Open-heavy flavor tetraquark states in a mass splitting model

满自龙

兰州大学

Based on

Shi-Yuan Li, Yan-Rui Liu, Zi-Long Man, Zong-Guo Si, and Jing Wu PRD 110 (2024) 094044

Shi-Yuan Li, Yan-Rui Liu, Zi-Long Man, Cheng-Rui Shu, Zong-Guo Si, and Jing Wu Symmetry 12 (2025) 170

This article belongs to the Special Issue

第八届强子谱和强子结构研讨会
桂林　2025年7月13日

Outlines

1. Background

2. Formalism

3. Results for $QQ\overline{qq}$, $Qq\overline{qq}$, and $QQ\overline{Q}\overline{q}$

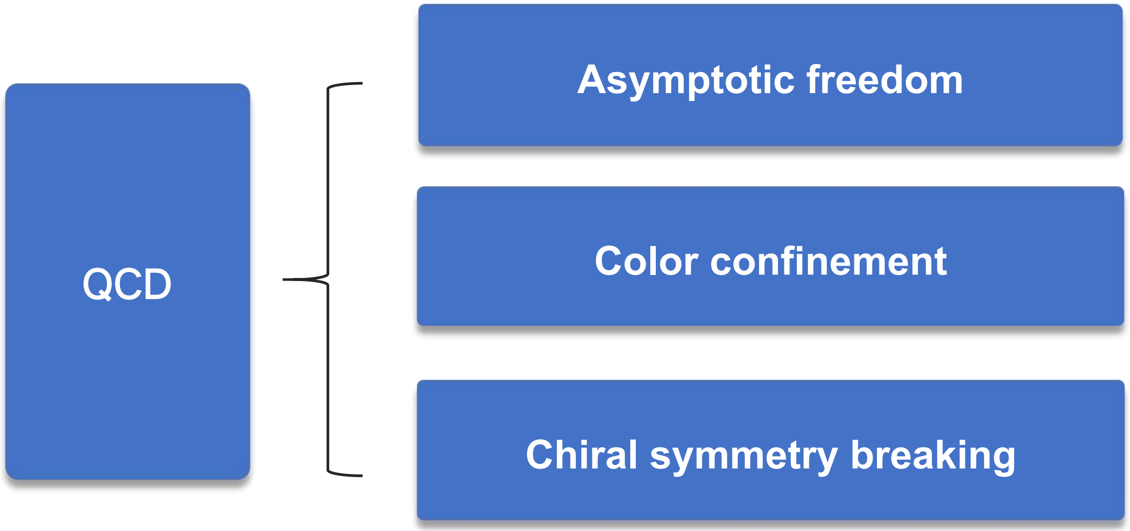
4. Summary

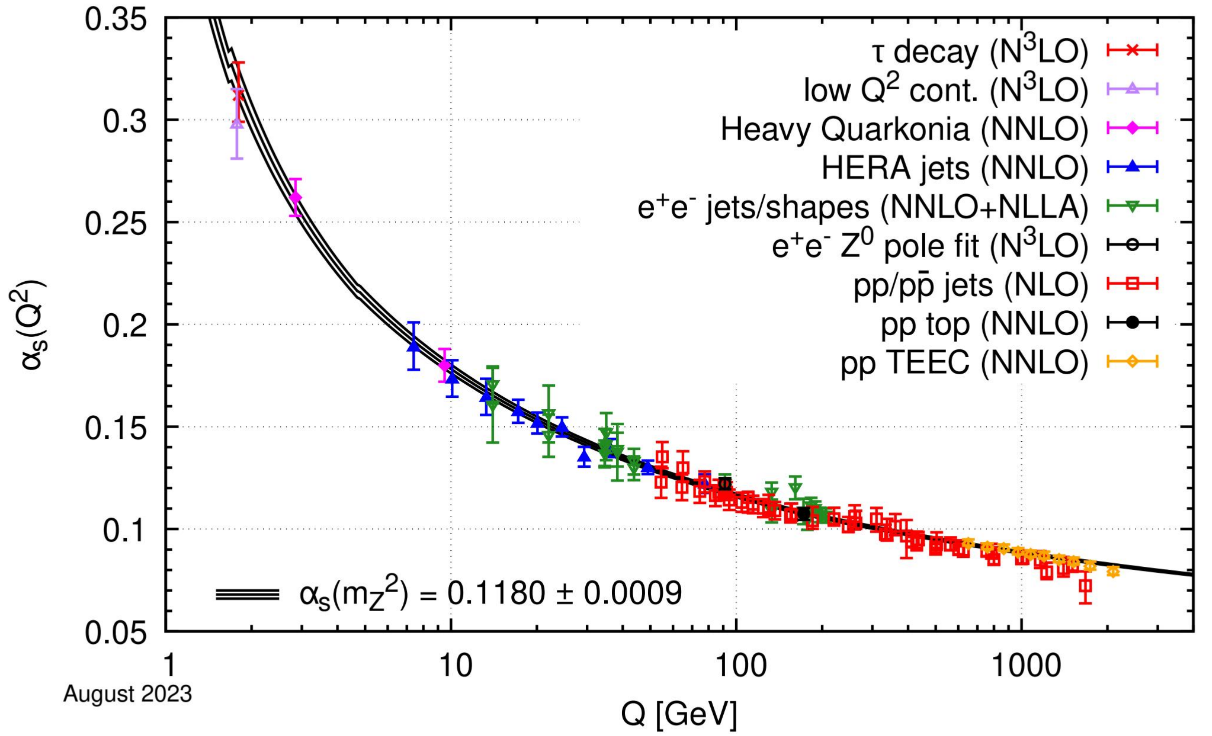
1

PART

Background

Hadron properties difficult to derive from QCD





Particle Data Group, Phys. Rev. D 110, 030001 (2024)

High energy: asymptotic freedom

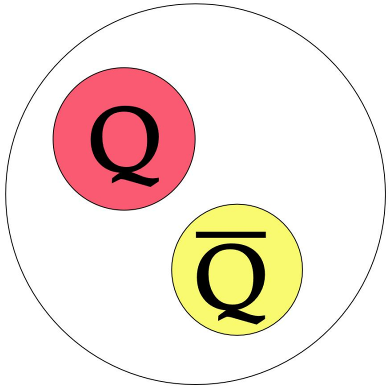
Low energy: non-perturbative important

Hadron properties difficult to derive from QCD

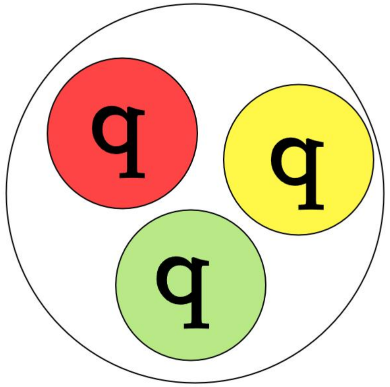
Methods: Lattice QCD, Quark Model, Effective field theory et al.

