南開大學
Nankai University

The Study of Pole Trajectory within a bare state in the coupled channel model

arXiv: 2506.02526

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广西师范大学•桂林

Outline

1. Motivation
2. HEFT Method
3. Discussion
4. Summary

Motivation

➢ One channel

Non-relativistic:

$$E = \frac{p^2}{2\mu} \qquad \mu = \frac{m_1 + m_2}{m_1\, m_2}$$

- Physical sheet

Bound state: Poles on the real E axis

- Unphysical sheet

Resonance(Area 8): Poles on the lower half plane and $ReE_{pole} > E_{th}$

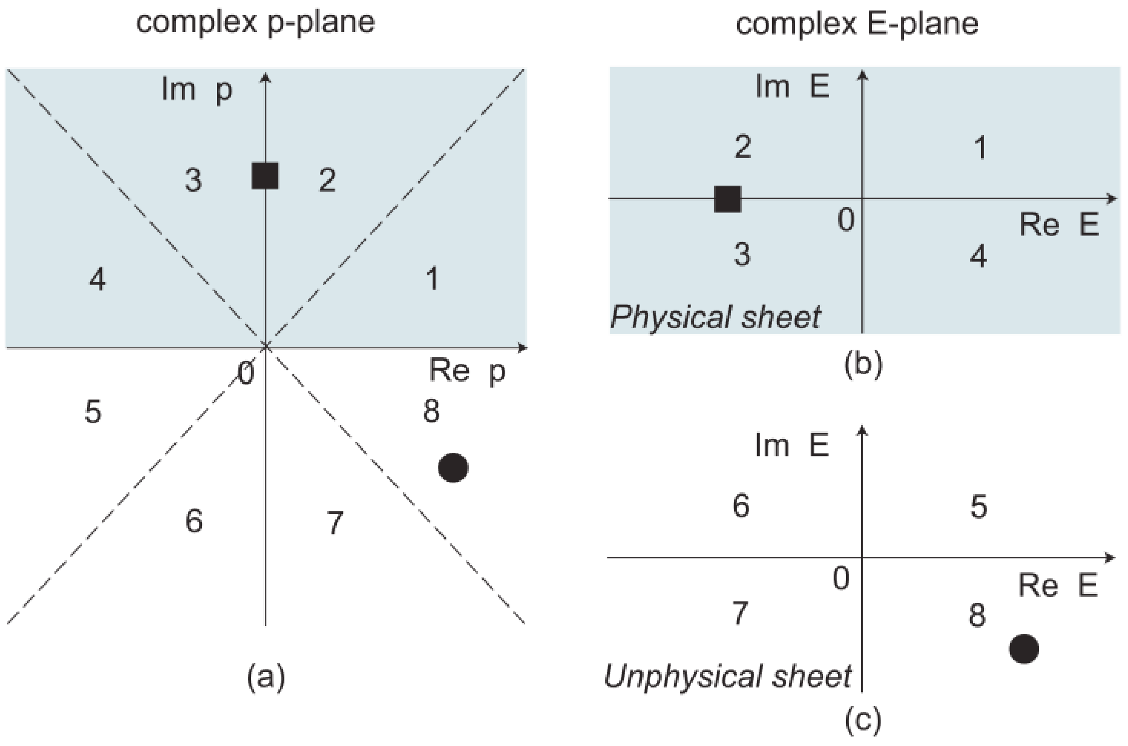


FIG. 1. (a) The complex momentum p-plane and its corresponding complex energy E-plane, which has (b) a physical sheet and (c) an unphysical sheet. Their correspondence is indicated by the same number. Solid squares (circles) represent the bound state (resonance) poles.

N. Suzuki and T. Sato, PRC 79 (2009) 025205.

Motivation

Relativistic:

$$E = \sqrt{m_1^2 + k^2} + \sqrt{m_2^2 + k^2}$$

- Physical sheet

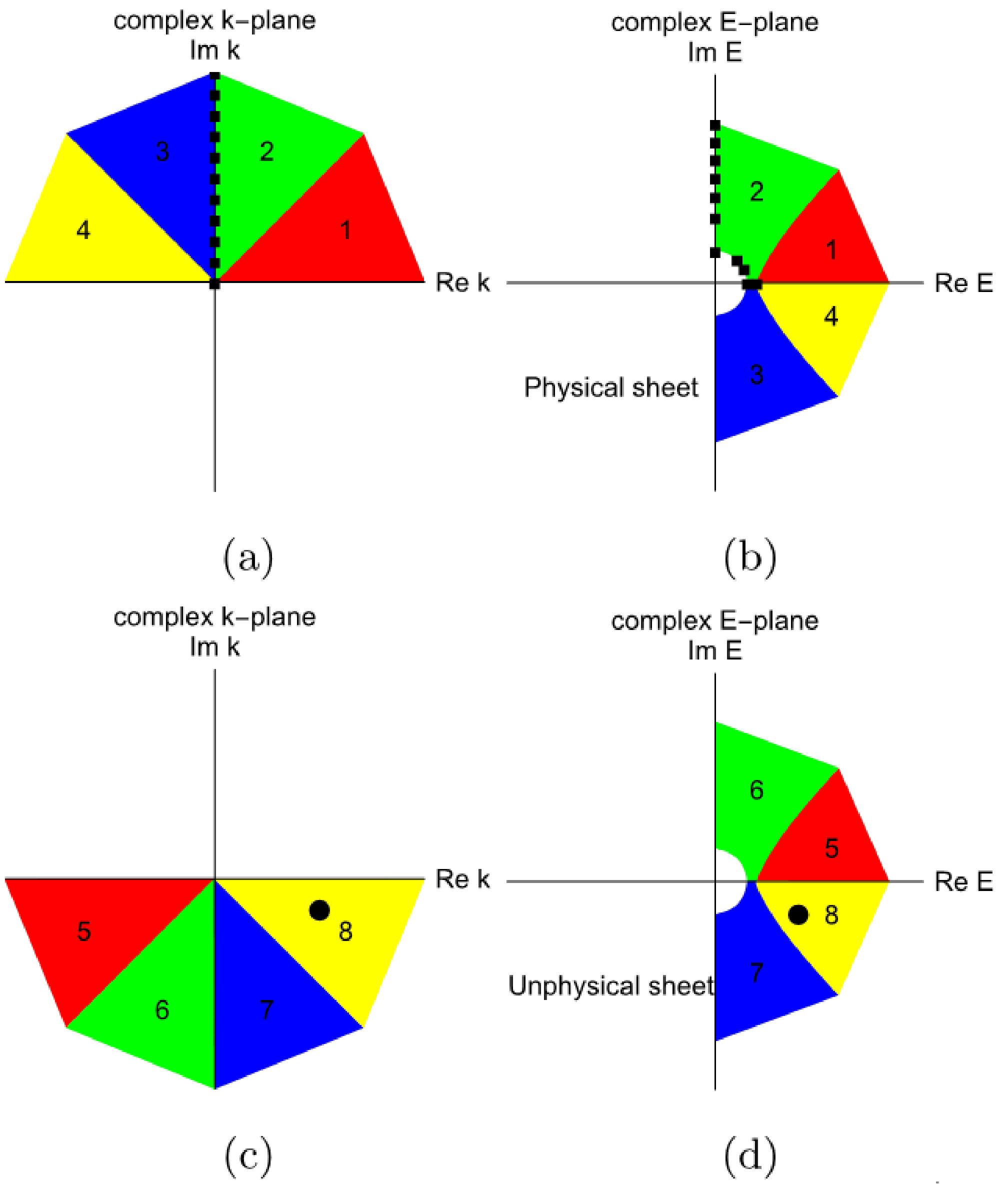
Bound state: Poles on the real E axis

- Unphysical sheet

Resonance(Area 7,8): Poles on the lower half plane and $ReE_{pole} > E_{th}$

Crazy resonance: C. Hanhart, J. R. Pelaez, and G. Rios, PLB 739 (2014) 375-382. Y-F Wang, D-L Yao, and H-Q Zheng, EPJC 78 (2018) 7, 543.

