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Theoretical study of $N(1535)$ and $a_0(980)$ in the process $\Lambda_c^+ \to \pi^+\eta n$

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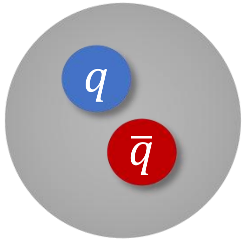
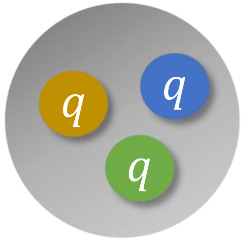
第八届强子谱和强子结构研讨会@广西桂林 2025年7月14日

Introduction

✓ Conventional hadrons

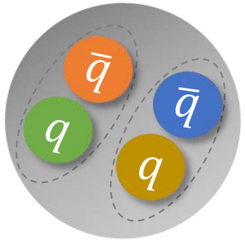


Meson

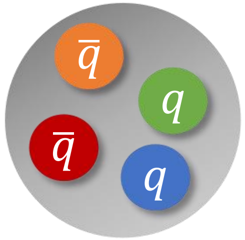


Baryon

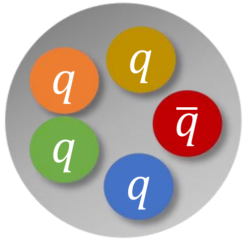
✓ Exotic hadrons



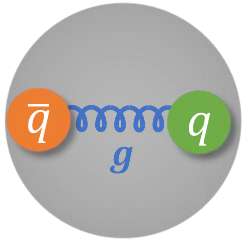
Molecule



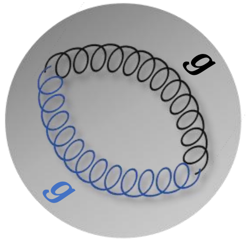
Tetraquark



Pentaquark



Hybird



Glueball

……

Introduction

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$$N(1535)\ \ I(J^P) = 1/2(1/2^-)$$

✓ Mass reverse problem

$N(1535)\ \ J^P = 1/2^-$ n=1 L=1

$N(1440)\ \ J^P = 1/2^+$ n=2 L=0

$N(1535)\ \ J^P = 1/2^-$

95 MeV

$N(1440)\ \ J^P = 1/2^+$

High mass vs. $N(1440)$

✓ Coupling to channels with strangeness

$N(1535) \xrightarrow{\text{couples strongly to}} \eta N, K\Lambda$ and $K\Sigma$

✓ Theoretical interpretation

- admixing of the $[ud][us]\bar{s}$ pentaquark component:
  - C. Helminen and D. O. Riska, Nucl. Phys. A699, 624 (2002)
  - B. S. Zou, Eur. Phys. J. A 35, 325 (2008)
- three-quark core:
  - Z. W. Liu et al. Phys. Rev. Lett. 116, 082004 (2016)
  - C. D. Abellet al. Phys. Rev. D 108, 094519 (2023)
- dynamically generated state:
  - P. C. Bruns et al. Phys. Lett. B 697, 254 (2011)
  - K. P. Khemchandani et al. Phys. Rev. D 88, 114016 (2013)
  - Y. Li, S. W. Liu et al. Phys. Rev. D 110 (2024) 7, 074010

Introduction

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$$a_0(980)\ \ I^G(J^{PC}) = 1^-(0^{++})$$

✓ **Flavor-wave functions**

$$f_0(500) = 1/\sqrt{2}(u\bar{u} + d\bar{d})$$

$$a_0(980) = 1/\sqrt{2}(u\bar{u} - d\bar{d})$$

$$K_0^*(700) = d\bar{s}$$

$$f_0(980) = c_1(u\bar{u} + d\bar{d}) + c_2(s\bar{s})$$

✓ **Conventional quark model**

$$f_0(500) \approx a_0(980) < K_0^*(700) < f_0(980)$$

✓ **Experiment**

$$f_0(500) < K_0^*(700) < a_0(980) < f_0(980)$$

✓ **Theoretical interpretation**

- $K\bar{K}$ molecular state:

  J. D. Weinstein and N. Isgur, Phys. Rev. Lett. 48 (1982) 659

  J. R. Pelaez, Phys. Rev. Lett. 92 (2004) 102001

- Compact tetraquark state:

  J. R. Pelaez, Phys. Rev. Lett. 92 (2004) 102001

  H. J. Lee, Eur. Phys. J. A 30 (2006) 423-426

  S. Stone and L. Zhang, Phys. Rev. Lett. 111 (2013) 6, 062001

- Dynamically generated states:

