



Two-pseudoscalar-meson decay of resonances dynamically generated by two vector mesons

Eur.Phys.J.A 61 (2025) 3, 42.
[arXiv:2409.05302v1 \[hep-ph\]](https://arxiv.org/abs/2409.05302v1)

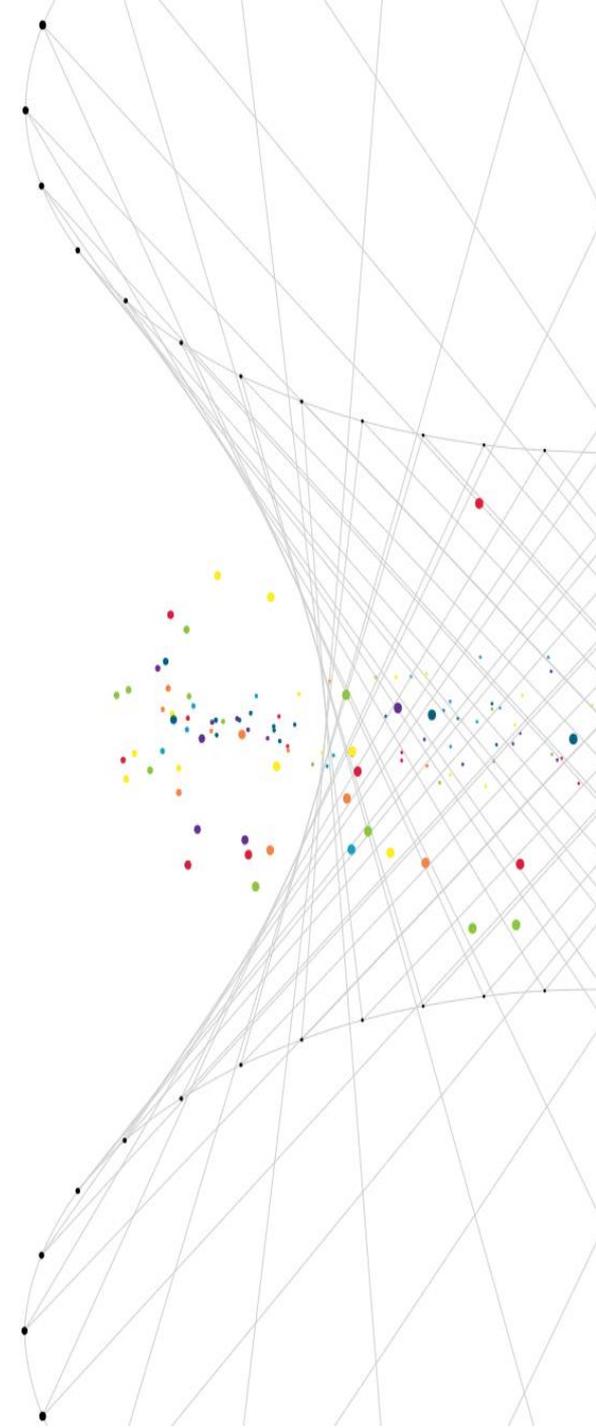
Speaker: Qing-Hua Shen (沈清华)

