湖南師範大學
HUNAN NORMAL UNIVERSITY

第八届强子谱强子结构研讨会

Emergence of charm-strange dibaryons with negative parity via baryon-baryon interactions

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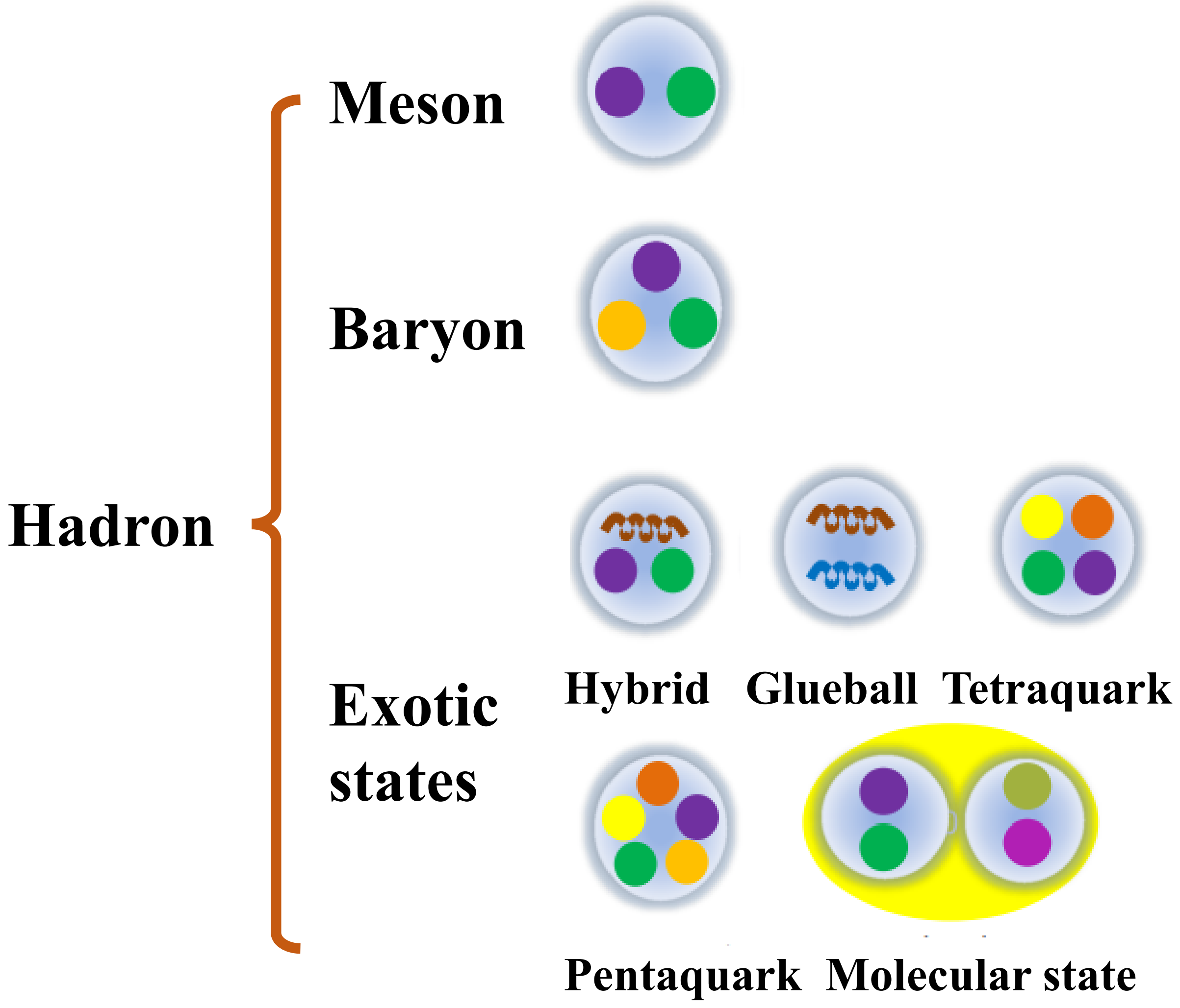
Outline

1.1 QCD color singlet



- Experiments： reported a series of new hadron structures.

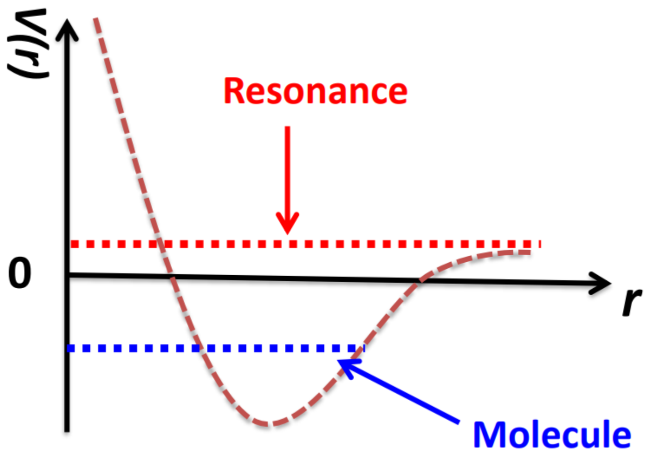
- Two key characteristics:
  1. Inconsistent with the predictions of the conventional quark mode.
  2. Close to the threshold of a pair of traditional hadrons.

