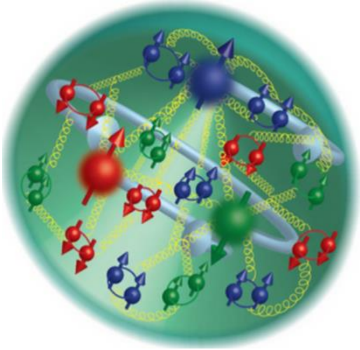


Relativistic electromagnetic structure of a moving spin-1/2 hadron

Yi Chen

Based on:
[PRD 106, 116024 (2022)]
[PRD 107, 096003 (2023)]
[arXiv: 25xx.xxxx (to appear)]

thanks to:
Cédric Lorcé, Qun Wang, Yang Li, Bing-Song Zou, …

第8届 | 强子谱和强子结构研讨会
The 8th Symposium on Hadron Spectrum and Hadron Structure
2025年7月11--16日 中国·桂林

Outline

1. Introduction and motivations

2. Quantum phase-space approach and relativistic spatial distributions

3. Relativistic electromagnetic structure of a moving spin-1/2 hadron

4. Summary and outlook

致谢：
特别感谢“第八届强子谱和强子结构研讨会”组委会给我一个展示近期研究工作的机会！谢谢各位老师！

This Symposium: presentations on hadron structures

■ 2025年7月12日

14:30–15:00	Dispersive Determination of Pion and Nucleon Gravitational Form Factors	姚德良(湖南大学)
16:30–17:00	Production, Decay and CP violation of baryon–antibaryon pairs	曹须(近代物理研究所)

■ 2025年7月13日

8:30–8:55	轻赝标介子的形状因子及其精细结构研究	程山(湖南大学)
9:20–9:40	Electromagnetic form factors of Ω^-	付东彦(近代物理研究所)
8:30–8:55	重味介子结构的格点 QCD 研究	张其安(北京航空航天大学)
8:55–9:20	Light cone structure of light meson and light baryon on Lattice QCD	华俊(华南师范大学)
9:20–9:40	Form factors in semileptonic decay $D_s\to\phi\ell\nu$ from lattice QCD	樊高峰(南京大学)

...

Please see also their nice presentations!

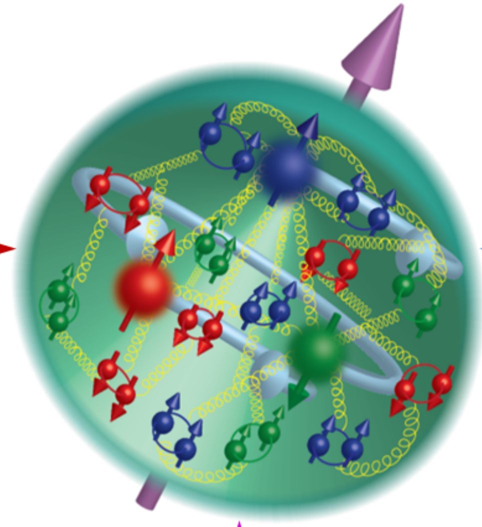
(I sincerely apologize if I may have missed listing your nice presentation.)

Probing the internal structures of a hadron

EM probes

Virtual photon γ*

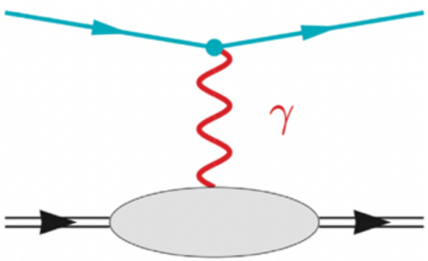
Vector



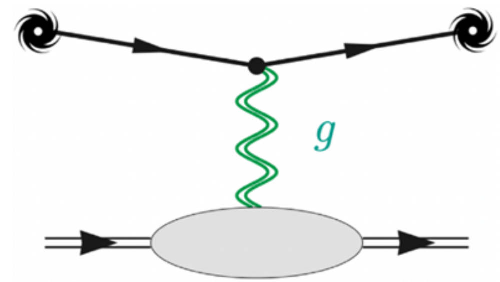
Tensor

Gravity-like probes

Virtual gluons g*



Axial-vector



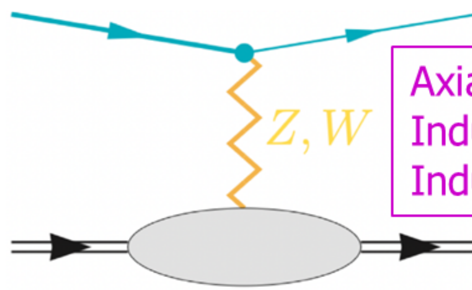
Electric charge: $Q=G_E(0)$
Magnetic moment: $\mu=G_M(0)$

$$Q_p = 1, \quad Q_n = 0$$
$$\mu_p \approx 2.79, \quad \mu_n \approx -1.91$$

Weak probes

Virtual weak bosons $Z^0{}^*/W^{\pm}{}^*$

Mass: M, $A(0)=1$
Spin: J, $J(0)=1/2$
D-term: $D(0)$



Axial charge: $G_A(0)$
Induced pseudoscalar charge: $G_P(0)$
Induced pseudotensor charge: $G_T(0)$

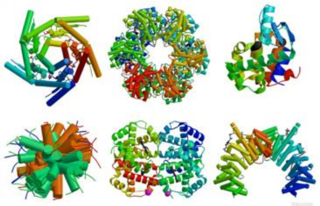
[M. Polyakov & P. Schweitzer, IJMPA 33(2018)1830025]

Structure dictates properties

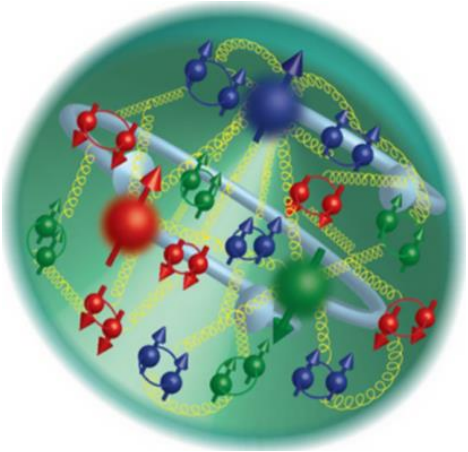
- **Analogy:**

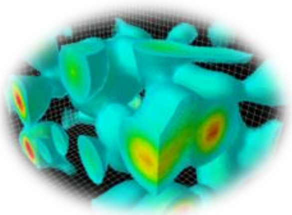
"Symmetry dictates interactions"

"Structure dictates properties"



different proteins





QCD vacuum

- **Hadron structures are highly non-trivial and complicated!**

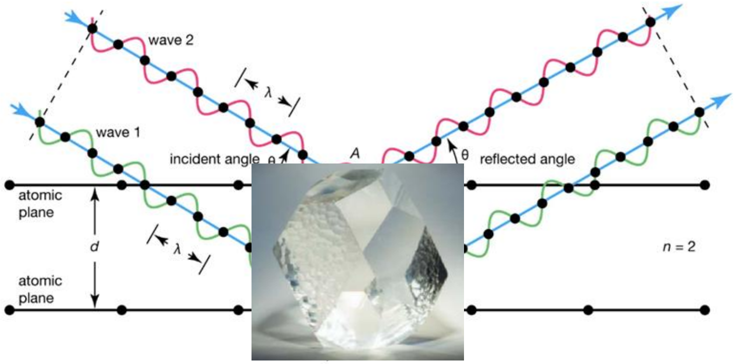
(1). Hadron structures are closely associated with the nonperturbative QCD dynamics between the internal quark and gluon degrees of freedom.

(2). The QCD vacuum itself is also highly non-trivial, due to quantum fluctuations (loop effects, pair creations and annihilations), non-trivial topologies, instanton/sphaleron transitions, and etc.

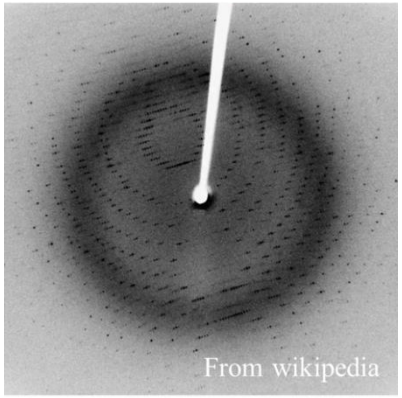
(3). On top of the QCD dynamics and non-trivial QCD vacuum structure, hadron structures are also affected by electromagnetic and weak interactions.

How to probe the internal structure of a system?

- **Classically, e.g. using the x-ray diffraction for crystals**



Diffraction pattern

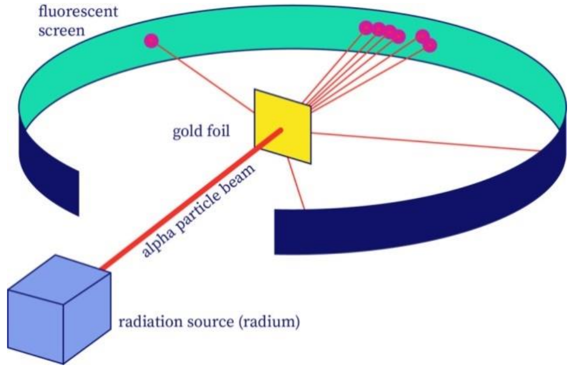


$\propto |\mathcal{A}_{\rm scatt}|^2$

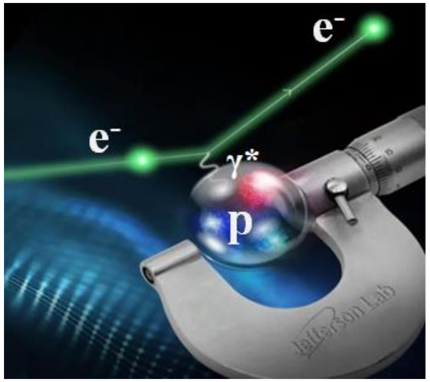
Scattering amplitude: $\mathcal{A}_{\rm scatt} \propto F(\boldsymbol{q}) = \int \mathrm{d}^3 r \, e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \rho(\boldsymbol{r})$ $\quad \boldsymbol{q} = \boldsymbol{k} - \boldsymbol{k}'$

Form factor **Target distribution**

- **Modern Rutherford scattering:**

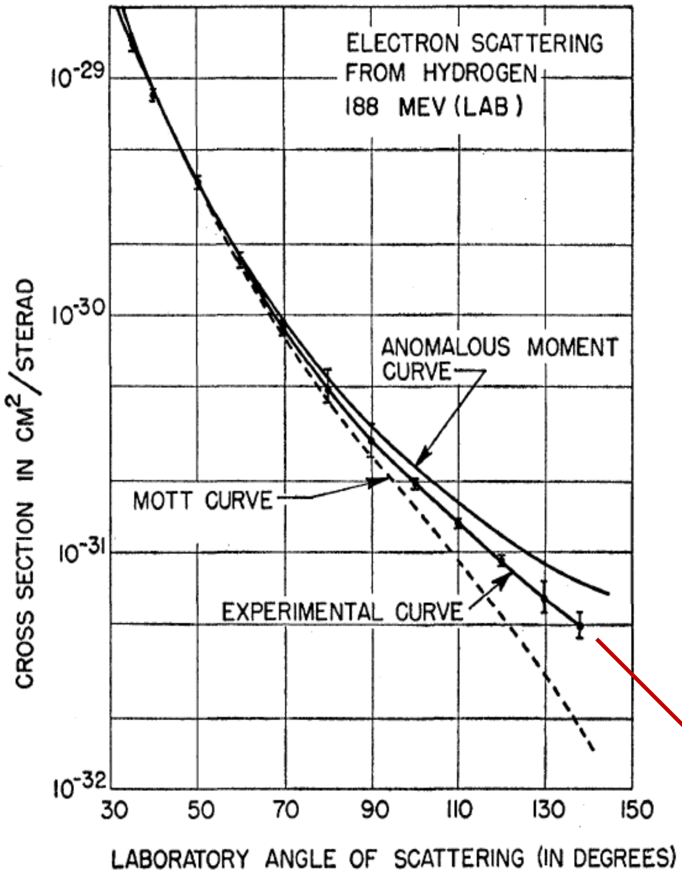


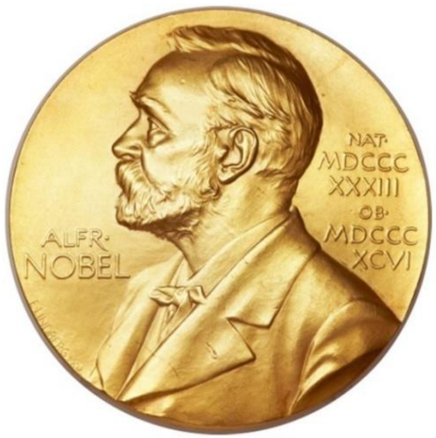
Rutherford alpha scattering experiment (1909)



Jefferson lab (JLab) e-p scattering exp.

