Nucleon-Hyperon Interaction from Lattice QCD

核子-超子相互作用的格点QCD研究

上海师范大学 刘航

合作者：王伟，谭金鑫，朱潜腾，刘柳明等

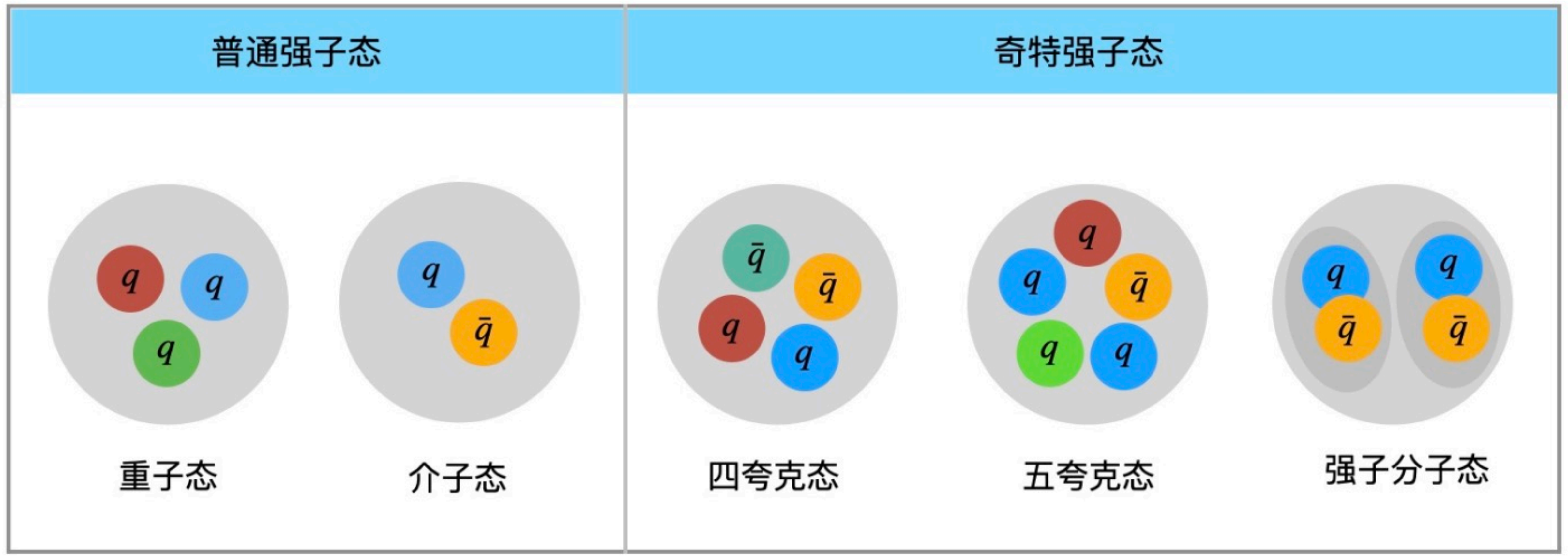
第八届强子谱和强子结构研讨会

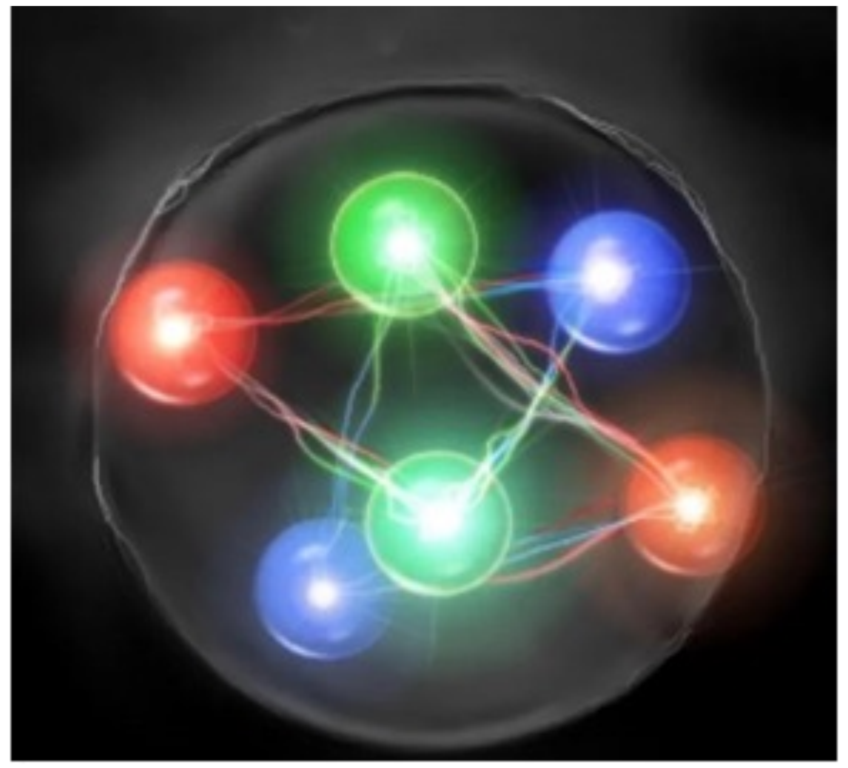
Outline

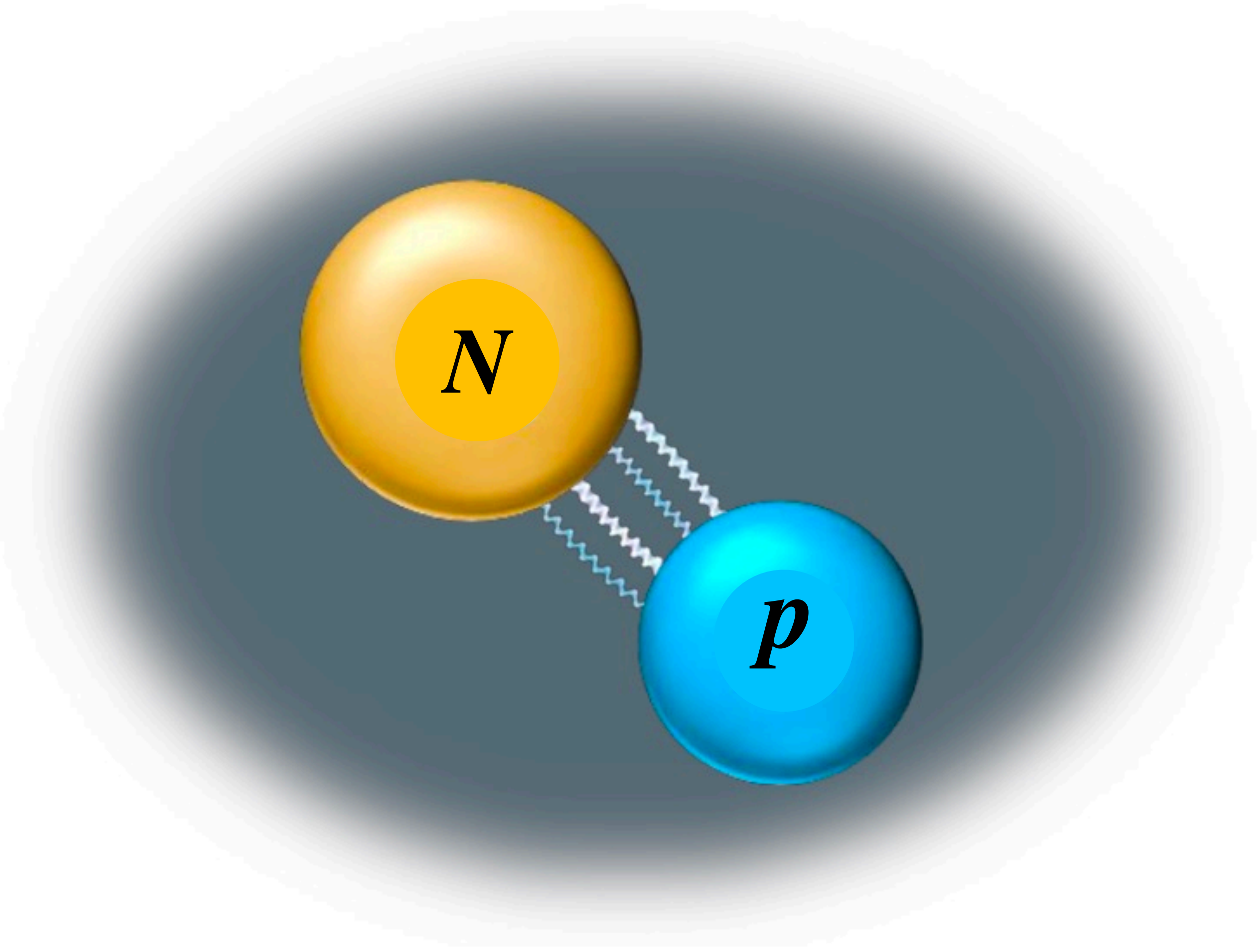
- Motivation
- proton – Λ interaction from the HALQCD approach
- proton – Λ scattering from the Lüscher's finite volume method
- Summary and Prospect

Motivation

exotic hadron







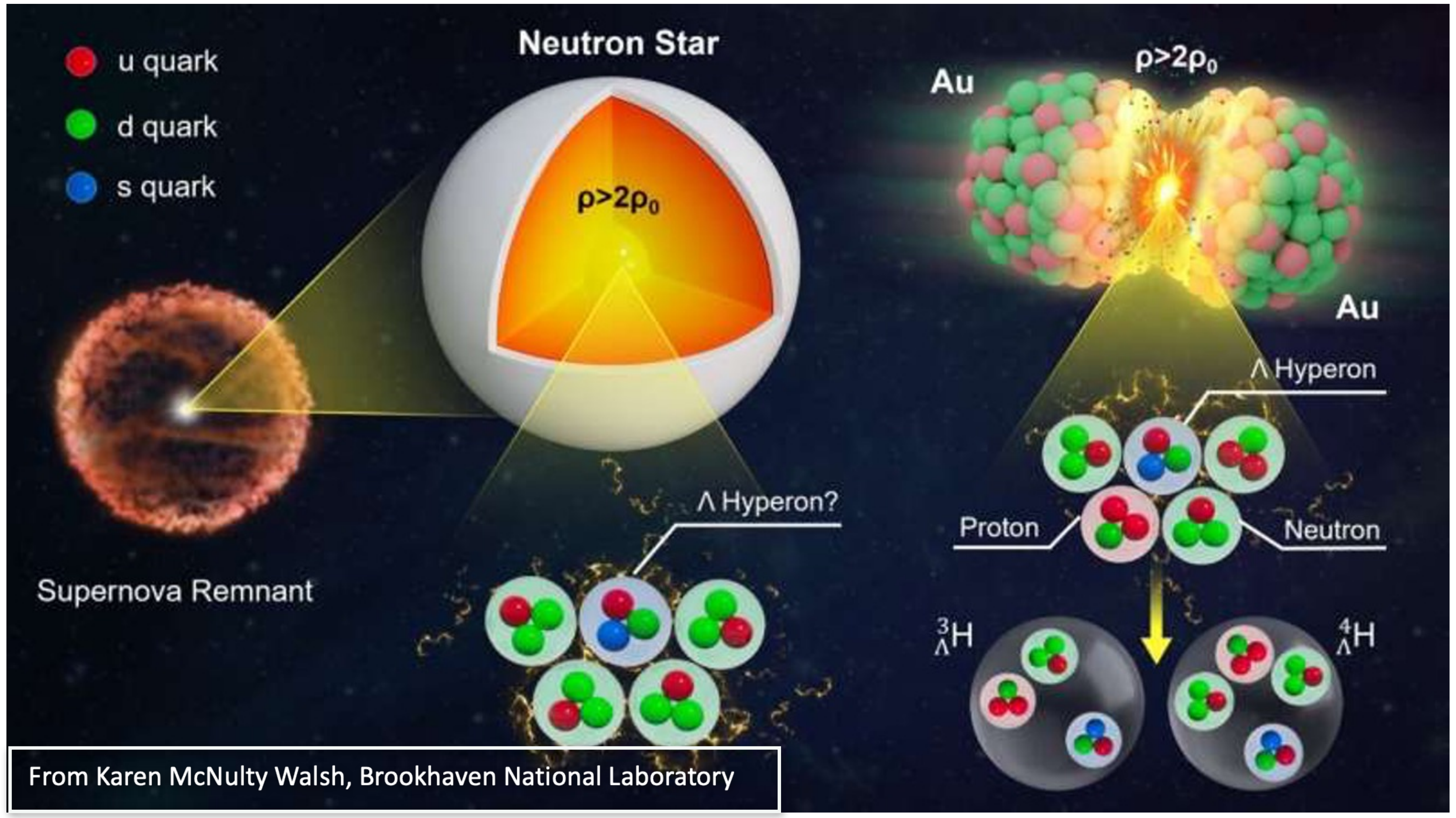
- ✓ hadronic molecule?
- ✓ multiquark state?
- ✓ the effective potential ?
- ✓ the binding energy?