1.2 Why do we study P system?

Predicting possible molecular states of nucleons with $\Xi_c N$, $\Xi'_c N, \Xi^*_c N$

[Phys. Rev. D 110 (2024) 5, 054040]

- **S-D wave interactions ;coupled channel effects.**
- **Result : Search for hadronic molecule candidates and predict the existence of resonance states.**
- **P− wave systems:** $\frac{l(l+1)}{r^2}$



[Phys. Rev. Lett. 133 (2024) 24, 241903]

- **The paper identifies G(3900) as the first P-wave $D\bar{D}^*/\bar{D}D^*$ molecular resonance.**

Outline

2.1 Effective potentials

P-wave interactions ; coupled-channel effects.

Systems: $\Xi_c N, \Lambda_c \Sigma, \Xi_c' N, \Sigma_c \Lambda, \Xi_c^* N, \Sigma_C^* \Lambda, \Sigma_C \Sigma, \Sigma_C^* \Sigma$.

- **For the interactions between the light baryons and the light mesons:**

$$\mathcal{L}_{BB\sigma} = g_{BB\sigma} \bar{B} \sigma B, \quad (4)$$

$$\mathcal{L}_{BBP} = \frac{g_{BBP}}{m_P} \bar{B} \gamma^5 \gamma^\mu \partial_\mu P B, \quad (5)$$

$$\mathcal{L}_{BBV} = g_{BBV} \bar{B} \gamma^\mu V_\mu B - \frac{f_{BBV}}{2m_B} \bar{B} \sigma^{\mu\nu} \partial_\nu V_\mu B. \quad (6)$$

$g_{\sigma NN}$=8.46,
$g_{\pi NN}$=13.07,
$g_{\rho NN}$ =3.25,
$f_{\rho NN}$=19.82.

Phys. Rept.149, 1 (1987).
Phys.Rev.C63, 024001(2001).
Phys. Rev. C 81, 065201 (2010)

➢ S U(3) symmetry

- **Effective Lagrangians for the interaction between hadrons and light mesons:**

$$\mathcal{L}_{\mathcal{B}_{\bar{3}}} = l_B \langle \bar{\mathcal{B}}_{\bar{3}} \sigma \mathcal{B}_{\bar{3}} \rangle + i\beta_B \langle \bar{\mathcal{B}}_{\bar{3}} v^\mu (\mathcal{V}_\mu - \rho_\mu) \mathcal{B}_{\bar{3}} \rangle, \quad (1)$$

$$\mathcal{L}_{\mathcal{B}_6} = l_S \langle \bar{\mathcal{S}}_\mu \sigma \mathcal{S}^\mu \rangle - \frac{3}{2} g_1 \varepsilon^{\mu\nu\lambda\kappa} v_\kappa \langle \bar{\mathcal{S}}_\mu \mathcal{A}_\nu \mathcal{S}_\lambda \rangle + i\beta_S \langle \bar{\mathcal{S}}_\mu v_\alpha (\mathcal{V}^\alpha - \rho^\alpha) \mathcal{S}^\mu \rangle + \lambda_S \langle \bar{\mathcal{S}}_\mu F^{\mu\nu}(\rho) \mathcal{S}_\nu \rangle, \quad (2)$$

$$\mathcal{L}_{\mathcal{B}_{\bar{3}}\mathcal{B}_6} = i g_4 \langle \bar{\mathcal{S}}^\mu \mathcal{A}_\mu \mathcal{B}_{\bar{3}} \rangle + i\lambda_I \varepsilon^{\mu\nu\lambda\kappa} v_\mu \langle \bar{\mathcal{S}}_\nu F_{\lambda\kappa} \mathcal{B}_{\bar{3}} \rangle + h.c.. \quad (3)$$

$$l_s = -2l_B = -\frac{2}{3} g_{\sigma NN}, \; g_1 = \frac{2\sqrt{2}}{3} g_4 = \frac{2\sqrt{2} f_\pi g_{\pi NN}}{5M_N}, \; \beta_s g_V = -2\beta_B g_V = -4 g_{\rho NN},$$

$$\lambda_s g_V = -\sqrt{8} \lambda_I g_V = -\frac{6(g_{\rho NN} + f_{\rho NN})}{5M_N}$$

Estimated from the quark model

➢ heavy quark symmetry; chiral symmetry; hidden local symmetry.