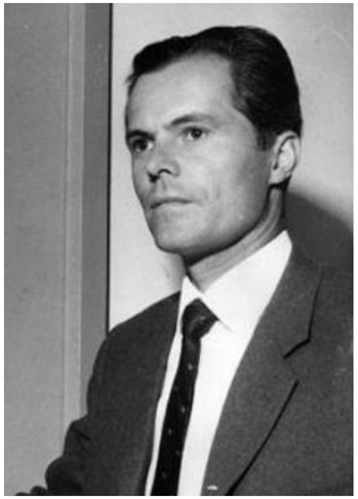
Electron-nucleon elastic scattering





Nobel prize in physics (1961)

Robert Hofstadter



Rudolf Mössbauer

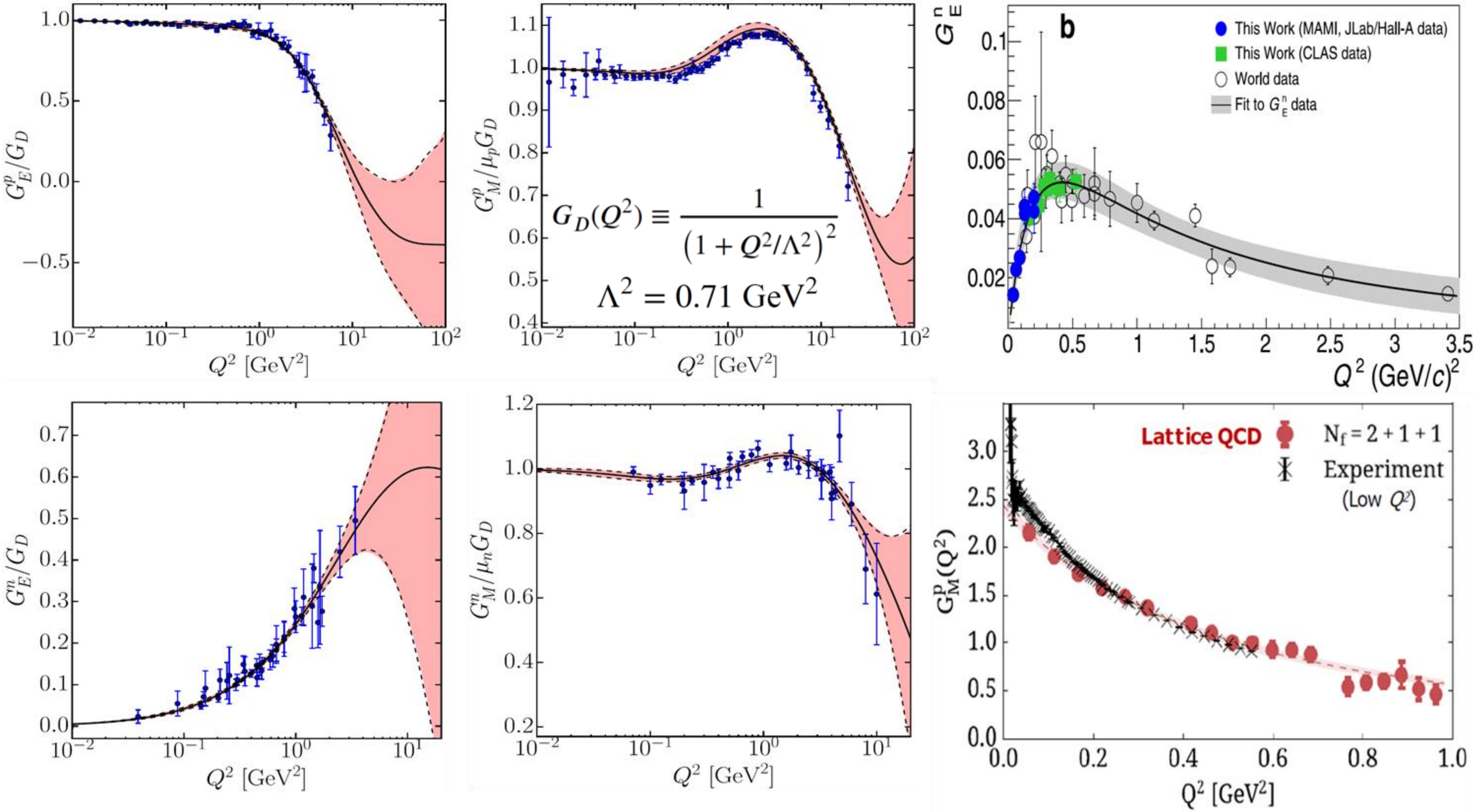
➡ **Proton is not a point-like particle!**

$r_E^p \approx 0.78\ \mathrm{fm}$ (**Sachs electric charge radius**)

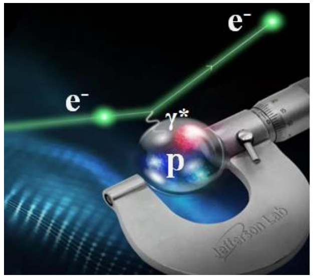
$$\boxed{r_E = \sqrt{\langle \boldsymbol{r}_E^2 \rangle}} \qquad \langle \boldsymbol{r}_E^2 \rangle \equiv -\frac{6}{G_E(0)} \frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2}\bigg|_{Q^2=0}$$

[R. Hofstadter and R. W. Mcallister. “Electron Scattering from the Proton”, PR. 98, 217 (1955)]
[Ernst, Sachs, and Wali, PR 119, 1105 (1960); Sachs, PR 126, 2256 (1962)]
[W. Xiong et al. (PRad), Nature (London) 575, 147 (2019)] $r_E^p \approx (0.831 \pm 0.014)\ \mathrm{fm}$

Nucleon electromagnetic form factors (FFs)



[Z. Ye et al., PLB777, 8 (2018)]
[Bradford et al., Nucl. Phys. B, Proc. Suppl. 159, 127 (2006)]
[Atac et al., Nat. Commun. (2021) 12, 1759]
[A1 Collaboration, PRL 105, 242001 (2010)]
[Alexandrou et al., PRD 100, 014509 (2019)]
[W. Xiong et al. (PRad), Nature 575, 147 (2019)] …



$$J^\mu(\boldsymbol{r}) = ?$$

Different methods/formalisms for density interpretation

- **Traditional/tilted light-front formalism:**

  [M. Burkardt, Int. J. Mod. Phys. A 18, 173 (2003)]...
  [M. Burkardt, PRD 62 (2000) 071503(R) [erratum: PRD 66 (2002) 119903(E)]...
  [G. Miller, PRL 99 (2007) 112001; PRC 79 (2009) 055204; Rev. Nucl. Part. Sci. 60, 1 (2010); PRC 99 (2019) 035202]...
  [Carlson & Vanderhaeghen, PRL 100 (2008) 032004]
  [Y. Guo, X. Ji and K. Shiells, NPB 969 (2021) 115440]
  [A. Freese and G. Miller, PRD 105 (2022) 014003; PRD 108 (2023) 034008; PRD 107 (2023) 074036; PRD 108 (2023) 094026]...

- **Dimensional counting+ZAMF (zero average momentum frame):**

  [G. N. Feming, Phys. Reality Math. Descrip. 357 (1974)]
  [Epelbaum, Gegelia, Lange, Meißner & Polyakov, PRL 129 (2022) 012001]
  [Panteleeva, Epelbaum, Gegelia & Meißner, PRD 106 (2022) 056019; EPJC 83 (2023) 617; JHEP 07 (2023) 237; arXiv: 2412.05050 [hep-ph]]
  [C. E. Carlson, arXiv:2208.00826 [hep-ph]]
  [Alharazin, Sun, Epelbaum, Gegelia & Meißner, JHEP 02, 163 (2023)]]...

- **Quantum phase-space approach/Wigner distribution:**

  [Belitsky, Ji & Yuan, PRD 69 (2004) 074014]
  [Cédric Lorcé, PRL 125 (2020) 232002; PRD105 (2022) 096032]
  [J.-Y. Kim & Hy.-Ch. Kim, PRD104 (2021) 074003; ......]
  [YC, Cédric Lorcé, PRD106 (2022) 116024; PRD107 (2023) 096003]
  [YC, Yang Li, Cédric Lorcé, Qun Wang, PRD 110, L091503 (2024); JHEP 04(2025)232]...

- **Traditional/covariant moments expansion:**

  [X. Ji and Y. Liu, PRD 106 (2022) 034028]
  [Yang Li, Wen-bo Dong, Yi-liang Yin, Qun Wang & James Vary, PLB 838 (2023) 137676; arXiv: 2405.06892 [hep-ph]]...

- **Light-front Wigner distribution:** [Y. Han, T. Liu & B. Ma, PLB 830 (2022) 137127]...

Quantum phase-space approach

- **Phase-space representation:**

$$\boldsymbol{P} = (\boldsymbol{p}' + \boldsymbol{p})/2 \qquad \boldsymbol{R} = (\boldsymbol{r} + \boldsymbol{r}')/2$$

$$\langle \Psi | \hat{O}(x) | \Psi \rangle = \sum_{s',s} \int \frac{\mathrm{d}^3 P}{(2\pi)^3} \mathrm{d}^3 R \, \rho_\Psi^{s's}(\boldsymbol{R}, \boldsymbol{P}) \, \langle \hat{O} \rangle_{\boldsymbol{R},\boldsymbol{P}}^{s's}(x)$$

$$\widetilde{\Psi}(\boldsymbol{p}, s) \equiv \frac{\langle p, s | \Psi \rangle}{\sqrt{2p^0}}$$

- **Wigner distribution:**

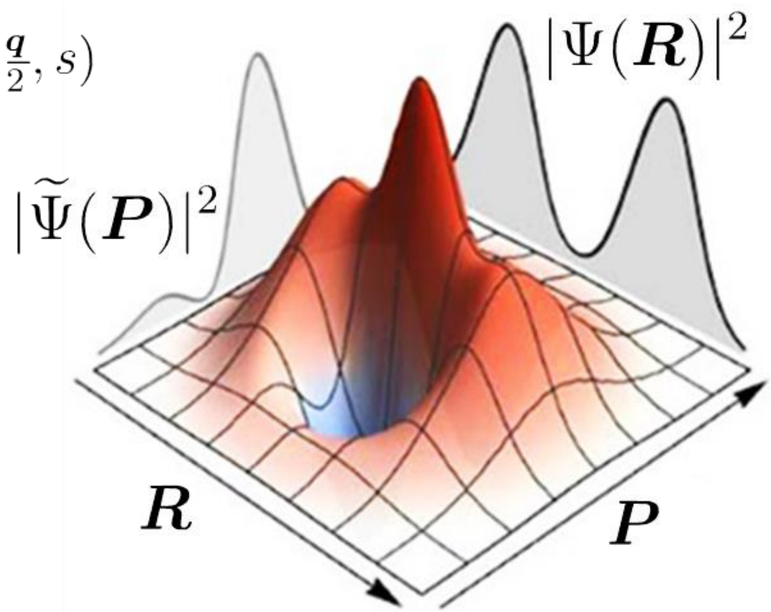
$$\rho_\Psi^{s's}(\boldsymbol{R}, \boldsymbol{P}) \equiv \int \mathrm{d}^3 z \, e^{-i\boldsymbol{P}\cdot\boldsymbol{z}} \, \Psi^*(\boldsymbol{R} - \tfrac{\boldsymbol{z}}{2}, s') \Psi(\boldsymbol{R} + \tfrac{\boldsymbol{z}}{2}, s)$$

$$= \int \frac{\mathrm{d}^3 q}{(2\pi)^3} e^{-i\boldsymbol{q}\cdot\boldsymbol{R}} \, \widetilde{\Psi}^*(\boldsymbol{P} + \tfrac{\boldsymbol{q}}{2}, s') \widetilde{\Psi}(\boldsymbol{P} - \tfrac{\boldsymbol{q}}{2}, s)$$

- **Quasi-probabilistic densities:**

$$\int \mathrm{d}^3 R \, \rho_\Psi^{s's}(\boldsymbol{R}, \boldsymbol{P}) = \widetilde{\Psi}^*(\boldsymbol{P}, s') \widetilde{\Psi}(\boldsymbol{P}, s),$$

$$\int \frac{\mathrm{d}^3 P}{(2\pi)^3} \rho_\Psi^{s's}(\boldsymbol{R}, \boldsymbol{P}) = \Psi^*(\boldsymbol{R}, s') \Psi(\boldsymbol{R}, s).$$



[Wigner, PR40 (1932) 749]
[Hillery, O'Connell, Scully, Wigner, PR106 (1984) 121]
[Bialynicki-Birula, Gornicki, Rafelski, PRD 44 (1991) 1825]
[Belitsky, Ji, and Yuan, Phys. Rev. D 69, 074014 (2004)]
[Cédric Lorcé, PRL 125 (2020) 232002]
[YC & Cédric Lorcé, PRD 107, 096003 (2023)]

Relativistic spatial distributions

- **Internal distribution** (for a state « localized » in phase-space) $\boldsymbol{r} = \boldsymbol{x} - \boldsymbol{R} = (\boldsymbol{b}_\perp, r_z)$

$$\langle \hat{O} \rangle^{s's}_{\boldsymbol{R},\boldsymbol{P}}(t, \boldsymbol{r}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{i\Delta^0 t - i\boldsymbol{\Delta}\cdot\boldsymbol{r}}\, \frac{\langle P + \frac{\Delta}{2}, s' | \hat{O}(0) | P - \frac{\Delta}{2}, s \rangle}{2\sqrt{p'^0 p^0}}$$

- **Elastic condition:**

$$\Delta^0 = p'^0 - p^0 = \frac{\boldsymbol{\Delta}\cdot\boldsymbol{P}}{P^0} = 0$$

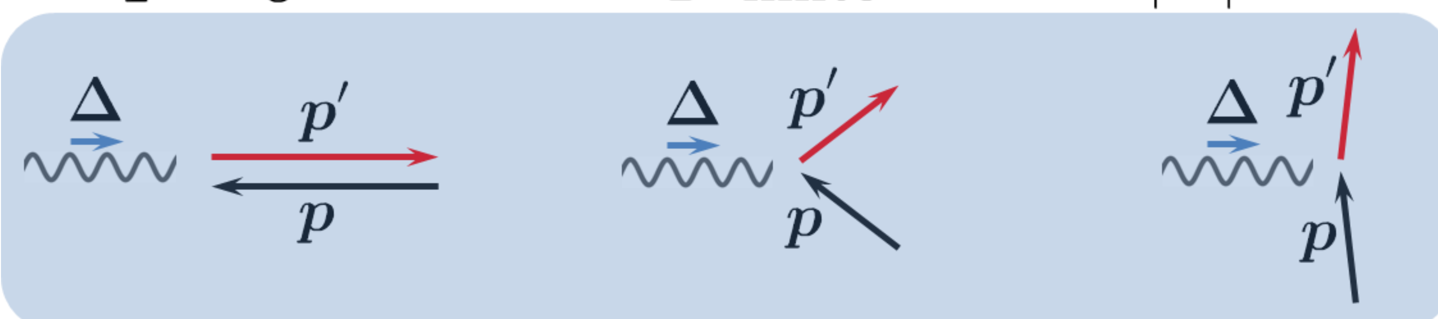
→ **No energy transfer** → **Time independence**

$$\Delta = p' - p \qquad P = (p' + p)/2$$

[Lorcé, Mantovani, Pasquini, PLB776 (2018) 38]
[Lorcé, EPJC78 (2018) 9, 785]
[Lorcé, Moutarde, Trawinski, EPJC79 (2019) 89]
[YC & Cédric Lorcé, PRD 107, 096003 (2023)]

- **Three cases:**

$\boldsymbol{P} = 0$ $\quad$ $\boldsymbol{P}$ finite $\quad$ $|\boldsymbol{P}| \to \infty$



BF (Breit frame) | **EF** (Elastic frame) | **IMF** (Infinite momentum frame) $\overset{\approx}{\longleftrightarrow}$ **LF** (Light-front)

Relativistic BF, EF & LF distributions

- **Breit frame (BF) distributions:**

$$O_B(\boldsymbol{r}) \equiv \langle \hat{O} \rangle_{\mathbf{0},\mathbf{0}}^{s'_B s_B}(\boldsymbol{r}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \frac{\langle p'_B, s'_B | \hat{O}(0) | p_B, s_B \rangle}{2P_B^0}\bigg|_{\boldsymbol{P}_B=\mathbf{0}}$$

The standard definition frame for different 3D mean-square radii.

