Understanding two puzzling phenomena under the unquenched charmonium spectroscopy

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Phys. Rev. D 109, 094048 (2024)
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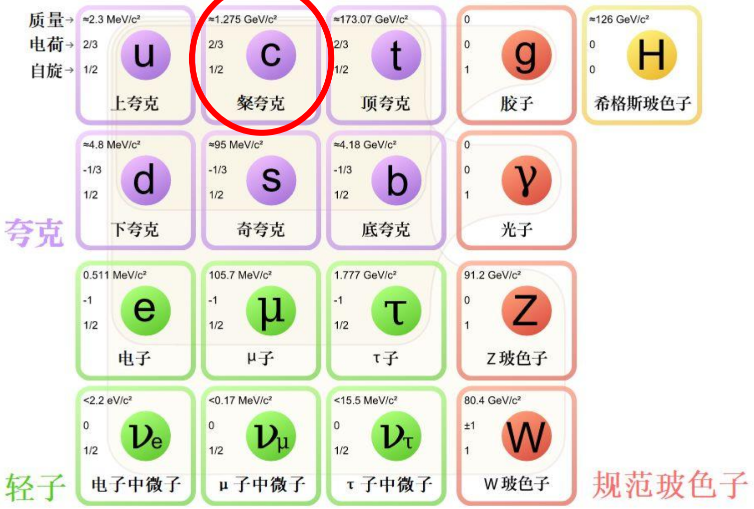
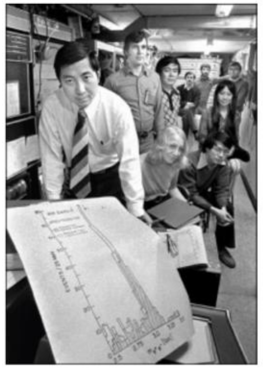
2025年7月11日-15日

Contents

In the quark model, the traditional baryon consists of three quarks, and the meson consists of two quarks.





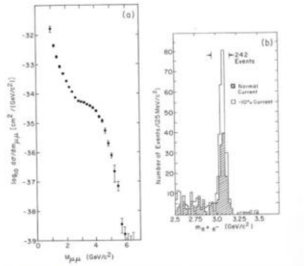
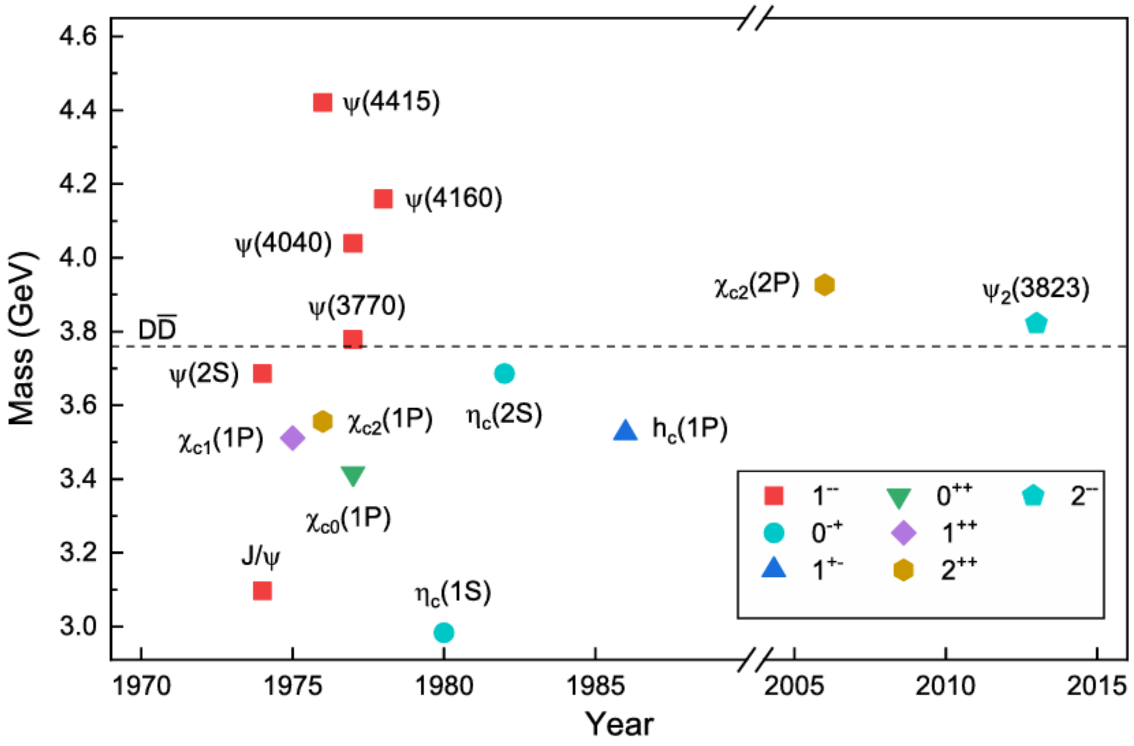


Fig. 1 - (left) Yield of the dimuons from the 1968 experiments by Lederman and his group. (left) Dielectron yield from Ting's 1974 experiment. Source: Lederman L. (1997) The discovery of the Upsilon, Bottom Quark, and B Mesons. In Hoddeson, L. Brown, L. M., Riordan, M. & Dresden, M., (1997) The Rise of the Standard Model: Particle Physics in the 1960s and the 1970s. Cambridge University Press, Cambridge. p. 101-113, on p. 104.

Most of the charmonium states listed in PDG



I. Background : Development of charmonium mass spectrum

PHYSICAL REVIEW D VOLUME 17, NUMBER 11 1 JUNE 1978

Charmonium: The model

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(Received 9 February 1978)

A comprehensive treatment of the charmonium model of the ψ family is presented. The model's basic assumption is a flavor-symmetric instantaneous effective interaction between quark color densities. This interaction describes both quark-antiquark binding and pair creation, and thereby provides a unified approach for energies below and above the threshold for charmed-meson production. If coupling to decay channels is ignored, one obtains the "naive" model wherein the dynamics is completely described by a single charmed-quark pair. A detailed description of this "naive" model is presented for the case where the instantaneous potential is a superposition of a linear and Coulomb term. A far more realistic picture is attained by incorporating those terms in the interaction that couple charmed quarks to light quarks. The coupled-channel formalism needed for this purpose is fully described. Formulas are given for the inclusive e^+e^- cross section and for e^+e^- annihilation into specific charmed-meson pairs. The influence of closed decay channels on ψ states below charm threshold is investigated, with particular attention to leptonic and radiative widths.

$$V(r) = -\frac{\kappa}{r} + \frac{r}{a^2}\,.$$ **Cornell potential**

PHYSICAL REVIEW D VOLUME 21, NUMBER 1 1 JANUARY 1980

Charmonium: Comparison with experiment

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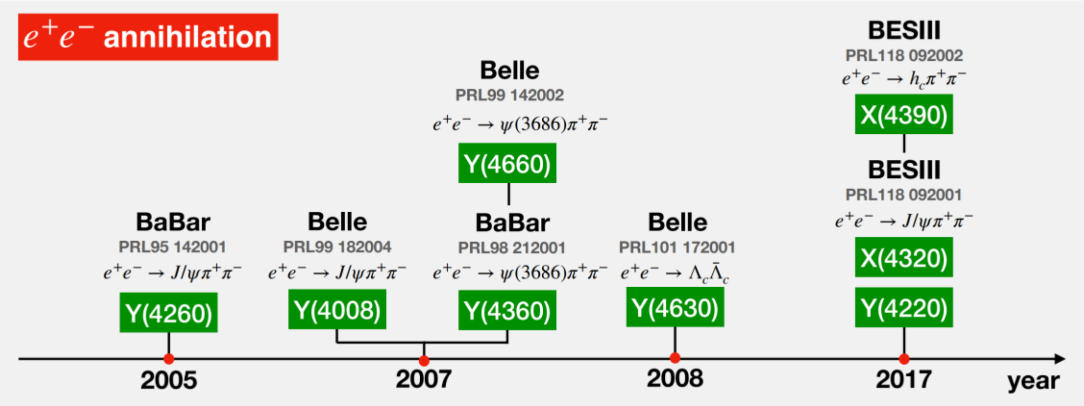
(Received 25 June 1979)

TABLE II. $c\bar{c}$ bound states in naive model, and their properties. Parameters used are $m_c = 1.84$ GeV, $a = 2.34$ GeV^{-1}, and $\kappa = 0.52$.

State	Mass (GeV)	Γ_{ee} (keV) [b]	$\langle v^2/c^2 \rangle$	$\langle r^2\rangle^{1/2}$ (fm)	Candidate
1S	3.095 [a]	4.8	0.20	0.47	$\psi(3095)$
1P	3.522 [a]		0.20	0.74	$\chi_{0,1,2}(3522\pm5)$
2S	3.684 [a]	2.1	0.24	0.96	$\psi'(3684)$
1D	3.81		0.23	1.0	$\psi'(3772)$ [c]
3S	4.11	1.5	0.30	1.3	$\psi(4028)$
2D	4.19		0.29	1.35	$\psi(4160)$ [d]
4S	4.46	1.1	0.35	1.7	$\psi(4414)$
5S	4.79	0.8	0.40	2.0	



So many $Y(1^{--})$ states have been reported!

