Radiative Decay of the $D_{s0}(2317)$: $^3P_0$ Model and Molecular Contributions

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Outlook

Introduction

Experimentally

- D_{s0} ($J^P = 0^+$) charm-strange state: First observed very narrow peak near 2320 MeV in the $D_s^+ \to K^+K^-\pi^+$ and $D_s^+ \to K^+K^-\pi^+\pi^0$ channels by BaBar.

$$m_{D_{s0}} - (m_D + m_K) \sim 40 MeV$$

- $D_{sJ} \to$ Exotic $c\bar{s}$ structure?
- $D_{s0}(2317) \to$ Its mass is significantly lower than early quark model predictions ($\sim$ 2.48 GeV)

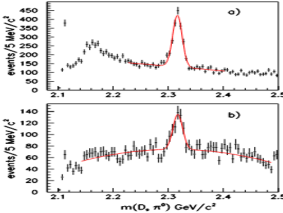
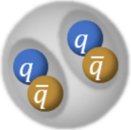


Figure: Babar: PRL 90, 242001(2003)

Theoretically

- Tetraquark/Molecular state: A bound state of DK (D^+K^0 or D^0K^+) near threshold.
- Strongly coupled to DK threshold: Its mass is just below the DK threshold ($\sim$2360 MeV), suggesting a hadronic molecule or dynamically generated state.



A proposed unified framework: Quark model + Effective Hadron interactions

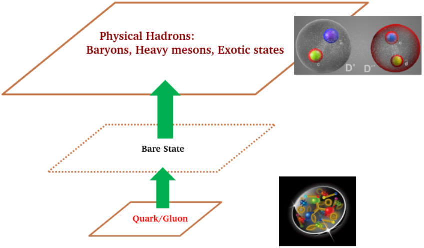
D_{sJ} properties: Motivation

There is important differences between the Quark Model (QM) and the experimental data.

D_{sJ} properties

J^P	State	M (MeV)	Γ (MeV)	QM	Γ
0^+	$D_{s0}^*(2317)$	2317.7±0.6	<3.8	2400∼ 2510	large
$1'^+$	$D'_{s1}(2460)$	2459.6±0.6	<3.5	2528∼ 2536	large
1^+	$D_{s1}(2536)$	2535.10±0.06	0.92±0.05	2543∼ 2605	small
2^+	$D_{s2}^*(2573)$	2569.1±0.8	16.9±0.8	2569∼ 2581	small

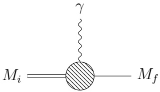
Motivation



Strong interaction scale:
What is the structure of a particle?
–What is inside? Low energy strong scale (Large scale–colorless Hadron interactions).
How affect this strong interactions to the masses, widths, the EM processes, etc,...?

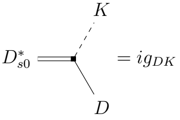
2. Theoretical framework

$D_{s0} \to D_s^* + \gamma$ Amplitude structure

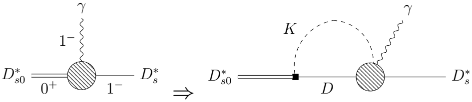


$$\langle f|V|i\rangle = \langle M_f,\gamma| - \int d^3x A_\mu(x) J^\mu(x) |M_i\rangle$$
$$= \sum\!\!\!\!\!\!\!\int_{l,q} \underbrace{\langle M_f,\gamma|\mathcal{H}_1(x)|l,q\rangle}_{\text{EM}-\text{Strong couplings}} \underbrace{\langle l,q|\mathcal{H}_2(x)|M_i\rangle}_{\textit{Strong couplings}}$$
$$= \sum_l \epsilon_\mu^* \mathcal{H}_l^\mu$$

The D_{s0} couples strongly only to its molecular components DK in a S-wave approximation of this channel. EM interaction interacts through molecular components.



$$\mathcal{L}^D_{\text{Mol}} = g_{DK} D^*_{s0}(D^0K^+ + D^+K^0) + \ldots$$

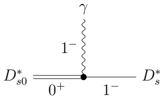


The tree and molecular contributions

In the molecular model a value for $Z_{DK} \sim 0.4$ suggest a contribution from also compact bare state.

