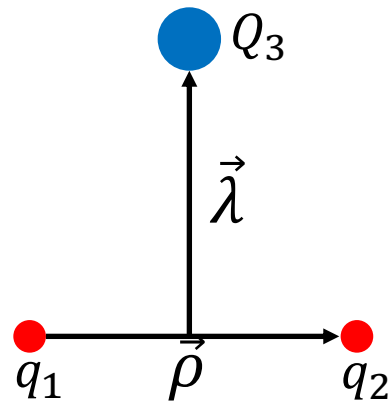
Q. F. Song, Q. F. Lu and A. Hosaka, arXiv:2308.03261 [hep-ph].

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Mass spectrum

Potential model

Gaussian expansion method



Jacobi coordinate

Gaussian base

$$\phi_{nlm}(\mathbf{r}) = \phi_{nl}(r)Y_{lm}(\hat{\mathbf{r}})$$

$$\phi_{nl}(r) = N_{nl}r^l e^{-\nu_n r^2}$$

$$N_{nl} = \sqrt{\frac{2^{l+2}(2\nu_n)^{l+\frac{3}{2}}}{\sqrt{\pi}(2l+1)!!}}$$

$$\nu_n = \frac{1}{r_n^2} \quad r_n = r_1 a^{n-1} \quad (n = 1 - n_{\max})$$

$$\vec{\rho} = \vec{r}_2 - \vec{r}_1$$

$$\vec{\lambda} = \vec{r}_3 - \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

近年来

$$\Psi^{\text{total}} = \phi^{\text{color}} \times \phi^{\text{flavor}} \times \phi^{\text{spin}} \times \phi^{\text{orbit}}$$

$$\phi^{\text{orbit}} = \sum_{n_\rho n_\lambda} C_{n_\rho n_\lambda} \sum_{m_\rho m_\lambda} \langle l_\rho m_\rho; l_\lambda m_\lambda | LM \rangle \phi_{n_\rho l_\rho m_\rho}(\rho) \phi_{n_\lambda l_\lambda m_\lambda}(\lambda)$$



Rayleigh-Ritz variational method

E. Hiyama, Y. Kino and M. Kamimura, Prog. Part. Nucl. Phys. 51, 223-307 (2003)

计算精度提升下的粲重子谱研究

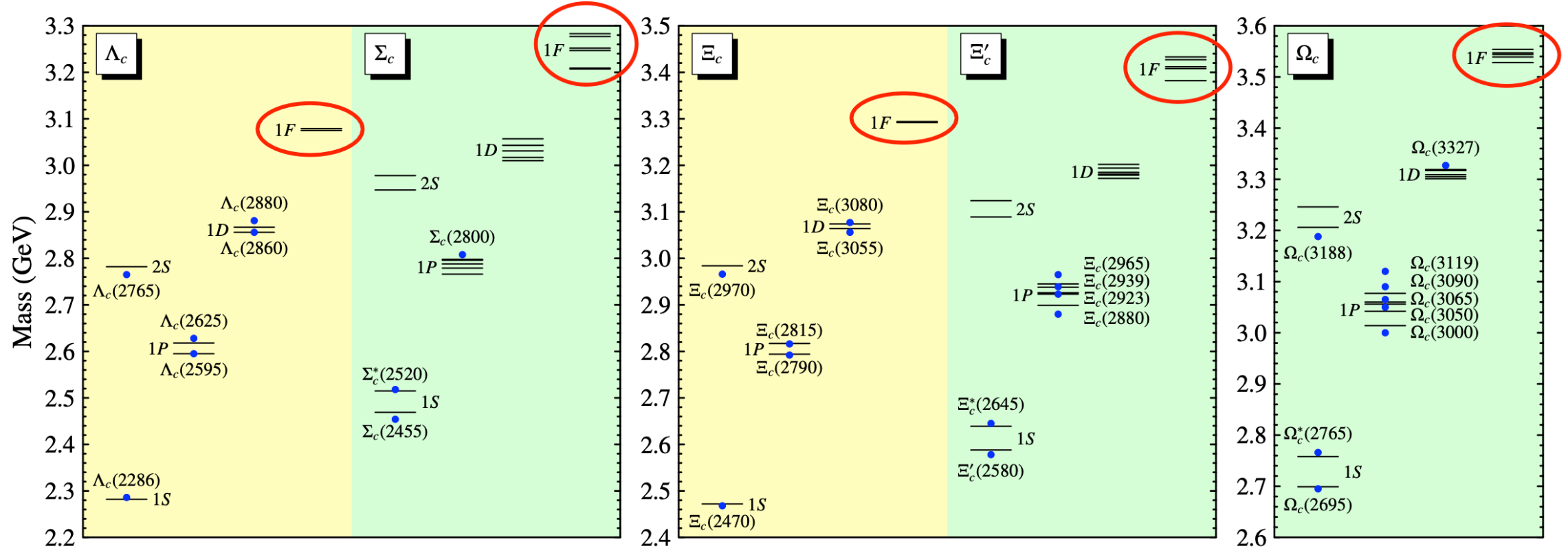


TABLE III. A comparison of predicted masses for F -wave singly charmed baryons from various studies. Here, the listed masses are in units of MeV.