Hadronic structure

meson

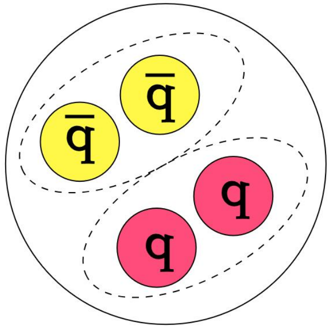


baryon

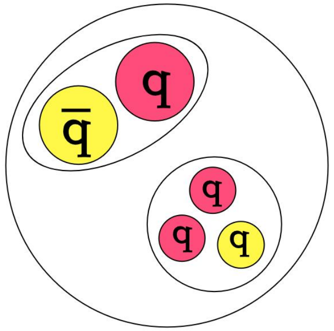


Traditional hadronic states

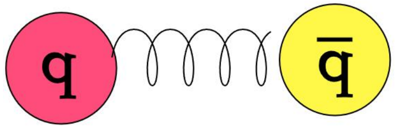
compact picture



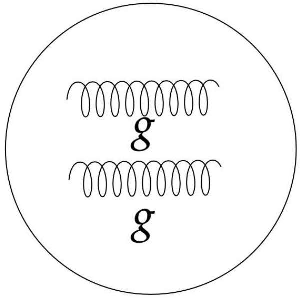
molecule picture



hybrid state

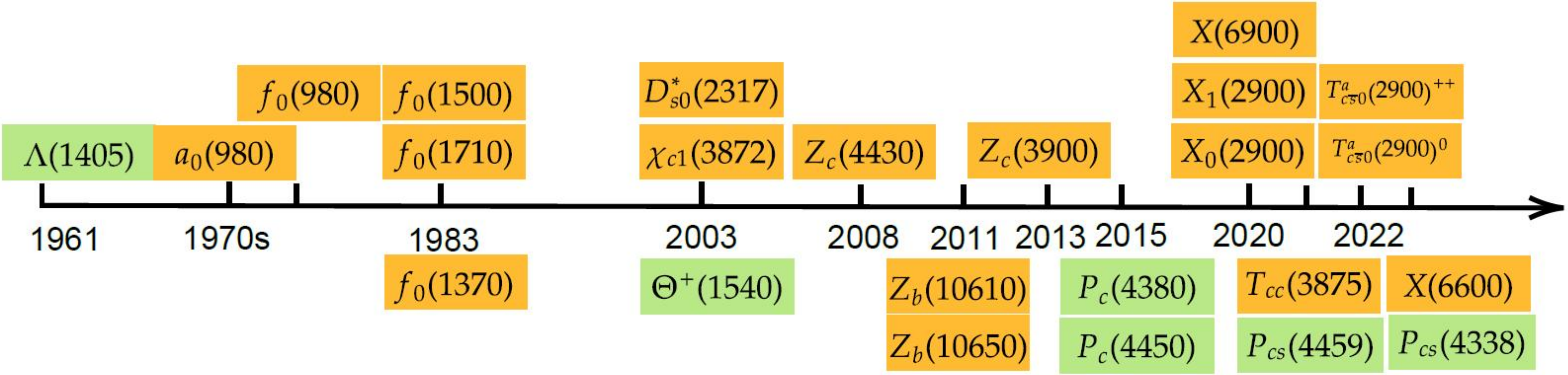


glueball



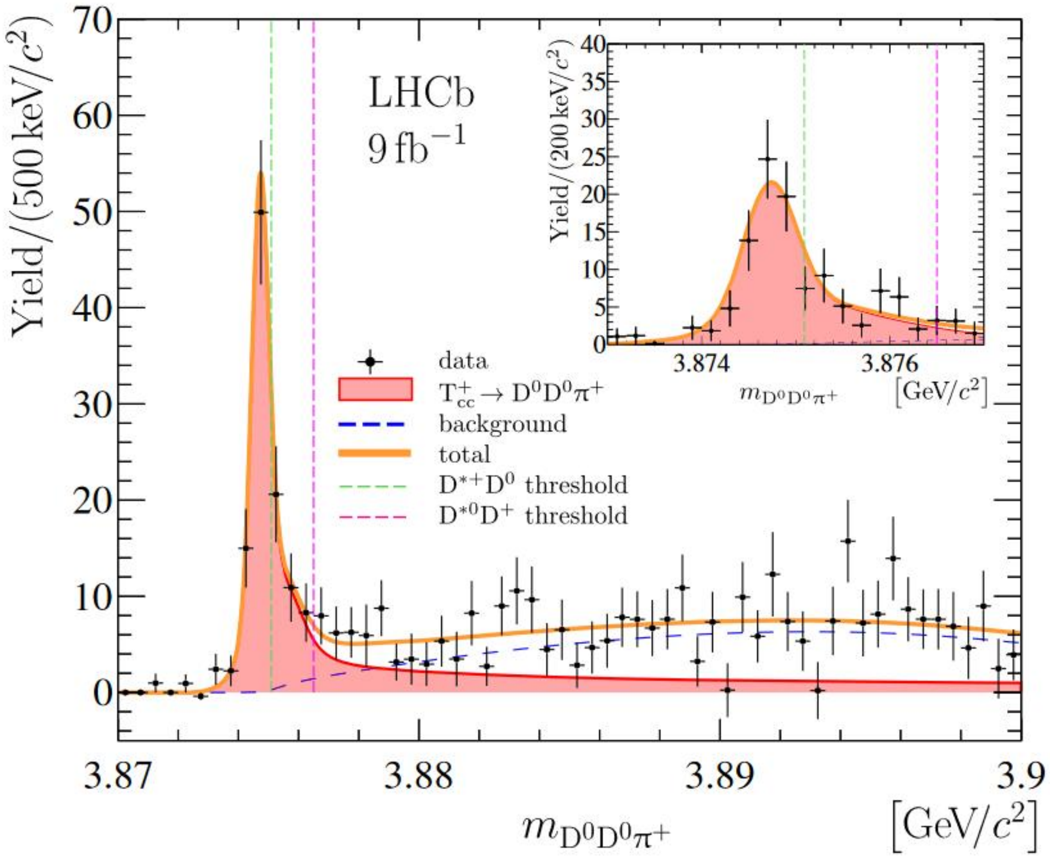
Exotic hadronic states

Experimental progress on the exotic hadronic states



T_{cc}^{+}

$$T_{cc}^{+} \to D^0 D^0 \pi^+$$



$$J^P = 1^+ \quad cc\bar{u}\bar{d}$$

Breit-Wigner parameterization

$$\delta m_{BW} = m_{T_{cc}} - m_{D^{*+}D^0}$$
$$= -273 \pm 61 \pm 5^{+11}_{-14} \text{ keV}$$
$$\Gamma_{BW} = 410 \pm 165 \pm 43^{+18}_{-38} \text{ keV}$$

LHCb, Nature Phys18, 751-754 (2022)

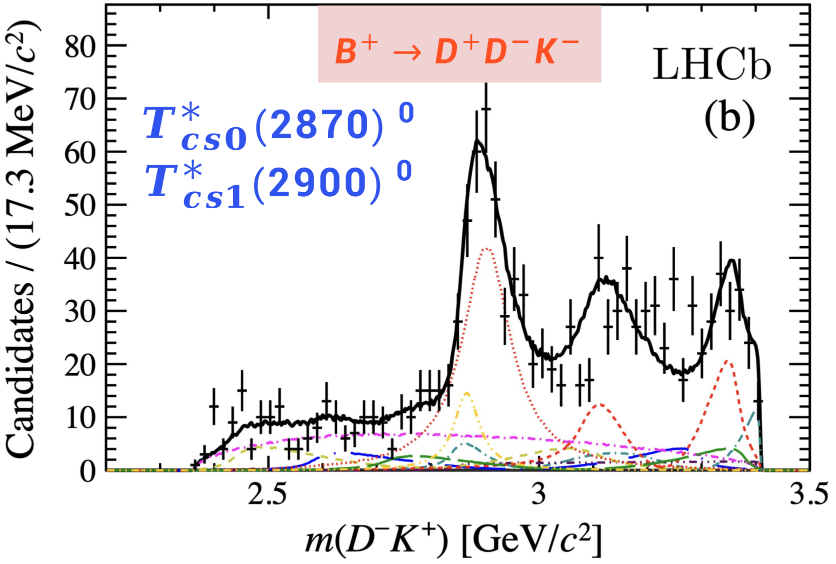
A unitarised three body Breit-Wigner function

$$\delta m_U = m_{T_{cc}} - m_{D^{*+}D^0}$$
$$= -360 \pm 40^{+4}_{-0} \text{ keV}$$
$$\Gamma_U = 48 \pm 2^{+0}_{-14} \text{ keV}$$

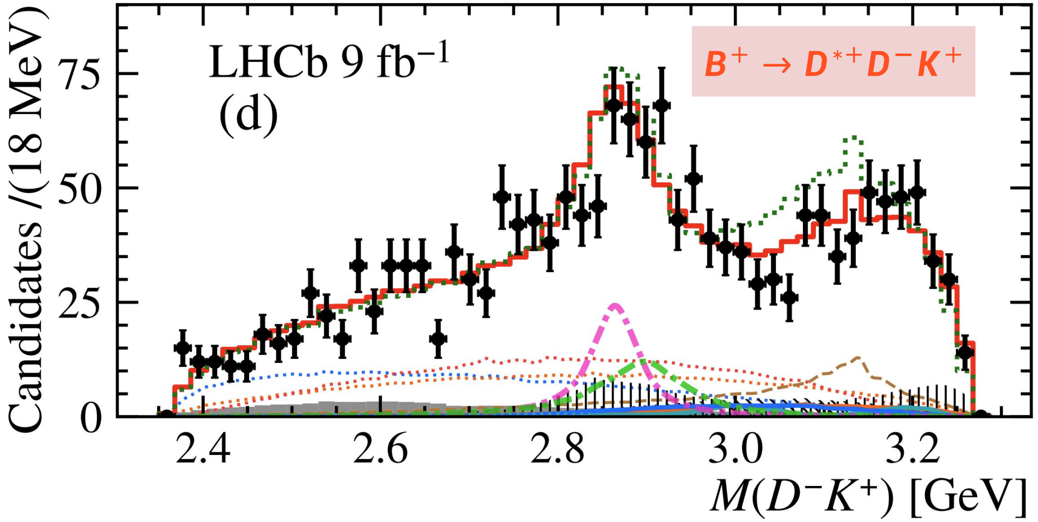
LHCb, Nature Commun. 13, 3351 (2022)