2.1 Effective potentials

$$\mathcal{B}_{\bar{3}} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix} \quad P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega'}{\sqrt{6}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega'}{\sqrt{6}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\sqrt{\frac{2}{3}}\omega' \end{pmatrix}$$

$$\mathcal{B}_6^{(*)} = \begin{pmatrix} \Sigma_c^{(*)++} & \frac{\Sigma_c^{(*)+}}{\sqrt{2}} & \frac{\Xi_c^{(\prime,*)+}}{\sqrt{2}} \\ \frac{\Sigma_c^{(*)+}}{\sqrt{2}} & \Sigma_c^{(*)0} & \frac{\Xi_c^{(\prime,*)0}}{\sqrt{2}} \\ \frac{\Xi_c^{(\prime,*)+}}{\sqrt{2}} & \frac{\Xi_c^{(\prime,*)0}}{\sqrt{2}} & \Omega_c^{(*)0} \end{pmatrix} B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

- **P and V denote the pseudoscalar meson matrix and vector meson matrix under SU(3) symmetry, respectively.**
- **B denotes the SU(3) baryon octet;**
- **B_3 represents the triplet heavy-flavor baryons;**
- **B_6 represents the sextet heavy-flavor baryons.**

2.2 One-boson-exchange (OBE) model

Scattering amplitude

$M(\mathrm{a}+\mathrm{b}\to\mathrm{c}+\mathrm{d})$

Interaction Lagrangian

Feynman Rules

→

Effective potential in momentum space

$V(\vec{q}) \approx -M(a+b\to c+d)/\sqrt{2m_a 2m_b 2m_c 2m_d}$

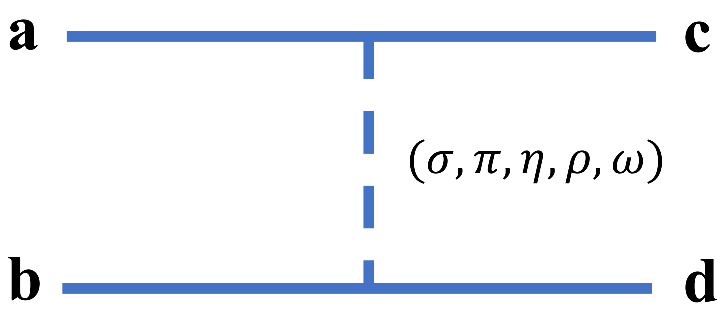
Breit approximation

→

Effective potential in coordinate space

$V(r)=\int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} V(\vec{q}) F^2(q^2, m_E^2)$

Fourier transformation



Form factor $F(q^2, m_E^2) = \frac{\Lambda^2-m^2}{\Lambda^2-q^2}$ Λ (One free parameter $< 2G$eV), m and q are the cutoff, mass and four-momentum of the exchanged meson, respectively.

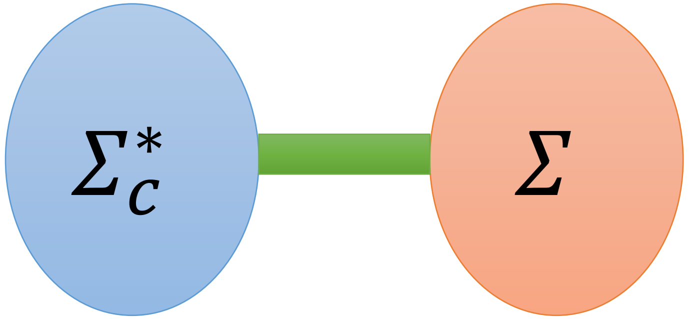
Outline

1. **Background**
2. **The one-boson-exchange model**
3. **Results and discussions**
4. **Summary**

3.11 Hadronic molecular states

Loosely bound molecular states in the single system

- Method：the multi-Gaussian expansion method.
- $I(J^p) = 0(0^-)$ The $\Sigma_C^*\Sigma$ single-channel system;
- $\boldsymbol{R} = \boldsymbol{R}_{\Sigma_C^*} + \boldsymbol{R}_{\Sigma}$ （the size of the system should be larger than the size of all component hadrons）.
- Mass $\boldsymbol{M} = M_{\Sigma_C^*} + M_{\Sigma}-|E|$,E: binding energy.



$I(J^P)$	Λ	E	r_{RMS}	$\Sigma_c^*\Sigma(^3P/^5P)$
$0(0^-)$	1.050	−1.15	1.98	100/–
	1.060	−3.95	1.46	100/–
	1.070	−7.21	1.26	100/–

3.12 $\Xi_c N$ System

- Single-channel system :

  No bound state solution.
- Coupled channel bound states:
  - The obtained binding energies are very sensitive with the cutoff value.
  - The dominant channels are not the lowest channel $\Xi_c N$, which leads to the small size of these bound states.
  - Thus, we cannot recommend these coupled channel bound states as prime molecular candidates.

TABLE VI: The bound state solutions (the binding energy E, the root-mean-square radius r_{RMS}, and the probabilities p_i for all the discussed channels) for the $\Xi_c N$ systems with $I(J^P) = 0(0^-), 0(1^-), 0(2^-), 1(0^-), 1(1^-)$, and $1(2^-)$ after considered the coupled channel effects. Here, the unites for the cutoff Λ, the binding energy E, and the root-mean-square radius r_{RMS} are GeV, MeV, and fm, respectively.

