Form factors in semileptonic decay $D_s \to \phi l \nu$ from lattice QCD

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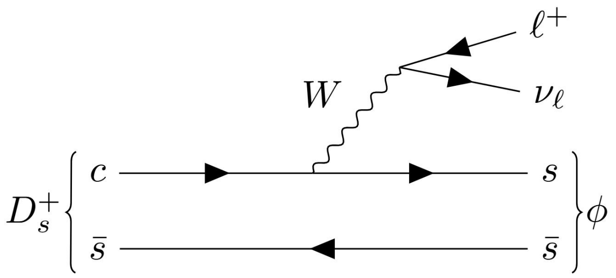
Based on arXiv:250x.xxxx

第八届强子谱和强子结构研讨会

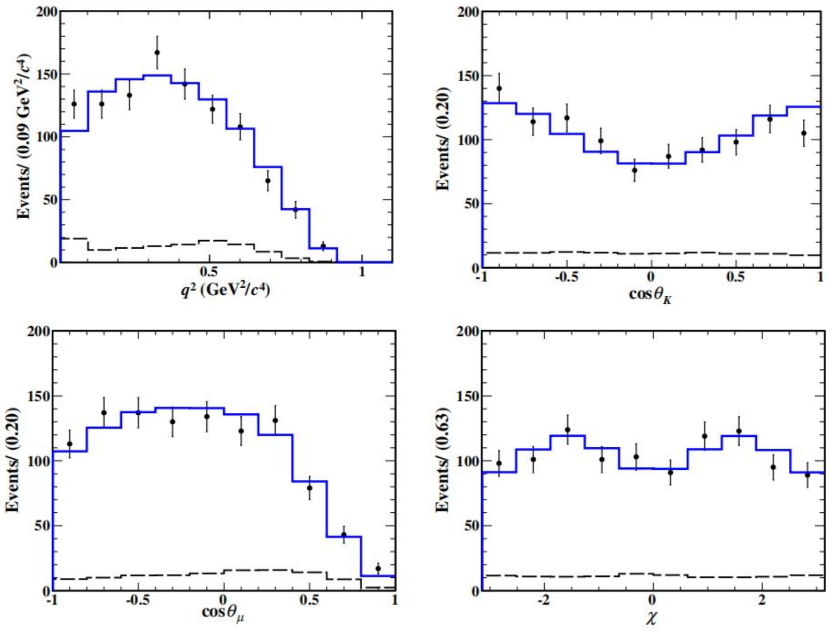
Outline

- Motivations
- Introduction to lattice QCD
- Lattice set up
- Method
- Results
- Summary

Motivations



- Semi-leptonic decays offer an ideal place to deeply understand hadronic transitions in the nonperturbative region of QCD, and can help to explore the weak and strong interactions in charm sector
- Vector meson decay makes this transition notoriously difficult to model due to theoretical complexity
- Combining with the experimental data, the CKM matrix element can be extracted, and it helps to test unitarity of CKM matrix and search for new physics beyond SM
- Calculating branching fractions helps to test $\mu - e$ lepton flavor universality



[BESIII, JHEP 12, 072 (2023)]

SM parameter

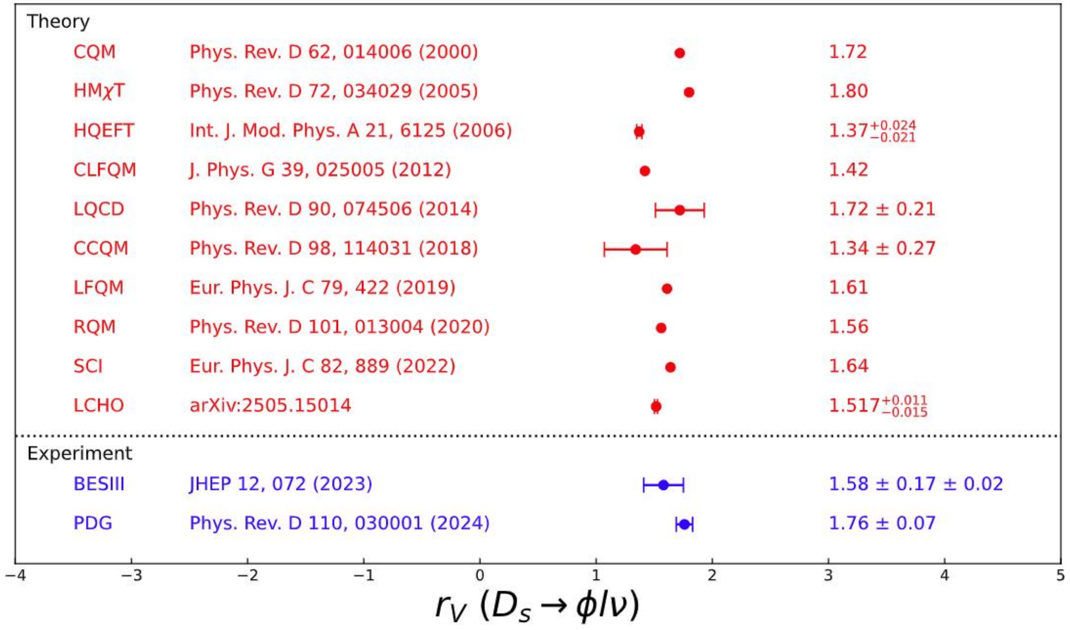
$$\boxed{\frac{d\Gamma(D_s \to \phi \ell \nu)}{dq^2}} = \frac{G_F^2 \boxed{|V_{cs}|^2} |\boldsymbol{p}_\phi| q^2}{96\pi^3 M_{D_s}^2} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) \boxed{\left(|H_+|^2 + |H_-|^2 + |H_0|^2\right)} + \frac{3m_\ell^2}{2q^2} \boxed{|H_t|^2}\right]$$

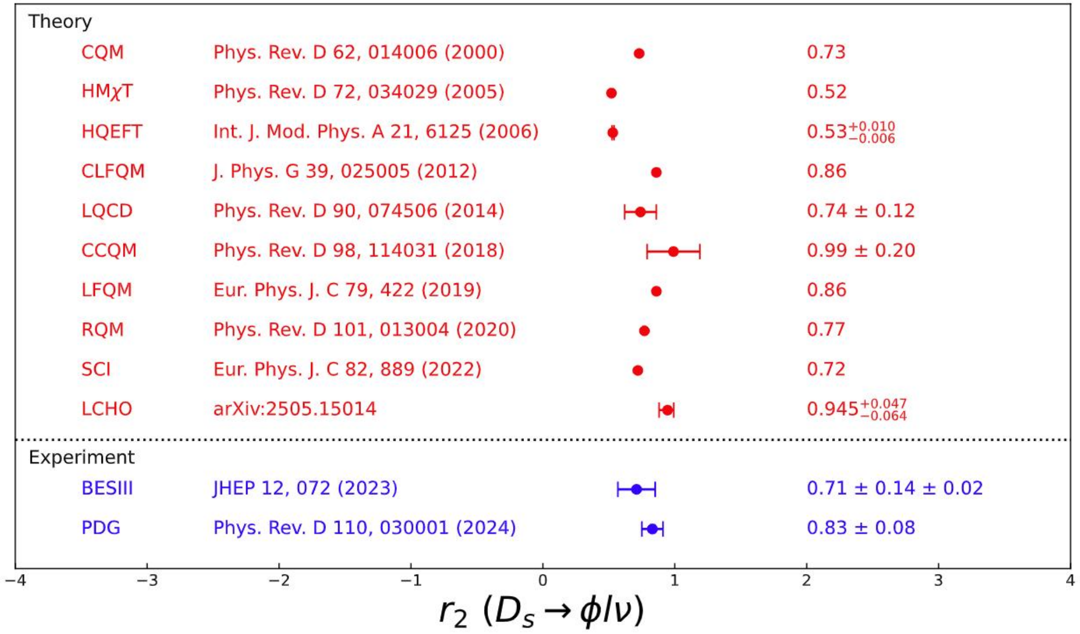
Experimental data

non-perturbative

Motivations

- Status of theoretical and experimental studies





A precise lattice calculation is important!

