Light cone structures, from light meson to light baryon on Lattice QCD

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第八届强子谱和强子结构研讨会

2025 07/13 @广西师范大学

OUT LINE

Motivation About Meson & Baryon

- **Meson — ubiquitous messengers**

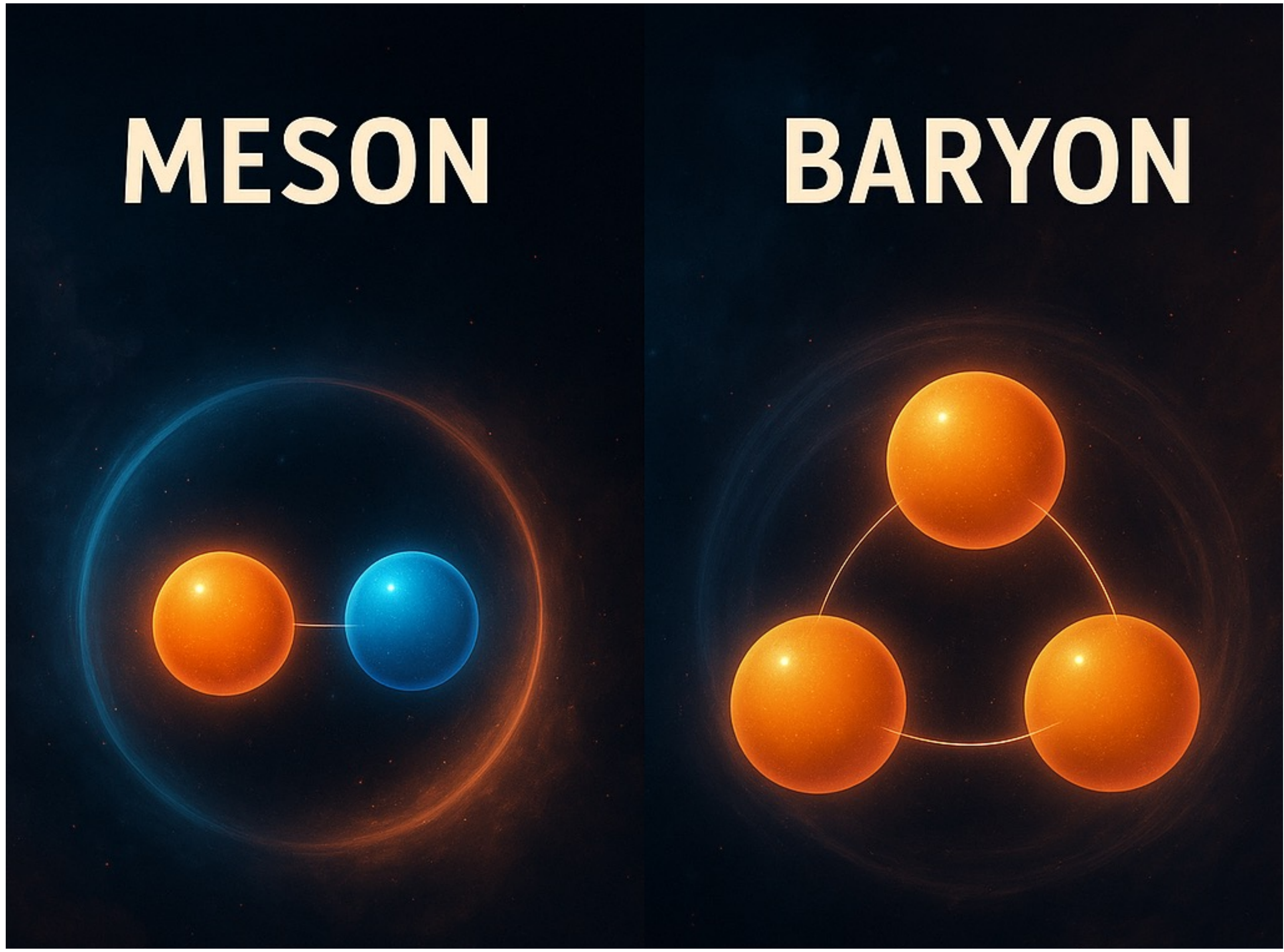
  Light Meson $SU(3)_{flavor}$ $8 \otimes 1$

  Heavy Meson $SU(4)_{flavor}$ …

- **Baryon — cornerstone particles**

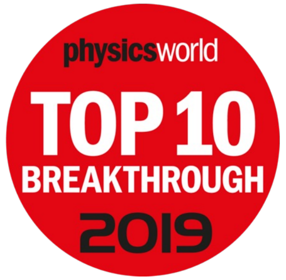
  Octet baryons

  Decuplet baryons



Meson and Baryon in Flavor Physics

- 1956, Parity violation in weak interaction
- 1964, Observation of CP violation in **Kaon meson**
- 1973, Kobayashi-Maskawa mechanism
- 2004, Observation of direct CPV in **B meson**
- 2019, Observation of direct CPV in **D meson**
- 2025, Observation of direct CPV in **Baryon $\Lambda_b^0 \to p$**

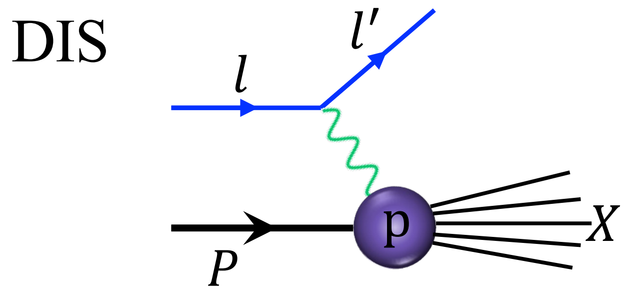


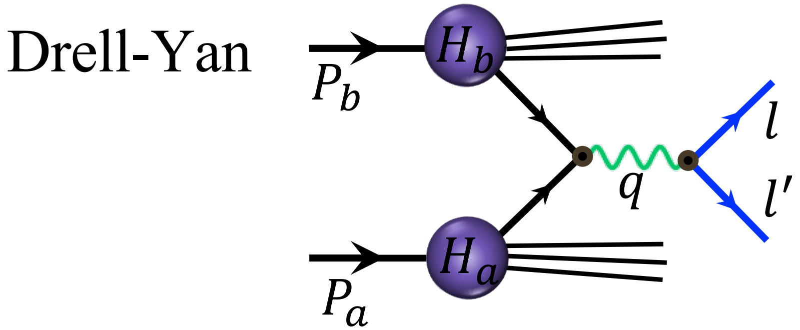
LHCb, arXiv: 2503.16954; Theoretical: *J.J.Han, et.al. 2409.02821*

Motivation About LCDA

- One try to probe internal structure of nucleons → **Inclusive beam-target collision**

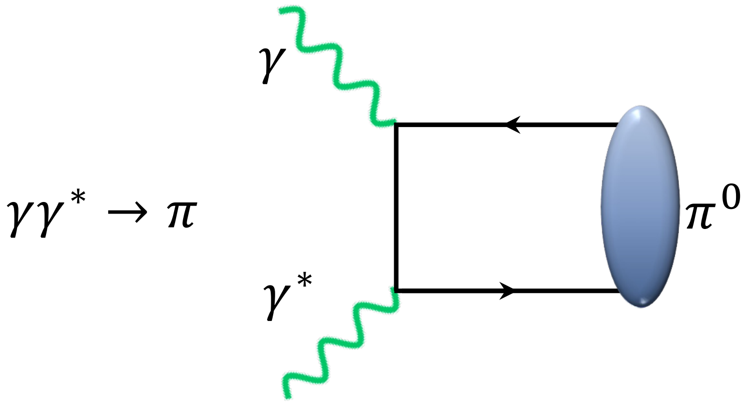
Defined PDFs

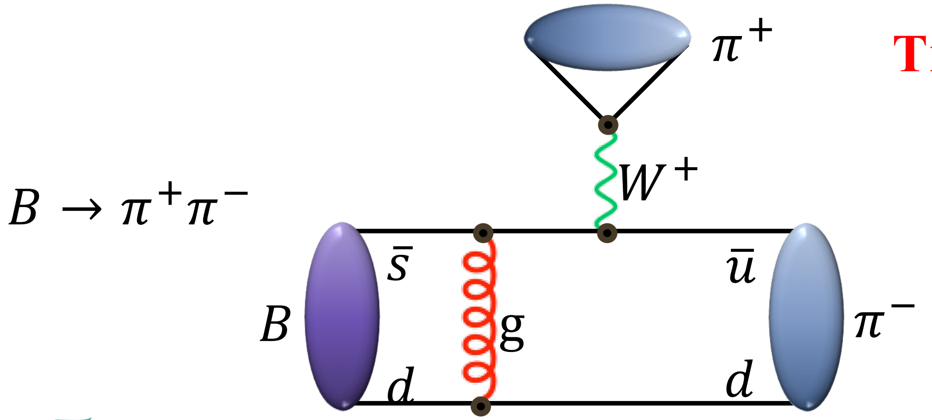




- One try to obtain richer QCD dynamical information → **Exclusive scattering**

Defined LCDAs





Try Baryon $\Lambda_b^0 \to p$

Motivation About LCDA

Light Meson LCDAs: (1977 - now)