Can the poles be explained as traditional resonances or bound states when they are located at certain special positions (such as 7 area)?



(a) (b) (c) (d)

Motivation

➢One channel with one bare state

- **$\Lambda(1405)$:** much lower than its nucleon counterpart $N(1535)$, mass splitting between the $\Lambda(1405)$ and its spin partner $\Lambda(1520)$ is much larger than that in the nucleon sector

$\bar{K}N$

Y-B He, X-H Liu, L-S Geng, F-K Guo, J-J Xie, 2407.13486.
E. Oset, A. Ramos, C. Bennhold, PLB 527 (2002) 99-105.
F-K Guo, Y. Kamiya, M. Mai, Ulf-G. Meißner, PLB 846 (2023) 138264.
J-M Xie, J-X Lu, L-S Geng, B-S Zou, PRD 108 (2023) 11, L111502.
L-L Liu, E Wang, J-J Xie, K-L Song, J-Y Zhu, PRD 98(2018) 11,114017.

$\bar{K}N-(uds)$ Jonathan M. M. H., Waseem K., Derek B. L., Benjamin J. M., Benjamin J. O., PRL 114 (2015) 13, 132002.

- **$D_{s0}^*(2317)$:** 160 MeV lower than GI model prediction S. Godfrey, N. Isgur, PRD 32 (1985) 189-231.

$DK-(c\bar{s})$, 70% DK component

T. Barnes, F.E. Close, H.J. Lipkin, PRD 68 (2003) 054006.
T E. Browder, S Pakvasa, A A. Petrov, PLB 578 (2004) 365-368.
Harry J. Lipkin, PLB 580 (2004) 50-53.

$c\bar{s}$ O. Lakhina, E. S. Swanson, PLB 650 (2007) 159-165.

Molecules and tetraquark state

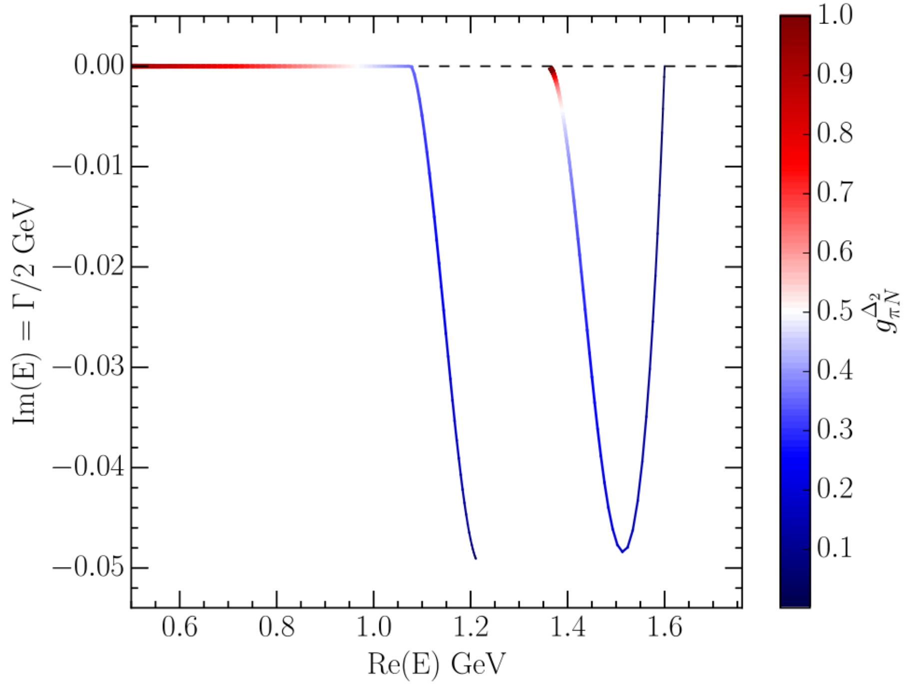
F-K Guo, Pos LATTICE2022, 232(2023).
S-Q Luo, B Chen, X Liu, T. Matsuki, PRD 103 (2021) 7, 074027.
Z Yang, G-J Wang, J-J Wu, M. Oka, S-L Zhu, PRL 128 (2022) 11, 112001.
Z Yang, G-J Wang, J-J Wu, M. Oka, S-L Zhu, PRL. 128 (2022) 11, 112001.
Y Li, J-J Wu, PRD 105 (2022) 11, 116024.

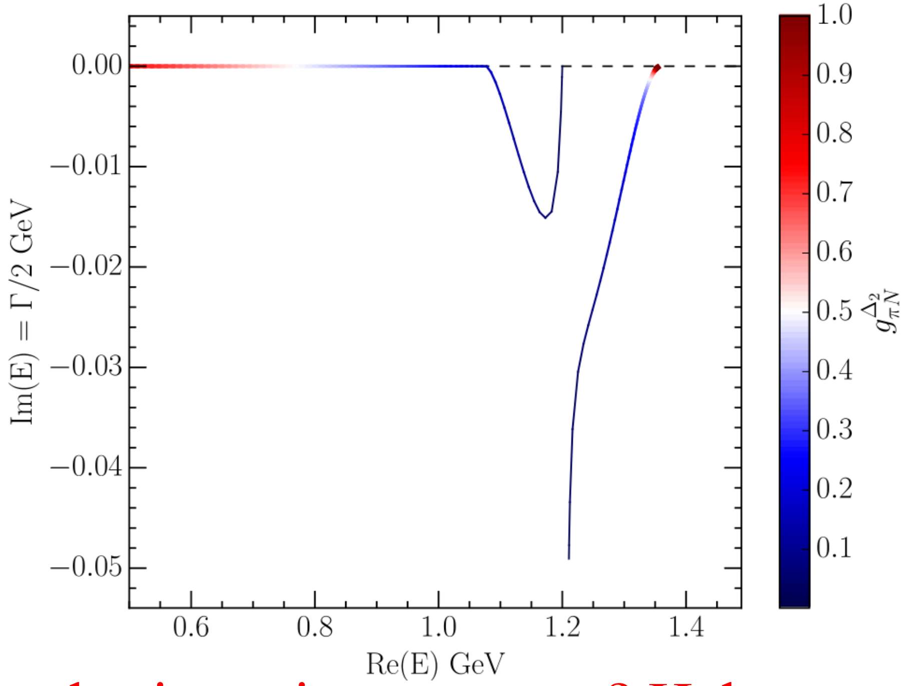
Motivation

- **One channel with two bare state** $\Delta^*(1232)$, $\Delta^*(1600)$ ----case 1
  $N^*(1535)$, $N^*(1650)$----case 2

πN-$\Delta_1\Delta_2$, $\Delta_1 = 1.359$ GeV, $\Delta_2 = 1.6$ GeV and 1.2 GeV

C. D. Abell, D. B. Leinweber, A. W. Thomas, J-J Wu, AOP 459(2023) 169531.





How to systematically study the distribution of poles in various systems? Helps to understand the origin of physical states (i.e., bound states, resonances, and virtual states)

HEFT Method

➢ **The Hamiltonian effective field theory (HEFT)** is a powerful theoretical tool to study resonance positions, partial decay widths, scattering phase shifts which is related to experimental observation

$$H = H_0 + H_I$$

$$H_0 = \sum_{i=1,n} |B_i\rangle m_i \langle B_i| + \sum_{\alpha} |\alpha(k_\alpha)\rangle \left[\sqrt{m_{\alpha 1}^2 + k_\alpha^2} + \sqrt{m_{\alpha 2}^2 + k_\alpha^2}\right] \langle \alpha(k_\alpha)|$$