  G. Janssen, B. C. Pearce et al. Phys. Rev. D 52, 2690 (1995)

  J. J. Xie, L. R. Dai and E. Oset, Phys. Lett. B 742, 363 (2015)

  R. Molina, J. J. Xie et al. Phys. Lett. B 803 (2020) 135279

  X. C. Feng, L. L. Wei et al. Phys. Lett. B 846 (2023) 138185

  M. Y. Duan, W. T. Lyu et al.Phys. Rev. D 111 (2025) 1, 016004

Introduction

✓ Charm baryon decay

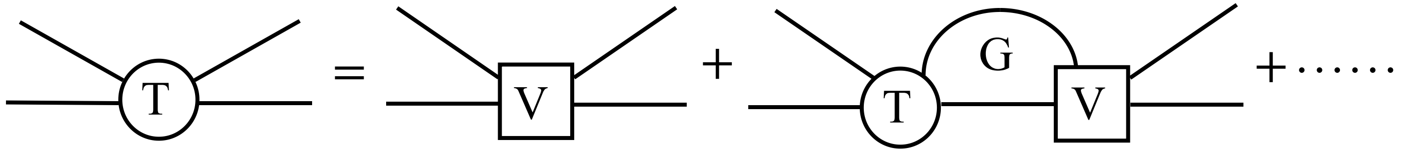
CS mode	$10^4\mathcal{B}$	CS mode	$10^4\mathcal{B}$
$\Lambda^0\pi^0K^+$	13.6 ± 2.5	$p\eta^0\eta^0$	4.46 ± 1.18
$\Lambda^0K^+\eta^0$	0.59 ± 0.24	$p\pi^+\pi^-$	46.4 ± 3.0
$\Lambda^0\pi^+K^0$	39.3 ± 7.6	pK^+K^-	4.82 ± 1.05
$\Sigma^0\pi^0K^+$	8.24 ± 1.41	$pK^0\bar{K}^0$	3.53 ± 1.68
$\Sigma^0K^+\eta^0$	$(4.29\pm1.23)\times10^{-2}$	$n\pi^+\pi^0$	25.3 ± 5.5
$\Sigma^0\pi^+K^0$	3.31 ± 0.97	$n\pi^+\eta^0$	45.2 ± 12.1
$\Sigma^-\pi^+K^+$	3.73 ± 1.11	nK^+K^0	6.54 ± 2.17
$\Xi^-K^+K^+$	$(3.33^{+4.23}_{-3.33})\times10^{-3}$	$\Sigma^+\pi^0K^0$	3.46 ± 1.01
$\Xi^0K^+K^0$	$(1.24^{+1.62}_{-1.24})\times10^{-3}$	$\Sigma^+K^0\eta^0$	$(8.38\pm2.40)\times10^{-2}$
$p\pi^0\pi^0$	39.6 ± 9.2	$\Sigma^+\pi^-K^+$	16.1 ± 3.0
$p\pi^0\eta^0$	25.8 ± 6.1		

TABLE I. The experimental data from Refs. [1–5,7–14,44–50] and reproductions for $\mathcal{B}(\mathbf{B}_c\to\mathbf{B}_nPP')$.