States	Our	Ref. [81]	Ref. [80]	States	Our	Ref. [81]	Ref. [82]	States	Our	Ref. [81]	Ref. [80]
$\Lambda_c(1F, 5/2^-)$	3075	3097	3104	$\Xi_c(1F, 5/2^-)$	3292	3278	3289				
$\Lambda_c(1F, 7/2^-)$	3079	3078	3111	$\Xi_c(1F, 7/2^-)$	3295	3292	3294				
$\Sigma_{c2}(1F, 3/2^-)$	3276	3288	3299	$\Xi'_{c2}(1F, 3/2^-)$	3427	3418	3424	$\Omega_{c2}(1F, 3/2^-)$	3540	3533	3525
$\Sigma_{c2}(1F, 5/2^-)$	3283	3254	3304	$\Xi'_{c2}(1F, 3/2^-)$	3433	3394	3428	$\Omega_{c2}(1F, 5/2^-)$	3547	3515	3528
$\Sigma_{c3}(1F, 5/2^-)$	3247	3283	3299	$\Xi'_{c3}(1F, 3/2^-)$	3408	3408	3424	$\Omega_{c3}(1F, 5/2^-)$	3532	3522	3525
$\Sigma_{c3}(1F, 7/2^-)$	3252	3227	3305	$\Xi'_{c3}(1F, 3/2^-)$	3412	3373	3428	$\Omega_{c3}(1F, 7/2^-)$	3537	3498	3529
$\Sigma_{c4}(1F, 7/2^-)$	3207	3253	3299	$\Xi'_{c4}(1F, 3/2^-)$	3382	3393	3423	$\Omega_{c4}(1F, 7/2^-)$	3521	3514	3524
$\Sigma_{c4}(1F, 9/2^-)$	3209	3209	3305	$\Xi'_{c4}(1F, 3/2^-)$	3383	3357	3428	$\Omega_{c4}(1F, 9/2^-)$	3520	3485	3529

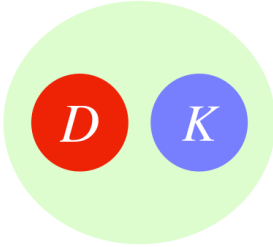
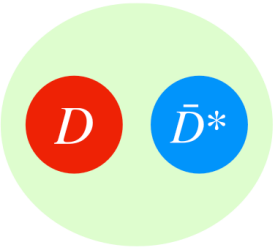
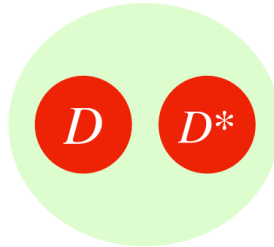
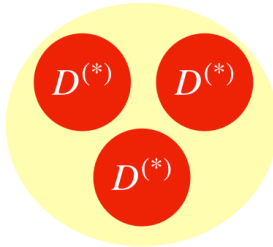
S.Q. Luo and X. Liu, PRD 108, 034002 (2023)

一类由粲介子构成的三体系统

Triple-charm molecular states composed of D^*D^*D and $D^*D^*D^*$

Si-Qiang Luo,^{1,2} Tian-Wei Wu,³ Ming-Zhu Liu,^{4,5} Li-Sheng Geng,^{5,6,7,8,*} and Xiang Liu^{1,8,9,10,†}

Inspired by the newly observed T_{cc}^+ state, we systematically investigate the S -wave triple-charm molecular states composed of D^*D^*D and $D^*D^*D^*$. We employ the one-boson-exchange model to derive the interactions between $D(D^*)$ and D^* and solve the three-body Schrödinger equations with the Gaussian expansion method. The S - D mixing and coupled channel effects are carefully assessed in our study. Our results show that the $I(J^P) = \frac{1}{2}(0^-, 1^-, 2^-)D^*D^*D$ and $I(J^P) = \frac{1}{2}(0^-, 1^-, 2^-, 3^-)D^*D^*D^*$ systems could form bound states, which can be viewed as three-body hadronic molecules. We present not only the binding energies of the three-body bound states, but also the root-mean-square radii of D - D^* and D^* - D^* , which further corroborate the molecular nature of these states. These predictions could be tested in the future at LHC or HL-LHC.

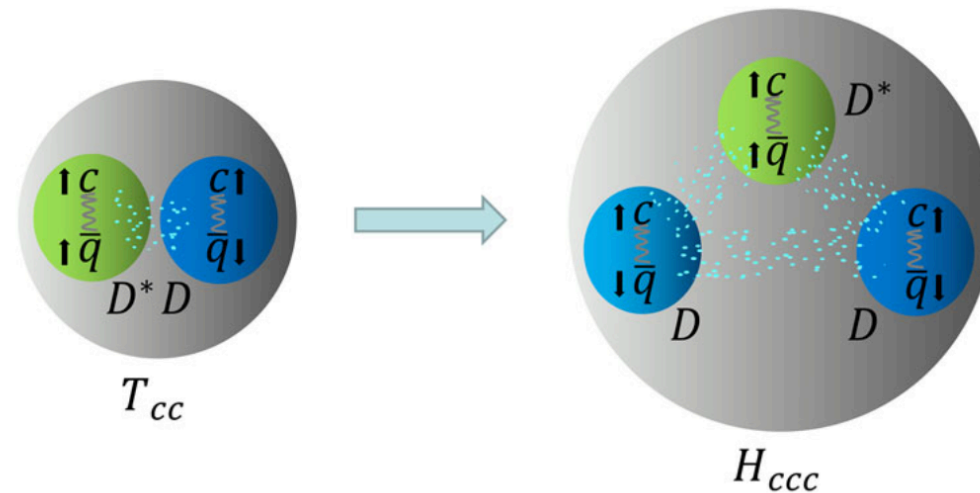
Single-charm	Hidden-charm	Double-charm	Triple-charm
 $D_{s0}^*(2327)$	 $X(3872)$	 T_{cc}	 ?

一类由粲介子构成的三体系统

Discovery of the doubly charmed T_{cc}^+ state implies a triply charmed H_{ccc} hexaquark state

Tian-Wei Wu,^{1,2} Ya-Wen Pan,¹ Ming-Zhu Liu^{3,1}, Si-Qiang Luo,⁴
Li-Sheng Geng^{1,5,6,7,*} and Xiang Liu^{4,8,7,†}

The doubly charmed exotic state T_{cc} recently discovered by the LHCb Collaboration could well be a DD^* molecular state long predicted in various theoretical models, in particular, the DD^* isoscalar axial vector molecular state predicted in the one-boson-exchange model. In this work, we study the DDD^* system in the Gaussian expansion method with the DD^* interaction derived from the one-boson-exchange model and constrained by the precise binding energy of 273 ± 63 keV of T_{cc} with respect to the $D^{*+}D^0$ threshold. We show the existence of a DDD^* state with a binding energy of a few hundred keV, isospin $1/2$, and spin-parity 1^- . Its main decay modes are $DDD\pi$ and $DDD\gamma$. The existence of such a state could in principle be confirmed with the upcoming LHC data and will unambiguously determine the nature of the T_{cc}^+ state and of the many exotic states of similar kind, thus deepening our understanding of the nonperturbative strong interaction.