$$\left|D_{s0}^{*+}(2317)\right\rangle = \cos\theta \left[\frac{Z_{DK}^{1/2}}{\sqrt{2}}|D^+K^0 + D^0K^+\rangle\right] + \sin\theta|c\bar{s}\rangle$$

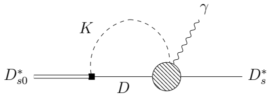
Bare tree EM transition



- Transition occurs throughout the EM E1 radiative decay for $0^+ \to 1^- + \gamma$
- We obtain the coupling of this transition from the relativized QM wave functions [Godfrey, Phys . Lett . B 568 (2003) 254 - 260]

$$\Gamma_{E1} = \frac{4}{3}\langle e_Q\rangle^2 \alpha k_\gamma C_{if} |\langle f|r|i\rangle|^2 \frac{E_f}{M_i}$$

Molecular EM transition



- **Strong interaction** : We implement an **HEFT** to fit the $D_{s0} - DK$ coupling by computing the residue of the T-matrix for a $DK - DK$ bound state
- **EM interaction**: We introduce **chiral** DK loops. We use a low energy EFT approach: Photon low energy coupling to the DK bound and final D_s^* states.

Hadron interaction Lagrangian terms

The effective Lagrangians responsible of the $\mathcal{D}\mathcal{D}^*\mathcal{V}$, $\mathcal{D}^*\mathcal{D}^*\mathcal{V}$, $\mathcal{D}\mathcal{D}^*\mathcal{P}$ and $\mathcal{D}^*\mathcal{D}^*\mathcal{P}$ interactions are [**Faessler:2007gv,Chen:2010re,Chen:2014sra**]

$$\begin{aligned}\mathcal{L}_{\mathcal{D}^{(*)}\mathcal{D}^{(*)}\mathcal{P}} &= i g_{\mathcal{D}^*_a\mathcal{D}_b P}\mathcal{D}^{*\mu\dagger}\mathcal{D}\overleftrightarrow{\partial}_\mu\mathcal{P} \\ &- g_{\mathcal{D}^*\mathcal{D}^*P}\epsilon_{\mu\nu\alpha\beta}\mathcal{D}^{*\nu\dagger}(\partial^\beta\mathcal{D}^{*\alpha})\partial^\mu\mathcal{P} + H.c..\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\mathcal{D}^{(*)}\mathcal{D}^{(*)}\mathcal{V}} &= -i g_{\mathcal{D}\mathcal{D}\mathcal{V}}\mathcal{D}_i^\dagger\overleftrightarrow{\partial}_\mu\mathcal{D}^j(\mathcal{V}^\mu)^i_j \\ &- 2f_{\mathcal{D}^*\mathcal{D}\mathcal{V}}\epsilon_{\mu\nu\alpha\beta}(\partial^\mu\mathcal{V}^\nu)^i_j\left(\mathcal{D}_i^\dagger\overleftrightarrow{\partial}^\alpha\mathcal{D}^{*\beta j} - \mathcal{D}_i^{*\beta\dagger}\overleftrightarrow{\partial}^\alpha\mathcal{D}^j\right) \\ &+ i g_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}\mathcal{D}_i^{*\nu\dagger}\overleftrightarrow{\partial}\mathcal{D}_\nu^{*j}(\mathcal{V}^\mu)^i_j \\ &+ 4i f_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}\mathcal{D}^{*\dagger}_{i\mu}(\partial^\mu\mathcal{V}^\nu)^i_j\mathcal{D}_\nu^{*j},\end{aligned}$$

The above coupling constants are given by

$$\begin{aligned}g_{D^*DP} &= \frac{g}{f_\pi}\sqrt{m_{D^*}m_D}, \qquad g_{D^*D^*P} = \frac{2g}{f_\pi}, \qquad f_{D^*DV} = \frac{\lambda g_V}{\sqrt{2}}, \\ f_{D^*D^*V} &= \frac{\lambda g_V}{\sqrt{2}}m_{D^*}, \qquad g_{DDV} = g_{D^*D^*V} = \frac{\beta g_V}{\sqrt{2}},\end{aligned}$$

with $\lambda = 0.56$ [**Chen:2010re**] and $g = 0.16$ [**Xiao:2016mho**] the gauge couplings. $g_V = m_\rho/f_\pi = 5.8$, $\beta = 0.9$ given the vector meson dominance.