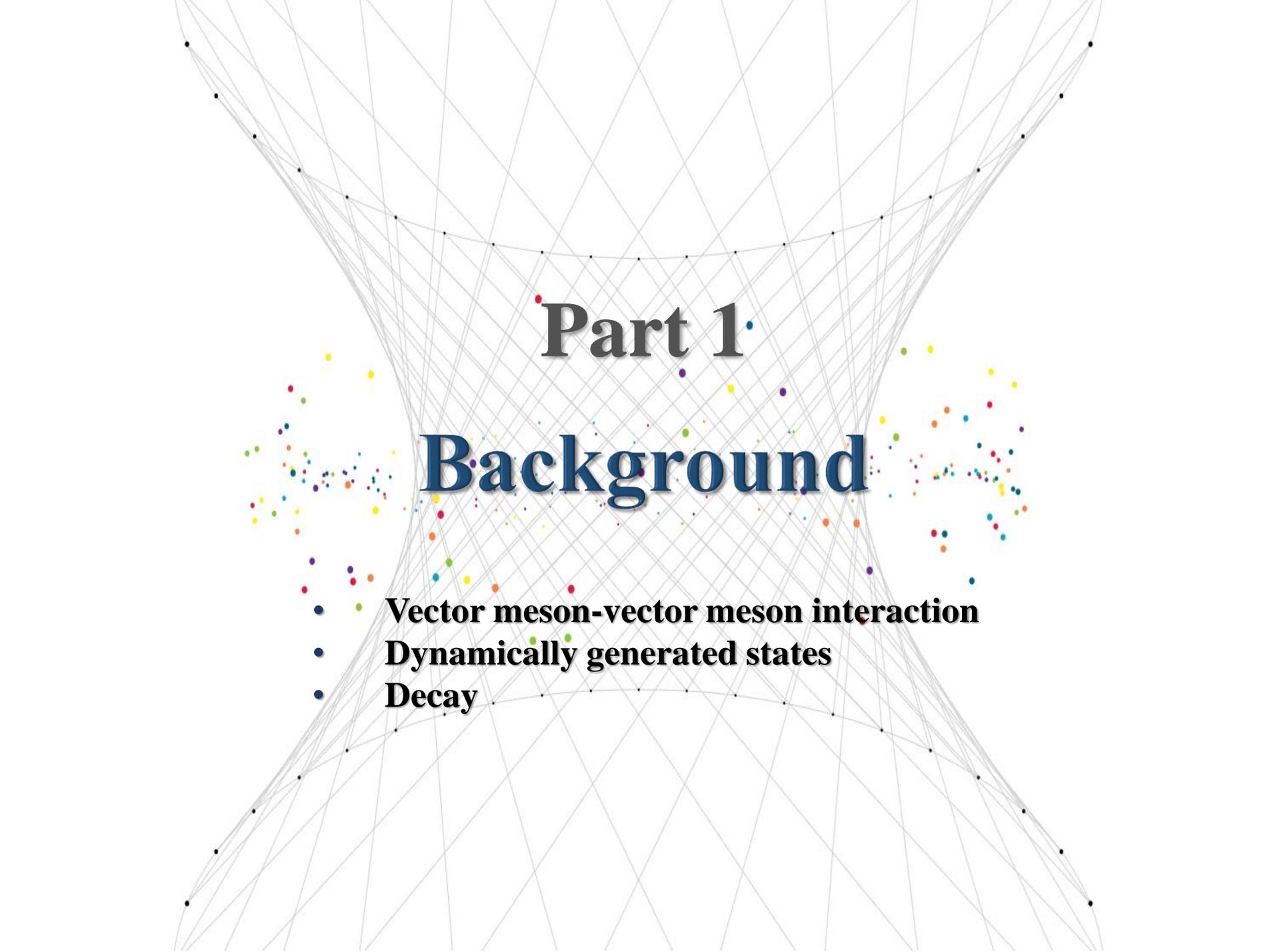
Co-authors: Li-Sheng Geng, Ju-Jun Xie

Thanks: Xiang Liu, Eulogio Oset

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- **Background**
- **Formalism**
- **Results**
- **Summary**





Part 1

Background

- **Vector meson-vector meson interaction**
- **Dynamically generated states**
- **Decay**



Background

Vector meson-vector meson interaction

L. S. Geng and E. Oset. Phys. Rev. D 79 (2009) 7, 074009

Hidden-gauge Lagrangian

$$\mathcal{L} = -\frac{1}{4} \langle \bar{V}_{uv} \bar{V}^{uv} \rangle + \frac{1}{2} M_V^2 \langle [V_u - \frac{i}{g} \Gamma_u]^2 \rangle$$

where

$$\bar{V}_{uv} = \partial_u V_v - \partial_v V_u - ig [V_u, V_v]$$

$$\Gamma_u = \frac{1}{2} \{ u^+ [\partial_u - i(v_u + a_u)] u + u [\partial_u - i(v_u - a_u)] u^+ \}$$

$$u^2 = U = e^{\frac{i\sqrt{2}P}{f}} \quad g = \frac{M_V}{2f}$$

$$V_u = \begin{pmatrix} \frac{\omega + \rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega - \pi^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \Phi \end{pmatrix}_u$$

$$P = \begin{pmatrix} \frac{\eta}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

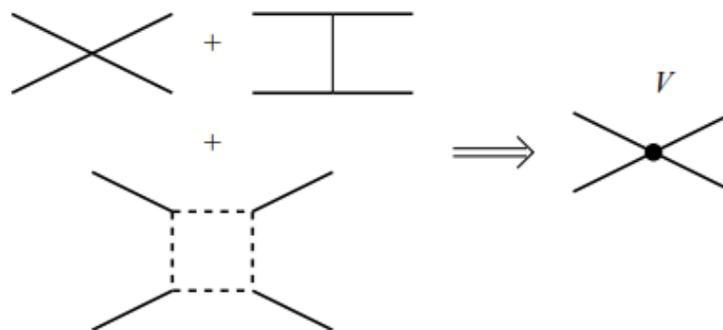
$$\mathcal{L}_{VVV} = \frac{1}{2} g^2 \langle [V_u, V_v] V^u V^v \rangle$$

$$\mathcal{L}_{VVV} = ig \langle (V^u \partial_v V_u - \partial_v V_u V^u) V^v \rangle$$