- Asymptotic form

  Chernyak, Zhitnitsky, 1977; Lepage, Brodsky, 1979; ……
- QCD Sum rules

  Chernyak, Zhitnitsky, 1982; Braun, Filyanov, 1989; ……
- Dyson-Schwinger Equation

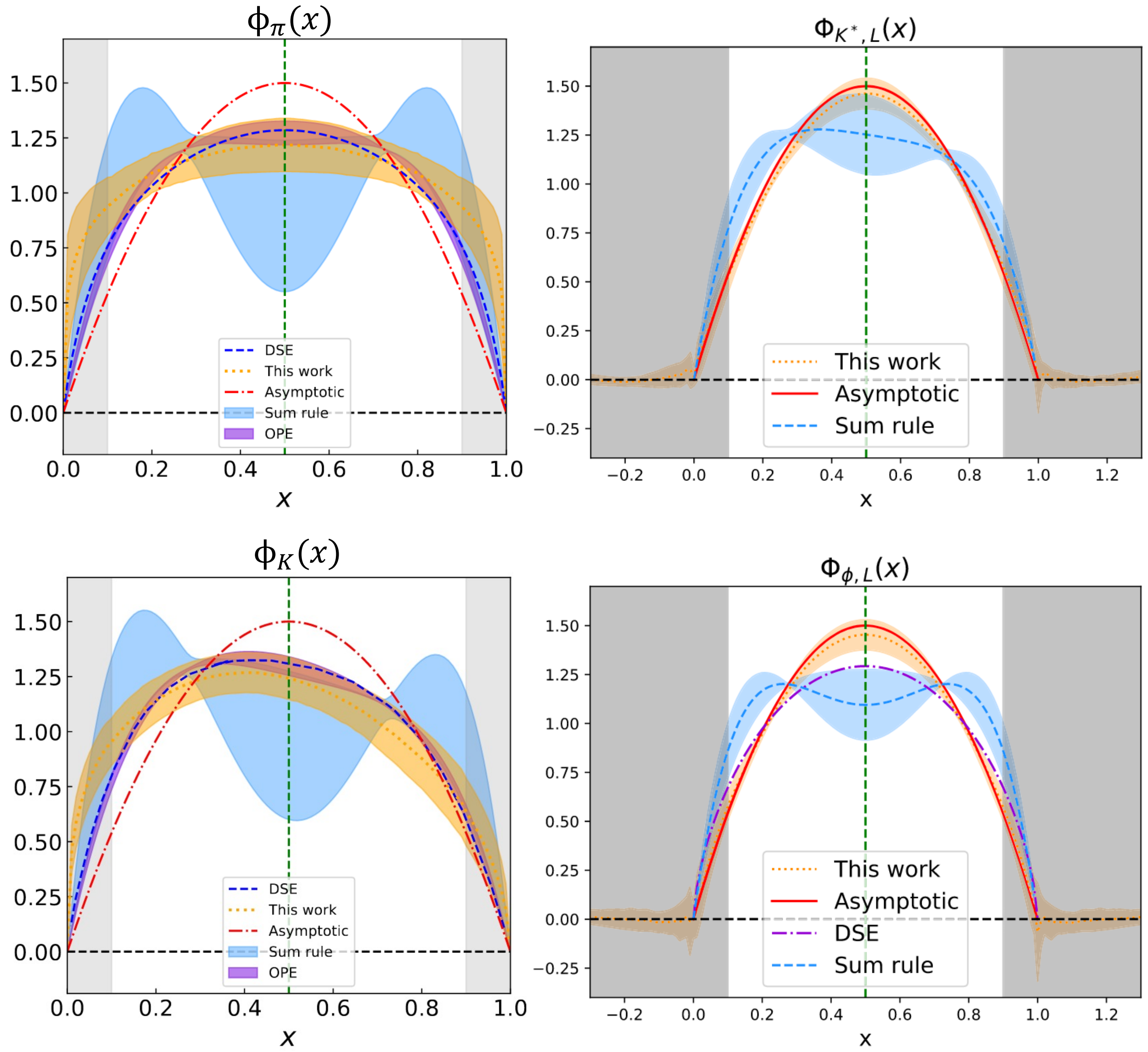
  Chang, Cloet, et.al, 2013; Gao, Chang, et.al, 2014; ……
- Global Fits

  Cheng, et.al, 2020; Hua, Li, Lu, Wang, Xing, 2021; ……
- Models

  Arriola, Broniowski, 2002; Zhong, Zhu, et.al, 2021; ……
- Lattice QCD with OPE

  Braun, et al., 2016; RQCD collaboration, 2019, 2020; ……
- Lattice QCD with LaMET

  LP3, 2019; LPC, 2021, 2022; ……



Motivation About LCDA

Light Baryon LCDAs: (1983 - now)

- Asymptotic form

  Chernyak, Zhitnitsky, 1983 ; ……
- QCD Sum rules

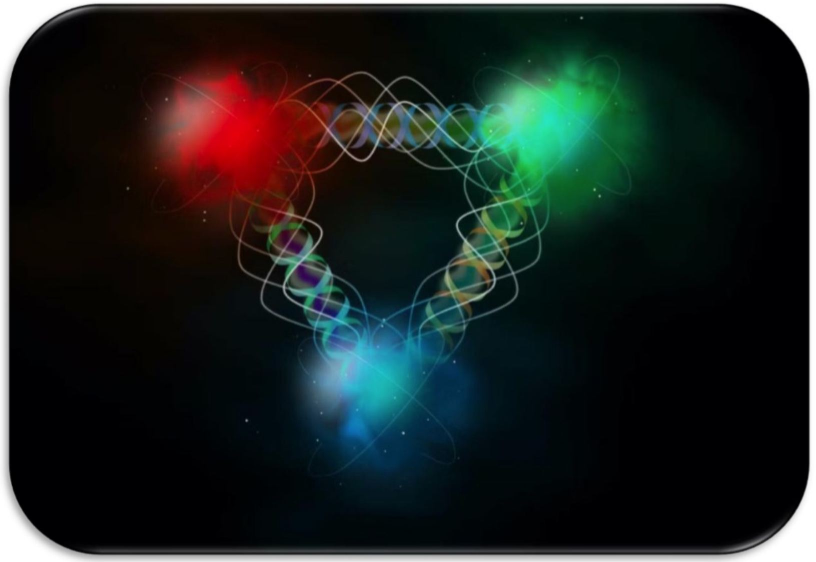
  King, Sachrajda,1987; Stefanis, Bergmann, 1993; Braun, et.al, 2000,2006; ……
- Models parametrization

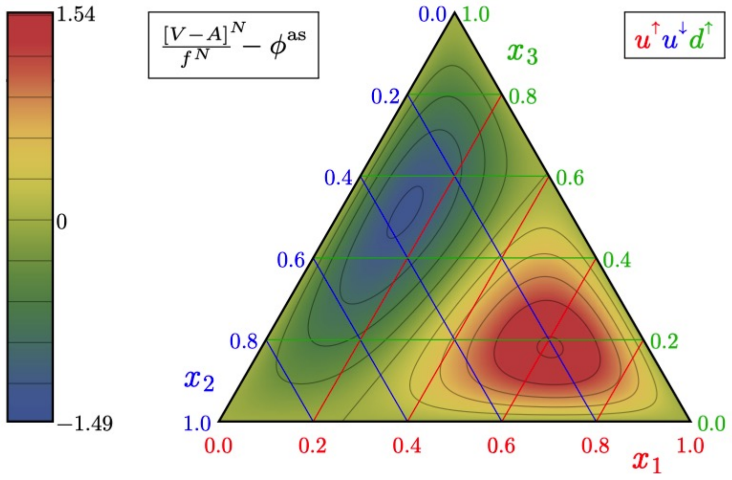
  Bell, Feldmann, Wang, Matthew 2013; ……
- Lattice QCD with OPE

  QCDSF, 2008, 2009; RQCD, 2016, 2019(2025) ……
- Lattice QCD with LaMET

  LPC, 2025; ……

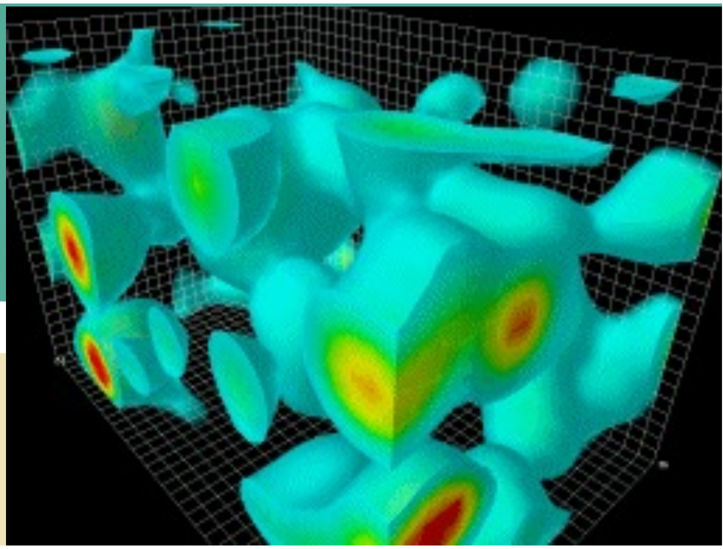
  PRD 111, 034510 (2025)





Lattice QCD formulate a Feynman path integral on discrete 4D Euclidean grid.