Channels	Data	Our fittings	Channels	Data	Our fittings
$10^2\mathcal{B}(\Lambda_c^+\to p\pi^+K^-)$	3.4 ± 0.4	3.4 ± 0.4	$10^2\mathcal{B}(\Lambda_c^+\to\Sigma^+\pi^0\pi^0)$	1.3 ± 0.1	1.3 ± 0.1
$10^3\mathcal{B}(\Lambda_c^+\to\Lambda^0K^+\bar{K}^0)$	5.6 ± 1.1	5.9 ± 1.0	$10^4\mathcal{B}(\Lambda_c^+\to p\pi^-K^+)$	1.0 ± 0.1	1.0 ± 0.1
$10^2\mathcal{B}(\Lambda_c^+\to\Lambda^0\pi^+\eta^0)$	1.8 ± 0.3	1.9 ± 0.3	$10^4\mathcal{B}(\Lambda_c^+\to nK^+\bar{K}^0)$	$8.6^{+3.8}_{-3.0}$	6.5 ± 2.2
$10^3\mathcal{B}(\Lambda_c^+\to\Lambda^0\pi^0K^+)$	1.5 ± 0.3	1.4 ± 0.3	$10^2\mathcal{B}(\Lambda_c^+\to p\pi^0K_S^0)$	1.9 ± 0.1	1.9 ± 0.1
$10^2\mathcal{B}(\Lambda_c^+\to\Sigma^+\pi^+\pi^-)$	2.9 ± 0.5	2.8 ± 0.5	$10^2\mathcal{B}(\Lambda_c^+\to n\pi^+K_S^0)$	1.9 ± 0.1	1.9 ± 0.1
$10^2\mathcal{B}(\Lambda_c^+\to\Sigma^-\pi^+\pi^+)$	1.9 ± 0.2	2.0 ± 0.2	$10^2\mathcal{B}(\Xi_c^+\to\Sigma^+\pi^+K^-)$	2.6 ± 1.2	3.9 ± 0.4
$10^2\mathcal{B}(\Lambda_c^+\to\Sigma^0\pi^+\pi^0)$	2.2 ± 0.8	1.0 ± 0.1	$10^2\mathcal{B}(\Xi_c^+\to\Xi^0\pi^+\pi^0)$	6.7 ± 3.5	1.0 ± 0.3
$10^3\mathcal{B}(\Lambda_c^+\to\Sigma^0\pi^+\eta^0)$	8.2 ± 0.9	8.3 ± 0.8	$10^3\mathcal{B}(\Xi_c^+\to\Sigma^+\pi^+\pi^-)$	14.0 ± 8.0	6.5 ± 1.6
$10^3\mathcal{B}(\Lambda_c^+\to\Sigma^+\pi^-K^+)$	2.0 ± 0.4	1.6 ± 0.3	$10^3\mathcal{B}(\Xi_c^+\to\Sigma^-\pi^+\pi^+)$	5.1 ± 3.4	6.9 ± 2.3
$10^3\mathcal{B}(\Lambda_c^+\to\Xi^-\pi^+K^+)$	3.3 ± 0.9	1.5 ± 0.5	$10^3\mathcal{B}(\Xi_c^+\to\Sigma^+K^+K^-)$	4.2 ± 2.5	0.4 ± 0.2
$10^3\mathcal{B}(\Lambda_c^+\to p\pi^+\pi^-)$	4.7 ± 0.3	4.6 ± 0.3	$10^2\mathcal{B}(\Xi_c^0\to\Lambda^0\pi^+K^-)$	1.2 ± 0.4	1.3 ± 0.3
$10^4\mathcal{B}(\Lambda_c^+\to pK^+K^-)$	5.2 ± 1.2	4.8 ± 1.0	$10^4\mathcal{B}(\Xi_c^0\to\Lambda^0K^+K^-)$	5.1 ± 1.9	4.5 ± 0.7
$10^2\mathcal{B}(\Lambda_c^+\to p\bar{K}^0\eta^0)$	0.8 ± 0.2	0.7 ± 0.1	$10^3\mathcal{B}(\Xi_c^0\to\Xi^0K^+K^-)$	0.7 ± 0.2	1.0 ± 0.1
$10^3\mathcal{B}(\Lambda_c^+\to\Xi^0\pi^0K^+)$	7.8 ± 1.6	8.0 ± 1.5	$10^2\mathcal{B}(\Xi_c^+\to\Xi^-\pi^+\pi^+)$	2.9 ± 1.3	4.0 ± 1.1

C. Q. Geng, C. W. Liu and S. L. Liu, Phys. Rev. D 109 (2024) 9, 093002

✓ Chiral unitary approach ----Coupled channel Bethe-Salpeter equation

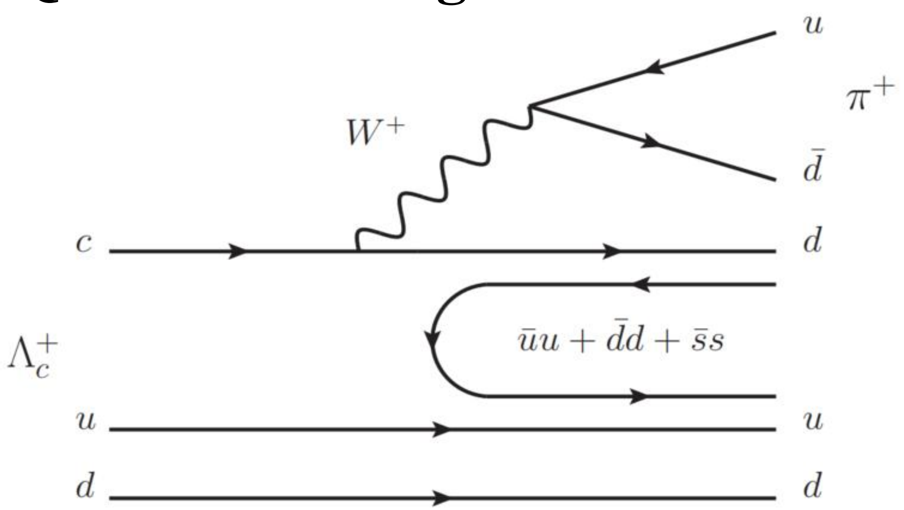


$$T=[1-VG]^{-1}V$$

Formalism for the process $\Lambda_c^+ \to \pi^+ \eta n$

$N(1535)$--dynamically generated by the S-wave meson-baryon interaction

✓ **Quark level diagram**



$$M=\begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}$$

$$=\begin{pmatrix} \frac{\eta}{\sqrt{3}}+\frac{\pi^0}{\sqrt{2}}+\frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta}{\sqrt{3}}-\frac{\pi^0}{\sqrt{2}}+\frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}}+\frac{\sqrt{6}\eta'}{3} \end{pmatrix}$$