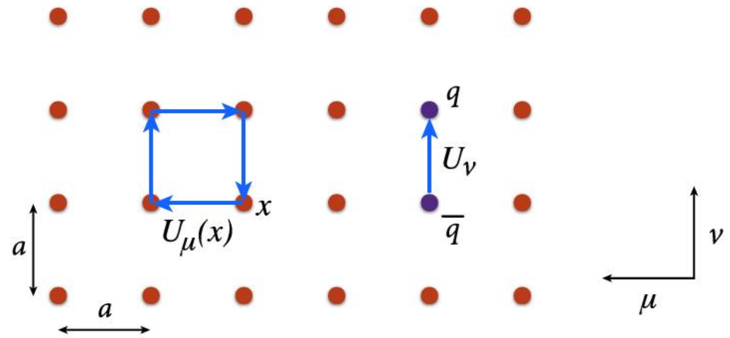
- Provide lattice QCD input to investigate the SU(3) symmetry (by combining with $D \to K^* l \nu$ calculation)

Introduction to lattice QCD

- Path integral in discrete Euclidean space

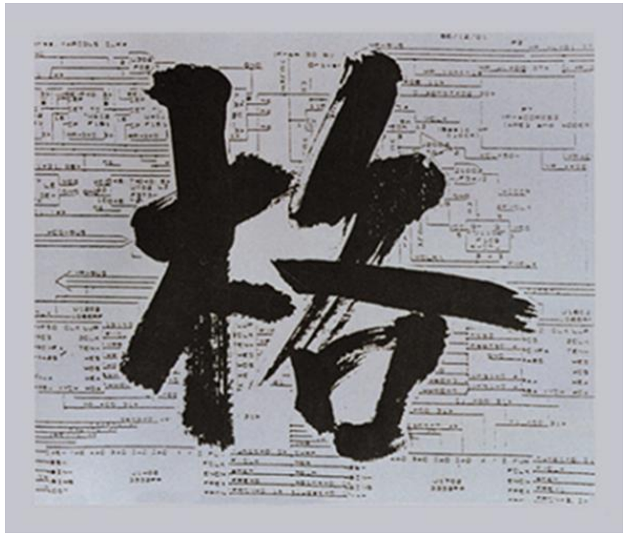
$$Z = \int [dU] \prod_f [dq_f][d\bar{q}_f] e^{-S_g[U] - \sum_f \bar{q}_f (D[U]+m_f) q_f}$$

$$Z = \int [dU] e^{-S_g[U]} \prod_f \det(D[U] + m_f)$$



- Expectation values of gauge-invariant operators, also known as "correlation functions"

$$\langle \mathcal{O}(U, q, \bar{q}) \rangle = (1/Z) \int [dU] \prod_f [dq_f][d\bar{q}_f] \mathcal{O}(U, q, \bar{q}) e^{-S_g[U] - \sum_f \bar{q}_f (D[U]+m_f) q_f}$$

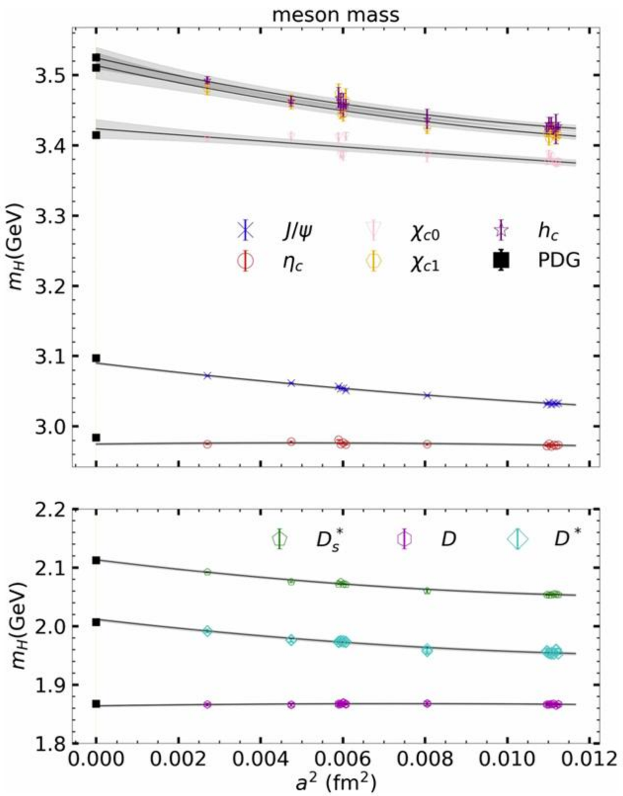


- Monte-Carlo method and data analysis

Lattice set up

- (2+1)-flavor Wilson-clover gauge ensembles [CLQCD, PRD 111, 054504 (2025)]
- Computer resources: "SongShan" supercomputer at Zhengzhou University

Ensemble	C24P29	C32P23	F32P30	F48P21	H48P32
a (fm)	0.10524(05)(62)	0.10524(05)(62)	0.07753(03)(45)	0.07753(03)(45)	0.05199(08)(31)
$\tilde{m}_s^b$	−0.2400	−0.2400	0.2050	0.2050	0.1700
$\tilde{m}_l^b$	−0.2770	−0.2790	0.2295	0.2320	0.1850
$\tilde{m}_c^b$	0.4159(07)	0.4190(07)	0.1974(05)	0.1997(04)	0.0551(07)
$L^3 \times T$	$24^3 \times 72$	$32^3 \times 64$	$32^3 \times 96$	$48^3 \times 96$	$48^3 \times 144$
$N_{\rm cfg} \times N_{\rm src}$	450×72	200×64	180×96	150×96	150×144
m_π (MeV)	292.3(1.0)	227.9(1.2)	300.4(1.2)	207.5(1.1)	316.6(1.0)
t	$2-17$	$2-20$	$4-22$	$4-26$	$8-30$
Z_V^s	0.85184(06)	0.85350(04)	0.86900(03)	0.86880(02)	0.88780(01)
Z_V^c	1.57353(18)	1.57644(12)	1.30566(07)	1.30673(04)	1.12882(11)
Z_A/Z_V	1.07244(70)	1.07375(40)	1.05549(54)	1.05434(88)	1.03802(28)



Method (correlation function formulism)

- 2-point correlation function (2pt),

$$C^{(2)}(\vec{p}, t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle \mathcal{O}_h(\vec{x}, t)\mathcal{O}_h^{\dagger}(0)\rangle$$

- 3-point correlation function (3pt) ,

$$\begin{aligned} C_{\mu\nu}(\vec{x}, t, t_s) &= \langle \mathcal{O}_{\phi_\nu}(t) J_{\mu}^{W}(0)\mathcal{O}_{D_s}^{\dagger}(-t_s)\rangle \\ &= \langle \bar{s}(t)\gamma_\nu s(t)\bar{s}(0)\gamma_\mu(1-\gamma_5)c(0)\bar{c}(-t_s)\gamma_5 s(-t_s)\rangle \\ &= \langle \mathrm{Tr}[\gamma_5\gamma_5 S_{-s}^{\dagger}(t, -t_s)\gamma_5\gamma_\nu S_s(t, 0)\gamma_\mu(1-\gamma_5)S_c(0, -t_s)]\rangle \end{aligned}$$