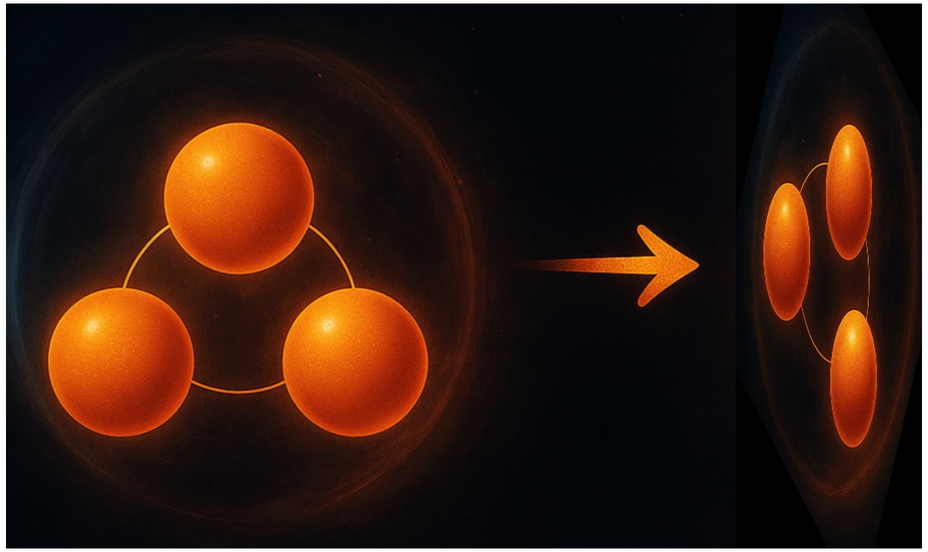
Numerical simulations based on a QCD Lagrangian with discrete from:



$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi - \frac{1}{4}F^{a}_{\mu\nu}F^{a,\mu\nu} \xrightarrow{t = i\tau} \mathcal{L}^{E} = \bar{\psi}(i\gamma^{\mu}D_{\mu} + m)\psi + \frac{1}{4}F^{a}_{\mu\nu}F^{a,\mu\nu}$$

$$S_E^{\mathrm{latt}} = -\sum_{\square}\frac{6}{g^2}\mathrm{Retr}_N\left(U_{\square,\mu\nu}\right) - \sum_{q}\bar{q}\left(D_{\mu}^{\mathrm{lat}}\gamma_{\mu} + am_q\right)q$$

- Lost of real-time correlation !
- Longitudinal correlations compress to a point



Motivation Moments & LaMET

Operator Product Expansion (OPE): the local moments are consistent between light-cone coordinate or Euclidean space

Limited for only few moments:

- First two moments for light mesons
- First moments for light baryons

Inverse problem from moments to LCDA/PDF

Light meson:

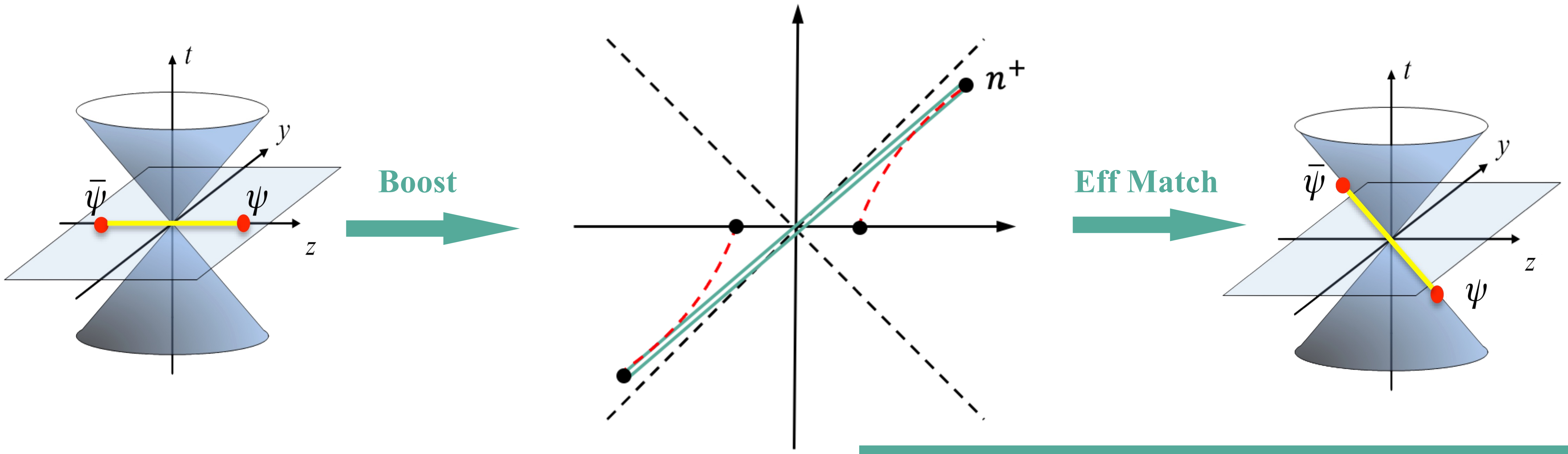
Braun, et al., 2016; RQCD collaboration, 2019, 2020; ……

Light baryon:

QCDSF, 2008, 2009; RQCD, 2016, 2019(2025) ……

Motivation Moments & LaMET

Large momentum effective theory (LaMET): the light-cone non-local operator correlated to Euclidean non-local operator with a large momentum boost



LaMET factorization:

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C(x, y, P^z, \mu)\, q(y, \mu) + \mathcal{O}\left(\frac{\Lambda^2, M^2}{(P^z)^2}\right)$$

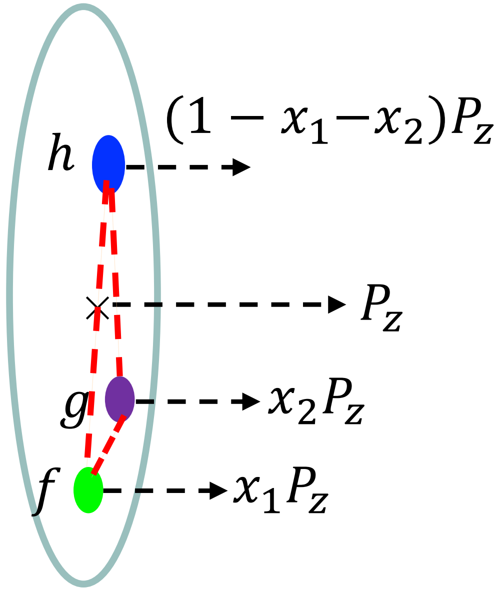
- Light meson: LP3, 2019; LPC, 2021, 2022; ANL,2024; ……
- Heavy meson: LPC, 2024,2025; ……
- Light baryon: LPC, 2025; ……

Baryon LCDA on Lattice Quasi

- **Definition of baryon LCDA:**

$$M_L(\xi_1,\xi_2;P) = \langle 0|\epsilon^{ijk} f_\alpha^{i'}(\xi_1 n) W_{i'i}(\xi_1,\xi_0) g_\beta^{j'}(\xi_2 n) W_{j'j}(\xi_2,\xi_0) h_\gamma^k(\xi_0 n)|B(P,\lambda)\rangle$$

$$\Phi(x_1,x_2,\mu) = \int \frac{\mathrm{d}P^+\xi_1}{2\pi}\frac{\mathrm{d}P^+\xi_2}{2\pi} \mathrm{e}^{\mathrm{i}x_1P^+\xi_1+\mathrm{i}x_2P^+\xi_2} \frac{M_L(\xi_1,\xi_2;P,\mu)}{M_L(0,0;P,\mu)}$$



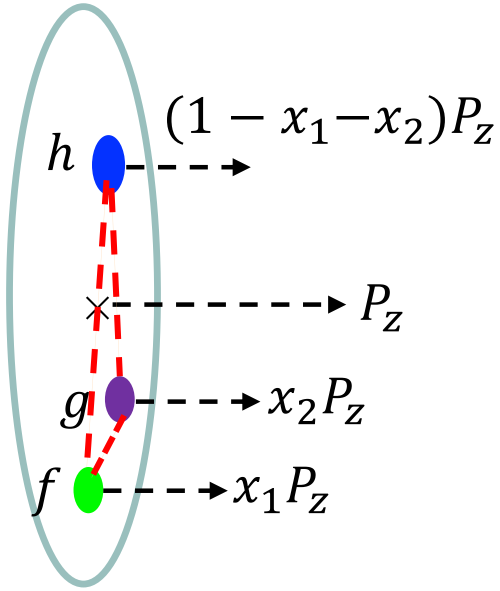
C.Han et.al. JHEP 07019 (2024);
V.L.C & I.R.Z NPB 24652(1984); G.R.Farrar et.al. NPB 311585(1989)

- **Leading twist octet baryon LCDA:**

$$\langle 0|f_\alpha(z_1 n) g_\beta(z_2 n) h_\gamma(z_3 n)|B(P_B,\lambda)\rangle$$
$$= \frac{1}{4} f_V \left[(\not{P}_B C)_{\alpha\beta}(\gamma_5 u_B)_\gamma V^B(z_i n\cdot P_B) + (\not{P}_B\gamma_5 C)_{\alpha\beta}(u_B)_\gamma A^B(z_i n\cdot P_B)\right]$$
$$+ \frac{1}{4} f_T (i\sigma_{\mu\nu}P_B^\nu C)_{\alpha\beta}(\gamma^\mu\gamma_5 u_B)_\gamma T^B(z_i n\cdot P_B),$$

