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宁波大学
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The spectrum of $bc\bar{b}\bar{c}$ tetraquark state from a diquark-antiquark perspective

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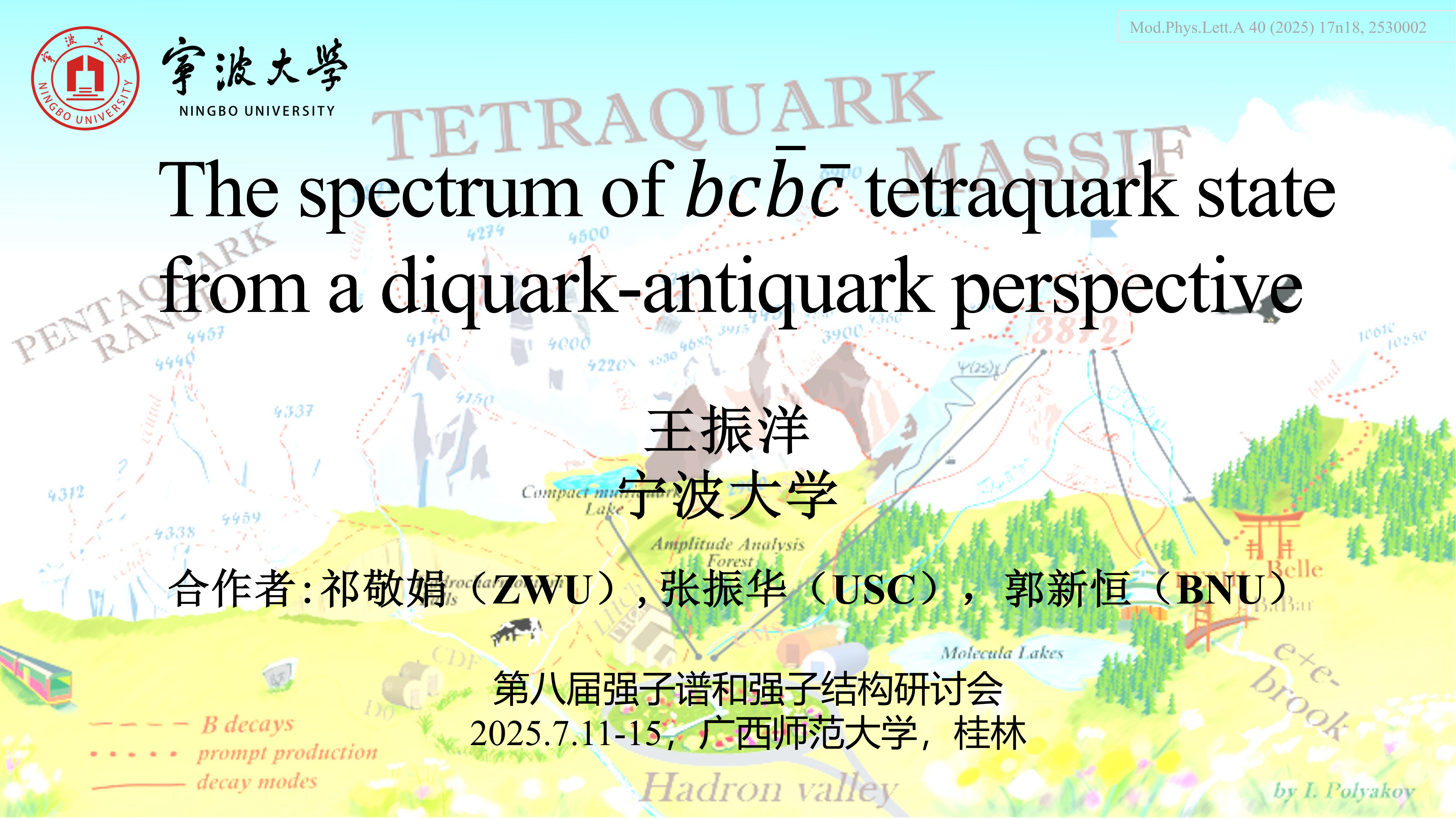
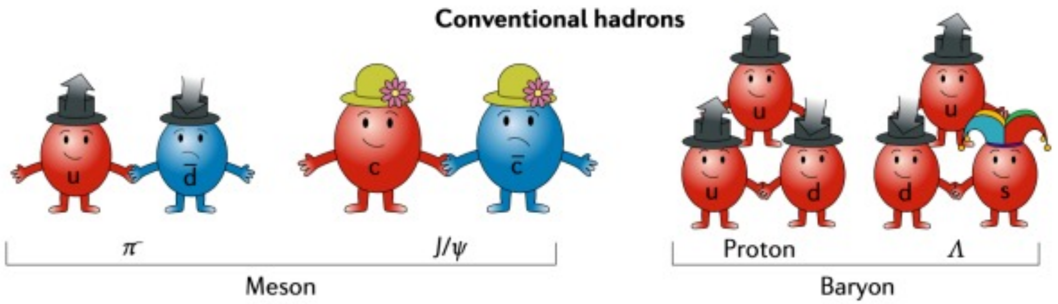
第八届强子谱和强子结构研讨会
2025.7.11-15，广西师范大学，桂林

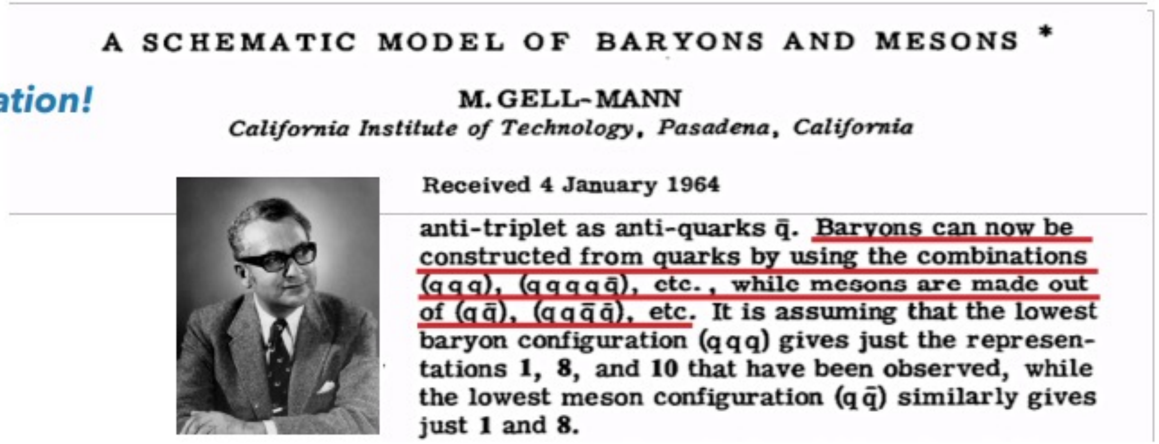


Hadrons

- **Quark Model** [1964 by Gell-Mann and Zweig]

Lowest Configuration!





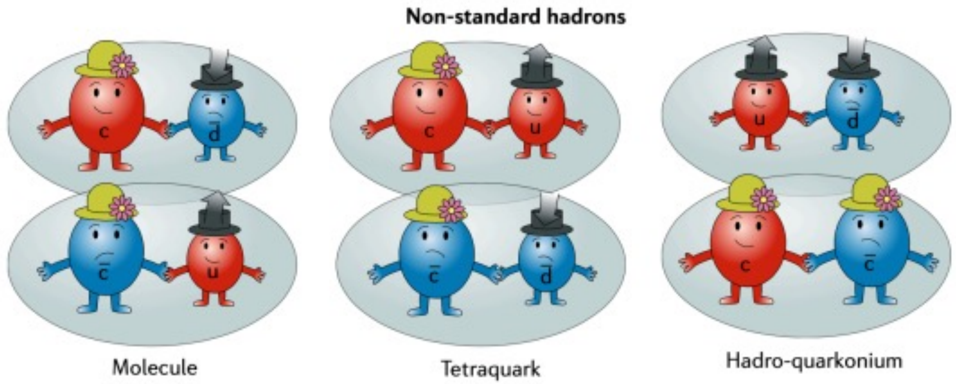
A SCHEMATIC MODEL OF BARYONS AND MESONS *

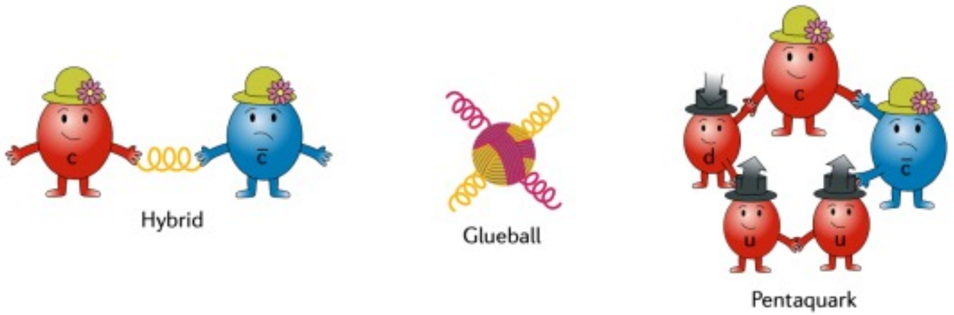
M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

anti-triplet as anti-quarks q̄. Baryons can now be constructed from quarks by using the combinations (q q q), (q q q q q̄), etc., while mesons are made out of (q q̄), (q q q̄ q̄), etc. It is assuming that the lowest baryon configuration (q q q) gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration (q q̄) similarly gives just **1** and **8**.

