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Heavy Meson Profiles from First-Principle: Challenges, Advances, and Implementations

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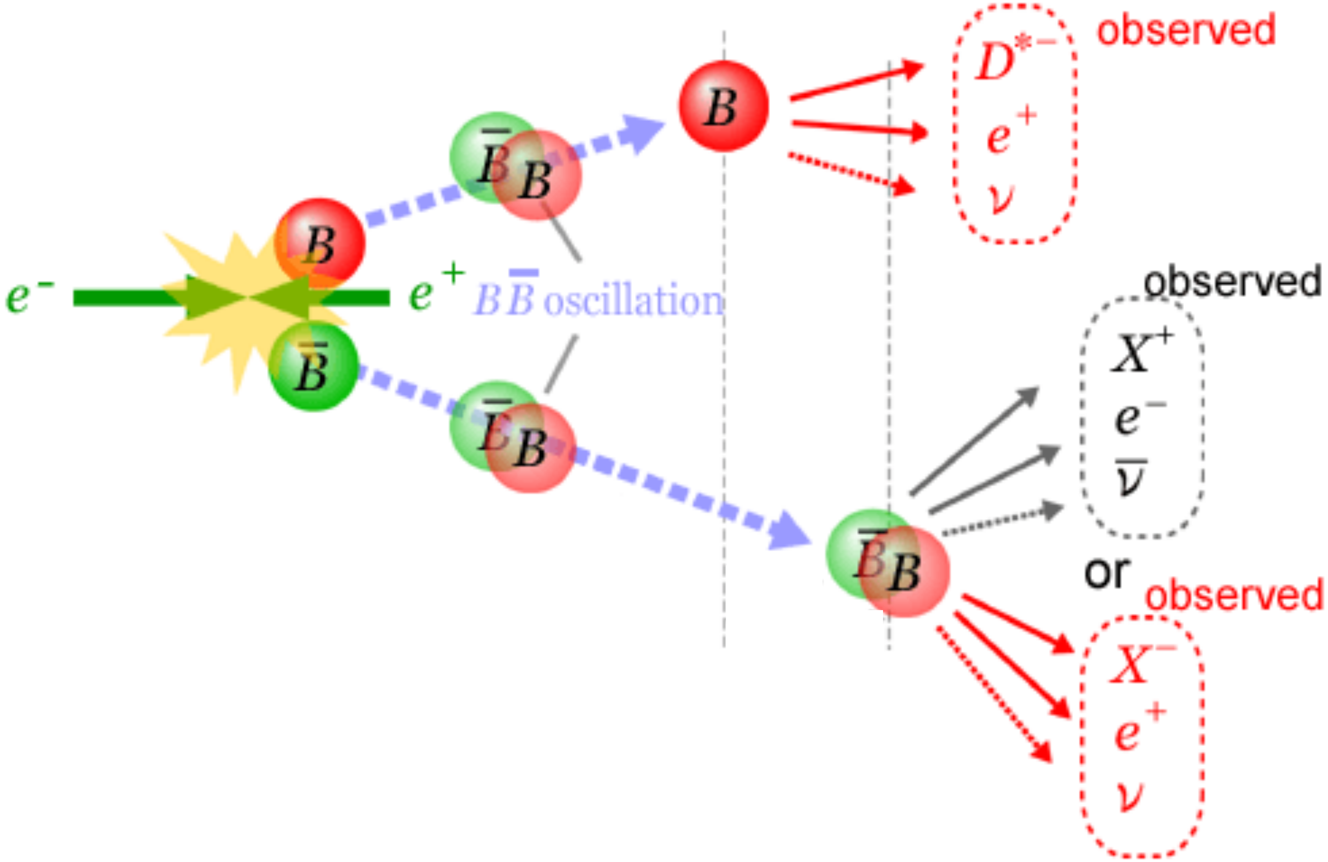
07/13/2025 @ 第八届强子谱和强子结构研讨会, 桂林

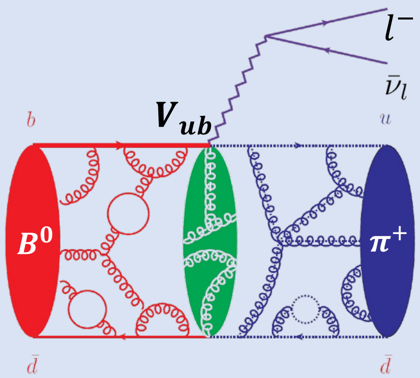
Outline

- Motivation
- Challenges in profiling the heavy mesons
- Sequential Effective Theory
- Heavy Meson Light-Cone Distribution Amplitudes
- Heavy Meson Shape Functions
- Summary and Outlook