Method (scalar function)

- The parameterization for $P \to V$ semileptonic matrix element

$$\langle \phi_\sigma(\vec{p})|J^W_\mu(0)|D_s(p')\rangle = \frac{F_0(q^2)}{Mm}\epsilon_{\mu\sigma\alpha\beta}p'^\alpha p^\beta + F_1(q^2)\delta_{\mu\sigma} + \frac{F_2(q^2)}{Mm}p_\mu p'_\sigma + \frac{F_3(q^2)}{M^2}p'_\mu p'_\sigma$$

$$\langle \phi(\varepsilon,\vec{p})|J^W_\mu(0)|D_s(p')\rangle = \varepsilon^*_\nu \varepsilon_{\mu\nu\alpha\beta}p'_\alpha p_\beta \frac{2V}{m+M} + (M+m)\varepsilon^*_\mu A_1 + \frac{\varepsilon^*\cdot q}{M+m}(p+p')_\mu A_2 - 2m\frac{\varepsilon^*\cdot q}{Q^2}q_\mu(A_0 - A_3)$$

- Correlation functions $\longrightarrow$ Scalar functions $\longrightarrow$ Form factors

$\langle \phi_\sigma(\vec{p})|J^W_\mu(0)|D_s(p')\rangle$ $\qquad \tilde{\mathcal{I}}_j$ $\qquad V, A_0, A_1, A_2$

- Relationship with the form factor

$$V = \frac{(m+M)}{2mM}F_0,$$
$$A_1 = \frac{F_1}{M+m},$$
$$A_2 = \frac{M+m}{2mM^2}(MF_2 + mF_3),$$
$$A_0 - A_3 = Q^2\left(\frac{F_2}{4m^2M} - \frac{F_3}{4mM^2}\right).$$

A_3 is not an independent form factor

$$A_3(q^2) = \frac{M+m}{2m}A_1(q^2) - \frac{M-m}{2m}A_2(q^2)$$

A_0 is then

$$A_0(q^2) = \frac{F_1}{2m} + \frac{m^2 - M^2 + Q^2}{4m^2M}F_2 + \frac{m^2 - M^2 - Q^2}{4mM^2}F_3$$

$A_0(0) = A_3(0)$ is automatically perserved

Method (scalar function)

- A similar scalar function scheme has been used for high-precision calculation
  - $\Gamma(\eta_c \to 2\gamma) = 6.67(16)_{\mathrm{stat}}(6)_{\mathrm{syst}}\ \mathrm{keV}$ [Y. M et al, Science Bulletin 68, 1880 (2023)]
  - $\Gamma(D_s^* \to \gamma D_s) = 0.0549(54)\ \mathrm{keV}$ [Y. M et al, PRD 109, 074511 (2024)]
  - $\mathrm{Br}(J/\psi \to De\nu_e) = 1.21(11) \times 10^{-11}$
    $\mathrm{Br}(J/\psi \to D\mu\nu_\mu) = 1.18(11) \times 10^{-11}$
    $\mathrm{Br}(J/\psi \to D_s e\nu_e) = 1.90(8) \times 10^{-10}$
    $\mathrm{Br}(J/\psi \to D_s\mu\nu_\mu) = 1.84(8) \times 10^{-10}$ [Y. M et al, PRD 110, 074510 (2024)]
  - $\mathrm{Br}(J/\psi \to \gamma\eta_c) = 2.49(11)_{\mathrm{lat}}(5)_{\mathrm{exp}}\%$ [Y. M et al, PRD 111, 014508 (2025)]

Results (2-point function fitting)

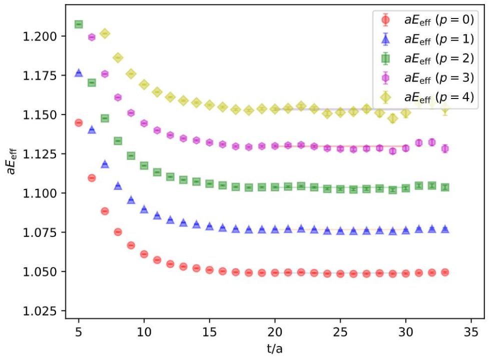
- Least χ^2 fitting considering covariance matrix between configurations and time

fit function for $\boldsymbol{D_s}$ is

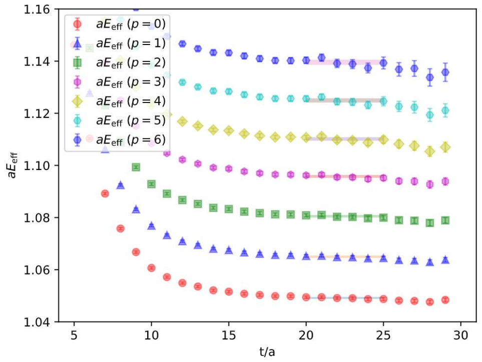
$$C^{(2)}(\vec{p}, t) = \frac{Z_h^2}{2E_h}\left(e^{-E_h t} + e^{-E_h(T-t)}\right)$$

- There should be a plateau when meson ground states are dominant

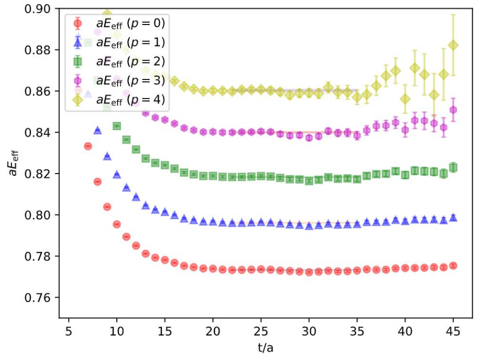
C24P29



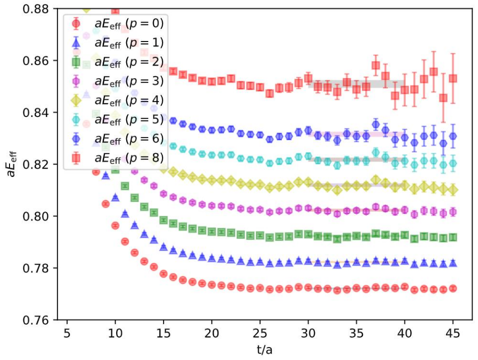
C32P23



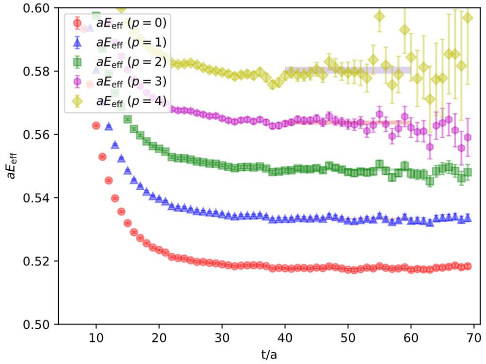
F32P30



F48P21



H48P32



Results (2-point function fitting)

- Least χ^2 fitting considering covariance matrix between configurations and time

fit function for $\boldsymbol{\phi}$ is $$C^{(2)}(\vec{p},t) = \left(-1 - \frac{|\vec{p}|^2}{3m_h^2}\right)\frac{Z_h^2}{2E_h}\left[e^{-E_h t} + e^{-E_h(T-t)}\right]$$