How to understand these *Y* states?

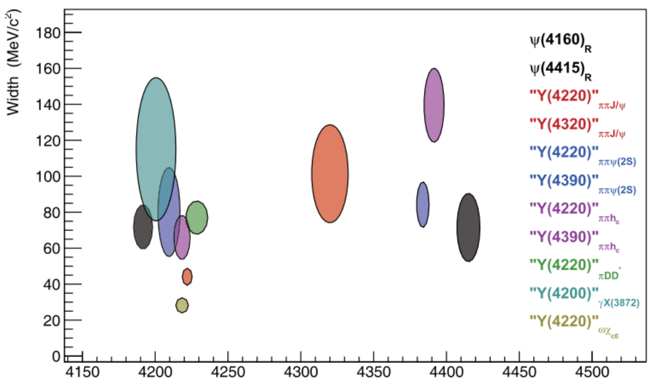
Chinese Physics C Vol. 44, No. 4 (2020)

Future Physics Programme of BESIII*

(1) The *Y* problem

Exclusive e^+e^- cross-sections have shown surprisingly complex behavior as a function of cms energy. The $Y(4260)$ is more complex than a single ordinary resonance, as shown by the complicated line shape of the $e^+e^- \to \pi^+\pi^- J/\psi$ cross-section in Fig. 3.10(e); the $Y(4360)$ and $Y(4660)$ are seen in $e^+e^- \to \pi^+\pi^-\psi(3686)$; two other peaks are seen in $e^+e^- \to \pi^+\pi^- h_c$ in Fig. 3.10(f); the $Y(4220)$ is seen in $e^+e^- \to \omega\chi_{c0}$ in Fig. 3.10(g), etc. A summary of the masses and widths of resonances extracted from recent BESIII results is shown in Fig. 3.11.

Parameters of the Peaks in e^+e^- Cross Sections

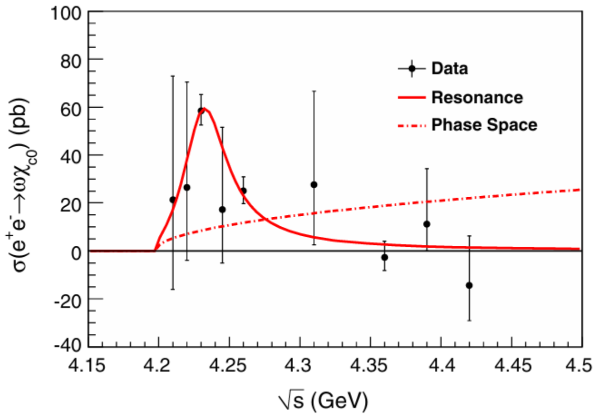


I. Background : $Y\,(4220)$ / $\psi(4S)$

PRL **114**, 092003 (2015) PHYSICAL REVIEW LETTERS week ending 6 MARCH 2015

Study of $e^+e^- \to \omega\chi_{cJ}$ at Center of Mass Energies from 4.21 to 4.42 GeV

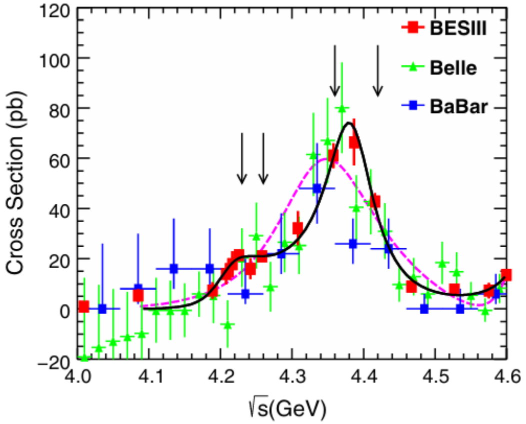
Based on data samples collected with the BESIII detector at the BEPCII collider at nine center of mass energies from 4.21 to 4.42 GeV, we search for the production of $e^+e^- \to \omega\chi_{cJ}$ ($J = 0, 1, 2$). The process $e^+e^- \to \omega\chi_{c0}$ is observed for the first time, and the Born cross sections at $\sqrt{s} = 4.23$ and 4.26 GeV are measured to be $(55.4 \pm 6.0 \pm 5.9)$ and $(23.7 \pm 5.3 \pm 3.5)$ pb, respectively, where the first uncertainties are statistical and the second are systematic. The $\omega\chi_{c0}$ signals at the other seven energies and the $e^+e^- \to \omega\chi_{c1}$ and $\omega\chi_{c2}$ signals are not significant, and the upper limits on the cross sections are determined. By examining the $\omega\chi_{c0}$ cross section as a function of center of mass energy, we find that it is inconsistent with the line shape of the $Y(4260)$ observed in $e^+e^- \to \pi^+\pi^- J/\psi$. Assuming the $\omega\chi_{c0}$ signals come from a single resonance, we extract the mass and width of the resonance to be $(4230 \pm 8 \pm 6)$ MeV/c^2 and $(38 \pm 12 \pm 2)$ MeV, respectively, and the statistical significance is more than 9σ.



PHYSICAL REVIEW D **96**, 032004 (2017)

Measurement of $e^+e^- \to \pi^+\pi^-\psi(3686)$ from 4.008 to 4.600 GeV and observation of a charged structure in the $\pi^\pm\psi(3686)$ mass spectrum

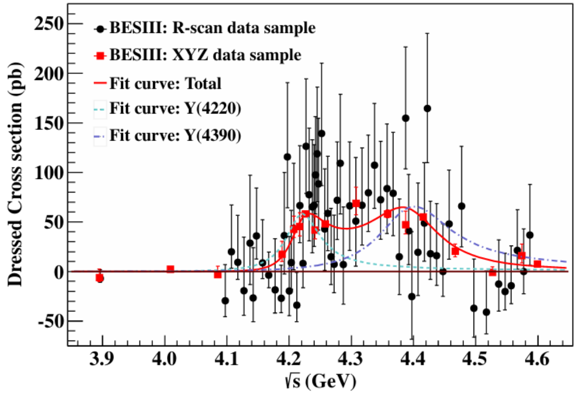
We study the process $e^+e^- \to \pi^+\pi^-\psi(3686)$ using 5.1 fb^{-1} of data collected at 16 center-of-mass energy ($\sqrt{s}$) points from 4.008 to 4.600 GeV by the BESIII detector operating at the BEPCII collider. The measured Born cross sections for $e^+e^- \to \pi^+\pi^-\psi(3686)$ are consistent with previous results, but with much improved precision. A fit to the cross section shows contributions from two structures: the first has $M = 4209.5 \pm 7.4 \pm 1.4$ MeV/c^2 and $\Gamma = 80.1 \pm 24.6 \pm 2.9$ MeV, and the second has $M = 4383.8 \pm 4.2 \pm 0.8$ MeV/c^2 and $\Gamma = 84.2 \pm 12.5 \pm 2.1$ MeV, where the first errors are statistical and the second systematic. The lower-mass resonance is observed in the process $e^+e^- \to \pi^+\pi^-\psi(3686)$ for the first time with a statistical significance of 5.8σ. A charged charmoniumlike structure is observed in the $\pi^\pm\psi(3686)$ invariant mass spectrum for data at $\sqrt{s} = 4.416$ GeV. A fit with an S-wave Breit-Wigner function yields a mass $M = 4032.1 \pm 2.4$ MeV/c^2, where the errors are statistical only. However, there are still unresolved discrepancies between the fit model and data. The width of the intermediate state varies in a wide range for different kinematic regions within the data set. Therefore, no simple interpretation of the data has been found, and a future data sample with larger statistics and more theoretical input will be required to better understand this issue.



PRL **118**, 092002 (2017) PHYSICAL REVIEW LETTERS week ending 3 MARCH 2017

Evidence of Two Resonant Structures in $e^+e^- \to \pi^+\pi^-h_c$

The cross sections of $e^+e^- \to \pi^+\pi^-h_c$ at center-of-mass energies from 3.896 to 4.600 GeV are measured using data samples collected with the BESIII detector operating at the Beijing Electron Positron Collider. The cross sections are found to be of the same order of magnitude as those of $e^+e^- \to \pi^+\pi^-J/\psi$ and $e^+e^- \to \pi^+\pi^-\psi(2S)$, but the line shape is inconsistent with the Y states observed in the latter two modes. Two structures are observed in the $e^+e^- \to \pi^+\pi^-h_c$ cross sections around 4.22 and 4.39 GeV/c^2, which we call $Y(4220)$ and $Y(4390)$, respectively. A fit with a coherent sum of two Breit-Wigner functions results in a mass of $(4218.4^{+5.5}_{-4.5} \pm 0.9)$ MeV/c^2 and a width of $(66.0^{+12.3}_{-8.3} \pm 0.4)$ MeV for the $Y(4220)$, and a mass of $(4391.5^{+6.3}_{-6.8} \pm 1.0)$ MeV/c^2 and a width of $(139.5^{+16.2}_{-20.6} \pm 0.6)$ MeV for the $Y(4390)$, where the first uncertainties are statistical and the second ones systematic. The statistical significance of $Y(4220)$ and $Y(4390)$ is 10σ over one structure assumption.