- **Exotic hadrons:**

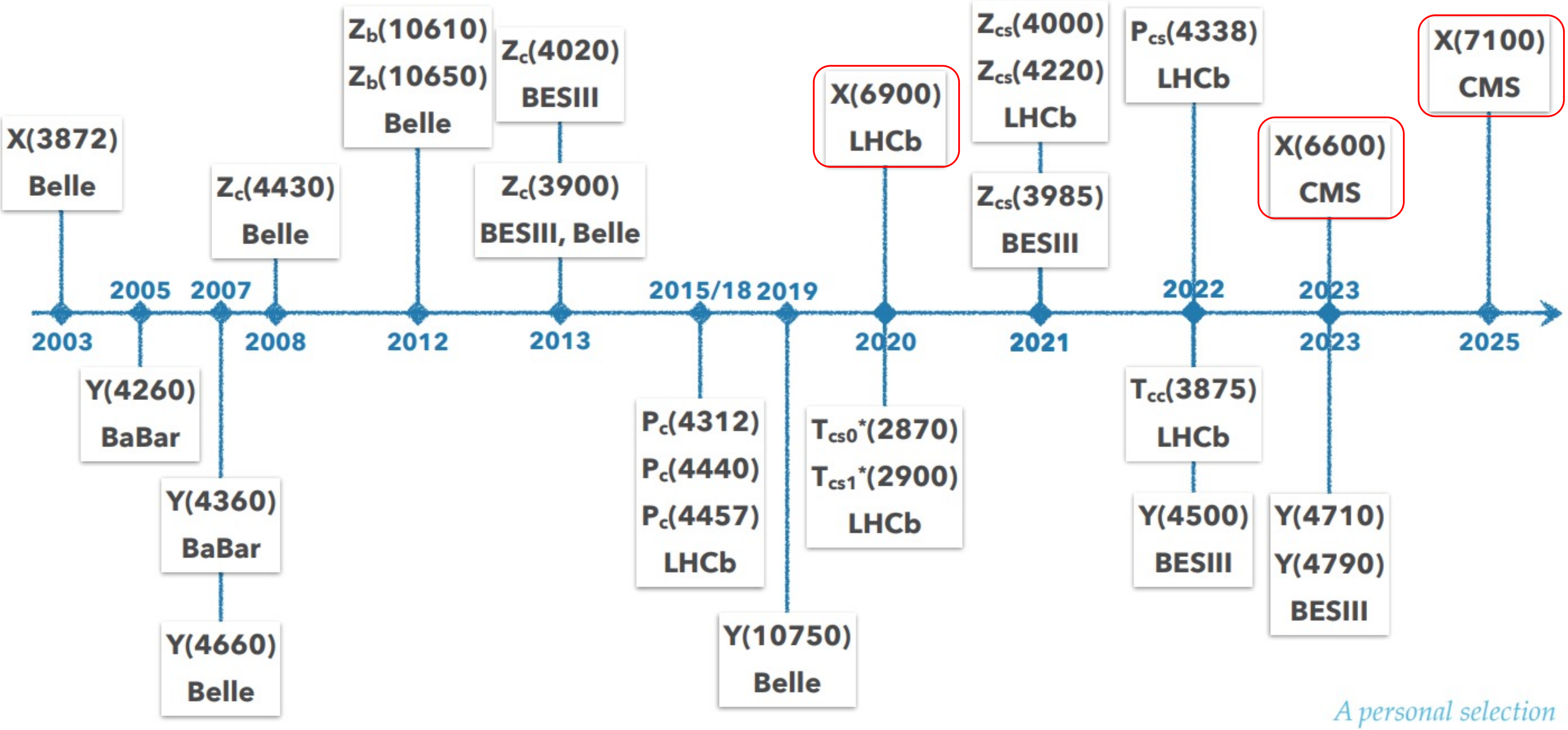




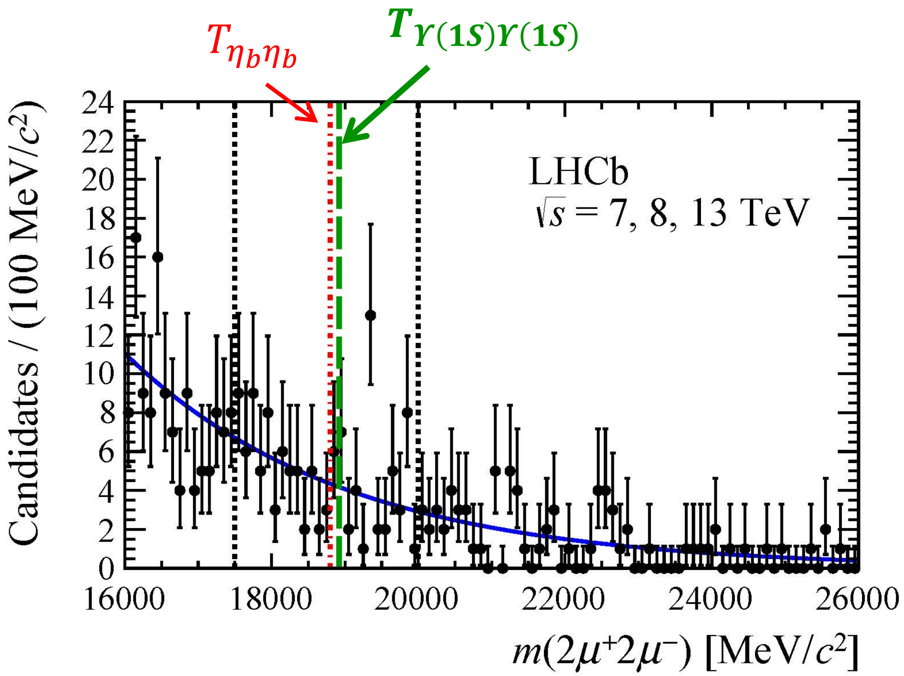
C. Z. Yuan, S. L. Olsen, Nature Reviews Physics 1, 480 (2019)