Octet	n	p	Λ
fgh	d d u	u u d	u d s

Baryon LCDA on Lattice Quasi



- **Definition Baryon Quasi-DA on Euclidean lattice:**

$$M(z_1, z_2; P^z) = \langle 0| \epsilon^{ijk} f_\alpha^{i'}(z_1) W_{i'i}(z_1, 0) g_\beta^{j'}(z_2) W_{j'j}(z_2, 0) h_\gamma^k(0) |B(P^z, \lambda)\rangle$$

$$\tilde{\Phi}(x_1, x_2, P^z, \mu) = (P^z)^2 \int \frac{dz_1}{2\pi} \frac{dz_2}{2\pi} e^{-x_1 P^z z_1 - x_2 P^z z_2} \frac{M(z_1, z_2; P^z, \mu)}{M(0, 0; P^z, \mu)}$$

- **The Leading twist (V,A,T terms) for octet baryon:**

$$\left\langle 0 \left| f^T(z_1 n)(C\not{n}) g(z_2 n) h(z_3 n) \right| B \right\rangle = -f_V V^B(z_i n \cdot P_B) P_B^+ \gamma_5 u_B,$$

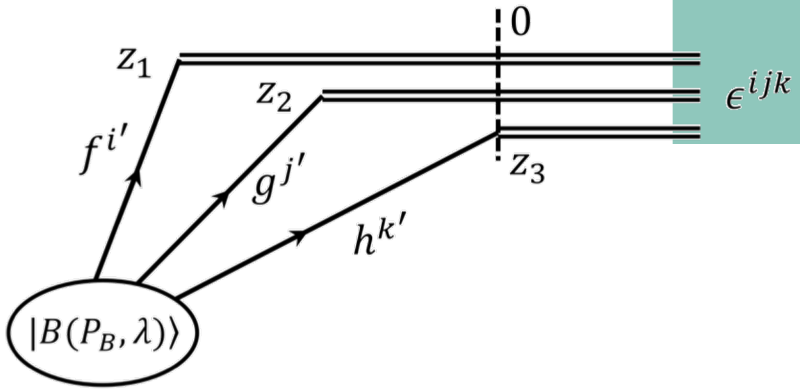
LPC, PRD 111, 034510 (2025) — Λ

$$\left\langle 0 \left| f^T(z_1 n)(C\gamma_5\not{n}) g(z_2 n) h(z_3 n) \right| B \right\rangle = f_V A^B(z_i n \cdot P_B) P_B^+ u_B,$$

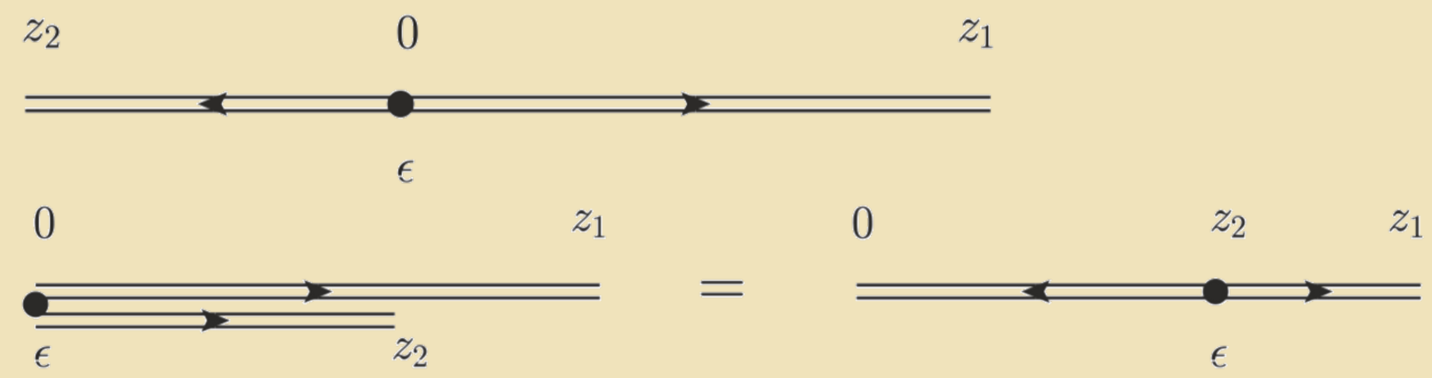
$$\left\langle 0 \left| f^T(z_1 n)(iC\sigma_{\mu\nu} n^\nu) g(z_2 n) \gamma^\mu h(z_3 n) \right| B \right\rangle = 2 f_T T^B(z_i n \cdot P_B) P_B^+ \gamma_5 u_B,$$

Baryon LCDA on Lattice Quasi

- **Nonlocal three-quark operators at light-like separations:**



$$H(z_1,z_2,z_3)_{\alpha\beta\gamma} = \epsilon^{ijk}\langle 0| f_\alpha^{i'}(z_1 n) W^{i'i}(z_1 n, z_0 n) \times g_\beta^{j'}(z_2 n) W^{j'j}(z_2 n, z_0 n) \times h_\gamma^{k'}(z_3 n) W^{k'k}(z_3 n, z_0 n) \,|B(P_B,\lambda)\rangle$$



- **Effective length of Wilson Line**

$$\tilde{z} = \begin{cases} |z_1 - z_2|, & z_1 z_2 < 0 \\ \max(|z_1|, |z_2|), & z_1 z_2 \geq 0. \end{cases}$$

C.Han et.al. JHEP 12044 (2023); LPC, PRD 111, 034510 (2025)

Baryon LCDA on Lattice Symmetries

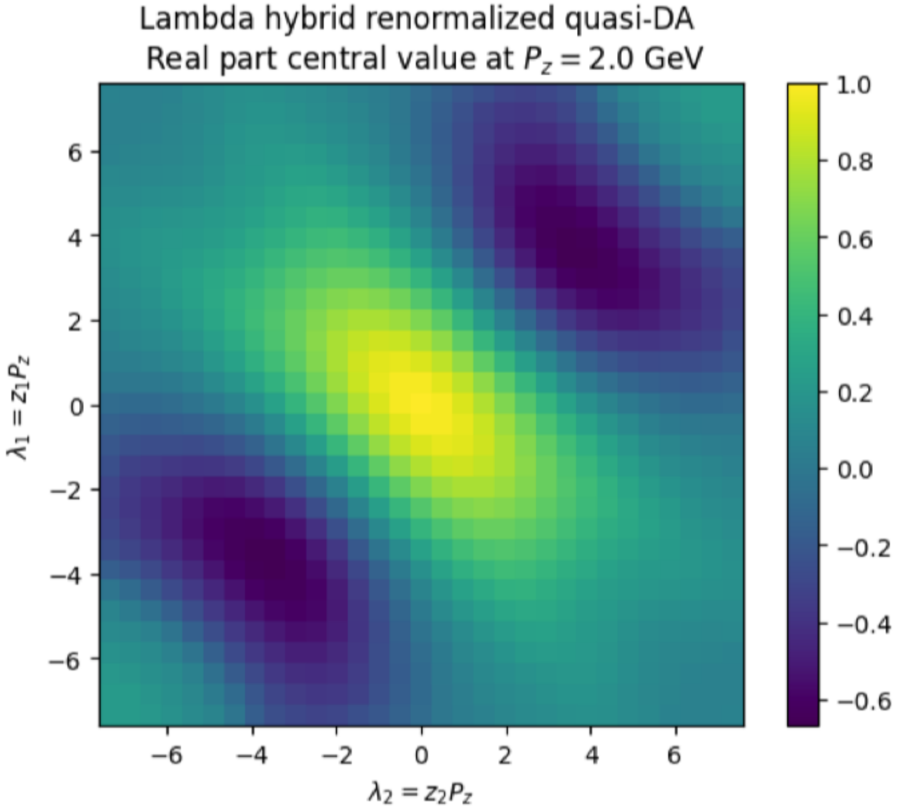
- **Two symmetries of quasi-DAs in coordinate space**

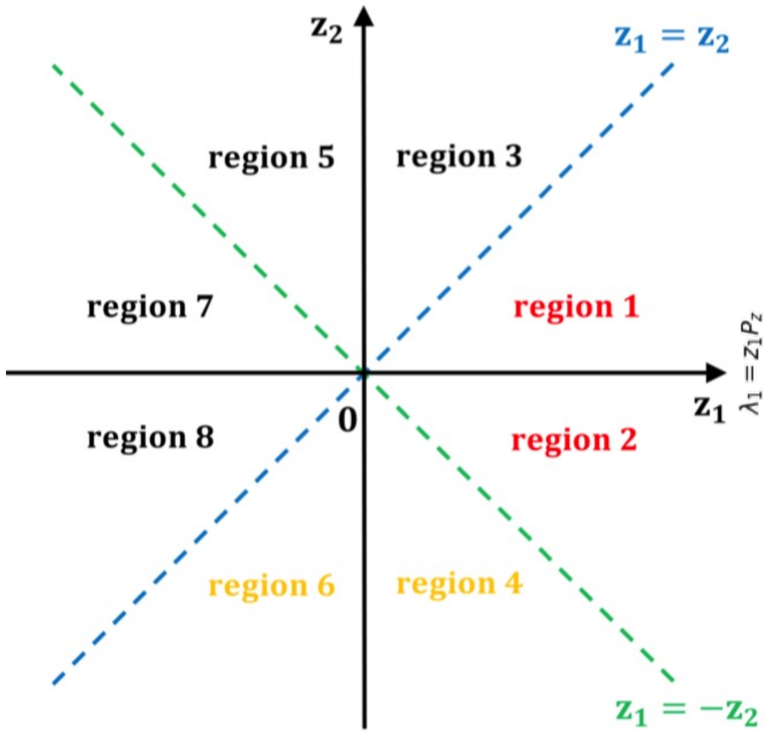
Identical symmetry: $n(ddu), p(uud)$ ...

