Nonrelativistic QCD as a novel probe to fully-heavy exotic hadrons

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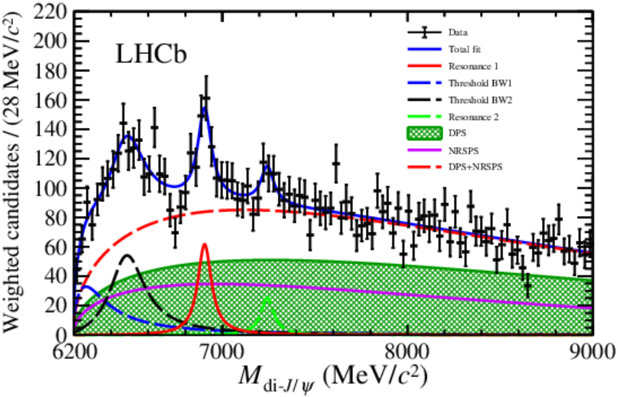
Background

Background

Double J/ψ Resonances

- Double J/ψ resonances observed at the LHC



Possible candidates (S-wave)

$T_{cc\bar{c}\bar{c}}$	${}^1S_0({}^1S_0, {}^1S_0)$	${}^1S_0({}^3S_1, {}^3S_1)$	${}^5S_2({}^3S_1, {}^3S_1)$
$M_{J/\psi J/\psi}$	${}^1S_0$	${}^5S_2$	

- Mass splitting: $m_c v^4 \approx 100\mathrm{MeV}$

Questions

- How to determine its J^{PC}? (Solved!)
  - From first principle
  - Robust!
- How to explore the structures of these resonances?
  - Compact tetraquarks or molecules?

NRQCD Framework

NRQCD Framework

NRQCD Factorization

- Cross Section Factorization

$$\mathrm{d}\sigma(H) = \sum_n \mathrm{d}\hat{\sigma}(n)\langle\mathcal{O}^H(n)\rangle$$

- n: intermediate state
- $\hat{\sigma}(n)$: Short-distance coefficient
- $\langle\mathcal{O}^H(n)\rangle$: Long-distance Matrix Element

Replacement

- For J/ψ

$$v_i(p_b)\overline{u}_j(p_a) \to \Pi_{J/\psi} \equiv \frac{1}{2\sqrt{2}}\frac{1}{\sqrt{N_c}}\delta_{ij}\not{\epsilon}(\not{p} + m_{J/\psi})$$

$$\mathrm{d}\Phi_a \mathrm{d}\Phi_b \to \mathrm{d}\Phi_{J/\psi} \equiv \frac{\mathrm{d}^3 p}{(2\pi)^3 2p_0}\frac{2}{m_{J/\psi}}\left|\psi_{J/\psi}(0)\right|^2$$

Replacement

- For Molecule

$$\epsilon^{\mu} \otimes \epsilon^{\nu} \to \varepsilon^{\mu\nu}_{ss_z}$$

$$\varepsilon^{\mu\nu}_{00} = \sqrt{\frac{1}{3}}(-g^{\mu\nu} + \frac{P^{\mu}P^{\nu}}{M^2}) \equiv \sqrt{\frac{1}{3}}\Pi^{\mu\nu}$$

$$\sum_{s_z} \varepsilon^{\mu\nu}_{2s_z}\varepsilon^{\alpha\beta*}_{2s_z} = \frac{1}{2}(\Pi^{\mu\alpha}\Pi^{\nu\beta} + \Pi^{\mu\beta}\Pi^{\nu\alpha}) - \frac{1}{3}\Pi^{\mu\nu}\Pi^{\alpha\beta}$$

Replacement

- For Genuine Tetraquark
  - CParity Transformation:

$$\overline{u}_i^{\alpha}(p_b)(\not{k}_1+m_1)\ldots(\not{k}_n+m_n)v_j^{\beta}(p_{(c,d)})$$
$$=\overline{u}_j^{\beta}(p_{(c,d)})(-\not{k}_n+m_n)\ldots(-\not{k}_1+m_1)v_i^{\alpha}(p_b)$$