$e^+e^- \to \pi^+\pi^- J/\psi$ [1, 2]
$e^+e^- \to \chi_{c0}\omega$ [3, 4]
$e^+e^- \to h_c\pi^+\pi^-$ [5]
$e^+e^- \to \pi^+\pi^-\psi(3686)$ [6]
$e^+e^- \to D^0D^{*-}\pi^+$[7]
$e^+e^- \to \pi^0\pi^0 J/\psi$ [8]
$e^+e^- \to K^+K^- J/\psi$[9]
$e^+e^- \to \eta J/\psi$ [10]

[1] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **95**, 142001 (2005).
[2] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **118**, 092001 (2017).
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[9] M. Ablikim *et al.* (BESIII Collaboration), Chin. Phys. C **46**, 111002 (2022).
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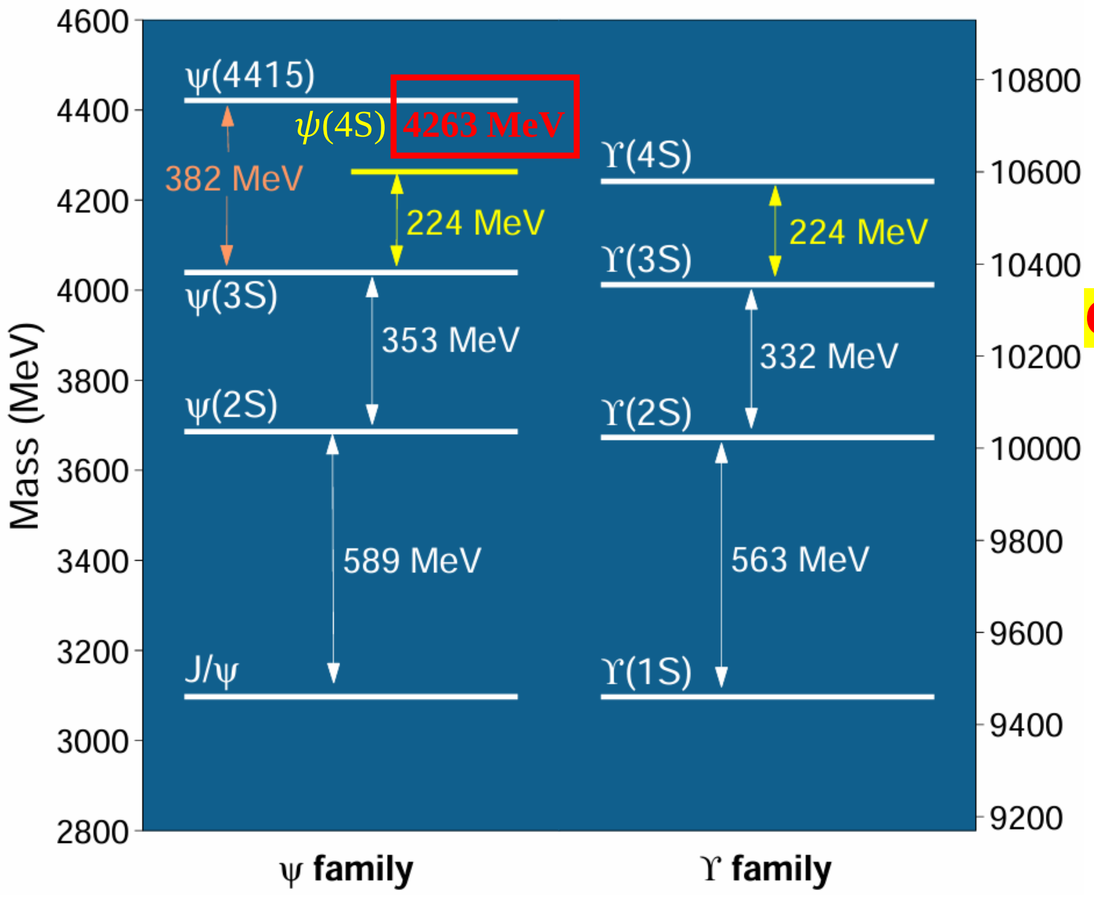
I. Background : Y (4220) / $\psi(4S)$

Screening Potential

The predicted $\psi(4S)$ EPJC 74, (2014) 3208

The similarity between charmonium and bottomium



Consistent

PHYSICAL REVIEW D VOLUME 49, NUMBER 3 1 FEBRUARY 1994

Leptonic decay of charmonium

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(Received 8 June 1993)

The leptonic decay of charmonium is studied by using an error function confinement potential in which the color-screening effect is considered. The relativistic correction of the quark-antiquark potential is also included in the calculation.

$M(\psi_{4S}) = 4273$ MeV

PHYSICAL REVIEW D VOLUME 51, NUMBER 9 1 MAY 1995

Possible effects of color screening and large string tension in heavy quarkonium spectra

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(Received 8 July 1994)

Possible effects of the color-screened confinement potential are investigated. A color-screened linear potential with a large string tension $T = (0.26\text{–}0.32)$ GeV^2 is suggested by a study of the $c\bar{c}$ and $b\bar{b}$ spectra. The $\psi(4160)$ and $\psi(4415)$ are, respectively, assigned as the $\psi(4S)$-dominated and the $\psi(5S)$ $c\bar{c}$ states. Satisfactory results for the masses and leptonic widths (with QCD radiative corrections) of $c\bar{c}$ and $b\bar{b}$ states are obtained.

$M(\psi_{4S}) = 4262$ MeV

PHYSICAL REVIEW D **79**, 094004 (2009)

Higher charmonia and X, Y, Z states with screened potential

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(Received 31 March 2009; published 7 May 2009)

We incorporate the color-screening effect due to light quark pair creation into the heavy quark-antiquark potential, and investigate the effects of screened potential on the spectrum of higher charmonium. We calculate the masses, electromagnetic decays, and E1 transitions of charmonium states in the screened potential model, and propose possible assignments for the newly discovered charmonium or charmoniumlike "X, Y, Z" states. We find the masses of higher charmonia with screened potential are considerably lower than those with unscreened potential. The $\chi_{c2}(2P)$ mass agrees well with that of the $Z(3930)$, and the mass of $\psi(4415)$ is compatible with $\psi(5S)$ rather than $\psi(4S)$. In particular, the discovered four Y states in the initial state radiation process, i.e., $Y(4008)$, $Y(4260)$, $Y(4320/4360)$, $Y(4660)$ may be assigned as the $\psi(3S)$, $\psi(4S)$, $\psi(3D)$, $\psi(6S)$ states, respectively. The $X(3940)$ and $X(4160)$ found in the double charmonium production in e^+e^- annihilation may be assigned as the $\eta_c(3S)$ and $\chi_{c0}(3P)$ states. Based on the calculated E1 transition widths for $\chi_{c1}(2P) \to \gamma J/\psi$ and $\chi_{c1}(2P) \to \gamma\psi(2S)$ and other results, we argue that the $X(3872)$ may be a $\chi_{c1}(2P)$ dominated charmonium state with some admixture of the $D^0\bar{D}^{*0}$ component. Possible problems encountered in these assignments and comparisons with other interpretations for these X, Y, Z states are discussed in detail. We emphasize that more theoretical and experimental investigations are urgently needed to clarify these assignments and other interpretations.

$M(\psi_{4S}) = 4274$ MeV

I. Background : Unquenched charmonium spectrum

Depicting the charmonimum spectrum
with unquenched potential model

PHYSICAL REVIEW D **99,** 114003 (2019)

Constructing J/ψ family with updated data of charmoniumlike Y states

Jun-Zhang Wang,[1,2,*] Dian-Yong Chen,[3,†] Xiang Liu,[1,2,‡] and Takayuki Matsuki[4,5,§]

$$\tilde{H} = (p^2 + m_c^2)^{1/2} + (p^2 + m_{\bar{c}}^2)^{1/2} + \tilde{V}_{eff}(\mathbf{p}, \mathbf{r})$$