$I(J^P)$	Λ	E	r_{RMS}	$\Xi_c N(^1P/^3P)$	$\Lambda_c \Sigma(^1P/^3P)$	$\Xi'_c N(^1P/^3P)$	$\Sigma_c \Lambda(^1P/^3P)$	$\Xi^*_c N(^3P/^5P)$	$\Sigma^*_c \Lambda(^3P/^5P)$	$\Sigma_c \Sigma(^1P/^3P)$	$\Sigma^*_c \Sigma(^3P/^5P)$
$0(0^-)$	1.108	−1.70	1.23	–/**53.90**	–/–	–/8.08	–/–	**37.95**/–	–/–	–/0.07	0.01/–
	1.110	−5.04	0.93	–/**50.56**	–/–	–/8.36	–/–	**41.00**/–	–/–	–/0.07	0.01/–
	1.112	−8.63	0.82	–/**48.24**	–/–	–/8.48	–/–	**43.20**/–	–/–	–/0.07	0.01/–
$0(1^-)$	1.074	−0.14	2.05	**31.59/20.52**	–/–	**31.55**/3.88	–/–	**12.13**/0.02	–/–	0.13/0.13	0.05/ ∼0.00
	1.076	−3.22	0.97	**29.14/18.84**	–/–	**34.20**/4.24	–/–	**13.19**/0.02	–/–	0.15/0.15	0.06/ ∼0.00
	1.078	−6.52	0.83	**27.93/18.05**	–/–	**35.44**/4.43	–/–	**13.74**/0.02	–/–	0.16/0.17	0.06/ ∼0.00
$0(2^-)$	1.108	−0.05	2.53	–/**52.40**	–/–	–/6.50	–/–	**41.00**/0.09	–/–	–/0.01	0.01/0.01
	1.110	−3.57	0.92	–/**47.62**	–/–	–/7.10	–/–	**45.18**/0.08	–/–	–/0.01	0.01/0.01
	1.112	−7.35	0.78	–/**45.45**	–/–	–/7.34	–/–	**47.11**/0.07	–/–	–/0.01	0.01/0.01
$1(0^-)$	1.103	−1.84	0.55	–/5.66	–/**34.88**	–/0.09	–/3.76	2.45/–	**20.31**/–	–/4.40	**28.45**/–
	1.104	−4.71	0.52	–/5.38	–/**34.74**	–/0.09	–/3.76	2.48/–	**20.48**/–	–/4.40	**28.67**/–
	1.105	−7.61	0.50	–/5.20	–/**34.57**	–/0.09	–/3.75	2.50/–	**20.63**/–	–/4.40	**28.86**/–
$1(1^-)$	1.092	−3.83	0.51	4.21/0.01	**29.70**/ ∼0.00	2.68/ ∼0.00	**25.58**/ ∼0.00	∼0.00/0.03	∼0.00/0.96	**36.72**/ ∼0.00	∼0.00/0.11
	1.093	−6.68	0.50	4.09/ ∼0.01	**29.57**/ ∼0.00	2.70/ ∼0.00	**25.68**/ ∼0.00	∼0.00/0.03	∼0.00/0.97	**36.85**/ ∼0.00	∼0.00/0.10
	1.094	−9.56	0.49	4.00/0.01	**29.44**/ ∼0.00	2.71/ ∼0.00	**25.76**/ ∼0.00	∼0.00/0.03	∼0.00/0.98	**36.96**/ ∼0.00	∼0.00/0.10
$1(2^-)$	1.103	−1.73	0.54	–/5.31	–/**33.80**	–/0.28	–/3.15	2.31/0.02	**21.29**/0.01	–/4.23	**29.57**/0.03
	1.104	−4.64	0.51	–/5.07	–/**33.69**	–/0.28	–/3.16	2.33/0.02	**21.42**/0.01	–/4.25	**29.74**/0.03
	1.105	−7.58	0.49	–/4.93	–/**33.55**	–/0.29	–/3.17	2.35/0.02	**21.53**/0.01	–/4.26	**29.88**/0.03

3.13 $\Xi_c^* N$ System

- In the cutoff $\Lambda \le 2.00$ GeV, we can obtain the bound state solutions for the single $\Xi_c^* N$ states with $0(0^-)$, $0(1^-)$, and $0(2^-)$.
- For other quantum numbers, we did not find hadronic molecular states and thus made corresponding selections.

TABLE X: The bound state solutions (the binding energy E, the root-mean-square radius r_{RMS}, and the probabilities p_i for all the discussed channels) for the $\Xi_c^* N$ systems with $I(J^P) = 0(0^-), 0(1^-), 0(2^-), 1(0^-), 1(1^-)$, and $1(2^-)$. Here, the unites for the cutoff Λ, the binding energy E, and the root-mean-square radius r_{RMS} are GeV, MeV, and fm, respectively.