- Replacement

$$v_{i_b}(p_b)\overline{u}_{i_a}(p_a)\otimes v_{i_{(c,d)}}(p_{(c,d)})\overline{u}_{i_{(d,c)}}(p_{(d,c)})$$
$$\rightarrow\frac{1}{32}\varepsilon_{ss_z}^{\mu\nu}\frac{\delta_{i_ai_c}\delta_{i_bi_d}-\delta_{i_ai_d}\delta_{i_bi_c}}{\sqrt{12}}\times\gamma^{\mu}(\not{P}+M)\otimes\gamma^{\nu}(\not{P}+M)$$
$$v_{i_b}(p_b)\overline{u}_{i_a}(p_a)\otimes v_{i_{(c,d)}}(p_{(c,d)})\overline{u}_{i_{(d,c)}}(p_{(d,c)})$$
$$\rightarrow\frac{1}{32}sgn\frac{\delta_{i_ai_c}\delta_{i_bi_d}+\delta_{i_ai_d}\delta_{i_bi_c}}{\sqrt{24}}\times\gamma^{5}(\not{P}+M)\otimes\gamma^{5}(\not{P}+M)$$

Prediction of the Spin

Prediction of the Spin

Quantity to explore the spin

- Ratio (R) of cross sections for spin-2 to spin-0

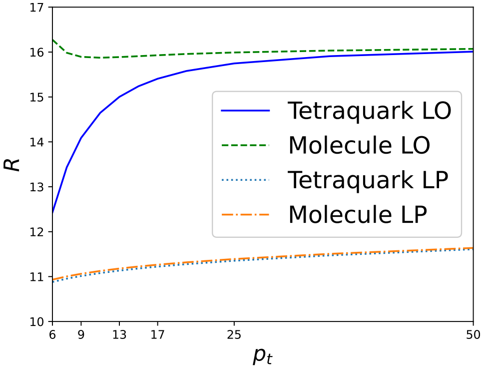
$$R(T) \equiv \frac{\mathrm{d}\sigma(T_{cc\bar{c}\bar{c}}[2^{++}])}{\mathrm{d}\sigma(T_{cc\bar{c}\bar{c}}[0^{++}])} = \frac{5\mathrm{d}\hat{\sigma}(cc\bar{c}\bar{c}[2^{++}])}{\mathrm{d}\hat{\sigma}(cc\bar{c}\bar{c}[0^{++}])}$$

$$R(M) \equiv \frac{\mathrm{d}\sigma(M_{\psi\psi}[2^{++}])}{\mathrm{d}\sigma(M_{\psi\psi}[0^{++}])} = \frac{5\mathrm{d}\hat{\sigma}(\psi\psi[2^{++}])}{\mathrm{d}\hat{\sigma}(\psi\psi[0^{++}])}$$

- Free of nonperturbative parameters!

Ratio versus Transverse Momentum

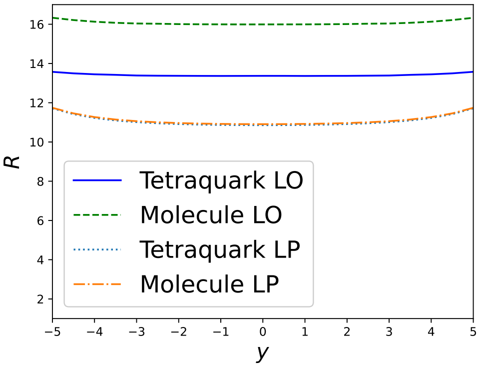
- Ratio of cross sections for spin-2 to spin-0[1]



[1] HFZ, Y.-Q. Ma, W.-L. Sang, Sci. Bull. 70, 1915 (2025)

Ratio versus Rapidity

- Ratio of cross sections for spin-2 to spin-0[2]



[2] HFZ, Y.-Q. Ma, W.-L. Sang, Sci. Bull. 70, 1915 (2025)

Conclusion

- The observed resonances are likely spin-2
- Confirmed by the CMS measurement

Ratio to Explore the Structure

Ratio to Explore the Structure

Ratio to distinguish between compact tetraquarks and molecules

- Ratio $\mathcal{R}$ to distinguish between compact tetraquarks and molecules

$$\mathcal{R} \equiv \frac{\mathcal{B}(T_{cc\bar{c}\bar{c}} \to c\bar{c})}{\mathcal{B}(T_{cc\bar{c}\bar{c}} \to J/\psi\gamma)}$$

- Free of nonperturbative parameters!

Results of $\mathcal{R}$

- Results of $\mathcal{R}$ for compact tetraquarks and molecules:

$$\mathcal{R}(T) \approx 3.4$$
$$\mathcal{R}(M) \approx 780$$

- The two structures can be perfectly distinguished!
- The structures of the resonances can be explored even by measuring the upper or lower limit of $\mathcal{R}$.

Conclusion

Conclusion

Conclusion

- NRQCD successfully predicted the spin of the di-J/ψ resonances.
- NRQCD can access the structure of the fully-heavy exotic states.