一类由粲介子和核子构成的三体系统

Double-charm heptaquark states composed of two charmed mesons and one nucleon

Si-Qiang Luo,^{1,2,3,*} Li-Sheng Geng,^{4,5,6,7,†} and Xiang Liu^{1,7,3,8,‡}

Inspired by the experimental discoveries of T_{cc} , $\Sigma_c(2800)$, and $\Lambda_c(2940)$ and the theoretical picture where they are DD^* , DN , and D^*N molecular candidates, we investigate the double-charm heptaquark system of DD^*N . We employ the one-boson-exchange model to deduce the pairwise D - D^* , D - N , and D^* - N potentials and then study the DD^*N system with the Gaussian expansion method. We find two good hadronic molecular candidates with $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ and $\frac{1}{2}(\frac{3}{2}^+)$ DD^*N with only S -wave pairwise interactions. The conclusion remains unchanged even taking into account the S - D mixing and coupled channel effects. In addition to providing the binding energies, we also calculate the root-mean-square radii of the DD^*N system, which further support the molecular nature of the predicted states. They can be searched for at the upcoming LHC run 3 and run 4.

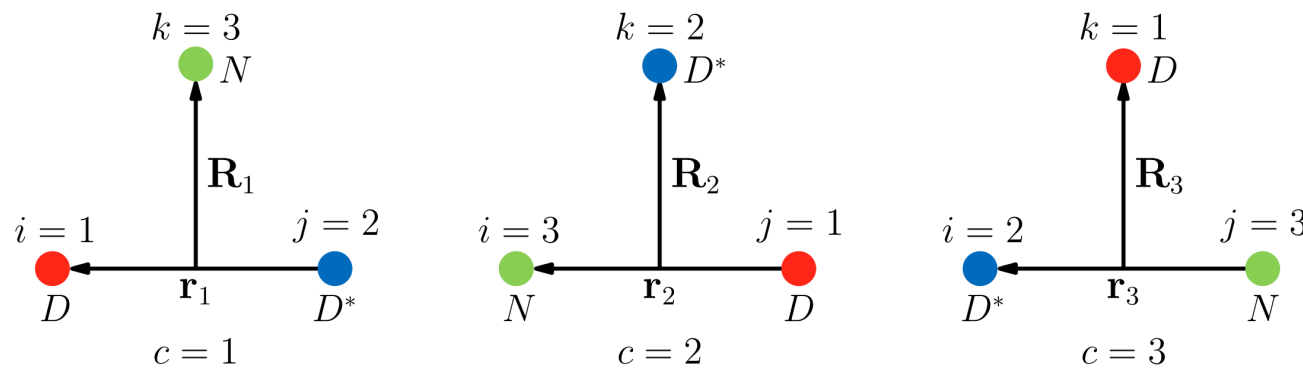
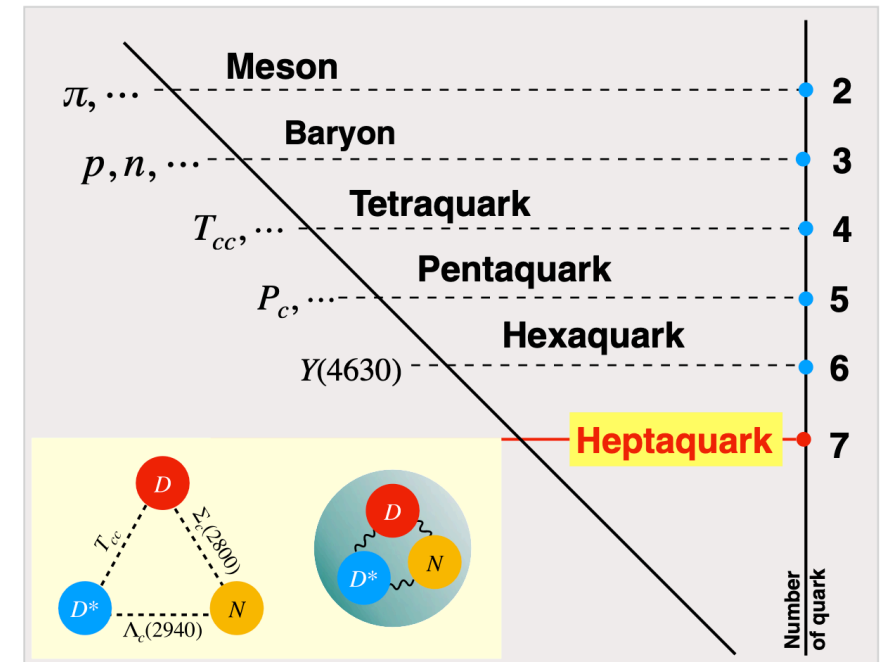


FIG. 2. Jacobi coordinates of the DD^*N system.