The linear confining $br + c$ including in the potential is modified as

$$S^{scr}(r) = \frac{b(1 - e^{-\mu r})}{\mu} + c$$

Screening potential

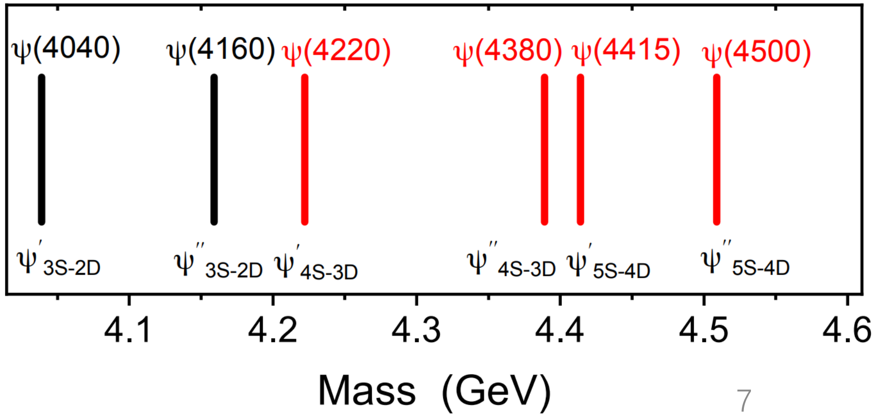
- Reproduce the masses of observed states
- $Y(4220)$ is an important scaling point

$\psi(4S)$

State	Mass	Expt. [9]	State	Mass	Expt. [9]
$\eta_c(1^1S_0)$	2981	2983.9 ± 0.5	$\psi(1^3D_1)$	3830	3778.1 ± 1.2
$\psi(1^3S_1)$	3096	3096.9 ± 0.006	$\psi_2(1^3D_2)$	3848	3822.2 ± 1.2
$\eta_c(2^1S_0)$	3642	3637.6 ± 1.2	$\psi_3(1^3D_3)$	3859	⋯
$\psi(2^3S_1)$	3683	3686.097 ± 0.01	$\eta_{c2}(2^1D_2)$	4137	⋯
$\eta_c(3^1S_0)$	4013	⋯	$\psi(2^3D_1)$	4125	4159 ± 20
$\psi(3^3S_1)$	4035	4039 ± 1	$\psi_2(2^3D_2)$	4137	⋯
$\eta_c(4^1S_0)$	4260	⋯	$\psi_3(2^3D_3)$	4144	⋯
$\psi(4^3S_1)$	4274	4230 ± 8	$\eta_{c2}(3^1D_2)$	4343	⋯
$\eta_c(5^1S_0)$	4433	⋯	$\psi(3^3D_1)$	4334	⋯
$\psi(5^3S_1)$	4443	⋯	$\psi_2(3^3D_2)$	4343	⋯
$h_c(1^1P_1)$	3538	3525.38 ± 0.11	$\psi_3(3^3D_3)$	4348	⋯
$\chi_{c0}(1^3P_0)$	3464	3414.71 ± 0.3	$\eta_{c2}(4^1D_2)$	4490	⋯
$\chi_{c1}(1^3P_1)$	3530	3510.67 ± 0.05	$\psi(4^3D_1)$	4484	⋯
$\chi_{c2}(1^3P_2)$	3571	3556.17 ± 0.07	$\psi_2(4^3D_2)$	4490	⋯
$h_c(2^1P_1)$	3933	⋯	$\psi_3(4^3D_3)$	4494	⋯
$\chi_{c0}(2^3P_0)$	3896	3918.4 ± 1.9	$h_{c3}(1^1F_3)$	4074	⋯
$\chi_{c1}(2^3P_1)$	3929	-	$\chi_{c2}(1^3F_2)$	4070	⋯
$\chi_{c2}(2^3P_2)$	3952	3927.2 ± 2.6	$\chi_{c3}(1^3F_3)$	4075	⋯
$h_c(3^1P_1)$	4200	⋯	$\chi_{c4}(1^3F_4)$	4076	⋯
$\chi_{c0}(3^3P_0)$	4177	⋯	$h_{c3}(2^1F_3)$	4296	⋯
$\chi_{c1}(3^3P_1)$	4197	⋯	$\chi_{c2}(2^3F_2)$	4293	⋯
$\chi_{c2}(3^3P_2)$	4213	⋯	$\chi_{c3}(2^3F_3)$	4297	⋯
$h_c(4^1P_1)$	4389	⋯	$\chi_{c4}(2^3F_4)$	4298	⋯
$\chi_{c0}(4^3P_0)$	4374	⋯	$\eta_{c4}(1^1G_4)$	4250	⋯
$\chi_{c1}(4^3P_1)$	4387	⋯	$\psi_3(1^3G_3)$	4252	⋯
$\chi_{c2}(4^3P_2)$	4398	⋯	$\psi_4(1^3G_4)$	4251	⋯
$\eta_{c2}(1^1D_2)$	3848	⋯	$\psi_5(1^3G_5)$	4249	⋯

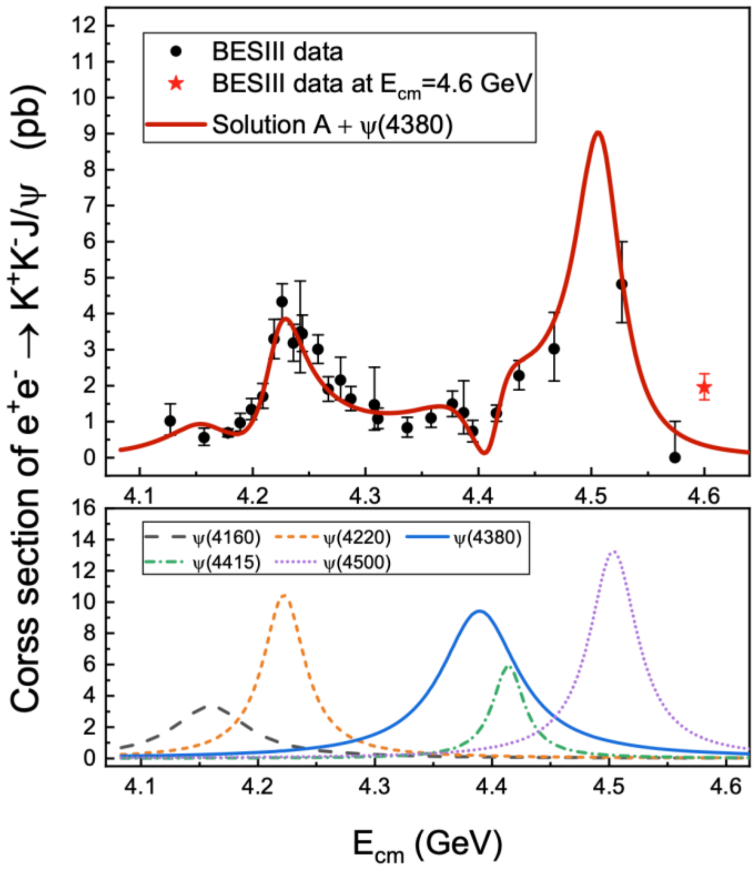
Characteristic spectrum of charmonium

$S - D$ mixing



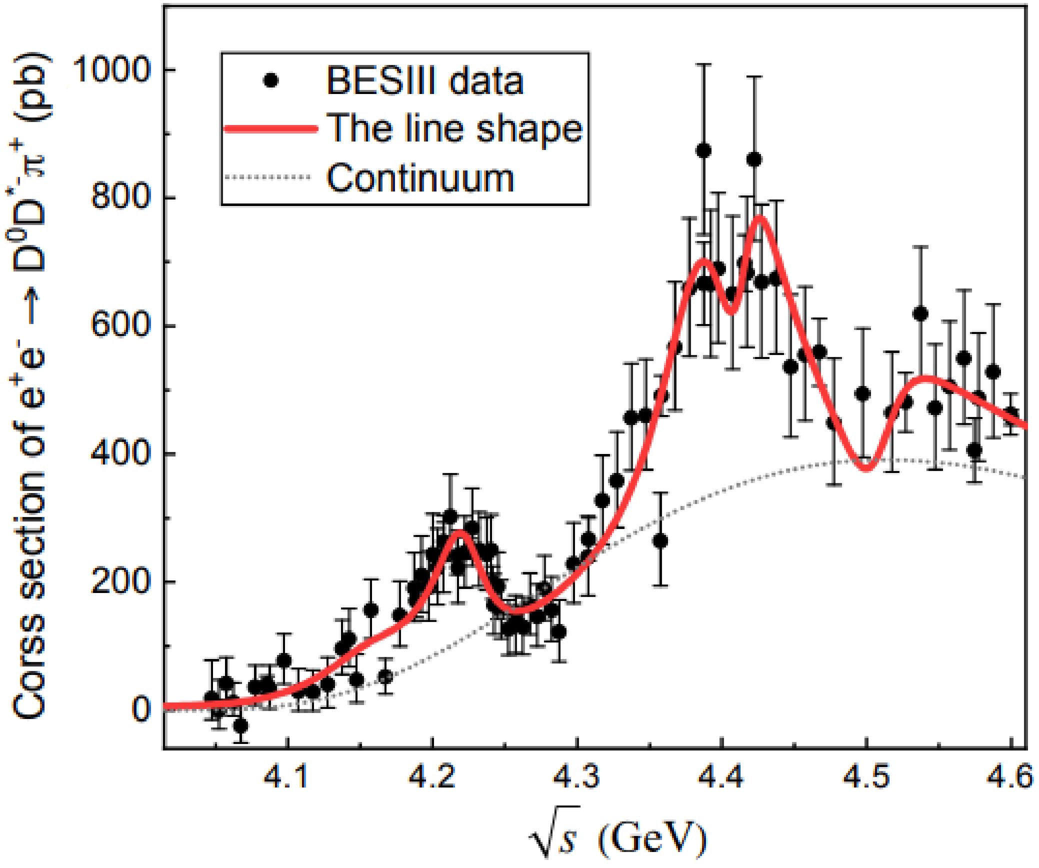
I. Background : Unquenched charmonium spectrum