Thanks

Thanks!

Appendix

Appendix: New Method for Solving Multibody Systems

Hartree-Fock Method

- Wave function ansatz: Slater determinant

$$\begin{vmatrix} \psi_1(x_1) & \psi_2(x_1) & \dots & \psi_n(x_1) \\ \psi_1(x_2) & \psi_2(x_2) & \dots & \psi_n(x_2) \\ \dots & & & \\ \psi_1(x_n) & \psi_2(x_n) & \dots & \psi_n(x_n) \end{vmatrix}$$

- Hartree-Fock Equation

$$\begin{aligned} &-\frac{\hbar^2}{2m_i}\nabla_i\psi_i(\boldsymbol{x_i}) + \int \mathrm{d}^3x_1 \dots \mathrm{d}^3x_{i-1}\mathrm{d}^3x_{i+1}\dots\mathrm{d}^3x_n \\ &\times |\psi_1(\boldsymbol{x}_1)|^2 \dots |\psi_{i-1}(\boldsymbol{x_{i-1}})|^2|\psi_{i+1}(\boldsymbol{x_{i+1}})|^2 \dots |\psi_n(\boldsymbol{x_n})|^2 \\ &\times V(\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x_n})\psi_i(\boldsymbol{x_i}) = E_i\psi_i(\boldsymbol{x_i}) \end{aligned}$$

Drawbacks of the Hartree-Fock Method

- Incapable of strongly correlated systems
  - Slater determinant ansatz
- Low precision
  - fail in solving hydrogen ground-state energy
- Kinetic energy of the center of mass cannot be naturally eliminated
  - each ψ_i has kinetic energy

New Ansatz

- Construct an eigenstate of angular momentum
- Spacial wave function

$$\psi = \sum_{mm' m_1 m_2} C_{LM}^{mm'} Y_{lm}(\theta, \vartheta) R_{nl}(r) C_{l'm'}^{m_1 m_2} \times Y_{l_1 m_1}(\theta_1, \vartheta_1) R_{1 n_1 l_1}(r_1) Y_{l_2 m_2}(\theta_2, \vartheta_2) R_{2 n_2 l_2}(r_2)$$

- Functions to be solved: R_{nl}, $R_{1 n_1 l_1}$, $R_{2 n_2 l_2}$

Hartree-Fock Equations

- Hartree-Fock Equation via new ansatz

$$\left[-\frac{\hbar^2}{2m_q}\left(\frac{1}{r}\frac{\mathrm{d}^2}{\mathrm{d}r^2}r-\frac{l(l+1)}{r^2}\right)+V_0(r)-\mathcal{E}_0\right]R_{nl}(r)=0,$$
$$\left[-\frac{\hbar^2}{m_q}\left(\frac{1}{r_1}\frac{\mathrm{d}^2}{\mathrm{d}r_1^2}r_1-\frac{l_1(l_1+1)}{r_1^2}\right)+V_{qq}(r_1)+V_1(r_1)-\mathcal{E}_1\right]R_{1n_1l_1}(r_1)=0,$$
$$\left[-\frac{\hbar^2}{m_q}\left(\frac{1}{r_2}\frac{\mathrm{d}^2}{\mathrm{d}r_2^2}r_2-\frac{l_2(l_2+1)}{r_2^2}\right)+V_{qq}(r_2)+V_2(r_2)-\mathcal{E}_2\right]R_{2n_2l_2}(r_2)=0$$

$$V_{q\bar{q}}(\boldsymbol{r},\boldsymbol{r}_1,\boldsymbol{r}_2)=\sum_{lml_1m_1l_2m_2}V_{lml_1m_1l_2m_2}(r,r_1,r_2)Y_{lm}(\theta,\vartheta)Y_{l_1m_1}(\theta_1,\vartheta_1)Y_{l_2m_2}(\theta_2,\vartheta_2)$$

$$V_0(r)=\int \mathrm{d}r_1\mathrm{d}r_2\,\varrho_1(r_1)\varrho_2(r_2)V_{lml_1m_1l_2m_2}(r,r_1,r_2),$$

$$V_1(r_1)=\int \mathrm{d}r\mathrm{d}r_2\,\varrho(r)\varrho_2(r_2)V_{lml_1m_1l_2m_2}(r,r_1,r_2),$$

$$V_2(r_2)=\int \mathrm{d}r\mathrm{d}r_1\,\varrho(r)\varrho_1(r_1)V_{lml_1m_1l_2m_2}(r,r_1,r_2)$$

Thanks!