New type of hydrogenlike charm-pion or charm-kaon matter

Si-Qiang Luo,^{1,2,3,4,5,*} Zhan-Wei Liu,^{1,3,4,5,†} and Xiang Liu^{1,3,4,5,‡}

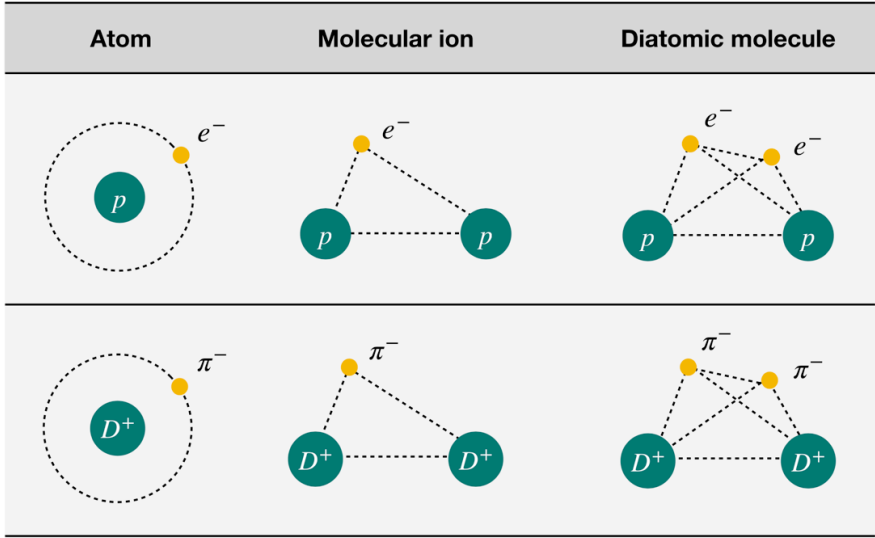