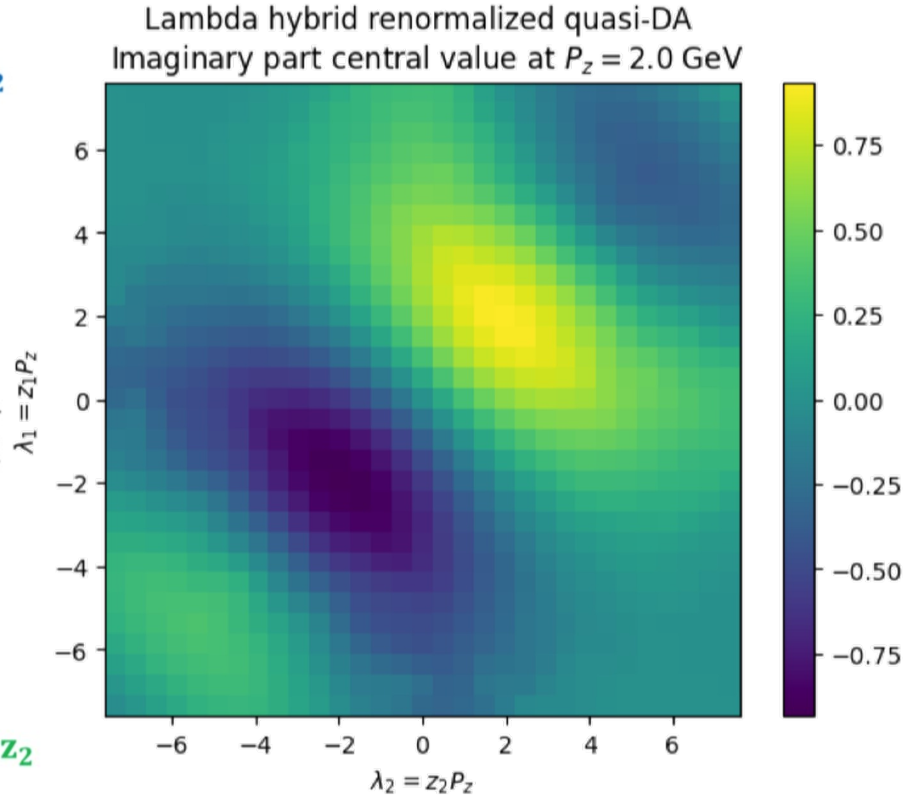
Isospin symmetry: $\Lambda(uds), \Sigma^0(uds+dus)$

$$\tilde{\Phi}\left(z_1, z_2, \mu\right)=\int_0^1 d x_1 \int_0^1 d x_2 e^{i\left(x_1 z_1 P^z+x_2 z_2 P^z\right)} \times \tilde{\phi}\left(x_1, x_2, \mu\right)$$

Pure real







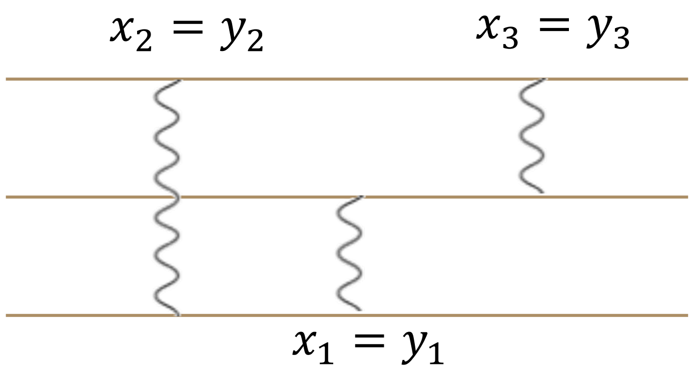
Baryon LCDA on Lattice Matching

- **2D matching for baryon LCDA**

- **Meson:** $\tilde{\phi}(x) = \int_0^1 dy C(x,y)\phi(y) + \mathcal{O}\left(\frac{1}{(xP^z)^2}, \frac{1}{[(1-x)P^z]^2}\right)$
- **Baryon:** $\tilde{\phi}(x_1,x_2) = \int_0^1 dy_1 \int_0^{1-y_1} dy_2 C(x_1,x_2,y_1,y_2)\phi(y_1,y_2) + \mathcal{O}\left(\frac{1}{(x_1P^z)^2}, \frac{1}{(x_2P^z)^2}, \frac{1}{[(1-x_1-x_2)P^z]^2}\right)$

$$
\begin{aligned}
C(x_1,x_2,y_1,y_2,\mu) = {} & \delta(x_1-y_1)\,\delta(x_2-y_2) \\
& + \frac{\alpha_s C_F}{2\pi}\left[\left(\frac{1}{4}C_2(x_1,x_2,y_1,y_2) - \frac{7}{8}\frac{-1}{|x1-y1|}\right)\delta(x_2-y_2)\right. \\
& + \left(\frac{1}{4}C_2(x_2,x_1,y_2,y_1) - \frac{7}{8}\frac{-1}{|x2-y2|}\right)\delta(x_1-y_1) \\
& \left. + \left(\frac{1}{4}C_3(x_1,x_2,y_1,y_2) + \frac{1}{4}C_3(x_2,x_1,y_2,y_1) - \frac{3}{4}\frac{-2}{|x_1-y_1-x_2+y_2|}\right)\delta(x_1+x_2-y_1-y_2)\right]_{\oplus},
\end{aligned}
$$

Double plus function



LPC, PRD 111, 034510 (2025)

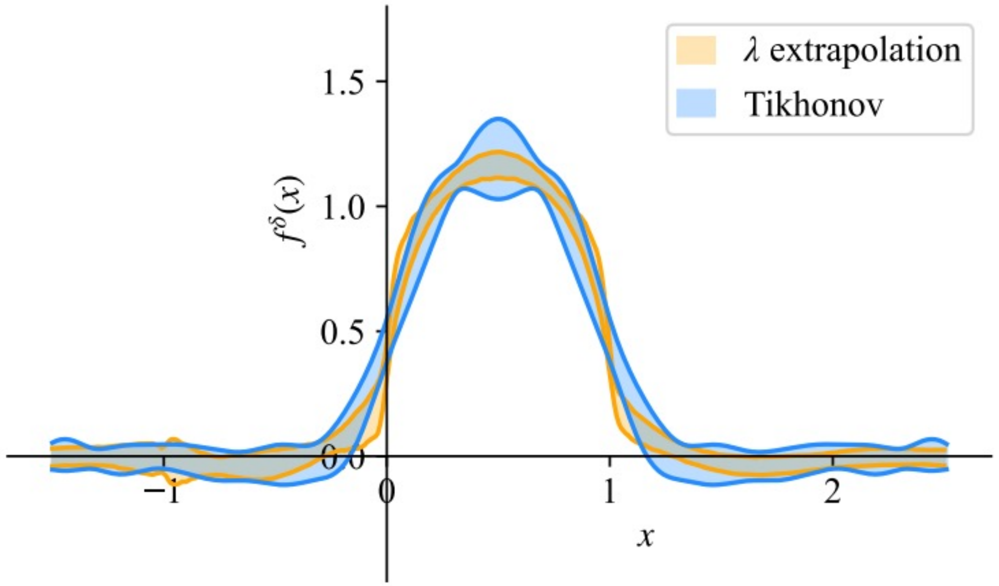
Baryon LCDA on Lattice FT

- **Quasi-DA in momentum space related to matrix elements in coordinate space with a limited Fourier transform**

$$\tilde{\psi}(x,\mu) \equiv \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{ix\lambda} \tilde{h}(\lambda,\mu) \quad \rightarrow \quad \tilde{\psi}(x,\mu) \equiv \int_{-\lambda_{cut}}^{\lambda_{cut}} \frac{d\lambda}{2\pi} e^{ix\lambda} \tilde{h}(\lambda,\mu)$$

<u>Inverse problem ?</u>

H.Dutrieux et.al, arXiv: :2504.17706;
H.Dutrieux et.al, arXiv: 2506.24037;
J.W.Chen et.al. arXiv:2505.14619; — can be circumvented
A.S.Xiong et.al. arXiv:2506.16689; — can be addressed