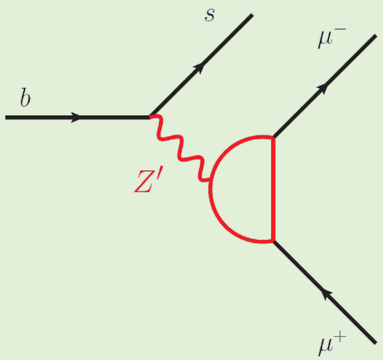
Motivation

- **Rich physics in heavy meson decays:**

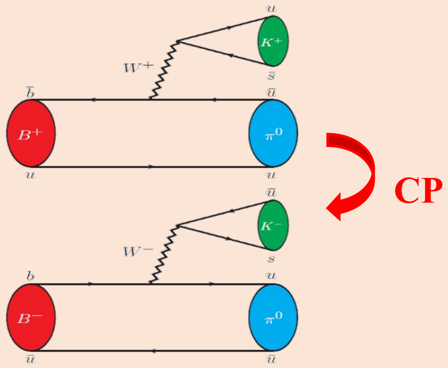




Precisely testing standard model



Indirect search for new physics



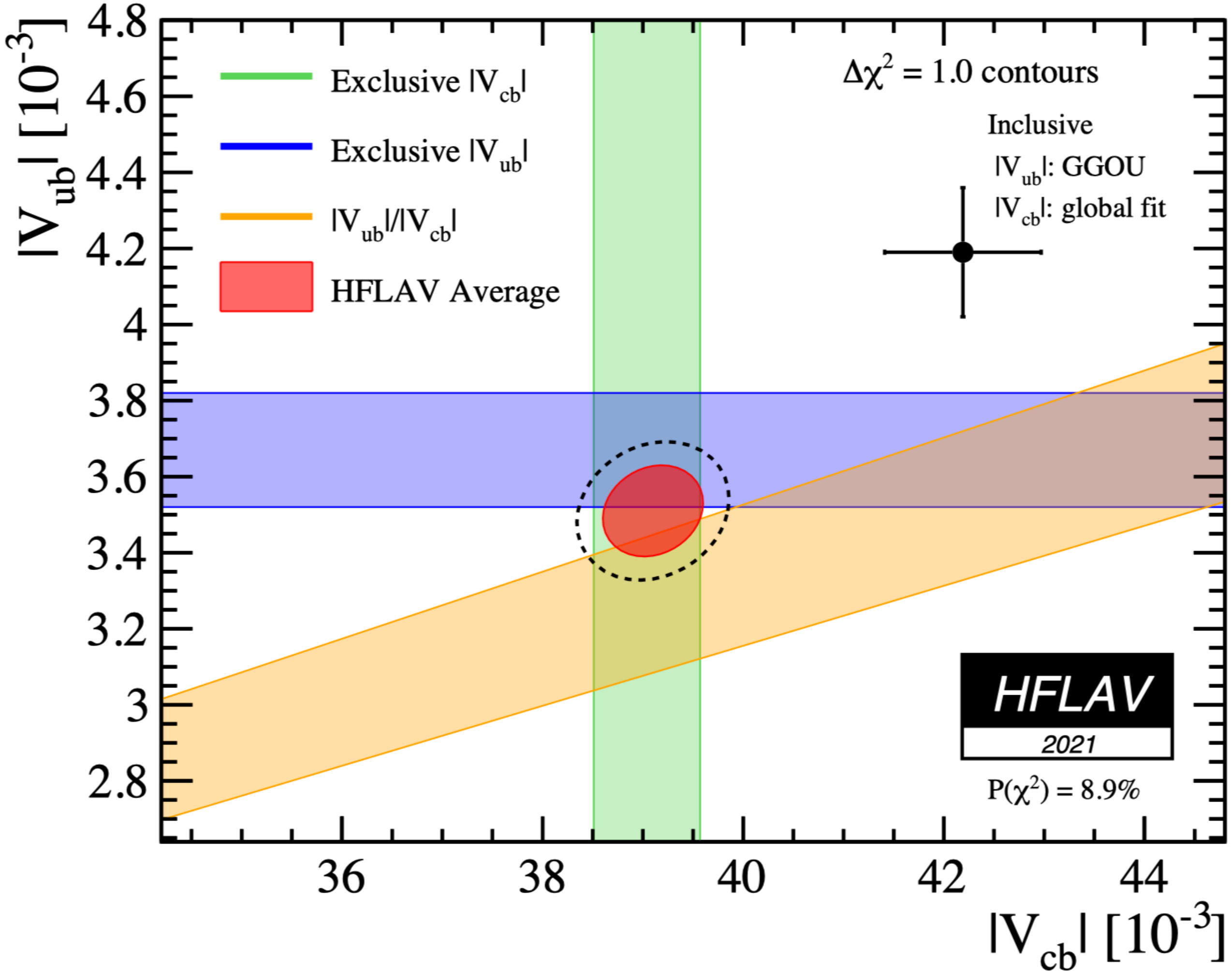
Study on CP violation

$|V_{xb}|$ Puzzle

	$\|V_{ub}\|$ $(\cdot\ 10^{-3})$	$\|V_{cb}\|$ $(\cdot\ 10^{-3})$
Exclusive	3.51 ± 0.12	39.10 ± 0.50
Inclusive	4.19 ± 0.17	42.19 ± 0.78

- **Inclusive / exclusive discrepancies for $|V_{ub}|$ and $|V_{cb}|$**
- **Current tensions stand at $\approx$ 3.3σ**

HFLAV, PRD 107, 052008 (2023)