$$\mathcal{L}_{VPP} = -ig \langle V_u [P, \partial^u P] \rangle$$

Background

Dynamically generated states

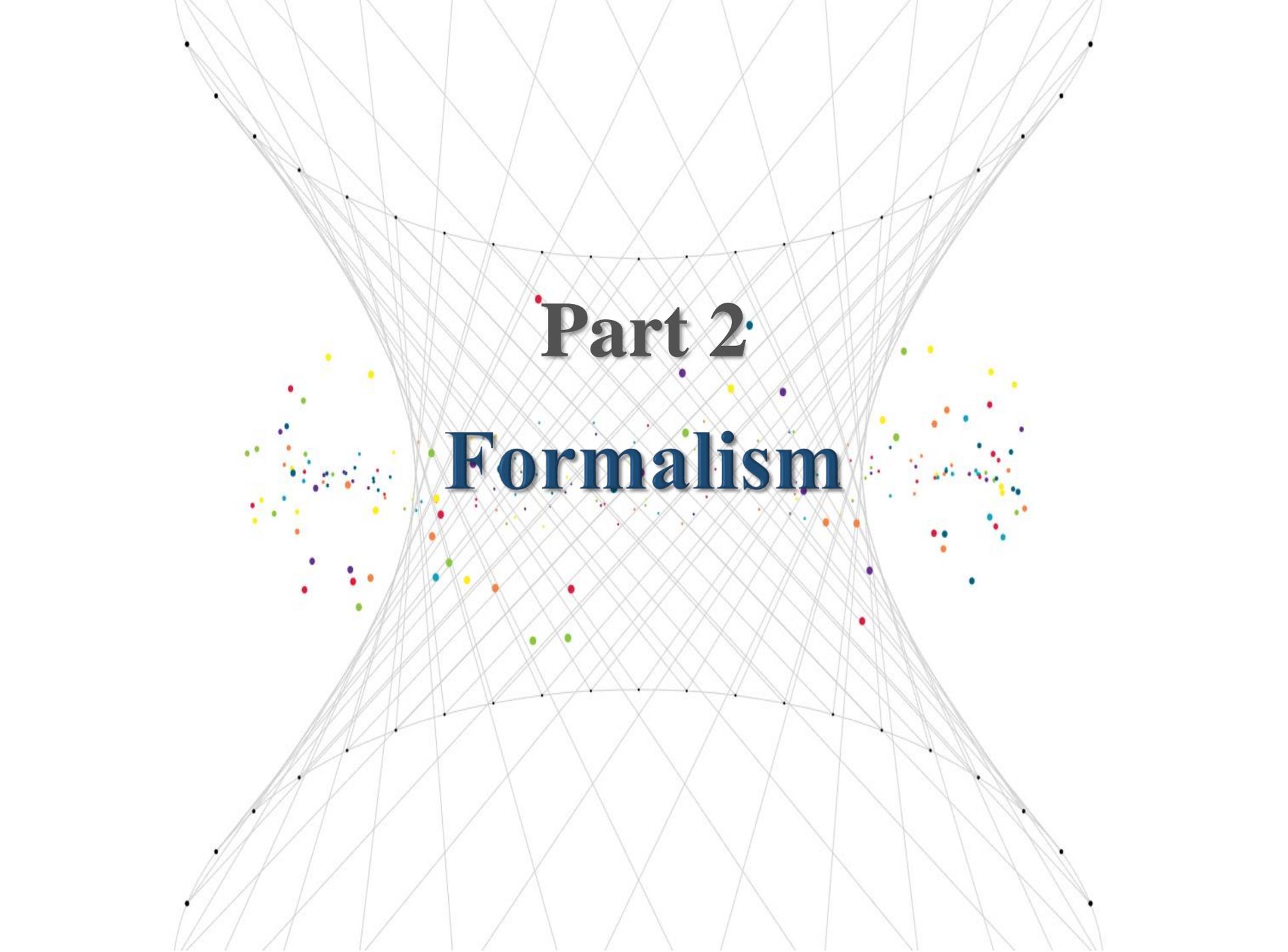


$I^G(J^{PC})$	Theory			PDG data		
	Pole position	Real axis		Name	Mass	Width
		$\Lambda_b = 1.4 \text{ GeV}$	$\Lambda_b = 1.5 \text{ GeV}$			
$0^+(0^{++})$	(1512,51)	(1523,257)	(1517,396)	$f_0(1370)$	1200~1500	200~500
$0^+(0^{++})$	(1726,28)	(1721,133)	(1717,151)	$f_0(1710)$	1724 ± 7	137 ± 8
$0^-(1^{+-})$	(1802,78)	(1802,49)		h_1		
$0^+(2^{++})$	(1275,2)	(1276,97)	(1275,111)	$f_2(1270)$	1275.1 ± 1.2	$185.0^{+2.9}_{-2.4}$
$0^+(2^{++})$	(1525,6)	(1525,45)	(1525,51)	$f_2'(1525)$	1525 ± 5	73^{+6}_{-5}
$1^-(0^{++})$	(1780,133)	(1777,148)	(1777,172)	a_0		
$1^+(1^{+-})$	(1679,235)	(1703,188)		b_1		
$1^-(2^{++})$	(1569,32)	(1567,47)	(1566,51)	$a_2(1700)??$		
$1/2(0^+)$	(1643,47)	(1639,139)	(1637,162)	K_0^*		
$1/2(1^+)$	(1737,165)	(1743,126)		$K_1(1650)?$		
$1/2(2^+)$	(1431,1)	(1431,56)	(1431,63)	$K_2^*(1430)$	1429 ± 1.4	104 ± 4



Background decay

Process	Reference	Process	Reference
$R \rightarrow \gamma\gamma$	Phys. Rev. D 81, 054037 (2010)	$J/\psi \rightarrow VR$	Phys. Lett. B 680 (2009) 310-315
$R \rightarrow \gamma P$	Phys. Rev. D 81, 054037 (2010)		Eur. Phys. J. A 56 (2020) 6, 173
	Phys. Rev. D 83 (2011) 094030		Phys. Lett. B 843 (2023) 137999
$R \rightarrow V\gamma P$	Phys. Lett. B 690 (2010) 376-381	$J/\psi \rightarrow \gamma R$	Eur. Phys. J. A 44 (2010) 305-311
$\bar{B}_0/\bar{B}_s^0 \rightarrow J/\Psi R$	Phys. Rev. D 90 (2014) 9, 094006	$\chi_{c1} \rightarrow PR$	Phys. Rev. D 100 (2019) 11, 114011
$\Psi(nS)/\Upsilon(nS) \rightarrow VR$	Eur. Phys. J. A 49 (2013) 130	$\gamma p \rightarrow RB$	Phys. Rev. C 93 (2016) 2, 025202
	Phys. Rev. D 91 (2015) 9, 094013		Eur. Phys. J. C 82 (2022) 3, 225
$\Psi(nS)/\Upsilon(nS) \rightarrow \gamma R$	Phys. Rev. D 91 (2015) 9, 094013	$D_s^+ \rightarrow \pi^+ K_s^0 K_s^0$	Phys. Rev. D 105 (2022) 11, 116010
$\eta_c \rightarrow \bar{K}^0 K^+ \pi^-$	Phys. Rev. D 108 (2023) 11, 114004	$D_s^+ \rightarrow K_s^0 K^+ \pi^-$	Phys. Rev. D 107 (2023) 11, 116018

The background features a complex network of thin grey lines connecting black dots, forming a hyperboloid-like shape that narrows in the center. Scattered throughout this structure are numerous small, multi-colored dots in shades of red, yellow, green, blue, and purple. The text is centered over this graphic.