$e^+e^- \to K^+K^-J/\psi$



Phys. Rev. D 107, 054016 (2023)

$e^+e^- \to D^0D^{*-}\pi^+$



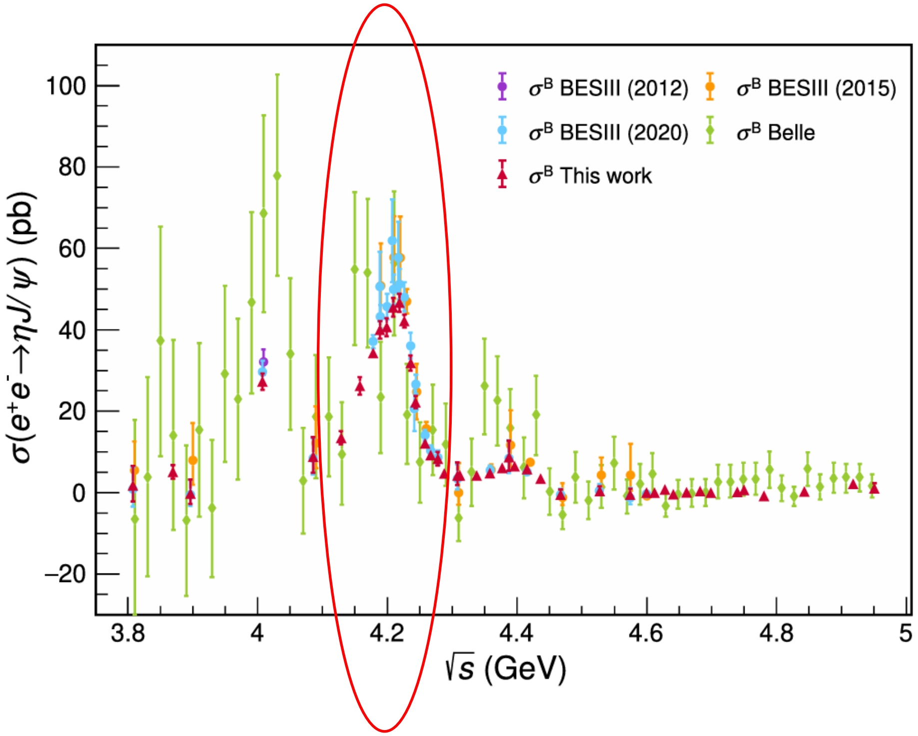
Phys. Lett. B 849, 138456 (2024)

II. The puzzling line shape in $e^+e^- \to \eta J/\psi$ process

PHYSICAL REVIEW D **109,** 092012 (2024)

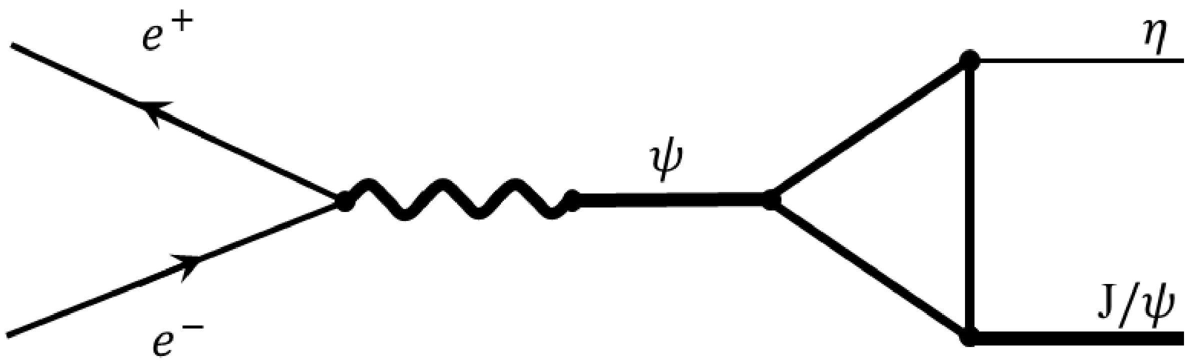
Measurement of $e^+e^- \to \eta J/\psi$ cross section from $\sqrt{s} = 3.808$ GeV to 4.951 GeV

Using data samples with an integrated luminosity of 22.42 fb^{-1} collected by the BESIII detector operating at the BEPCII storage ring, we measure the cross sections of the $e^+e^- \to \eta J/\psi$ process at center-of-mass energies from 3.808 to 4.951 GeV. Three structures are observed in the line shape of the measured cross sections. A maximum-likelihood fit with $\psi(4040)$, two additional resonances, and a nonresonant component are performed. The mass and width of the first additional state are $(4219.7 \pm 2.5 \pm 4.5)$ MeV/c^2 and $(80.7 \pm 4.4 \pm 1.4)$ MeV, respectively, consistent with the $\psi(4230)$. For the second state, the mass and width are $(4386 \pm 13 \pm 17)$ MeV/c^2 and $(177 \pm 32 \pm 13)$ MeV, respectively, consistent with the $\psi(4360)$. The first uncertainties are statistical, and the second ones are systematic. The statistical significance of $\psi(4040)$ is 8.0σ and those for $\psi(4230)$ and $\psi(4360)$ are more than 10.0σ.

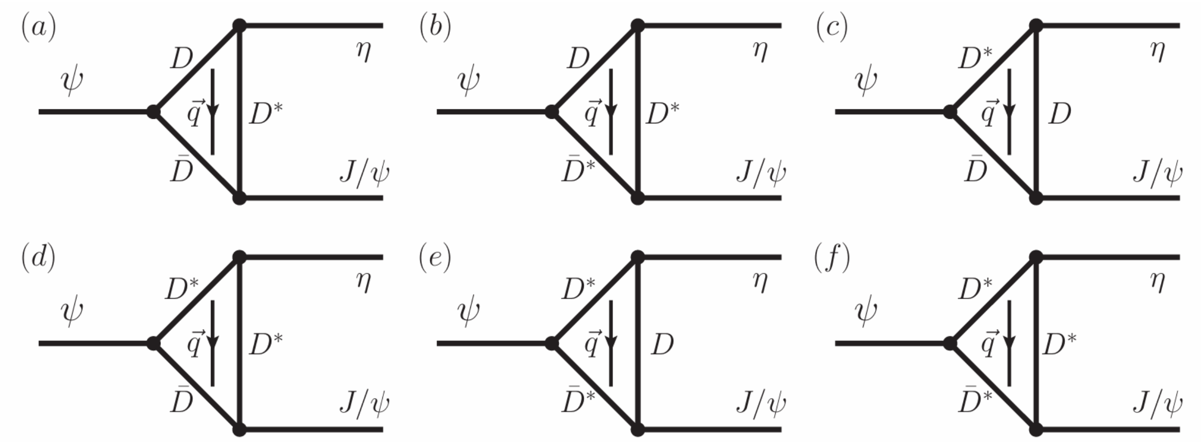


II. The puzzling line shape in $e^+e^- \to \eta J/\psi$ process

Hidden charm decay



Hadron loop



$$\mathcal{M}^{(a)}_{\psi(4040)} = \int \frac{d^4q}{(2\pi)^4} g_{\psi(4040)DD} \times \epsilon^{\mu}_{\psi}(q_{2\mu} - q_{1\mu})(-g_{D^*D\eta}) p_1^{\alpha} g_{J/\psi DD^*} \varepsilon_{\lambda\gamma\beta\rho} p_2^{\lambda} \epsilon^{\gamma *}_{J/\psi}(q_2^{\rho} - q^{\rho}) \times \frac{1}{q_1^2 - m_D^2} \frac{1}{q_2^2 - m_D^2} \frac{-g^{\beta}_{\alpha} + q_{\alpha} q^{\beta}/m^2_{D^*}}{q^2 - m^2_{D^*}} \mathcal{F}^2(q^2, m^2_{D^*}),$$

Branching ratios : $\mathcal{BR}[\psi \to J/\psi\eta] = \frac{1}{3}\frac{|\vec{p}_1|}{8\pi m_\psi^2} \sum_{spin} \left|\mathcal{M}_{\psi}^{\text{Total}}\right|^2 / \Gamma_\psi$

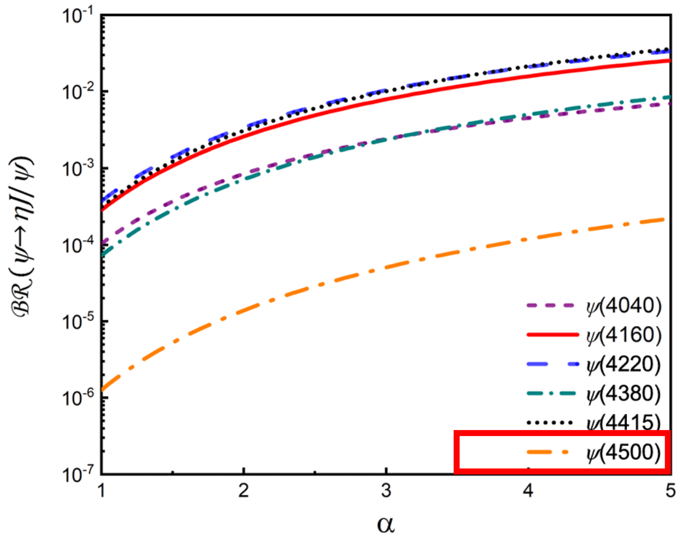


FIG. 2: The α parameter dependence of the predicted branching ratios, including $\mathcal{BR}(\psi(4040) \to \eta J/\psi)$, $\mathcal{BR}(\psi(4160) \to \eta J/\psi)$, $\mathcal{BR}(\psi(4220) \to \eta J/\psi)$, $\mathcal{BR}(\psi(4380) \to \eta J/\psi)$, $\mathcal{BR}(\psi(4415) \to \eta J/\psi)$, and $\mathcal{BR}(\psi(4500) \to \eta J/\psi)$.