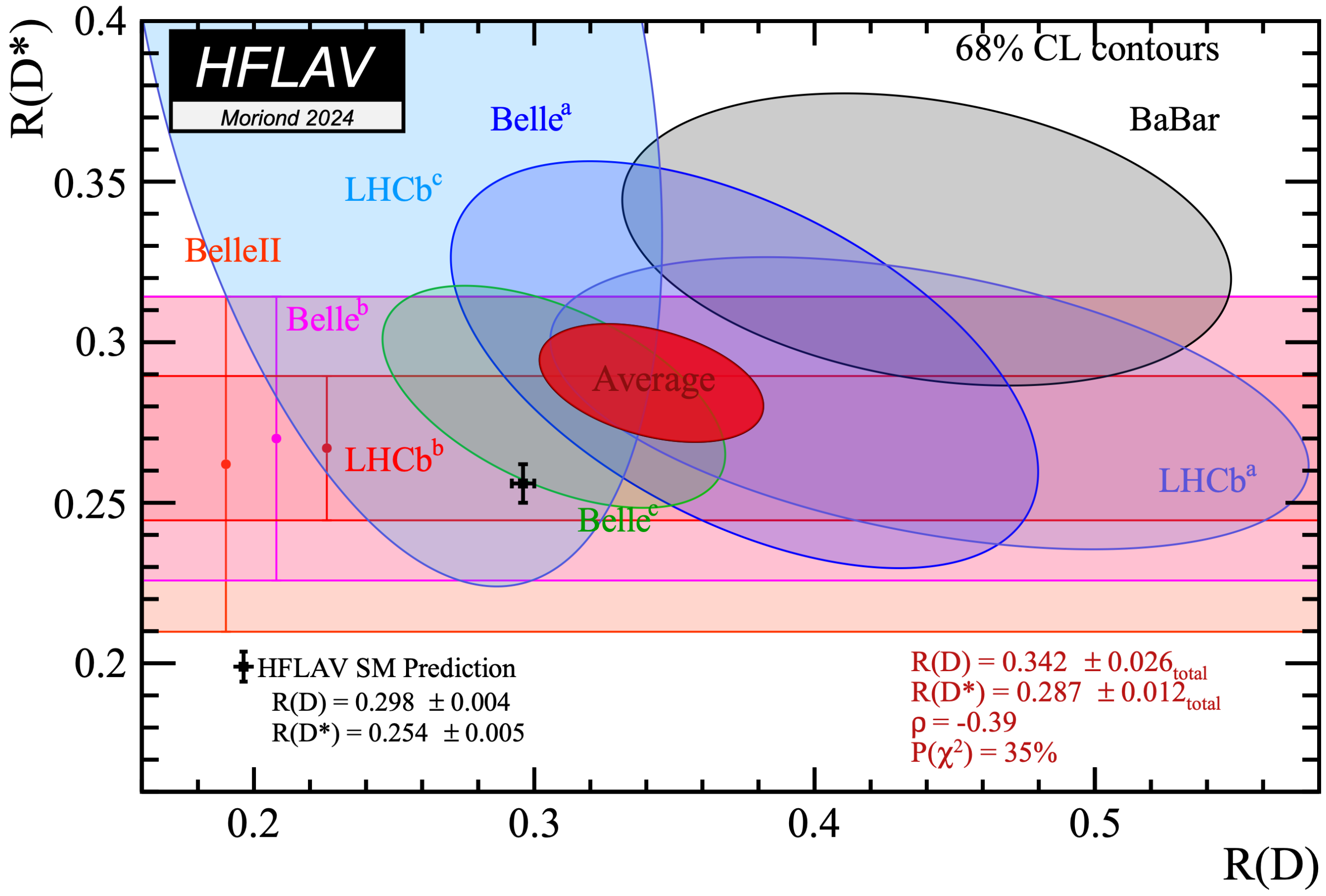
$\mathcal{R}(D)$ and $\mathcal{R}(D^*)$

$$\mathcal{R}(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu_\ell)}$$

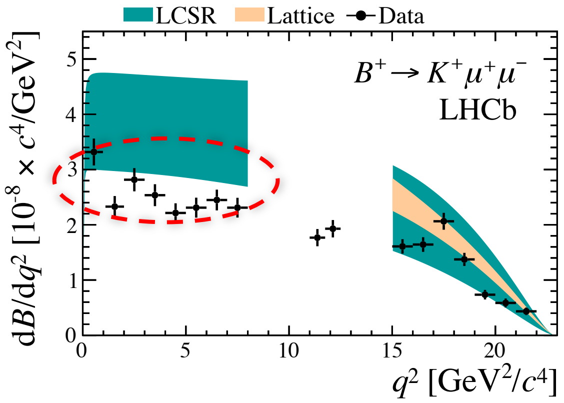
	R(D)	R(D*)
Exp	0.342 ± 0.026	0.287 ± 0.012
SM	0.298 ± 0.004	0.254 ± 0.005

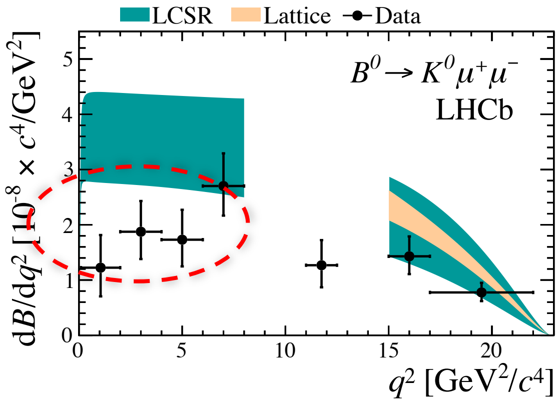
- **Current combined tensions at ≈ 3.31σ**

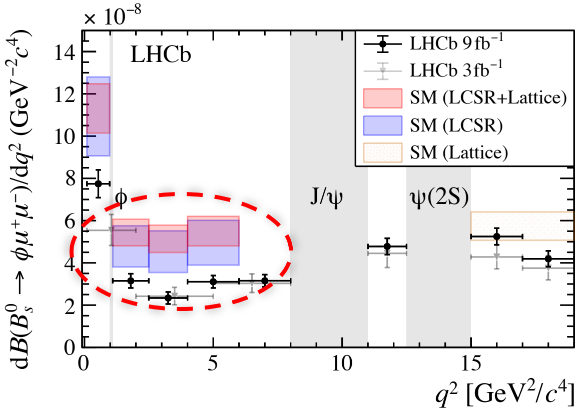
HFLAV, preliminary average for Moriond 2024

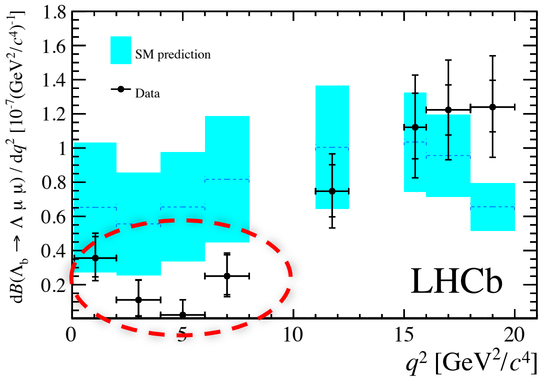


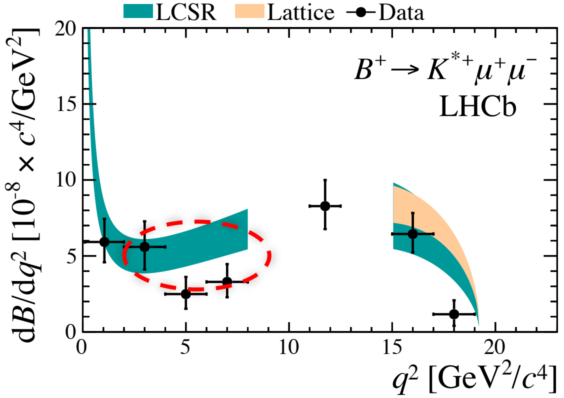
FCNC processes: $b \to s\ell\ell$ rare decays