✓ **Flavor-wave functions of the baryon**

$$p=\frac{u(ud-du)}{\sqrt{2}} \qquad n=\frac{d(ud-du)}{\sqrt{2}}$$

$$\Lambda=\frac{u(ds-sd)+d(us-su)-2s(ud-du)}{\sqrt{2}}$$

✓ **Hadronization**

$$\Lambda_c^+ = \frac{1}{\sqrt{2}}c(ud-du)$$

$$\Rightarrow \frac{1}{\sqrt{2}}\pi^+\sum_i d(\bar{u}u+\bar{d}d+\bar{s}s)(ud-du)$$

$$=\frac{1}{\sqrt{2}}\pi^+\sum_i M_{2i}q_i\,(ud-du)$$

Formalism for the process $\Lambda_c^+ \to \pi^+ \eta n$

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$N(1535)$--dynamically generated by the S-wave meson-baryon interaction

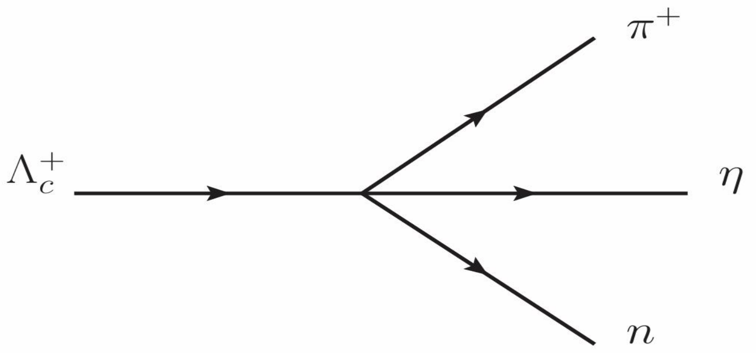
✓ Components of the final states

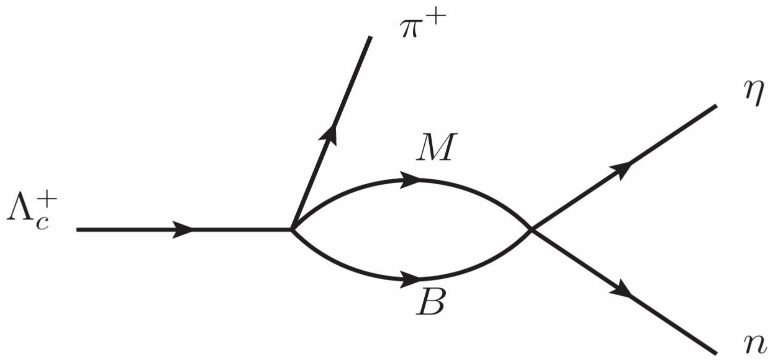
$$\Lambda_c^+ = \pi^+(\pi^- p - \frac{\sqrt{2}}{2}\pi^0 n + \frac{\sqrt{3}}{3}\eta n - \frac{\sqrt{6}}{3}K^0\Lambda)$$

isospin basis

$$\Lambda_c^+ = \pi^+(-\frac{\sqrt{6}}{2}\pi N^{I=\frac{1}{2}} + \frac{\sqrt{3}}{3}\eta N^{I=\frac{1}{2}} - \frac{\sqrt{6}}{3}K\Lambda^{I=\frac{1}{2}})$$

✓ Final states interaction





✓ Amplitude

$$\mathcal{M}^{Tree} = h_{\eta n}$$

$$\mathcal{M}^{N(1535)} = h_{\pi N} G_{\pi N} t_{\pi N \to \eta n} + h_{\eta n} G_{\eta N} t_{\eta N \to \eta n} + h_{K\Lambda} G_{K\Lambda} t_{K\Lambda \to \eta n}$$

Formalism for the process $\Lambda_c^+ \to \pi^+ \eta n$

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$N(1535)$—dynamically generated by the S-wave meson-baryon interaction

✓ **Loop function**

$$G_i = i\int \frac{d^4q}{(2\pi)^4} \frac{2M_i}{(P-q)^2 - M_i^2 + i\varepsilon} \frac{1}{q^2 - m_i^2 + i\varepsilon}$$

$$G(s) = \frac{2M}{16\pi^2 s}\{\sigma(arctan\frac{s+\Delta}{\sigma\lambda_1} + arctan\frac{s-\Delta}{\sigma\lambda_2})$$
$$-[(s+\Delta)ln\frac{q_{max}(1+\lambda_1)}{m_1}$$
$$+[(s-\Delta)ln\frac{q_{max}(1+\lambda_2)}{m_2}]\}$$

$$\sigma = [-(s-(M_i+m_i)^2)(s-(M_i-m_i)^2)]^{1/2}$$

$$\lambda_1 = \sqrt{1+\frac{M_i^2}{q_{max}^2}} \qquad \lambda_2 = \sqrt{1+\frac{m_i^2}{q_{max}^2}}$$

$$\Delta = M_i^2 - m_i^2 \qquad q_{max} = 1150 \text{ MeV}$$

✓ **Bethe-Salpetere quation**

$$T = [1 - VG]^{-1}V$$

$$V_{ij} = -C_{ij}\frac{1}{4f^2}(2\sqrt{s} - M_i - M_j) \times (\frac{M_i + E_i}{2M_i})^{1/2}(\frac{M_j + E_j}{2M_j})^{1/2}$$

TABLE I. The S-wave meson-baryon scattering coefficients [54].