II. The puzzling line shape in $e^+e^- \to \eta J/\psi$ process

The total cross section of $e^+e^- \to \eta J/\psi$ can be written as

$$\sigma(s) = |\mathcal{M}_{\mathrm{Total}}(s)|^2$$

The total amplitude is

$$\mathcal{M}_{\mathrm{Total}}(s) = \mathcal{M}_0(s) + \sum_i e^{i\phi_i}\mathcal{M}_{\psi_i}(s)$$

An exponentially parametrized background

$$\mathcal{M}_0(s) = g(\sqrt{s} - m_\eta - m_{J/\psi})^2 e^{-a(\sqrt{s}-m_\eta-m_{J/\psi})^2}$$

The intermediate charmonium can be described Breit-Wigner function

$$\mathcal{M}_\psi(s) = \frac{\sqrt{12\pi\Gamma_\psi^{e^+e^-}\mathcal{B}(\psi\to\eta J/\psi)\Gamma_\psi}}{s - m_\psi^2 + im_\psi\Gamma_\psi} \times \sqrt{\frac{\Phi_{2\to2}(s)}{\Phi_{2\to2}(m_\psi^2)}},$$

Scheme I

With the same cut off parameter α (in Form factor) for each sate

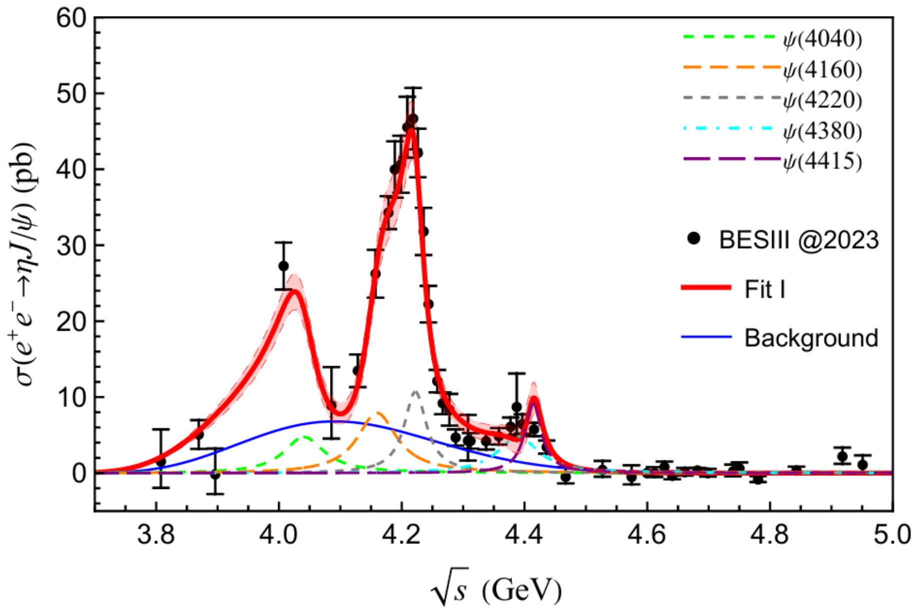


FIG. 3: Our fit to the higher vector charmonium contribution in the cross section distribution of $e^+e^- \to \eta J/\psi$ within scheme I. Here, the data points are from BESIII measurement [10], the five dashed lines represent the contributions of the higher charmonium states, the blue line represents the background, and the red line with a band represents the total contribution and uncertainties.

II. The puzzling line shape in $e^+e^- \to \eta J/\psi$ process

Scheme II

With cut off parameter α from 1 to 5

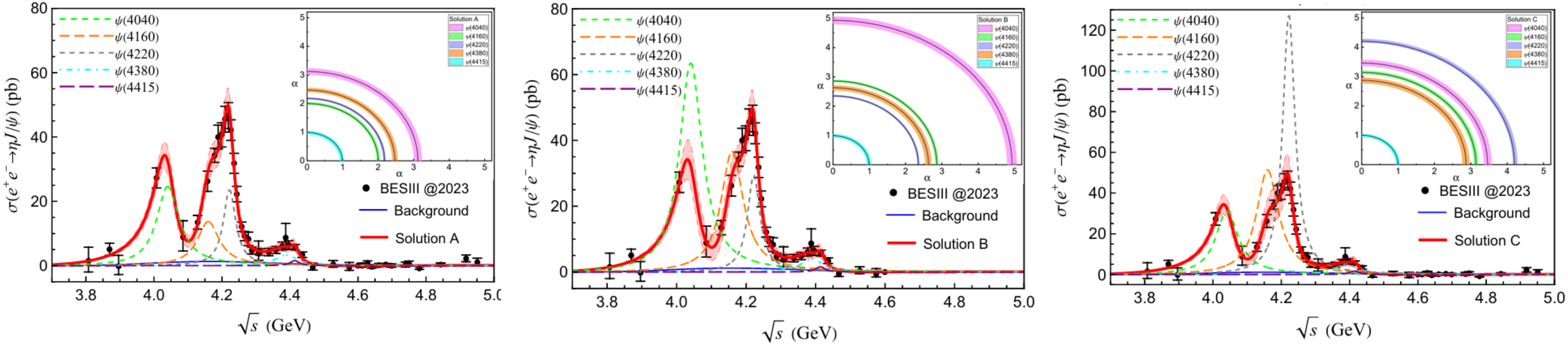
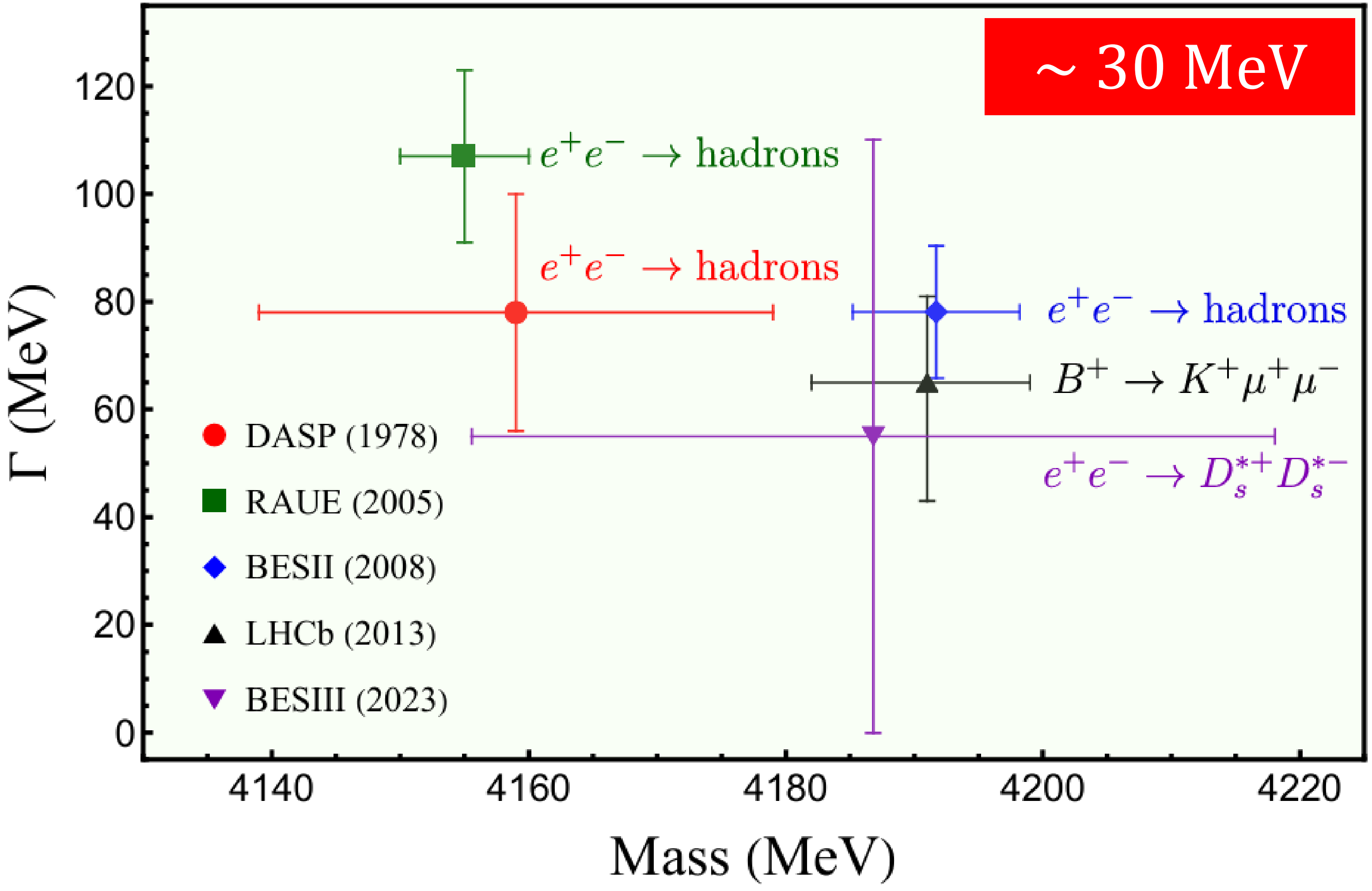