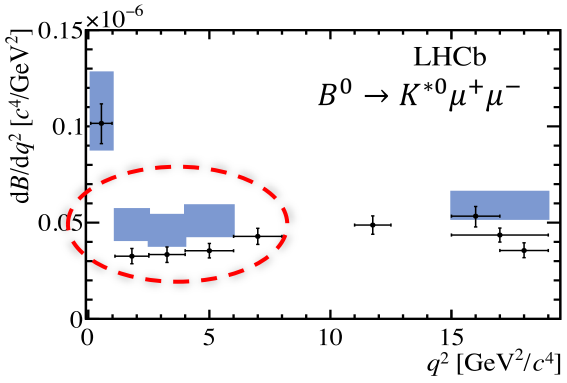


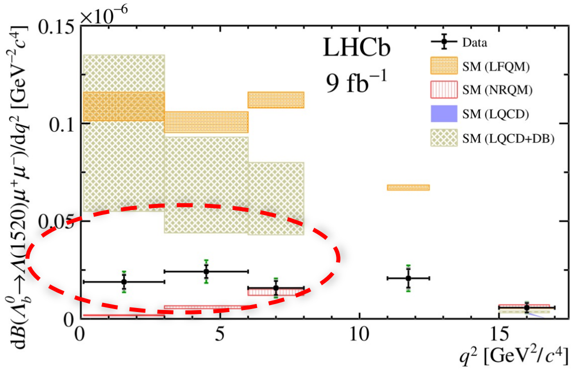






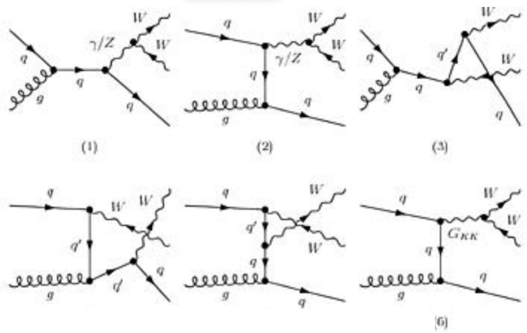
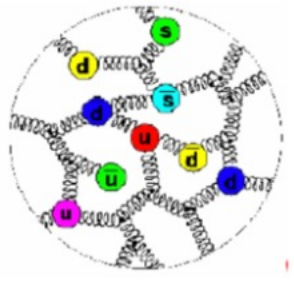






- **Significant deviations between experiment data and theoretical predictions**
- **Ubiquitous:** $B \to K\mu\mu$, $B \to K^*\mu\mu$, $B_s \to \phi\mu\mu$, $\Lambda_b \to \Lambda\mu\mu$

Factorization Theorem

Perturbative	Nonperturbative
High energy Small distance; Asymptotic freedom	Low energy Large distance; Confinement
 << 0.1 fm	 > 1 fm

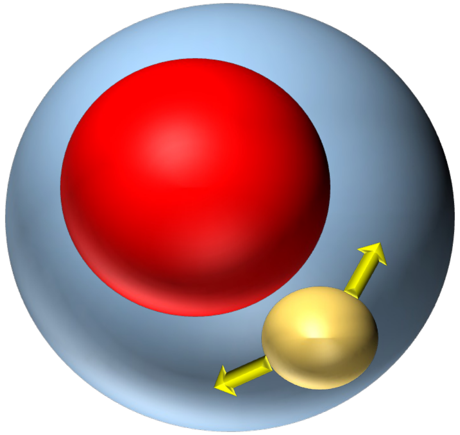
- Semileptonic decays $B \to M\ell\bar{\nu}$:
- FCNC processes $B^0 \to K^*\ell\ell$:
- Nonleptonic decays $B \to \pi\pi, \pi K$:

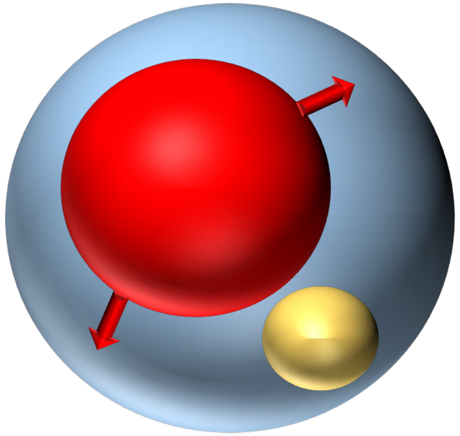
Hard kernel ⊗ **Form factor** ⊗ Light-cone distribution amplitudes (LCDAs)

- Inclusive decays $\bar{B} \to X_s\gamma, X_u\ell\bar{\nu}$: Hard function ⊗ Jet function ⊗ Shape function

Heavy Meson Profiles: Properties of heavy mesons as non-point particles

- A heavy flavor meson consists of a pair of heavy and light quarks.





LCDAs describe the momentum distribution amplitude of the light quark.

Shape function characterizes the momentum distribution function of the heavy quark.

$$\varphi^+(\omega,\mu) = \frac{1}{i\,\tilde{f}_{H_Q} m_{H_Q} n_+\cdot v}\int\frac{dt}{2\pi}e^{-i\omega t n_+\cdot v} \times\langle 0\,|\,\bar{q}(tn_+)\not{n}_+\gamma_5\,W_c(tn_+,0)h_v(0)\,|\,H_Q(v)\rangle$$

$$S(\omega,\mu) = \frac{1}{2m_{H_Q}}\int\frac{dt}{2\pi}e^{-i\omega t n_+\cdot v} \times\langle H_Q(v)|\,\bar{h}_v(tn_+,0)W_c(tn_+,0)h_v(0)\,|H_Q(v)\rangle$$

- **They provide the most essential information about the profile of heavy mesons.**

What we know?

- **Uncertainties dominate** the errors in theoretical prediction.

  - *e.g.*: $B \to \pi,\ K^*$ form factors from LCSRs:

Gao, Lu, Shen, Wang, Wei, PRD 101 (2020) 074035

Cui, Huang, Shen, Wang, JHEP 03 (2023) 140

$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

$$f^+_{B\to\pi}(0) = 0.122 \times \Big[1 \pm 0.07\Big|_{S_0^\pi} \pm 0.11\Big|_{\Lambda_q} \pm 0.02\Big|_{\lambda_E^2/\lambda_H^2}{}^{+0.05}_{-0.06}\Big|_{M^2} \pm 0.05\Big|_{2\lambda_E^2+\lambda_H^2}$$
$$\qquad {}^{+0.06}_{-0.10}\Big|_{\mu_h} \pm 0.04\Big|_{\mu}{}^{+1.36}_{-0.56}\Big|_{\lambda_B}{}^{+0.25}_{-0.43}\Big|_{\sigma_1,\sigma_2}\Big].$$

One of the most important non-perturbative input parameters, and currently the least understood.

What we know?