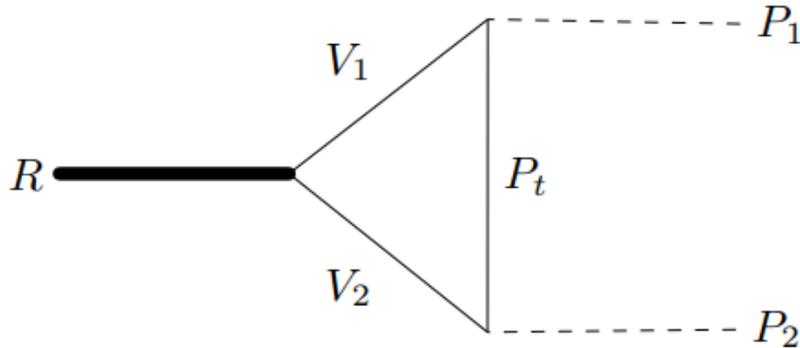
Part 2:

Formalism



Formalism

$$R \rightarrow PP$$



RVV vertex:

$$\mathcal{L}_{RVV} = g_{RVV} \mathcal{P}^{(s)}$$

VPP vertex:

$$\mathcal{L}_{VPP} = -ig \langle V_u [P, \partial^u P] \rangle$$

form factor:

$$F(t^2) = \frac{\Lambda^2 - m_t^2}{\Lambda^2 - t^2}$$

projection operator:

$$\mathcal{P}^{(1)} = \frac{1}{\sqrt{3}} \epsilon_i \epsilon_i \quad \mathcal{P}^{(2)} = \frac{1}{2} [\epsilon_i \epsilon_j - \epsilon_j \epsilon_i] - \frac{1}{3} \epsilon_m \epsilon_m \delta_{ij}$$

Phys.Rev.D 78 (2008) 114018

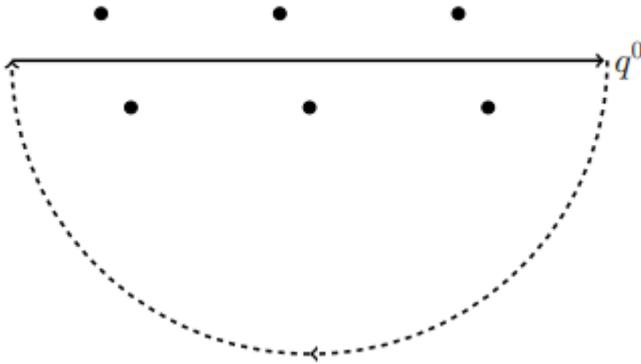
Phys.Rev.D 79 (2009) 074009

Eur.Phys.J.C 83 (2023) 12, 1106

Phys.Rev.D 80 (2009) 014025

Phys.Rev.D 80 (2009) 114013

PhD thesis of Feng-Kun Guo



$$t_{V_1 P_1 P_t} = -igC_2 \epsilon(V_1) \cdot (2k_1 - q)$$

$$t_{V_2 P_2 P_t} = -igC_3 \epsilon(V_2) \cdot (k_2 - k_1 + q)$$



$$t_a = \mathcal{P}_J^*(V_1 V_2) \epsilon(V_1) \cdot (2k_1 - q) \epsilon(V_2) \cdot (k_2 - k_1 + q)$$

$$t = Cg_{RV_1 V_2} g^2 \int \frac{d^4 q}{(2\pi)^4} t_a F^2 \frac{1}{q^2 - M_{V_1}^2 + i\epsilon} \frac{1}{(P - q)^2 - M_{V_2}^2 + i\epsilon} \frac{1}{(q - k_1)^2 - m_{P_t}^2 + i\epsilon}$$

$$\Gamma_{R \rightarrow P_1 P_2} = \frac{1}{2J + 1} \frac{|\vec{k}_1|}{8\pi M^2} \sum_{pol} |t|^2$$

$$\Lambda = 1350 (\pm 100) \text{ MeV}$$

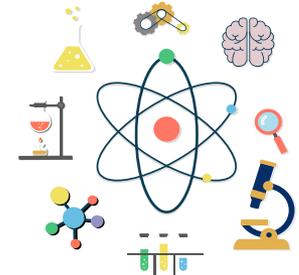
Width of vector meson:

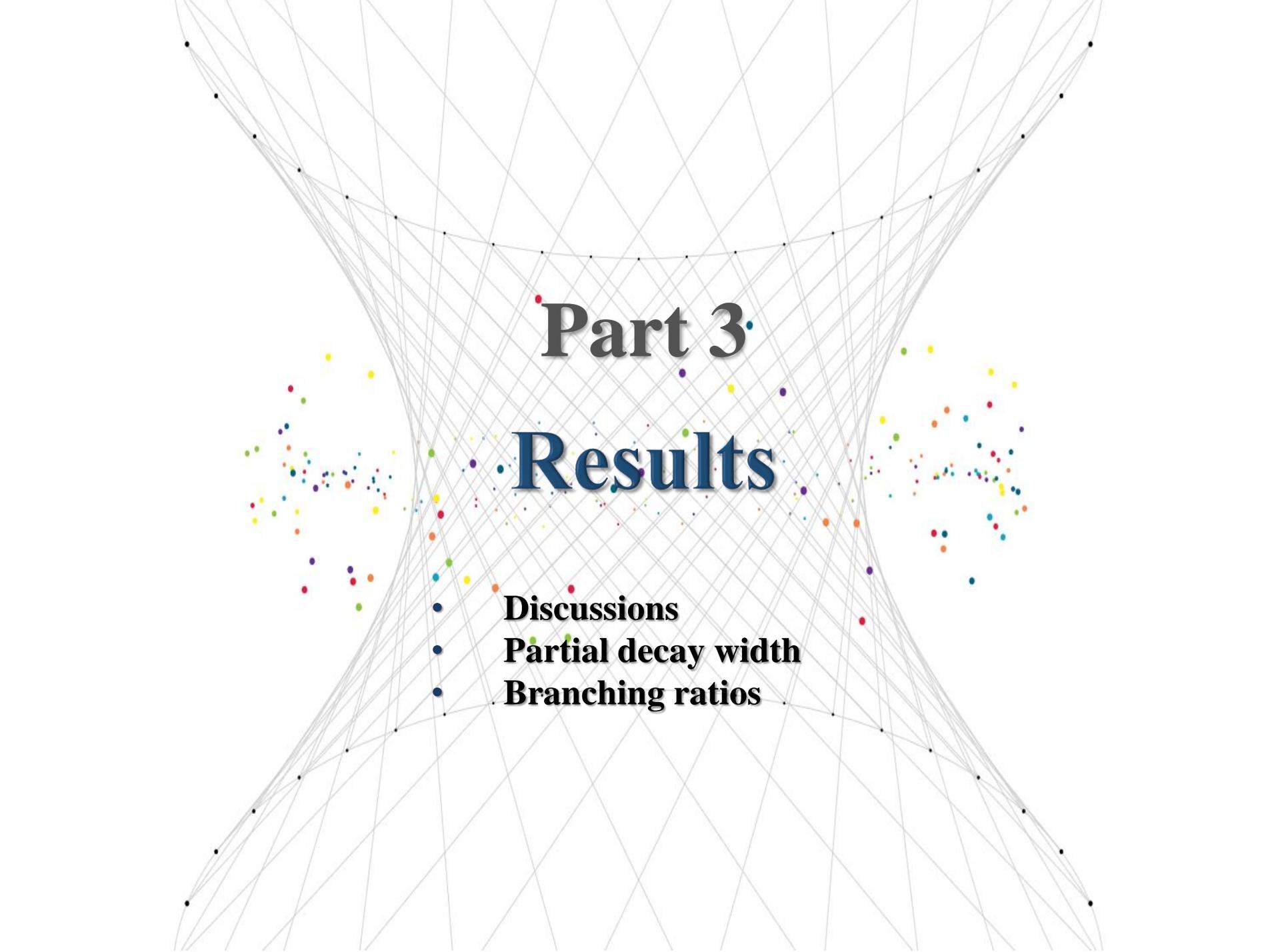
$$\omega_{V_1} \rightarrow \omega_{V_1} + \frac{\Gamma_{V_1}}{4}$$

$$\omega_{V_2} \rightarrow \omega_{V_2} - \frac{\Gamma_{V_2}}{4}$$

[Phys.Rev.D 78 \(2008\) 114018](#)

[Phys.Rev.D 79 \(2009\) 074009](#)





Part 3

Results

- **Discussions**
- **Partial decay width**
- **Branching ratios**



Results

Discussions

$f_2(1270)$

This work	
$f_2(1270) \rightarrow \pi\pi$	$141.1^{+68.8}_{-48.0}$
$f_2(1270) \rightarrow K\bar{K}$	$15.63^{+10.29}_{-6.68}$
$f_2(1270) \rightarrow \eta\eta$	$0.87^{+0.61}_{-0.39}$
$R_{f_2(1270)}^{K\bar{K}/\pi\pi}$	0.11 ± 0.02
$R_{f_2(1270)}^{\eta/\pi\pi}$	0.006 ± 0.001

The Review of Particle Physics

	$f_2(1270) \rightarrow \pi\pi$	$f_2(1270) \rightarrow K\bar{K}$	$f_2(1270) \rightarrow \eta\eta$	$R_{f_2(1270)}^{K\bar{K}/\pi\pi}$	$R_{f_2(1270)}^{\eta/\pi\pi}$
Value (average/fit)	$157.2^{+4.0}_{-1.1}$	8.6 ± 0.8	0.75 ± 0.14	$0.041^{+0.004}_{-0.005}$	
Exp.	$157.0^{+6.0}_{-1.0}$	$9.0^{+0.7}_{-0.3}$	1.0 ± 0.1	0.02 - 0.06	0.003 ± 0.001
	152 ± 8	7.5 ± 2.0	1.8 ± 0.4		



Results

Discussions

$f_2'(1525)$

This work	
$f_2'(1525) \rightarrow \pi\pi$	$5.5^{+1.0}_{-1.0}$
$f_2'(1525) \rightarrow K\bar{K}$	$17.6^{+10.1}_{-6.9}$
$f_2'(1525) \rightarrow \eta\eta$	$6.6^{+4.3}_{-2.9}$
$R_{f_2'(1525)}^{\pi\pi/K\bar{K}}$	$0.31^{+0.11}_{-0.08}$
$R_{f_2'(1525)}^{\eta\eta/K\bar{K}}$	$0.37^{+0.03}_{-0.02}$

The Review of Particle Physics

	$f_2'(1525) \rightarrow \pi\pi$	$f_2'(1525) \rightarrow K\bar{K}$	$f_2'(1525) \rightarrow \eta\eta$	$R_{f_2'(1525)}^{\pi\pi/K\bar{K}}$	$R_{f_2'(1525)}^{\eta\eta/K\bar{K}}$
Value (average/fit)	0.71 ± 0.14	75 ± 4	9.9 ± 1.9	0.0092 ± 0.0018	0.115 ± 0.028
Exp.	$1.4^{+1.0}_{-0.7}$	63^{+6}_{-5}	24^{+3}_{-1}	0.075 ± 0.035	$0.119 \pm 0.015 \pm 0.036$
	$0.2^{+1.0}_{-0.2}$		5.0 ± 8		0.11 ± 0.04