Single channel	$I(J^P)$	Λ	E	r_{RMS}	$\Xi_c^* N(^3P/^5P)$	$I(J^P)$	Λ	E	r_{RMS}	$\Xi_c^* N(^3P/^5P)$
	$0(0^-)$	1.285	−1.37	1.59	100/–	$0(1^-)$	1.340	−2.22	1.31	91.29/8.70
		1.290	−3.90	1.22	100/–		1.345	−6.50	1.00	93.08/6.91
		1.295	−6.71	1.06	100/–		1.350	−11.30	0.86	94.25/5.74
	$0(2^-)$	1.320	−0.21	2.99	74.75/25.24	$1(0^-)$	–/–	–/–	–/–	–/–
		1.327	−4.27	1.27	86.21/13.78	$1(1^-)$	–/–	–/–	–/–	–/–
		1.334	−9.47	0.98	91.19/8.80	$1(2^-)$	–/–	–/–	–/–	–/–

Coupled channel	$I(J^P)$	Λ	E	r_{RMS}	$\Xi_c^* N(^3P/^5P)$	$\Sigma_c^*\Lambda(^3P/^5P)$	$\Sigma_c\Sigma(^1P/^3P)$	$\Sigma_c^*\Sigma(^3P/^5P)$
	$0(0^-)$	1.280	−0.06	3.35	**94.47**/–	–/–	–/ ~0.00	5.52/–
		1.290	−5.11	1.13	**93.03**/–	–/–	–/~0.00	6.96/–
		1.300	−11.21	0.93	**92.15**/–	–/–	–/~0.00	7.83/–
	$0(1^-)$	1.339	−1.73	1.41	**87.64**/8.94	–/–	0.29/0.09	2.75/0.26
		1.342	−4.08	1.13	**88.60**/7.68	–/–	0.32/0.10	2.99/0.28
		1.345	−6.62	1.00	**89.24**/6.79	–/–	0.34/0.12	3.19/0.29
	$0(2^-)$	1.323	−0.59	2.24	**71.52**/22.25	–/–	–/0.07	5.77/0.36
		1.329	−4.01	1.28	**78.44**/13.63	–/–	–/0.09	7.37/0.44
		1.335	−8.18	1.02	**81.63**/9.32	–/–	–/0.10	8.45/0.49
	$1(0^-)$	1.276	−1.25	0.60	6.28/–	**43.15**/–	–/4.83	**45.74**/–
		1.279	−5.72	0.55	5.90/–	**43.20**/–	–/4.95	**45.94**/–
		1.282	−10.33	0.53	5.73/–	**43.18**/–	–/5.06	**46.02**/–
	$1(1^-)$	1.604	−2.21	0.53	0.17/0.75	0.87/**29.66**	**54.39**/4.13	0.06/9.96
		1.607	−5.84	0.51	0.15/0.68	0.88/**29.91**	**54.02**/4.10	0.07/10.18
		1.610	−9.56	0.50	0.13/0.63	0.88/**30.15**	**53.64**/4.06	0.07/10.40
	$1(2^-)$	1.277	−1.24	0.59	5.12/0.23	**39.99**/0.52	14.43/–	**37.56**/2.16
		1.280	−6.04	0.54	4.87/0.17	**40.13**/0.45	14.44/–	**37.83**/2.07
		1.283	−11.01	0.53	4.79/0.14	**40.22**/0.40	14.43/–	**38.03**/2.00

3.14 $\Sigma_C\Sigma$ System

- **For the single-channel system of $\Sigma_C\Sigma$ with $I(J^P)0(1^-)$ bound state solutions exist. The coupled-channel effect has a positive effect on this molecular state.**
- **The $\Sigma_C\Sigma/\Sigma_C^*\Sigma$ coupled molecular state with $1(1^-)$.**

TABLE XII: The bound state solutions (the binding energy E, the root-mean-square radius r_{RMS}, and the probabilities p_i for all the discussed channels) for the $\Sigma_c\Sigma$ systems with $I(J^P) = 0(0^-), 0(1^-), 0(2^-), 1(0^-), 1(1^-)$, and $1(2^-)$. Here, the unites for the cutoff Λ, the binding energy E, and the root-mean-square radius r_{RMS} are GeV, MeV, and fm, respectively.

Single channel	$I(J^P)$	Λ	E	r_{RMS}	$\Sigma_c\Sigma(^3P/^5P)$	
	$0(1^-)$	1.100	−0.10	2.86	100/0.00	
		1.110	−4.57	1.19	100/0.00	
		1.120	−9.84	1.00	100/0.00	
Couple channel	$I(J^P)$	Λ	E	r_{RMS}	$\Sigma_c\Sigma(^3P/^5P)$	$\Sigma_c^*\Sigma(^3P/^5P)$
	$0(0^-)$	1.150	−2.90	0.68	–/1.63	**98.36**/–
		1.155	−7.47	0.67	–/1.70	**98.30**/–
		1.160	−12.18	0.65	–/1.76	**98.23**/–
	$0(1^-)$	0.930	−0.04	5.15	**29.25/68.48**	0.37/1.89
		0.950	−4.90	1.65	**38.55/57.89**	0.72/2.82
		0.970	−11.83	1.30	**43.64/52.22**	1.05/3.07
	$0(2^-)$	1.165	−2.19	0.77	–/13.70	**82.27**/4.03
		1.170	−6.89	0.69	–/12.71	**83.44**/3.83
		1.175	−11.79	0.67	–/12.21	**84.18**/3.62
	$1(0^-)$	–/–	–/–	–/–	–/–	–/–
	$1(1^-)$	1.850	−0.37	2.80	5.41/**91.23**	0.22/3.12
		1.900	−3.68	1.57	6.25/**89.31**	0.24/4.18
		1.950	−7.86	1.28	6.81/**88.00**	0.23/4.94
	$1(2^-)$	–/–	–/–	–/–	–/–	–/–