Exotic Hadron Candidates

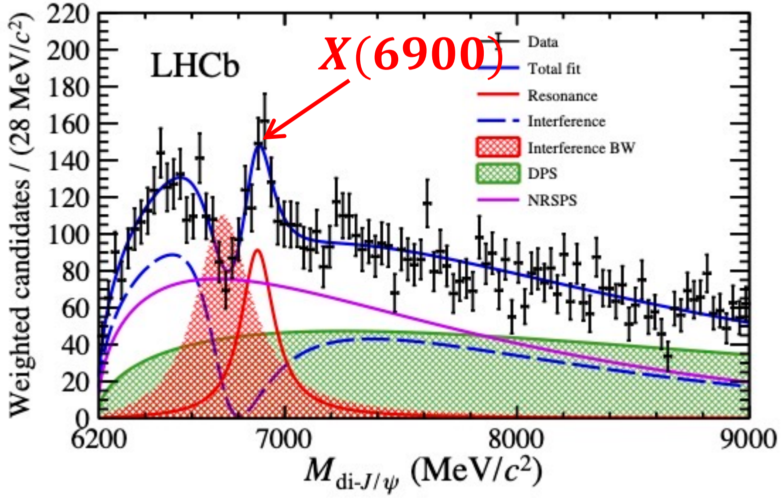
Yuping Guo @ 第二十届全国中高能核物理大会



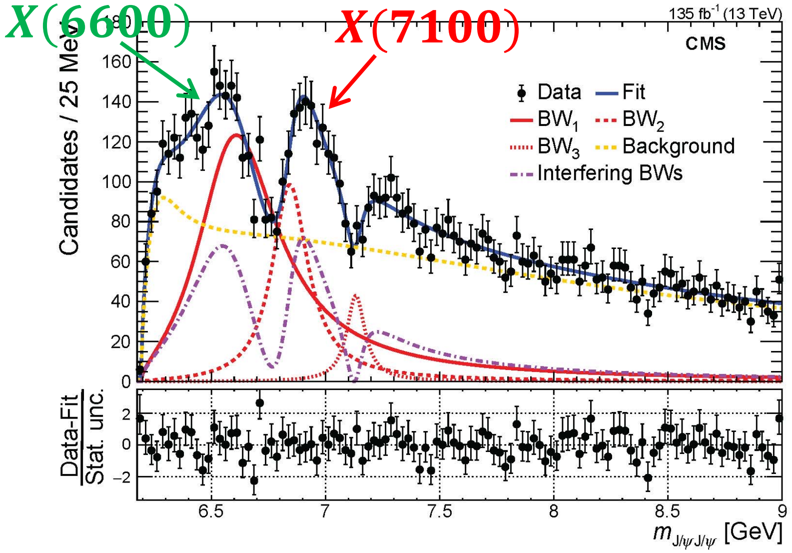
Experiment data



Distributions of $m(2\mu^+2\mu^-)$ [JHEP 10 (2018) 086]

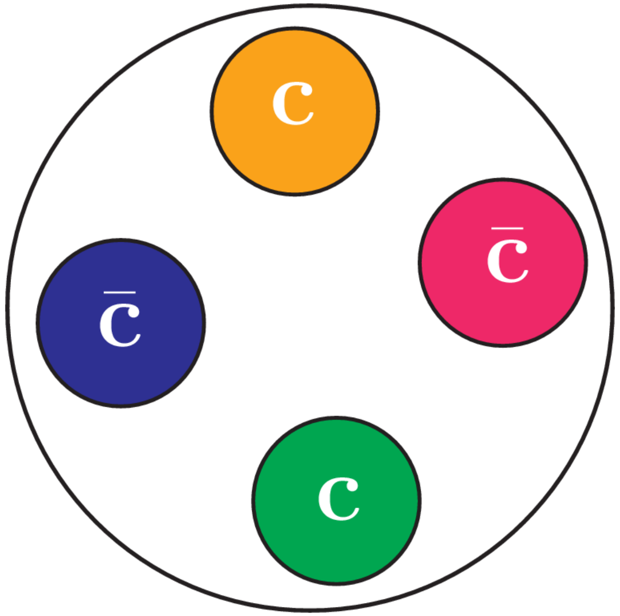


Invariant mass spectra of di-J/ψ [Sci.Bull. 65 (2020) 23, 1983-1993]

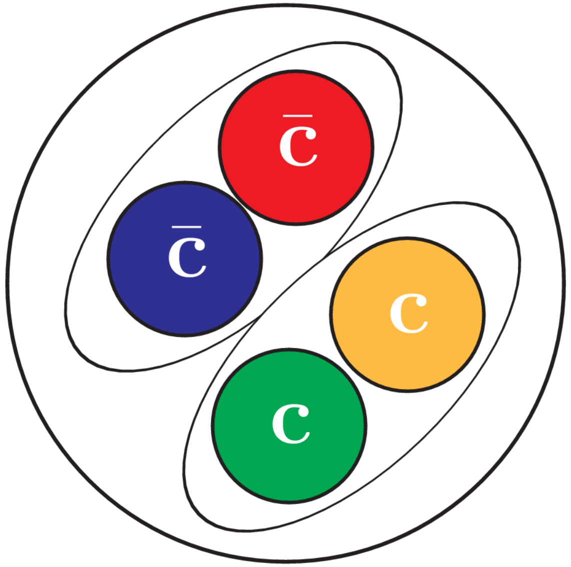


The $J/\psi J/\psi$ invariant mass spectrum [Phys.Rev.Lett. 132 (2024) 11, 111901]

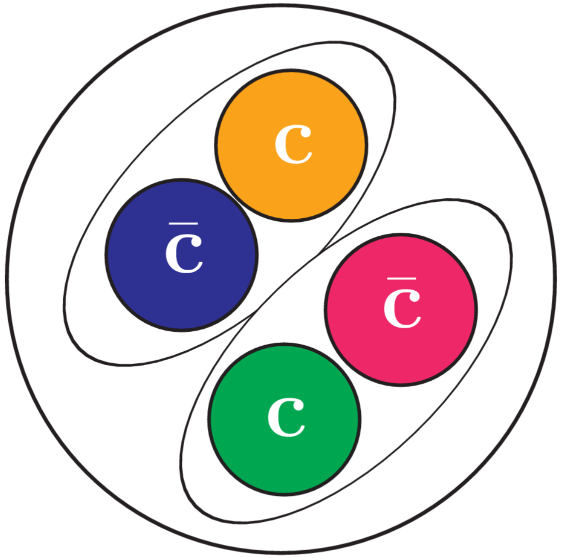
Idealized models



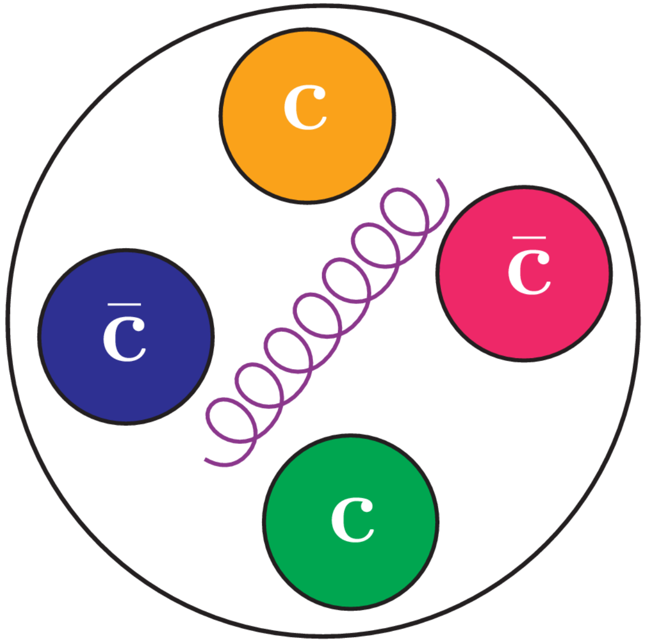
Compact tetraquark



Diquark-antidiquark



Molecule



Hybrid

Spectra of $QQ\bar{Q}\bar{Q}$ in the Bethe-Salpeter equation

The results are in units of GeV

m_{cc}	m_{bb}		$m_{cc\bar{c}\bar{c}}$		$m_{bb\bar{b}\bar{b}}$	
3.23	9.8	$\mathbf{0^{++}}$	6.201-6.270	6.419	19.302-19.429	19.205
3.303	9.816	$\mathbf{1^{+-}}$	6.369-6.424	6.456	19.409-19.557	19.221
		$\mathbf{2^{++}}$	6.391-6.424	6.516	19.409-19.557	19.253

[Eur.Phys.J.C 81 (2021) , 427] [Phys.Rev.D 104 (2021) , 014018]

$m_{\eta_c\eta_c}$ = 5.9678 GeV, $m_{J/\psi J/\psi}$= 6.1938 GeV, $m_{\eta_b\eta_b}$= 18.798 GeV, $m_{\Upsilon(1S)\Upsilon(1S)}$= 18.9206 GeV

- ✓ $X(6900)$ is less likely to be the ground states of compact $cc\bar{c}\bar{c}$ tetraquarks
- ✓ $X(6900)$ might be the radially excited states
- ✓ The masses of the ground states are above the threshold of the lowest quarkonium pair
- ✓ Thus these ground states are expected to be broad

The wave function of $bc\bar{b}\bar{c}$ in diquark and antidiquark picture

$$\psi = \psi_{space} \otimes \psi_{flvour} \otimes \psi_{spin} \otimes \psi_{color}$$