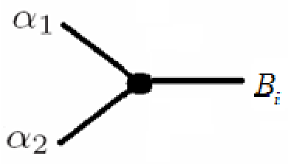
$|B_i >$ bare state, bare mass m_i ⟷ Interactions at the quark gluon level

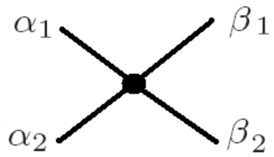
$|\alpha(k_a) >$ non-interaction channels ⟷ Hadron level interaction

$$H_I = \hat{g} + \hat{v}$$

$$\hat{g} = \sum_{\alpha} \sum_{i=1,n} \left[|\alpha(k_\alpha)\rangle g_{i,\alpha}^{+} \langle B_i| + |B_i\rangle g_{i,\alpha} \langle \alpha(k_\alpha)| \right]$$

$$\hat{v} = \sum_{\alpha,\beta} |\alpha(k_\alpha)\rangle v_{\alpha,\beta} \langle \beta(k_\beta)|$$





Resonance

(mass, width, pole position, Coupling strength)

↑

HEFT

↙ ↘

T Matrix
(Phase shift, non-elastic coefficient)

Lattice energy spectrum

Discussion

➢The number of pole .vs. Form factor

Form factor: $f_\alpha(k) = \dfrac{2}{(1+(k/\Lambda)^2)^n}$

$\omega_\alpha(k) = m_{\alpha_N} + m_{\alpha_N} + \dfrac{k^2}{2m_{\alpha_N}} + \dfrac{k^2}{2m_{\alpha_m}}$

Pole position: $1 = v_{\alpha\alpha} \int dq q^2 \dfrac{f_\alpha^2(q)}{E - \omega_\alpha(q) + i\epsilon}$

$m_{\alpha N} = 1$ GeV, $m_{\alpha N} = 1$ GeV, $\Lambda = 1$, $v_{\alpha\alpha} = -1$

◆$n = 1$

$$\int dq q^2 \frac{f_\alpha^2(q)}{E - \omega_\alpha(q) + i\epsilon} = \frac{2\pi i\sqrt{\mu}\sqrt{\frac{k_0^2}{\mu} + k_0^2 - 1}}{(k_0^2+1)^2}$$

$$k_0 = \sqrt{2\mu(E - m_{\alpha_N} - m_{\alpha_M})} \qquad \mu = \frac{m_{\alpha_N} m_{\alpha_m}}{\Lambda^2(m_{\alpha_N} + m_{\alpha_M})}$$

Pole position: $E = 1.403$ GeV

Discussion

◆ $n = 2$

$$\int dq q^2 \frac{f_\alpha^2(q)}{E-\omega_\alpha(q)+i\epsilon}$$

$$= \frac{\pi\mu(k_0^6+5k_0^4-16i\sqrt{\mu}\sqrt{\frac{k_0^2}{\mu}}+15k_0^2-5)}{4(k_0^2+1)^4}$$

$$k_0 = \sqrt{2\mu(E-m_{\alpha_N}-m_{\alpha_M})} \qquad \mu = \frac{m_{\alpha_N} m_{\alpha_m}}{\Lambda^2(m_{\alpha_N}+m_{\alpha_M})}$$

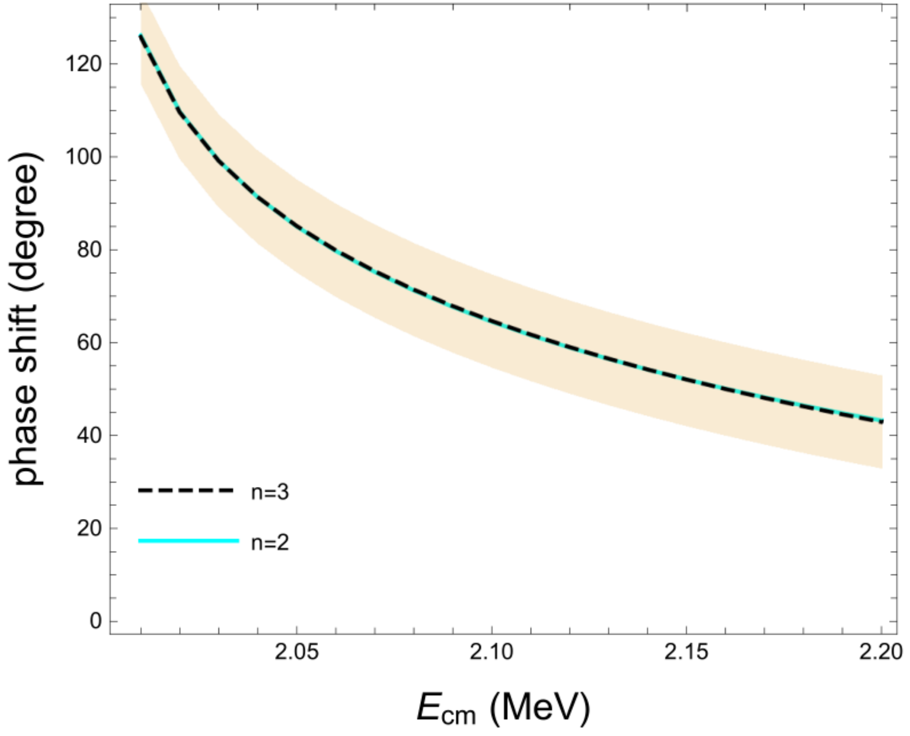
Pole position: $E = 1.255 - 2.105i$ GeV, $E = 1.942$ GeV

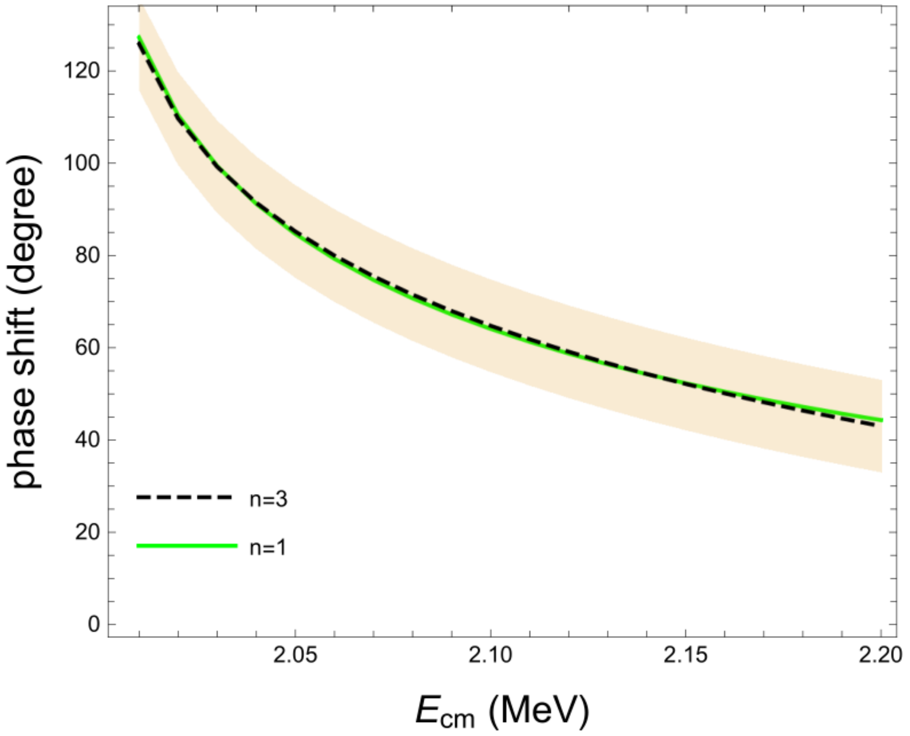
Conclusion: if n=1, 2, 3,etc., the number of poles will be 1, 2, 3, etc.