T_{cs} and $T_{c\bar{s}}$

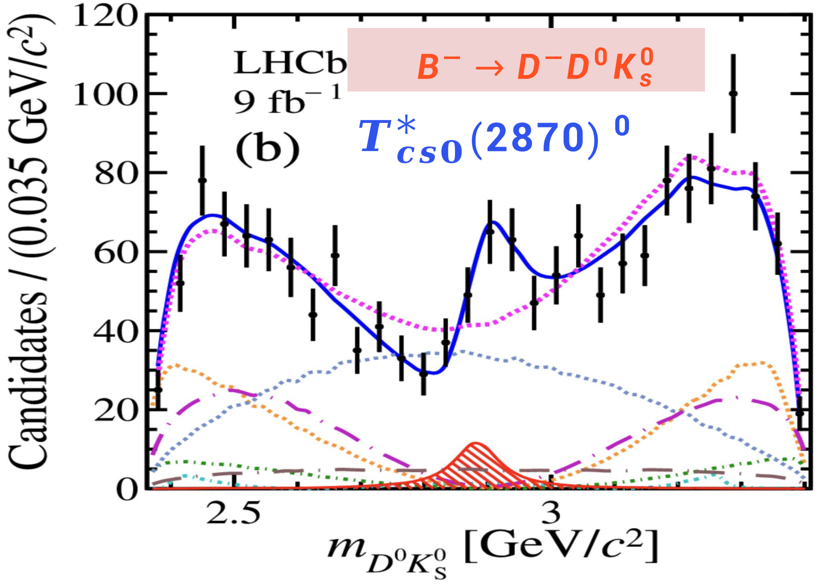
LHCb, Phys. Rev. Lett 125, 242001 (2020)



LHCb, Phys. Rev. Lett 133, 131902 (2024)



LHCb, Phys. Rev. Lett 134, 101901 (2025)



$cs\bar{u}\bar{d}$ and $cu\bar{d}\bar{s}$

State	Mass(MeV)	Γ (MeV)	Observed channels
$T^*_{cs0}(2870)^0$	$2866 \pm 7 \pm 2$	$57 \pm 12 \pm 4$	$B^+ \to D^+D^-K^+$
$T^*_{cs1}(2900)^0$	$2904 \pm 5 \pm 1$	$110 \pm 11 \pm 4$	$B^+ \to D^+D^-K^+$
$T^a_{c\bar{s}0}(2900)^0$	$2892 \pm 21 \pm 2$	119 ± 29	$B^0 \to \bar{D}^0D^+_s\pi^-$
$T^a_{c\bar{s}0}(2900)^{++}$	$2921 \pm 23 \pm 2$	137 ± 35	$B^+ \to D^-D^+_s\pi^+$

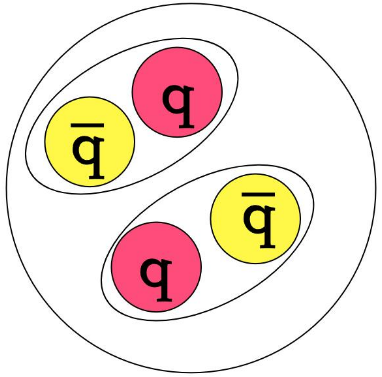
LHCb, Phys. Rev. D 102, 112003 (2020)

LHCb, Phys. Rev. Lett 131, 041902 (2023)

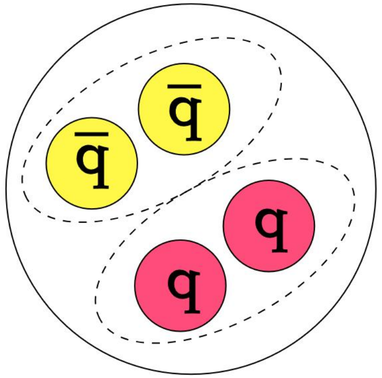
LHCb, Phys. Rev. D 108, 012017 (2023)

Theoretical explanations

$$(3_q \otimes 3^*_{\bar{q}})_{1c} \otimes (3_q \otimes 3^*_{\bar{q}})_{1c} \to 1_{1c}$$



$$(6_{qq} \otimes 6^*_{\bar{q}\bar{q}})_{1c} \to 1_{1c}, (3_{\bar{q}\bar{q}} \otimes 3^*_{qq})_{1c} \to 1_{1c}$$



➢ Review

- H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, Phys. Rept. 639, 1 (2016)
- H.X. Chen, W. Chen, X. Liu, Y.R. Liu, and S.L. Zhu, Rept. Prog. Phys. 80, 076201 (2017)
- F. K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, and B.-S. Zou, Rev. Mod. Phys. 90, 015004 (2018)
- Y. R. Liu, H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, Prog. Part. Nucl. Phys. 107, 237 (2019)
- N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C.-P. Shen, C. E. Thomas, A. Vairo, and C.-Z. Yuan, Phys. Rept. 873, 1 (2020)
- H. X. Chen, W. Chen, X. Liu, Y. R. Liu, and S. L. Zhu, Rept. Prog. Phys. 86, 026201 (2023)
- M. Z. Liu, Y. W. Pan, Z. W. Liu, T. W. Wu, J. X. Lu, and L. S. Geng, Phys. Rept. 1108,1 (2025)

2

PART

Formalism

Mass splitting model

$$H = \sum_i m_i + \sum_i \frac{\vec{p}_i^{\,2}}{2m_i} + \sum_{i<j} \frac{\vec{\lambda}_i \vec{\lambda}_j}{4}\left(\frac{\alpha_s}{r_{ij}} - \frac{3}{4} b r_{ij} - \frac{8\pi\alpha_s}{3m_i m_j} S_i \cdot S_j e^{-\sigma^2 r^2} \frac{\sigma^3}{\pi^{3/2}}\right)$$

$$H = \sum_i m_i + H_{\mathrm{CMI}} = \sum_i m_i - \sum_{i<j} C_{ij} \lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j$$

m_i : the effective quark mass

Hadron	$\langle H_{CMI} \rangle$	Hadron	$\langle H_{CMI} \rangle$	C_{ij}
N	$-8C_{nn}$	Δ	$8C_{nn}$	$C_{nn} = 18.3$
Σ	$\frac{8}{3}C_{nn} - \frac{32}{3}C_{ns}$	Σ^*	$\frac{8}{3}C_{nn} + \frac{16}{3}C_{ns}$	$C_{ns} = 12.1$
Σ_c	$\frac{8}{3}C_{nn} - \frac{32}{3}C_{cn}$	Σ_c^*	$\frac{8}{3}C_{nn} + \frac{16}{3}C_{cn}$	$C_{cn} = 4.0$
Ξ_c'	$\frac{8}{3}C_{ns} - \frac{16}{3}C_{cn} - \frac{16}{3}C_{cs}$	Ξ_c^*	$\frac{8}{3}C_{ns} + \frac{8}{3}C_{cn} + \frac{8}{3}C_{cs}$	$C_{cs} = 4.4$
η_c	$-16C_{c\bar{c}}$	J/ψ	$\frac{16}{3}C_{c\bar{c}}$	$C_{c\bar{c}} = 5.3$
D_s	$-16C_{c\bar{s}}$	D_s^*	$\frac{16}{3}C_{c\bar{s}}$	$C_{c\bar{s}} = 6.7$
D	$-16C_{c\bar{n}}$	D^*	$\frac{16}{3}C_{c\bar{n}}$	$C_{c\bar{n}} = 6.6$
Ω	$8C_{ss}$			$C_{ss} = 6.5$
π	$-16C_{n\bar{n}}$	ρ	$\frac{16}{3}C_{n\bar{n}}$	$C_{n\bar{n}} = 29.8$
K	$-16C_{n\bar{s}}$	K^*	$\frac{16}{3}C_{n\bar{s}}$	$C_{n\bar{s}} = 18.7$
B	$-16C_{b\bar{n}}$	B^*	$\frac{16}{3}C_{b\bar{n}}$	$C_{b\bar{n}} = 2.1$
B_c	$-16C_{b\bar{c}}$	B_c^*	$\frac{16}{3}C_{b\bar{c}}$	$C_{b\bar{c}} = 3.3$
B_s	$-16C_{b\bar{s}}$	B_s^*	$\frac{16}{3}C_{b\bar{s}}$	$C_{b\bar{s}} = 2.3$
η_b	$-16C_{b\bar{b}}$	Υ	$\frac{16}{3}C_{b\bar{b}}$	$C_{b\bar{b}} = 2.9$
Σ_b	$\frac{8}{3}C_{nn} - \frac{32}{3}C_{bn}$	Σ_c^*	$\frac{8}{3}C_{nn} + \frac{16}{3}C_{bn}$	$C_{bn} = 1.3$
Ξ_b'	$\frac{8}{3}C_{ns} - \frac{16}{3}C_{bn} - \frac{16}{3}C_{bs}$	Ξ_b^*	$\frac{8}{3}C_{ns} + \frac{8}{3}C_{cn} + \frac{8}{3}C_{bs}$	$C_{bs} = 1.3$

$$\frac{C_{cc}}{C_{c\bar{c}}} = \frac{C_{bb}}{C_{b\bar{b}}} = \frac{C_{bc}}{C_{b\bar{c}}} = \frac{C_{nn}}{C_{n\bar{n}}} \approx \frac{2}{3}$$

$m_n = 361.8$ MeV, $m_s = 542.4$ MeV,
$m_c = 1724.1$ MeV, $m_b = 5054.4$ MeV.