- **Elastic frame (EF) distributions:**

$$O_{\mathrm{EF}}(\boldsymbol{b}_\perp; P_z) \equiv \int \mathrm{d}r_z \, \langle \hat{O} \rangle_{\mathbf{0},\boldsymbol{P}}^{s' s}(\boldsymbol{r})$$

$$= \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \frac{\langle p', s' | \hat{O}(0) | p, s \rangle}{2P^0}\bigg|_{\Delta_z = |\boldsymbol{P}_\perp| = 0}$$

$$O_{\mathrm{EF}}(\boldsymbol{b}_\perp; 0) = \int \mathrm{d}r_z \, O_B(\boldsymbol{r}) \qquad O_{\mathrm{IMF}}(\boldsymbol{b}_\perp) = \lim_{P_z\to\infty} O_{\mathrm{EF}}(\boldsymbol{b}_\perp; P_z)$$

- **Light-front (LF) distributions:**

$$O_{\mathrm{LF}}(\boldsymbol{b}_\perp; P^+) \equiv \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \frac{{}_{\mathrm{LF}}\langle p', \lambda' | \hat{O}(0) | p, \lambda \rangle_{\mathrm{LF}}}{2P^+}\bigg|_{\Delta^+ = |\boldsymbol{P}_\perp| = 0}$$

$$a^\pm = (a^0 \pm a^3)/\sqrt{2}, \qquad a^\mu = [a^+, a^-, \boldsymbol{a}_\perp]$$

Matrix elements of a general spin-1/2 hadron

◆ **Hadronic matrix elements:**

$$\langle p', s'|\hat{j}^\mu(0)|p, s\rangle = e\bar{u}(p', s')\Gamma^\mu(p, p')u(p, s)$$

Vertex function: $$\Gamma^\mu(p', p) = \gamma^\mu \underbrace{F_1(Q^2)}_{\text{Dirac}} + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}\underbrace{F_2(Q^2)}_{\text{Pauli}}$$

$\Delta = p' - p$

$P = (p' + p)/2$

$$= \frac{MP^\mu}{P^2}\underbrace{G_E(Q^2)}_{\text{Electric}} + \frac{i\epsilon^{\mu\alpha\beta\lambda}\Delta_\alpha P_\beta \gamma_\lambda \gamma^5}{2P^2}\underbrace{G_M(Q^2)}_{\text{Magnetic}}$$

(1). Electric charge:

$q_e = F_1(0) = G_E(0)$

(2). Magnetic moment:

$\mu = G_M(0) \quad \kappa = F_2(0)$

$= q_e + \kappa$

◆ **Sachs electromagnetic FFs:**

$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2) \qquad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

◆ **Classical electromagnetic four-current:**

$$J^\mu = J^\mu_c + J^\mu_P, \qquad J^\mu_P = \partial_\alpha P^{\alpha\mu}$$

Electric Magnetic

$$J^0 = \rho_c + \rho_P$$

$$\boldsymbol{J} = \rho_c \boldsymbol{v} + \boldsymbol{\nabla} \times \boldsymbol{M} + \partial_0 \boldsymbol{\mathcal{P}}$$

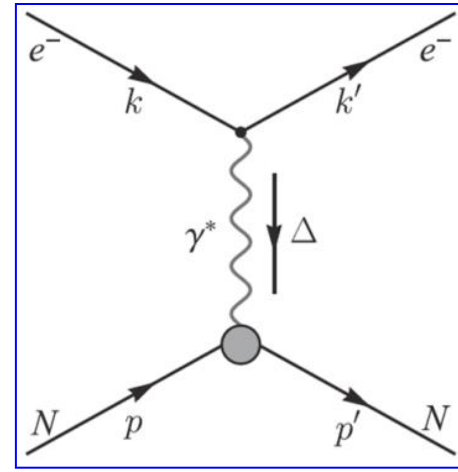
$$\rho_P = -\boldsymbol{\nabla} \cdot \boldsymbol{\mathcal{P}}$$

$$\rho_M = -\boldsymbol{\nabla} \cdot \boldsymbol{M}$$

[Cédric Lorcé, PRL125 (2020) 232002]
[Yang Li, Wen-bo Dong, Yi-liang Yin, Qun Wang & James Vary, PLB 838 (2023) 137676]
[YC & Cédric Lorcé, PRD 106, 116024 (2022); PRD 107, 096003 (2023)]

Elastic electron-hadron scattering

- **A typical reaction of elastic electron-hadron scattering:**



$$e^-(k, r) + h(p, s) \to e^-(k', r') + h(p', s')$$

Four-momentum transfer: $\Delta = p' - p = q = (\Delta^0, \boldsymbol{\Delta})$

$$Q^2 = -q^2 = -\Delta^2 \geq 0$$

- **Spin-0 hadron:**

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) \Big/ \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} = F^2(Q^2)$$

Breit frame $\longrightarrow$ $\Delta^0 = 0$, $Q^2 = \boldsymbol{q}^2$

$$F(\boldsymbol{q}^2) = \int \mathrm{d}^3 r \, e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \rho_e(\boldsymbol{r})$$

- **Spin-1/2 hadron:**

$$\tilde{\epsilon} \to \epsilon = [1 + 2(1+\tau)\tan^2\frac{\theta}{2}]^{-1}$$

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) \Big/ \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} = \frac{1}{1+\tau}\left[G_E^2(Q^2) + \frac{\tau}{\tilde{\epsilon}} G_M^2(Q^2)\right] \qquad \tau \equiv \frac{Q^2}{4M^2}$$

Breit frame $\longrightarrow$ $\Delta^0 = 0$, $Q^2 = \boldsymbol{q}^2$

$$\frac{G_E(\boldsymbol{q}^2)}{\sqrt{1+\tau}} = \int \mathrm{d}^3 r \, e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \rho_e(\boldsymbol{r})$$

A natural expectation

[Rosenbluth, PR79 (1950) 615]
[Hofstadter, RMP28 (1956) 214]
[Yennie, Lévy, Ravenhall, RMP 29 (1957) 144]
[Gao, Vanderhaeghen, RMP 94 (2022) 015002]

Relativistic Breit frame (BF) interpretation

◆ **BF charge and current density:**

recoil factor $\dfrac{1}{\sqrt{1+\tau}} \approx 1 - \dfrac{\tau}{2} + \cdots$

$$J_B^0(\boldsymbol{r}) = e\delta_{s'_B s_B} \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \frac{G_E(\boldsymbol{\Delta}^2)}{\sqrt{1+\tau}}$$

$$\boldsymbol{J}_B(\boldsymbol{r}) = \boldsymbol{\nabla} \times \frac{e(\boldsymbol{\sigma})_{s'_B s_B}}{2M} \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \frac{G_M(\boldsymbol{\Delta}^2)}{\sqrt{1+\tau}}$$

$$J^0 = \rho_c + \cancel{\rho_P}$$

$$\boldsymbol{J} = \cancel{\rho_c \boldsymbol{v}} + \boldsymbol{\nabla} \times \boldsymbol{M} + \cancel{\partial_0 \boldsymbol{\mathcal{P}}}$$

In the differential cross section, the **same factor** $1/\sqrt{1+\tau}$ is naturally there!

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) \Big/ \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Mott}} = \frac{1}{1+\tau}\Big[\underbrace{G_E^2(Q^2)}_{\text{Electric}} + \frac{\tau}{\tilde{\epsilon}}\underbrace{G_M^2(Q^2)}_{\text{Magnetic}}\Big]$$

$$= \overline{G}_E^2(Q^2) + \frac{\tau}{\tilde{\epsilon}}\overline{G}_M^2(Q^2)$$

$$G_{E,M} \to \bar{G}_{E,M} = G_{E,M}/\sqrt{1+\tau}$$

◆ **3D relativistic mean-square charge radius:**

$$\langle \boldsymbol{r}_{\text{ch}}^2 \rangle = \frac{\int \mathrm{d}^3 r\, r\, \boldsymbol{r}^2 J_B^0(\boldsymbol{r})}{\int \mathrm{d}^3 r J_B^0(\boldsymbol{r})} = \langle \boldsymbol{r}_E^2 \rangle + \frac{3}{4M^2}$$

$$\frac{3\bar{\lambda}^2}{4}$$

$$= -\frac{6}{\overline{G}_E(0)} \frac{\mathrm{d}\overline{G}_E(Q^2)}{\mathrm{d}Q^2}\bigg|_{Q^2=0}$$

$$\langle \boldsymbol{r}_E^2 \rangle \equiv -\frac{6}{G_E(0)} \frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2}\bigg|_{Q=0}$$

traditional/standard mean-square charge radius

Darwin-Foldy term (relativistic correction)

[Foldy and Wouthuysen, PR 78, 29 (1950)]
[Foldy, PR 87, 688 (1952)]
[Friar, et al., PRA 56, 4579 (1997)]

[YC & Cédric Lorcé, PRD 107, 096003 (2023)]
[YC, Y. Li, C. Lorcé, & Q. Wang. PRD 110, L091503 (2024)]

Relativistic Breit frame (BF) interpretation

- **The relativistic recoil factor comes from relativistic normalization of Dirac spinors. It essentially ensures that the total charge is a Lorentz scalar!**

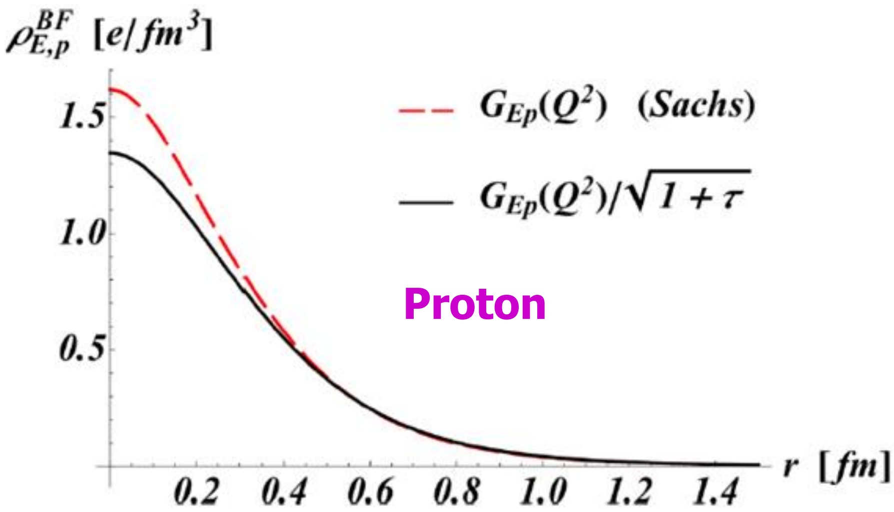
$$\int d^3r\, \langle \hat{j}^0 \rangle_{\boldsymbol{R},\boldsymbol{P}}(\boldsymbol{r}) = \frac{\langle P, s|\hat{j}^0(0)|P, s\rangle}{2P^0} = eG_E(0)$$