- Only focus on the ground *S*-wave fully heavy tetraquarks, the spatial wave function is symmetric
- Without the Pauli principle, the J^P of bc diquark could be 0^+ or 1^+
- The diquark can exist in either the $\bar{3}_c$ (attractive) or 6_c (repulsive) color group representations

[Phys.Rev.D 100 (2019), 016006, Phys.Rev.D 97 (2018) , 094015]

	$\|SS\rangle$	$\|AS\rangle$	$\|AA\rangle$
0^{++}	$\|[bc]_0^{\bar{3}}[\bar{b}\bar{c}]_0^{3}\rangle_0$	$\cdots$	$\|\{bc\}_1^{\bar{3}}\{\bar{b}\bar{c}\}_1^{3}\rangle_0$
1^{+-}	$\cdots$	$\frac{1}{\sqrt{2}}(\|[bc]_0^{\bar{3}}\{\bar{b}\bar{c}\}_1^{3}\rangle_1 - \|\{bc\}_0^{\bar{3}}[\bar{b}\bar{c}]_1^{3}\rangle)$	$\|\{bc\}_1^{\bar{3}}\{\bar{b}\bar{c}\}_1^{3}\rangle_1$
1^{++}	$\cdots$	$\frac{1}{\sqrt{2}}(\|[bc]_0^{\bar{3}}\{\bar{b}\bar{c}\}_1^{3}\rangle_1 + \|\{bc\}_0^{\bar{3}}\{\bar{b}\bar{c}\}_1^{3}\rangle)$	$\cdots$
2^{++}	$\cdots$	$\cdots$	$\|\{bc\}_1^{\bar{3}}\{\bar{b}\bar{c}\}_1^{3}\rangle_2$

Bethe-Salpeter equation ($|SS\rangle$)

- The Bethe-Salpeter wave function for this state is expressed as

$$\chi_P(x_1, x_2) = \langle 0|T\phi(x_1)\bar{\phi}\,(x_2)|P\rangle = e^{-iPX}\int \frac{d^4p}{(2\pi)^4} e^{-ipx}\chi_P(p),$$

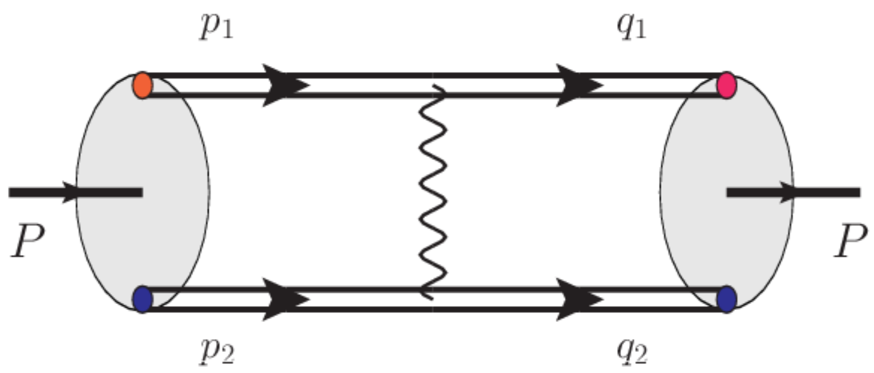
where $\phi(x_1)$ and $\bar{\phi}(x_2)$ denote the field operators for the diquark and antidiquark, respectively. $X = \lambda_1 x_1 + \lambda_2 x_2$ and $x = x_1 - x_2$ with $\lambda_{1(2)} = \frac{m_{1(2)}}{m_1+m_2}$.

- The relative momentum p and the total momentum P of the tetraquark bound state are defined by

$$p = \lambda_2 p_1 - \lambda_1 p_2, \quad P = p_1 + p_2 = Mv,$$

or inversely

$$p_1 = \lambda_1 P + p, \quad p_2 = \lambda_2 P - p.$$



Bethe-Salpeter equation ($|SS\rangle$)

- The Bethe-Salpeter equation for this state in momentum space takes the following form:

$$\chi_P(p) = S(p_1)\int \frac{d^4q}{(2\pi)^4} G(P,p,q)\chi_P(q)S(p_2)$$

- The scalar diquark propagators $S(p_1)$ and $S(p_2)$ in the leading order of $\frac{1}{m_Q}$ expansion,

  can be expressed as:

$$S(p_1) = \frac{i}{2w_1(p_l + \lambda_1 M - w_1 + i\epsilon)},$$

  and

$$S(p_2) = \frac{i}{2w_2(p_l - \lambda_2 M + w_2 - i\epsilon)},$$

  where $w_{1(2)} = \sqrt{m_{1(2)}^2 - p_t^2}$.

- The interaction kernel: $-iG(P,p,q) = 4m_1m_2 I \otimes I\, V_1 - \Gamma_\mu \otimes \Gamma^\mu\, V_2$

  For convenience, $p_l = p \cdot v$ and $p_t^\mu = p^\mu - p_l\, v^\mu$

Bethe-Salpeter equation ($|AS\rangle$)

- The Bethe-Salpeter wave function for the tetraquark composed of an axial-vector diquark and a scalar antidiquark

$$\chi_P^\mu(x_1,x_2) = \langle 0|TA^\mu(x_1)\bar{\phi}\,(x_2)|P\rangle = e^{-iPX}\int \frac{d^4p}{(2\pi)^4} e^{-ipx}\chi_P^\mu(p),$$

- $\chi_P^\mu(p)$ satisfies the following Bethe-Salpeter equation

$$\chi_P^\mu(p) = S^{\mu\nu}(p_1)\int \frac{d^4q}{(2\pi)^4} G_{\nu\alpha}(P,p,q)\chi_P^\alpha(q)S(p_2),$$

- The propagator of the axial-vector diquark $S^{\mu\nu}(p_1)$ in the leading order of a $1/m_Q$ expansion:

$$S^{\mu\nu}(p_1) \quad = -i\frac{g^{\mu\nu} - p_1^\mu\, p_1^\nu/m_1^2}{2w_1(p_l + \lambda_1 M - w_1 + i\epsilon)},$$

- The kernel $G_{\nu\alpha}(P,p,q)$ for the BS equation is specified by

$$iG_{\nu\alpha}(P,p,q) = g_{\nu\alpha}4m_1m_2 I \otimes\; I\, V_1 - \Gamma_{\alpha\nu\beta} \otimes \Gamma^\beta\, V_2,$$

Bethe-Salpeter equation ($|AA\rangle$)

- The Bethe-Salpeter wave function for the tetraquark composed of an axial-vector diquark and a scalar antidiquark

$$\chi_P^{\mu\nu}(x_1, x_2) = \langle 0|TA^\mu(x_1)\bar{A}^\nu\,(x_2)|P\rangle = e^{-iPX}\int \frac{d^4p}{(2\pi)^4} e^{-ipx}\chi_P^{\mu\nu}(p),$$

- $\chi_P^{\mu\nu}(p)$ satisfies the following Bethe-Salpeter equation

$$\chi_P^{\mu\nu}(p) = S^{\mu\alpha}(p_1)\int \frac{d^4q}{(2\pi)^4} G_{\alpha\beta\kappa\lambda}(P,p,q)\chi_P^{\kappa\lambda}(q)S^{\nu\beta}(p\_2),$$

- The propagator of the axial-vector antidiquark $S^{\mu\nu}(p_2)$ in the leading order of a $1/m_Q$ expansion:

$$S^{\mu\nu}(p_2) \quad = -i\frac{g^{\mu\nu} - p_2^\mu\, p_2^\nu/m_2^2}{2w_2(p_l - \lambda_2 M + w_2 - i\epsilon)},$$

- The kernel $G_{\nu\alpha}(P,p,q)$ for the BS equation is specified by

$$-iG_{\alpha\beta\kappa\lambda} = 4\, m_1 m_2 g_{\alpha\kappa} g_{\beta\lambda} I \otimes\; I\, V_1 - \Gamma_{\alpha\kappa\gamma} \otimes \Gamma_{\beta\lambda}^{\gamma}\, V_2$$

Interaction Vertex

- The vertex of a gluon with two scalar diquarks: $ig_s \frac{\lambda_a}{2}(p_{1(2)} + q_{1(2)})^\mu F_S(Q^2)$