Interaction Lagrangian terms

The Lagrangians containing the relevant electromagnetic couplings are [**Dong:2009uf,Faessler:2008vc**]

$$\mathcal{L}_{\mathcal{D}\mathcal{D}\gamma} = ieA_\mu D^- \overleftrightarrow{\partial^\mu} D^+ + ieA_\mu D_s^- \overleftrightarrow{\partial^\mu} D_s^+ ,$$

$$\begin{aligned} \mathcal{L}_{\mathcal{D}^*\mathcal{D}\gamma} &= \left(\frac{e}{4} g_{D^{*+}D^+\gamma} \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} D^{*+}_{\alpha\beta} D^- \right. \\ &\quad \left. + \frac{e}{4} g_{K^{*0}K^0\gamma} \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} K^{*0}_{\alpha\beta} \bar{K}^0 \right) + H.c., \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\mathcal{D}^*\mathcal{D}^*\gamma} &= -ieA_\mu \left(g^{\alpha\beta} D^{*-}_\alpha \overleftrightarrow{\partial^\mu} D^{*+}_\beta - g^{\mu\beta} D^{*-}_\alpha \partial^\alpha D^{*+}_\beta + g^{\mu\alpha} \partial^\beta D^{*-}_\alpha D^{*+}_\beta \right) \\ &\quad -ieA_\mu \left(g^{\alpha\beta} D^{*-}_{s\alpha} \overleftrightarrow{\partial^\mu} D^{*+}_{s\beta} - g^{\mu\beta} D^{*-}_{s\alpha} \partial^\alpha D^{*+}_{s\beta} + g^{\mu\alpha} \partial^\beta D^{*-}_{s\alpha} D^{*+}_{s\beta} \right) , \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\mathcal{K}^*\mathcal{K}\gamma} &= \left(\frac{e}{4} g_{K^{*+}K^+\gamma} \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} K^{*+}_{\alpha\beta} K^- \right. \\ &\quad \left. + \frac{e}{4} g_{D^{*0}D^0\gamma} \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} D^{*0}_{\alpha\beta} \bar{D}^0 \right) + H.c., \end{aligned}$$

$$\mathcal{L}_{\mathcal{K}\mathcal{K}\gamma} = ieA_\mu K^- \overleftrightarrow{\partial^\mu} K^+ ,$$

All the relevant coulplings involving $D^{(*)}K^{(*)}$-γD_s^* channels are already fixed by other physical quantities and processes
The electromagnetic coupling constants $g_{D^*D\gamma}$ are fixed to the experimental widths $\Gamma_\gamma(D^{*+} \to D^+\gamma) = 1.54$ keV [**PDG**] and $\Gamma_\gamma(D^{*0} \to D^0\gamma) = 26.04$ keV [PDG],[**Dong:2008gb**] respectively. These are

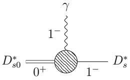
$$
\begin{aligned}
|g_{D^{*+}D^+\gamma}| &= 0.5\,\mathrm{GeV}^{-1} \\
|g_{D^{*0}D^0\gamma}| &= 2.0\,\mathrm{GeV}^{-1}
\end{aligned}
$$

The electromagnetic coupling constants $g_{K^*K\gamma}$ are fixed to the experimental widths $\Gamma_\gamma(K^{*+} \to K^+\gamma) = 50.29$ keV and $\Gamma_\gamma(K^{*0} \to K^0\gamma) = 116.29$ keV [PDG][**Faessler:2008vc**]. These are

$$
\begin{aligned}
|g_{K^{*+}K^+\gamma}| &= 0.836\,\mathrm{GeV}^{-1} \\
|g_{K^{*0}K^0\gamma}| &= 1.267\,\mathrm{GeV}^{-1}
\end{aligned}
$$