CLQCD

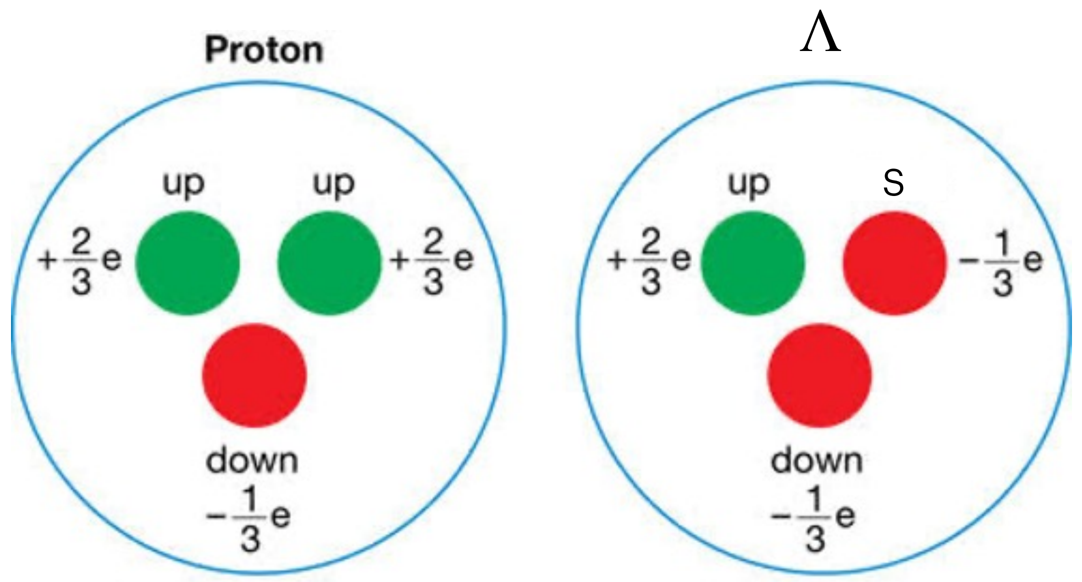
- np scattering
- $\Lambda\Lambda - N\Xi$ scattering
- $\Lambda_c\Lambda_c$ scattering

Liuming Liu, et al.
Chin.Phys.C 49 (2025) 6, 063107
and some in progress

Motivation



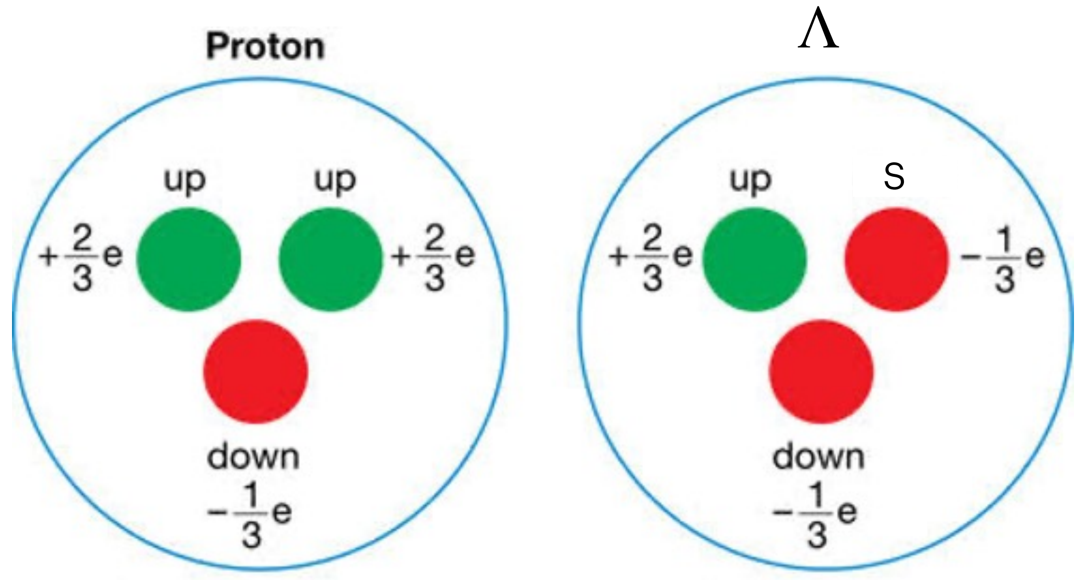
- Hyperon-Nucleon interactions



Also concerned

- ✓ hadronic molecule?
- ✓ multiquark state?
- ✓ the effective potential ?
- ✓ the binding energy?
- ✓ "hyperon puzzle" in neutron stars

Motivation



Hep-ex:

✓ YN correlation functions in heavy-ion collisions:

J. Adams et al. [STAR Collaboration], Phys. Rev. C 74, 064906 (2006)

J. Adam et al. [STAR Collaboration], Phys. Lett. B 790, 490 (2019)

S. Acharya et al. [ALICE Collaboration], Phys. Rev. Lett. 123, 112002 (2019)

S. Acharya et al. [ALICE Collaboration], Nature 588, 232 (2020)

✓ hypernuclei:

[J-PARC E07 Collaboration], Phys. Rev. Lett. 126, 062501 (2021)

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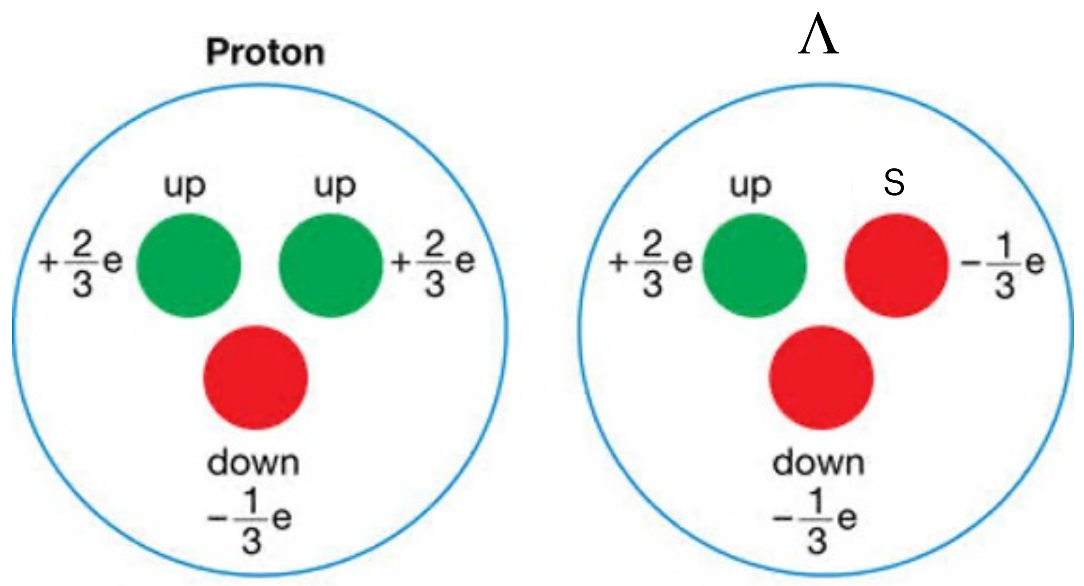
BESIII Collaboration,PhysRevLett.132.231902(2024)

Motivation



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$$C(\boldsymbol{k}^*) \approx 1 + \frac{|f(k)|^2}{2\boldsymbol{R_G}^2} F(d_0) + \frac{2\mathrm{Re}f(k)}{\sqrt{\pi}\boldsymbol{R_G}} F_1(2kR) - \frac{\mathrm{Im}f(k)}{\boldsymbol{R_G}} F_2(2k\boldsymbol{R_G})$$

$$\frac{1}{f(k)} \approx \frac{1}{\boldsymbol{f_0}} + \frac{\boldsymbol{d_0} k^2}{2} - ik$$

Different f_0 and d_0 for different spin states

B. Sechi-Zorn, et al. Phys. Rev. 175, 1735 (1968)

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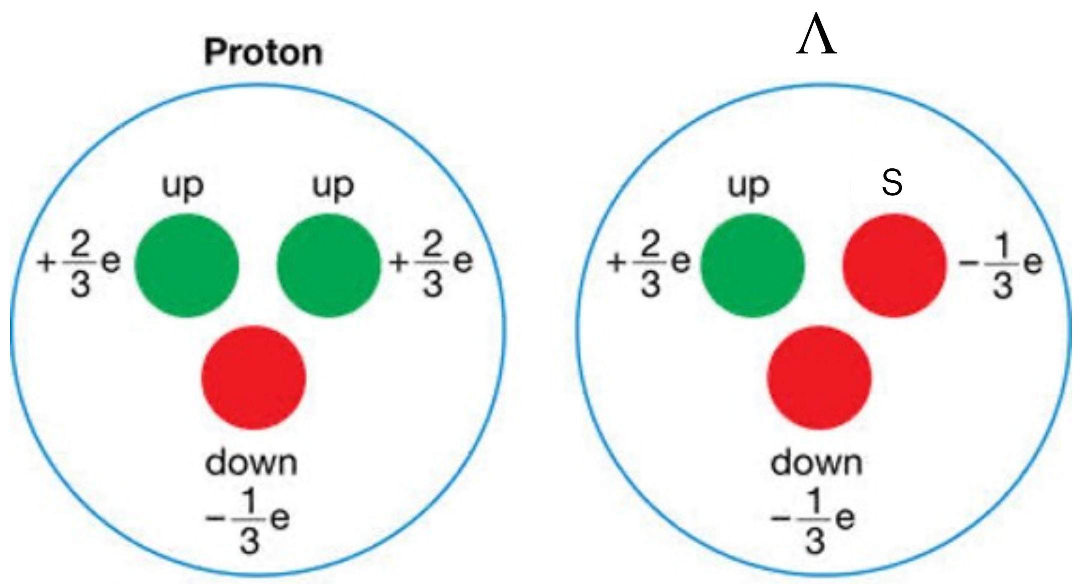

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[J-PARC E07 Collaboration], Phys. Rev. Lett. 126, 062501 (2021)


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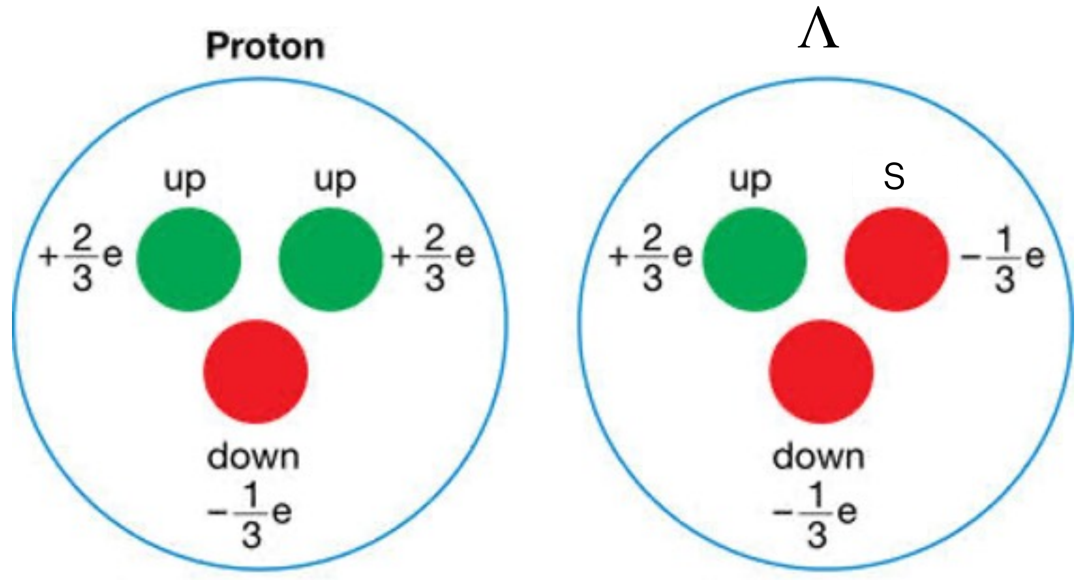
Observation of Coulomb-Assisted Nuclear Bound State of $\Xi^- - {}^{14}N$ System