✓ $\Gamma^\mu = (p_{1(2)} + q_{1(2)})^\mu F_S(Q^2)$

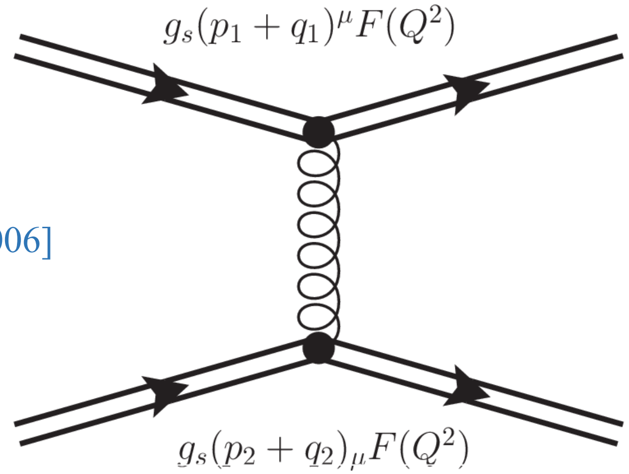
- The vertex of a gluon with two axial-vector diquarks: $ig_s \frac{\lambda_a}{2}[g^{\alpha\beta}(p_{1(2)} + q_{1(2)})^\mu F_{V1}(Q^2) - (p^\beta_{1(2)} g^{\mu\alpha} + q^\alpha_{1(2)} g^{\mu\beta}) F_{V2}(Q^2) + p^\alpha_{1(2)} p^\beta_{1(2)} (p_{1(2)} + q_{1(2)})^\mu F_{V3}(Q^2)]$ [Z.Phys.C 36 (1987) 89]

➢ The high momentum powers multiplied by $F_{V3}(Q^2)$ suppress its contribution at small and intermediate Q^2

➢ $F_{V2}(Q^2) = 0$ in the leading order of an expansion $1/m_Q$

[Phys.Rev.D 83 (2011) 056006]

✓ $\Gamma^{\alpha\beta\mu} = g^{\alpha\beta}(p_{1(2)} + q_{1(2)})^\mu F_V(Q^2)$



Form factor

① The form factors are unknown

② Dependence on Q^2 $(Q = p_1 - q_1)$

③ A possible parametrization is obtained from the asymptotic behaviour

✓ $Q^2 \to \infty$, the diquarks dissolve into quarks

- $F_S(Q^2) = F_V(Q^2) = F(Q^2) = \frac{\alpha_s Q_0^2}{Q^2+Q_0^2}$ [Z.Phys.C 36 (1987) 89]

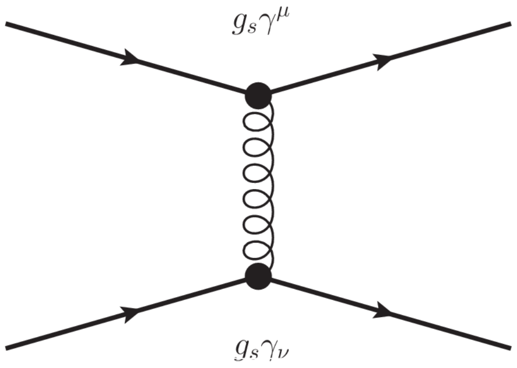
➢ Q_0 is a parameter

➢ $Q^2 \to 0$, Q_0^2 freezes $F(Q^2)$

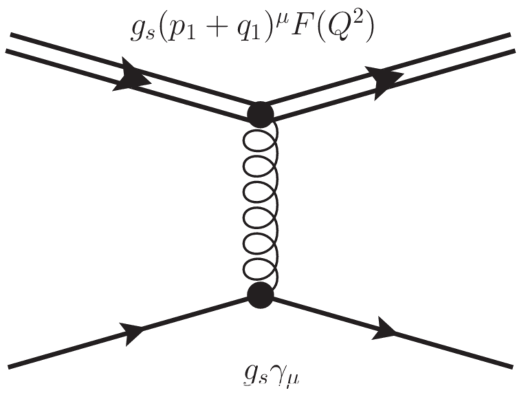
➢ $Q^2 \to \infty$, the form factor is proportional to $\frac{1}{Q^2}$, which is consistent with perturbative QCD calculations

Note: The form factors of diquarks composed of different quark combinations exhibit differences, but the forms of the form factors are similar. Moreover, research has found that the results are not strongly dependent on Q_0^2.

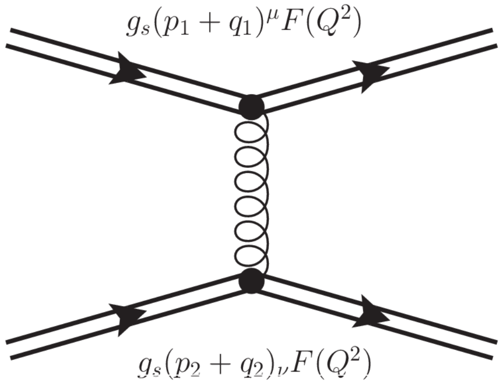
Potential



Meson



Baryon



Tetraquark

Scalar confinement term [Z. Phys. C 56(1992) 707, Phys. Rev. D 53(1996) 1153]

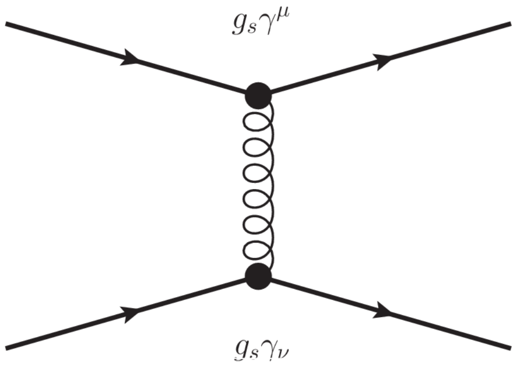
$$V_1 = \frac{8\pi\kappa}{[(p_t-q_t)^2+\mu^2]^2} - (2\pi)^3\delta^3(p_t-q_t)\int \frac{d^3k}{(2\pi)^3}\frac{8\pi\kappa}{[k^2+\mu^2]^2}$$

One-gluon-exchange term

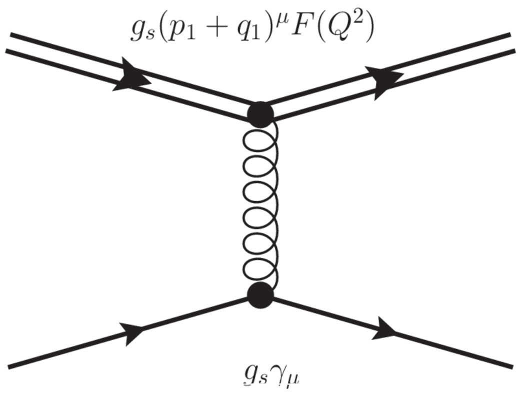
$$V_2 = -\frac{16\pi}{3}\frac{\alpha_s}{(p_t-q_t)^2+\mu^2}$$

The dimension of κ (κ is around 0.2) is Two!

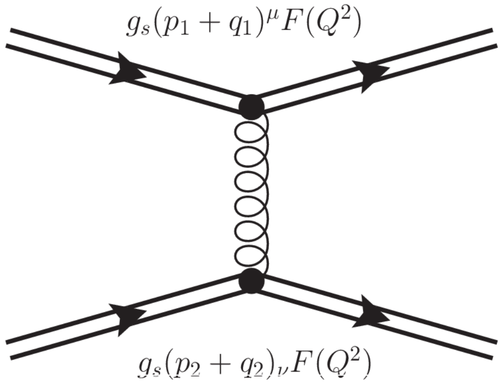
Potential



Meson



Baryon



Tetraquark

Scalar confinement term

$$V_1 = \frac{8\pi\kappa'}{[(p_t-q_t)^2+\mu^2]^2} - (2\pi)^3\delta^3(p_t-q_t)\int \frac{d^3k'}{(2\pi)^3}\frac{8\pi\kappa}{[k^2+\mu^2]^2}$$

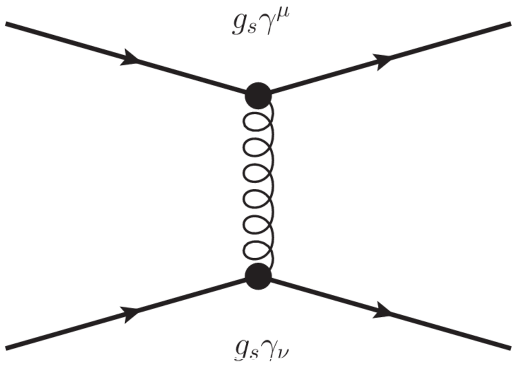
One-gluon-exchange term

$$V_2 = -\frac{16\pi}{3}\frac{\alpha_s}{(p_t-q_t)^2+\mu^2}$$

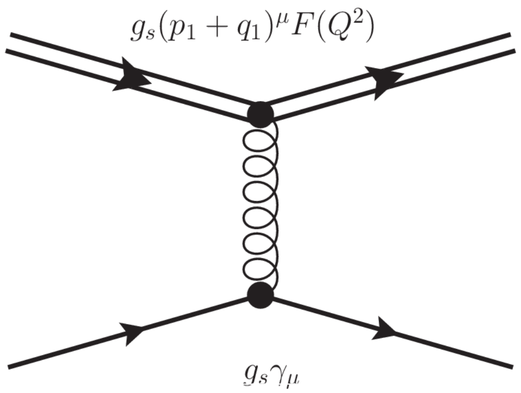
The dimension of κ' ($\kappa' \sim \Lambda_{\mathrm{QCD}}\kappa$, vary in the range 0.02 GeV^3 to 0.1 GeV^3) is Three!