As is suggested in [**Faessler:2008vc**] one should consider the above coupling constants positively, excepting $g_{K^{*0}K^0\gamma} = -1.267\,\mathrm{GeV}^{-1}$

Gauge Invariant amplitude



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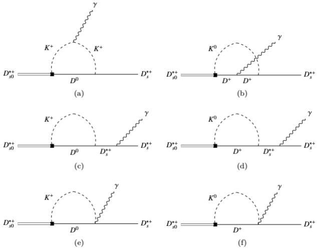

Feynman diagrams including DK loops

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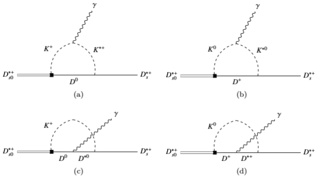
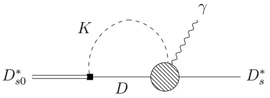


One-loop diagrams from $D^*D\gamma$ and $K^*K\gamma$ interaction couplings.

Splitting of Molecular coupling $D_{s0} - DK$



For the process $D^*_{s0}(2317) \to D^*_s \gamma$ the effective transition amplitude, $\mathcal{M}^m_{\mathrm{Total}}$, takes the general form

$$\mathcal{M}^m_{\mathrm{Mol}} = i\epsilon^{*\mu}_{\gamma} \mathcal{M}_{\mu\nu} \epsilon^{\nu}_{D^*_s},$$
$$\mathcal{M}_{\mu\nu} = eG_{\mathrm{loop}} \left(g_{\mu\nu} p_f \cdot k - p_{f,\mu} k_\nu \right)$$

The $D_{s0} - DK$ strong S-wave coupling, g_{DK}, simply factorizes from the EM decay amplitude

$$G_{loop} = i g_{DK} f(A_0[m_K^2], A_0[m_D^2], B_0[m_K^2, ...], C_0[m_D^2, ...]...)$$

in terms of well-known 1, 2 and 3 point one loop Passarino-Veltman integral expressions $\Longrightarrow$ Systematically UV regularized

Strong coupling $D_{s0} - DK$

HEFT $\Rightarrow$ describe the $D_s0(2317)$ as bare $c\bar{s}$ and bound hadron states

$$H = H_0 + H_I$$

$$H_0 = \sum_i |B_i\rangle m_i \langle B_i| + \sum_\alpha |\alpha(k_\alpha)\rangle \left[\sqrt{m_{\alpha_1}^2 + k_\alpha} + \sqrt{m_{\alpha_2}^2 + k_\alpha}\right] \langle \alpha(k_\alpha)|$$

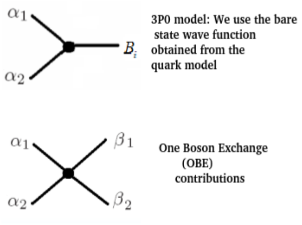
$|B_i\rangle$ bare state, bare mass m_i $\longleftrightarrow$ Quark-gluon based interactions

$|\alpha(k_\alpha)\rangle$ non-interaction channels $\longleftrightarrow$ Strong Hadron level interactions

$$H_I = \hat{g} + \hat{v}$$

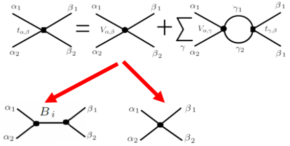
$$\hat{g} = \sum_\alpha \sum_i \left[|\alpha(k_\alpha)\rangle g_{i,\alpha}^\dagger \langle B_i| + |B_i\rangle g_{i,\alpha} \langle \alpha(k_\alpha)|\right]$$

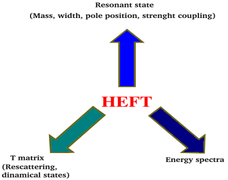
$$\hat{v} = \sum_{\alpha,\beta} |\alpha(k_\alpha)\rangle v_{\alpha\beta} \langle \beta(k_\beta)|$$