	πN	ηn	$K\Lambda$	$K\Sigma$
πN	2	0	3/2	−1/2
ηn		0	−3/2	−3/2
$K\Lambda$			0	0
$K\Sigma$				2

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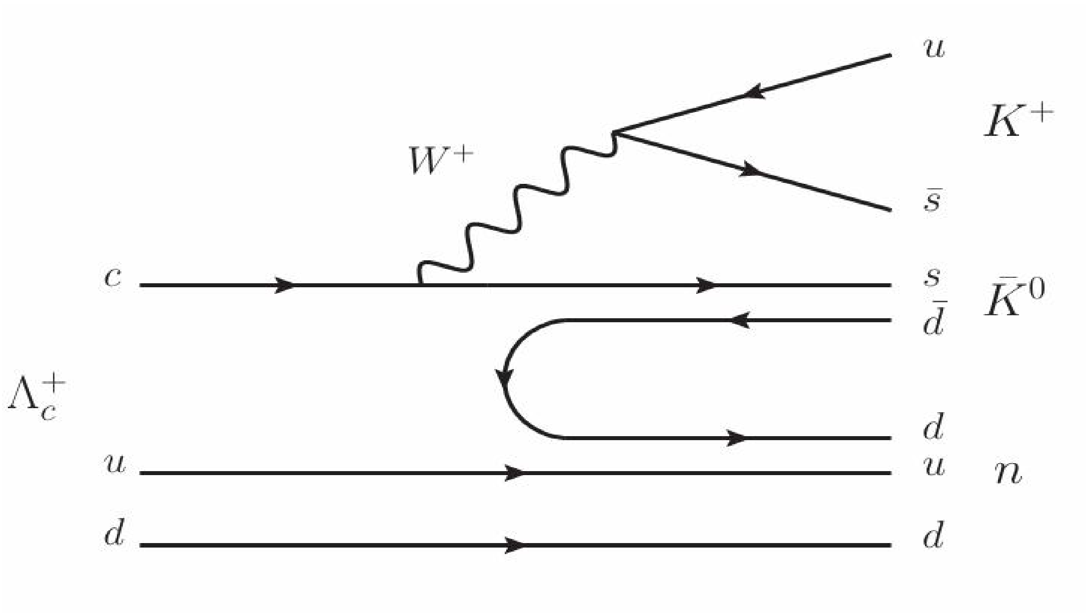
Formalism for the process $\Lambda_c^+ \to \pi^+ \eta n$

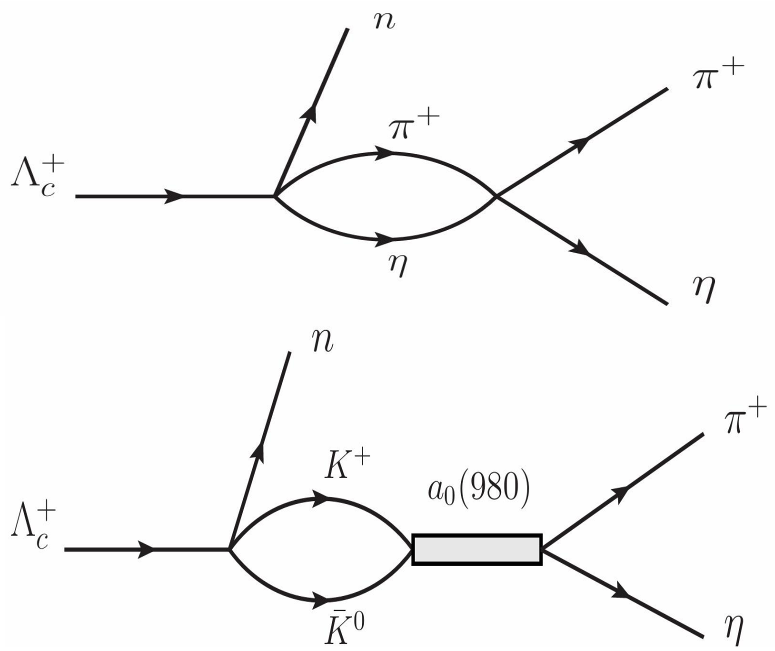
$a_0(980)$--dynamically generated by the S-wave meson-meson interaction

✓ **Quark level diagram**



$$\Lambda_c^+ \Longrightarrow \pi^+\left(-\frac{\sqrt{6}}{2}\pi N + \frac{\sqrt{3}}{3}\eta N - \frac{\sqrt{6}}{3}K\Lambda\right)$$

✓ **Final states interaction**



✓ **Amplitude**

$$\mathcal{M}^{a_0(980)} = h_{\pi^+\eta}\, G_{\pi^+\eta}\, t_{\pi^+\eta\to\pi^+\eta} + h_{K^+\bar{K}^0} G_{K^+\bar{K}^0} t_{K^+\bar{K}^0\to\pi^+\eta}$$