FIG. 4: Our fit to the cross section of the $e^+e^- \to \eta J/\psi$ process between $E_{cm} = 3.808$ to 4.600 GeV by scheme II. Here, the five dashed lines represent the contributions of the higher charmonium states, the blue line represents the background, and the red line with a band represents the total contribution and uncertainties. The insets show the branching ratios for each state with the central values and uncertainties in the fit as shown in Table IV and are represented by the corresponding α values in the hadronic loop mechanism [in Eq. (4)] as shown in Fig. 2, where the central values and the errors are represented by the solid black lines and the colored bands, respectively.

The mass changes from 4160 MeV to 4190 MeV



III. Reevaluating the $\psi(4160)$ resonance parameters

PRL **111,** 112003 (2013) PHYSICAL REVIEW LETTERS week ending 13 SEPTEMBER 2013

Observation of a Resonance in $B^+ \to K^+\mu^+\mu^-$ Decays at Low Recoil

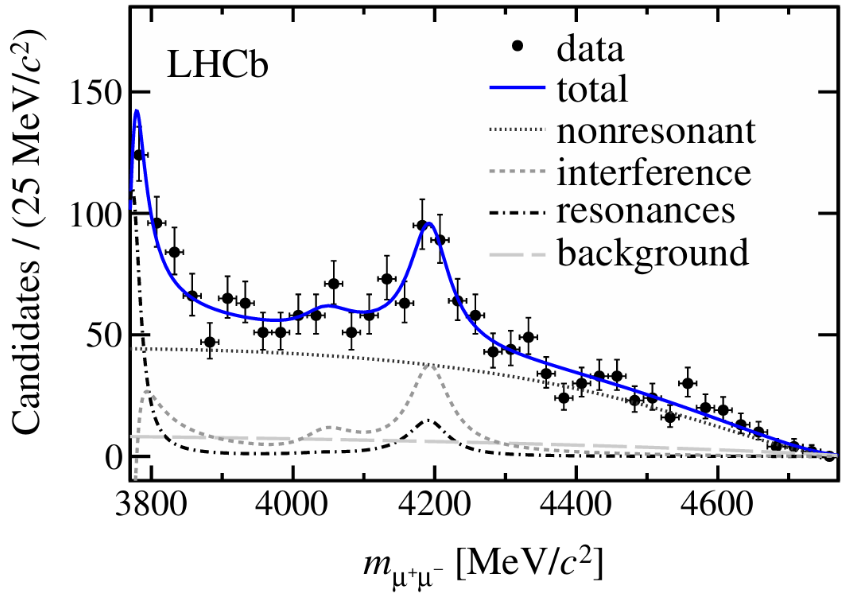


FIG. 1 (color online). Dimuon mass distribution of data with fit results overlaid for the fit that includes contributions from the nonresonant vector and axial vector components, and the $\psi(3770)$, $\psi(4040)$, and $\psi(4160)$ resonances. Interference terms are included and the relative strong phases are left free in the fit.

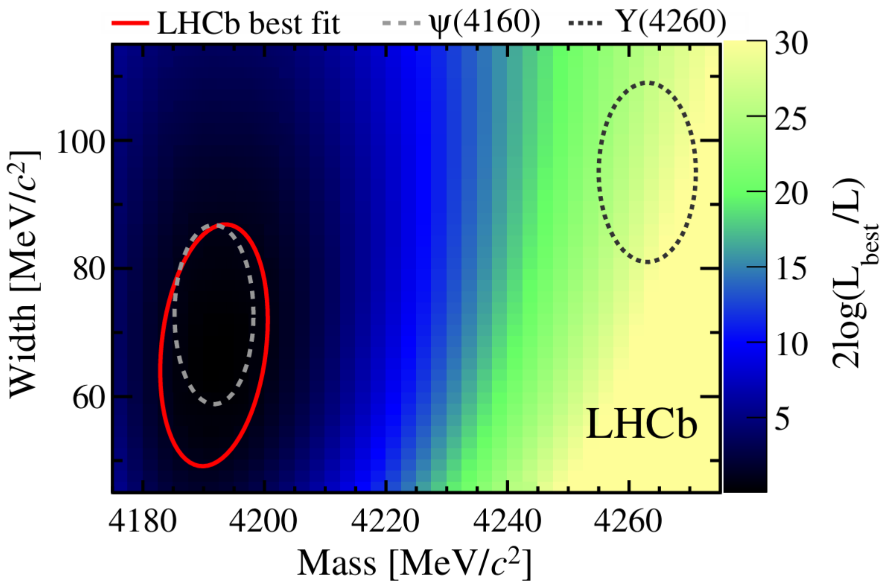


FIG. 3 (color online). Profile likelihood as a function of mass and width of a fit with a single extra resonance. At each point all other fit parameters are reoptimized. The three ellipses are (red, solid line) the best fit and previous measurements of (gray, dashed line) the $\psi(4160)$ [4] and (black, dotted line) the $Y(4260)$ [21] states.

III. Reevaluating the $\psi(4160)$ resonance parameters

A puzzling phenomenon, where the measured mass of the $\psi(4160)$ is pushed higher, presents a challenge to current theoretical models of hadron spectroscopy. This study suggests that the issue arises from analyses based on the outdated quenched charmonium spectrum. In the past two decades, the discovery of new hadronic states has emphasized the importance of the unquenched effect. Under the unquenched picture, six vector charmonium states—$\psi(4040)$, $\psi(4160)$, $\psi(4220)$, $\psi(4380)$, $\psi(4415)$, and $\psi(4500)$—are identified in the $4 \sim 4.5$ GeV range, contrasting with the three states predicted in the quenched model. We reevaluate the resonance parameters of the $\psi(4160)$ using the di-muon invariant mass spectrum of $B^+ \to K^+\mu^+\mu^-$ and unquenched charmonium spectroscopy. Our analysis indicates previous experimental overestimations for the mass of the $\psi(4160)$. This conclusion is supported by analyzing $e^+e^- \to D_s\bar{D}_s^*$, which finds the $\psi(4160)$ mass at 4145.76 ± 4.48 MeV. Our findings have significant implications for both hadron spectroscopy and search for new physics signals by R_K.

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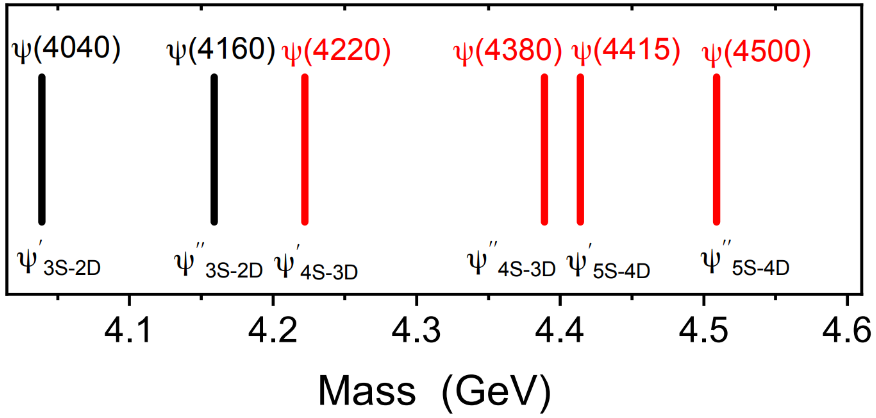