T matrix in HEFT

$$T(E,p_i,p_f) = V(E,p_i,p_f) + \int q^2 dq\, V(E,p_i,q) G(E,q) T(E,q,p_f)$$





$$V(E,p_i,p_f) = \frac{g(p_i)g(p_f)}{E - m_B} + f(p_i,p_f)$$

$$T(E,p_i,p_f) = t_{res}(E,p_i,p_f) + t_{bg}(E,p_i,p_f)$$

$$= \frac{\Gamma(E,p_i)\overline{\Gamma}(E,p_f)}{E - m_B - \Sigma(E)} + t_{bg}(E,p_i,p_f)$$

$$t_{bg}(E,p_i,p_f) = f(p_i,p_f) + \int q^2 dq\, f(p_i,q) G(E,q) t_{bg}(E,q,p_f)$$

If we can find t_{bg} everything is OK!

$$t_{bg} = (1 - fw)^{-1} f$$

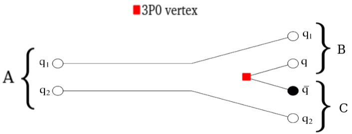
$$\Gamma(E,p_i) = g(p_i) + \int_0^\infty q^2 dq\, t_{bg}(E,p_i,q) G(E,q) g(q)$$

$$\overline{\Gamma}(E,p_f) = g(p_f) + \int_0^\infty q^2 dq\, g(q) G(E,q) t_{bg}(E,q,p_f)$$

$$\Sigma(E) = \int_0^\infty q^2 dq\, g(p_i) G(E,q) \overline{\Gamma}(E,p_f)$$

$$G(E,q) = \frac{1}{E - \sqrt{m_D^2 + q^2} - \sqrt{m_K^2 + q^2} + i\epsilon}$$

Bare strong coupling $D_{s0} - DK$: $^3P_0$ Model



$$g(|\vec{K}|) = \langle BC, \vec{K} l J | T^\dagger | A \rangle$$

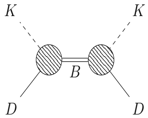
$$\begin{aligned} T^\dagger &= T^\dagger(^3P_0) \\ &= -3 \sum_{ij} \int d\vec{p}_i d\vec{p}_j \delta(\vec{p}_i + \vec{p}_j) C_{ij} F_{ij} V(\vec{p}_i - \vec{p}_j) [\chi_{ij} \times \mathcal{Y}_1(\vec{p}_i - \vec{p}_j)]^{(0)} b_i^\dagger(\vec{p}_i) d_j^\dagger(\vec{p}_j). \end{aligned}$$

This is the quark-pair creation operator of the $^3P_0$ model which considers the quantum numbers of vacuum (Micu, 1969). $V(\vec{p}_i - \vec{p}_j) = \gamma e^{-r_q^2(\vec{p}_i - \vec{p}_j)^2/6}$, where γ is a free parameter containing the creation probability of the quark-antiquark pair: coupling constant between the $|A\rangle$ states $|BC\rangle$. It can be fited from the strong decay of D_s states.

$$g(|\vec{K}|) = \gamma I_{A,[BC]}(|\vec{K}|) e^{-K^2/2\Lambda'^2}$$

Strong coupling $D_{s0} - DK$: Resonant molecular state

The pole positions of bound states or resonances are obtained by searching for the poles of the T -matrix in the complex plane.



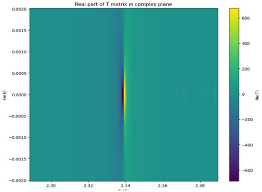
$$T(E, p_i, p_f) = \frac{\Gamma(E, p_i)\bar{\Gamma}(E, p_f)}{E - m_B - \Sigma(E)} + t_{bg}(E, p_i, p_f)$$

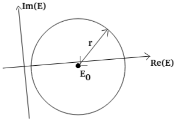
$$E_0 - m_B - \Sigma(E_0) = 0, \quad \text{pole at } E_0 = M_{D_{s0}} - i\frac{\Gamma_{\text{width}}}{2} = 2338.9 MeV < m_D + m_K$$

[Yang, Zhi, et. al. Phys.Rev.Lett. 128 (2022) 11, 112001]