S. H. Hayakawa *et al.* (J-PARC E07 Collaboration)
Phys. Rev. Lett. **126**, 062501 – Published 11 February 2021

Motivation



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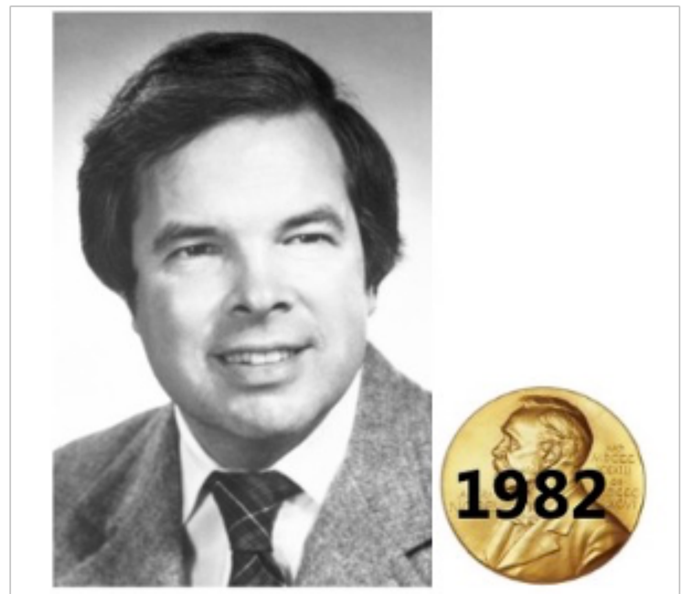
BESIII Collaboration,PhysRevLett.132.231902(2024)

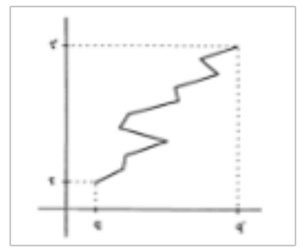
large uncertainty due to short-lifetime of hyperon beams

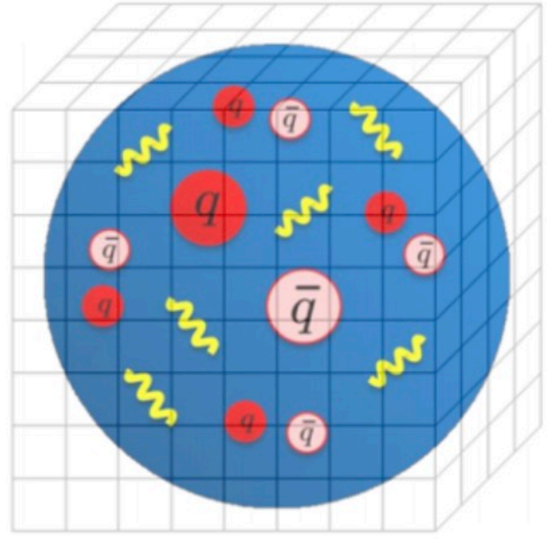
Lattice QCD

LatticeQCD(Wilson,1974): the ab-initio non-perturbative method



K. G. Wilson



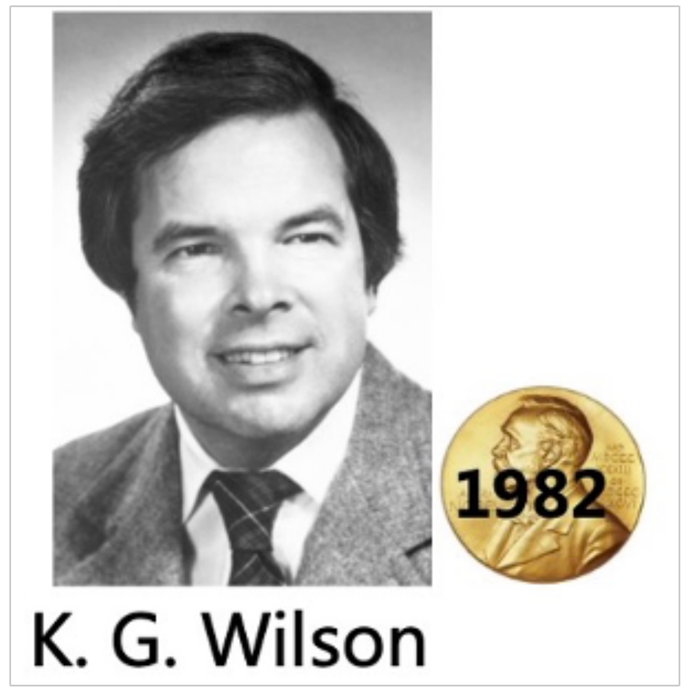


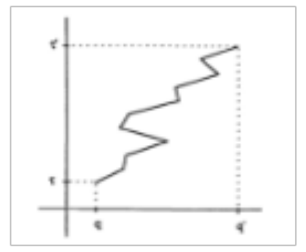
$> 2^{1000000}$

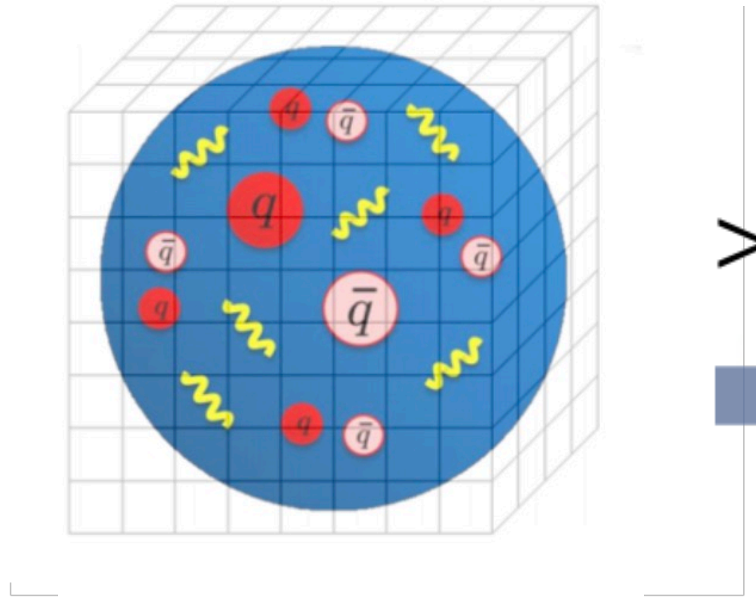
Lattice QCD

Supercomputer

Lattice QCD







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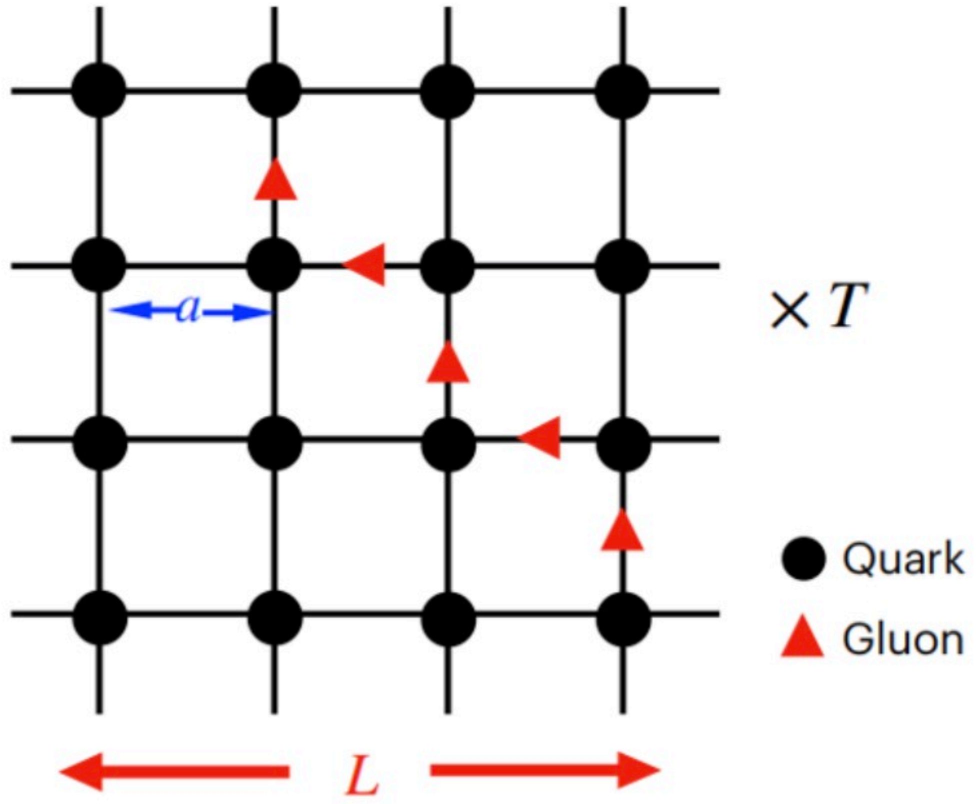
Lattice QCD