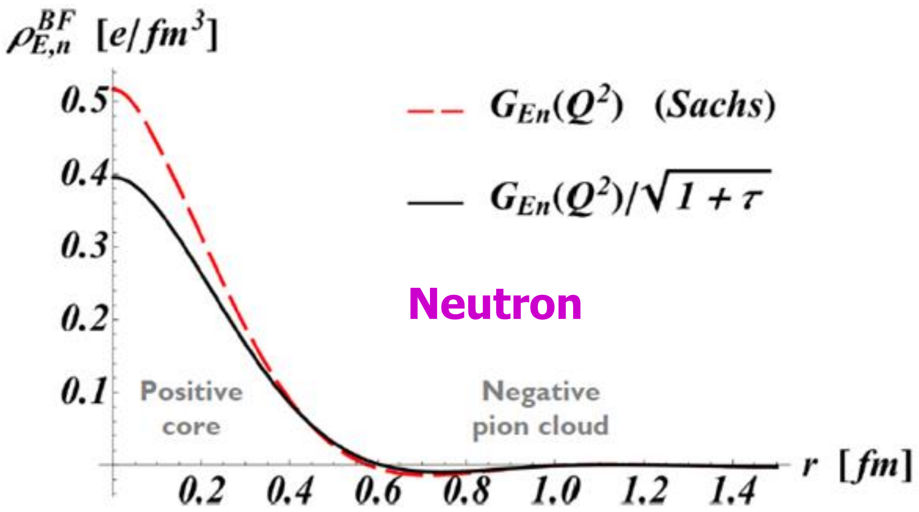
[Foldy and Wouthuysen, PR 78, 29 (1950)]
[Foldy, PR 87, 688 (1952)]
[Friar, et al., PRA 56, 4579 (1997)]

- **Taylor expansion of $1/\sqrt{1+\tau}$:**

$$\frac{1}{\sqrt{1+\tau}} \approx 1 - \frac{\tau}{2} + \frac{3\tau^2}{8} + \cdots \qquad -\frac{\tau}{2} = \boxed{-\frac{Q^2}{8M^2}} \rightarrow \textbf{Darwin-Foldy term}$$

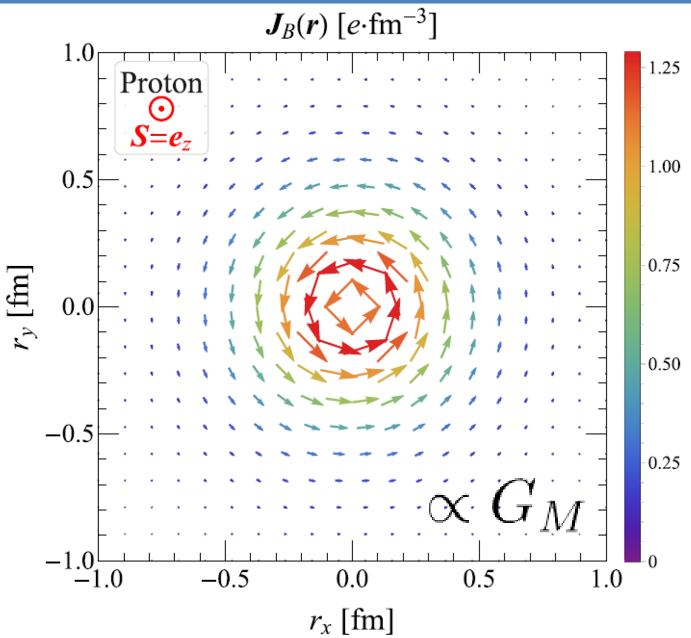
- **Breit frame charge density of the nucleon:**

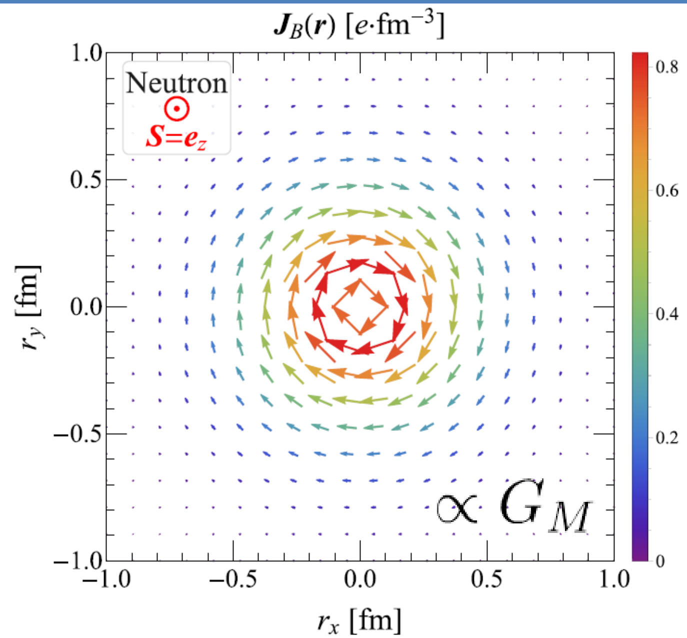




[Cédric Lorcé, PRL 125, 232002 (2020)]
[YC & Cédric Lorcé, PRD 106, 116024 (2022)]
[YC & Cédric Lorcé, PRD 107, 096003 (2023)]

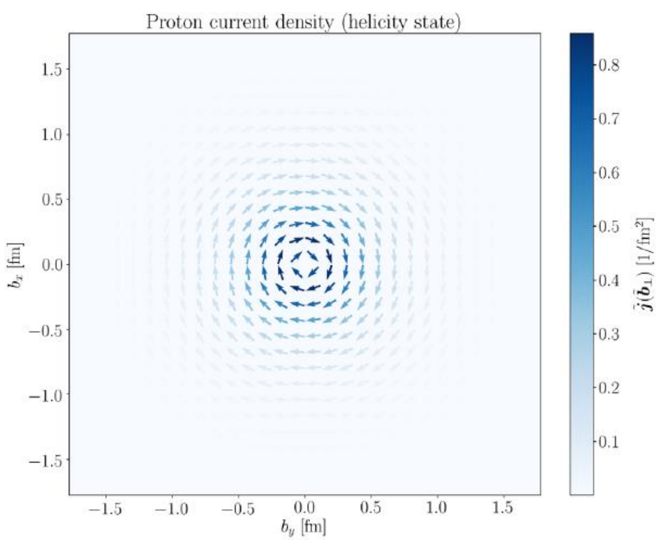
Relativistic Breit frame (BF) interpretation

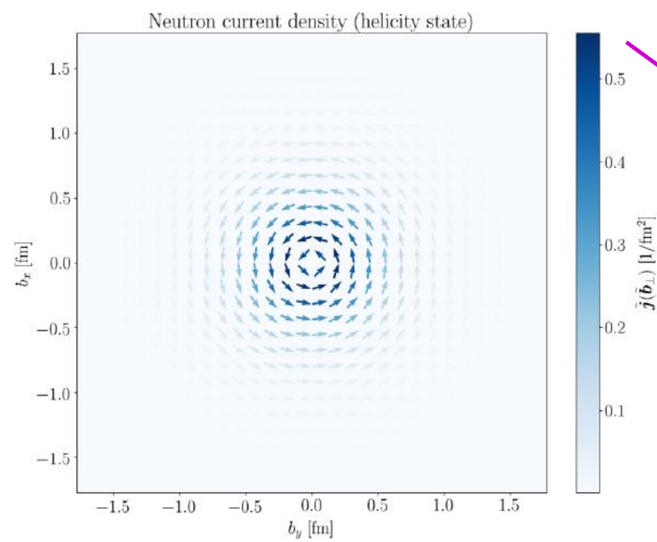




$$\mu_p = G_M^p(0) \approx 2.79$$

$$\mu_n = G_M^n(0) \approx -1.91$$





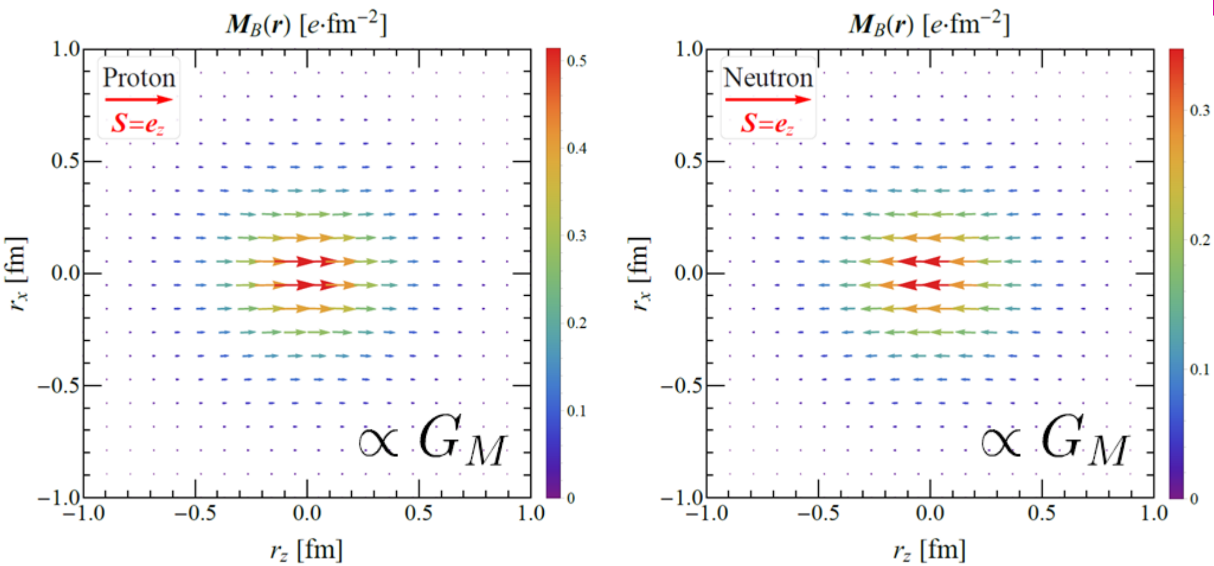
Our results were later on confirmed by Freese and Miller via the tilted light-front formalism.

[YC & Cédric Lorcé, PRD 106, 116024 (2022)]
[A. Freese & G. Miller, PRD 107, 074036 (2023)]

Relativistic Breit frame (BF) interpretation

◆ **Breit frame magnetization density:**

$$\boldsymbol{M}_B(\boldsymbol{r}) = \frac{e}{2M}\int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \left[\boldsymbol{\sigma} - \frac{\boldsymbol{\Delta}(\boldsymbol{\Delta}\cdot\boldsymbol{\sigma})}{4P_B^0(P_B^0+M)}\right] \frac{M}{P_B^0} G_M(\boldsymbol{\Delta}^2)$$



Polarization-magnetization tensor

$$P^{\mu\nu} = \begin{pmatrix} 0 & \mathcal{P}_x & \mathcal{P}_y & \mathcal{P}_z \\ -\mathcal{P}_x & 0 & -M_z & M_y \\ -\mathcal{P}_y & M_z & 0 & -M_x \\ -\mathcal{P}_z & -M_y & M_x & 0 \end{pmatrix}$$

$$\boldsymbol{\mathcal{P}}, \quad \boldsymbol{M}$$

◆ **Relativistic mean-square magnetization radius:**

$$\langle r^2_{\text{magn}}\rangle = \frac{\int d^3r\, r^2\, \hat{\boldsymbol{s}}\cdot\boldsymbol{M}_B(\boldsymbol{r})}{\int d^3r\, \hat{\boldsymbol{s}}\cdot\boldsymbol{M}_B(\boldsymbol{r})} = \langle r^2_M\rangle + \frac{3}{2M^2}$$

by analogy with "mean-square spin radius"

$$\langle r^2_M\rangle \equiv -\frac{6}{G_M(0)}\frac{dG_M(Q^2)}{dQ^2}\bigg|_{Q=0}$$

p MAGNETIC RADIUS

This is the rms magnetic radius, $\sqrt{\langle r^2_M\rangle}$.

VALUE (fm)	*DOCUMENT ID*		*TECN*
0.851±0.026	[1] LEE	15	

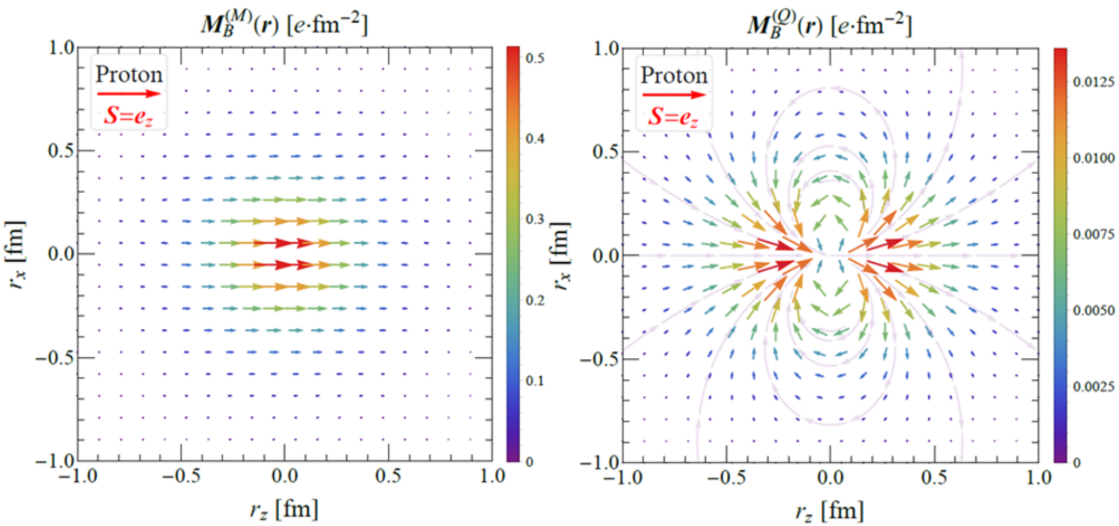
[YC & Cédric Lorcé, PRD 107, 096003 (2023)]
[YC, Y. Li, C. Lorcé, & Q. Wang. PRD 110, L091503 (2024); JHEP04, 132 (2025)]

Relativistic Breit frame interpretation

- **Effective magnetic charge distribution:** $\rho_M \equiv -\boldsymbol{\nabla}\cdot\boldsymbol{M}$

$$\rho_{M,B}(\boldsymbol{r}) = \frac{e}{2M}\int\frac{\mathrm{d}^3\Delta}{(2\pi)^2}e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}}\underbrace{(i\boldsymbol{\Delta}\cdot\boldsymbol{\sigma})}_{\text{P-odd}}\frac{G_M(\boldsymbol{\Delta}^2)}{1+\tau}$$