The strong effective coupling constant is obtained from the residue of $T(E, p_i, p_f)$ for $p_i = p_f$ around E_0

$$Res(T(E, p_i, p_i)) = \frac{1}{2\pi i}\oint_C T(E(l), p_i, p_i)dl$$
$$= \frac{r}{2\pi}\int_0^{2\pi} T(E_0 + re^{i\theta})e^{i\theta}d\theta$$





Relating HEFT and Lagrangian EFT strong coupling g_{DK}

HEFT

- $^3P_0$ amplitude for bare $c\bar{s}$ D_{s0} state

$$A_{D_{s0}\to DK} = g(p_0) = \langle DK|\, T^\dagger(^3P_0)\, \left| D^*_{s0} \right\rangle$$

- The decay width in multipolar expansion for a $^3P_0$-like amplitude is given by

$$\Gamma = 2\pi \frac{E_D E_K}{M_{D_{s0}}} p_0 \left| A_{D_{s0}\to DK} \right|^2$$

$\mathcal{L}^D_{Mol}$ S-wave effective coupling

- The invariant amplitude given by the $\mathcal{L}^D_{Mol}$ term is

$$i\mathcal{M} = \langle DK|\, i\mathcal{L}^D_{\mathrm{Mol}} \left| D^*_{s0} \right\rangle = i g_{DK}\,.$$

- Thus, the strong decay width (standard PDG formula) is

$$\Gamma = \frac{p_0 g^2_{DK}}{8\pi M^2_{D_{s0}}}$$

Matching the both decay widths we get that the effective g_{DK} is related to the HEFT matrix element through

$$\boxed{g^2_{DK} = 16\pi^2 M_{D_{s0}} E_D E_K \left| A_{D_{s0}\to DK} \right|^2}$$

In the molecular model, the strong transition in the HEFT approach is the residue of the T-matrix in the physical pole, E_0

$$|A_{D_{s0}\to DK}|^2 = Res(T)$$

3. Results

Radiative decays $D_{s0} \to D_s^* + \gamma$: Molecular picture

Table: EM decay widths in units of keV (g_{DK} here represents only the absolute values in GeV) for the different contributions and Renormalization schemes.

Renor. scheme	**Only DK loops**	**Only $DK(K^*)$ and $D(D^*)K$ loops**	**All loops**
$\overline{\mathrm{MS}} - 1$	$0.0097 g_{DK}^2$	$0.25 g_{DK}^2$	$0.36 g_{DK}^2$
$\overline{\mathrm{MS}}$	$0.0097 g_{DK}^2$	$0.14 g_{DK}^2$	$0.22 g_{DK}^2$

- The only loops dependent on the renormalization scheme are those with $DD^*\gamma$ and $KK^*\gamma$ transition vertices.
- The set of pure DK loops and loops with $PV^*\gamma$ verices iterfere positively
- The main contribution to the EM transition comes from the transitions $P \to V\gamma$ in the molecular components. Individual DK loops are of similar size but they cancel among themselves.

Effective molecular coupling $D_{s0} - DK$

Table: Effective couplings for the $D_{s0} - DK$ interaction in the HEFT and the corresponding Lagrangian molecular S-wave approach

Case	**HEFT**	**Lagrangian EFT**
Bare $^3P_0$ model	$g(p_0) = -0.53\ GeV^{-1/2}$	$g_{DK} = 10.34\ GeV?$
Molecular model	$\widetilde{\Gamma}(E_0) = 0.587\ GeV^{-1/2}$	$g_{DK} = 10.62\ GeV$

$$\widetilde{\Gamma}(E_0) = \sqrt{Res(T)} \approx \mathcal{Z}_{D_{s0}}^{1/2}\Gamma(E_0)$$

$$\mathcal{Z}_{D_{s0}} = (1 - \frac{d\Sigma(E_0)}{dE})^{-1} = 0.41$$

Other study on molecular picture include:

- $g_{DK} = 9.0 \pm 0.5$ GeV **[Cleven M., et al. EPJ A 50 (2014) 149]**
- $g_{DK} = 10.77 \pm 13$ GeV **[Gil F., Molina R. PRD 109 (2024) 9, 096002]**

See also [Faessler A, et al. PRD 76 (2007) 014005] $\longmapsto$

Table 1. Coupling constant $g_{D_{s0}^* DK}$. The range of values for our results is due to the variation of $\Lambda_{D_{s0}^*}$ from 1 to 2 GeV.