- There should be a plateau when meson ground states are dominant

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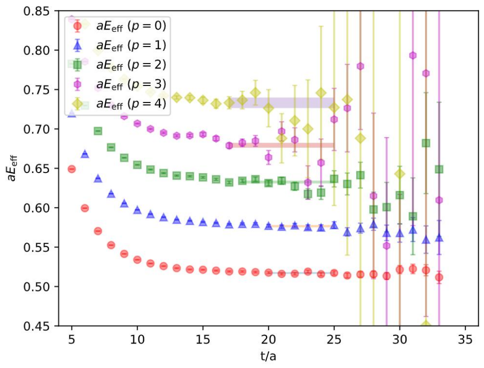

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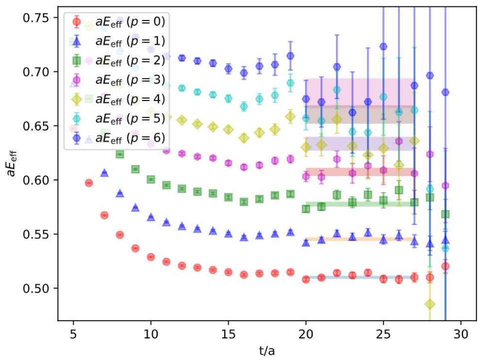

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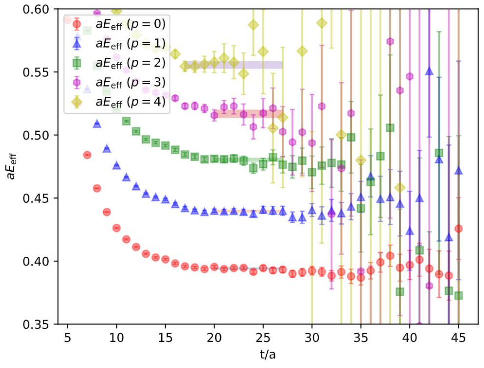

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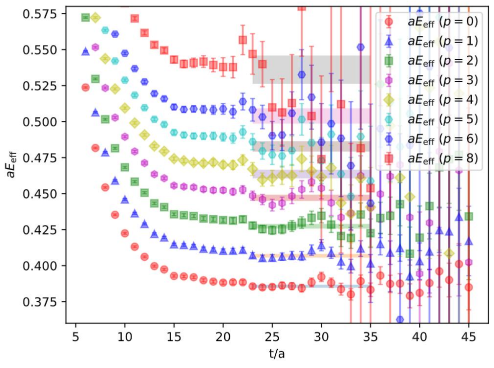

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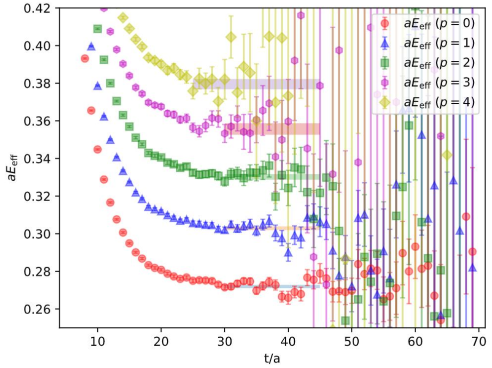


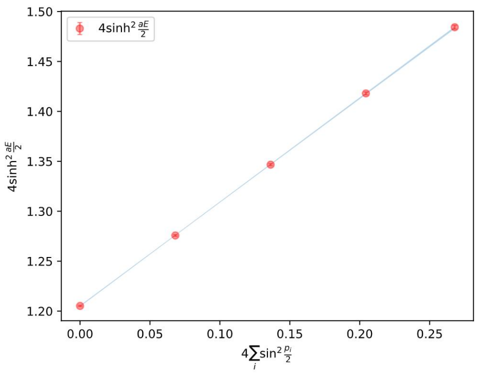
Results (dispersion relation)

- We checked the dispersion relation of $\boldsymbol{D_s}$ meson at different momenta
- Use a discrete dispersion relation as the fitting function

$$4\sinh^2\frac{E_h}{2} = 4\sinh^2\frac{m_h}{2} + \mathcal{Z}^h_{\mathrm{latt}}\cdot 4\sum_i \sin^2\frac{p_i}{2}$$

$\mathcal{Z}_{\mathrm{latt}}$ is 1.0407(54), 1.0447(80), 1.0384(72), 1.032(10), 1.029(10)

C24P29



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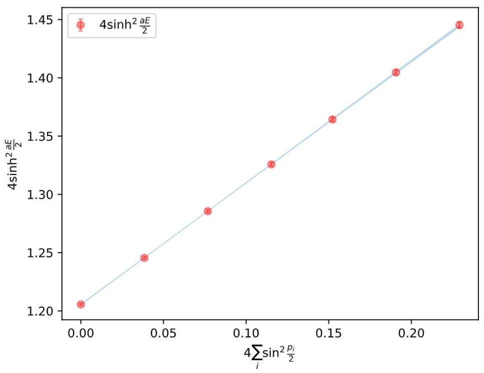

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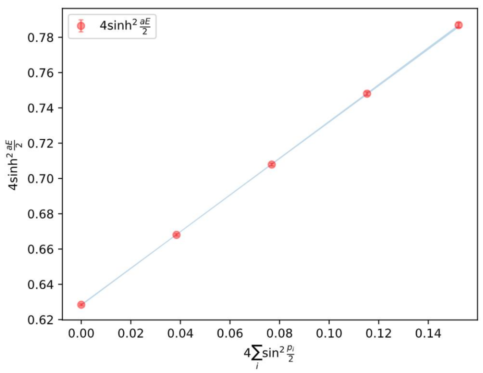

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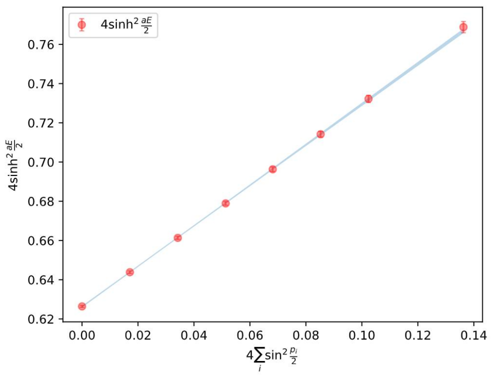

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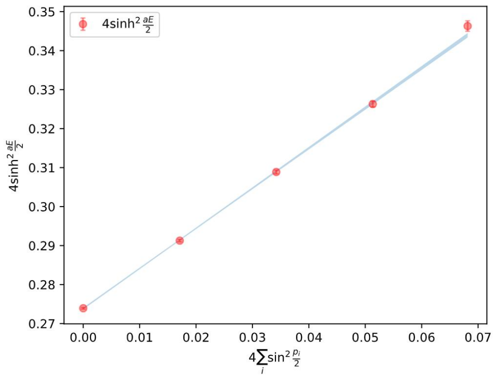

Results (dispersion relation)

- We checked the dispersion relation of ϕ meson at different momenta
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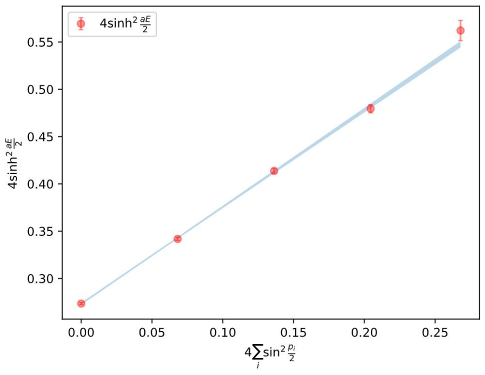
$$4\sinh^2\frac{E_h}{2} = 4\sinh^2\frac{m_h}{2} + \mathcal{Z}^h_{\mathrm{latt}}\cdot 4\sum_i \sin^2\frac{p_i}{2}$$