Discussion

➢ Fitting phase shift

Not all of the poles solved from T-matrix have physical meanings, How to filter them?





$n = 3$: 1.98433 GeV, 1.14228-2.36899i GeV, 2.22393−1.26388i GeV

$n = 2$: 1.98336 GeV, 1.79308 − 1.39633i GeV

$n = 1$: 1.97886 GeV

$$f_\alpha(k) = \frac{2}{(1+(k/\Lambda)^2)^n}$$

Discussion

➢ One channel without bare state

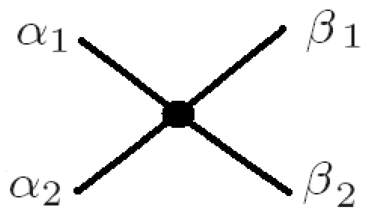
S- wave interaction

$$u(k) = (1 + \frac{k^2}{\Lambda^2})^{-2}$$

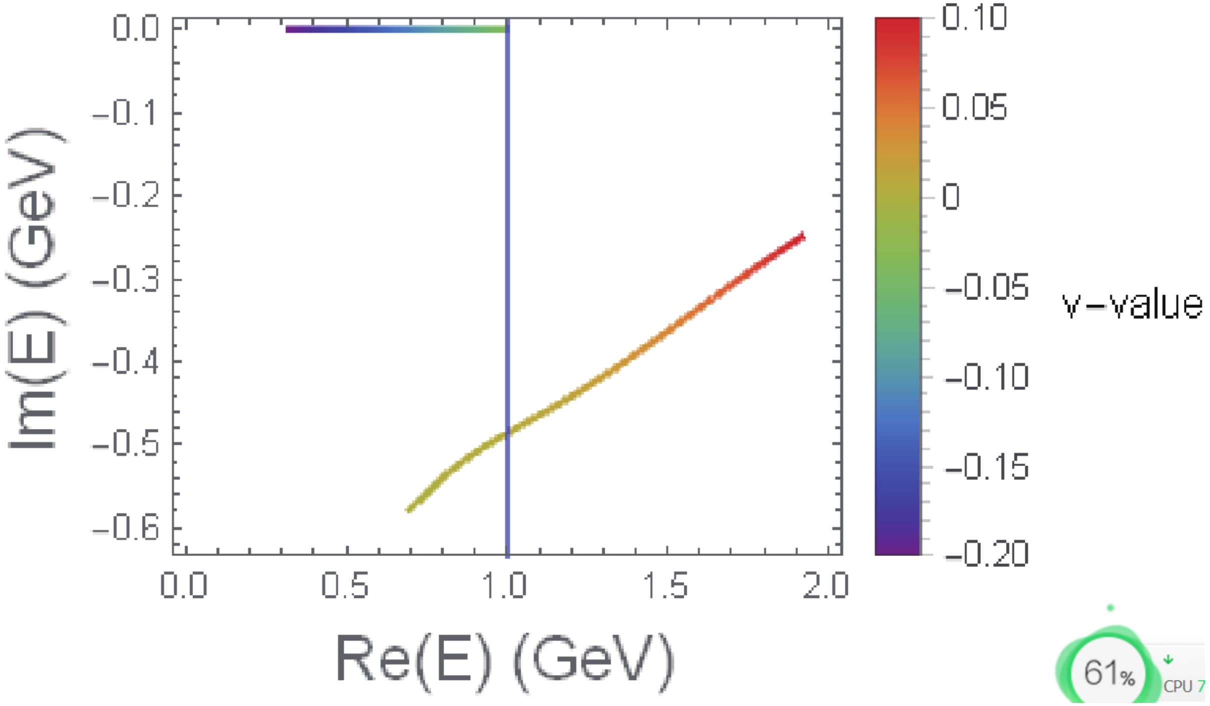
$$f(k) = \frac{1}{m_M} \frac{k}{\omega_M(k)} u(k)$$

$$0 = \det \left\{ \delta_{\alpha\beta} - v_{\alpha\beta} \int dq q^2 \frac{f_\beta^2(q)}{E - \omega_\beta(q) + i\epsilon} \right\}$$