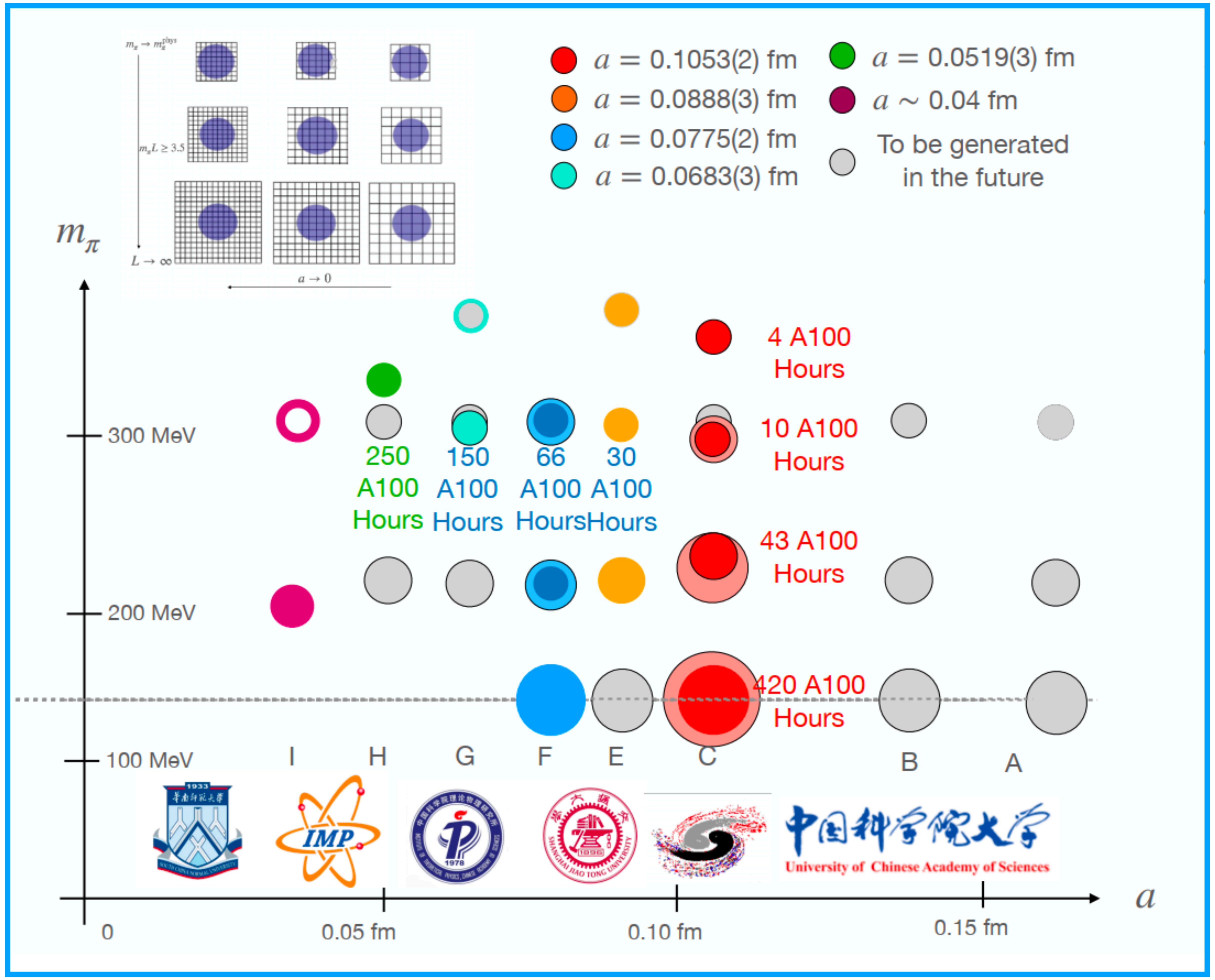
Supercomputer



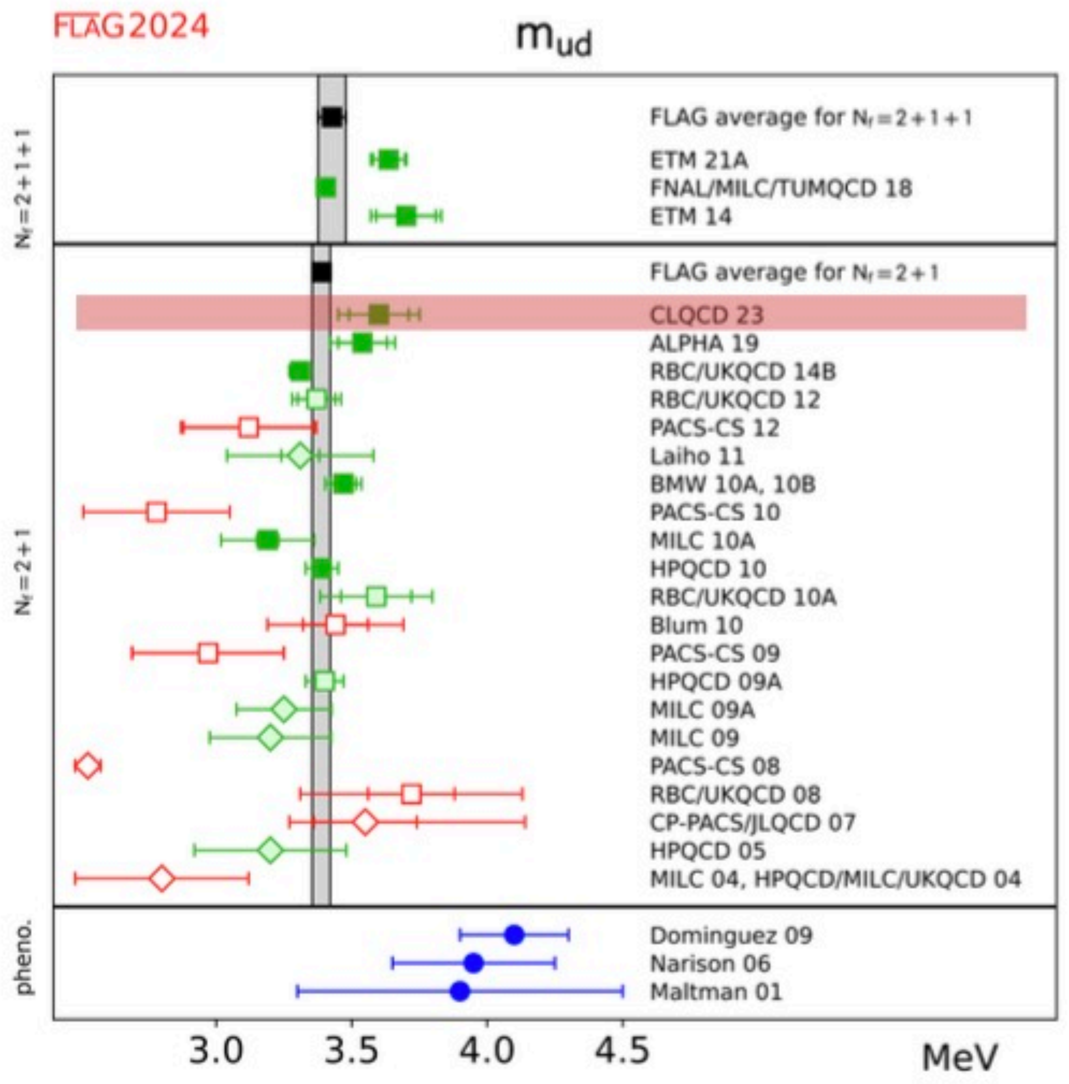
- Quark on discrete lattice:
  consider both IR and UV effects:

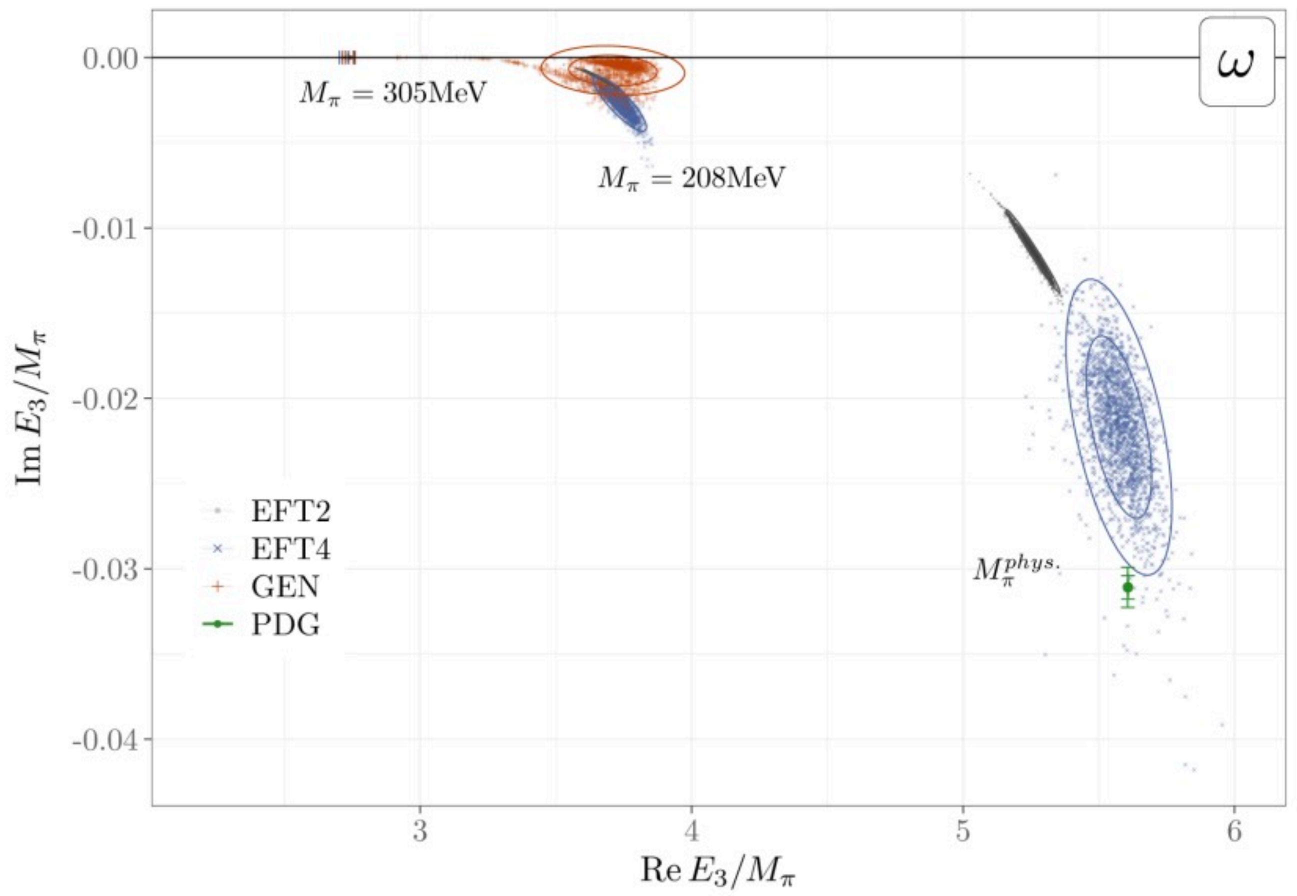
$$m_\pi L \gtrsim 4, \quad \text{and} \quad a^{-1} \gg \text{mass scale}$$

New Lattice QCD configurations



Hu, et.al., PRD 109, 054507 (2024)





Hao-bo Yan, et al. Phys.Rev.Lett. 133 (2024) 21, 211906

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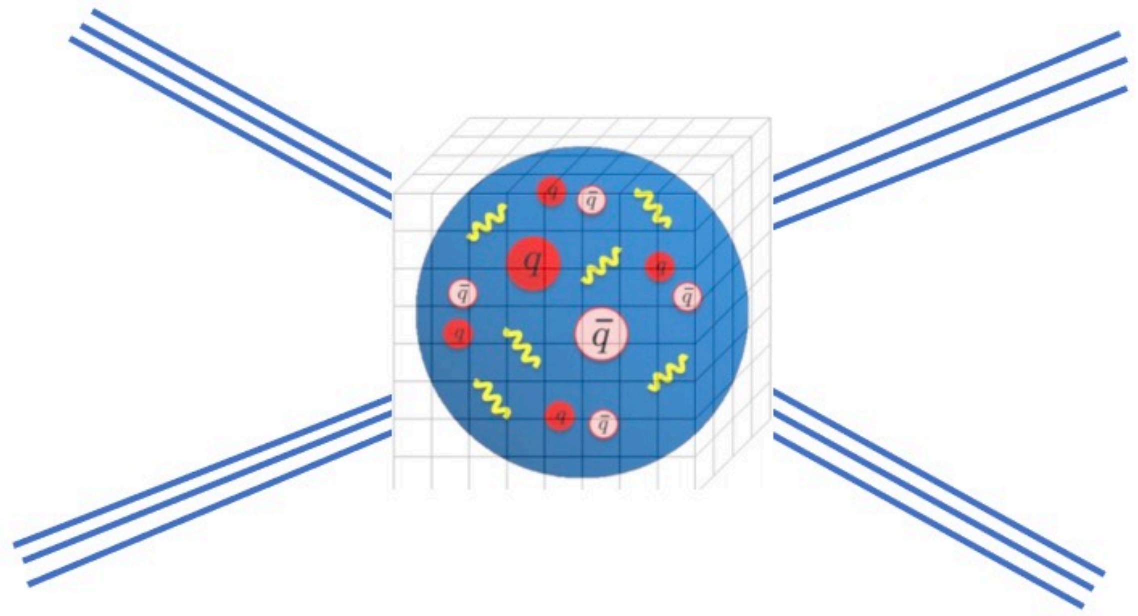
The HALQCD method

$$\Psi^{W_n}(\vec{r})e^{-W_n t}$$

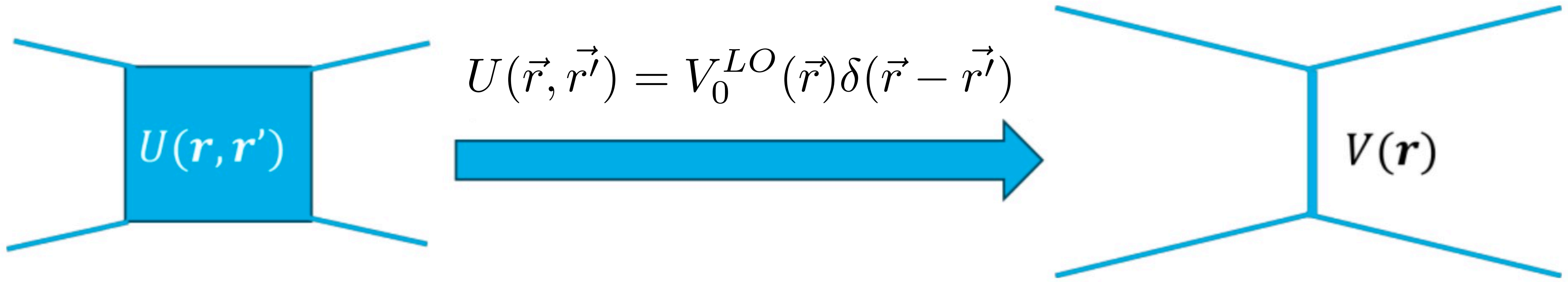
Nambu-Bethe-Salpeter wave function

$$C_{p\Lambda}(\vec{r},t) = \sum_{\vec{x}} \langle 0 | p(\vec{x},t)\Lambda(\vec{x}+\vec{r},t)\bar{J}_{p\Lambda}(0) | 0 \rangle$$