$\mathcal{Z}_{\mathrm{latt}}$ is 1.024(15), 0.988(26), 1.023(15), 1.026(23), 1.045(21)

C24P29



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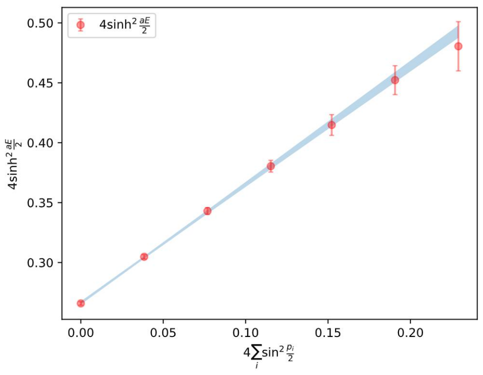

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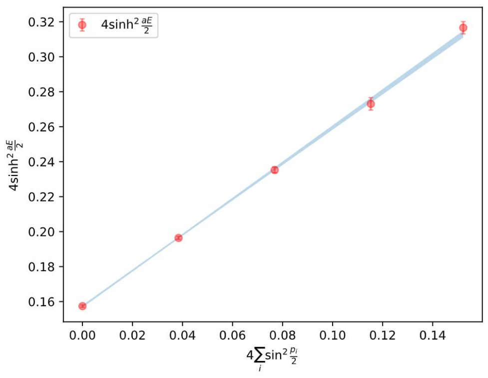

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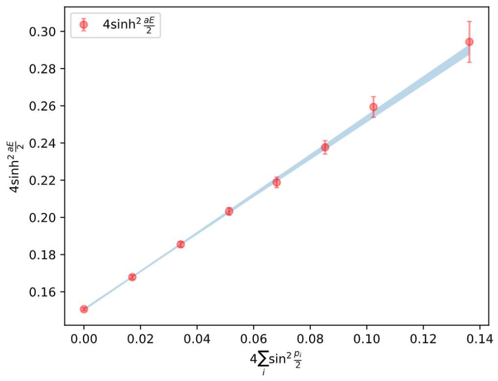

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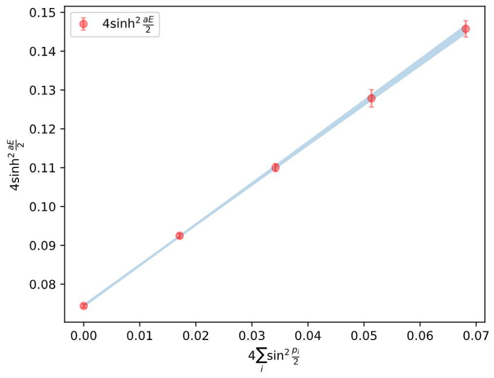


Results (form factor)

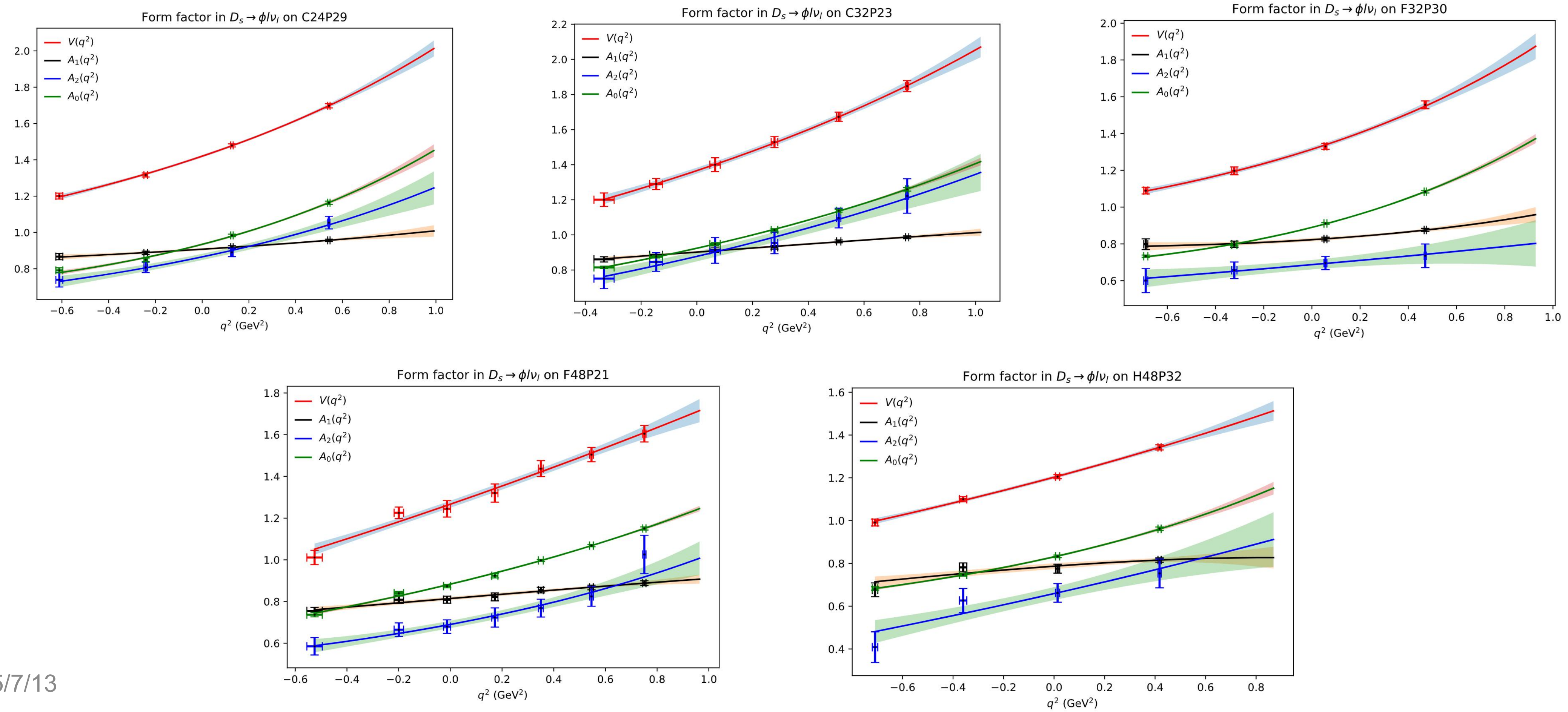
$$z\left(q^2, t_0\right) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

where $t_+ = (m_{D_s} + m_\phi)^2,\ t_0 = 0$

$$V = \frac{1}{1 - q^2/m_{D_s^*}^2}\left(a_0 + a_1 z + a_2 z^2\right)$$

$$A_{0,1,2} = \frac{1}{1 - q^2/m_{D_{s1}}^2}\left(a_0 + a_1 z + a_2 z^2\right)$$

- The results have been multiplied by the renormalization constant, data point errors from jackknife analysis



Results (global fit)