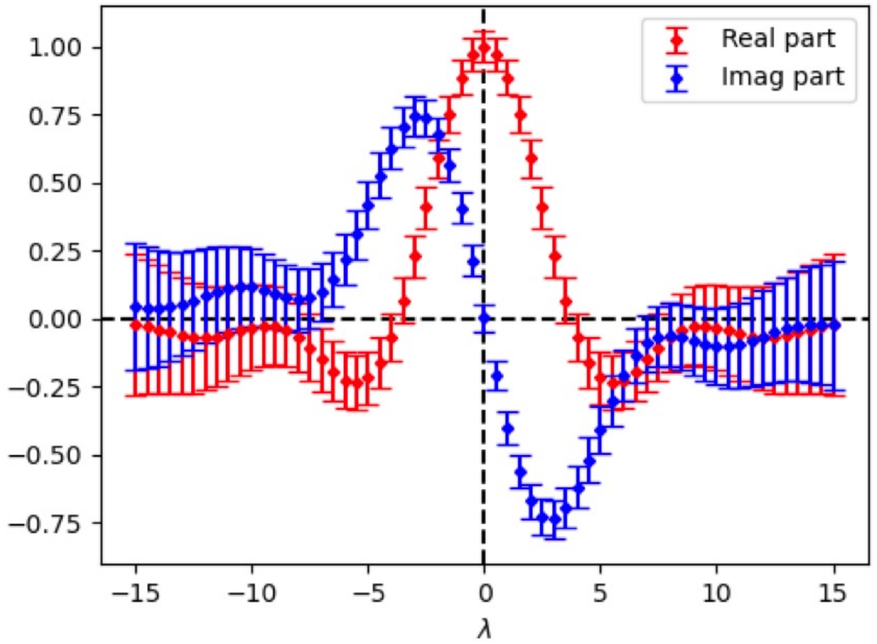


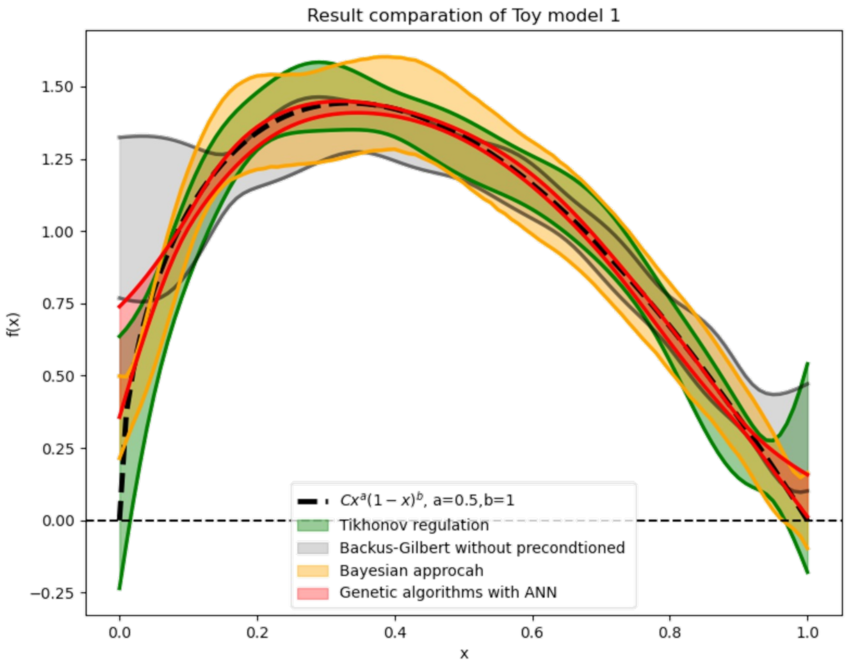
Baryon LCDA on Lattice FT

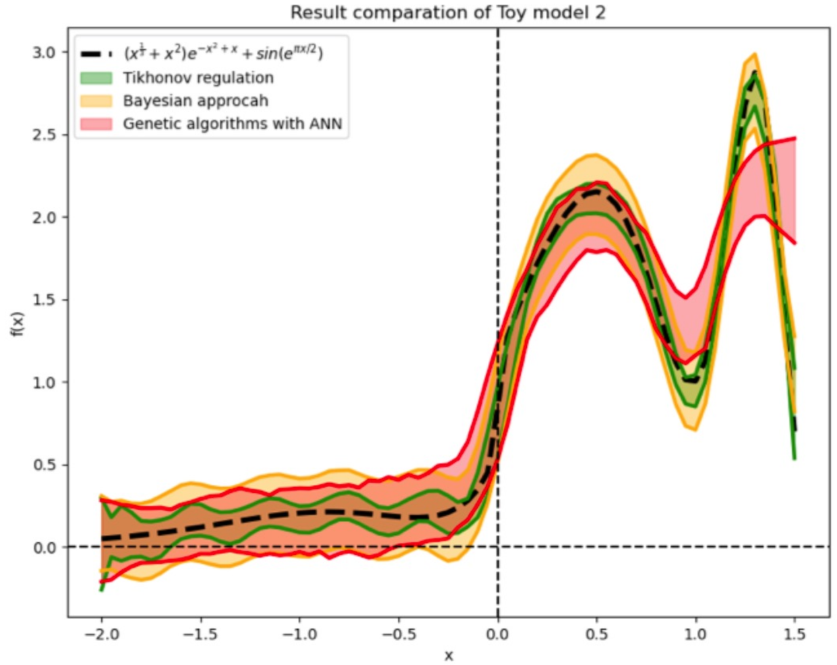
- **Baryon LCDA is more complicated 2D FT**

More alternatives: *— in 2 months*

- **Tikhonov Regularization**
- **Backus-Gilbert Method**
- **Bayesian Approach**
- **Genetic Algorithms with ANNs**







Improvement Lattice Simulation

- **Based on CLQCD Ensembles**
  - Three lattice spacing for $a \to 0$ limit
  - Three momentum for $P_z \to \infty$ limit

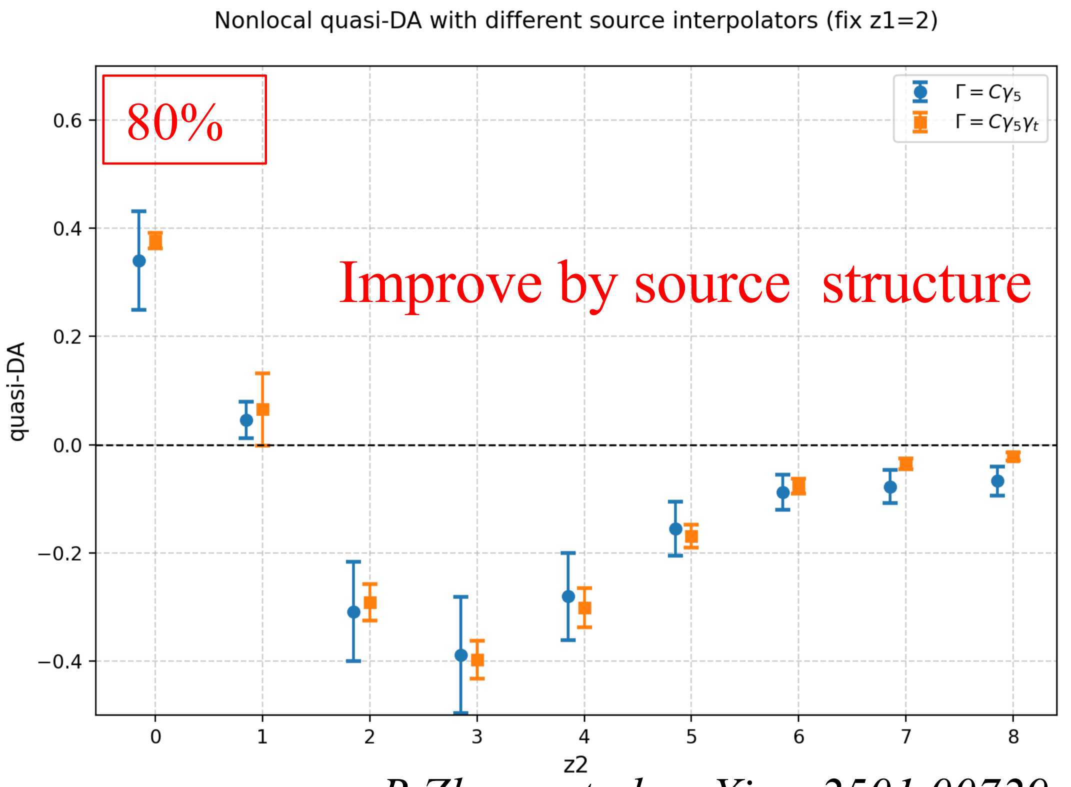
Ensemble	Volume	Lattice spacing	π mass	measurement	P^z
C24P29	$24^3\times72$	0.105 fm	293 MeV	864*4*8	1.96, 2.45, 2.94, 3.43 GeV
F32P30	$32^3\times96$	0.077 fm	303 MeV	777*4*8	1.99, 2.50, 2.99, 3.49 GeV
G36P29	36^3x108	0.068 fm	295MeV	656*6*9	2.01, 2.53, 3.03, 3.54 GeV
H48P32	$48^3\times144$	0.052 fm	317 MeV	550*6*8	1.99, 2.48, 2.98, 3.48GeV