Usually, such bound state is recognized as hadronic molecular state



$$v_{\alpha\beta}(k, k') = v_{\alpha\beta} f_\alpha(k) f_\beta(k')$$

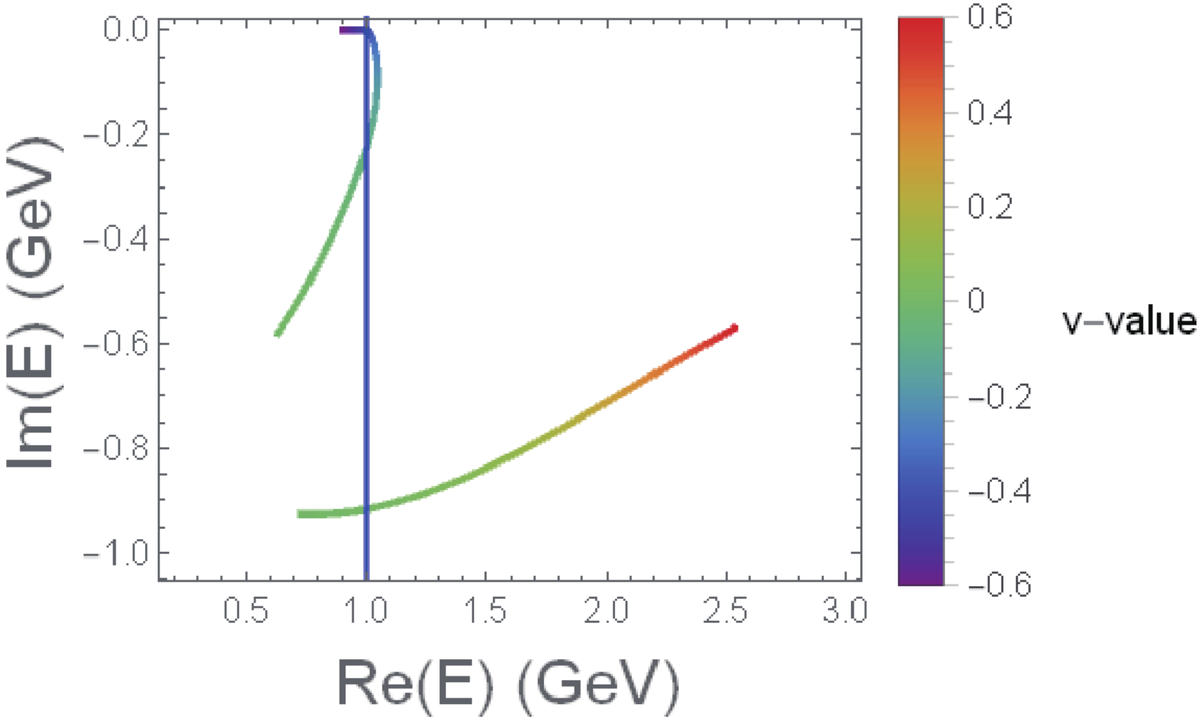


The more attractive, the deeper bound state

Discussion

➢ One channel without bare state

P- wave interaction



P-wave nucleon-nucleon interaction

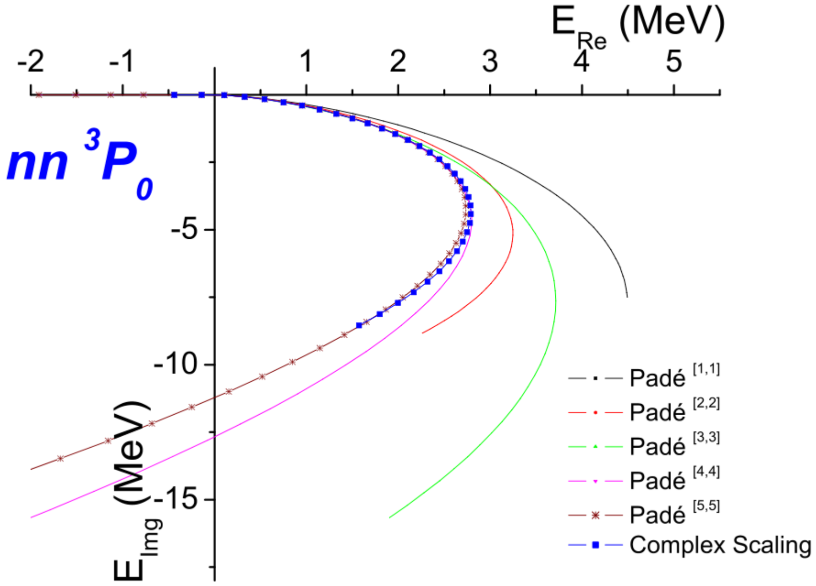
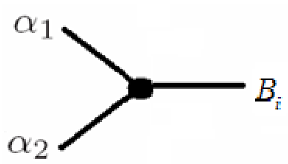
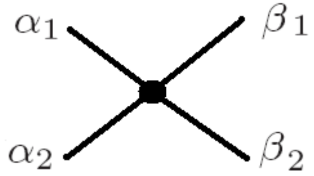


FIG. 3: Comparison between ACCC and CS methods for $^3P_0$ *nn* resonance trajectories with Reid93 potential. ACCC results with several Padé orders [N,M] and γ=6.1 to 1.0 are represented by solid lines. CS results are denoted by squares and correspond to γ from 6.1 to 2.7 by steps of 0.1. Star points correspond to [5,5] Padé approximant used in ACCC. They are already very close to CS results: by adding a few terms in the approximant a perfect agreement is reached.

R. Lazauskas, J. Carbonell, PRC 71(2005) 044004.

➢ One channel with a bare state



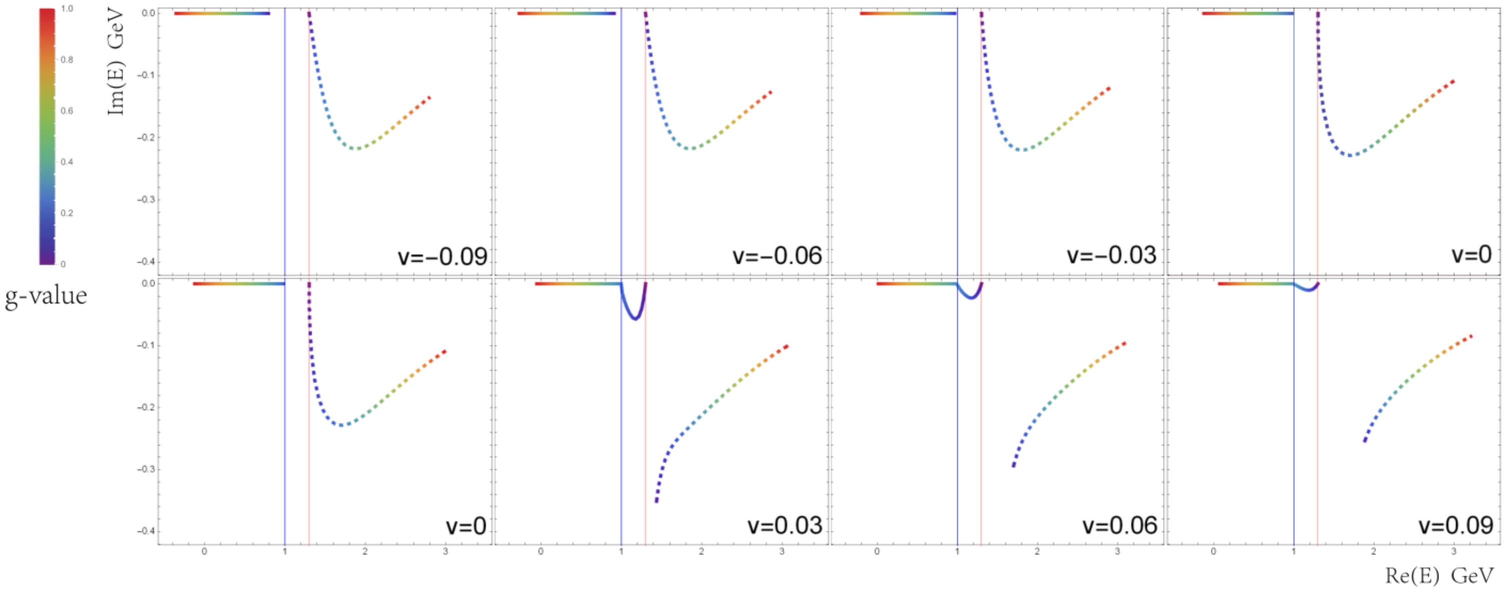


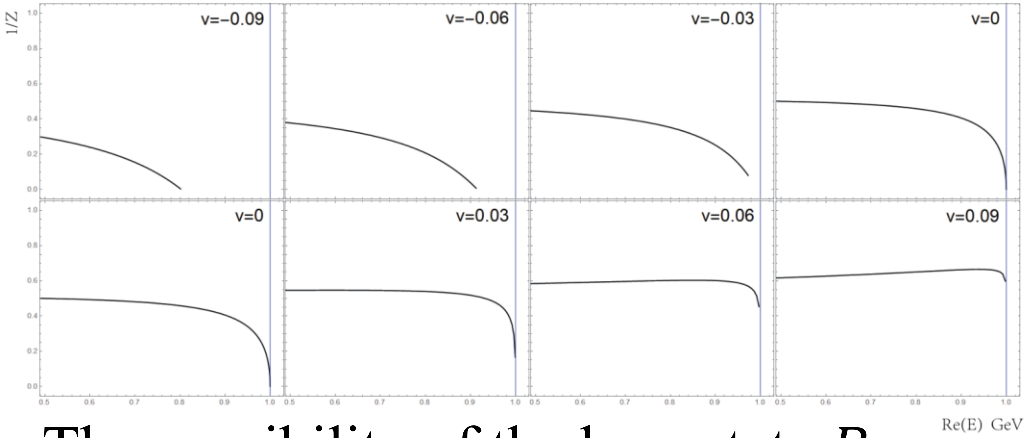
S-wave, m_B above channel threshold

$$G^{B}_{MN}(k) = \frac{g^{B}_{MN}}{m_M} \frac{k}{\sqrt{\omega_M(k)}} u(k) \qquad v_{\alpha\beta}(k,k') = v_{\alpha\beta} f_\alpha(k) f_\beta(k')$$

$$\bar{\Sigma}_{BB'}(E) = \Sigma_{BB'}(E) + \Sigma^{I}_{BB'}(E)$$

$$0 = \det\{\delta_{BB'}(E - m_B) - \bar{\Sigma}_{BB'}(E)\}$$





The possibility of the bare state B component in the dressed state $\bar{B}$

$$|\bar{B}\rangle = \frac{1}{Z^{1/2}} \left[|B\rangle + \int k^2 dk a(k) |k\rangle \right]$$