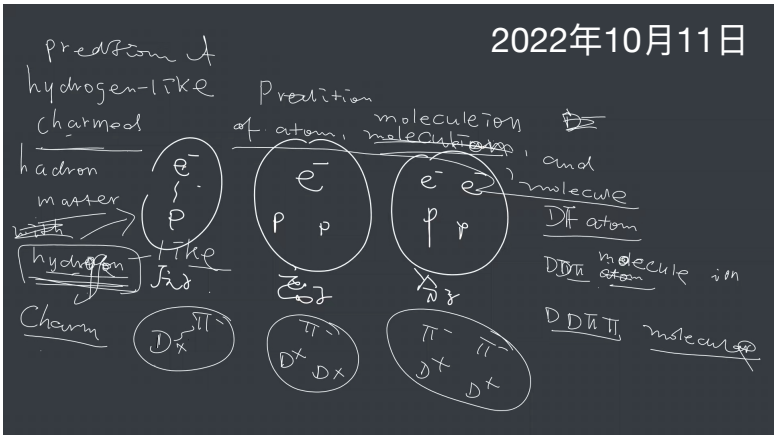
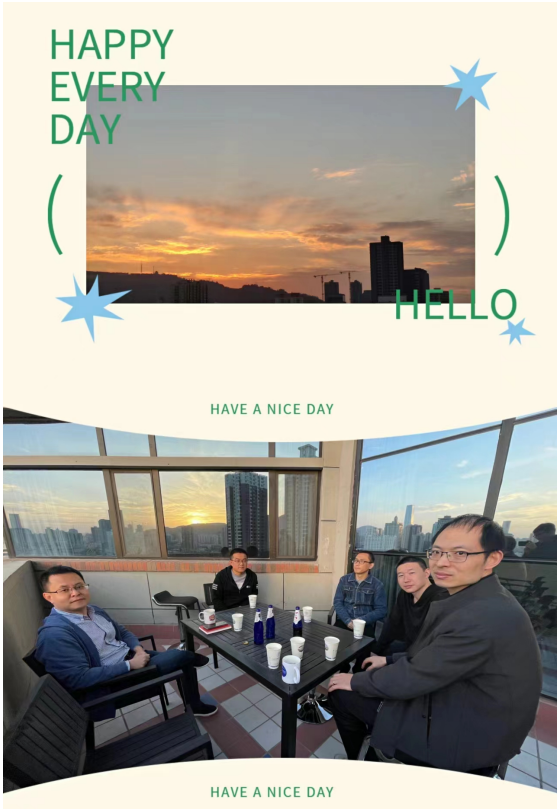
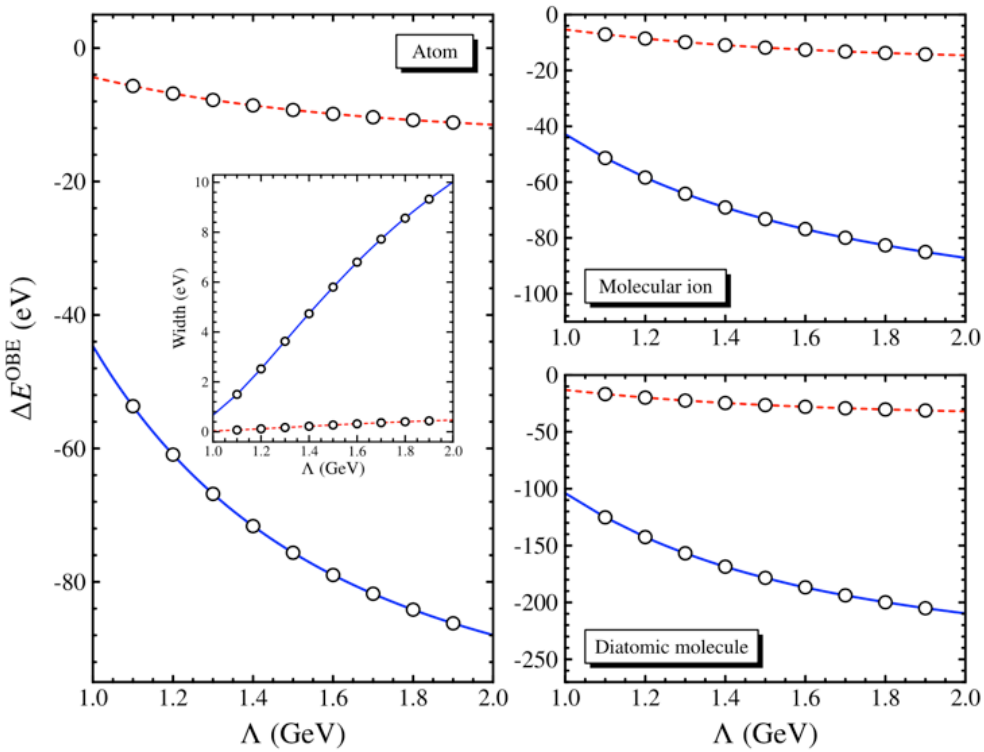
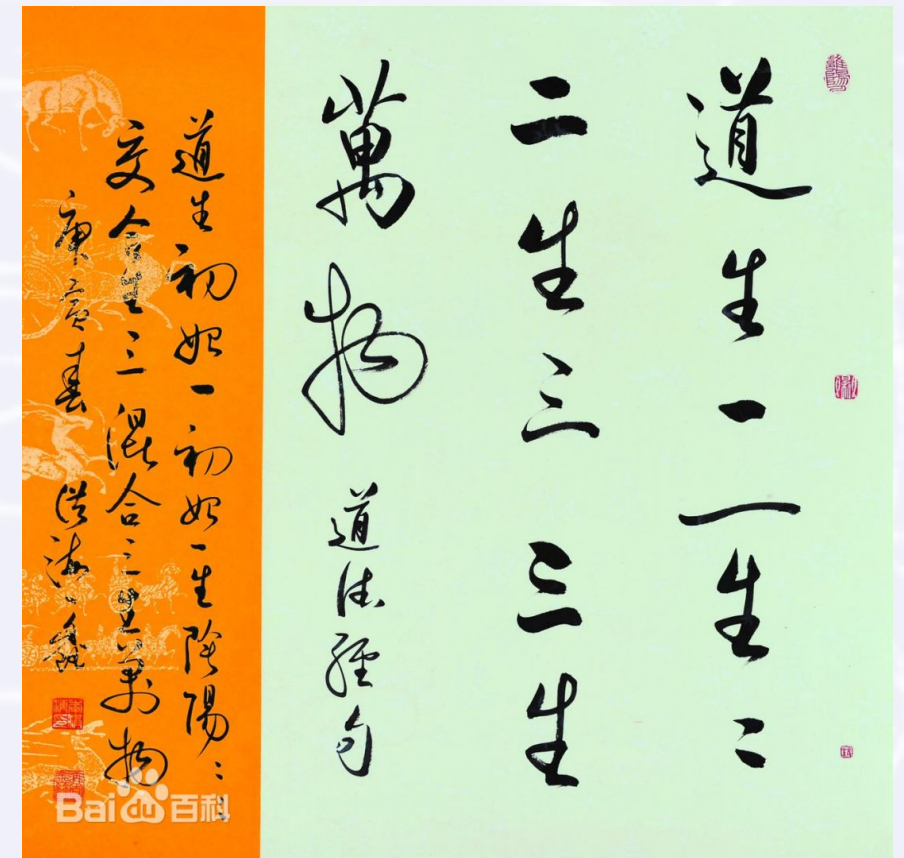
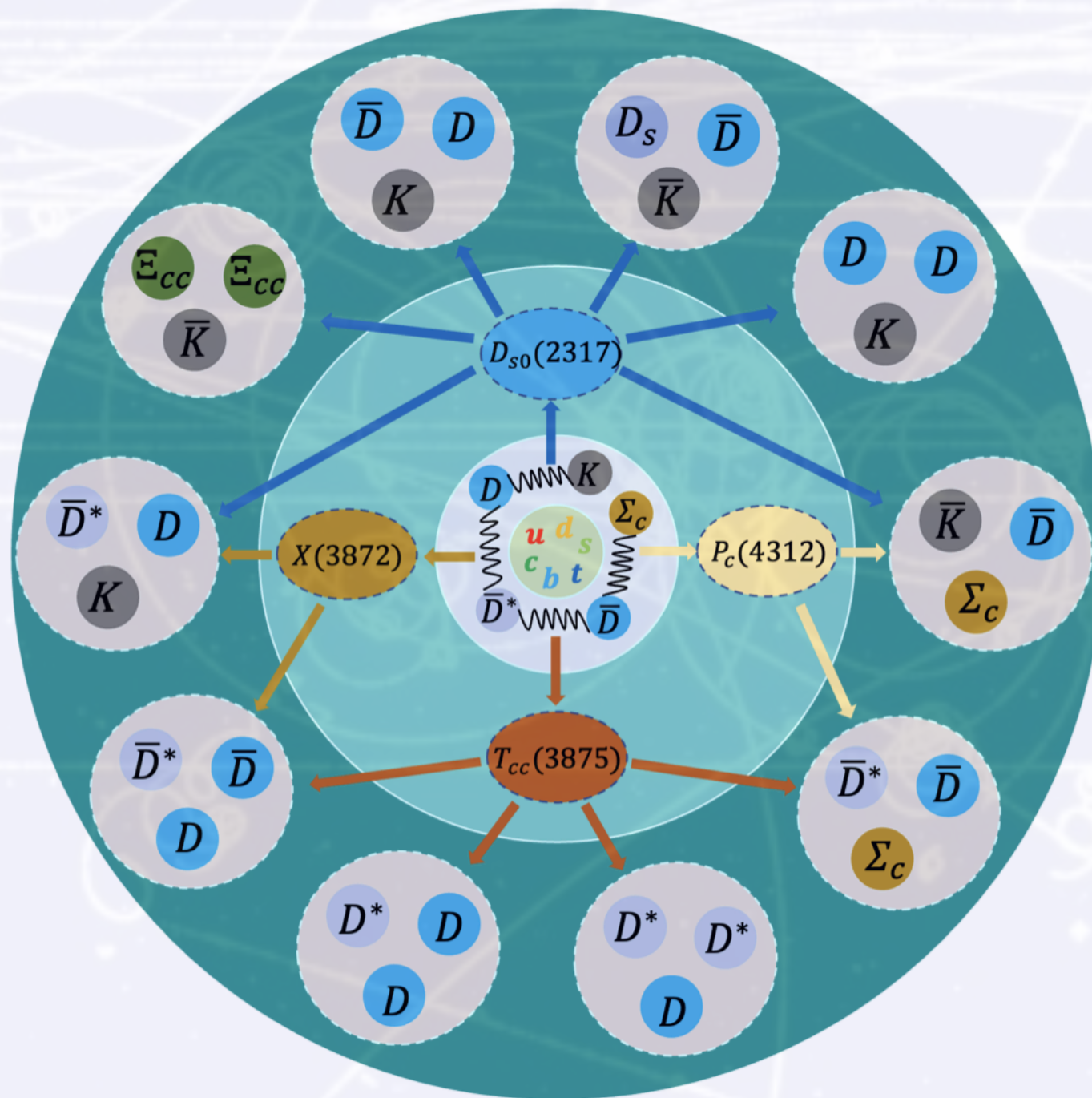