$\longrightarrow$ **No 3D mean-square effective magnetic charge radius**



Multipole decomposition:

$$\boldsymbol{M}_B(\boldsymbol{r}) = \boldsymbol{M}_B^{(M)}(\boldsymbol{r}) + \boldsymbol{M}_B^{(Q)}(\boldsymbol{r})$$

NB: BF polarization density violates CP symmetry.

$$\boldsymbol{\mathcal{P}}_B(\boldsymbol{r}) = \boldsymbol{0} \longrightarrow \rho_{P,B}(\boldsymbol{r}) = -\boldsymbol{\nabla}\cdot\boldsymbol{\mathcal{P}}_B = 0 \longrightarrow$$

well consistent with expectation

- **Electric dipole moment (EDM):**

$$\boldsymbol{d}_B = \int\mathrm{d}^3r\,\boldsymbol{r}J_B^0(\boldsymbol{r}) = \int\mathrm{d}^3r\,\boldsymbol{r}\rho_{P,B}(\boldsymbol{r}) = \int\mathrm{d}^3r\,\boldsymbol{\mathcal{P}}_B(\boldsymbol{r}) = \boldsymbol{0}.$$

- **Magnetic dipole moment (MDM):**

$$\boldsymbol{\mu}_B = \int\mathrm{d}^3r\,\boldsymbol{M}_B(\boldsymbol{r}) = \int\mathrm{d}^3r\,\boldsymbol{r}\rho_{M,B}(\boldsymbol{r}) = \int\mathrm{d}^3r\,\frac{\boldsymbol{r}\times\boldsymbol{J}_B(\boldsymbol{r})}{2} = \underbrace{\boldsymbol{\sigma}G_M(0)\frac{e}{2M}}_{\text{well consistent with QM}}$$

[YC & Cédric Lorcé, PRD 107, 096003 (2023)]

Relativistic Light-front (LF) interpretation

PRL **99,** 112001 (2007) PHYSICAL REVIEW LETTERS week ending 14 SEPTEMBER 2007

Charge Densities of the Neutron and Proton

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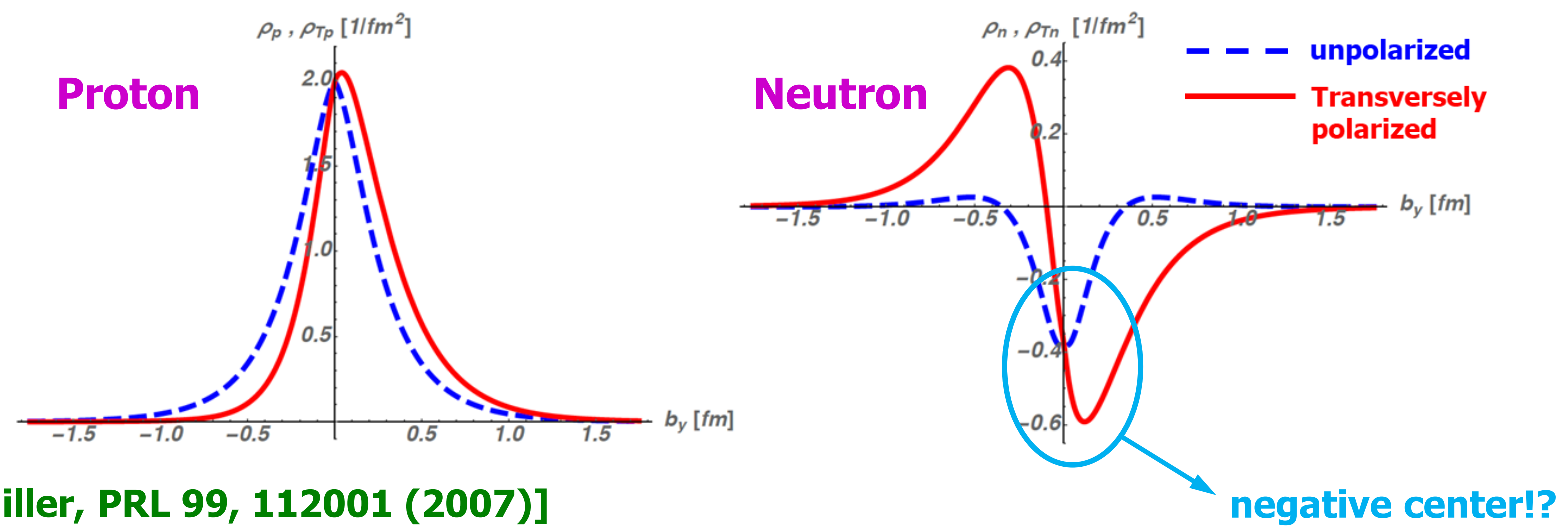
(Received 18 May 2007; published 13 September 2007)

A model-independent analysis of the infinite-momentum-frame charge density of partons in the transverse plane is presented for the nucleon. We find that the neutron-parton charge density is negative at the center, so that the square of the transverse charge radius is positive, in contrast with many expectations. Additionally, the proton's central *d* quark charge density is larger than that of the *u* quark by about 30%. The proton (neutron) charge density has a long range positively (negatively) charged component.

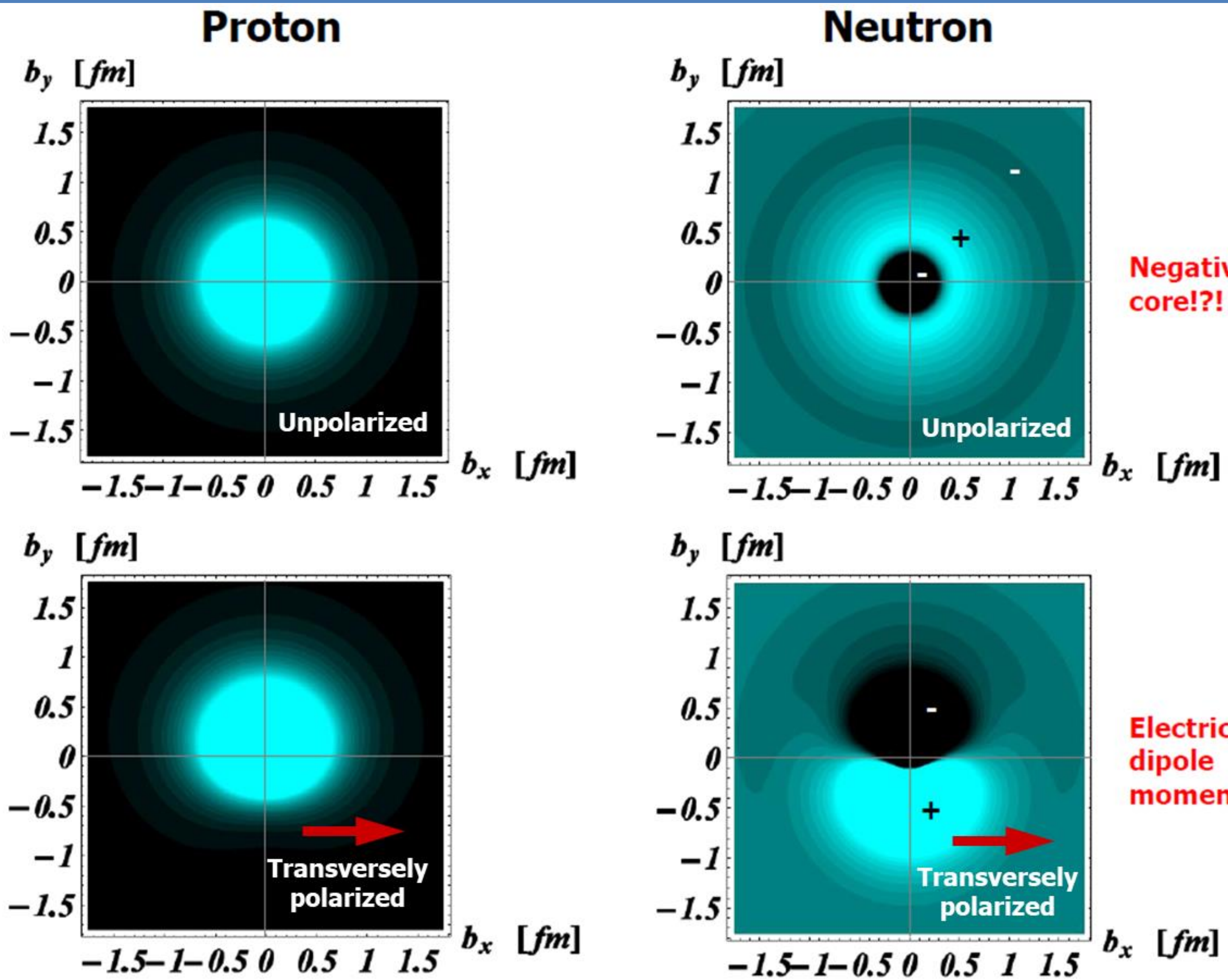
- **Generic LF distributions (strict probabilistic!!!):**

$$O_{\rm LF}(\boldsymbol{b}_\perp; P^+) \equiv \int \frac{d^2\Delta_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\, \frac{{}_{\rm LF}\langle p',\lambda'|\hat{O}(0)|p,\lambda\rangle_{\rm LF}}{2P^+}\Bigg|_{\Delta^+=|\boldsymbol{P}_\perp|=0}$$



[G. A. Miller, PRL 99, 112001 (2007)]
[Carlson & Vanderhaeghen, PRL 100, 032004 (2008)]

Some unusual observations on the LF

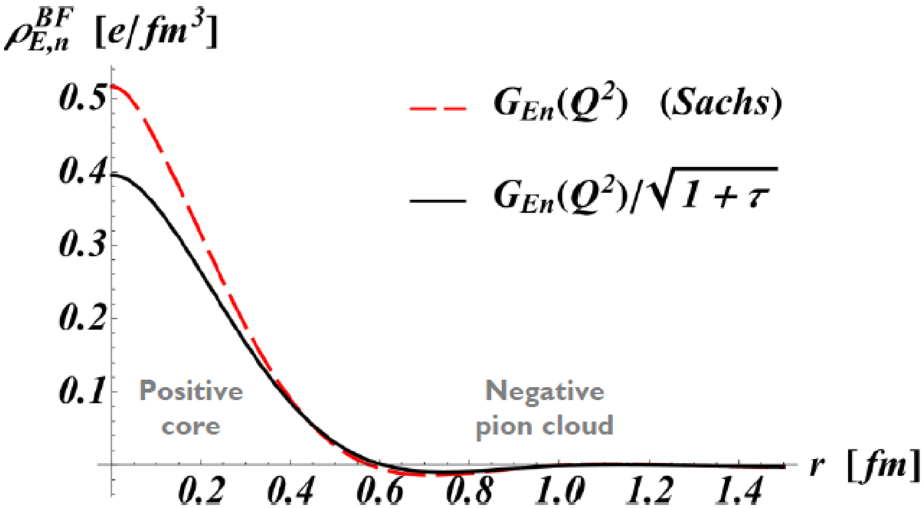


Negative core!?!

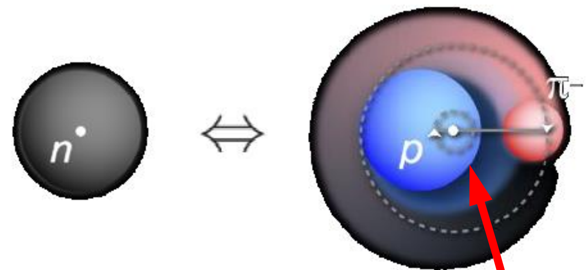
Electric dipole moment!?!

[G. Miller, PRL 99, 112001 (2007)]
[Carlson & Vanderhaeghen, PRL 100, 032004 (2008)]

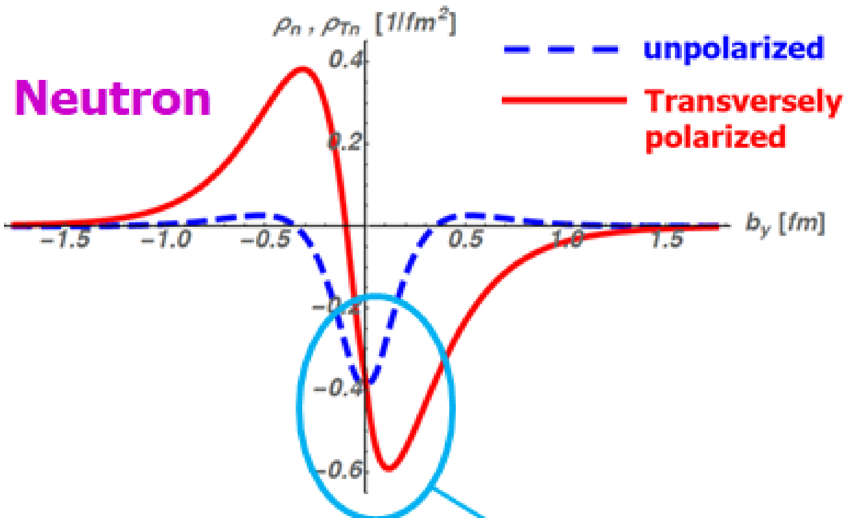
Neutron: 3D positive center → 2D negative center?!