Model I: $M_1 = \sum_i m_i + E_{\mathrm{CMI}}$

Hadron	Th	Ep	Th-Ep	Hadron	Th	Ep	Th-Ep
π	246.8	139.6	107.2	ρ	882.5	775.3	107.2
K	605.0	493.7	111.3	K^*	1003.9	891.8	112.1
D	1980.3	1869.7	110.6	D^*	2121.1	2010.3	110.8
D_s	2159.3	1968.4	191.0	D_s^*	2301.7	2010.3	291.4
η	3363.4	2983.9	379.5	J/ψ	3476.5	3096.9	379.6
Σ	1185.7	1189.4	3.7	Σ^*	1379.3	1282.8	3.5
Ξ_c'	2616.3	2578.2	38.1	Ξ_c^*	2682.7	2645.1	37.6

Overestimated theoretical masses → Upper limits

Mass splitting model

$$M = \left[M_{ref} - (E_{CMI})_{ref}\right] + E_{CMI}$$

Model II: meson-meson threshold → underestimated

Lower limit

Model III: chose the exotic hadron as the reference state

$$M = M_{X(4140)} - (E_{CMI})_{X(4140)} + E_{CMI} + \sum_{ij} n_{ij}(m_i - m_j)$$

$$\tilde{m} = M_{X(4140)} - (E_{CMI})_{X(4140)}$$

$$M = \tilde{m} + E_{CMI} + \sum_{ij} n_{ij}\Delta_{ij}$$

$\Delta_{ij} = m_i - m_j$: the effective mass gap

$\Delta_{bc} = 3340.2$ MeV, $\Delta_{cn} = 1280.7$ MeV, $\Delta_{sn} = 90.6$ MeV

$\Delta_{cs} = 1180.6$ MeV, $\Delta_{bs} = 4520.2$ MeV

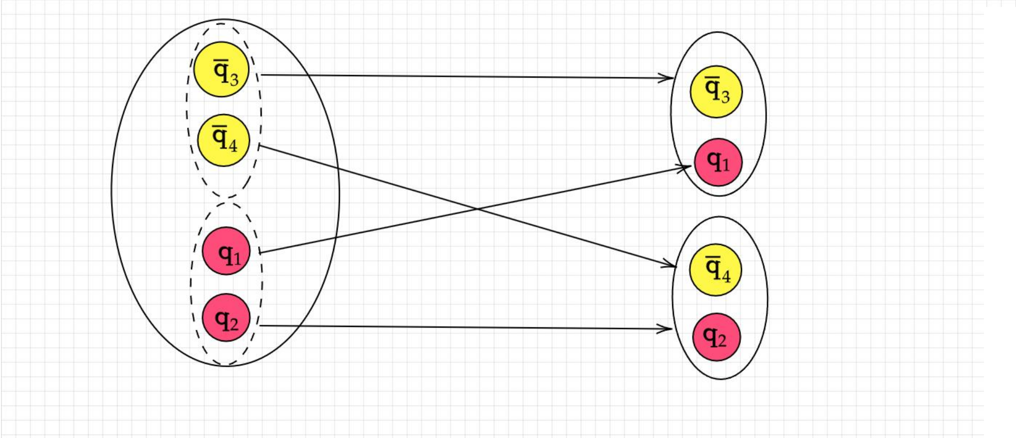
$$\Delta_{cn} = \Delta_{cs} + \Delta_{sn}, \Delta_{bs} = \Delta_{bc} + \Delta_{cs}$$

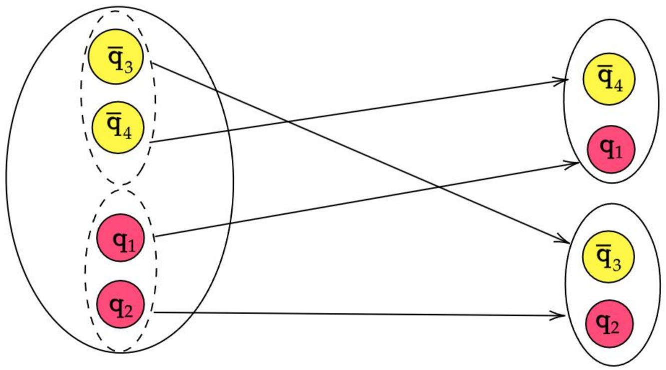
Hadron	Hadron	Δ_{bc}	Hadron	Hadron	Δ_{cn}	Hadron	Hadron	Δ_{sn}
B (B^*)	D (D^*)	3340.2 (3340.1)	D (D^*)	π (ρ, ω)	1356.4 (1357.6, 1350.2)	K (K^*)	π (ρ, ω)	178.4 (178.1, 170.7)
B_s (B_s^*)	D_s (D_s^*)	3328.2 (3326.7)	D_s (D_s^*)	K (K^*)	1280.7 (1282.6)	ϕ	K^*	175.9
B_c	η_c	3259.0	η_c (J/ψ)	D (D^*)	1095.9 (1095.3)	D_s (D_s^*)	D (D^*)	102.7 (103.1)
η_b	B_c	3117.7	B_c	B	1014.6	B_s (B_s^*)	B (B^*)	90.6 (89.7)
Λ_b	Λ_c, Σ_c	3333.1, 3318.6	Λ_c	N	1347.5	Λ	N	176.8
Σ_b (Σ_b^*)	Σ_c (Σ_c^*)	3331.1 (3329.9)	Σ_c (Σ_c^*)	N (Δ)	1362.1 (1362.4)	Σ (Σ^*)	N (Δ)	187.0 (186.2)
Σ_b	Λ_c	3345.6	Ξ_c'	Λ	1328.8	Ξ	Λ	169.0
Ξ_b	Ξ_c, Ξ_c'	3325.1, 3299.6	Ξ_c' (Ξ_c^*)	Σ (Σ^*)	1318.6 (1319.8)	Ξ (Ξ^*)	Σ (Σ^*)	158.7 (159.3)
Ξ_b' (Ξ_b^*)	Ξ_c' (Ξ_c^*)	3323.9 (3323.2)	Ξ_c	Λ, Σ	1303.3, 1293.0	Ω	Ξ^*	172.7
Ξ_b'	Ξ_c	3349.4	Ω_c (Ω_c^*)	Ξ (Ξ^*)	1295.9 (1273.0)	Ξ_c'	Λ_c	158.0
Ω_b	Ω_c	3313.6				Ξ_c' (Ξ_c^*)	Σ_c (Σ_c^*)	143.5 (143.5)
						Ξ_c	Λ_c, Σ_c	132.5, 118.0
						Ω_c	Ξ_c	161.6
						Ω_c (Ω_c^*)	Ξ_c' (Ξ_c^*)	136.0 (135.7)
						Ξ_b'	Λ_b	148.8
						Ξ_b' (Ξ_b^*)	Σ_b (Σ_b^*)	136.3 (136.8)
						Ξ_b	Λ_b, Σ_b	124.5, 112.0
						Ω_b	Ξ_b, Ξ_b'	150.1, 125.7
η_b	η_c	3188.4	η_c (J/ψ)	π (ρ, ω)	1226.1 (1226.4, 1222.7)	ϕ	ρ, ω	177.0, 173.3
			Ξ_{cc}	N	1287.2	Ξ (Ξ^*)	N (Δ)	172.9 (184.3)
						Ω	Δ, Σ^*	180.4, 177.5
						Ω_c	Λ_c	147.0
						Ω_c (Ω_c^*)	Σ_c (Σ_c^*)	139.8, 139.6
						Ω_b	Λ_b, Σ_b	137.3, 121.0