TABLE I. The binding energy E , root-mean-square radius R of the atom, molecular ion, and diatomic molecule type systems. For the charm-pion and charm-kaon atoms, the decay widths are also given.

		Hydrogen		Charm-pion		Charm-kaon	
		Experimental	Theoretical	Theoretical		Theoretical	
Atom	E (eV)	-13.6	-13.6	E^{QED} (keV)	-3.458	E^{QED} (keV)	-10.421
	R (nm)		0.09	R (fm)	360.6	R (fm)	119.6
				ΔE^{OBE} (eV)	-4.4 ~ -11.5	ΔE^{OBE} (eV)	-44.7 ~ -88.0
				Γ (eV)	0.03 ~ 0.47	Γ (eV)	0.7 ~ 10.0
Molecular ion	B (eV)	-16.25	-16.20	E^{QED} (keV)	-3.848	E^{QED} (keV)	-11.182
	R^{pp} (nm)		0.11	$R^{D^+D^+}$ (fm)	613.0	$R^{D^+D^+}$ (fm)	259.2
	R^{pe^-} (nm)		0.10	$R^{D^+\pi^-}$ (fm)	496.0	$R^{D^+K^-}$ (fm)	197.2
				ΔE^{OBE} (eV)	-5.3 ~ -14.6	ΔE^{OBE} (eV)	-42.8 ~ -87.2
Diatomic molecule	E (eV)	-31.65	-31.60	E^{QED} (keV)	-7.517	E^{QED} (keV)	-21.889
	R^{pp} (nm)		0.127	$R^{D^+D^+}$ (fm)	574.3	$R^{D^+D^+}$ (fm)	187.3
	$R^{e^-e^-}$ (nm)		0.076	$R^{\pi^-\pi^-}$ (fm)	435.4	$R^{K^-K^-}$ (fm)	214.3
	R^{pe^-} (nm)		0.094	$R^{D^+\pi^-}$ (fm)	433.8	$R^{D^+K^-}$ (fm)	164.7
				ΔE^{OBE} (eV)	-13.1 ~ -32.0	ΔE^{OBE} (eV)	-103.9 ~ -209.7