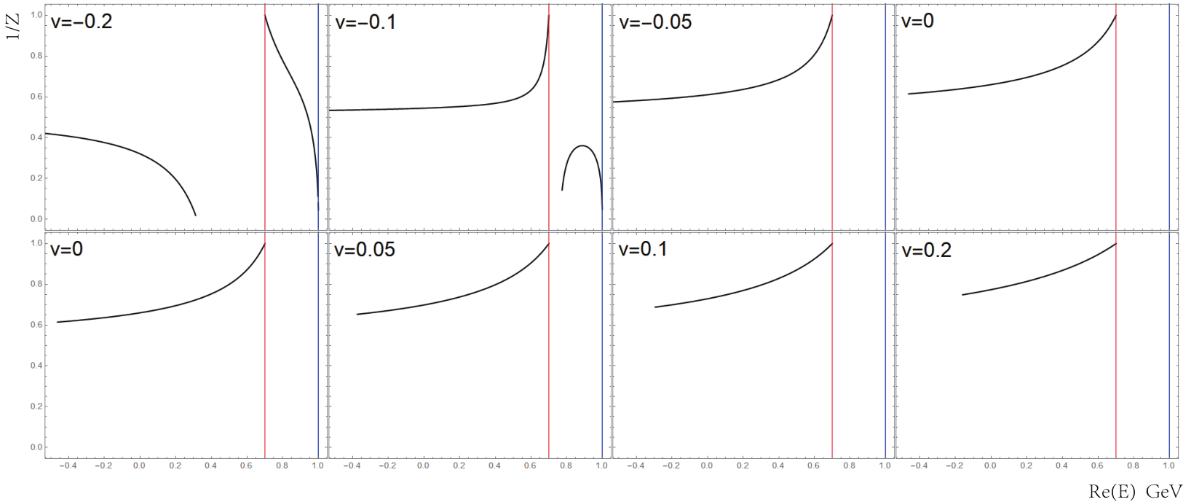
We suggest looking for cases where two particles are repulsive and there is a bare state predicted by traditional quark models near the threshold

Discussion

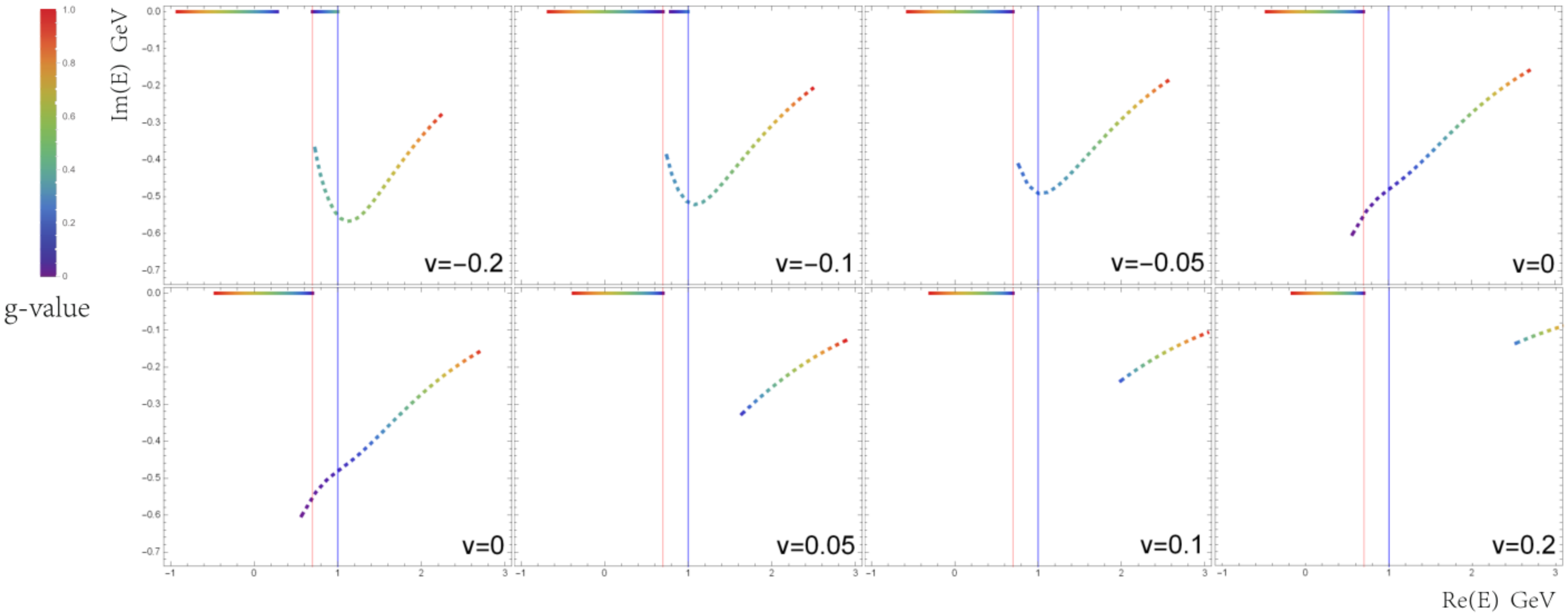
S-wave, m_B below channel threshold

$$0 = \det\{\delta_{BB'}(E - m_B) - \bar{\Sigma}_{BB'}(E)\}$$

$$\bar{\Sigma}_{BB'}(E) = \Sigma_{BB'}(E) + \Sigma^I_{BB'}(E)$$



$$|\bar{B}\rangle = \frac{1}{Z^{1/2}}\left[|B\rangle + \int k^2 dk a(k)|k\rangle\right]$$