Pion cloud picture (rest frame)



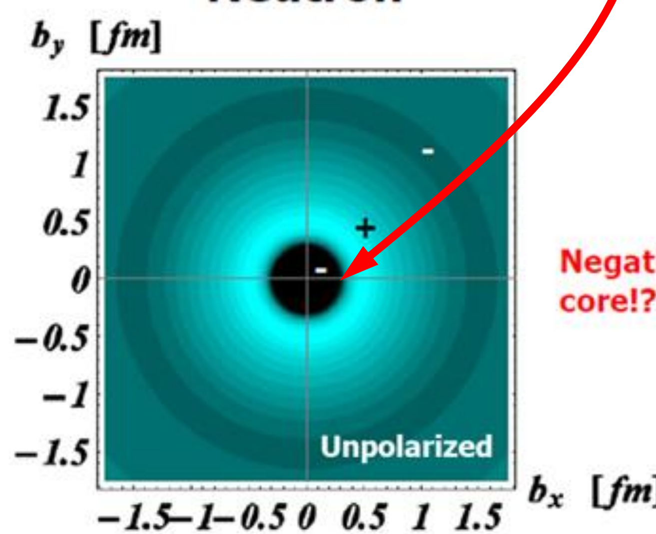
Neutron



negative center!?

tension?

Neutron



Negative core!?!

[Cédric Lorcé, PRL 125, 232002 (2020)]
[Carlson & Vanderhaeghen, PRL 100, 032004 (2008)]

Relativistic elastic frame (EF) interpretation

◆ Poincaré symmetry:

$$\langle p', s'|\hat{j}^\mu(0)|p, s\rangle = \sum_{s'_B, s_B} \underbrace{D^{\dagger(j)}_{s's'_B}(p'_B, \Lambda) D^{(j)}_{s_B s}(p_B, \Lambda)}_{\text{Wigner rotation}} \underbrace{\Lambda^\mu{}_\nu \langle p'_B, s'_B|\hat{j}^\nu(0)|p_B, s_B\rangle}_{\text{Lorentz mixing}}$$

(1). Lorentz mixing

$$\begin{pmatrix} \tilde{J}^0_{\mathrm{EF}} \\ \tilde{J}^z_{\mathrm{EF}} \end{pmatrix} = \overbrace{\begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix}}^{\text{boost } \Lambda^\mu{}_\nu} \begin{pmatrix} \tilde{J}^0_B \\ \tilde{J}^z_B \end{pmatrix}$$

~ momentum-space amplitudes

(2). Wigner spin rotation

Key insight: boost generators do not commutate!

$$\left[\hat{K}^i,\ \hat{K}^j\right] = -i\epsilon^{ijk}\hat{J}^k$$

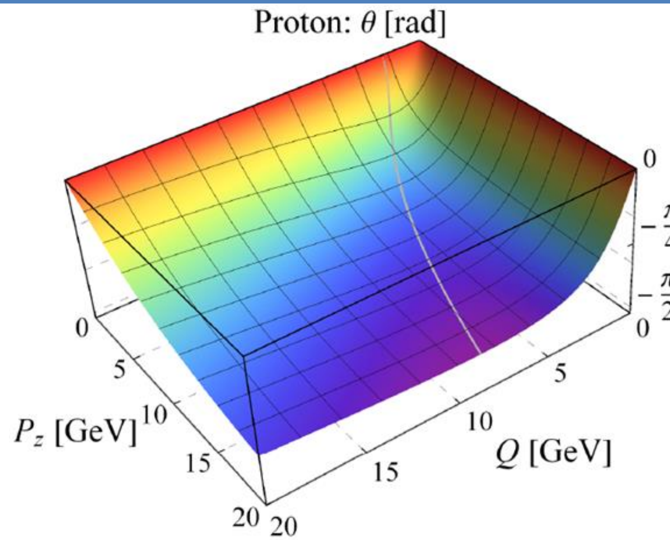
$$D^{(1/2)}(p_B, \Lambda) = \begin{pmatrix} \cos\frac{\theta}{2} & -e^{-i\phi_\Delta}\sin\frac{\theta}{2} \\ e^{i\phi_\Delta}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$

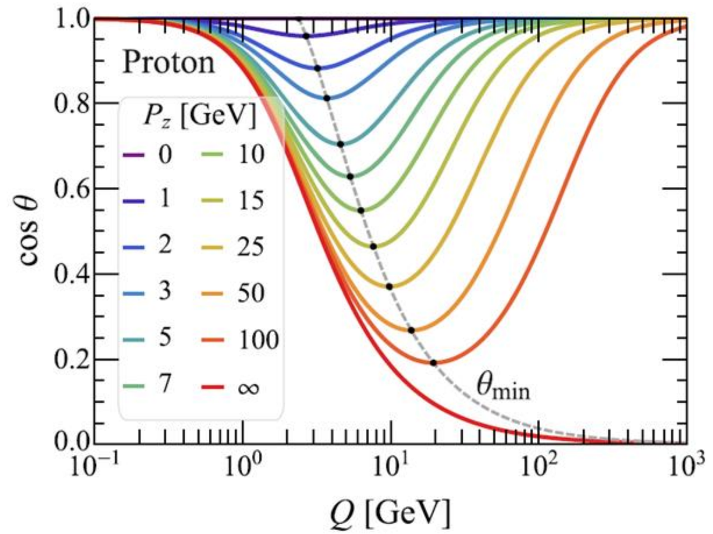
θ: Wigner rotation angle

$$\cos\theta = \frac{P^0 + M(1+\tau)}{(P^0 + M)\sqrt{1+\tau}}, \qquad \sin\theta = -\frac{\sqrt{\tau}P_z}{(P^0 + M)\sqrt{1+\tau}}$$

[Durand, De Celles, Marr, PR 126, 1882 (1962)]
[Cédric Lorcé, PRL 125, 232002 (2020)]
[YC & Cédric Lorcé, PRD 106, 116024 (2022)]
[YC & Cédric Lorcé, PRD 107, 096003 (2023)]

Relativistic elastic frame (EF) interpretation





$$-\frac{\pi}{2} \leq \theta \leq 0$$

◆ **Generic angular conditions for Wigner spin rotation:**

$$\cos\theta = \frac{P^0 + M(1+\tau)}{(P^0+M)\sqrt{1+\tau}}, \qquad \sin\theta = -\frac{\sqrt{\tau}P_z}{(P^0+M)\sqrt{1+\tau}}$$

(1). At fixed P_z:

$$\cos\theta_{\min} = \frac{1+2\tau_{\min}}{(1+\tau_{\min})^{3/2}} \longrightarrow \tau_{\min} = \frac{1}{2} + \sqrt{\frac{P_z^2}{M^2} + \frac{5}{4}}$$

(2). At $P_z \to \infty$:

$$\lim_{P_z\to\infty} \cos\theta = \frac{1}{\sqrt{1+\tau}}, \qquad \lim_{P_z\to\infty} \sin\theta = -\frac{\sqrt{\tau}}{\sqrt{1+\tau}}$$

[YC & Cédric Lorcé, PRD 106, 116024 (2022)]

Relativistic elastic frame (EF) interpretation

◆ **EF distributions (2D):**

Wigner spin rotation

$$J^0_{\mathrm{EF}}(\boldsymbol{b}_\perp;P_z) = e\int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\,e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[\delta_{s's}\cos\theta+\frac{(\boldsymbol{\sigma}_{s's}\times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}}\sin\theta\right]\frac{G_E(\boldsymbol{\Delta}^2_\perp)}{\sqrt{1+\tau}}$$
$$+\,e\int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\,e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\frac{P_z}{P^0}\left[-\delta_{s's}\sin\theta+\frac{(\boldsymbol{\sigma}_{s's}\times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}}\cos\theta\right]\frac{\sqrt{\tau}G_M(\boldsymbol{\Delta}^2_\perp)}{\sqrt{1+\tau}}$$

$$J_{z,\mathrm{EF}}(\boldsymbol{b}_\perp;P_z) = e\int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\,e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\frac{P_z}{P^0}\left[\delta_{s's}\cos\theta+\frac{(\boldsymbol{\sigma}_{s's}\times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}}\sin\theta\right]\frac{G_E(\boldsymbol{\Delta}^2_\perp)}{\sqrt{1+\tau}}$$
$$+\,e\int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\,e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[-\delta_{s's}\sin\theta+\frac{(\boldsymbol{\sigma}_{s's}\times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}}\cos\theta\right]\frac{\sqrt{\tau}G_M(\boldsymbol{\Delta}^2_\perp)}{\sqrt{1+\tau}}$$

$$\boldsymbol{J}_{\perp,\mathrm{EF}}(\boldsymbol{b}_\perp;P_z) = \delta_{s's}\frac{e\sigma_z}{2M}\int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\,e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\,(\boldsymbol{e}_z\times i\boldsymbol{\Delta})_\perp\frac{P^0_B}{P^0}\frac{G_M(\boldsymbol{\Delta}^2_\perp)}{\sqrt{1+\tau}}$$

free of Wigner spin rotation

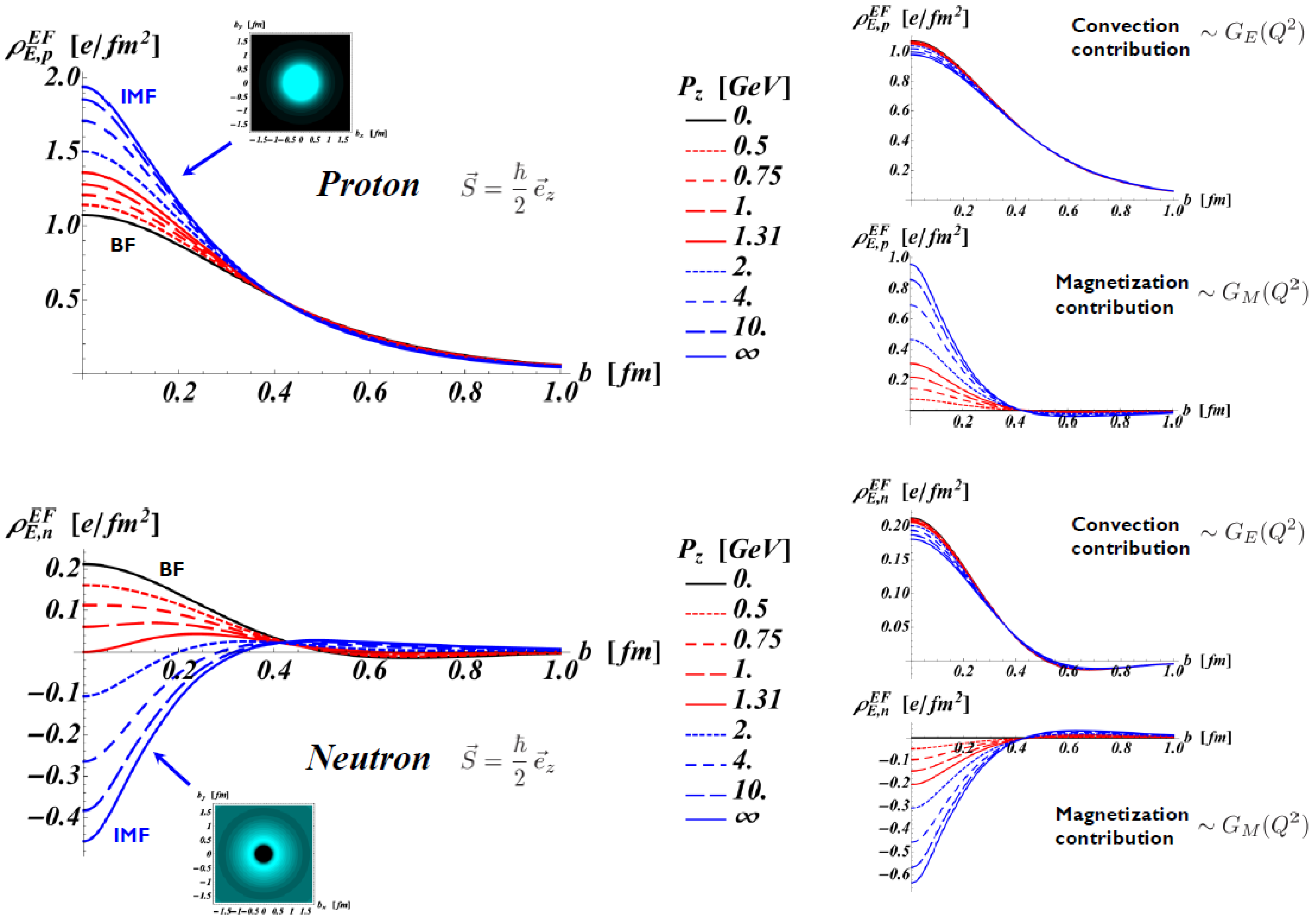
suppressed by Lorentz contraction

In conclusion: **Wigner spin rotation indeed plays a central role for the structures of moving spinning hadrons.**

[Cédric Lorcé, Pierre Wang, PRD 105, 096032 (2022)]
[YC, Cédric Lorcé, PRD106 (2022) 116024]
[YC, Cédric Lorcé, PRD107 (2023) 096003]