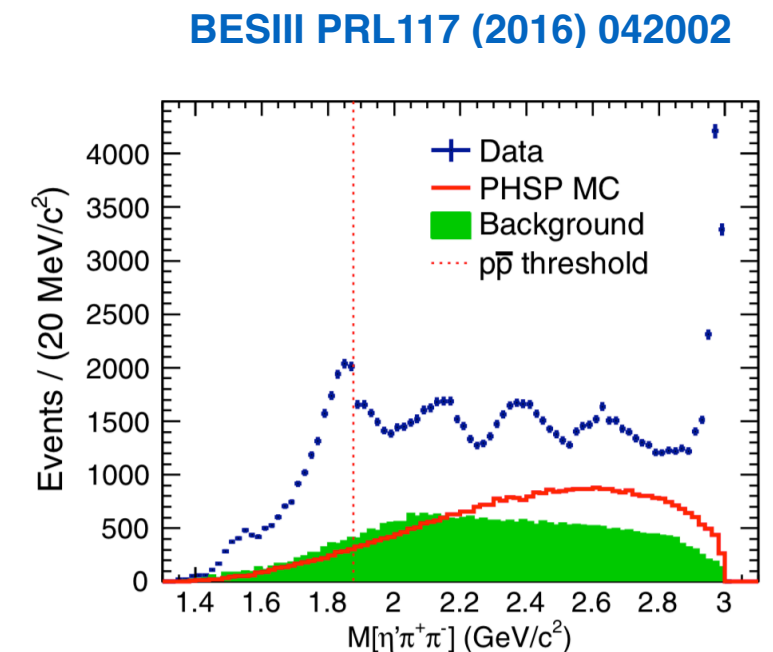
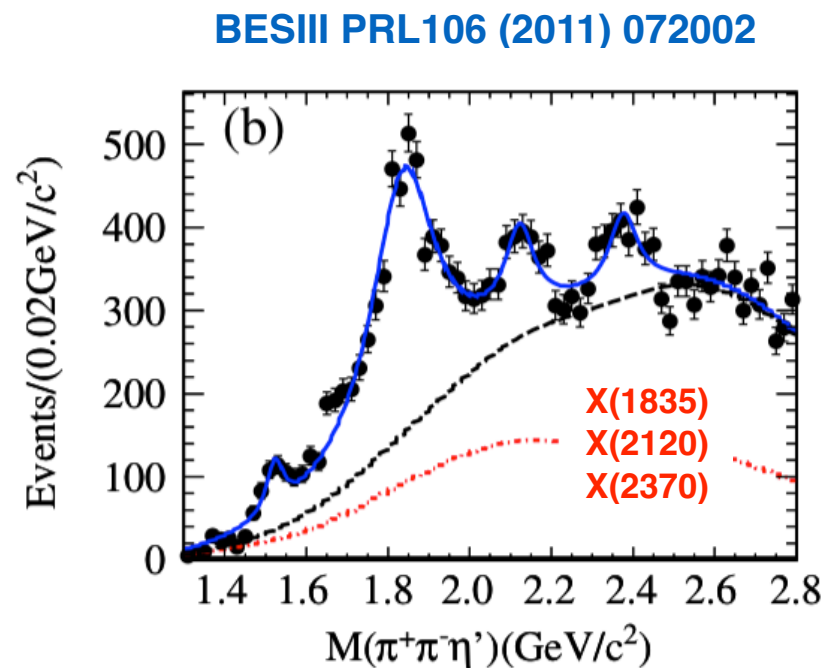
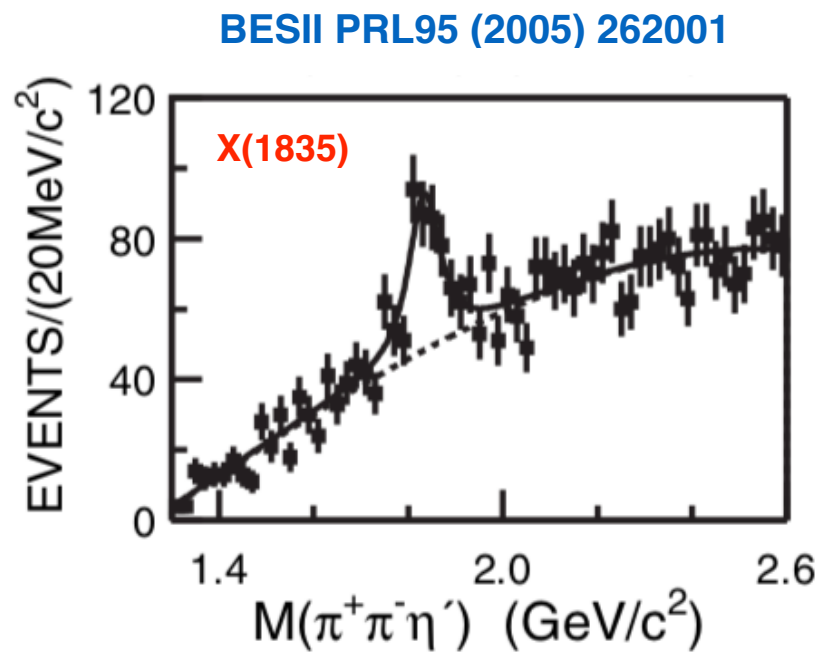
对实验精度提出了新要求



Tian-Wei Wu, Ya-Wen Pan, Ming-Zhu Liu, Li-Sheng Geng, Sci.Bull. 67 (2022), 1735-1738