Results

Discussions

$K_2^*(1430)$

This work	
$K_2^*(1430) \rightarrow K\pi$	$34.6^{+16.5}_{-11.7}$
$K_2^*(1430) \rightarrow K\eta$	$2.62^{+1.65}_{-1.09}$
$R_{K_2^*(1430)}^{K\eta/K\pi}$	0.075 ± 0.009

The Review of Particle Physics			
	$K_2^*(1430) \rightarrow K\pi$	$K_2^*(1430) \rightarrow K\eta$	$R_{K_2^*(1430)}^{K\eta/K\pi}$
Value (average/fit)	53.2 ± 2.9	$0.16^{+0.36}_{-0.11}$	$0.0030^{+0.0070}_{-0.0020}$
Exp.			0 ± 0.0056



Results

Discussions

$f_0(1710)$

This work	
$f_0(1710) \rightarrow \pi\pi$	$5.76^{+2.04}_{-1.31}$
$f_0(1710) \rightarrow K\bar{K}$	$57.9^{+28.8}_{-20.5}$
$f_0(1710) \rightarrow \eta\eta$	$32.7^{+18.8}_{-12.9}$
$R_{f_0(1710)}^{\pi\pi/K\bar{K}}$	0.10 ± 0.02
$R_{f_0(1710)}^{\eta\eta/K\bar{K}}$	0.56 ± 0.04

The Review of Particle Physics					
	$f_0(1710) \rightarrow \pi\pi$	$f_0(1710) \rightarrow K\bar{K}$	$f_0(1710) \rightarrow \eta\eta$	$R_{f_0(1710)}^{\pi\pi/K\bar{K}}$	$R_{f_0(1710)}^{\eta\eta/K\bar{K}}$
Value (average /fit)	$5.85^{+0.30}_{-4}$	54^{+18}_{-19}	33 ± 18	0.23 ± 0.05	
Exp.				$0.64 \pm 0.27 \pm 0.18$	0.48 ± 0.15
				$0.41^{+0.11}_{-0.17}$	$0.46^{+0.70}_{-0.38}$
				$0.2 \pm 0.024 \pm 0.036$	
				0.39 ± 0.14	
				0.32 ± 0.14	
				$5.8^{+9.1}_{-5.5}$	



Results

Discussions

$a_0(1710)$

This work	
$a_0(1710) \rightarrow K\bar{K}$	$40.2^{+26.8}_{-17.6}$
$a_0(1710) \rightarrow \pi\eta$	$38.4^{+19.9}_{-14.2}$
$R_{a_0(1710)}^{K\eta/K\pi}$	$0.96^{+0.14}_{-0.09}$

The Review of Particle Physics			
	$a_0(1710) \rightarrow K\bar{K}$	$a_0(1710) \rightarrow \pi\eta$	$R_{a_0(1710)}^{K\eta/K\pi}$
Value (average/fit)			
Exp.			



Results

Discussions

$f_0(1500)$?

This work

$f_0(1500) \rightarrow \pi\pi$	632^{+269}_{-198}
$f_0(1500) \rightarrow K\bar{K}$	$51.0^{+30.6}_{-20.5}$
$f_0(1500) \rightarrow \eta\eta$	$0.54^{+0.37}_{-0.24}$
$R_{f_0(1500)}^{K\bar{K}/\pi\pi}$	0.08 ± 0.01
$R_{f_0(1500)}^{\eta\eta/\pi\pi}$	$(8.6 \pm 1.5) \times 10^{-4}$

The Review of Particle Physics

Mass: 1522 ± 25 MeV Width: 108 ± 33 MeV

	$f_0(1500) \rightarrow \pi\pi$	$f_0(1500) \rightarrow K\bar{K}$	$f_0(1500) \rightarrow \eta\eta$	$R_{f_0(1500)}^{K\bar{K}/\pi\pi}$	$R_{f_0(1500)}^{\eta\eta/\pi\pi}$
Value (average/fit)	37 ± 12	9.2 ± 3.0	$0.54^{+0.37}_{-0.24}$	0.236 ± 0.026	0.175 ± 0.027
Exp.				0.25 ± 0.03	0.18 ± 0.03
				0.33 ± 0.03 ± 0.07	0.11 ± 0.03
				0.16 ± 0.05	0.157 ± 0.060
				0.19 ± 0.07	0.080 ± 0.033
				0.20 ± 0.08	0.078 ± 0.013
					0.230 ± 0.097



Results

Discussions

$f_0(1370)$?

This work	
$f_0(1370) \rightarrow \pi\pi$	632^{+269}_{-198}
$f_0(1370) \rightarrow K\bar{K}$	$51.0^{+30.6}_{-20.5}$
$f_0(1370) \rightarrow \eta\eta$	$0.54^{+0.37}_{-0.24}$
$R_{f_0(1370)}^{K\bar{K}/\pi\pi}$	0.08 ± 0.01
$R_{f_0(1370)}^{\eta/\pi\pi}$	$(8.6 \pm 1.5) \times 10^{-4}$

The Review of Particle Physics					
Mass: 1200 – 1500 MeV Width: 200 – 500 MeV					
	$R_{f_0(1370)}^{\pi\pi/Total}$	$R_{f_0(1370)}^{K\bar{K}/Total}$	$R_{f_0(1370)}^{\eta/Total}$	$R_{f_0(1370)}^{K\bar{K}/\pi\pi}$	$R_{f_0(1370)}^{\eta/\pi\pi}$
Value (average/fit)					
Exp.	0.26 ± 0.09	0.35 ± 0.13		0.91 ± 0.20	
				0.46 ± 0.15	
				± 0.11	
				0.12 ± 0.06	
				0.08 ± 0.08	