Lattice simulations

$$(E_k - H_0)\Psi^W(\vec{r}) = \int d^3\vec{r'}\, U(\vec{r},\vec{r'})\Psi^W(\vec{r'})$$



The HALQCD method

☆ To enhance the signal, the following ratio was attempted:

$$R_{p\Lambda}(\vec{r},t) = \frac{C_{p\Lambda}(\vec{r},t)}{C_p(t)C_\Lambda(t)} = \sum_n A_n' \Psi^{W_n}(\vec{r}) e^{-\Delta W_n t}$$

☆ Assume the nonlocal potential is energy independent

$$\left[-H_0 - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right] R(\vec{r},t) = \int d^3\vec{r'}\, U(\vec{r},\vec{r'}) R(\vec{r'},t)$$

☆ Then the effective potential leads to

$$V_0^{LO}(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2 R(\vec{r},t)}{R(\vec{r},t)} - \frac{(\partial/\partial t)R(\vec{r},t)}{R(\vec{r},t)} + \frac{1}{8\mu}\frac{(\partial^2/\partial t^2)R(\vec{r},t)}{R(\vec{r},t)}$$

Lattice setup for $p - \Lambda$

	β	$L^3 \times T$	a	m_π	N_{confs}
C24P29	6.20	$24^3 \times 72$	0.10530	292.7(1.2)	872
C32P29	6.20	$32^3 \times 64$	0.10530	292.4(1.1)	984
C48P23	6.20	$48^3 \times 96$	0.10530	225.6(0.9)	265
C48P14	6.20	$48^3 \times 96$	0.10530	135.5(1.6)	259
F48P21	6.41	$48^3 \times 96$	0.07746	207.2(1.1)	220
F48P30	6.41	$48^3 \times 96$	0.07746	303.4(0.9)	359
H48P32	6.72	$48^3 \times 144$	0.05187	317.2(0.9)	274

7 different ensembles: 3 different lattice spacings,
pion mass 140MeV~320MeV,
different volume

Results can be extend to the physical point and the continuum limit.

Two-point Correlation Function

One-particle operators and two-particle operators

$$p_\sigma = \epsilon^{abc}\frac{1}{\sqrt{2}}[u^a_\zeta(x)(C\gamma_5 P_+)_{\zeta\xi}d^b_\xi(x) - d^a_\zeta(x)(C\gamma_5 P_+)_{\zeta\xi}u^b_\xi(x)] \times [P_+(1-(-1)^\sigma i\gamma_1\gamma_2)]_{\sigma\rho}u^c_\rho(x)$$

$$\Lambda_\sigma = \epsilon^{abc}\frac{1}{\sqrt{2}}[d^a_\zeta(x)(C\gamma_5 P_+)_{\zeta\xi}u^b_\xi(x) - u^a_\zeta(x)(C\gamma_5 P_+)_{\zeta\xi}d^b_\xi(x)] \times [P_+(1-(-1)^\sigma i\gamma_1\gamma_2)]_{\sigma\rho}s^c_\rho(x)$$

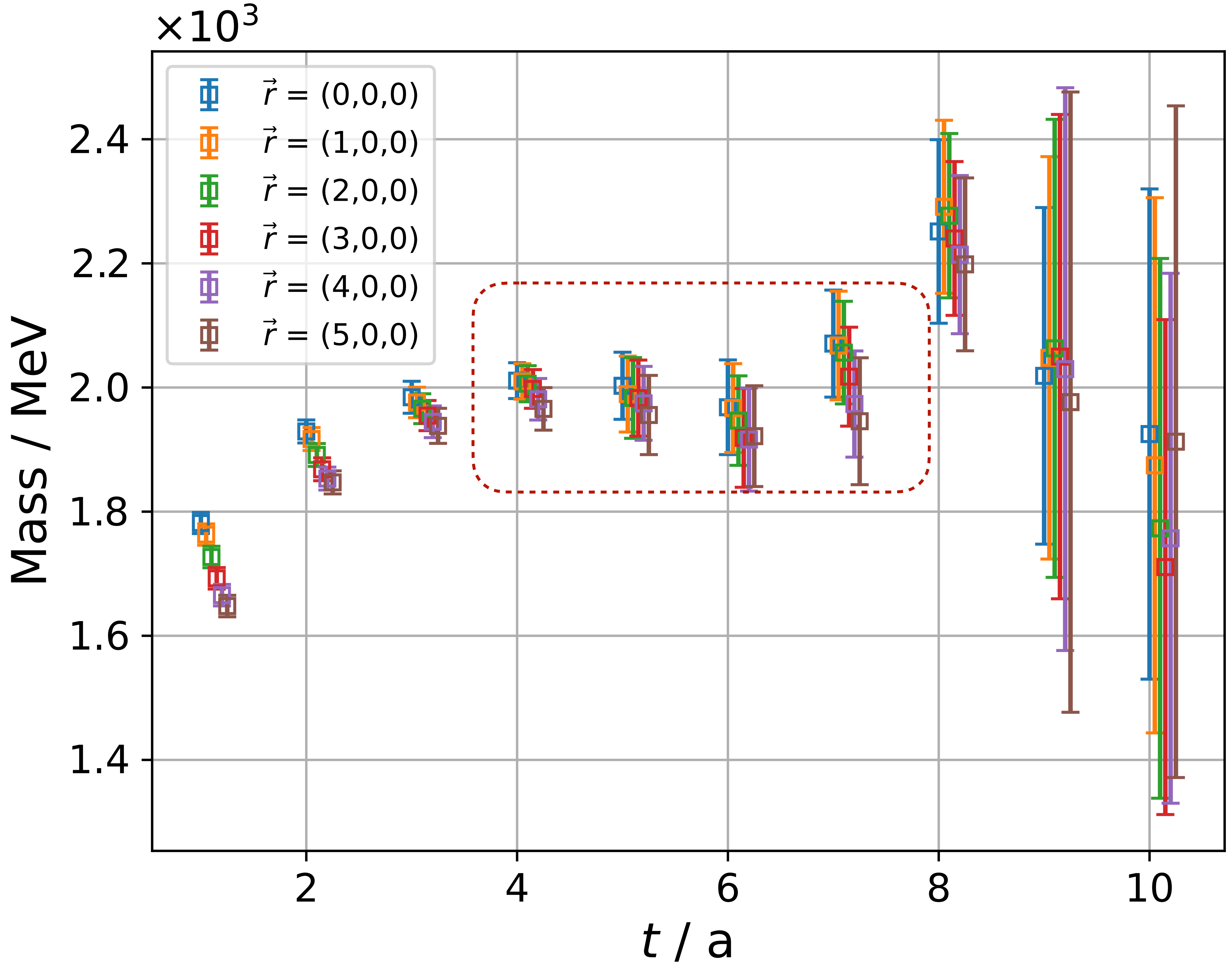
$$p\Lambda_{\rho\mathfrak{m}}(t) = \sum_{\vec{x}_1,\vec{x}_2\in\Lambda_S}\psi^{[D]}_{\mathfrak{m}}(\vec{x}_1,\vec{x}_2)\sum_{\sigma,\sigma'}v^\rho_{\sigma\sigma'} \frac{1}{\sqrt{2}}[p_\sigma(\vec{x}_1,t)\Lambda_{\sigma'}(\vec{x}_2,t) + (-1)^{1-\delta_{\rho 0}}\Lambda_\sigma(\vec{x}_1,t)p_{\sigma'}(\vec{x}_2,t)],$$

For the p-Λ system, correlation functions can be obtained

$$C_{p\Lambda}(\vec{r},t) = \sum_{\vec{x}}\langle 0|(p(\vec{x},t)\Lambda(\vec{x}+\vec{r},t))^D \bar{J}^H_{p\Lambda}(0)|0\rangle$$

The effective mass

We find the appropriate time slice for the ground state saturation of the system.



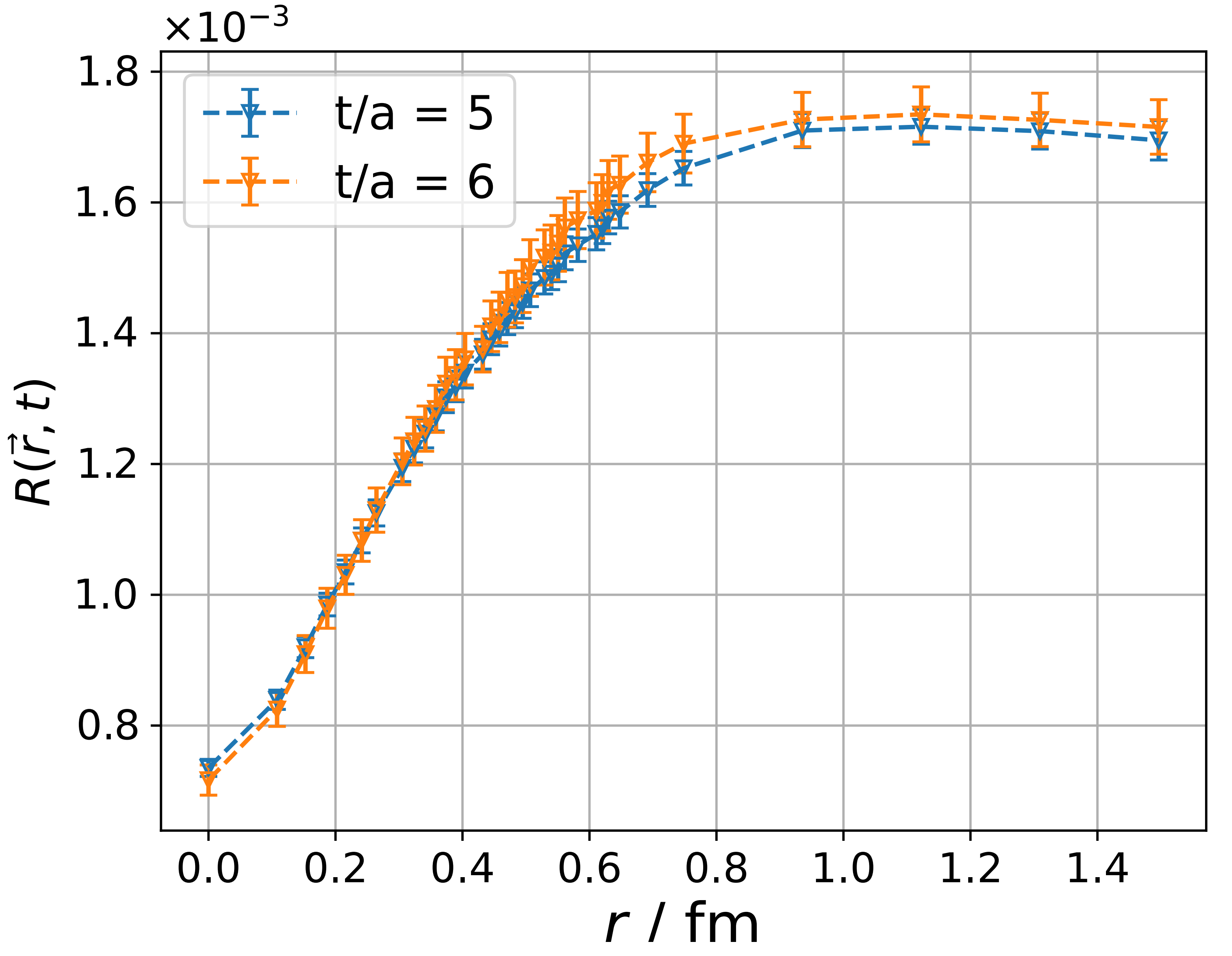
The effective mass of the dibaryon system