Approach	$g_{D_{s0}^* DK}$ (GeV)
Ref. [51]	2.5 - 3.8
Ref. [72]	5.068
Ref. [73]	5.5 ± 1.8
Ref. [67]	$5.9^{+1.7}_{-1.6}$
Ref. [38]	6.0 - 7.8
Ref. [49]	$9.3^{+2.7}_{-2.1}$
Ref. [74]	< 9.86
Ref. [11]	10.203
Our results:	
NC case	$9.90 - 11.26$
LC case	8.98
NCHQL case	$11.52 - 16.22$
LCHQL case	11.52

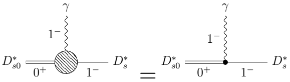
EM decays: Pure Molecular contributions

Kinematical inputs:
$M_{Ds0} = M_0 = 2.338$ GeV (T matrix pole)
$k_\gamma = 196$ MeV

Table: EM decay widths in units of keV

Renor. scheme	DK **loops**	$DK(K^*)$ **and** $D(D^*)K$ **loops**	**Full loops**
$\overline{\mathrm{MS}} - 1$	1.094	40.602	28.647
$\overline{\mathrm{MS}}$	1.094	24.813	15.564

Bare state radiative decay



Relativized QM

$$\Gamma_{E1} = \frac{4}{3}\langle e_Q\rangle^2 \alpha k_\gamma C_{if}\delta_{SS'}|\langle f|r|i\rangle|^2 \frac{E_f}{M_i}$$

Using the Godfrey-Isgur potential model

$$\langle f|r|i\rangle = \int dr r^2 R_f(r)^* r R_i(r) = 2.17\,\mathrm{GeV}^{-1}$$

[Godfrey, Phys.Lett.B 568 (2003) 254-260]

χEFT

$$\mathcal{L}_{ctc} = i\kappa F_{\mu\nu}\left(v^\mu D^*_{s0} D^{*\dagger}_{s\,\beta} + \cdots\right) + \mathrm{H.c.}$$

[Cleven, M. et al. Eur.Phys.J.A 50 (2014) 149] In the D^*_{s0} rest frame, $v = (1,0,0,0)$, the effective tree amplitude takes the form

$$\mathcal{M}_{ctc} = -i\kappa k_\gamma \epsilon^*_\gamma \epsilon^*_{D^*_s}$$

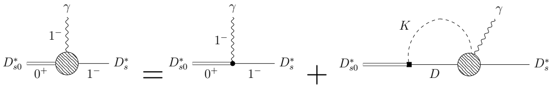
Thus, the decay amplitude (PDG)

$$\Gamma_{E1} = \frac{k_\gamma}{8\pi M_i^2} 2|ik_\gamma \kappa|^2$$

- Later on, in the interpretation of D^*_{s0} state as an hybrid ($c\bar{s} + DK$), we use $\mathcal{M}_{ctc}$ and fix κ from the GI wave functions

$$\kappa^2 = \frac{4\pi}{3}(2E_f)(2M_i)C_{if}\langle e_Q\rangle^2 \alpha\delta_{SS'}|\langle f|r|i\rangle|^2$$

EM Decay widths: Bare + molecular contributions



$$\mathcal{M}_{Full} = \mathcal{M}_{ctc} + \mathcal{M}_{\mathrm{Mol}}$$

D_{s0} **Interpretation**	Contributions	Γ_γ (keV)
Pure Bare state	$[c\bar{s}] \to D_s^* \gamma$	1.25
Pure Molecular	$[DK] \to D_s^* \gamma$	1.09
	$[DK] \to [D^{(*)} K^{(*)}]\gamma \to D_s^* \gamma$	15.59
	$[DK] \to$ All loops$\to D_s^* \gamma$	24.96
Bare+Molecular	$[c\bar{c} + DK] \to D_s^* \gamma$	0.005
	$[c\bar{s} + DK] \to [D^{(*)} K^{(*)}]\gamma \to D_s^* \gamma$	8.01
	$[c\bar{s} + DK] \to$ All loops$\to D_s^* \gamma$	15.04

Table: Radiative decay widths $D_{s0}^* \to D_s^* \gamma$ for all the models.