Formalism for the process $\Lambda_c^+ \to \pi^+ \eta n$

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$a_0(980)$--dynamically generated by the S-wave meson-meson interaction

$$\mathcal{M}^{a_0(980)} = h_{\pi^+\eta} G_{\pi^+\eta} t_{\pi^+\eta\to\pi^+\eta} + h_{K^+\bar{K}^0} G_{K^+\bar{K}^0} t_{K^+\bar{K}^0\to\pi^+\eta}$$

✓ **Loop function**

$$G = i\int \frac{d^4q}{(2\pi)^4} \frac{1}{(P-q)^2 - m_1^2 + i\varepsilon} \frac{1}{q^2 - m_2^2 + i\varepsilon}$$

$$= \int_0^{q_{max}} \frac{|\vec{q}|^2 d|\vec{q}|}{(2\pi)^2} \frac{\omega_1 + \omega_2}{\omega_1\omega_2[s - (\omega_1+\omega_2)^2 + i\varepsilon]}$$

q_{max} =600 MeV

✓ **Bethe-Salpetere quation**

$$T = [1 - VG]^{-1} V$$

$$V_{K^+\bar{K}^0\to K^+\bar{K}^0} = -\frac{s}{4f^2}$$

$$V_{K^+\bar{K}^0\to\pi^+\eta} = -\frac{3s - 2m_K^2 - m_\eta^2}{3\sqrt{3}f^2}$$

$$V_{\pi^+\eta\to\pi^+\eta} = -\frac{2m_K^2}{3f^2} \quad (f = 93\ \text{MeV})$$

J. J. Xie, L. R. Dai and E. Oset, Phys. Lett. B 742, 363 (2015)

Results

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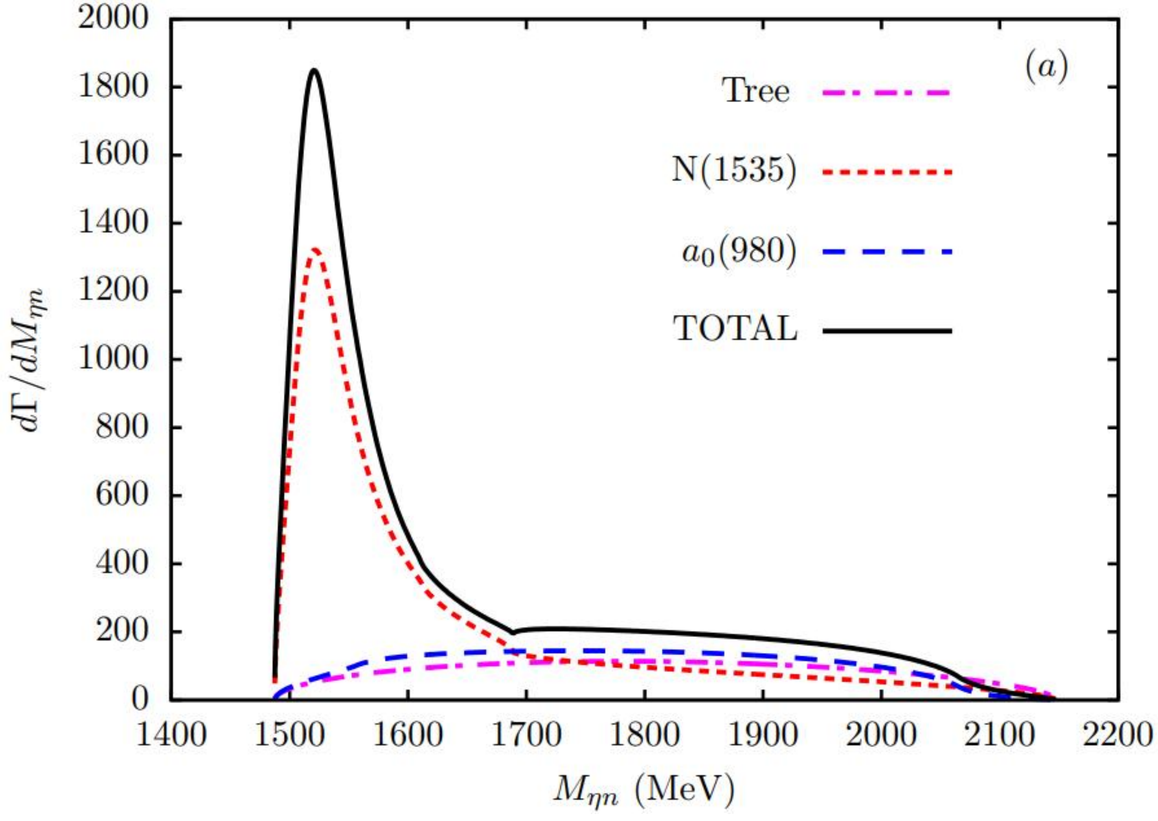
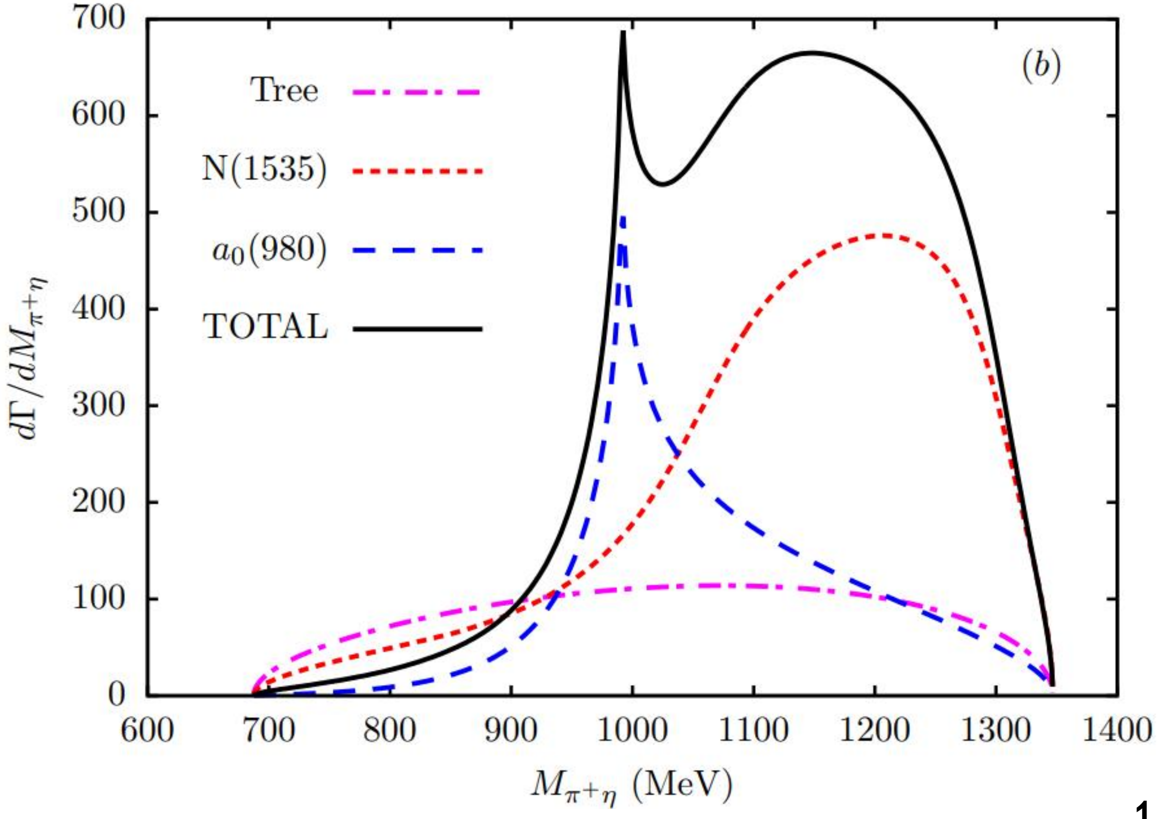
- ✓ **Total amplitude** $\mathcal{M} = \mathcal{M}^{Tree} + \mathcal{M}^{N(1535)} + \mathcal{M}^{a_0(980)}$
- ✓ **Double differential width** $\frac{d^2\Gamma}{d^2M_{\eta n}\, d^2M_{\pi^+\eta}} = \frac{1}{(2\pi)^3}\frac{4M_{\Lambda_c^+}M_n}{32M_{\Lambda_c^+}^3}|\mathcal{M}|^2$