The HALQCD method

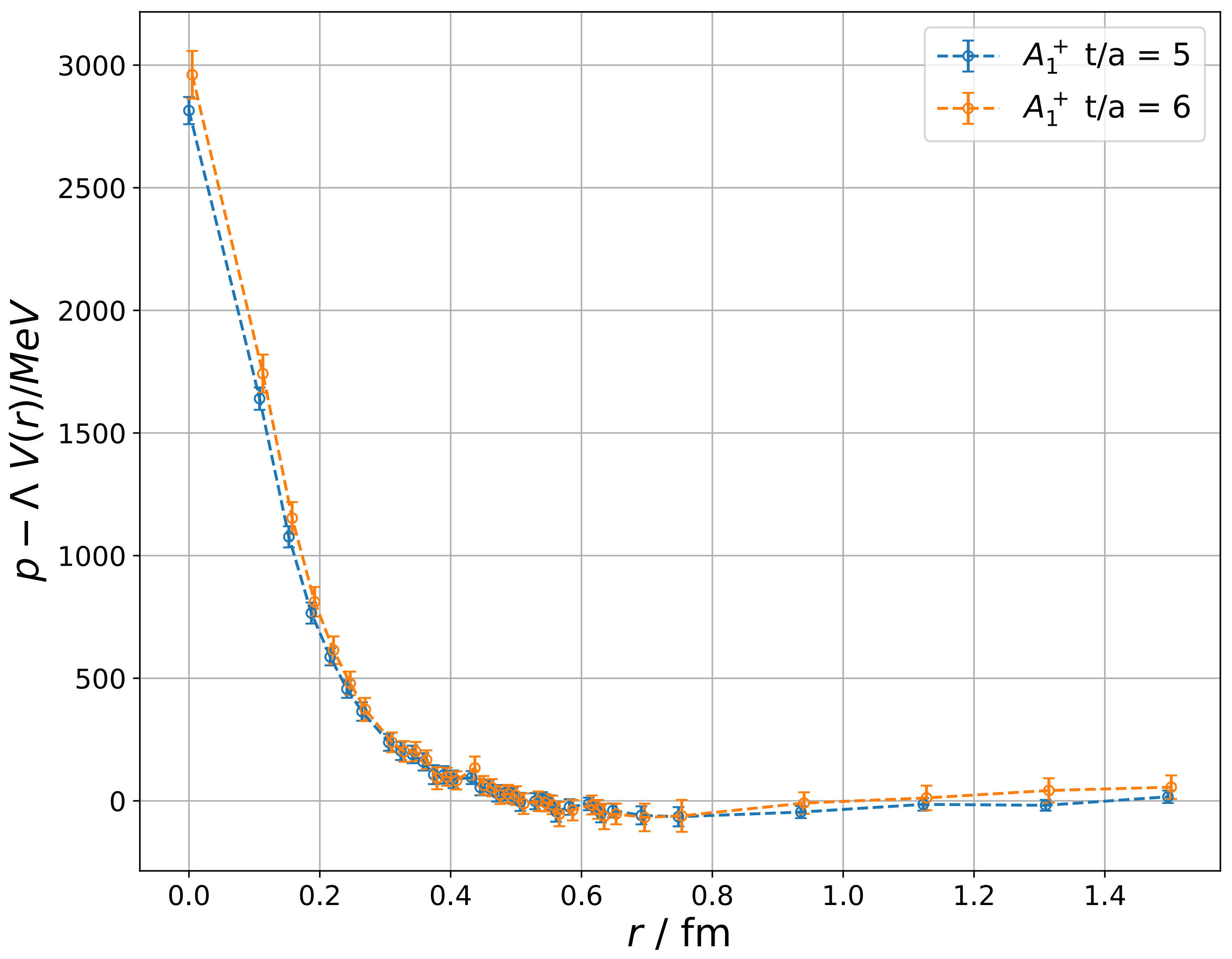
We extract the NBS wave function and the effective potential on the timeslice $t/a = 5{,}6$.

$$R_{p\Lambda}(\vec{r},t) = \frac{C_{p\Lambda}(\vec{r},t)}{C_p(t)C_\Lambda(t)} = \sum_n A_n' \Psi^{W_n}(\vec{r}) e^{-\Delta W_n t}$$

$$V_0^{LO}(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2 R(\vec{r},t)}{R(\vec{r},t)} - \frac{(\partial/\partial t)R(\vec{r},t)}{R(\vec{r},t)} + \frac{1}{8\mu}\frac{(\partial^2/\partial t^2)R(\vec{r},t)}{R(\vec{r},t)}$$



NBS wave function



The effective potential

The effective potential from the HALQCD method

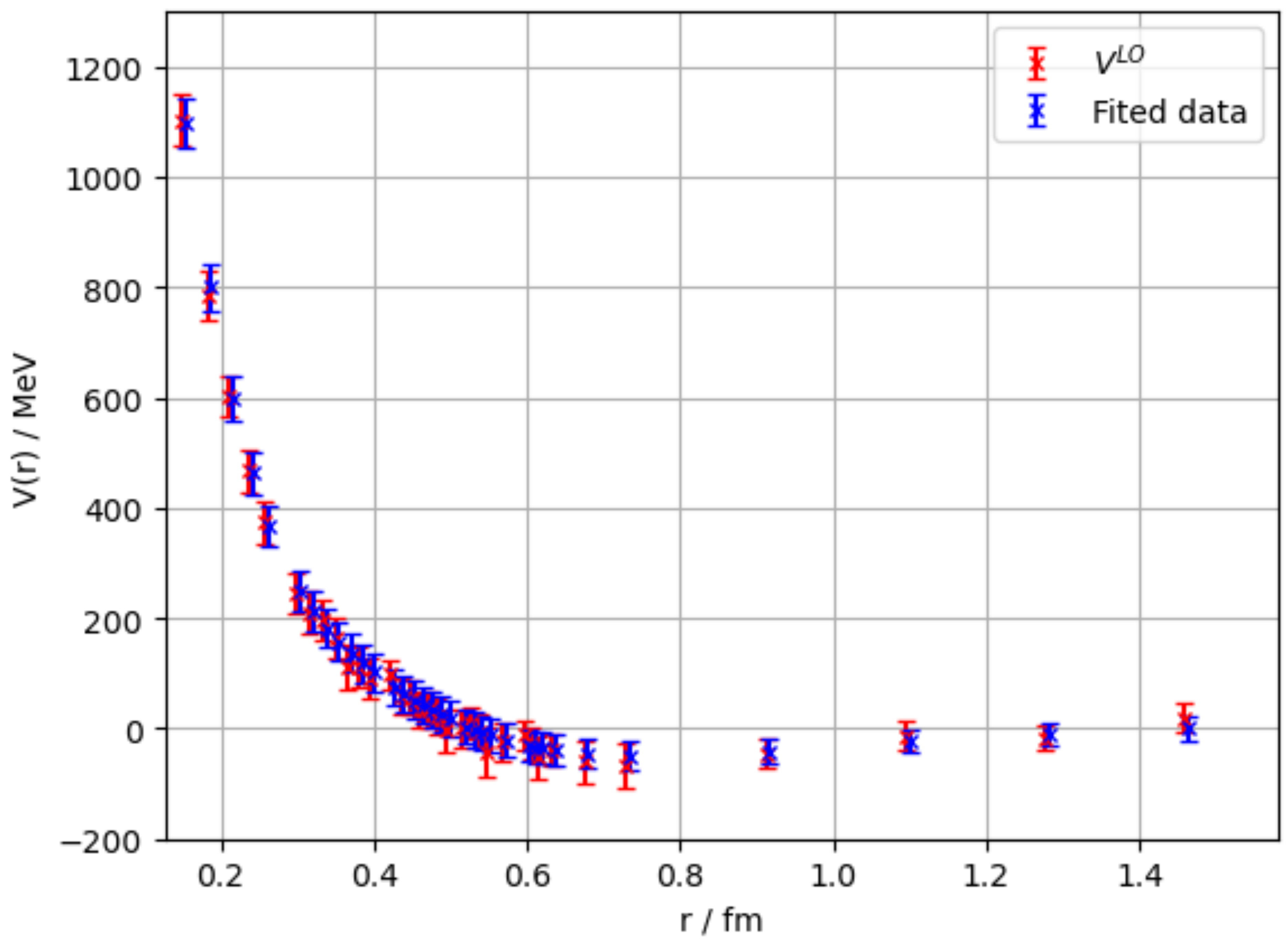
We parameterize the effective potential in this form

Argonne-type form factor

$$V(r) = v_{C1} e^{-\kappa_{C1} r^2} + v_{C2} e^{-\kappa_{C2} r^2} + v_{C3} \left(1 - e^{-\alpha_C r^2}\right)^2 \left(\frac{e^{-\beta_C r}}{r}\right)^2$$

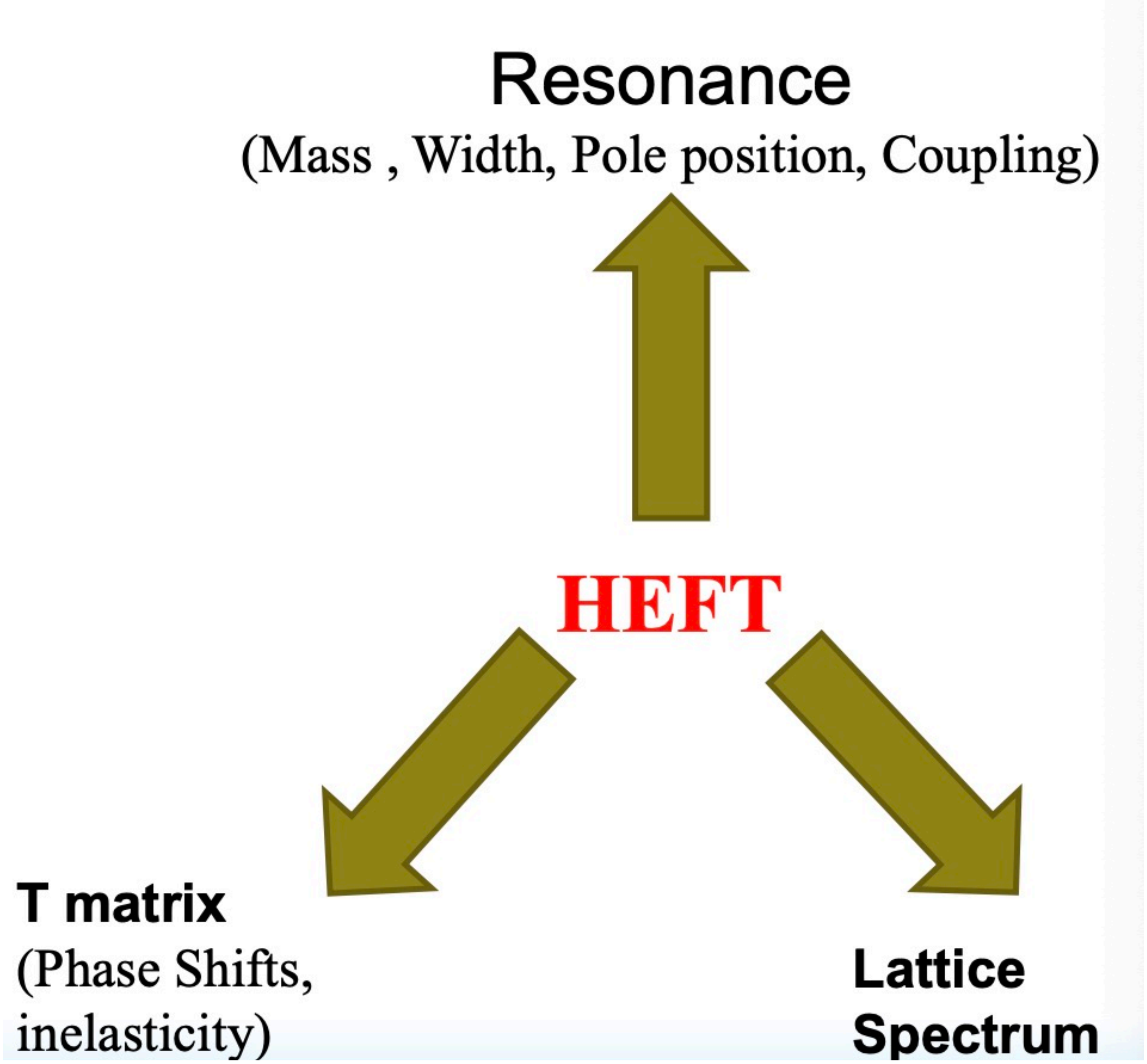
Gaussian form

two-pion exchange



The HALQCD method

$$V(r) = v_{C1}\mathrm{e}^{-\kappa_{C1}r^2} + v_{C2}\mathrm{e}^{-\kappa_{C2}r^2} + v_{C3}\left(1-\mathrm{e}^{-\alpha_C r^2}\right)^2\left(\frac{\mathrm{e}^{-\beta_C r}}{r}\right)^2$$