The exp. value is $\Gamma_\gamma^{exp} < 6\% \Gamma_{Full}$, where $\Gamma_{Full} < 3.8$ MeV. Then $\Gamma_\gamma^{exp} < 228$ keV

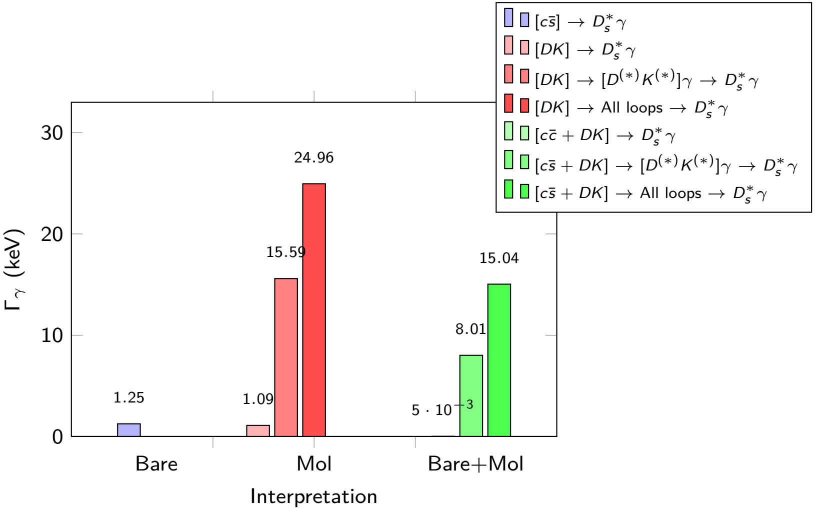


Figure: Radiative decay widths $D_{s0}^* \to D_s^*\gamma$ for different interpretations.

Comparison with other works

Approach	$\Gamma_\gamma(D_s0^* \to D_s^*\gamma)$ (keV)
Bare [PRD 69, 114008 (2004)]	0.2
Bare [PLB 570, 180 (2003)]	$\simeq 1$
Bare [Mol] PRD 72, 094004 (2005)	1 [25]
CQM **Bare [EPJ C 47, 445 (2006)]**	1.1
Bare [PRD 75, 034013 (2007)]	1.3-9.9
Bare [EPJ A 21, 501 (2004)]	$\lesssim 1.4$
Bare [PRD 68, 054024 (2003)]	1.74
Bare [S. Godfrey PLB 568, 254 (2003)]	1.9
Bare [Phys.Rev.D 72 (2005) 074004]	4-6
[AIP Conf. Proc. 717, 716 (2004)]	21
pure [DK] Mol. [PRD 76 (2007) 014005]	0.47 -1.41
unquenched frame **Mol. [Bare+Mol.] PRD 110 (2024) 9, 094037**	32.06 [4.27]
χEFT **[Bare+Mol.] EPJ A 50 (2014) 149**	9.4
χEFT **[Bare+Mol.] EPJ A 58 (2022)70**	3.7
Our results:	
Bare case	1.25
Molecular case	24.96
Bare + Mol.	15.04

4. Outlook and further studies

Summary and Outlook

- Molecular and bare components play a crucial role in explaining $D_{s0}(2317)$**mass below the DK threshold** due to self energy effects.
- Understanding the **strong coupling** of the bound state D_{s0} is a key aspect (scales as $\sim g_{DK}$) in the study of its radiative decay.
- Bare contributions and pure DK contibutions are of similar size. However they interfere destructively and the **main contribution comes from $PV\gamma$ couplings** in the molecular structure.
- The present approach can be extended to radiative decays for the D^*K bound state partner $D_{s1}(2460)$, and their bottom analogous B^*_{s0} and B_{s1} where radiative decays are supposed less suppressed.
- Results are far below the upper limit of the experimental value which is around 10 times larger in comparison. Branching ratios may offer a more accurate comparison for theoretical results.

Thank you!