- **Limited understanding** of the nonperturbative heavy meson LCDAs and SFs:

  - **Only models** for heavy meson LCDAs:

    Grozin, Neubert, 1997; Braun, Ivanov, Korchemsky, 2004; Beneke, Braun, Ji, Wei, 2018; ……

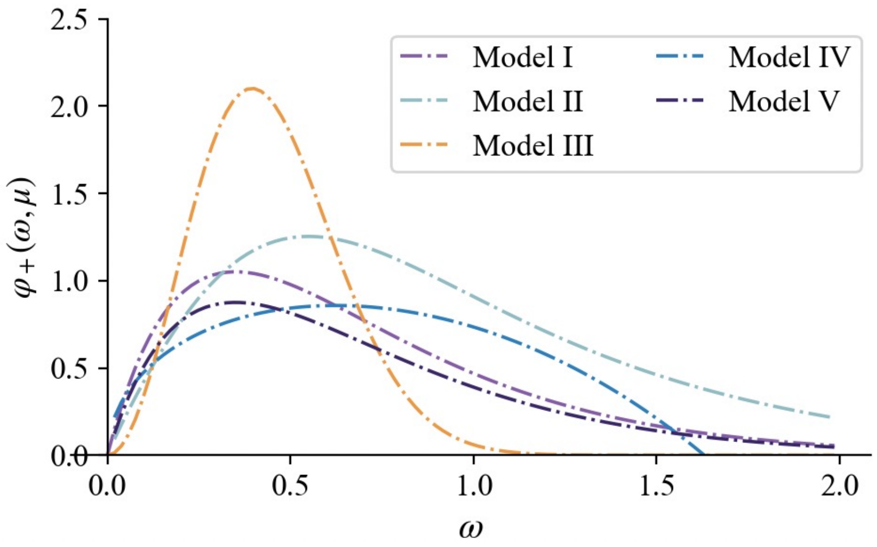
  - **Only models** for heavy meson SFs:

    Korchemsky, Sterman, 1994; Bauer, Luke, Mannel, 2001; Neubert, 2005; Lee, Ligeti, Stewart, Tackmann, 2006; ……

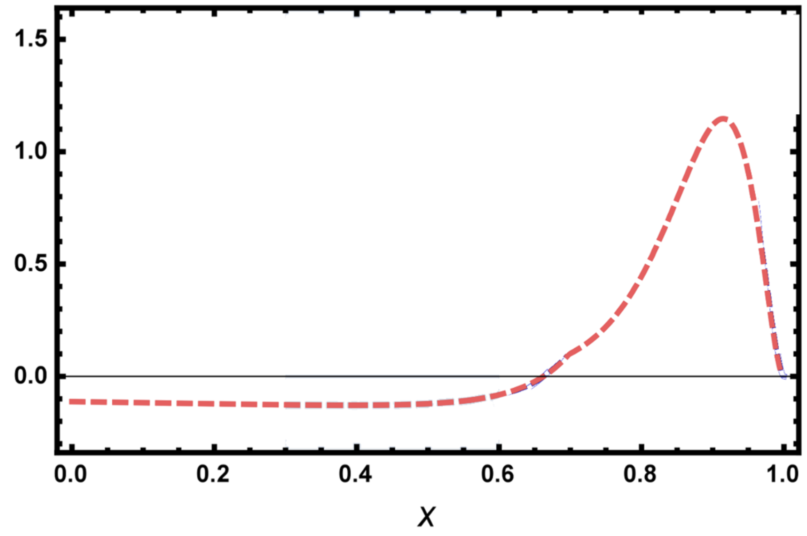
  - Relations between LCDAs and SFs?

    Yaouanc, Oliver, Raynal, 2008

Models for heavy meson LCDA



Model for heavy meson SF



Challenges in profiling the heavy mesons from first-principles

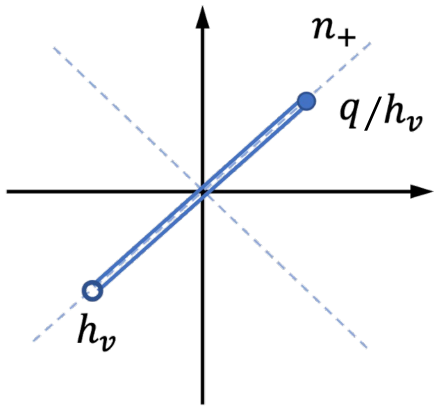
- Challenge 1: **light-like correlators**

  1. **OPE**: Expansion into **local** operators matrix elements

     QCD sum rule, Mellin moments from lattice QCD, ⋯

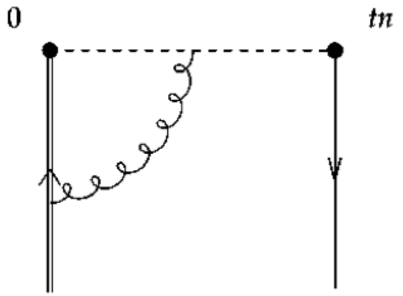
  2. **LaMET**: From equal-time correlation functions to light-cone variables

     Full distributions from lattice QCD



- Challenge 2: **Cusp divergence**

Braun, Ivanov, Korchemsky, PRD 69 (2004) 034014



$$O_v^{\rm ren}(t,\mu) = \frac{4}{\hat{\epsilon}}\underline{\ln(it\mu)}\,O_v^{\rm bare}(t) + \ldots \xrightarrow{t\to 0} \log 0!$$

NO Local Limit!

OPE Breakdown!

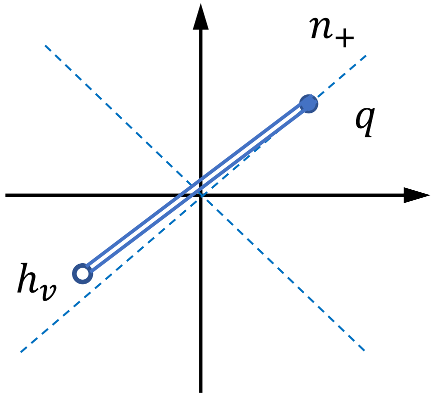
Challenges in profiling the heavy mesons from first-principles

- **OPE Breakdown** ⇒ **Can only rely on LaMET**

An intuitive approach: adopt **off light-cone Wilson line** to avoid cusp divergence

$$\langle H_Q(P_{H_Q})\,|\,\bar{q}(z)\not{n}_z\gamma_5\, W_c(z,0)h_v(0)\,|\,0\rangle \qquad \langle H_Q(v)\,|\,\bar{h}_v(z,0)W_c(z,0)h_v(0)\,|\,H_Q(v)\rangle$$

Wang, Wang, Xu, Zhao, PRD 102, 011502 (2020);
Xu, Zhang, PRD 106, 114019 (2022);
Hu, Xu, Zhao, EPJC 84, 502 (2024); ……