J. M. M. Hall etc. PRD 87(2013), 094510
J.-j. Wu etc. PRC90 (2014), 055206
Y. Li etc. PRD 101(2020), 114501
PRD 103(2021), 094518

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Lüscher′s **finite volume formula**

The direct method for scattering on the lattice: **Lüscher′s finite volume method**

M. Lüscher, Nucl. Phys. B354, 531(1991)

Finite volume energies

$$E_{1\cdot 2}(k) = \sqrt{m_1^2 + \vec{k}^2} + \sqrt{m_2^2 + \vec{k}^2}$$

$$k = n * 2\pi/L$$

格点计算

Lüscher formula

Infinite volume scattering

$$\delta_l \,.\, a_l \,.\, r_l$$

ERE(effective range expansion)

$$k^{2l+1} cot\delta_l(k) = a_l^{-1} + \frac{1}{2} r_l k^2 + \ldots$$

scattering length effective range

无限体积中的散射

Lüscher's **finite volume formula**

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Finite volume energies ⟷ Lüscher formula ⟷ **Infinite volume scattering**

$$E_{1\cdot 2}(k) = \sqrt{m_1^2 + \bar{k}^2} + \sqrt{m_2^2 + \bar{k}^2}$$

$$k = n * 2\pi/L$$

Due to the interaction, $\bar{k}^2$ differs from its free counter-part k^2

格点计算

$$q \cot \delta_0(q) = \frac{1}{\pi^{3/2}} Z_{00}(1; q^2)$$

$$q^2 = \bar{k}^2 * \frac{L^2}{(2\pi)^2}$$

$$\begin{cases} q^2 = n^2 \\ q^2 < n^2 \\ q^2 > n^2 \end{cases}$$

$$\delta_l \,.\, a_l \,.\, r_l$$

ERE(effective range expansion)

$$k^{2l+1} cot\delta_l(k) = a_l^{-1} + \frac{1}{2} r_l k^2 + \ldots$$

scattering length　effective range

无限体积中的散射

Lattice calculation

The two-particle operators for the $^3S_1(T_1^+)$ and $^1S_0(A_1^+)$ channels

$$
\begin{aligned}
&A_1^+ : \\
&\mathcal{O}_{A_1^+} = p_{\frac{1}{2}}(0)\Lambda_{-\frac{1}{2}}(0) - p_{-\frac{1}{2}}(0)\Lambda_{\frac{1}{2}}(0) \\
&\mathcal{O}'_{A_1^+} = p_{\frac{1}{2}}(e_x)\Lambda_{-\frac{1}{2}}(-e_x) - p_{-\frac{1}{2}}(e_x)\Lambda_{\frac{1}{2}}(-e_x) + p_{\frac{1}{2}}(-e_x)\Lambda_{-\frac{1}{2}}(e_x) - p_{-\frac{1}{2}}(-e_x)\Lambda_{\frac{1}{2}}(e_x) \\
&\quad + p_{\frac{1}{2}}(e_y)\Lambda_{-\frac{1}{2}}(-e_y) - p_{-\frac{1}{2}}(e_y)\Lambda_{\frac{1}{2}}(-e_y) + p_{\frac{1}{2}}(-e_y)\Lambda_{-\frac{1}{2}}(e_y) - p_{-\frac{1}{2}}(-e_y)\Lambda_{\frac{1}{2}}(e_y) \\
&\quad + p_{\frac{1}{2}}(e_z)\Lambda_{-\frac{1}{2}}(-e_z) - p_{-\frac{1}{2}}(e_z)\Lambda_{\frac{1}{2}}(-e_z) + p_{\frac{1}{2}}(-e_z)\Lambda_{-\frac{1}{2}}(e_z) - p_{-\frac{1}{2}}(-e_z)\Lambda_{\frac{1}{2}}(e_z)
\end{aligned}
$$

The correlation matrix of the form

$$C_i^{\alpha\beta}(t) = \langle 0 | \mathcal{O}_i^{\alpha}(t)\mathcal{O}_i^{\beta\dagger}(0) | 0 \rangle$$

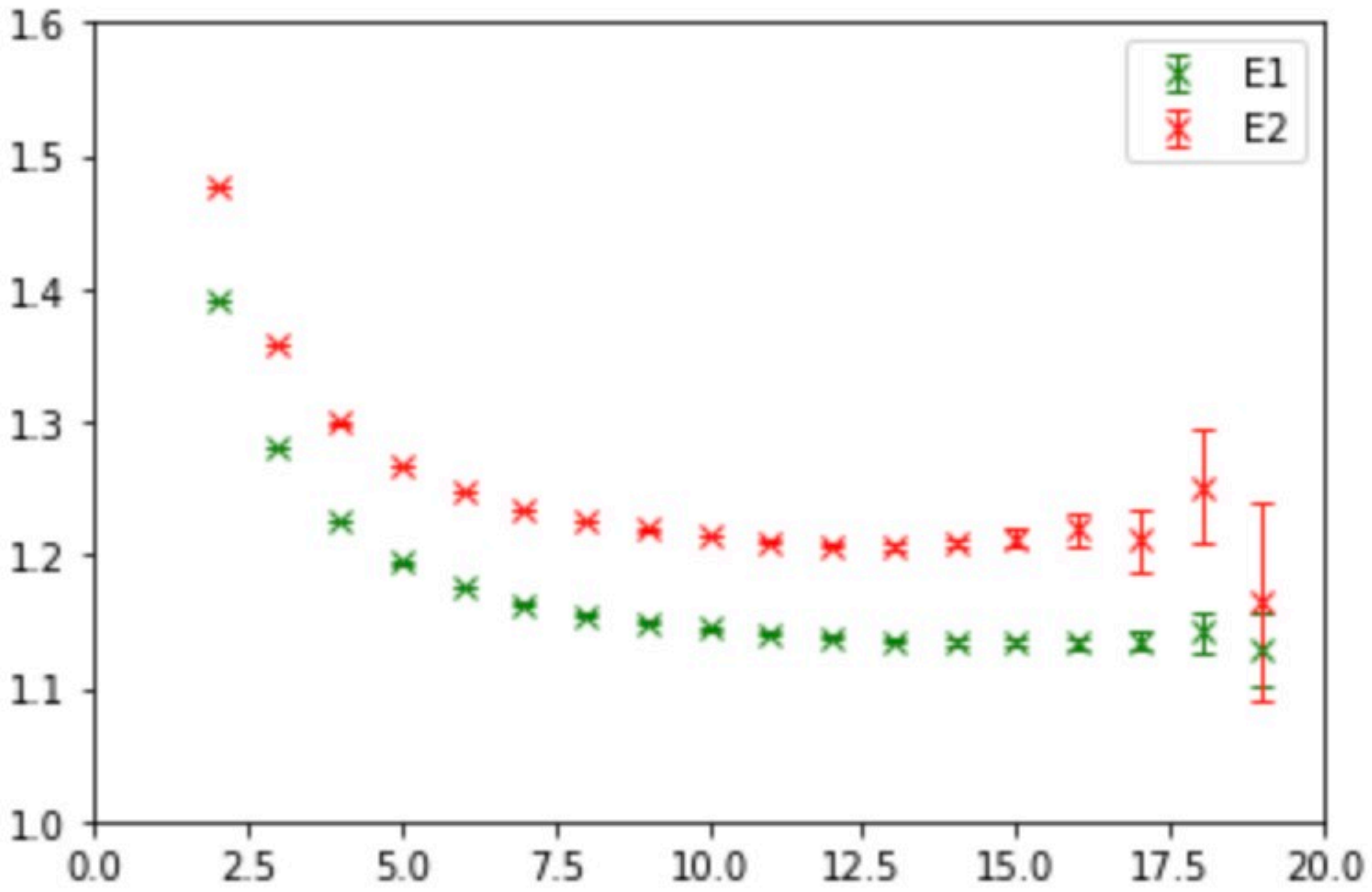
Solving the so-called Generalized Eigenvalue Problem(GEVP)

$$C(t)v_n(t) = \lambda_n(t)C(t_r)v_n(t)$$

$$\lambda_n(t) = c_0 e^{-E_n(t-t_r)}\left(1 + c_1 e^{-\Delta E(t-t_r)}\right)$$

The Effective mass of the YN system

$$\lambda_n(t) = c_0 e^{-E_n(t-t_r)} \left(1 + c_1 e^{-\Delta E(t-t_r)}\right)$$

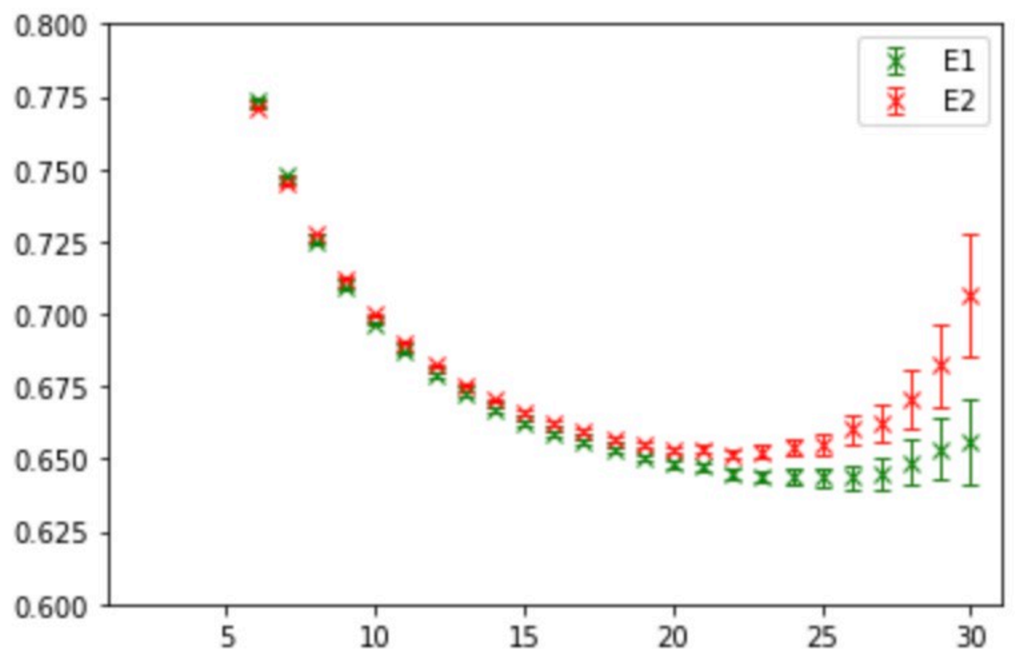


Preliminary

	C24P29	C32P29	C48P23	C48P14	F48P30	H48P32	F48P21
Irrep	E_0(MeV)	E_0(MeV)	E_0(MeV)	E_0(MeV)	E_0(MeV)	E_0(MeV)	E_0(MeV)
A_1^+	2150.25(12.18)	2114.84(6.00)	1993.89(2.89)	1910.20(4.10)	2239.17(2.80)	2352.90(7.40)	2053.49(5.60)
T_1^+	2148.33(12.81)	2126.46(4.3)	1994.03(2.85)	1911.44(4.45)	2238.41(3.31)	2348.40(7.60)	2045.01(8.14)

Scattering length & effective range

To get more information of the interaction, we also use some moving frames with non-zero total momentum



ERE(effective range expansion)

$$k^{2l+1} cot\delta_l(k) = a_l^{-1} + \frac{1}{2} r_l k^2 + \ldots$$

Close to the threshold, a_0 and r_0 can be determined by minimizing the χ^2

$$\chi^2 = \sum_{L,n,n'} [E_n(L) - E_n^{sol.}(L, a_0, r_0)] C_{nn'}^{-1} [E_{n'}(L) - E_{n'}^{sol.}(L, a_0, r_0)],$$

Working in progress!