- Extrapolate results to the physical pion mass and continuum limit using z-expansion

$$z\left(q^2, t_0\right) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

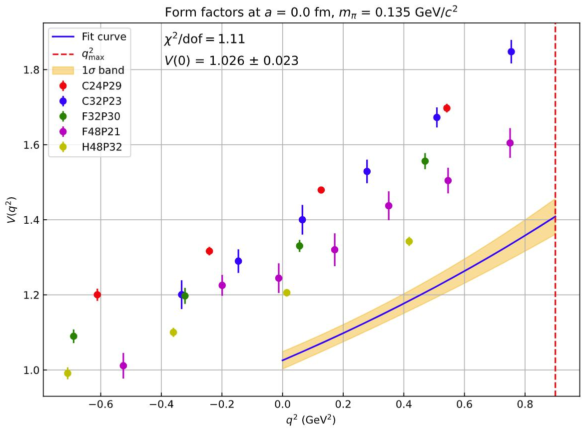
where $t_+ = (m_{D_s} + m_\phi)^2,\ t_0 = 0$

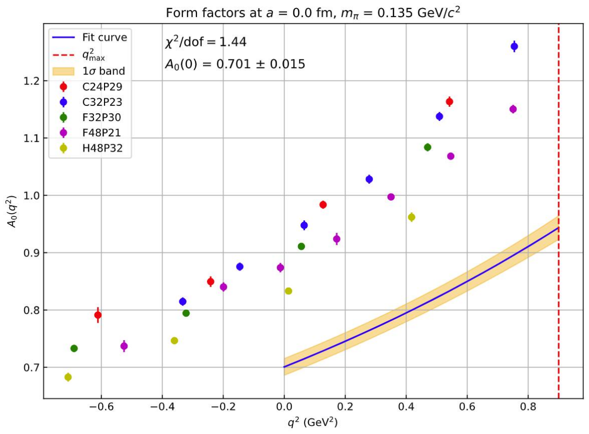
$$V\left(q^2, a, m_\pi\right) = \frac{1}{1 - q^2/m^2_{D_s^*}} \sum_{i=0}^{2} \left(c_i + d_i a^2\right) \left[1 + f_i\left(m_\pi^2 - m^2_{\pi,\text{phys}}\right) + g_i\left(m_\pi^2 - m^2_{\pi,\text{phys}}\right)^2\right] z^i$$

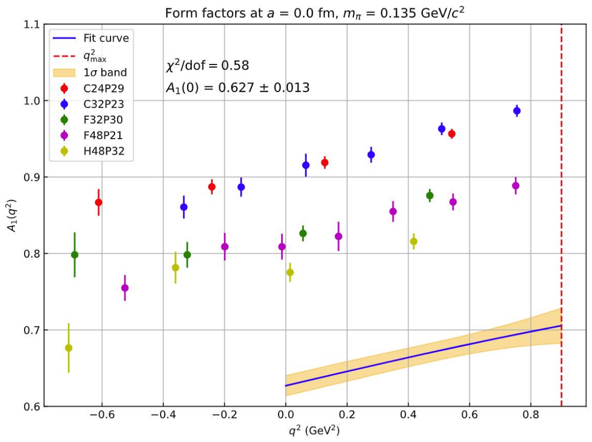
$$A_{0,1,2}\left(q^2, a, m_\pi\right) = \frac{1}{1 - q^2/m^2_{D_{s1}}} \sum_{i=0}^{2} \left(c_i + d_i a^2\right) \left[1 + f_i\left(m_\pi^2 - m^2_{\pi,\text{phys}}\right) + g_i\left(m_\pi^2 - m^2_{\pi,\text{phys}}\right)^2\right] z^i$$

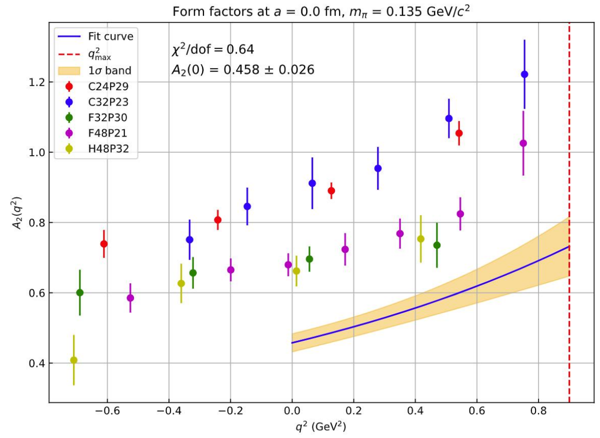
$$m^2_{\pi,\text{phys}} = 135.0\ \text{MeV}/c^2,\ m_{D_s^*} = 2112.2\ \text{MeV}/c^2,\ m_{D_{s1}} = 2459.5\ \text{MeV}/c^2$$

- $A_0(0) - A_3(0) =- 0.005(27)$, consistent with zero



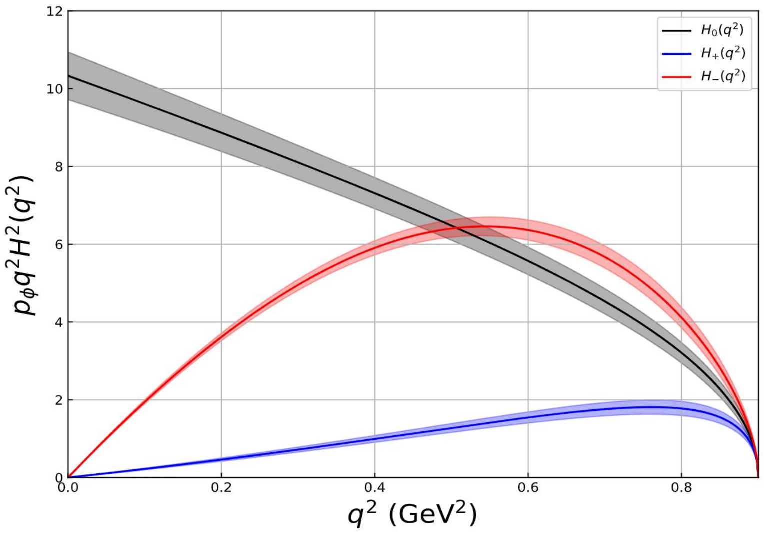




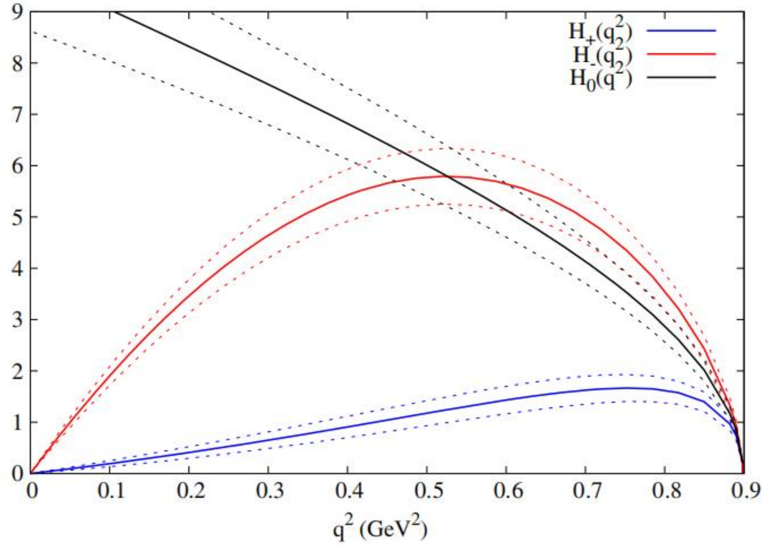


Results (H form factor)

$$H_{\pm}\left(q^2\right) = (M+m)\,A_1\left(q^2\right) \mp \frac{2Mp_{K^*}}{M+m}V\left(q^2\right),$$
$$H_0\left(q^2\right) = \frac{1}{2M\sqrt{q^2}} \times \left[\left(M^2-m^2-q^2\right)(M+m)\,A_1\left(q^2\right) - 4\frac{M^2p_{K^*}^2}{M+m}A_2\left(q^2\right)\right].$$