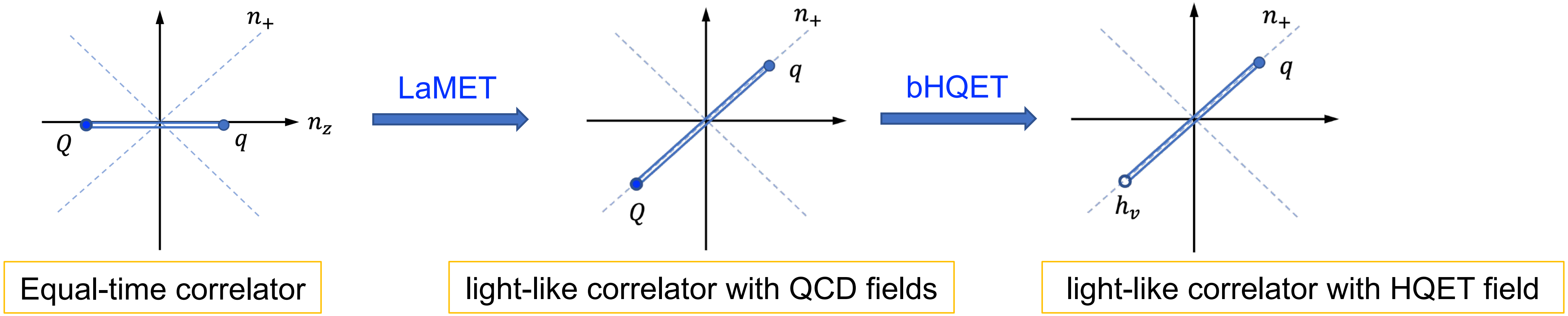
Need to realize the **boosted HQET fields on lattice**.

"We have tried, but failed."

Sequential Effective Theory

We propose a two-step factorization to combine LaMET and bHQET:

Han, QAZ, *et.al.*, PRD111 (2025) L111503



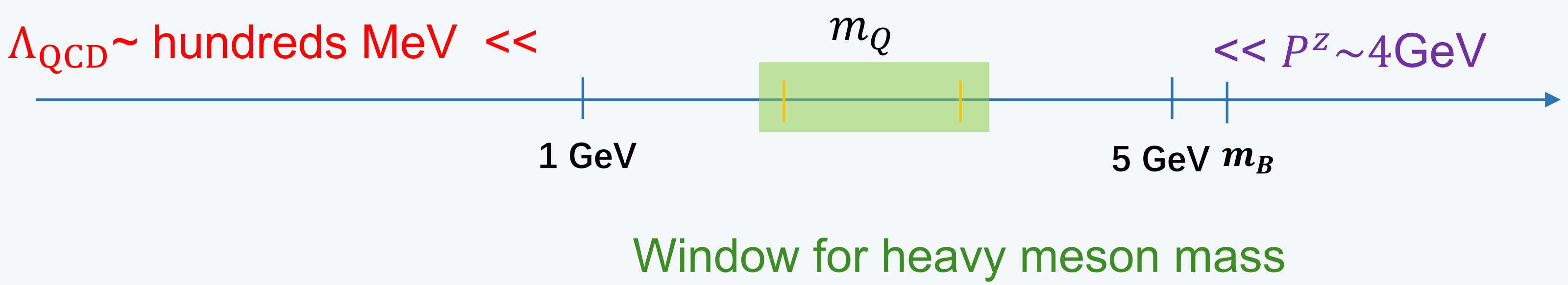
- **3 scales** in the equal-time correlator: $\boldsymbol{\Lambda_{QCD}, m_Q, P^z}$

- Effective theories:
  - LaMET: $\Lambda_{\rm QCD}, m_{\rm Q} \ll P^z$ and integrate out P^z
  - bHQET: $\Lambda_{\rm QCD} \ll m_{\rm Q}$ and integrate out $m_{\rm Q}$

$\Rightarrow$ Introduce a hierarchy $\boldsymbol{\Lambda_{QCD} \ll m_Q \ll P^z}$

Sequential Effective Theory on the Lattice

- **Lattice feasibility** at this stage:



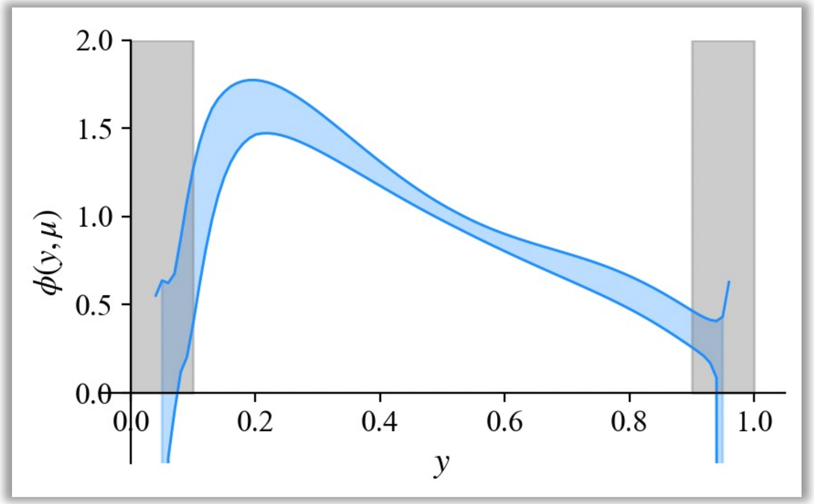
Only valid for *D* mesons, rather than *B* mesons?

✓ Heavy quark flavor symmetry ensures that the HQET measurement is independent of heavy quark mass;

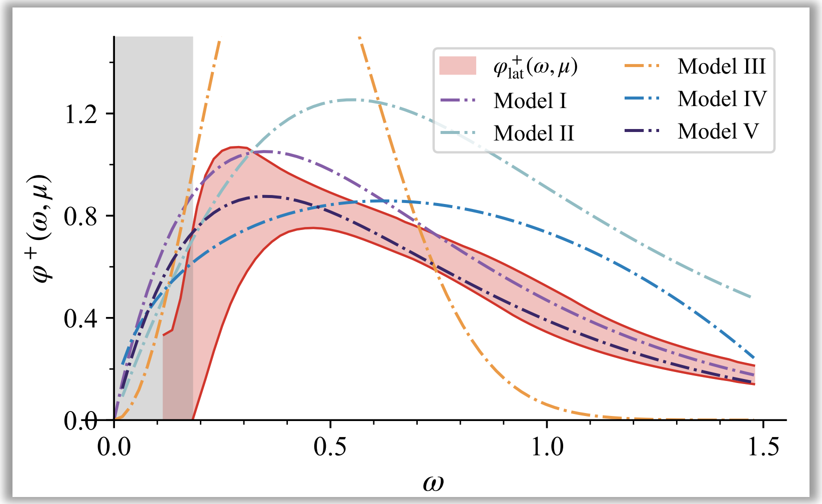
✓ m_Q (m_c or m_b) only contributes to the power corrections.