Results

Partial decay width

Tensor

Decay mode	This work	RPP	Exp.
$f_2(1270) \rightarrow \pi\pi$	$141.1^{+68.8}_{-48.0}$	$157.2^{+4.0}_{-1.1}$	$157.0^{+6.0}_{-1.0}$ Phys. Lett. B 177,223 (1986)
			152 ± 8 Eur. Phys. J. A 27,207 (2006)
$f_2(1270) \rightarrow K\bar{K}$	$15.63^{+10.29}_{-6.68}$	8.6 ± 0.8	$9.0^{+0.7}_{-0.3}$ Phys. Lett. B 177,223(1986)
			7.5 ± 2.0 Eur. Phys. J. A 27,207 (2006)
$f_2(1270) \rightarrow \eta\eta$	$0.87^{+0.61}_{-0.39}$	0.75 ± 0.14	1.0 ± 0.1 Phys. Lett. B 177,223(1986)
			1.8 ± 0.4 Eur. Phys. J. A 27,207 (2006)
$f_2'(1525) \rightarrow \pi\pi$	$5.5^{+1.0}_{-1.0}$	0.71 ± 0.14	$1.4^{+1.0}_{-0.7}$ Phys. Lett. B 177,223(1986)
			$0.2^{+1.0}_{-0.2}$ Eur. Phys. J. A 27,207 (2006)
$f_2'(1525) \rightarrow K\bar{K}$	$17.6^{+10.1}_{-6.9}$	75 ± 4	63^{+6}_{-5} Phys. Lett. B 177,223(1986)
$f_2'(1525) \rightarrow \eta\eta$	$6.6^{+4.3}_{-2.9}$	9.9 ± 1.9	24^{+3}_{-1} Phys. Lett. B 177,223(1986)
			5.0 ± 8 Eur. Phys. J. A 27,207 (2006)
$K_2^*(1430) \rightarrow K\pi$	$34.6^{+16.5}_{-11.7}$	53.2 ± 2.9	
$K_2^*(1430) \rightarrow K\eta$	$2.62^{+1.65}_{-1.09}$	$0.16^{+0.36}_{-0.11}$	



Results

Partial decay width

Scalar

Decay mode	This work	RPP	Exp.
$f_0(1710) \rightarrow \pi\pi$	$5.76^{+2.04}_{-1.31}$	$5.85^{+0.30}_{-4}$	
$f_0(1710) \rightarrow K\bar{K}$	$57.9^{+28.8}_{-20.5}$	54^{+18}_{-19}	
$f_0(1710) \rightarrow \eta\eta$	$32.7^{+18.8}_{-12.9}$	33 ± 18	
$a_0(1710) \rightarrow K\bar{K}$	$40.2^{+26.8}_{-17.6}$		
$a_0(1710) \rightarrow \pi\eta$	$38.4^{+19.9}_{-14.2}$		
$f_0(1500) \rightarrow \pi\pi$	632^{+269}_{-198}	37 ± 12	
$f_0(1500) \rightarrow K\bar{K}$	$51.0^{+30.6}_{-20.5}$	9.2 ± 3.0	
$f_0(1500) \rightarrow \eta\eta$	$0.54^{+0.37}_{-0.24}$	6.5 ± 2.2	



Results

Branching ratios

Tensor

Ratio		This work	RPP	EXP.
$f_2(1270)$	$R_{f_2(1270)}^{K\bar{K}/\pi\pi}$	0.11 ± 0.02	$0.041^{+0.004}_{-0.005}$	$0.02 - 0.06$
	$R_{f_2(1270)}^{\eta/\pi\pi}$	0.006 ± 0.001		0.003 ± 0.001 Phys. Lett. B 479, 59(2000)
$f_2'(1525)$	$R_{f_2'(1525)}^{\pi\pi/K\bar{K}}$	$0.31^{+0.11}_{-0.08}$	0.0092 ± 0.0018	0.075 ± 0.035 Z. Phys. C 36,369(1987)
	$R_{f_2'(1525)}^{\eta/K\bar{K}}$	$0.37^{+0.03}_{-0.02}$	0.115 ± 0.028	$0.119 \pm 0.015 \pm 0.036$ Phys.Atom.Nucl.70,1713(2007) 0.11 ± 0.04 Sov.Phys.Dokl.36 (1991) 155-158
$K_2^*(1430)$	$R_{K_2^*(1430)}^{K\eta/K\pi}$	0.075 ± 0.009	$0.0030^{+0.0070}_{-0.0020}$	0 ± 0.00561988 Phys. Lett. B 201,169(1988)



Results

Branching ratios

Scalar

Ratio		This work	RPP	Exp.	
$f_0(1710)$	$R_{f_0(1710)}^{\pi\pi/K\bar{K}}$	0.10 ± 0.02	0.23 ± 0.05	$0.64 \pm 0.27 \pm 0.18$	Phys. Rev. D 97, 112006(2018)
				$0.41_{-0.17}^{+0.11}$	Phys. Lett. B 642,441(2006)
				$0.2 \pm 0.024 \pm 0.036$	Phys. Lett. B 642, 462(1999)
				0.39 ± 0.14	Z. Phys. C 51, 351(1991)
				0.32 ± 0.14	Phys. Rev. Lett.101,252002.
	$5.8_{-5.5}^{+9.1}$	Phys. Atom. Nucl.65, 1545(2002)			
$R_{f_0(1710)}^{\eta\eta/K\bar{K}}$	0.56 ± 0.04			0.48 ± 0.15	Phys. Lett. B 479, 59(2000)
				$0.46_{-0.38}^{+0.70}$	Phys. Atom. Nucl.65, 1545(2002)
$a_0(1710)$	$R_{a_0(1710)}^{K\eta/K\pi}$	$0.96_{-0.09}^{+0.14}$			