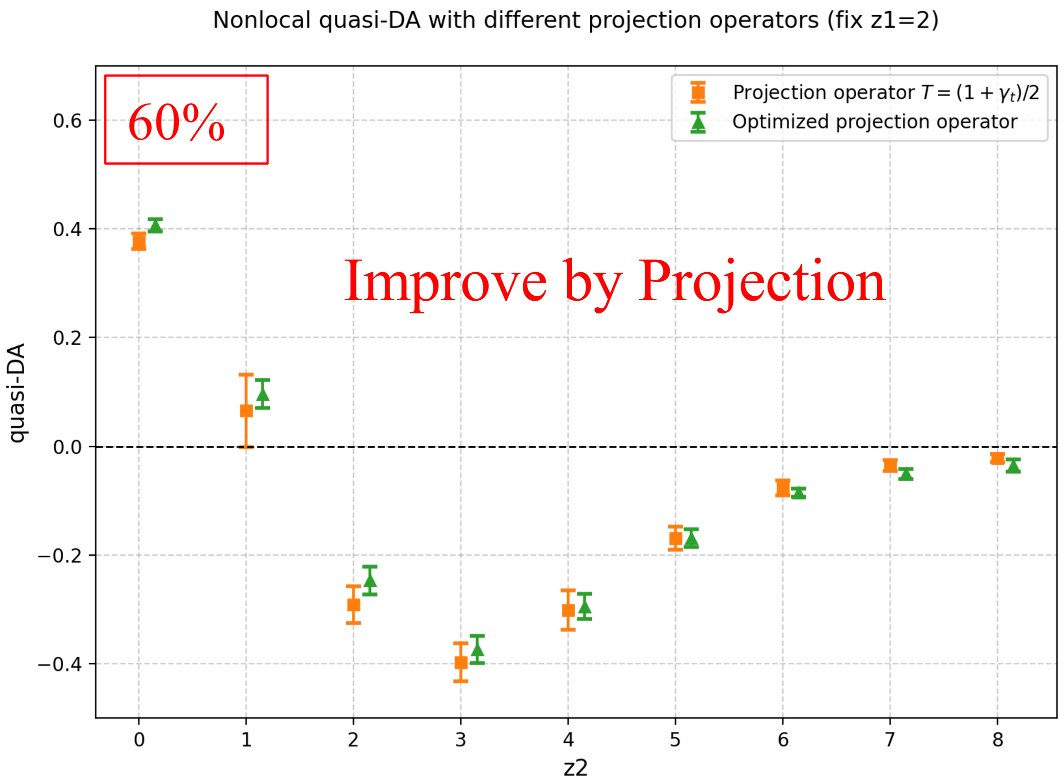
Improvement Lattice Simulation

- **Two point correlation of baryon**

$$C_2(z_a, z_2; t, \vec{P}) = \int \mathrm{d}^3x e^{-\mathrm{i}\vec{P}\cdot\vec{x}} \langle 0 | \mathcal{O}_{\mathrm{Sink}}(\vec{x}, t; z_1, z_2) \bar{\mathcal{O}}_{\mathrm{Src}}(0, 0; 0, 0) T | 0 \rangle$$

Determined by DA — Up to choice





R.Zhang et.al, arXiv: :2501.00729 ;

Improvement Renormalization

- **The challenge of renormalization in baryon quasi-DA (linear divergence)**

  - Self renormalization: 1) parameterize the matrix element to extract the linear divergence

    2) match with the $\overline{\mathrm{MS}}$ perturbative matrix element

I. **Parameterized form:**

$$\ln M\left(z_1, z_2; P_z=0; a\right)=\boxed{\frac{k}{a \ln \left(a \Lambda_{\mathrm{QCD}}\right)}} \tilde{z}+g\left(z_1, z_2\right)+f\left(z_1, z_2\right) a^2+\frac{\gamma_0}{b_0} \ln \frac{\ln \left(1 / a \Lambda_{\mathrm{QCD}}\right)}{\ln \left(\mu / \Lambda_{\overline{\mathrm{MS}}}\right)}+\ln \left[1+\frac{d}{\ln \left(a \Lambda_{\mathrm{QCD}}\right)}\right]$$

Linear divergence

II. **$\overline{\mathrm{MS}}$ perturbative matrix element:**

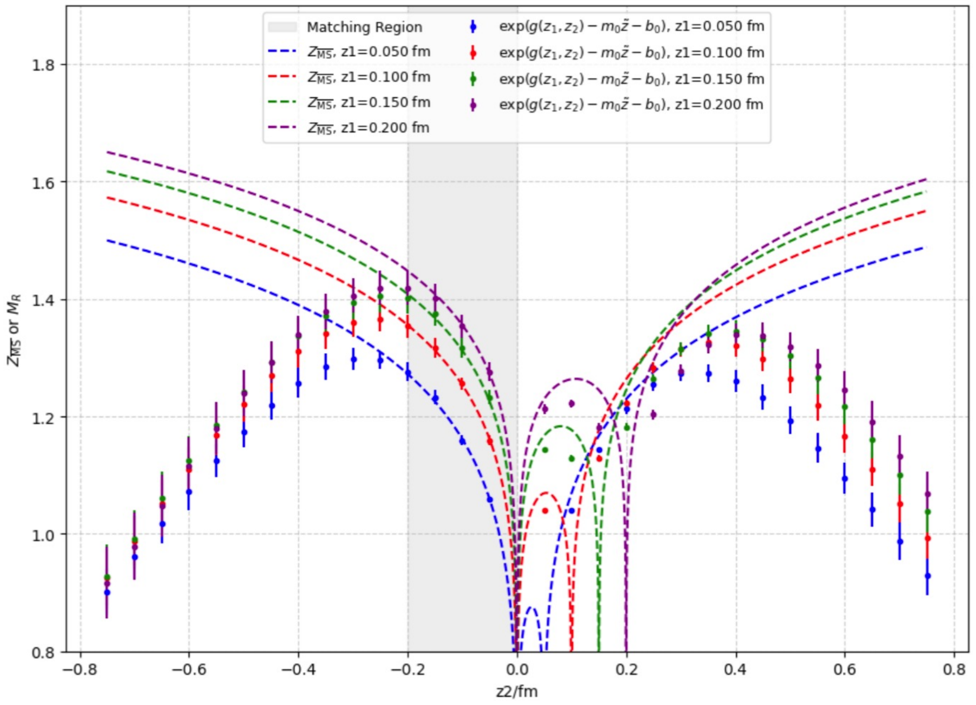
$$Z_{\overline{\mathrm{MS}}}\left(z_1, z_2; \mu, \Lambda_{\overline{\mathrm{MS}}}\right)=1+\frac{\alpha_s C_F}{2 \pi}\left[\frac{7}{8} \ln \frac{z_1^2 \mu^2 e^{2 \gamma_E}}{4}+\frac{7}{8} \ln \frac{z_2^2 \mu^2 e^{2 \gamma_E}}{4}+\frac{3}{4} \ln \frac{\boxed{\left(z_1-z_2\right)^2} \mu^2 e^{2 \gamma_E}}{4}+4\right]$$