J. Wu, X. Liu, Y. R. Liu, and S. L. Zhu Phys. Rev. D 99, 014037(2019)

Rearrangement decay

1. The decay Hamiltonian is described by a constant $H = \mathcal{C}$.
2. The total width is equal to the sum of two-body rearrangement decay widths $\Gamma_{\text{total}} \sim \Gamma_{\text{sum}}$





$$q_1 q_2 \bar{q}_3 \bar{q}_4 \to (q_1 \bar{q}_3)_{1c} + (q_2 \bar{q}_4)_{1c} \qquad q_1 q_2 \bar{q}_3 \bar{q}_4 \to (q_1 \bar{q}_4)_{1c} + (q_2 \bar{q}_3)_{1c}$$

$$|M|^2 = \mathcal{C}^2 |\sum_i X_i Y_i|^2$$

$$\psi_{tetraquark} = \sum_i X_i (q_1 q_2 \bar{q}_3 \bar{q}_4)$$

$$\psi_{final} = \sum_i Y_i (q_1 q_2 \bar{q}_3 \bar{q}_4)$$

$$\Gamma = |\mathcal{M}|^2 \frac{|\mathbf{P}|}{8\pi M^2_{QQ\bar{q}\bar{q}}}$$

3

PART

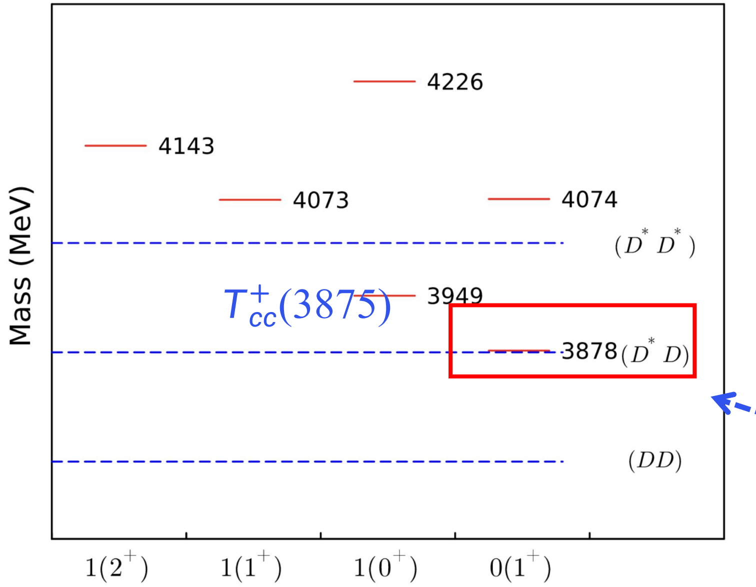
Results for $QQ\overline{qq}$, $Qq\overline{qq}$, and $QQ\overline{Q}\overline{q}$

T_{cc}^{+}

$$M_{cc\bar{n}\bar{n}} = \widetilde{m} + E_{CMI} - 2\Delta_{sn}$$

Assume $X(4140)$ to be the lowest $1^{++} cs\bar{c}\bar{s}$

$$\mathcal{C} = 7282.15 \text{ MeV}$$



$cc\bar{n}\bar{n}$				
$I(J^P)$	Mass	Channels		Γ
		D^*D^*		
$1(2^+)$	[4143.2]	[(33.3,20.8)]		[20.8]
		D^*D		
$1(1^+)$	[4072.8]	[(16.7,53.0)]		[53.0]
		D^*D^*	DD	
$1(0^+)$	[4225.9 3948.8]	[(55.7,43.2) (2.6,–)]	[(0.3,0.3) (41.4,35.9)]	[43.5 35.9]
		D^*D^*	D^*D	
$0(1^+)$	[4074.0 3878.2]	[(48.4,20.9) (1.6,–)]	[(6.2,19.8) (18.8,7.2)]	[40.7 7.2]

$$\Gamma = \int_0^{k_{\max}} dk \frac{\Gamma_{D^{*+}\to D^0\pi^+}}{(M_{T_{cc}^+} - E_{D^{*+}}(k) - E_{D^0}(k))^2 + \frac{1}{4}\Gamma_{D^{*+}}^2} \times \frac{k^2|\mathcal{M}|^2}{2(2\pi)^2 M_{T_{cc}^+} E_{D^{*+}}(k) E_{D^0}(k)}$$

$$\Gamma \sim 105 \text{ keV}$$

$$\delta m_{BW} = m_{T_{cc}} - m_{D^{*+}D^0} = -273 \pm 61 \pm 5^{+11}_{-14} \text{ keV}$$

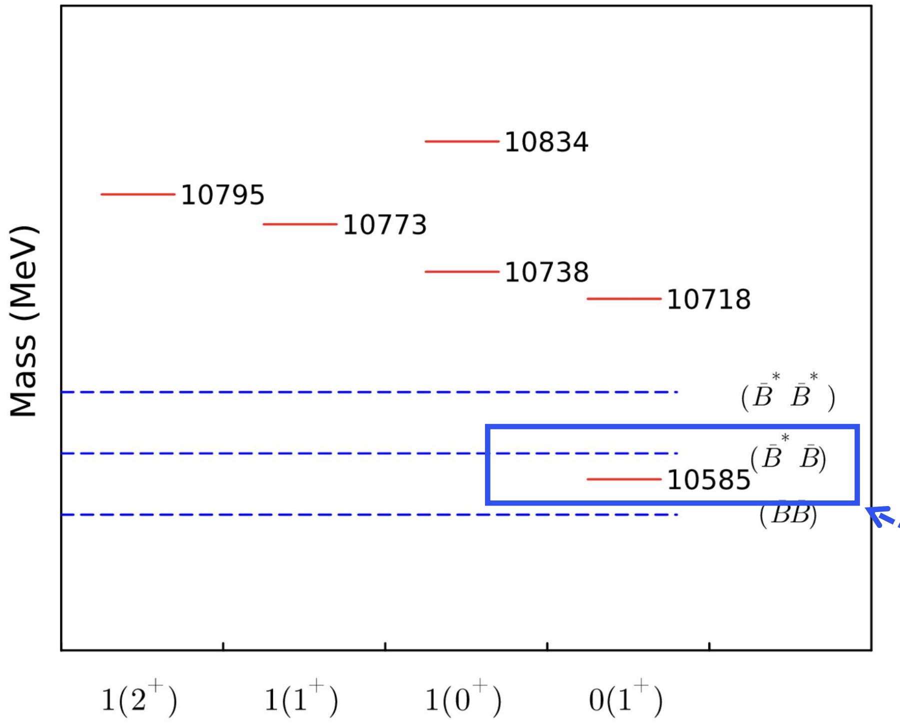
$$\Gamma_{BW} = 410 \pm 165 \pm 43^{+18}_{-38} \text{ keV}$$

- **$T_{cc}^{+}(3875)$ can be regarded as the lowest $0(1^+)$ $cc\bar{u}\bar{d}$**

$bb\bar{n}\bar{n}$ states

$$M_{bb\bar{n}\bar{n}} = \widetilde{m} + E_{CMI} - 2\Delta_{sn} + 2\Delta_{bc}$$

$$\mathcal{C} = 7282.15 \text{ MeV}$$



$bb\bar{n}\bar{n}$				
$I(J^P)$	Mass	Channels		Γ
		$\bar{B}^*\bar{B}^*$		
$1(2^+)$	[10795.3]	[(33.3,5.3)]		[5.3]
		$\bar{B}^*\bar{B}$		
$1(1^+)$	[10772.9]	[(16.7,11.5)]		[11.5]
		$\bar{B}^*\bar{B}^*$	$\bar{B}\bar{B}$	
$1(0^+)$	[10834.4; 10738.4]	[(57.4,10.3); (0.9,0.1)]	[(1.2,0.3); (40.5,7.2)]	[10.5; 7.4]
		$\bar{B}^*\bar{B}^*$	$\bar{B}^*\bar{B}$	
$0(1^+)$	[10717.8; 10584.5]	[(41.2,4.6); (8.8,–)]	[(12.2,7.0); (12.8,–)]	[11.6; 0.0]

- **The lowest $0(1^+)$ $bb\bar{u}\bar{d}$ is a stable tetraquark about 20 MeV below the $\bar{B}^*\bar{B}$ threshold**

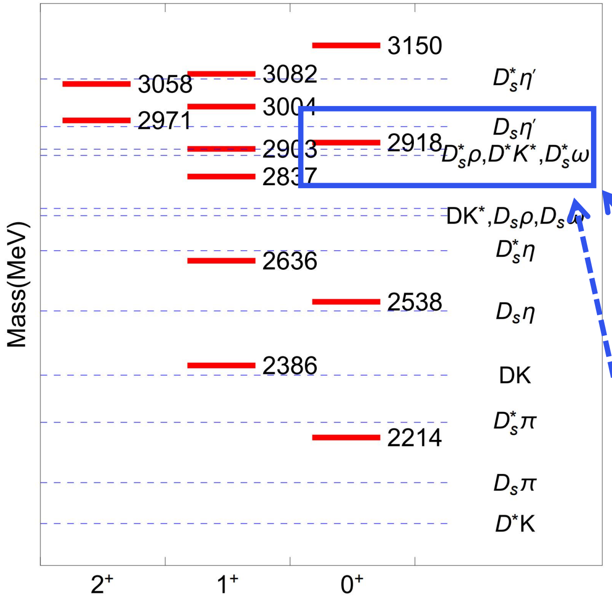
  Almost all theoretical studies favor this conclusion