3.21 Resonances

Resonances { Shape-type / Feshbach-type

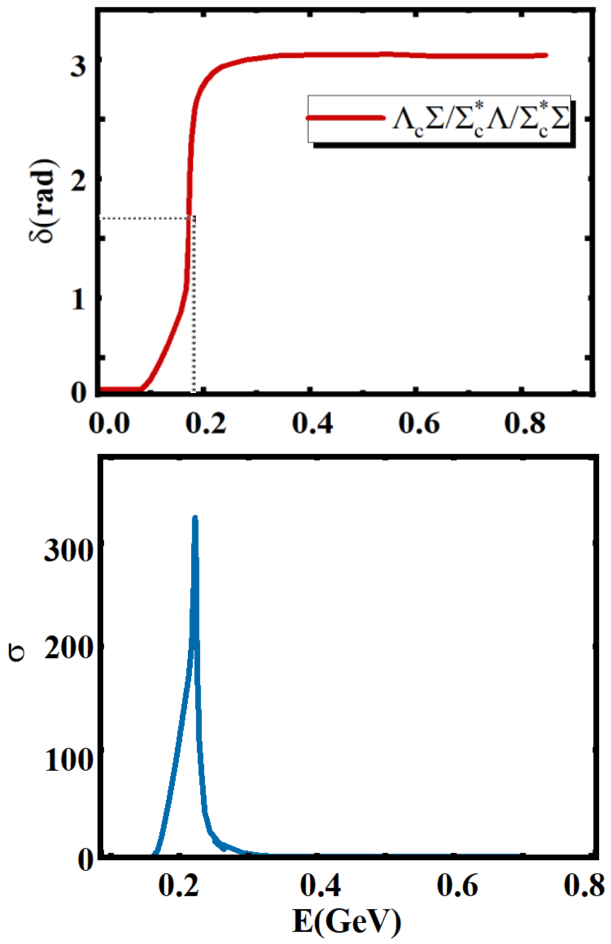
- For the :$\Lambda_c\Sigma, \Sigma_C^*\Lambda, \Sigma_c^*\Sigma$. interactions with $0(0-)$;
- This leads to themaximum of the scattering cross

section,

$$\sigma_t = \frac{4\pi}{2\mu E}\sum_{l=0}^{\infty}(2l+1)\sin^2\delta_l(E)$$

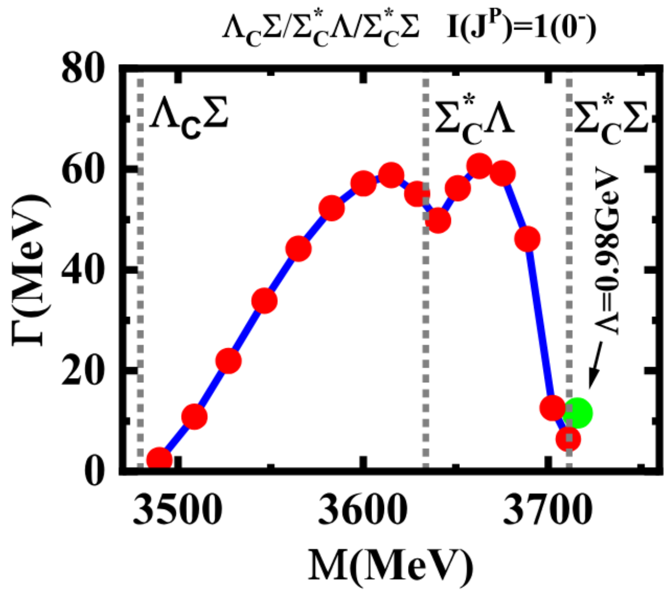
$$\delta_l(E_r) = (n+1/2)\pi, \quad n = 0, 1, 2\cdots$$

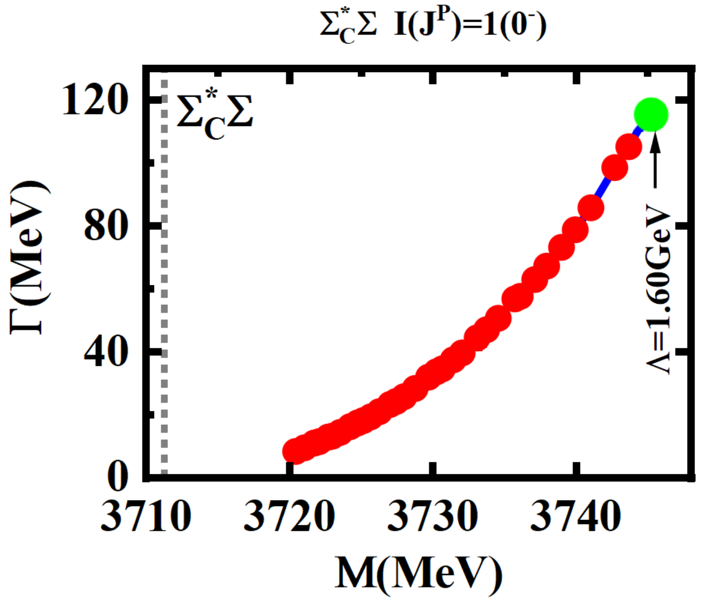
- The resonant width is defined as $\Gamma_r = 2/\left(\frac{d\delta}{dE}\right)_{E_r}$,
- Msaa: $M = M_{\Lambda_c} + M_{\Sigma} + E$。