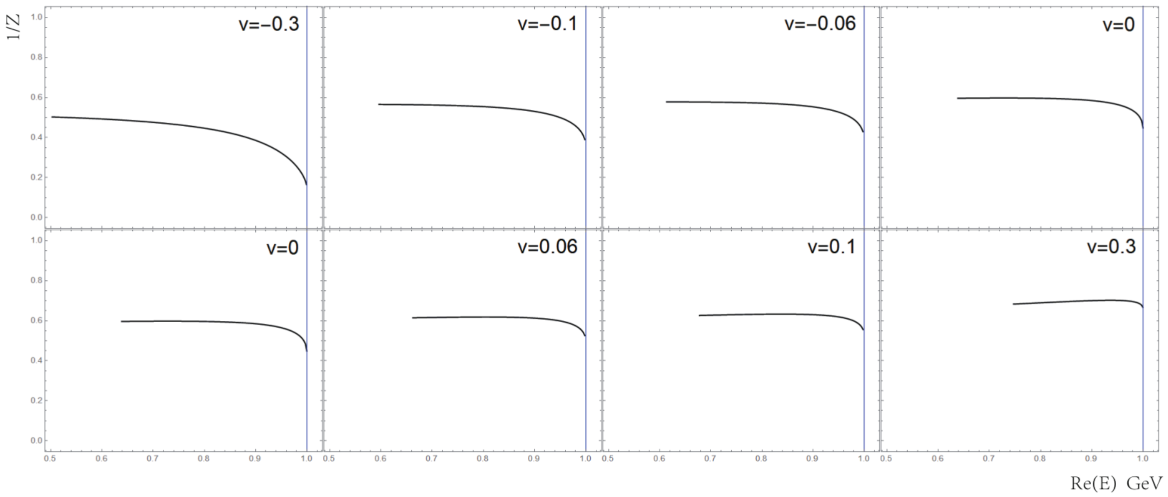


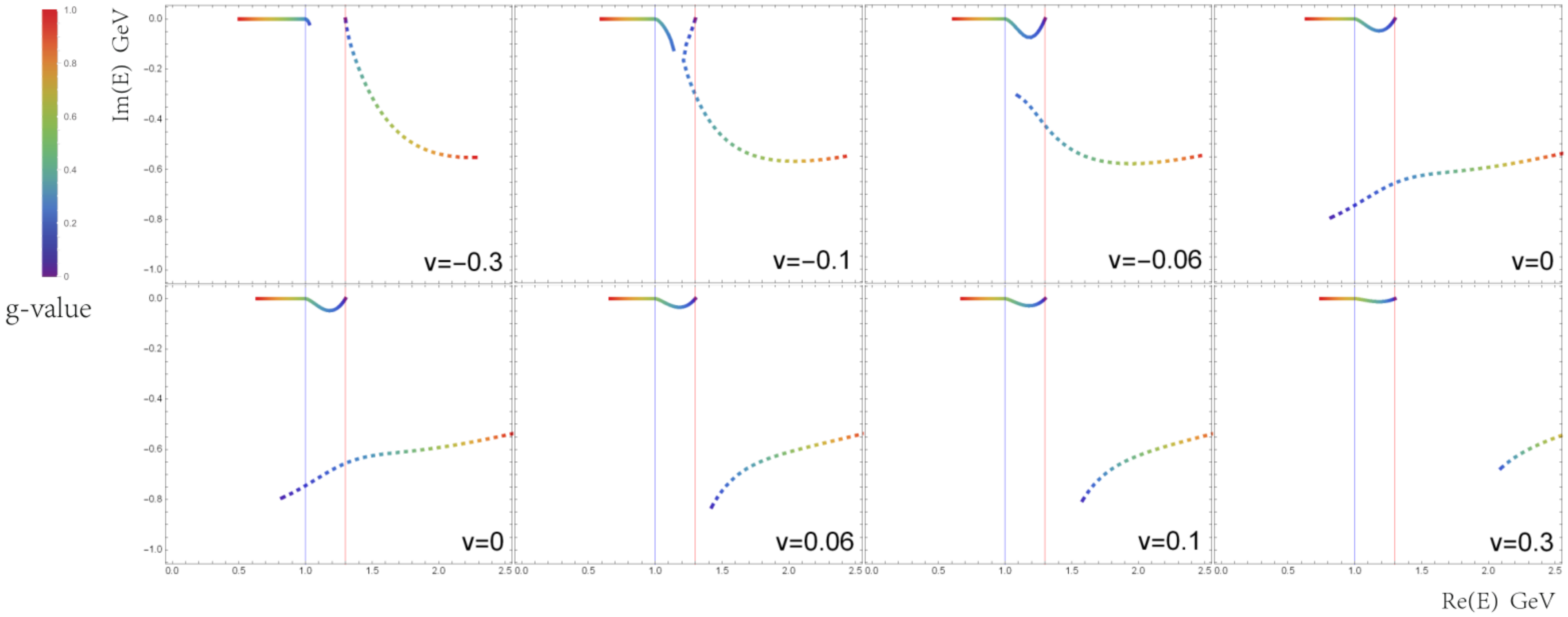
Discussion

P-wave, m_B above channel threshold

$$0 = \det\{\delta_{BB'}\,(E - m_B) - \bar{\Sigma}_{BB'}(E)\}$$

$$\bar{\Sigma}_{BB'}(E) = \Sigma_{BB'}(E) + \Sigma^{I}_{BB'}(E)$$





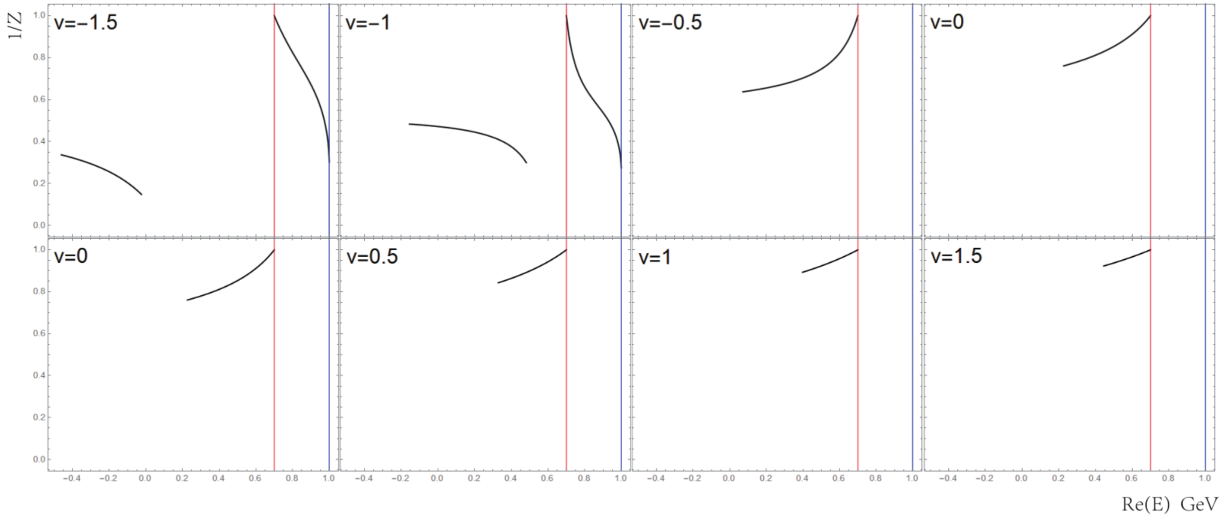
$$|\bar{B}\rangle = \frac{1}{Z^{1/2}}\left[|B\rangle + \int k^2 dk a(k)|k\rangle\right]$$