$T^{a}_{c\bar{s}0}(2900)$

$$M_{cn\bar{s}\bar{n}} = \tilde{m} + E_{CMI} - \Delta_{sn} - \Delta_{cn}$$

Assume $T^{a}_{c\bar{s}0}(2900)$ to be the $1(0^+)cn\bar{s}\bar{n}$

$\mathcal{C} = 13.577$ GeV



$cn\bar{s}\bar{n}$ states

The mass of I=1 and I=0 states are degenerate

$$B^0 \to \bar{D}^0 D_s^+ \pi^-$$
$$B^+ \to D^- D_s^+ \pi^+$$

$$M = 2.908 \pm 0.011 \pm 0.020 \text{ GeV}$$

$$\Gamma = 0.136 \pm 0.023 \pm 0.013 \text{ GeV}$$

$cn\bar{s}\bar{n}$

$I(J^P)$	Mass	Channels						Γ
		$D_s^*\rho$	D^*K^*					
$1(2^+)$	3058.0	(95.8, 340.4)	(26.9, 94.8)					435.2
	2971.3	(4.2, 10.9)	(73.1, 179.3)					190.1
		$D_s^*\rho$	$D_s^*\pi$	$D_s\rho$	D^*K^*	D^*K	DK^*	
$1(1^+)$	3082.1	(71.1, 266.6)	(0.0, 0.2)	(4.5, 22.7)	(22.5, 84.1)	(0.8, 4.6)	(9.3, 46.7)	424.9
	3004.4	(26.8, 80.9)	(0.3, 2.0)	(22.7, 103.6)	(61.8, 181.3)	(1.3, 7.5)	(1.3, 6.1)	381.4
	2902.7	(0.0, 0.0)	(0.7, 4.2)	(44.1, 166.0)	(12.6, 2.7)	(0.0, 0.2)	(12.4, 45.5)	218.5
	2837.3	(1.3, –)	(0.0, 0.0)	(26.8, 80.2)	(0.1, –)	(3.2, 16.5)	(70.9, 198.4)	295.1
	2636.4	(0.8, –)	(10.7, 51.5)	(1.8, –)	(1.5, –)	(72.7, 258.8)	(4.4, –)	310.3
	2385.9	(0.0, –)	(88.2, 253.0)	(0.0, –)	(1.4, –)	(22.1, –)	(1.7, –)	253.0
		$D_s^*\rho$	$D_s\pi$	D^*K^*	DK			
$1(0^+)$	3149.7	(63.3, 266.6)	(0.0, 0.3)	(46.3, 196.1)	(0.2, 1.7)			464.6
	2917.9	(34.1, 54.9)	(0.7, 5.1)	(47.4, 56.9)	(3.1, 20.1)			137.0
	2537.5	(2.5, –)	(17.3, 96.4)	(4.1, –)	(67.0, 297.1)			393.5
	2214.3	(0.1, –)	(81.9, 235.6)	(2.1, –)	(29.6, –)			235.6

$$\Gamma(D_s^*\rho) : \Gamma(D_s\pi) : \Gamma(D^*K^*) : \Gamma(DK) \simeq 10.8 : 1.0 : 11.2 : 3.9$$

$cn\bar{s}\bar{n}$

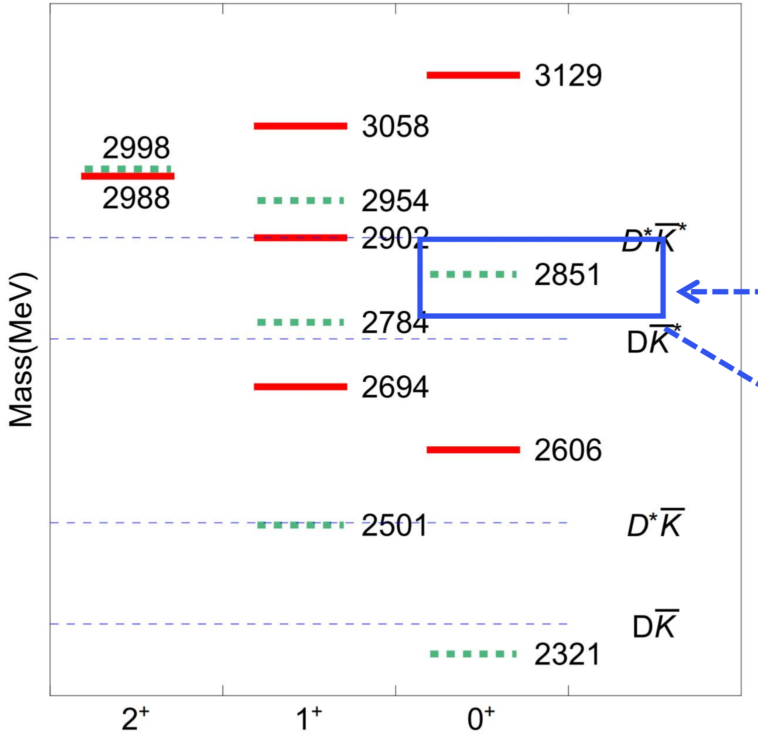
$I(J^P)$	Mass	Channels							Γ
		$D_s^*\omega$	D^*K^*						
$0(2^+)$	3058.0	(95.8, 333.6)	(26.9, 94.8)						428.4
	2971.3	(4.2, 10.4)	(73.1, 179.3)						189.7
		$D_s^*\omega$	$D_s^*\eta$	$D_s^*\eta'$	$D_s\omega$	D^*K^*	D^*K	DK^*	
$0(1^+)$	3082.1	(71.1, 262.0)	(0.0, 0.1)	(0.0, 0.0)	(4.5, 22.5)	(22.5, 84.1)	(0.8, 4.6)	(9.3, 46.7)	420.0
	3004.4	(26.8, 78.5)	(0.2, 0.8)	(0.2, –)	(22.7, 102.3)	(61.8, 181.3)	(1.3, 7.5)	(1.3, 6.1)	376.5
	2902.7	(0.0, 0.0)	(0.4, 1.6)	(0.3, –)	(44.1, 162.4)	(12.6, 2.7)	(0.0, 0.2)	(12.4, 45.5)	212.3
	2837.3	(1.3, –)	(0.4, 0.0)	(0.0, –)	(26.8, 77.1)	(0.1, –)	(3.2, 16.5)	(70.9, 198.4)	292.0
	2636.4	(0.8, –)	(5.7, –)	(5.0, –)	(1.8, –)	(1.5, –)	(72.7, 258.8)	(4.4, –)	258.8
	2385.9	(0.0, –)	(46.7, –)	(41.5, –)	(0.0, –)	(1.4, –)	(22.1, –)	(1.7, –)	–
		$D_s^*\omega$	$D_s\eta$	$D_s\eta'$	D^*K^*	DK			
$0(0^+)$	3149.7	(63.3, 263.3)	(0.0, 0.1)	(0.0, 0.1)	(46.3, 196.1)	(0.2, 1.7)			461.3
	2917.9	(34.1, 47.8)	(0.4, 2.2)	(0.4, 0.1)	(47.4, 56.9)	(3.1, 20.1)			127.1
	2537.5	(2.5, –)	(9.1, 14.1)	(8.1, –)	(4.1, –)	(67.0, 297.1)			311.2
	2214.3	(0.1, –)	(43.3, –)	(38.6, –)	(2.1, –)	(29.6, –)			4.0

$$\Gamma(D^*\omega) : \Gamma(D^*K^*) : \Gamma(DK) = 2.4 : 2.8 : 1.0.$$

$T^*_{cs0}(2870)$

$$M_{cs\bar{n}\bar{n}} = \tilde{m} + E_{CMI} - \Delta_{sn} - \Delta_{cn}$$

$\mathcal{C} = 13.577$ GeV



$I = 0/1\ cs\bar{n}\bar{n}$ states

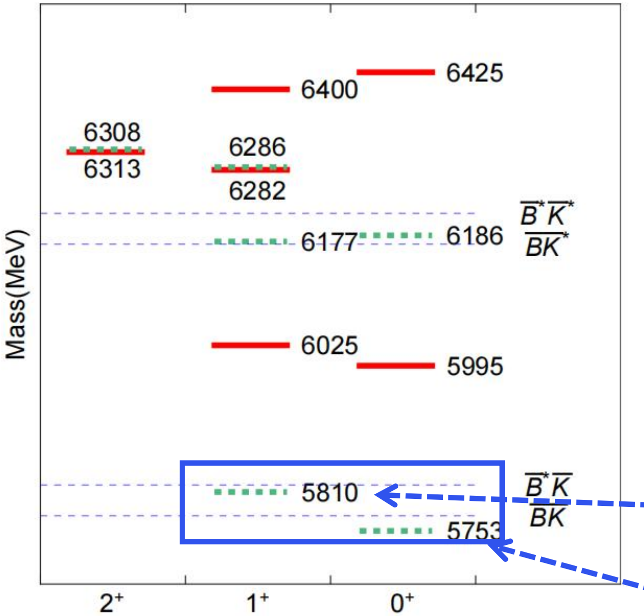
$I(J^P)$	Mass	Channels			Γ
		$D^*\bar{K}^*$			
$0(2^+)$	[2998.3]	[(66.7, 380.2)]			[380.2]
		$D^*\bar{K}^*$	$D^*\bar{K}$	$D\bar{K}^*$	
$0(1^+)$	2953.5	(47.6, 203.0)	(1.3, 14.5)	(10.6, 88.5)	306.1
	2784.3	(2.4, –)	(1.8, 17.8)	(47.6, 151.9)	169.7
	2500.6	(0.1, –)	(55.2, –)	(0.1, –)	–
		$D^*\bar{K}^*$	$D\bar{K}$		
$0(0^+)$	2850.5	(41.4, –)	(2.7, 34.4)		34.4
	2320.7	(0.3, –)	(55.6, –)		–