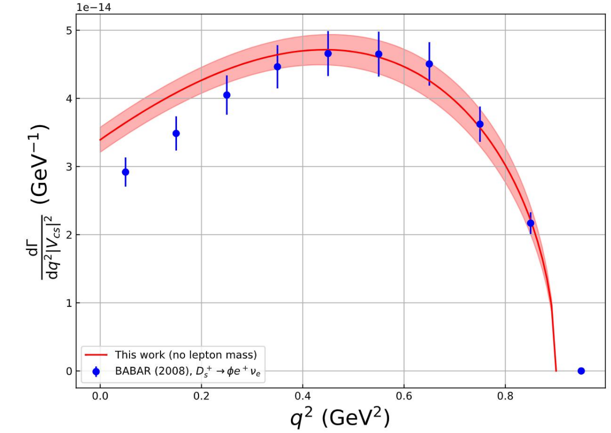
using z-expansion

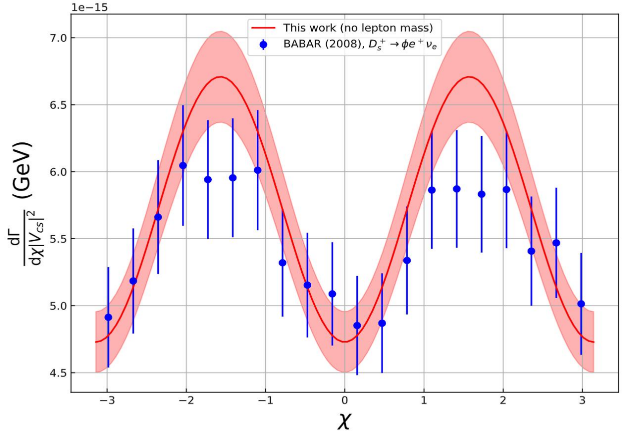


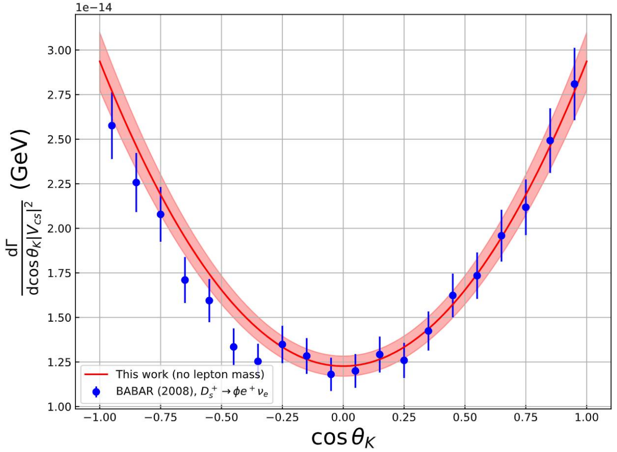
[HPQCD, PRD 90, 074506 (2014)]

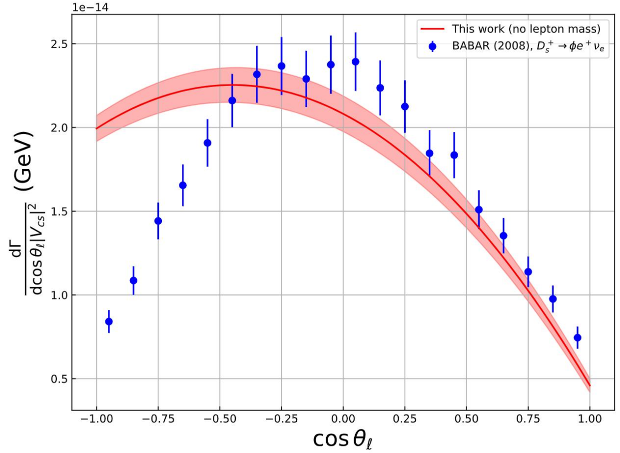
Results (differential decay width)

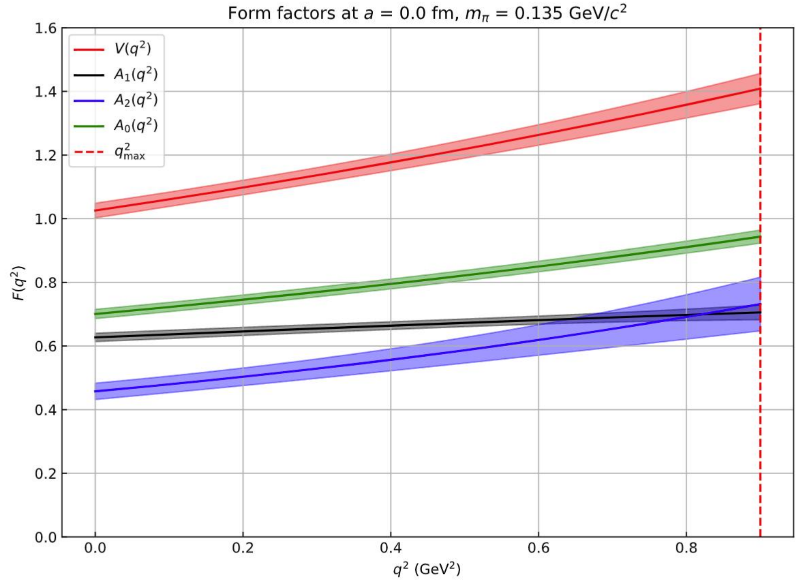
- Differential decay width using z-expansion, where the lepton mass is neglected [Rev. Mod. Phys 67, 893 (1995)]

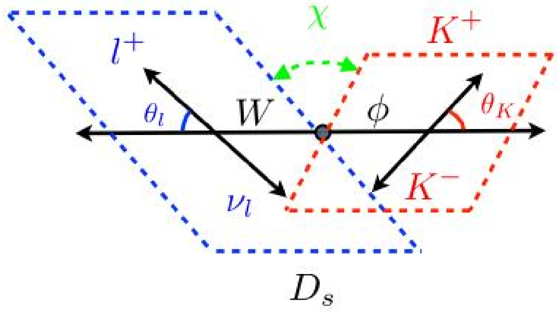












Results

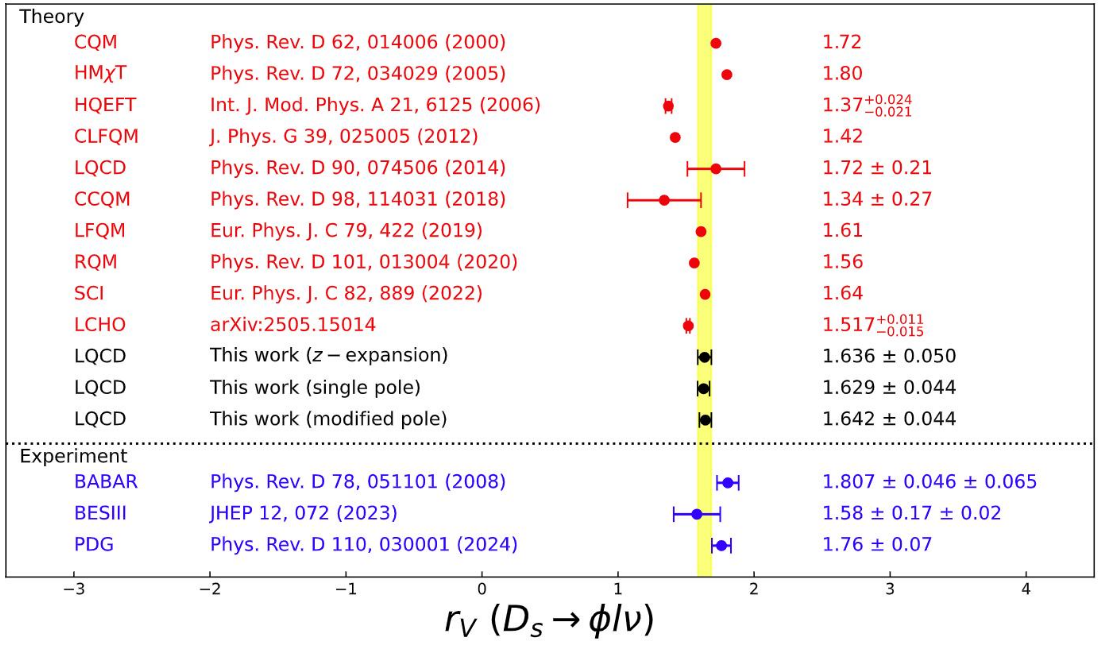
- Summary of preliminary results (form factor)