[Phys.Rev.D 61 (2000) 116015]

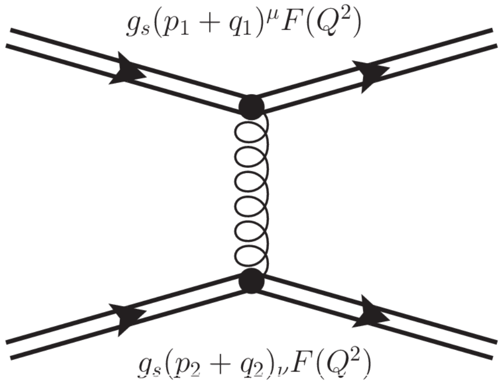
Potential



Meson



Baryon



Tetraquark

Scalar confinement term

$$V_1 = \frac{8\pi\kappa''}{[(p_t-q_t)^2+\mu^2]^2} - (2\pi)^3\delta^3(p_t-q_t)\int \frac{d^3k}{(2\pi)^3}\frac{8\pi\kappa''}{[k^2+\mu^2]^2}$$

One-gluon-exchange term

$$V_2 = -\frac{16\pi}{3}\frac{\alpha_s}{(p_t-q_t)^2+\mu^2}$$

The dimension of κ'' ($\kappa'' \sim 2m_1 * 2m_2\beta\kappa$, with β in the range (0.1,1.5)) is Four!

Parameterization of Bethe-Salpeter wave functions

- Constraints from PCT

$$\begin{aligned}\chi_{P\zeta}(x_1,x_2) &= \langle 0|T\phi(x_1)\,\bar{\phi}\,(x_2)|P\zeta\rangle \\ &= \left\langle 0\middle|\mathcal{P}^{-1}\mathcal{P}T\{\phi(x_1)\bar{\phi}(x_2)\}\mathcal{P}^{-1}\mathcal{P}\middle|P\zeta\right\rangle \\ &= \eta_P\left\langle 0\middle|\mathcal{P}T\{\phi(x_1)\bar{\phi}(x_2)\}\mathcal{P}^{-1}|P\zeta\right\rangle \\ &= \eta_P\left\langle 0\middle|T\{\phi(t_1,-\boldsymbol{x}_1)\bar{\phi}(t_2,-\boldsymbol{x}_2)\}\middle|E,-\boldsymbol{P},\zeta\right\rangle \\ &= \eta_P\chi_{E,-\boldsymbol{P},\zeta}(t_1,-\boldsymbol{x}_1,t_2,-\boldsymbol{x}_2),\end{aligned}$$

or

$$\chi_{P\zeta}(x) = \eta_P\chi_{E,-\boldsymbol{P},\zeta}(t,-\boldsymbol{x}),$$

Similarly

$$\chi_{P\zeta}(x) = \eta_C\,\chi_{P\zeta}(-x), \qquad \chi_P\,(x) = \eta_T\chi_P(-t,\boldsymbol{x}).$$

In momentum space,

$$\begin{aligned}\chi_{P\zeta}(p) &= \eta_P\chi_{E,-\boldsymbol{P},\zeta}(p_0,-\boldsymbol{p}), \\ \chi_{P\zeta}(p) &= \eta_C\chi_{P\zeta}(-p), \\ \chi_{P\zeta}(p) &= \eta_T\chi_{P\zeta}(-p_0,\boldsymbol{p}).\end{aligned}$$

Lorentz structure of the Bethe-Salpeter wave functions

$|SS\rangle$

$$|0^{++}\rangle \quad \chi_P(p) = s(p)$$

$|AS\rangle$

$$|1^{+-}\rangle \quad \chi_P^{\mu}(p) = a(p)\epsilon^{\mu\nu\alpha\beta}P_\nu p_\alpha \epsilon_\beta$$

$$|1^{++}\rangle \quad \chi_P^{\mu}(p) = b(p)\epsilon^{\mu\nu\alpha\beta}P_\nu p_\alpha P\cdot p\,\epsilon_\beta$$

$|AA\rangle$

$$|0^{++}\rangle \quad \chi_P^{\mu\nu}(p) = c_1(p)g^{\mu\nu} + c_2(p)P^\mu P^\nu + c_3(p)p^\mu p^\nu$$

$$|1^{+-}\rangle \quad \chi_P^{\mu\nu}(p) = d(p)\epsilon^{\mu\nu\alpha\beta}p_\alpha \epsilon_\beta$$

$$|2^{++}\rangle \quad \chi_P^{\mu\nu}(p) = e_1(p)\xi^{\mu\nu} + e_2(p)\xi^{\mu\sigma}p_\sigma p^\nu + e_3\xi^{\nu\sigma}p_\rho p^\mu + e_4\xi^{\rho\sigma}p_\rho p_\sigma g^{\mu\nu} + e_5(p)\xi^{\rho\sigma}p_\rho p_\sigma p^\mu p^\nu + e_6(p)\xi^{\rho\sigma}p_\rho p_\sigma P^\mu P^\nu$$

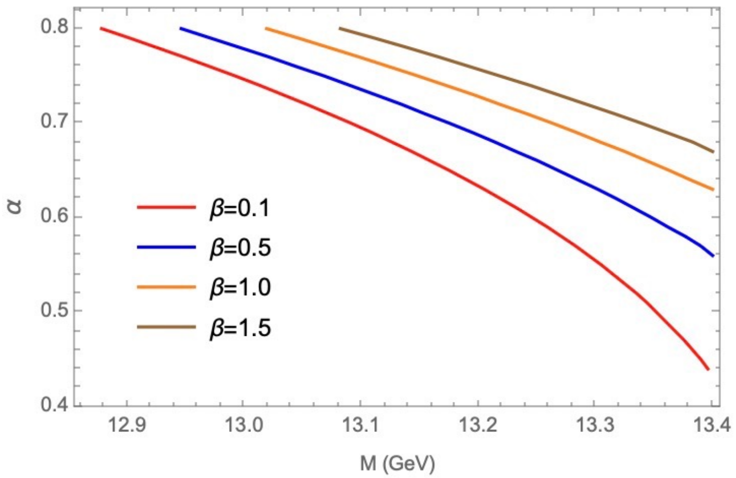
$$P_\mu \epsilon^\mu = 0, \qquad T^{\mu\nu} \equiv \sum_\epsilon \epsilon^\mu \epsilon^\nu = \frac{P^\mu P^\nu}{M^2} - g^{\mu\nu},$$

$$\xi^{\mu\nu} = \xi^{\nu\mu}, \qquad \xi^{\mu\nu}g_{\mu\nu} = 0, \qquad P_\mu \xi^{\mu\nu} = 0,$$