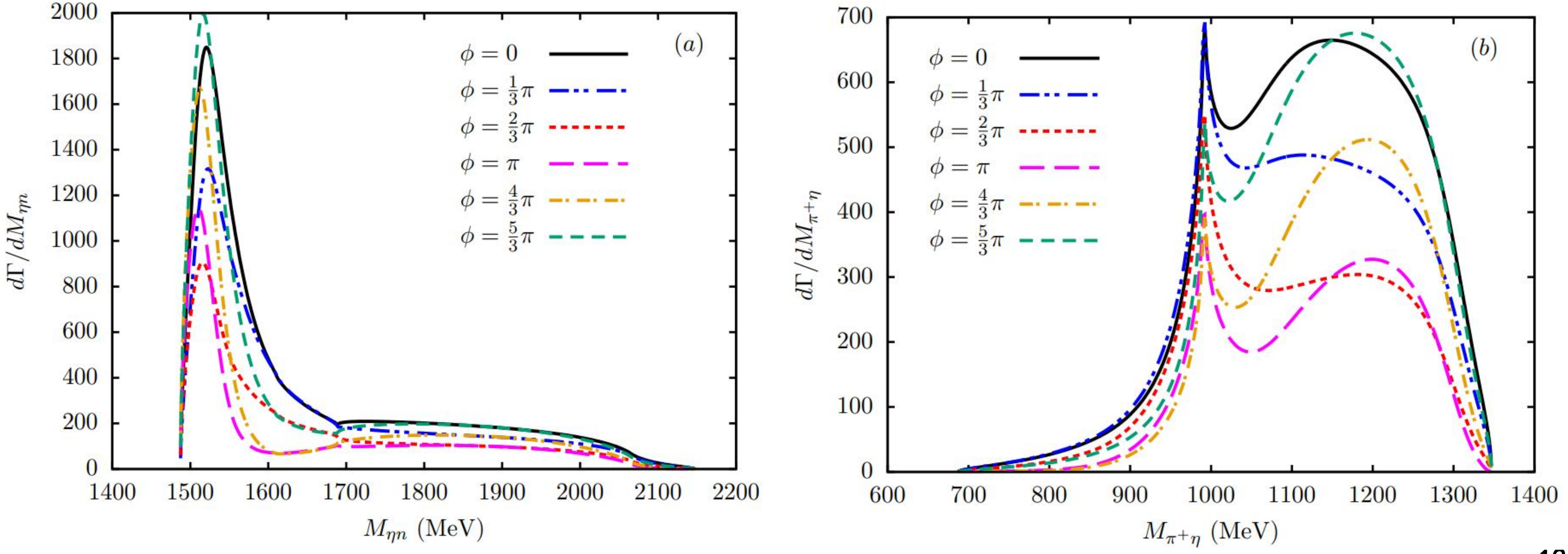




Results

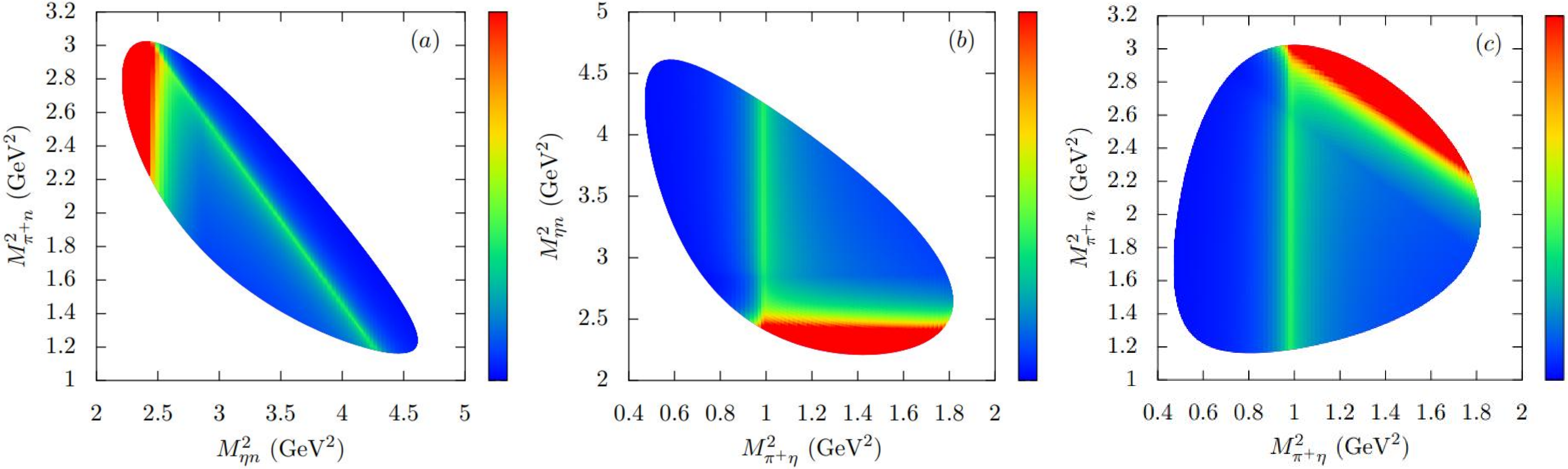
✓ The invariant mass distributions with phase interference

$$\mathcal{M} = \mathcal{M}^{Tree} + \mathcal{M}^{N(1535)} + \mathcal{M}^{a_0(980)} e^{i\phi}$$



Results

✓ The Dalitz plots



$M^2_{\eta n}$ vs $M^2_{\pi^+n}$

$M^2_{\pi^+\eta}$ vs $M^2_{\eta n}$

$M^2_{\pi^+\eta}$ vs $M^2_{\pi^+n}$

Summary

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- ✓ The significant peak structure in the ηn invariant mass distribution of the decay $\Lambda_c^+ \to \pi^+\eta n$ should be related to $N(1535)$.
- ✓ A cusp structure in the $\pi^+\eta$ invariant mass distribution of the decay $\Lambda_c^+ \to \pi^+\eta n$ should be related to $a_0(980)$.
- ✓ More precise measurement results from future experiments will help us to better understand the nature of $N(1535)$ and $a_0(980)$.

Thanks for your attention!