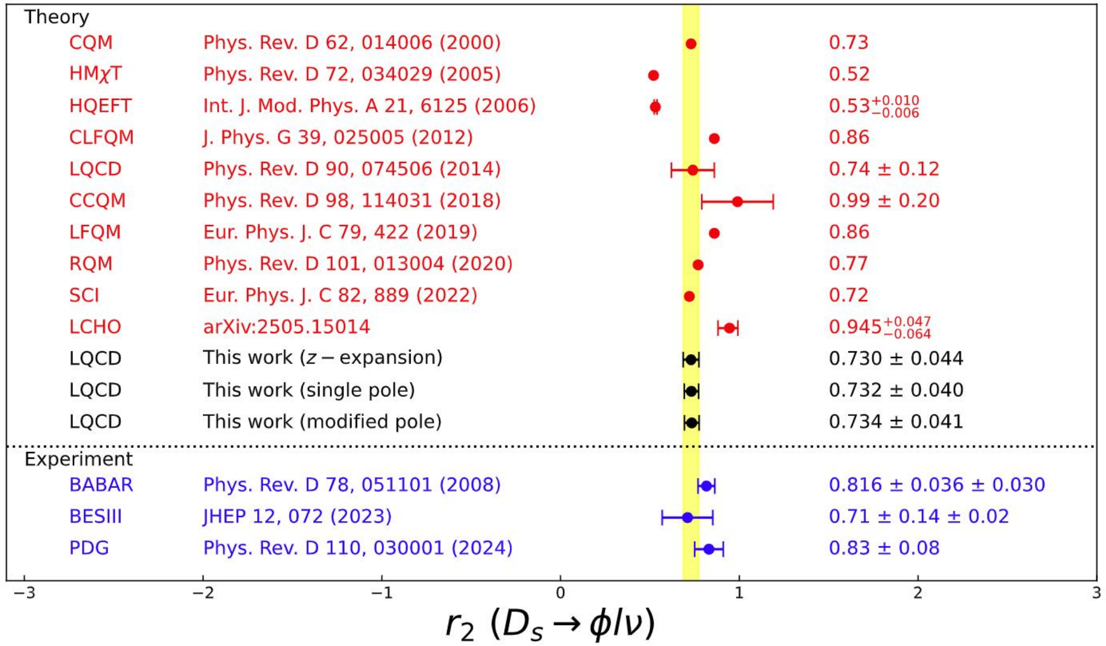
single pole

$$F\left(q^2, a, m_\pi\right) = \frac{1}{1 - q^2/h^2}\left(c + da^2\right)\left[1 + f\left(m_\pi^2 - m_{\pi,\text{phys}}^2\right) + g\left(m_\pi^2 - m_{\pi,\text{phys}}^2\right)^2\right]$$

modified pole

$$F\left(q^2, a, m_\pi\right) = \frac{1}{\left(1 - q^2/m_{\text{pole}}^2\right)\left(1 - \alpha q^2/m_{\text{pole}}^2\right)}\left(c + da^2\right)\left[1 + f\left(m_\pi^2 - m_{\pi,\text{phys}}^2\right) + g\left(m_\pi^2 - m_{\pi,\text{phys}}^2\right)^2\right]$$





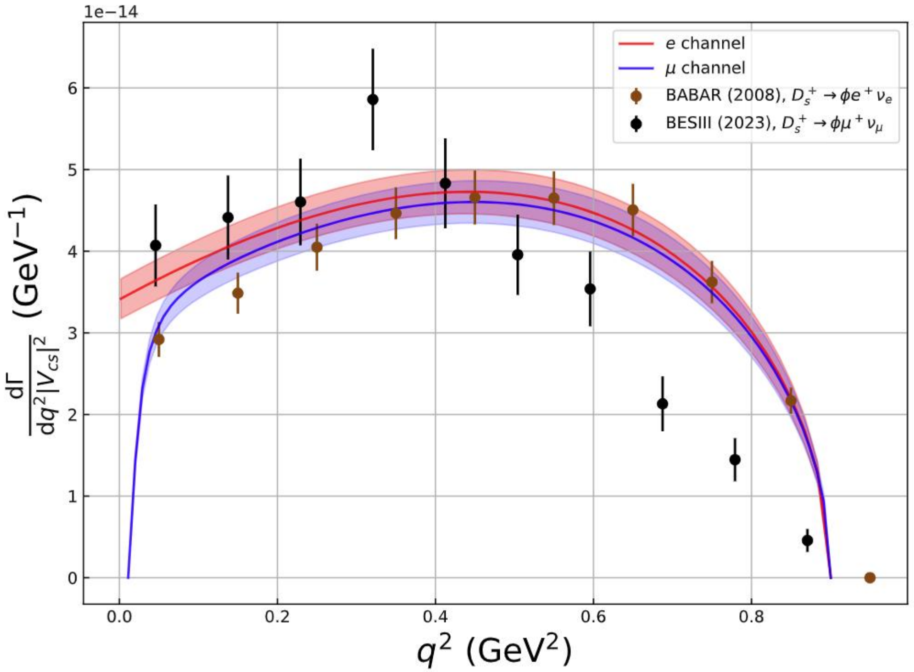
Results

- Summary of preliminary results (branching fraction)

PDG $|V_{cs}|$ as input

$$\frac{d\Gamma(D_s \to \phi\ell\nu)}{dq^2} = \frac{G_F^2 |V_{cs}|^2 |\boldsymbol{p}_\phi| q^2}{96\pi^3 M_{D_s}^2}\left(1-\frac{m_\ell^2}{q^2}\right)^2\left[\left(1+\frac{m_\ell^2}{2q^2}\right)\left(|H_+|^2+|H_-|^2+|H_0|^2\right)+\frac{3m_\ell^2}{2q^2}|H_t|^2\right]$$

$\mathcal{B}(D_s \to \phi\ell\nu) \times 10^{-2}$	e channel	μ channel	$\mathcal{R}_{\mu/e}$
z-expansion	2.61(15)	2.46(13)	0.9431(25)
single pole	2.56(10)	2.40(10)	0.9422(23)
modified pole	2.55(10)	2.41(10)	0.9428(24)
PDG	2.34(12)	2.24(11)	0.957(68)



Summary

- Dispersion relations of D_s meson and ϕ meson are calculated
- Form factors on five lattice sets with different q^2 are calculated
- Extrapolate form factors to the physical pion mass and continuum limit
- Differential decay width and branching fraction results are calculated
- Preliminary work on $D \to K^* l\nu$ form factors are ongoing

Thank you for your attention!