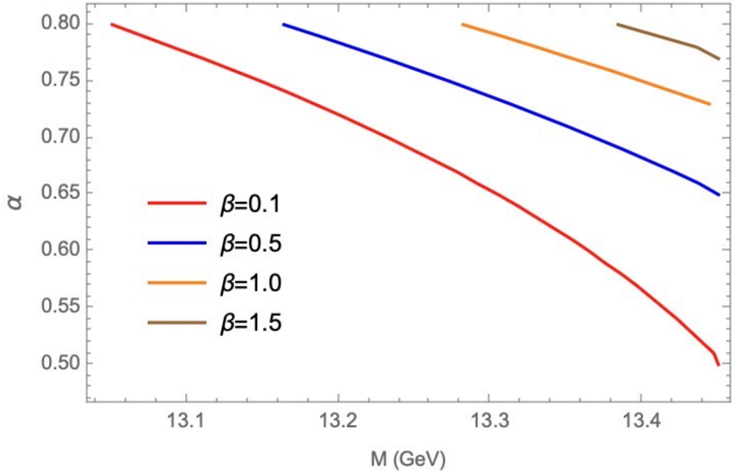
$$\sum_\xi \xi^{\mu\nu}\xi^{\alpha\beta} = \frac{1}{2}(T^{\mu\alpha}T^{\nu\beta} + T^{\mu\beta}T^{\nu\alpha}) - \frac{1}{3}T^{\mu\nu}T^{\alpha\beta}$$

Our results (Preliminary)

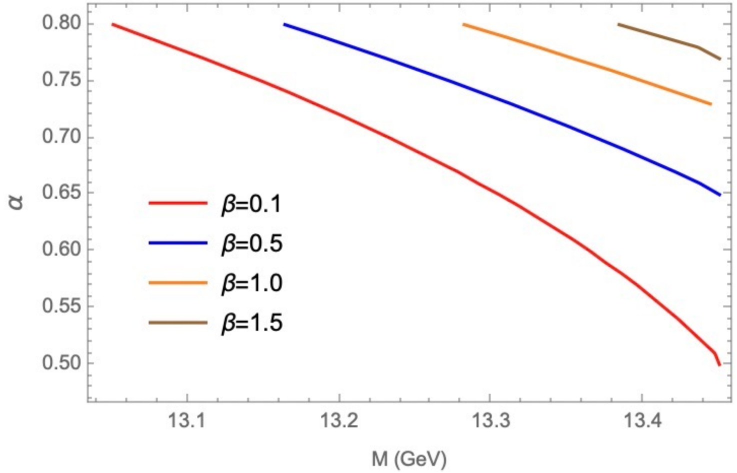
- $m_{bc}^{s} = 6.7\text{GeV}, m_{bc}^{a} = 6.75\ \text{GeV}$ [Nucl.Phys.B 947 (2019) 114727]



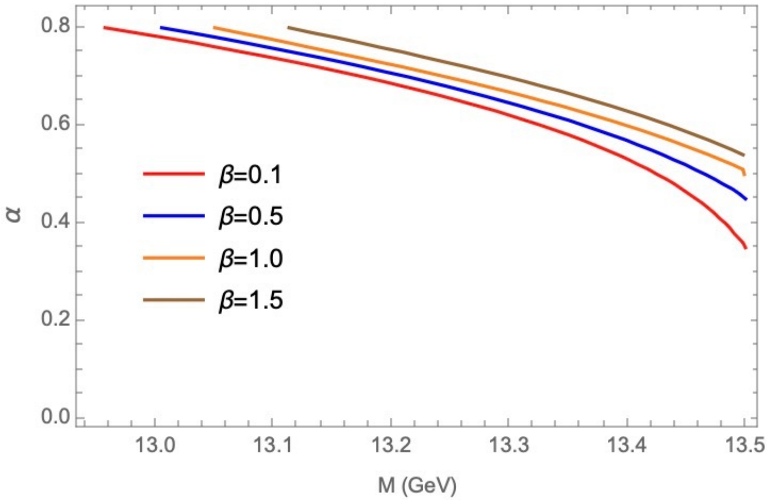
$|SS\rangle\ J^{PC} = 0^{++}$



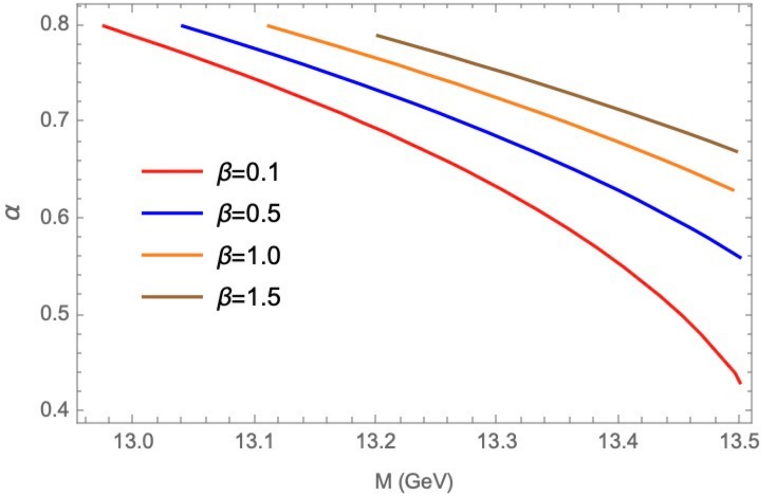
$|AS\rangle\ J^{PC} = 1^{+-}$



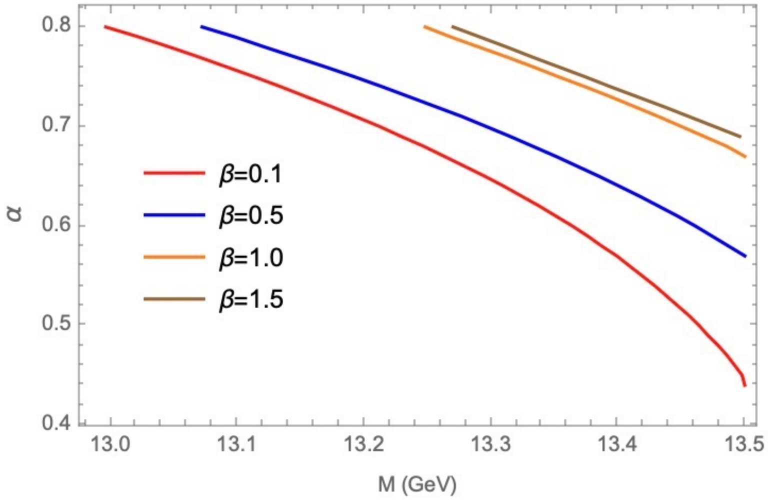
$|AS\rangle\ J^{PC} = 1^{++}$



$|AA\rangle\ J^{PC} = 0^{++}$



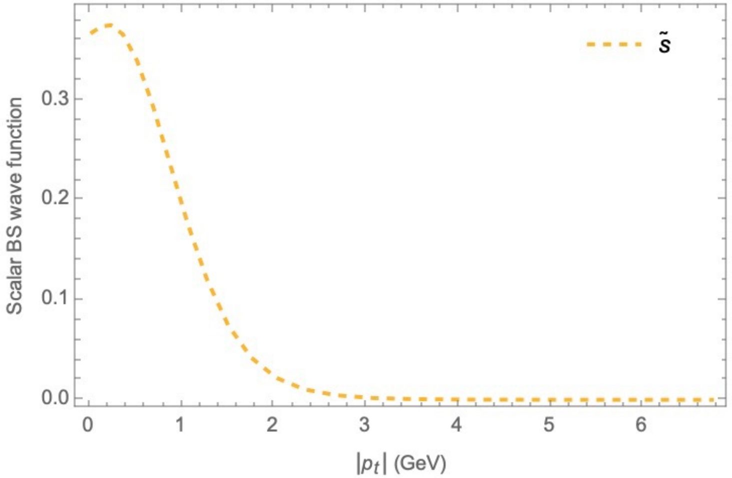
$|AA\rangle\ J^{PC} = 1^{+-}$



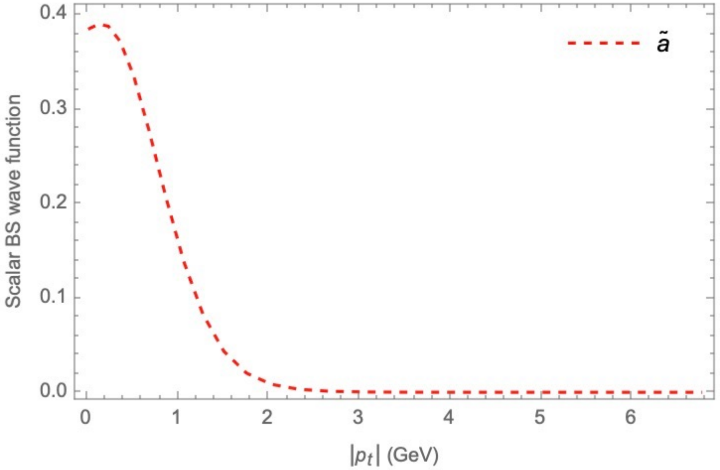
$|AA\rangle\ J^{PC} = 2^{++}$

Our results (Preliminary)