III. Reevaluating the $\psi(4160)$ resonance parameters

Peng, Bai, Wang, Liu, PRD 111 (2025) 054023

The decay process $B^+ \to K^+\mu^+\mu^-$ can occur through three distinct mechanisms, where the dimuon pair $(\mu^+\mu^-)$ couples to a Z^0 boson, a photon (γ), or a vector resonance. These contributions are represented by the amplitudes $\mathcal{A}^{\rm AV}_{\rm non-res}(Z^0)$, $\mathcal{A}^{\rm V}_{\rm non-res}(Z^0$ and $\gamma)$, and $\mathcal{A}^n_{\rm res}$. The subscripts AV and V are used to denote the first two terms, reflecting the axial-vector (AV) and vector (V) nature of the couplings involved. The total amplitude is

$$\mathcal{A}_{\rm Tot} = \mathcal{A}^{\rm AV}_{\rm non-res} + \mathcal{A}^{\rm V}_{\rm non-res} + \sum_n f_n e^{i\delta_n} \mathcal{A}^n_{\rm res}, \quad (2)$$

Vector Charmonium

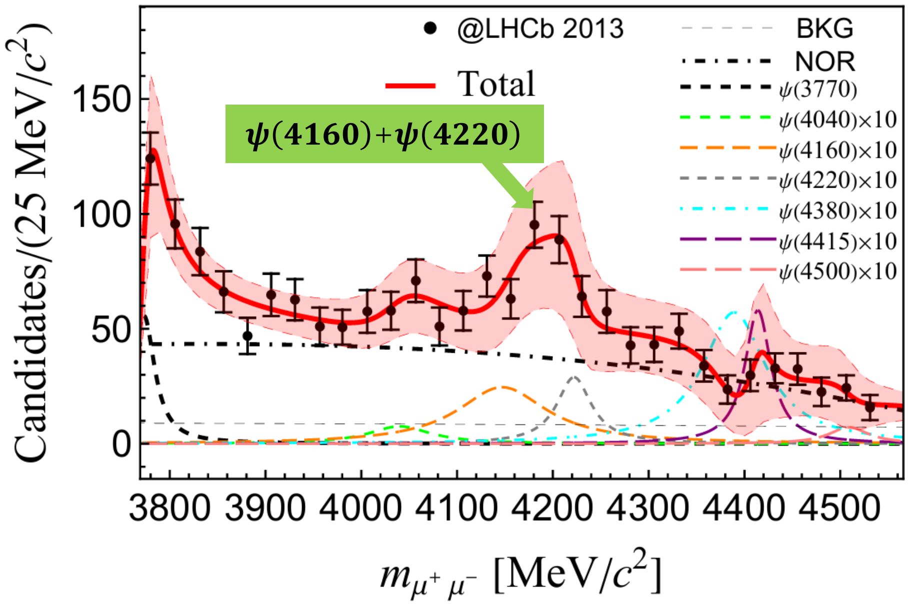


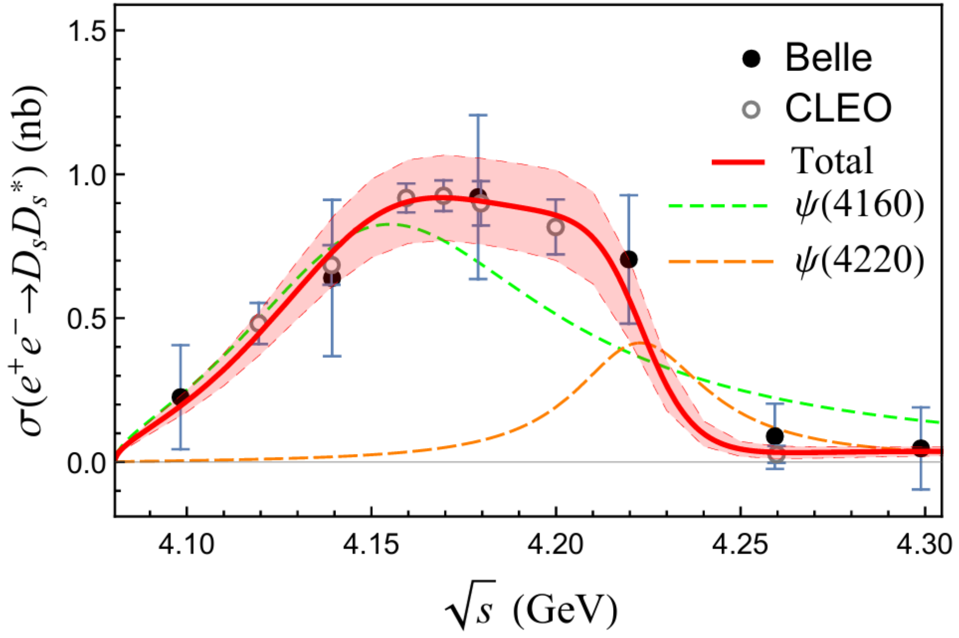
TABLE I: The masses and widths of higher charmonium states in the range of 4.0–4.5 GeV, which were obtained from the theoretical predictions [13–15], as well as some experimental values [3, 4].

States	Mass (MeV)	Γ (MeV)
$\psi(3770)$	3773.7 ± 0.7 [3]	27.2 ± 1.0 [3]
$\psi(4040)$	4040 ± 4 [3]	84 ± 12 [3]
$\psi(4160)$	4159 ± 22 [4]	78 ± 22 [4]
$\psi(4220)$	4222	44
$\psi(4380)$	4389	80
$\psi(4415)$	4414	33
$\psi(4500)$	4509	50

TABLE II: The parameter values obtained from fitting the experimental data are as follows. The factors f_i (in units of MeV) are chosen to ensure that the resonance amplitudes have the same dimensions as the nonresonance contribution. The phases δ_i (in radians) correspond to the seven ψ states: $\psi(3770)$, $\psi(4040)$, $\psi(4160)$, $\psi(4220)$, $\psi(4380)$, $\psi(4415)$, and $\psi(4500)$, listed in succession.

Parameters (MeV)	Value	Parameters (rad)	Value
f_1	46.37 ± 2.52	δ_1	0.95 ± 0.05
f_2	4.83 ± 0.33	δ_2	2.30 ± 0.17
f_3	7.12 ± 0.56	δ_3	1.67 ± 0.11
f_4	8.85 ± 0.56	δ_4	4.36 ± 0.32
f_5	9.87 ± 0.74	δ_5	5.66 ± 0.42
f_6	9.29 ± 0.49	δ_6	2.74 ± 0.20
f_7	3.57 ± 0.26	δ_7	5.00 ± 0.30
χ^2/d.o.f. = 0.90			

III. Reevaluating the $\psi(4160)$ resonance parameters



$\psi(4160)+\psi(4220)$

$$\sigma(s) = |\mathrm{BW}_1(s) + \mathrm{BW}_2(s)e^{i\phi}|^2$$

$$\mathrm{BW}(s) = \frac{\sqrt{12\pi\Gamma_\psi^{ee}\Gamma_{\mathrm{tot}}\mathcal{B}(\psi \to D_s\bar{D}_s^*)}}{s - M^2 + iM\Gamma_{\mathrm{R}}}\sqrt{\frac{\mathrm{PS}(\sqrt{s})}{\mathrm{PS}(M)}}$$

TABLE III: The fitting parameters $m_{\psi(4160)}$, $\Gamma_{\psi(4160)}$, Γ_ψ^{ee} and $\mathcal{B}_\psi$ represent mass, total width, di-lepton width and branch ratio of $\psi \to D_s\bar{D}_s^*$, respectively, and ϕ (rad) is the phase between the resonance amplitudes associated with the $\psi(4160)$ and $\psi(4220)$ in the $e^+e^- \to D_s\bar{D}_s^*$ cross section.

Parameters	Best fit	I	II	III	IV
$m_{\psi(4160)}$ (MeV)	4145.76 ± 5.48	4140 (fixed)	4150 (fixed)	4160 (fixed)	4170 (fixed)
$\Gamma_{\psi(4160)}$ (MeV)	104.83 ± 23.71	113.98 ± 24.01	108.78 ± 23.65	127.17 ± 17.65	143.37 ± 25.24
$\Gamma^{ee}_{\psi(4160)}\mathcal{B}_{\psi(4160)}$ (eV)	98.02 ± 26.88	108.43 ± 29.8	109.14 ± 30.04	168.42 ± 35.09	207.29 ± 25.91
$\Gamma^{ee}_{\psi(4220)}\mathcal{B}_{\psi(4220)}$ (eV)	22.09 ± 8.82	23.23 ± 10.00	21.32 ± 10.96	51.12 ± 34.72	66.81 ± 22.22
ϕ (rad)	2.91 ± 0.27	2.90 ± 0.23	3.01 ± 0.30	3.35 ± 0.23	3.25 ± 0.14
χ^2/d.o.f.	0.22	0.31	0.26	0.79	1.88

Summary

- Combining unquenched charmonium mass spectrum and hadronic mechanism, explains line shape asymmetry in $e^+e^- \to \eta J/\psi$ cross section.
- Taking into account the interference between the $\psi(4160)$ and $\psi(4220)$ under unquenched charmonium spectrum, solving the mass deviation of the $\psi(4160)$.

Thanks