Consistent

State	Mass(MeV)	Γ (MeV)	Observed channels
$T^*_{cs0}(2870)^0$	$2866 \pm 7 \pm 2$	$57 \pm 12 \pm 4$	$B^+ \to D^+D^-K^+$
$T^*_{cs1}(2900)^0$	$2904 \pm 5 \pm 1$	$110 \pm 11 \pm 4$	$B^+ \to D^+D^-K^+$
$T^a_{c\bar{s}0}(2900)^0$	$2892 \pm 21 \pm 2$	119 ± 29	$B^0 \to \bar{D}^0D^+_s\pi^-$
$T^a_{c\bar{s}0}(2900)^{++}$	$2921 \pm 23 \pm 2$	137 ± 35	$B^+ \to D^-D^+_s\pi^+$

- **$T^*_{cs0}(2870)$ can be regarded as the highest $0(0^+)\ cs\bar{n}\bar{n}$**
- **The lowest $0(0^+)$ and the lowest $0(1^+)\ cs\bar{n}\bar{n}$ is stable**

$b\,n\,\bar{s}\,\bar{n}$ and $b\,s\,\bar{n}\,\bar{n}$ states



$I = 0/1\ bs\bar{n}\bar{n}$ states

$$M_{bn\bar{s}\bar{n}} = \tilde{m} + E_{CMI} - \Delta_{sn} - \Delta_{cn} + \Delta_{bc},$$

$0(2^+)$	[6312.5]	$\bar{B}^*\bar{K}^*$ [(66.7, 95.0)]			[95.0]
$0(1^+)$	6286.0 6176.8 5810.1	$\bar{B}^*\bar{K}^*$ (34.1, 41.3) (15.8, –) (0.1, –)	$\bar{B}^*\bar{K}$ (0.2, 0.6) (2.9, 7.2) (55.2, –)	$\bar{B}\bar{K}^*$ (31.2, 49.2) (27.1, 7.8) (0.1, –)	91.0 14.9 –
$0(0^+)$	6186.4 5753.2	$\bar{B}^*\bar{K}^*$ (41.5, –) (0.2, –)	$\bar{B}\bar{K}$ (2.9, 7.8) (55.4, –)		7.8 –

$bn\bar{s}\bar{n}$									
$0(2^+)$	6373.6 6291.1	$\bar{B}_s^*\omega$ (97.1, 89.4) (2.9, 2.0)	$\bar{B}^*K^*$ (24.0, 21.8) (76.0, 47.7)						111.2 49.7
$0(1^+)$	6422.0 6344.0 6269.6 6237.6 5963.3 5698.5	$\bar{B}_s^*\omega$ (55.7, 57.7) (32.2, 27.1) (0.5, 0.3) (10.3, 4.6) (1.2, –) (0.0, –)	$\bar{B}_s^*\eta$ (0.0, 0.0) (0.0, 0.0) (0.1, 0.1) (0.1, 0.2) (6.7, 0.1) (45.9, –)	$\bar{B}_s^*\eta'$ (0.0, 0.0) (0.0, 0.0) (0.1, 0.1) (0.2, 0.2) (5.9, 0.1) (40.9, –)	$\bar{B}_s\omega$ (12.1, 14.0) (57.1, 56.1) (13.1, 10.2) (16.6, 11.1) (1.0, –) (0.0, –)	$\bar{B}^*K^*$ (27.8, 28.8) (23.8, 19.5) (37.0, 19.6) (7.3, 2.4) (2.4, –) (1.6, –)	$\bar{B}^*K$ (0.4, 0.6) (0.2, 0.3) (0.2, 0.3) (3.2, 4.2) (72.1, 56.7) (23.9, –)	$\bar{B}K^*$ (13.6, 15.6) (6.4, 6.2) (24.5, 17.9) (52.4, 31.4) (2.0, –) (1.0, –)	116.6 109.3 48.6 54.1 56.8 –
$0(0^+)$	6447.4 6252.1 5930.1 5640.8	$\bar{B}_s^*\omega$ (65.0, 70.8) (32.7, 16.9) (2.3, –) (0.1, –)	$\bar{B}_s\eta$ (0.0, 0.0) (0.3, 0.4) (7.6, 2.0) (45.0, –)	$\bar{B}_s\eta'$ (0.0, 0.0) (0.3, 0.0) (6.7, –) (40.0, –)	$\bar{B}^*K^*$ (44.7, 48.8) (48.7, 20.9) (4.2, –) (2.4, –)	$\bar{B}K$ (0.3, 0.5) (3.4, 4.9) (70.3, 58.4) (26.0, –)			120.1 43.1 60.3 –

Stable states

Channels

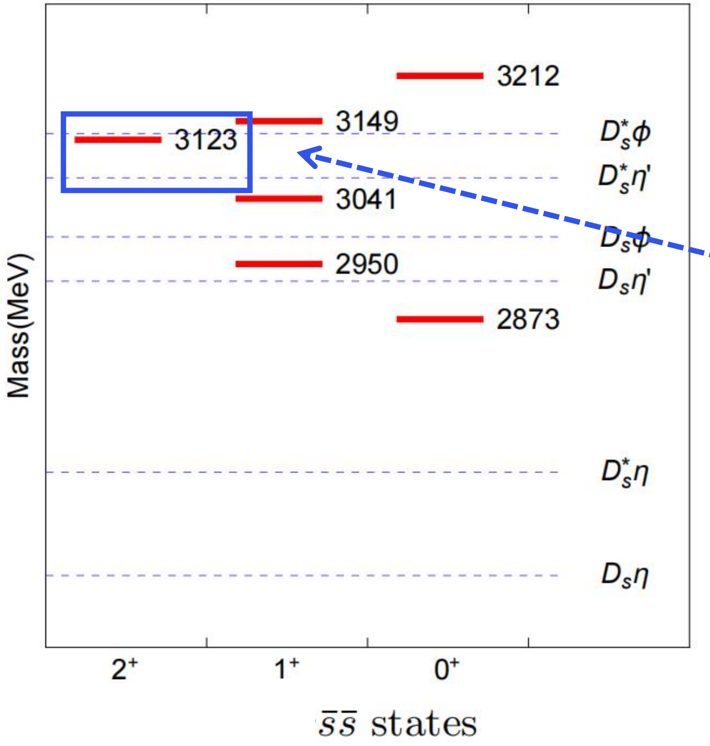
$\bar{B}_s\eta$

$\bar{B}\bar{K}, \bar{B}^*\bar{K}$

The lowest $0(0^+)$ and lowest $0(1^+)$ $bn\bar{s}\bar{n}$

The lowest $0(0^+)$ and lowest $0(1^+)$ $bs\bar{n}\bar{n}$

$cs\bar{s}\bar{s}$ states



$cs\bar{s}\bar{s}$						
J^P	Mass	Channels				Γ
2^+	3123.3	$D_s^*\phi$: (33.3, –)				–
1^+	3148.5	$D_s^*\phi$: (49.3, 111.2)	$D_s^*\eta$: (0.4, 4.6)	$D_s^*\eta'$: (0.5, 2.3)	$D_s\phi$: (3.3, 23.2)	141.3
	3041.2	(0.4, –)	(3.4, 34.0)	(3.8, –)	(26.1, 111.4)	145.5
	2949.7	(0.4, –)	(15.8, 143.9)	(17.7, –)	(12.3, –)	143.9
0^+	3212.0	$D_s^*\phi$: (54.9, 262.2)	$D_s\eta$: (0.1, 0.7)	$D_s\eta'$: (0.1, 0.5)		263.4
	2872.9	(3.4, –)	(19.6, 209.6)	(22.0, –)		209.6

- **There may exist a stable state with $J^P = 2^+$ for $cs\bar{s}\bar{s}$ case**