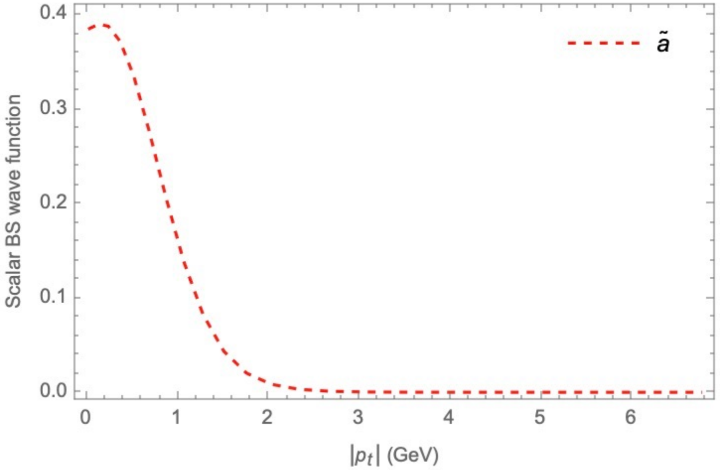
- The numerical Bethe-Salpeter wave function with $\kappa = 0.5$ and $\alpha = 0.65$



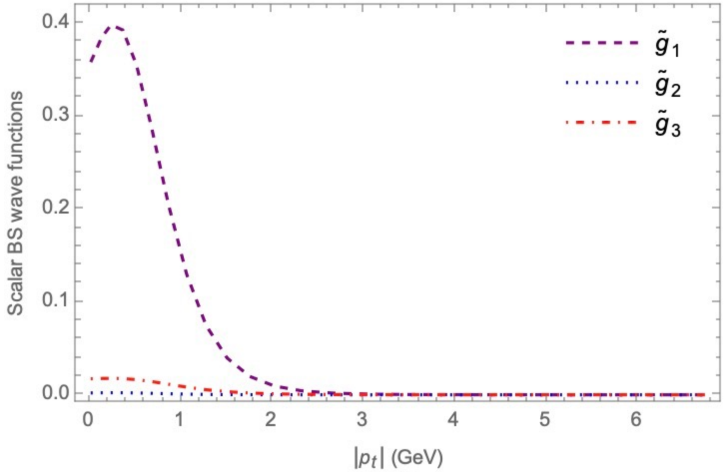
$|SS\rangle\, J^{PC} = 0^{++}$



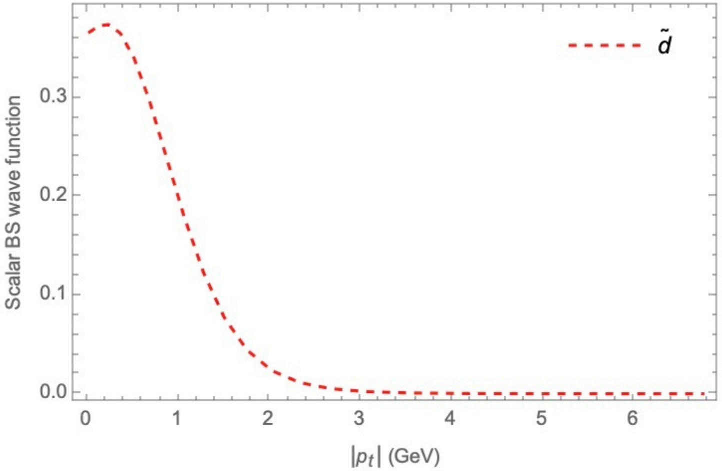
$|AS\rangle\, J^{PC} = 1^{+-}$



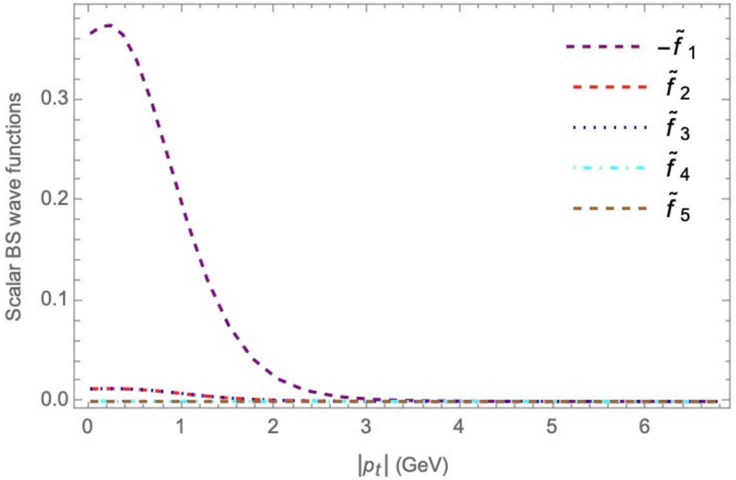
$|AS\rangle\, J^{PC} = 1^{++}$



$|AA\rangle\, J^{PC} = 0^{++}$



$|AA\rangle\, J^{PC} = 1^{+-}$



$|AA\rangle\, J^{PC} = 2^{++}$

Our results (Preliminary)

Configuration	$\lvert S\bar{S}\rangle$	$\frac{1}{\sqrt{2}}(\lvert A\bar{S}\rangle \pm \lvert S\bar{A}\rangle)$		$\lvert A\bar{A}\rangle$		
J^{PC}	0^{++}	1^{+-}	1^{++}	0^{++}	1^{+-}	2^{++}
Our results	13.268	13.450	13.450	13.294	13.365	13.385
Eur.Phys.J.C 80 (2020), 1004	12.521	12.533	12.533	12.374	12.491	12.576
Phys.Rev.D 102 (2020), 114030	12.824	12.831	12.831	12.813	12.826	12.849
Phys.Rev.D 103 (2021), 034001	12.747	12.744	12.703	12.682	12.720	12.755
Eur.Phys.J.C 82 (2022), 1126	12.837	12.886	12.850	12.790	12.794	12.896
Eur.Phys.J.C 82 (2022), 1126	13.035	12.964	12.938	12.850	12.835	12.964
Phys.Rev.D 105 (2022), 054024	12.359	12.896	12.155	12.503	12.016	12.897
Phys.Rev.D 104 (2021), 014003	$12.28^{+0.15}_{-0.14}$	$12.32^{+0.15}_{-0.13}$	$12.30^{+0.15}_{-0.14}$	$12.35^{+0.14}_{-0.12}$	$12.38^{+0.13}_{-0.12}$	$12.30^{+0.15}_{-0.14}$
Phys.Rev.D 100 (2019), 016006	13.050	13.052	13.056	13.035	13.047	13.070
Symmetry 14 (2022), 2504	12.856	12.863	12.863	12.838	12.855	12.883
Phys.Rev.D 97 (2018), 094015	13.553	13.592	13.510	13.483	13.520	13.590
Phys.Rev.D 86 (2012) 034004	12.471	12.488	12.485	12.359	12.424	12.566
Phys.Rev.D 100 (2019), 094009	···	···	12.804	12.746	12.776	12.809
Nucl.Phys.B 1018 (2025) 116977	···	···	12.810 ± 0.376	12.924 ± 0.478	11.982 ± 0.421	12.276 ± 0.329
Nucl.Phys.B 1018 (2025) 116977	···	···	12.947 ± 0.353	13.316 ± 0.498	13.165 ± 0.458	12.891 ± 0.283
Threshold	$\eta_c(1S)\eta_b(1S)$	$\eta_c(1S)\Upsilon(1S)$	$J/\psi(1S)\Upsilon(1S)$	$\eta_c(1S)\eta_b(1S)$	$\eta_c(1S)\Upsilon(1S)$	$J/\psi(1S)\Upsilon(1S)$
E_{th}	12.3828	12.4445	12.5573	12.3828	12.4445	12.5573

Summary

- ✓ The Bethe-Salpeter equation for the S-wave $bc\bar{b}\bar{c}$ tetraquark state was constructed within the diquark-antidiquark picture.
- ✓ The spectra of S-wave $bc\bar{b}\bar{c}$ tetraquark states are obtained.
- ✓ The spectra of S-wave $bc\bar{b}\bar{c}$ tetraquark states are above the threshold of the lowest quarkonium pair.

Thank you for your attention !