Relativistic elastic frame (EF) interpretation

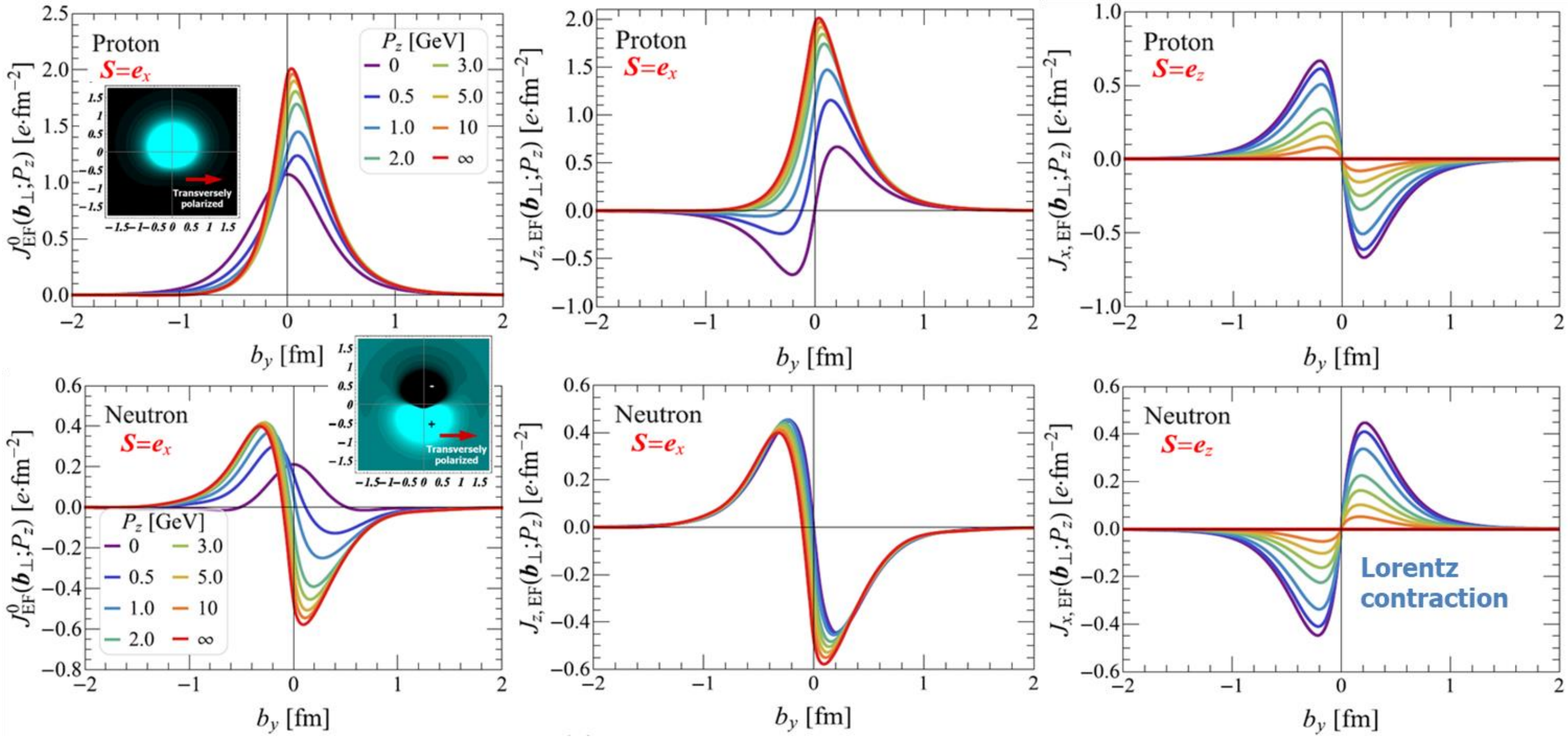
◆ **EF distributions (2D): unpolarized case**

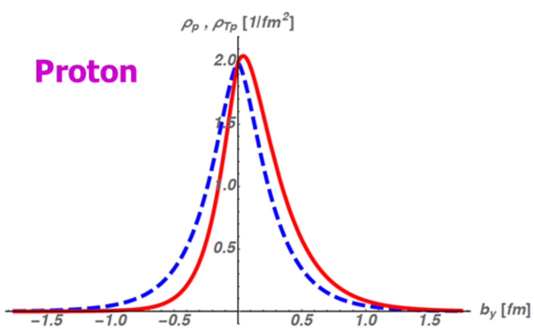


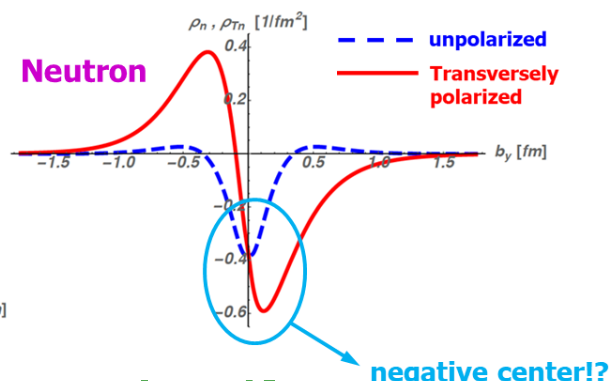
[Cédric Lorcé, PRL 125, 232002 (2020)]

Relativistic elastic frame (EF) interpretation

◆ EF distributions (2D): polarized case







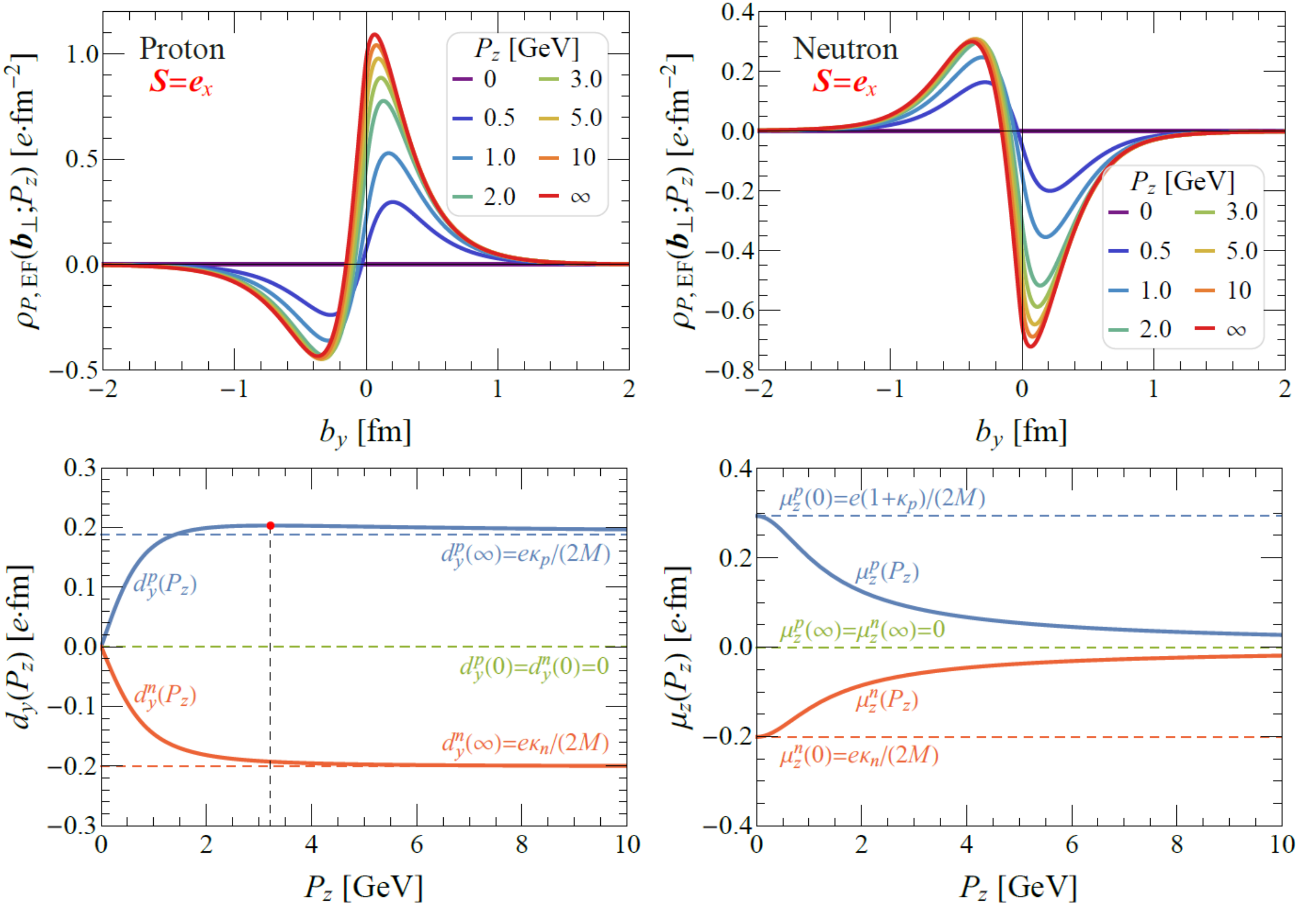
$$J^0 = \rho_c + \rho_P$$

$$\boldsymbol{J} = \rho_c \boldsymbol{v} + \boldsymbol{\nabla} \times M + \cancel{\partial_0 \mathcal{P}}$$

$$\rho_P = -\boldsymbol{\nabla} \cdot \boldsymbol{\mathcal{P}}$$

[Carlson & Vanderhaeghen, PRL 100, 032004 (2008)]
[YC & Cédric Lorcé, PRD 106, 116024 (2022); PRD 107, 096003 (2023)]

Relativistic elastic frame (EF) interpretation



Induced EDM: $\boldsymbol{d} = \boldsymbol{v} \times \boldsymbol{\mu}$ **~ manifestation of relativistic effect!**

Relativistic elastic frame (EF) interpretation

- **In the IMF limit ($P_z \to \infty$): why it becomes $F_1(Q^2)$ and $F_2(Q^2)$ for LF J^+?**

$$J^+_{\mathrm{LF}}(\boldsymbol{b}_\perp; P^+) = e \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \left[\delta_{\lambda'\lambda} F_1(\boldsymbol{\Delta}^2_\perp) + \frac{(\boldsymbol{\sigma}_{\lambda'\lambda}\times i\boldsymbol{\Delta})_z}{2M} F_2(\boldsymbol{\Delta}^2_\perp)\right]$$

$$\lim_{P_z\to\infty} \frac{\cos\theta G_E(Q^2) - \sin\theta\sqrt{\tau}G_M(Q^2)}{\sqrt{1+\tau}} = \frac{G_E(Q^2)+\tau G_M(Q^2)}{1+\tau} = F_1(Q^2)$$

unpolarized

$$\lim_{P_z\to\infty} \frac{\sin\theta G_E(Q^2) + \cos\theta\sqrt{\tau}G_M(Q^2)}{\sqrt{\tau}\sqrt{1+\tau}} = \frac{G_M(Q^2)-G_E(Q^2)}{1+\tau} = F_2(Q^2)$$

Transversely polarized

- **Besides, we found another analytic relations for LF J^-:**

$$J^-_{\mathrm{LF}}(\boldsymbol{b}_\perp; P^+) = e \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \frac{P^-}{P^+} \left[\delta_{\lambda'\lambda} G_1(\boldsymbol{\Delta}^2_\perp) + \frac{(\boldsymbol{\sigma}_{\lambda'\lambda}\times i\boldsymbol{\Delta})_z}{2M} G_2(\boldsymbol{\Delta}^2_\perp)\right]$$

$$\lim_{P_z\to\infty} \frac{\cos\theta G_E(Q^2) + \sin\theta\sqrt{\tau}G_M(Q^2)}{\sqrt{1+\tau}} = \frac{(1-\tau)F_1(Q^2) - 2\tau F_2(Q^2)}{1+\tau} \equiv G_1(Q^2)$$

unpolarized

$$\lim_{P_z\to\infty} \frac{\sin\theta G_E(Q^2) - \cos\theta\sqrt{\tau}G_M(Q^2)}{\sqrt{\tau}\sqrt{1+\tau}} = -\frac{2F_1(Q^2) + (1-\tau)F_2(Q^2)}{1+\tau} \equiv G_2(Q^2)$$

Transversely polarized

new

$$\longrightarrow\quad J^+_{\mathrm{LF}}(\boldsymbol{b}_\perp; P^+) = J^0_{\mathrm{EF}}(\boldsymbol{b}_\perp; \infty) = J^z_{\mathrm{EF}}(\boldsymbol{b}_\perp; \infty)$$ ~ **generic relation**

[G. A. Miller, PRL 99, 112001 (2007)]
[YC, Cédric Lorcé, PRD 106, 116024 (2022)]

LF interpretation vs. EF interpretation

◆ LF interpretation:

$$J^+_{\rm LF}(\boldsymbol{b}_\perp;P^+) = e\int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[\delta_{\lambda'\lambda}F_1(\boldsymbol{\Delta}^2_\perp)+\frac{(\boldsymbol{\sigma}_{\lambda'\lambda}\times i\boldsymbol{\Delta})_z}{2M}F_2(\boldsymbol{\Delta}^2_\perp)\right],$$

$$J^-_{\rm LF}(\boldsymbol{b}_\perp;P^+) = e\int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\frac{P^-}{P^+}\left[\delta_{\lambda'\lambda}G_1(\boldsymbol{\Delta}^2_\perp)+\frac{(\boldsymbol{\sigma}_{\lambda'\lambda}\times i\boldsymbol{\Delta})_z}{2M}G_2(\boldsymbol{\Delta}^2_\perp)\right],$$

$$\boldsymbol{J}_{\perp,\rm LF}(\boldsymbol{b}_\perp;P^+) = e(\sigma_z)_{\lambda'\lambda}\int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\frac{(\boldsymbol{e}_z\times i\boldsymbol{\Delta})_\perp}{2P^+}G_M(\boldsymbol{\Delta}^2{}_\perp).$$

In total,
we need **5** EMFFs.