$QQ\overline{Q}\overline{q}$ states

$$M_{cc\bar{c}\bar{n}} = \tilde{m} + \langle H_{CMI} \rangle + \Delta_{cs} - \Delta_{sn},$$
$$M_{cc\bar{c}\bar{s}} = \tilde{m} + \langle H_{CMI} \rangle + \Delta_{cs},$$
$$M_{cc\bar{b}\bar{n}} = \tilde{m} + \langle H_{CMI} \rangle + \Delta_{bs} - \Delta_{sn},$$
$$M_{cc\bar{b}\bar{s}} = \tilde{m} + \langle H_{CMI} \rangle + \Delta_{bs},$$
$$M_{bb\bar{c}\bar{n}} = \tilde{m} + \langle H_{CMI} \rangle + 2\Delta_{bs} - \Delta_{cn},$$
$$M_{bb\bar{c}\bar{s}} = \tilde{m} + \langle H_{CMI} \rangle + \Delta_{bc} + \Delta_{bs},$$
$$M_{bb\bar{b}\bar{n}} = \tilde{m} + \langle H_{CMI} \rangle + 2\Delta_{bs} + \Delta_{bc} - \Delta_{cn},$$
$$M_{bb\bar{b}\bar{s}} = \tilde{m} + \langle H_{CMI} \rangle + 2\Delta_{bc} + \Delta_{bs},$$
$$M_{bc\bar{c}\bar{n}} = M_{cc\bar{b}\bar{n}}, \quad M_{bc\bar{c}\bar{s}} = M_{cc\bar{b}\bar{s}},$$
$$M_{bc\bar{b}\bar{n}} = M_{bb\bar{c}\bar{n}}, \quad M_{bc\bar{b}\bar{s}} = M_{bb\bar{c}\bar{s}}.$$

Treat $X(6600)$ as the ground scalar $cc\overline{cc}$ tenraquark

↓

$\mathcal{C} = 14.955$ GeV

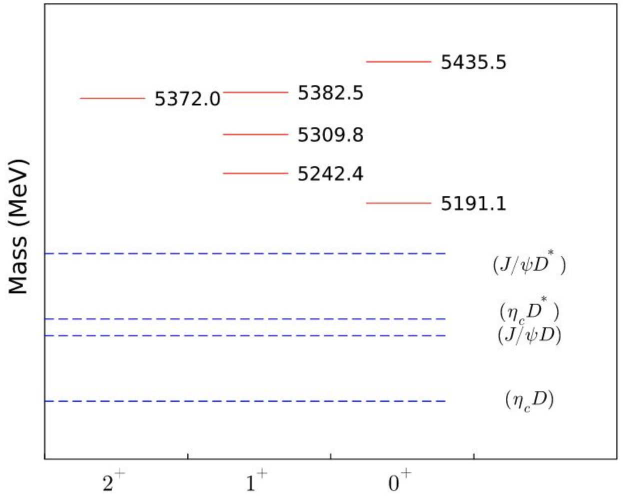
No stable $\mathbf{QQ\overline{Q}\overline{q}}$ state is found.

The mass spectrum of thiply theavy tetraquark ranges from 5.2 GeV-15.5 GeV

Most theoretical studies are in agreement with this conclusion

$QQ\bar{Q}\bar{q}$ states

$$M_{cc\bar{c}\bar{n}} = \tilde{m} + \langle H_{CMI} \rangle + \Delta_{cs} - \Delta_{sn}$$

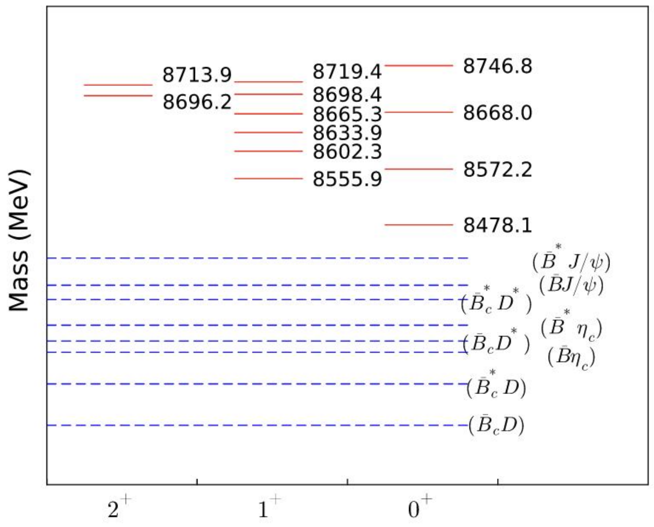


Six states : 5.19 GeV ~ 5.44 GeV

J^P	Mass	Channels			Γ_{sum}
		$cc\bar{c}\bar{n}$			
		$J/\psi D^*$			
2^+	5372.0	(33.3, 168.2)			168.2
		$J/\psi D^*$	$J/\psi D$	$\eta_c D^*$	
1^+	5382.5	(49.6, 254.5)	(1.3, 8.0)	(2.9, 17.6)	280.0
	5309.8	(0.2, 0.8)	(11.4, 66.0)	(21.8, 122.3)	189.1
	5242.4	(0.2, 0.7)	(29.0, 153.6)	(17.0, 86.4)	240.8
		$J/\psi D^*$	$\eta_c D$		
0^+	5435.5	(54.9, 302.0)	(0.1, 0.8)		302.8
	5191.1	(3.5, 10.5)	(41.6, 247.6)		258.1

Similar features are found in the $cc\bar{c}\bar{s}$, $cc\bar{b}\bar{n}$, $cc\bar{b}\bar{s}$, $bb\bar{c}\bar{n}$, $bb\bar{c}\bar{s}$, $bb\bar{b}\bar{n}$, and $bb\bar{b}\bar{s}$ states

$$M_{bb\bar{c}\bar{n}} = \tilde{m} + \langle H_{CMI} \rangle + 2\Delta_{bs} - \Delta_{cn}.$$



J^P	Mass	Channels						Γ_{sum}
		$bc\bar{c}\bar{n}$						
		$\bar{B}_c^* D^*$	$\bar{B}^* J/\psi$					
2^+	8713.9	(97.3, 245.8)	(21.8, 55.2)					301.1
	8696.2	(2.1, 5.3)	(77.7, 191.3)					196.6
		$\bar{B}_c^* D^*$	$\bar{B}_c^* D$	$\bar{B}_c D^*$	$\bar{B}^* J/\psi$	$\bar{B}^* \eta_c$	$\bar{B}_s J/\psi$	
1^+	8719.4	(78.2, 199.0)	(0.1, 0.2)	(2.1, 5.7)	(17.8, 45.6)	(1.2, 3.7)	(10.3, 28.2)	282.3
	8698.4	(19.4, 48.1)	(1.3, 3.8)	(7.6, 20.8)	(79.1, 195.6)	(1.0, 2.8)	(2.1, 5.6)	276.7
	8665.3	(2.1, 4.9)	(1.5, 4.4)	(16.7, 43.9)	(1.2, 2.8)	(3.8, 10.6)	(43.6, 111.1)	177.7
	8633.9	(0.3, 0.6)	(1.0, 2.8)	(64.7, 163.6)	(0.3, 0.8)	(0.1, 0.2)	(38.4, 92.8)	260.6
	8602.3	(0.5, 1.0)	(26.1, 68.7)	(8.4, 20.4)	(0.9, 1.7)	(51.0, 131.6)	(4.8, 10.6)	233.9
	8555.9	(0.1, 0.2)	(69.5, 173.3)	(0.4, 0.8)	(0.4, 0.8)	(42.8, 102.1)	(0.9, 1.7)	278.9
		$\bar{B}_c^* D^*$	$\bar{B}_c D$	$\bar{B}^* J/\psi$	$\bar{B}\eta_c$			
0^+	8746.8	(61.5, 161.5)	(0.1, 0.2)	(49.3, 130.8)	(0.3, 0.8)			293.2
	8668.0	(36.8, 87.5)	(1.2, 3.6)	(45.2, 106.1)	(3.6, 10.8)			208.0
	8572.2	(2.4, 4.8)	(23.4, 65.3)	(4.4, 8.2)	(60.8, 161.9)			240.2
	8478.1	(0.1, 0.1)	(75.9, 190.2)	(1.3, 1.5)	(35.4, 80.0)			271.8

Twelve states: 8.48 GeV ~ 8.75GeV

Similar features are found in the $bc\bar{c}\bar{s}$、$bc\bar{b}\bar{n}$, and $bc\bar{b}\bar{s}$ states

Summary

1. $QQ\bar{q}\bar{q}$ states, $T_{cc}^{+}(3875)$: the lowest $I(J^P) = 0(1^+)cc\bar{u}\bar{d}$,

   Stable: the lowest $0(1^+)bb\bar{u}\bar{d}$.

2. $Qq\bar{q}\bar{q}$ states, $T_{c\bar{s}0}^{a}(2900)^{++/0}$: $1(0^+)cn\bar{s}\bar{n}$, $T_{cs0}^{*}(2870)$: $0(0^+)cs\bar{n}\bar{n}$.

   Stable: the lowest $0(1^+)cn\bar{s}\bar{n}$, the lowest $0(0^+)$ and $0(1^+)cs\bar{n}\bar{n}$,

   the lowest $0(0^+)$and $0(1^+)bn\bar{s}\bar{n}$, the lowest $0(0^+)$ and $0(1^+)bs\bar{n}\bar{n}$.

3. $QQ\bar{Q}\bar{q}$ states, no stable tetraquark state is found.

Thank you for your attention !