First Lattice QCD Calculations of Heavy Meson LCDAs

D meson QCD LCDA



HQET LCDA φ^+



Han, QAZ, *et.al.* [LPC Collaboration], PRD111 (2025) 034503

- **Exploratory** calculations on the CLQCD ensembles, with single lattice spacing and non-physical hadron masses;
- The QCD LCDA of *D* meson, as well as the HQET LCDA for heavy meson, have been obtained **as expected**;
- Based on this results, we can obtain:

  More accurate extraction of the inverse moments;

  More precise prediction for the $B \to K^*$ form factors.

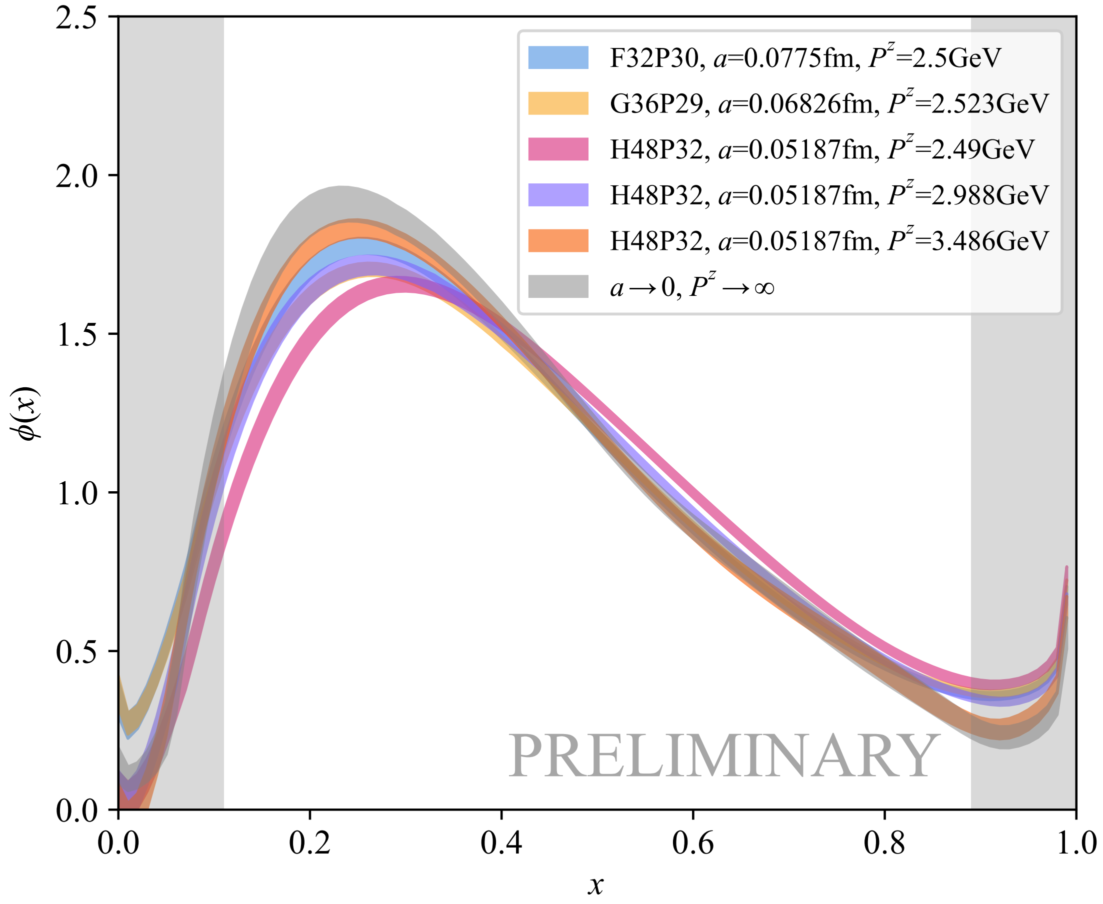
This result inspires us to conduct further exploration...

Towards precision calculation

QAZ, *et.al.*, 25xx.xxxxx

	Exploratory	Now
Action	Tadpole improved Wilson clover fermion action	
Improvement	No	Hyp smear for gauge link; Multi source enhancement
NPR	Simplified hybrid scheme	Hybrid scheme based on self renormalization
Continuum extrapolation	No, only $a = 0.0519\text{fm}$	Yes, with $a = (0.0775, 0.0683, 0.0519)\text{fm}$
Infinity momentum extrapolation	No, only $P^z \sim 3\ \text{GeV}$	Yes, with $P^z = (2.5, 3, 3.5)\text{GeV}$
Statistics	~5k	~100k

Predictions for *D* meson QCD LCDA and Gegenbauer moments



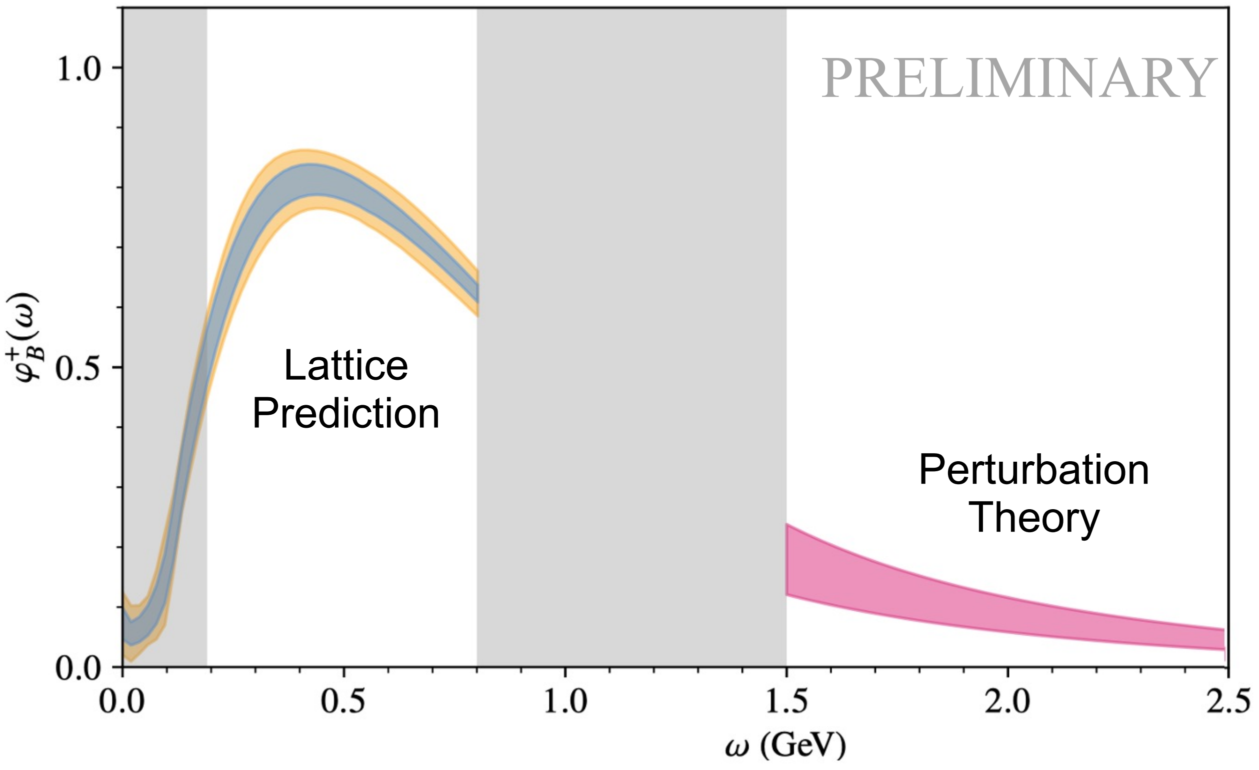
- QCD LCDA of *D* meson @ $\mu = m_D$.
- Gegenbauer moments of *D* meson:

(Need more systematic analysis)

n	a_1	a_2	a_3	a_4	a_5	a_6
2	-0.397(18)	0.118(10)				
4	-0.412(20)	0.134(16)	-0.016(11)	0.005(8)		
6	-0.391(22)	0.111(18)	0.026(20)	-0.019(15)	0.024(9)	-0.004(7)
8	-0.376(24)	0.109(19)	0.041(22)	-0.034(18)	0.041(11)	-0.012(9)

Predictions for heavy meson HQET LCDA and Inverse moments

- Predictions for the nonperturbtive HQET LCDA @ leading power.



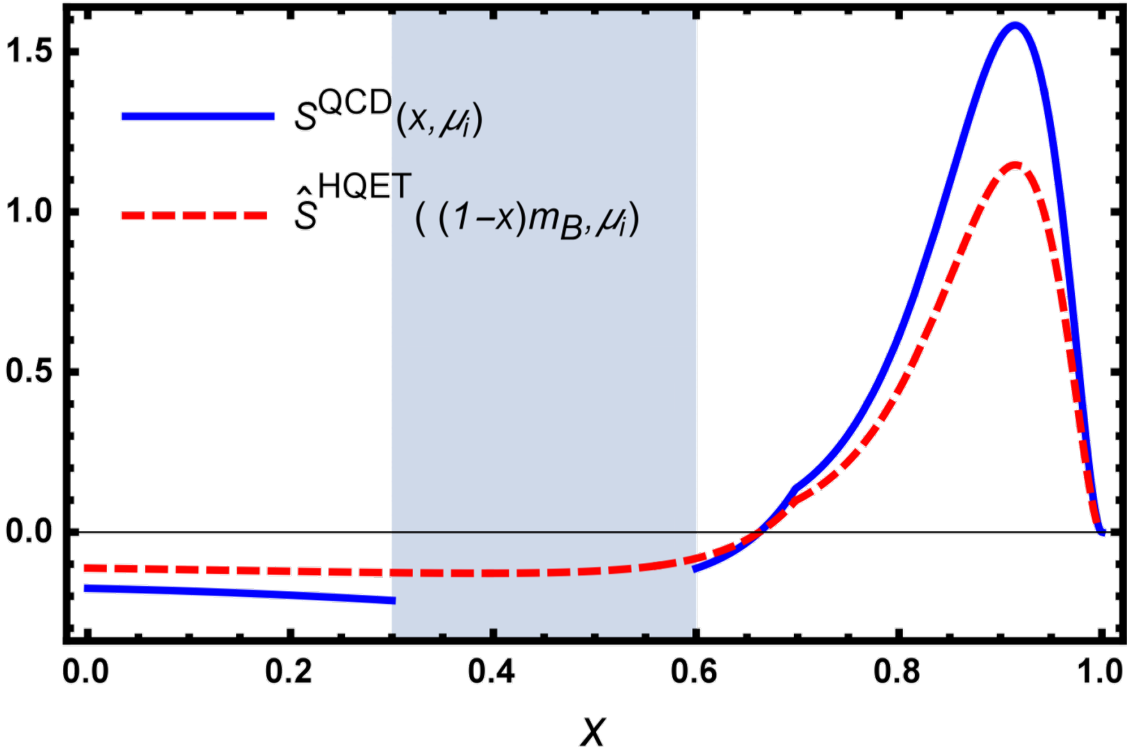
- Combining model-independent parameterization, predictions for the inverse moment @ μ = 1GeV.

	λ_B (GeV)	$\sigma_B^{(1)}$
Our work	0.421 (21)	1.470 (24)
PRD98 (2018)	> 0.24	
JHEP (2020)	0.383 (153)	
PRD72 (2005)	0.48 (11)	1.6 (2)
PRD69 (2004)	0.46 (11)	1.4 (4)
PRD55 (1997)	0.35 (15)	
PRD101 (2020)	$0.343^{+0.064}_{-0.079}$	
PLB848 (2024)	0.338 (68)	

Lattice QCD Calculations for Heavy Meson Shape Functions

Wang, Xu, **QAZ**, Zhao, arXiv:2504.18018

Illustration of the matching based on modelized QCD SF



- The QCD shape function (SF) is refers to the PDF of heavy meson.
- The theoretical framework for factorization has been established.
- Lattice QCD calculations are currently underway…

Summary and Outlook

- Our work provides an opportunity to explore the internal structures of heavy mesons using first-principles lattice QCD calculations for the first time, which is typically unattainable in traditional approaches due to the presence of cusp divergence.

- For Heavy meson LCDAs:
  - ✓ Factorization has been established;
  - ✓ Preliminary lattice QCD results are now available;
  - ✓ We are advancing toward precision calculation.

- For Heavy meson shape function:
  - ✓ Factorization has been proposed;
  - ✓ Expecting further validation in lattice QCD calculations.

The journey of discovery has just set sail, with uncharted horizons awaiting exploration.

Thanks for your attention!