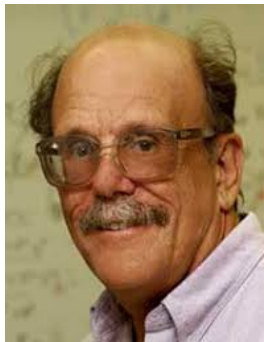
4. 小结

- 精度提升导致新发现



- 精度永远不够用

You never have enough J/ψ events



—The case for a J/ψ factory—

Stephen Lars Olsen

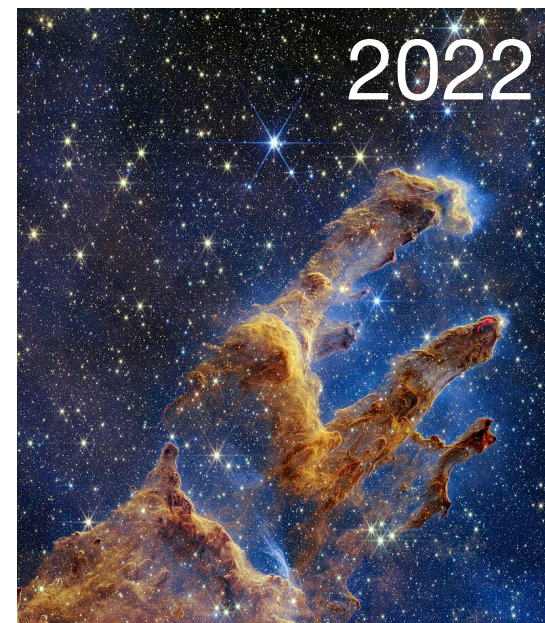
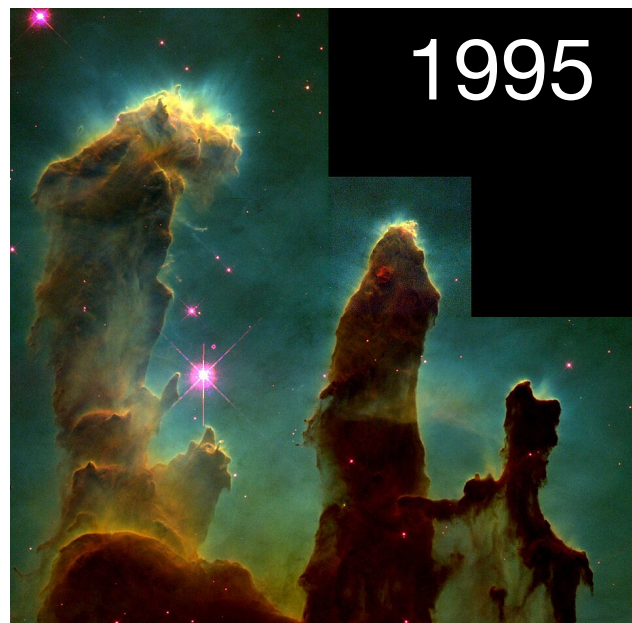
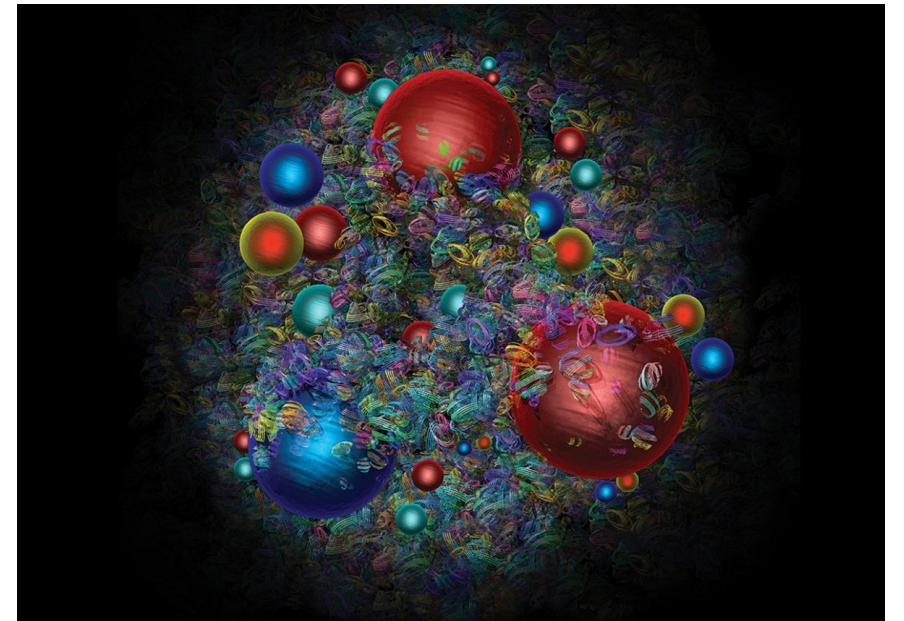
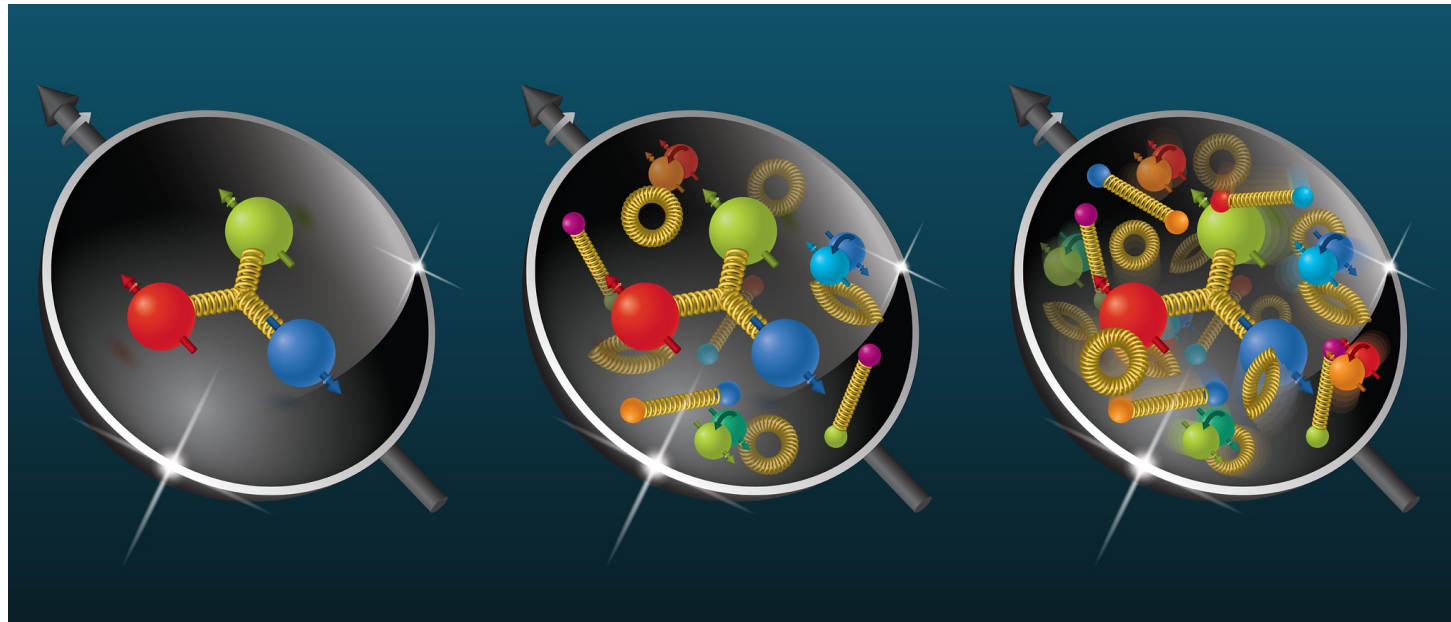
1 Introduction

“Why do we need more J/ψ events?” and “...don’t we already have enough?” are questions that have been frequently asked at collaboration meetings throughout the history of the BES, BESII and, now, BESIII experiments, usually by charm physics aficionados while they are clamoring for more data at the $D\bar{D}$ or $D_s\bar{D}_s$ thresholds, or, more recently, by XYZ -meson enthusiasts with ambitions to find yet another multiquark state. In the past, the answer has been primarily been that the J/ψ is a prolific source of light hadrons that are produced with low backgrounds and well defined quantum numbers that have supported numerous, and often unique, studies of the spectroscopy of light-quark hadrons and the dynamics of their production and decays. Maybe not the most glamorous physics, but still very interesting and useful.

arXiv: 2506.20975

4. 小结

- 精度提升也带来了挑战



遂古之初，
谁传道之？
上下未形，
何由考之？
——《天问》

行尽桂林山水 邂逅强子峰峦



行尽桂林山水 邂逅强子峰峦





**Thank you
for your
attention!**