$F_1,\ F_2$

$G_1,\ G_2$

G_M

◆ EF interpretation:

$$\begin{aligned}J^0_{\rm EF}(\boldsymbol{b}_\perp;P_z) &= e\int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[\delta_{s's}\cos\theta+\frac{(\boldsymbol{\sigma}_{s's}\times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}}\sin\theta\right]\frac{G_E(\boldsymbol{\Delta}^2_\perp)}{\sqrt{1+\tau}}\\ &+e\int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\frac{P_z}{P^0}\left[-\delta_{s's}\sin\theta+\frac{(\boldsymbol{\sigma}_{s's}\times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}}\cos\theta\right]\frac{\sqrt{\tau}G_M(\boldsymbol{\Delta}^2_\perp)}{\sqrt{1+\tau}}\end{aligned}$$

$$\begin{aligned}J_{z,\rm EF}(\boldsymbol{b}_\perp;P_z) &= e\int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\frac{P_z}{P^0}\left[\delta_{s's}\cos\theta+\frac{(\boldsymbol{\sigma}_{s's}\times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}}\sin\theta\right]\frac{G_E(\boldsymbol{\Delta}^2_\perp)}{\sqrt{1+\tau}}\\ &+e\int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[-\delta_{s's}\sin\theta+\frac{(\boldsymbol{\sigma}_{s's}\times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}}\cos\theta\right]\frac{\sqrt{\tau}G_M(\boldsymbol{\Delta}^2_\perp)}{\sqrt{1+\tau}}\end{aligned}$$

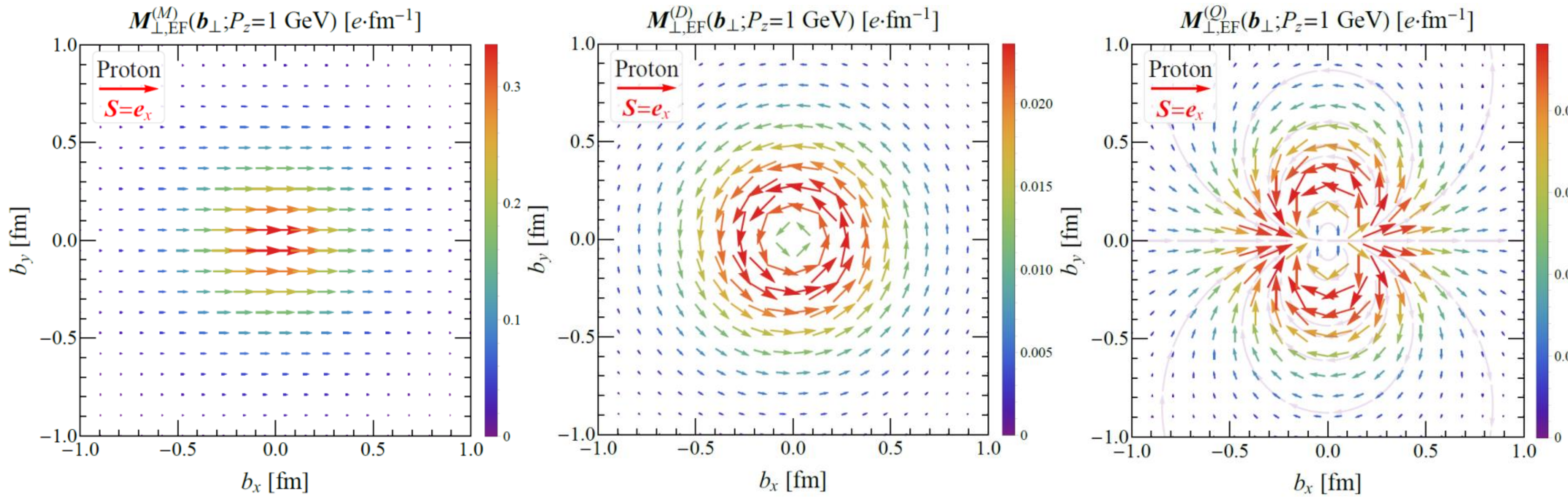
$$\boldsymbol{J}_{\perp,\rm EF}(\boldsymbol{b}_\perp;P_z) = \delta_{s's}\frac{e\sigma_z}{2M}\int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}(\boldsymbol{e}_z\times i\boldsymbol{\Delta})_\perp\frac{P^0_B}{P^0}\frac{G_M(\boldsymbol{\Delta}^2_\perp)}{\sqrt{1+\tau}}$$

In total,
we need **2** EMFFs
and **1** Wigner
rotation angle.

$G_E,\ G_M,\ \theta$

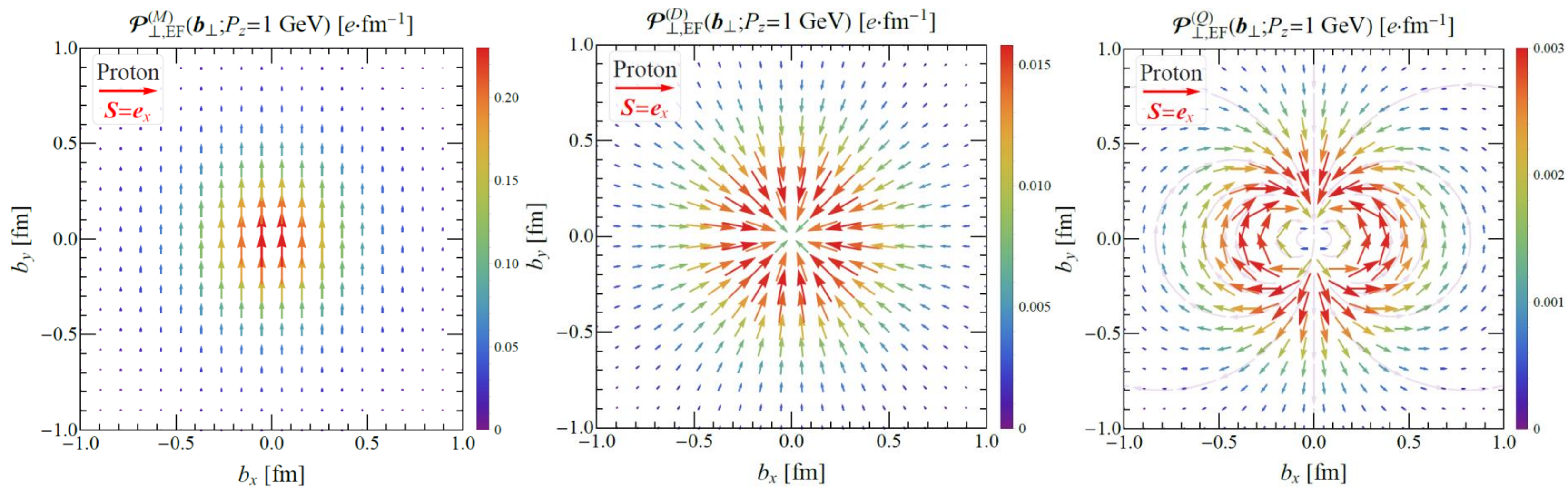
[Cédric Lorcé, PRL 125, 232002 (2020)]
[YC, Cédric Lorcé, PRD 106, 116024 (2022)]

Relativistic elastic frame (EF) interpretation



magnetization

monopole   dipole   quadrupole



polarization

$$\boldsymbol{d} = \boldsymbol{v} \times \boldsymbol{\mu} \quad \Longleftarrow \quad \widetilde{\mathcal{P}}^{(l)} = \boldsymbol{v} \times \widetilde{M}^{(l)}$$

classical level

~ explicit manifestation of Ehrenfest's theorem!

valid at quantum level (in momentum space) to all orders!

Summary and outlook

- **Quantum phase-space approach allows one to define relativistic spatial distributions inside a hadron with arbitrary spin and arbitrary average momentum.**

- **For a spin-1/2 hadron, its electromagnetic vertex function has an one-to-one correspondence with the classical electromagnetic four-current, where convection and magnetization contributions are associated with $G_E(Q^2)$ and $G_M(Q^2)$, respectively.**

- **Off all possible elastic frames, the Breit frame distributions lead to the simplest multipole structures, which strongly suggests that Sachs electromagnetic FFs $G_E(Q^2)$ and $G_M(Q^2)$ should be interpreted as more physical ones. In the moving case, polarization and magnetization contributions to the electromagnetic four-current distributions are crucial and significant.**

- **Any relativistic spatial distortions of a moving spinning hadron can be understood as a combination of Lorentz transformation and Wigner spin rotation. This is precisely protected by Poincaré symmetry.**

[YC & Cédric Lorcé, PRD 106, 116024 (2022)]
[YC & Cédric Lorcé, PRD 107, 096003 (2023)]
[YC et al., arXiv: 25xx.xxxx (to appear)]

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第8届 强子谱和强子结构研讨会

The 8th Symposium on Hadron Spectrum and Hadron Structure

2025年7月11--16日 中国·桂林

Comparison of elastic scattering at different scales

- Crystals & atoms:

$$d \approx 10^{-10}\ \mathrm{m} \quad \rightarrow \quad \hbar\omega \approx 10^{4}\ \mathrm{eV} \quad \rightarrow$$

Method (probe)

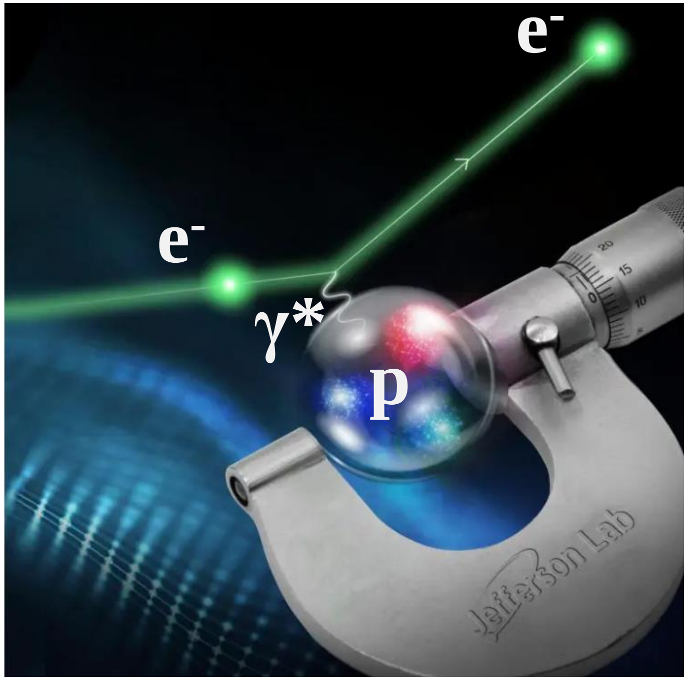
x-ray

- Nuclei & nucleon:

$$d \approx 10^{-15}\ \mathrm{m} \quad \rightarrow \quad \hbar\omega \approx 10^{9}\ \mathrm{eV} \quad \rightarrow$$

high-energy electron scattering

→ **Larger recoil for lighter targets** (e.g., nucleon, pion).



- **Elastic electron-hadron scattering:**

$$e^-(k,r) + h(p,s) \rightarrow e^-(k',r') + h(p',s')$$

$h \sim$ **hadron**

- **Spin-0**: e.g., pion, kaon, …
- **Spin-1/2**: e.g., nucleon, …
- **Spin-1**: e.g., deutron, ρ meson, …
- **Spin-3/2**: e.g., Δ baryon, …

Backup: Covariant Lorentz transformation -- two key effects

- **Four-vector case** (e.g., electromagnetic four-current):

$$\langle p', s'|\hat{j}^{\mu}(0)|p, s\rangle = \sum_{s'_B, s_B} \underbrace{D^{\dagger(j)}_{s's'_B}(p'_B, \Lambda) D^{(j)}_{s_B s}(p_B, \Lambda)}_{\text{Wigner rotation}} \underbrace{\Lambda^{\mu}{}_{\nu} \langle p'_B, s'_B|\hat{j}^{\nu}(0)|p_B, s_B\rangle}_{\text{Lorentz mixing}}$$

- **Axial four-vector case** (e.g., axial-vector four-current):

$$\langle p', s'|\hat{j}^{\mu}_5(0)|p, s\rangle = \sum_{s'_B, s_B} \underbrace{D^{\dagger(j)}_{s's'_B}(p'_B, \Lambda) D^{(j)}_{s_B s}(p_B, \Lambda)}_{\text{Wigner rotation}} \underbrace{\Lambda^{\mu}{}_{\nu} \langle p'_B, s'_B|\hat{j}^{\nu}_5(0)|p_B, s_B\rangle}_{\text{Lorentz mixing}}$$

- **Tensor case** (e.g., polarization-magnetization tensor):

$$\langle p', s'|\hat{P}^{\mu\nu}(0)|p, s\rangle = \sum_{s'_B, s_B} \underbrace{D^{\dagger(j)}_{s's'_B}(p'_B, \Lambda) D^{(j)}_{s_B s}(p_B, \Lambda)}_{\text{Wigner rotation}} \underbrace{\Lambda^{\mu}{}_{\alpha} \Lambda^{\nu}{}_{\beta} \langle p'_B, s'_B|\hat{P}^{\alpha\beta}(0)|p_B, s_B\rangle}_{\text{Lorentz mixing}}$$

(1). Lorentz mixing effect

$$\begin{pmatrix} \tilde{J}^0_{\mathrm{EF}} \\ \tilde{J}^z_{\mathrm{EF}} \end{pmatrix} = \underbrace{\begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix}}^{\text{boost } \Lambda^{\mu}{}_{\nu}} \begin{pmatrix} \tilde{J}^0_B \\ \tilde{J}^z_B \end{pmatrix}$$

~ momentum-space amplitudes

(2). Wigner rotation effect

Fundamental reason: boost generators of Lorentz group do not commutate!

$$\left[\hat{K}^i, \ \hat{K}^j\right] = -i\epsilon^{ijk}\hat{J}^k \longrightarrow \text{Wigner rotation } D^{(j)}_{s_B s}(p_B, \Lambda)$$

[Durand, De Celles, Marr, PR 126, 1882 (1962)]
[Cédric Lorcé, PRL 125, 232002 (2020)]
[YC & Cédric Lorcé, PRD 106, 116024 (2022)]
[YC & Cédric Lorcé, PRD 107, 096003 (2023)]
[YC, Y. Li, C. Lorcé, & Q. Wang. PRD 110, L091503 (2024); JHEP 04, 132 (2025)]