Λ=1.07GeV E= 0.14GeV Γ=0.06GeV

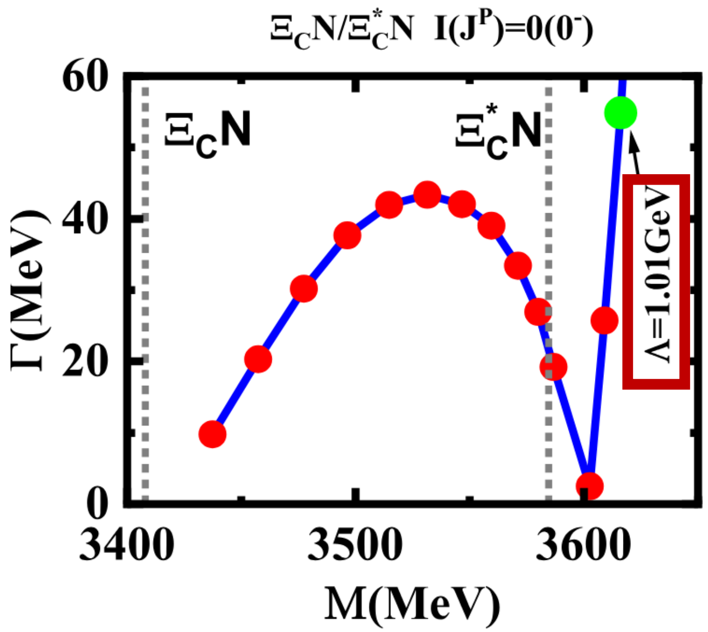
3.21 $I(J^P) = 1(0^-)$ $\Sigma_C^*\Sigma$ Resonances





- We can not obtain the loosely bound state solutions for the $\Sigma_C^*\Sigma$ molecule with $1(0^-)$ in the reasonable cutoff region.
- Thus , this state is a shape-type resonance dominated by the $\Sigma_C^*\Sigma$ channel.

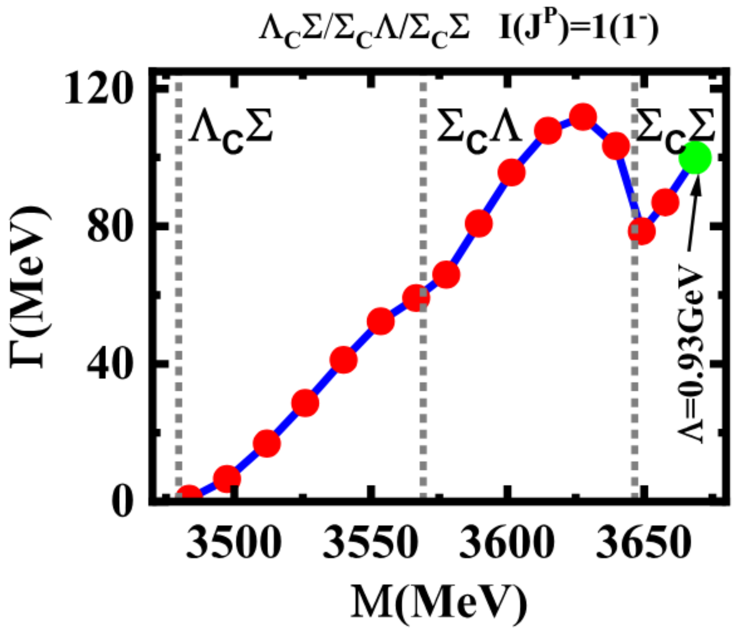
3.22 $I(J^P) = 0(0^-)$ $\Xi_c^* N$ bound state

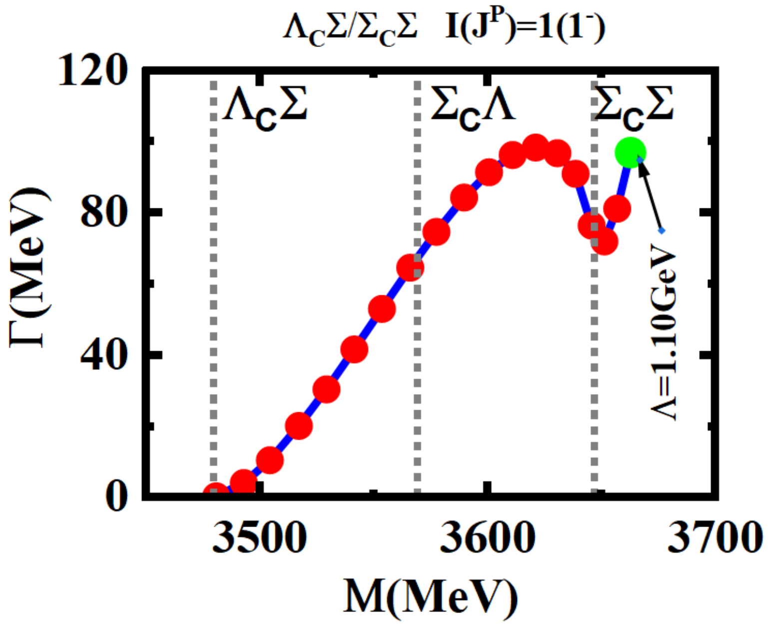


$I(J^P)$	Λ	E	r_{RMS}	$\Xi_c^* N(^3P/^5P)$
$0(0^-)$	1.285	−1.37	1.59	100/–
	1.290	−3.90	1.22	100/–
	1.295	−6.71	1.06	100/–