Pole in coordinate space

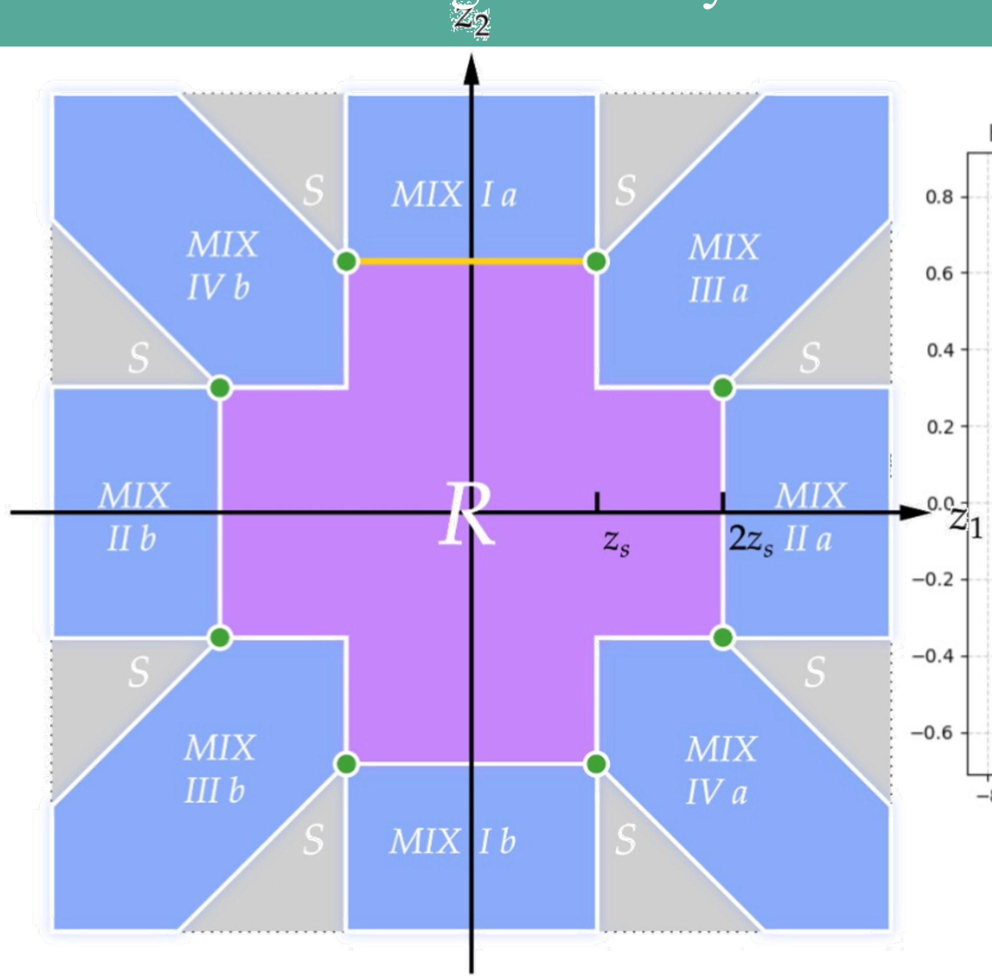
◉ Improvement Renormalization ◉

- **Hybrid (based on self renormalization) scheme for baryon quasi-DA**

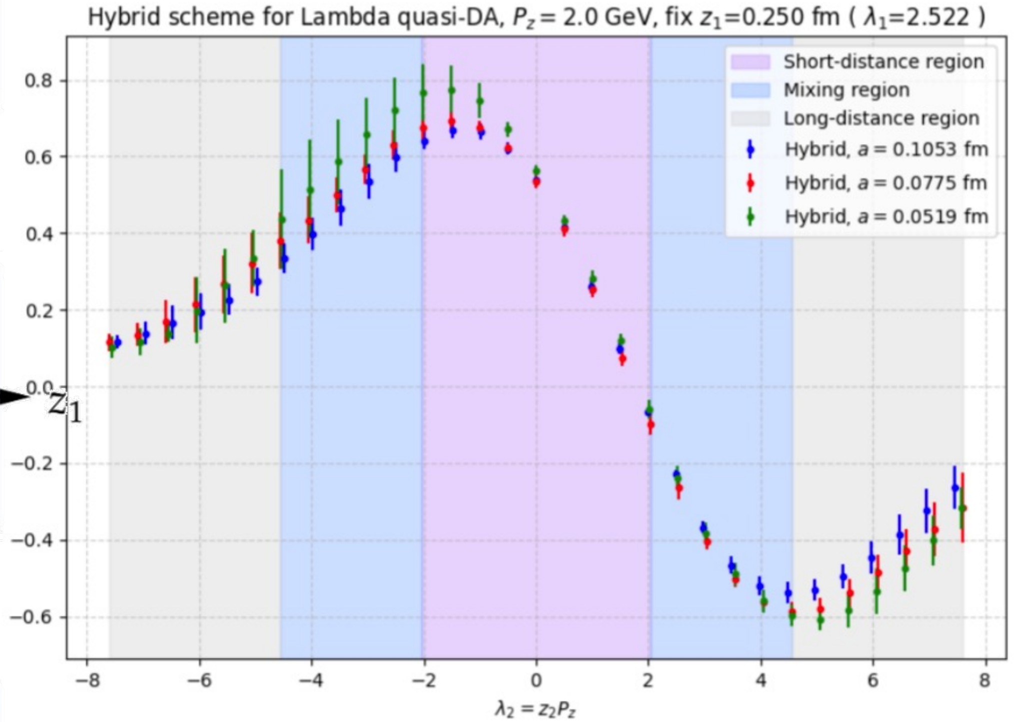
☐ Match with $\overline{\text{MS}}$ matrix element



☐ Different regions in Hybrid



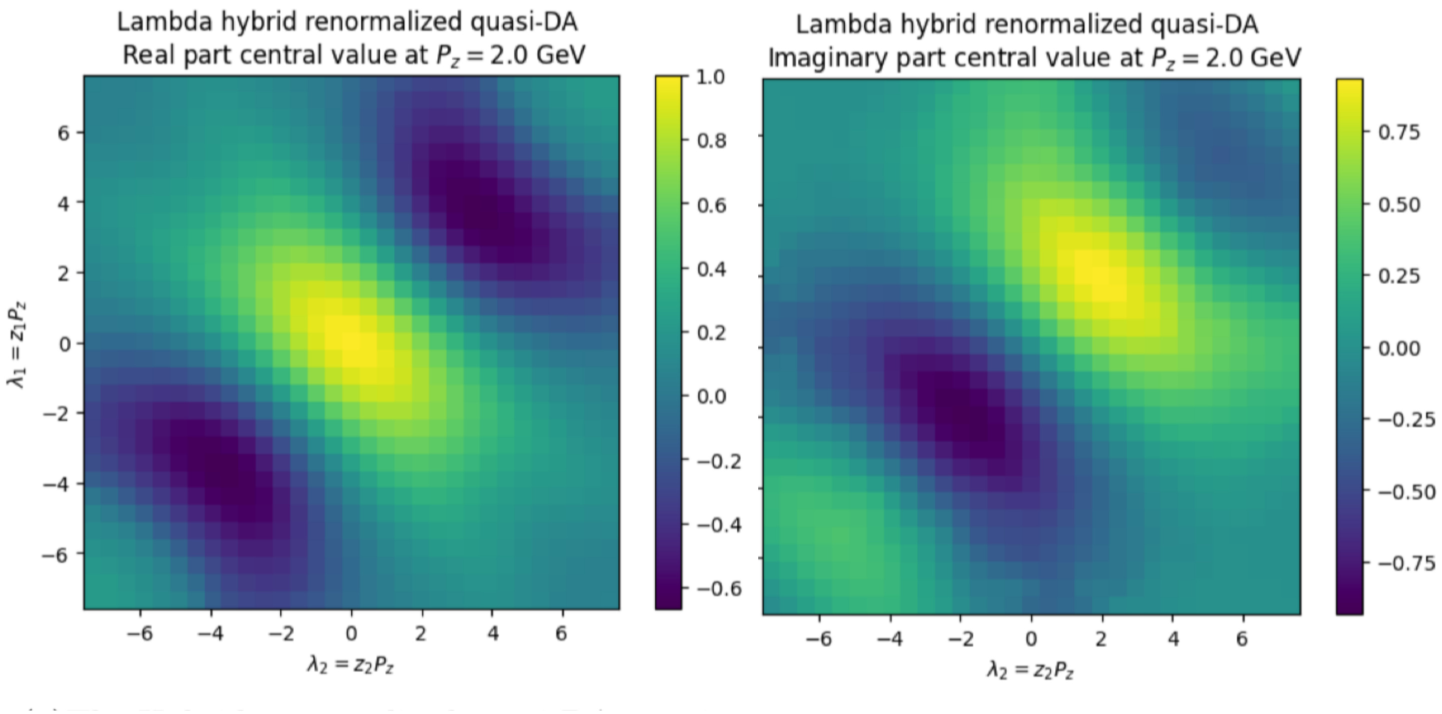
☐ Renormalized quasi-DA



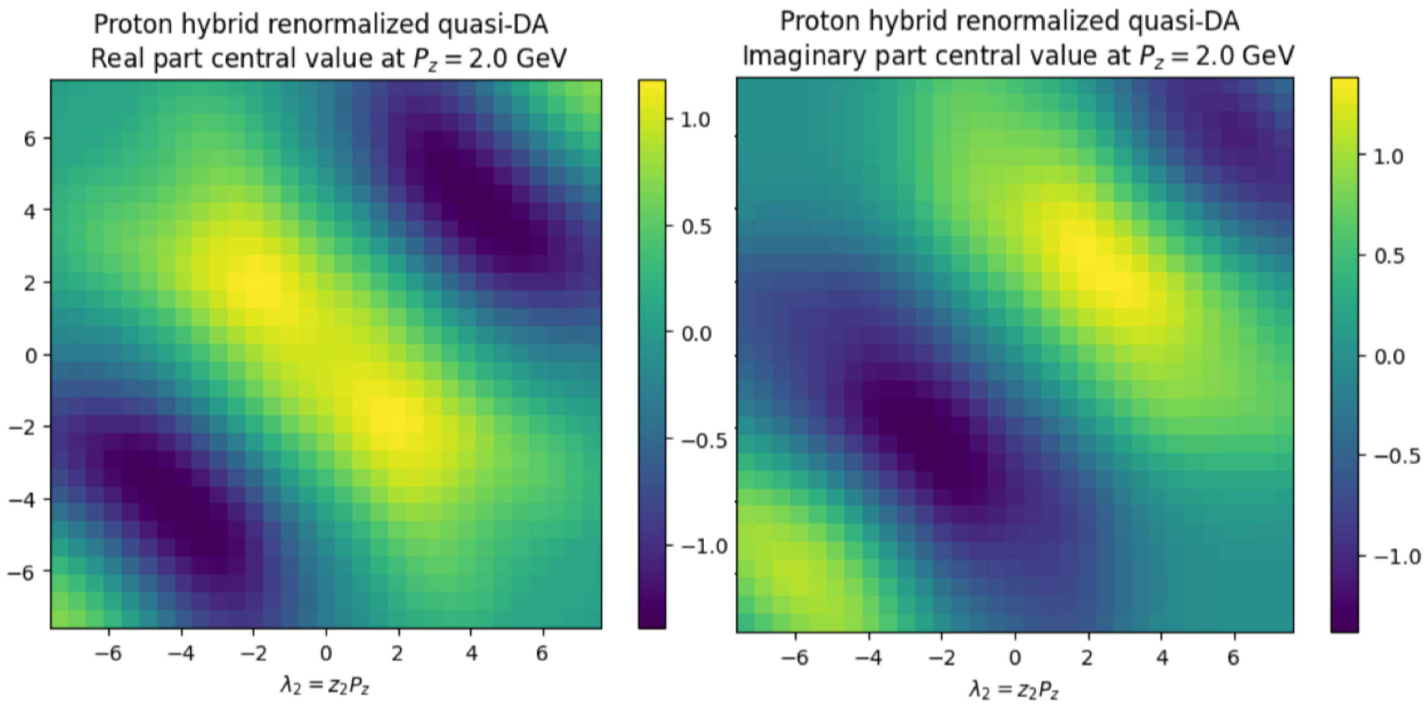
Improvement Renormalization

- **Hybrid (based on self renormalization) renormalized baryon quasi-DA**

Λ quasi-DA



Proton quasi-DA



Summary & Outlook

- We have extend the numerical computation on Lattice from light meson to light baryon in the LaMET framework.

- To calculate all leading twist structure Proton and Lambda LCDA, we improve:
  - lattice simulation $a \to 0$
  - source interpolator
  - projection operator
  - strategies for limited FT
  - renormalization scheme
  - matching scheme

◆ Please stay tuned our results for all leading twists LCDA of Proton and Lambda

◆ High twists will be the next

Thanks for Your Attention !