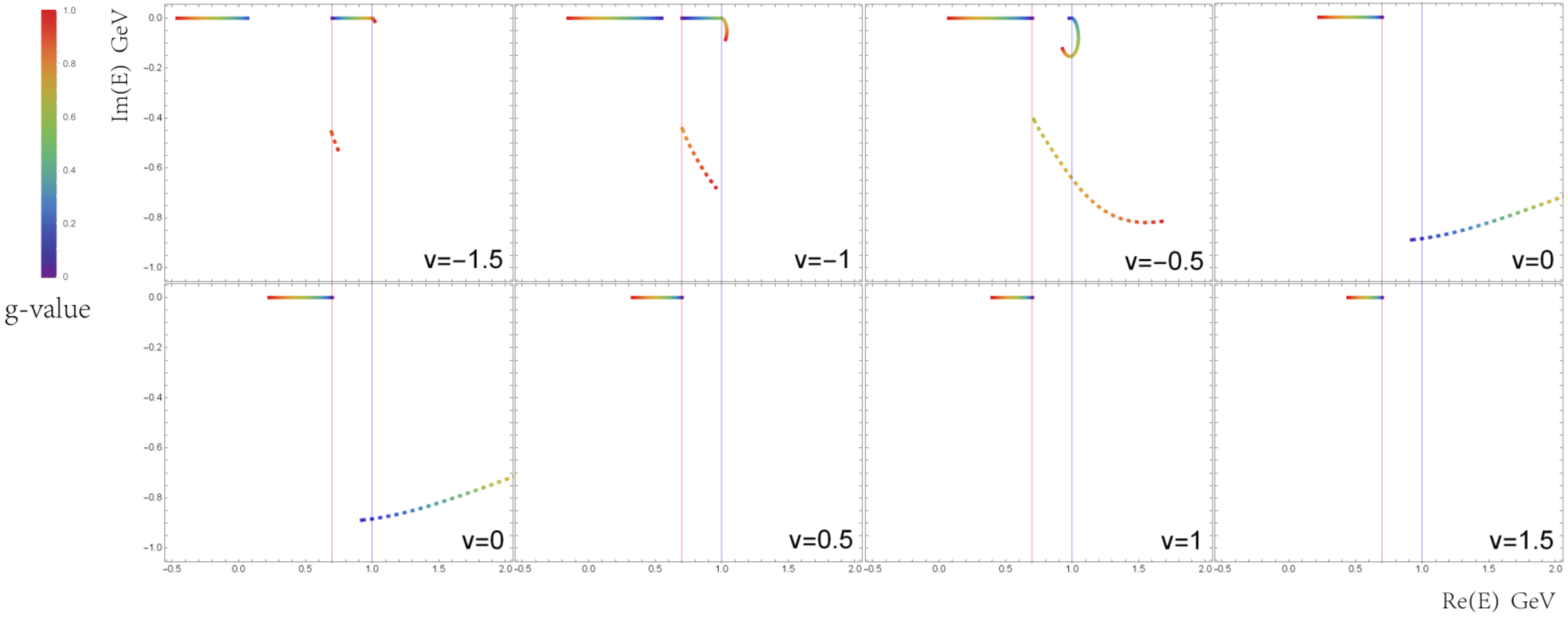
Discussion

P-wave, m_B below channel threshold

$$0 = \det\{\delta_{BB'}(E - m_B) - \bar{\Sigma}_{BB'}(E)\}$$

$$\bar{\Sigma}_{BB'}(E) = \Sigma_{BB'}(E) + \Sigma^{I}_{BB'}(E)$$





$$|\bar{B}\rangle = \frac{1}{Z^{1/2}}\left[|B\rangle + \int k^2 dk a(k)|k\rangle\right]$$

The existence of bare states has a significant impact on the distribution of poles

Summary

We systematically calculated the pole distribution of c-c system and b-c-c system.

- In relativistic cases, there is a significant difference in the distribution of resonance state poles and bound state poles, and the distribution of energy momentum on different Riemann sheet is also different.
- The form factor of the interaction influences the number of singularities in the scattering amplitude.
- The presence of a bare state can influence the distribution of two-body scattering poles, thereby altering their properties.

Thank you for your attention

For bare state $|B\rangle$ and coupled channel states $|\alpha(k)\rangle$, the eigenvalue equation of the Hamiltonian system can be written as

$$H_0|\alpha(k)> = (E_{\alpha_M}(k) + E_{\alpha_N}(k))|\alpha(k)>, \quad (25)$$

$$H_0|B> = m_B|B> . \quad (26)$$

For a bound state $\bar{B}$, the eigenvalue equation of the Hamiltonian is defined as

$$H|\bar{B}\rangle = E_{\bar{B}}|\bar{B}\rangle, \quad (27)$$

$$|\bar{B}\rangle = \frac{1}{Z^{1/2}}\left[\sum_B c_B|B\rangle + \sum_\alpha \int k^2 dk a_\alpha(k)|\alpha(k)\rangle\right], \quad (28)$$

where Z is normalization constant, c_B and $a_\alpha(k)$ are the bare state and $|\alpha(k)\rangle$ components of the bound state $\bar{B}$, respectively, and $E_{\bar{B}}$ is the mass of the bound state. In this paper, our aim is to find the nature of pole position related to bare states, thus, to simplify this model, we only consider one bare state and one two-body coupled channel. Then the expression of the bound state reads,

$$|\bar{B}\rangle = \frac{1}{Z^{1/2}}\left[|B\rangle + \int k^2 dk a(k)|k\rangle\right], \quad (29)$$

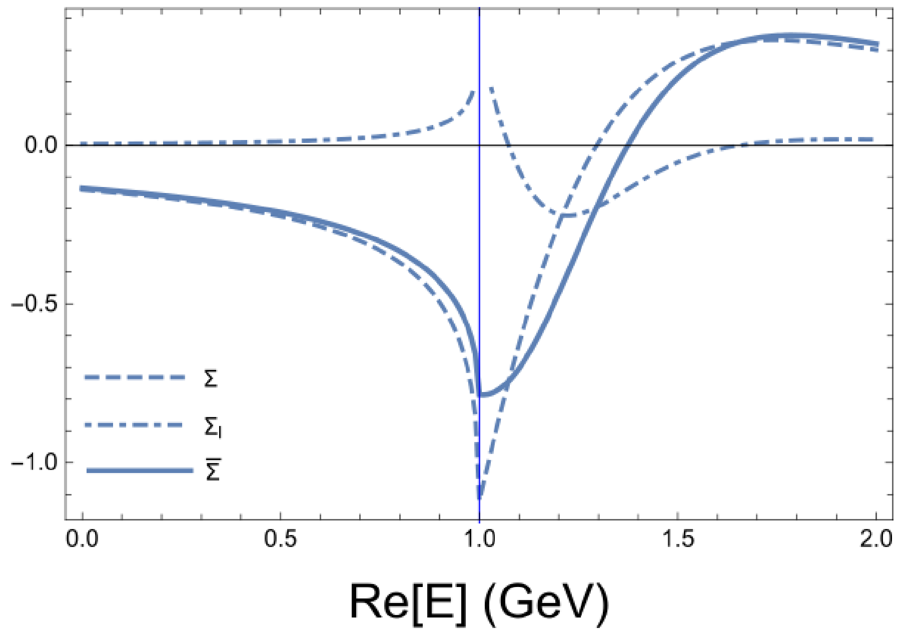


FIG. 12: The variation law of the real part $\Sigma_B(E)$ [Dashed line], $\Sigma_B^I(E)$ [DotDashed line] and $\bar{\Sigma}(E)$ [Solid line] with binding energy E. Hear we set $g = 0.3$, $v = 0.01$. The vertical axis is the real part of the $\Sigma_B(E)$, $\Sigma_B^I(E)$ and $\bar{\Sigma}(E)$.

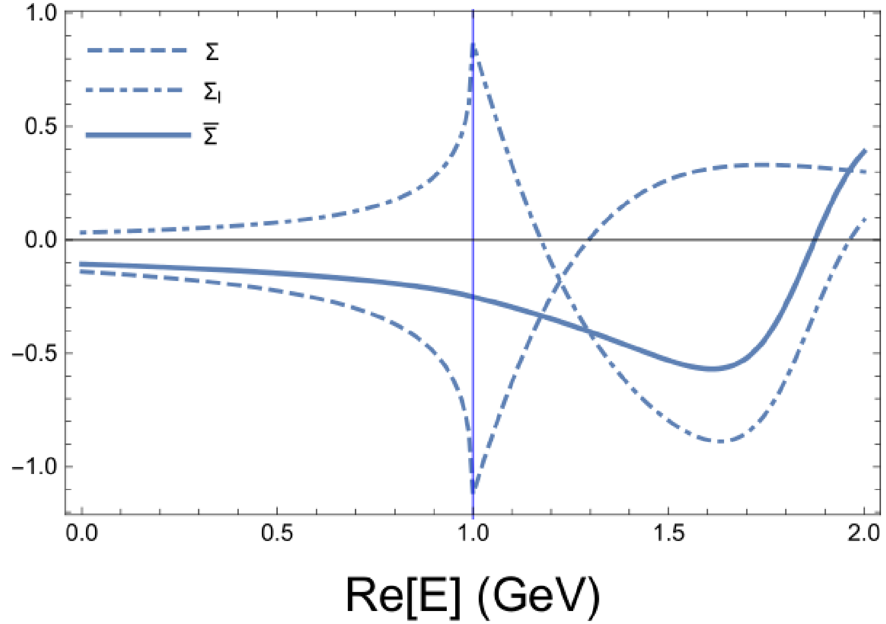


FIG. 13: The variation law of the real part $\Sigma_B(E)$ [Dashed line], $\Sigma_B^I(E)$ [DotDashed line] and $\bar{\Sigma}(E)$ [Solid line] with binding energy E. Hear we set $g = 0.3$, $v = 0.09$. The vertical axis is the real part of the $\Sigma_B(E)$, $\Sigma_B^I(E)$ and $\bar{\Sigma}(E)$.