- **The obtained resonance here is not an independent state, but corresponds to the $\Xi_c^* N$molecule with 0(0−).**
- **The current results can provide the important information of the total decay width for the $\Xi_c^* N$bound state.**

3.23 $I(J^P) = 1(1^-)\Lambda_C\Sigma\ /\Sigma_C\Sigma$ Feshbach-type resonance



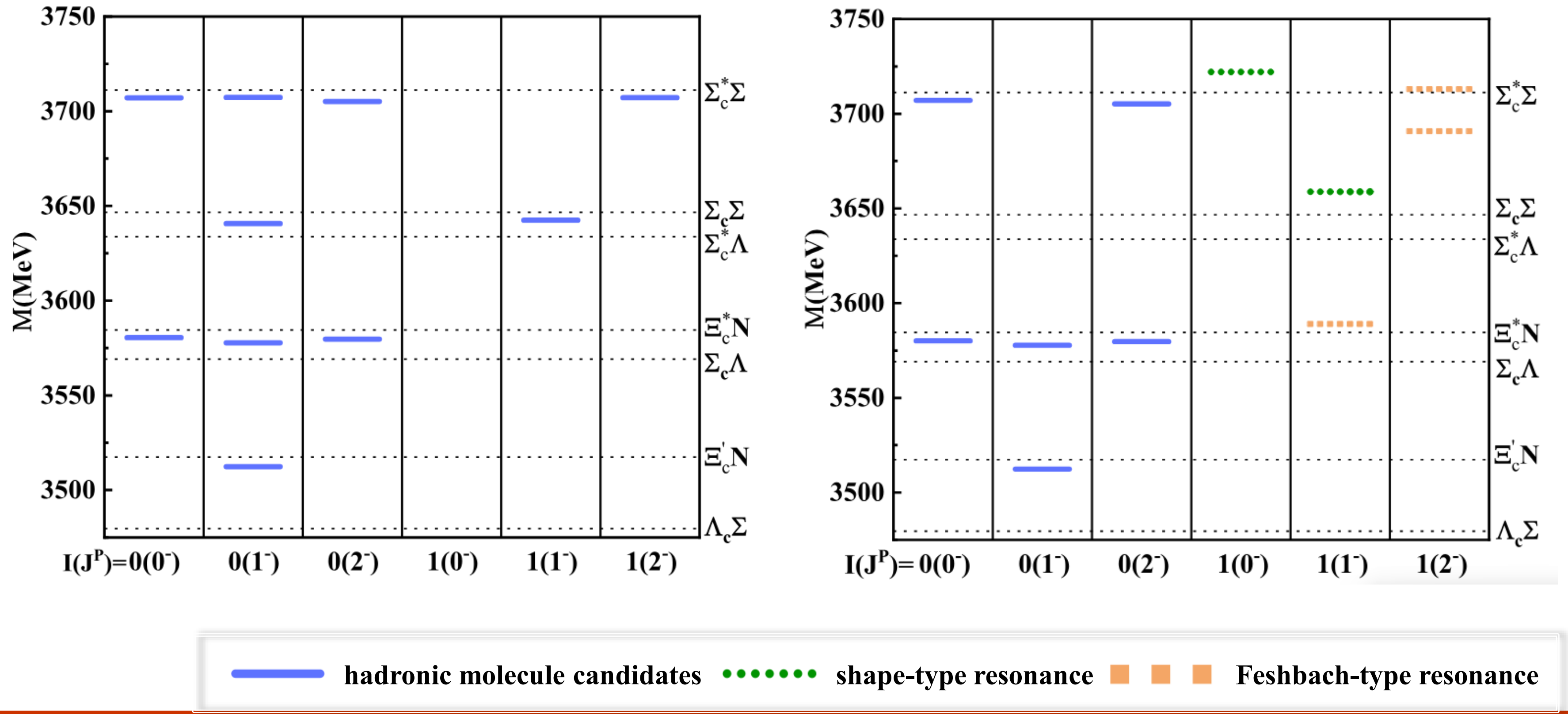


- The $\Lambda_C\Sigma$ / $\Sigma_C\Sigma$ coupled interactions and the $\Lambda_C\Sigma$ / $\Sigma_C\Lambda$/ $\Sigma_C\Sigma$ coupled interactions are very similar.
- This indicates that the resonance is a Feshbach-type resonance, where both the $\Lambda_C\Sigma$ and $\Sigma_C\Sigma$ channels play important roles.

Outline

4. Summary

In this work, we systematically study the P−wave interactions between the charmed and light baryons by using the OBE model. The research results are as follows：



Thanks for your attention