Summary and Prospect

- ✓ HALQCD method: p-Λ NBS wave function, the interaction potential;
- ✓ Lüscher′s finite volume method: preliminary results for finite volume energies and phase shift;

  Effective range expansion: scattering length and effective range;

- ➡ HALQCD method： HEFT
- ➡ Extrapolation: discretization error, pion mass, finite volume effect
- ➡ p-Λ: p-Σ coupled channel
- ➡ ${}^{3}_{\Lambda}H$: three-body problem

Summary and Prospect

- ✓ HALQCD method: p-Λ NBS wave function, the interaction potential;
- ✓ Lüscher′s finite volume method: preliminary results for finite volume energies and phase shift;

  Effective range expansion: scattering length and effective range;

- ➡ HALQCD method: HEFT
- ➡ Extrapolation: discretization error, pion mass, finite volume effect
- ➡ p-Λ: p-Σ coupled channel
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Thank you!

Back up

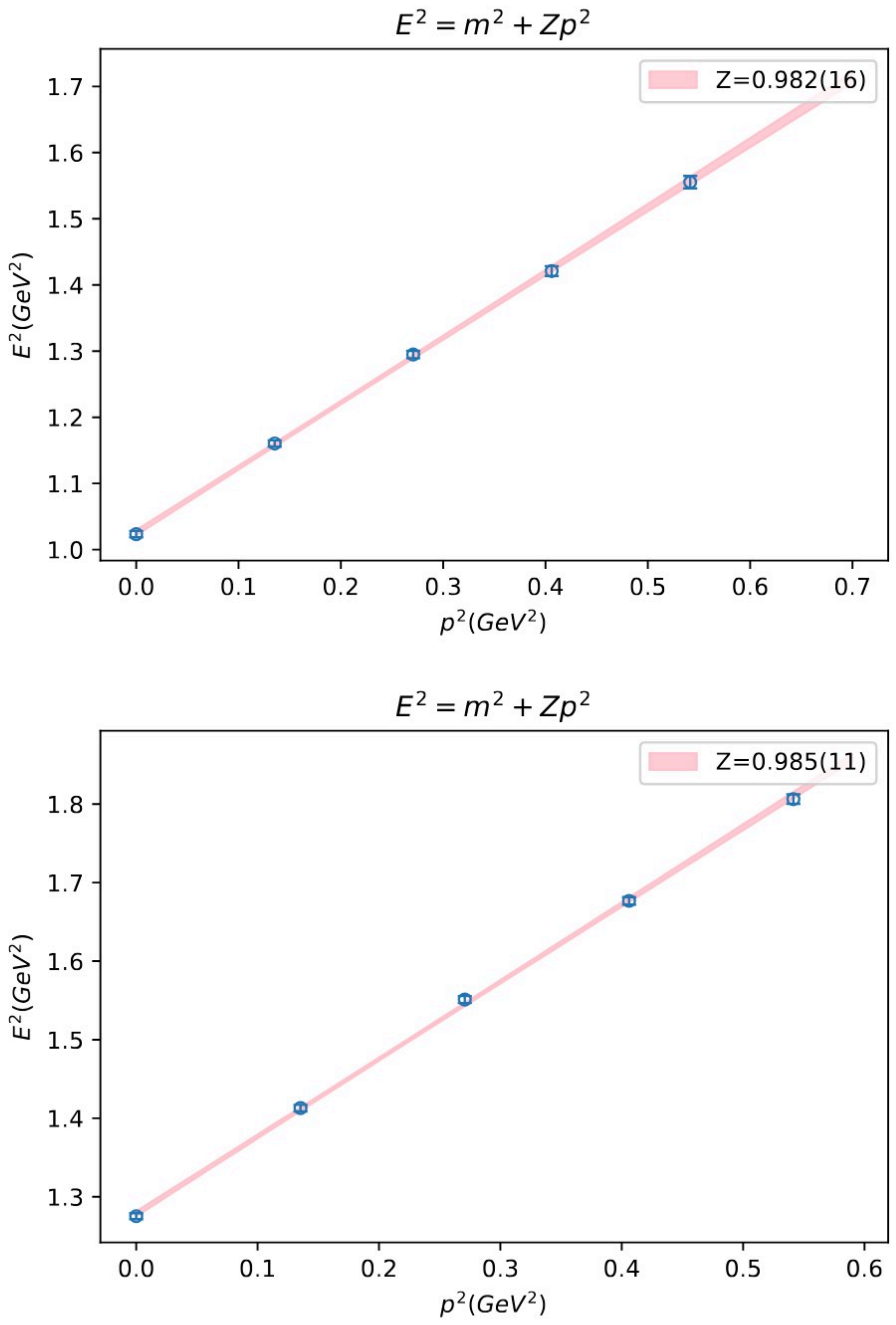


FIG. 1. Dispersion relation for proton (upper panel) and for Λ on the C32P29 ensembles(lower panel).

Back up

Deuteron: *pn* scattering

Scattering amplitude:

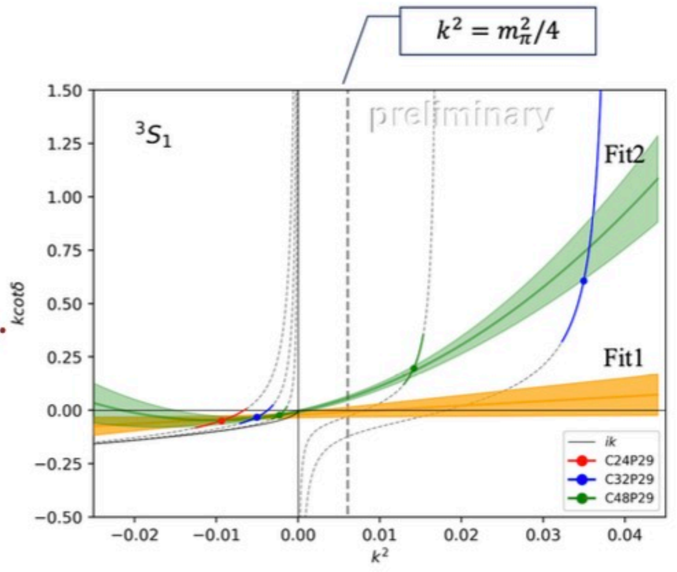
$$T \sim \frac{1}{k\cot\delta - ik}$$

Effective range expansion:

$$k\cot\delta(k) = \frac{1}{a_0} + \frac{1}{2}r_0 k^2 + c^3 k^4 + \cdots$$

S-wave Luscher's formula:

$$k\cot\delta(k) = \frac{2Z_{00}(1; (\frac{kL}{2\pi})^2)}{L\sqrt{\pi}}$$



	Fit 1	Fit 2
$1/a_0$ (fm^{-1})	-0.2(1)	-0.054(62)
r_0 (fm)	0.45(17)	1.67(23)
c (fm)		0.755(74)
χ^2/dof	0.70	0.59
Binding Energy (MeV)	2.4(1.9)	1.33(96)

Fit1: using three data points below the threshold, ERE up to k^2
Fit2: using five data points, ERE up to k^4

ΛΛ scattering

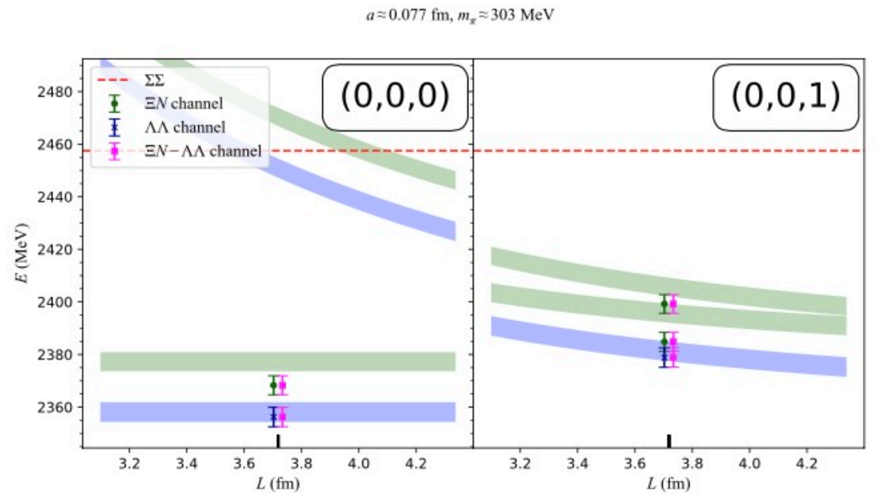
CLQCD

- We are intrested in the $I(J^P) = 0(0^+)$ ΛΛ scattering
- coupled channels: $\Lambda\Lambda, N\Xi, \Sigma\Sigma$
- To avoid the complexity of three coupled channels, we will keep the energy range below $\Sigma\Sigma$ threshold, and consider only $\Lambda\Lambda$ and $N\Xi$.

$$\mathcal{O}^{\mathbf{P}}(\Lambda\Lambda) = \Lambda(\mathbf{p_1})\Lambda(\mathbf{p_2})$$

$$\mathcal{O}^{\mathbf{P}}(N\Xi) = p(\mathbf{p_1})\Xi^-(\mathbf{p_2}) - n(\mathbf{p_1})\Xi^0(\mathbf{p_2})$$

$$\mathbf{P} = \mathbf{p_1} + \mathbf{p_2}, \quad \mathbf{P} = (0,0,0), (0,0,1)$$

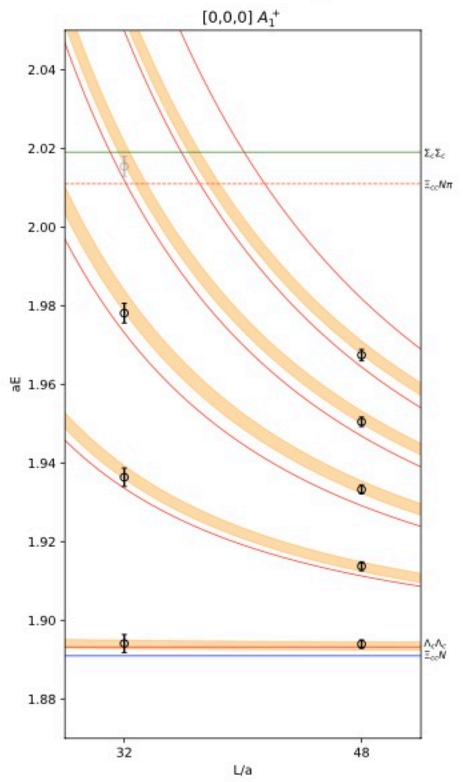


The energy levels obtained from GEVP using both ΛΛ and $N\Xi$ operators are almost the same as using them separately.

$\Lambda_c\Lambda_c$ scattering

CLQCD

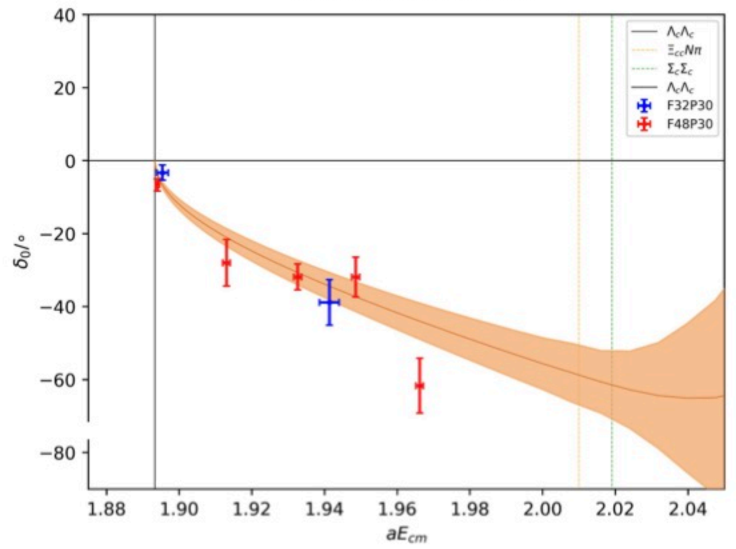
Finite-volume spectrum



effective range expansion:

$$p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2}r_0 p^2$$

Phase shift



$a_0 = -0.225(33)$ fm

$r_0 = 0.02(10)$ fm