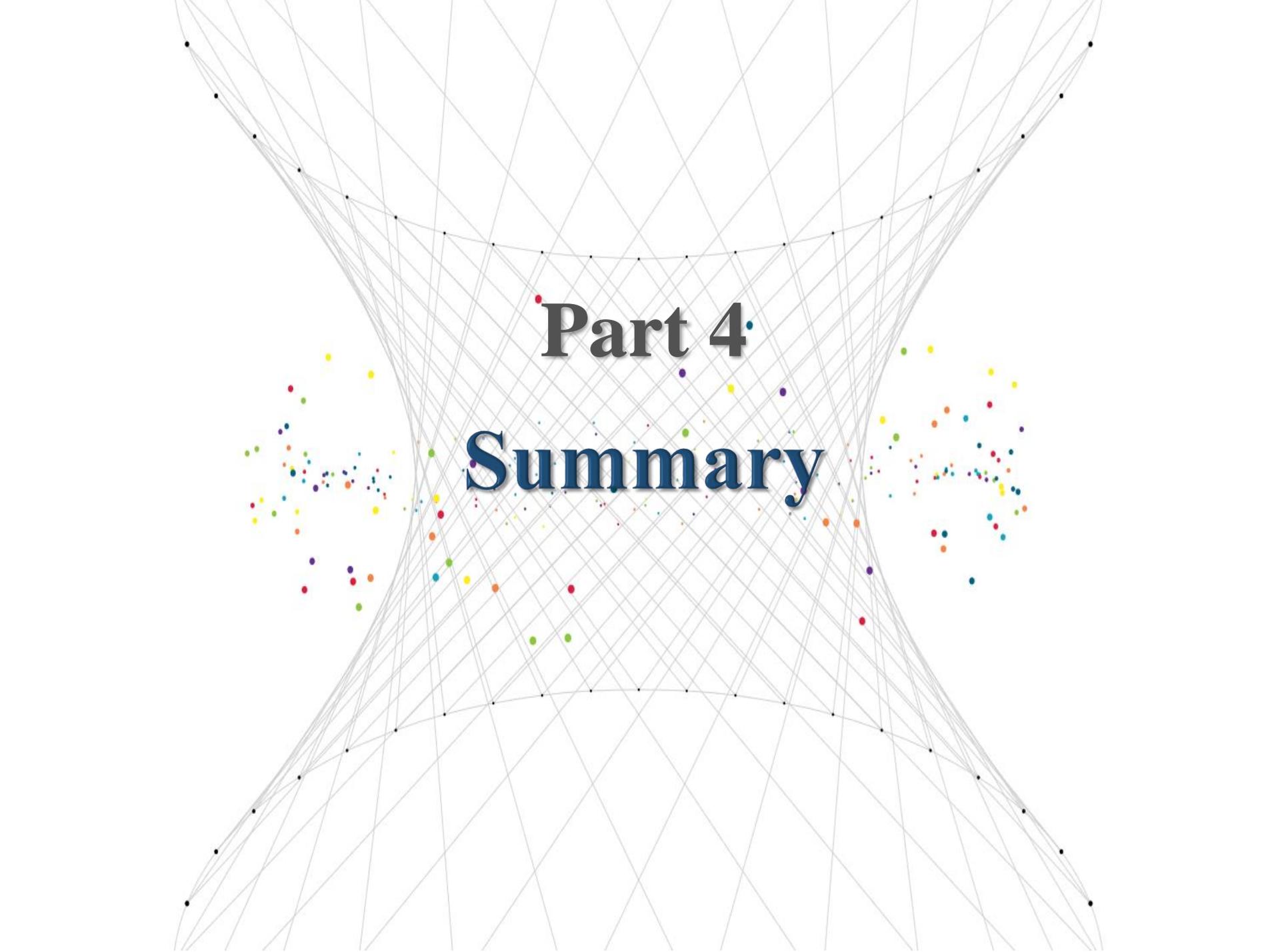


Results

Branching ratios

Scalar

Ratio		This work	RPP	Exp.	
$f_0(1500)$	$R_{f_0(1500)}^{K\bar{K}/\pi\pi}$	0.08 ± 0.01	0.236 ± 0.026	0.25 ± 0.03	Eur. Phys. J. C 26,371(2003)
				$0.33 \pm 0.03 \pm 0.07$	Phys. Lett. B 462,462 (1999)
				0.16 ± 0.05	Phys. Atom. Nucl. 65,1545(2002)
				0.19 ± 0.07	Phys. Rev. D 57,3860(1998)
				0.20 ± 0.08	Phys. Lett. B 385,425(1996)
	$R_{f_0(1500)}^{\eta\eta/\pi\pi}$	$(8.6 \pm 1.5) \times 10^{-4}$	0.175 ± 0.027	0.18 ± 0.03	Phys. Lett. B 479, 59(2000)
				0.11 ± 0.03	Phys. Atom. Nucl. 65,1545(2002)
				0.157 ± 0.060	Phys. Lett. B 355,425(1995)
				0.080 ± 0.033	Eur. Phys. J. C 23, 29(2002)
				0.078 ± 0.013	Nucl. Phys. A 609, 562 (1996)
			0.230 ± 0.097	Phys. Lett. B 353,571(1995)	

The background features a complex network of thin grey lines connecting black dots, forming a hyperboloid-like shape that narrows in the center. Scattered throughout this structure are numerous small, multi-colored dots in shades of red, yellow, green, blue, and purple. The text 'Part 4' and 'Summary' is centered over this network.

Part 4
Summary

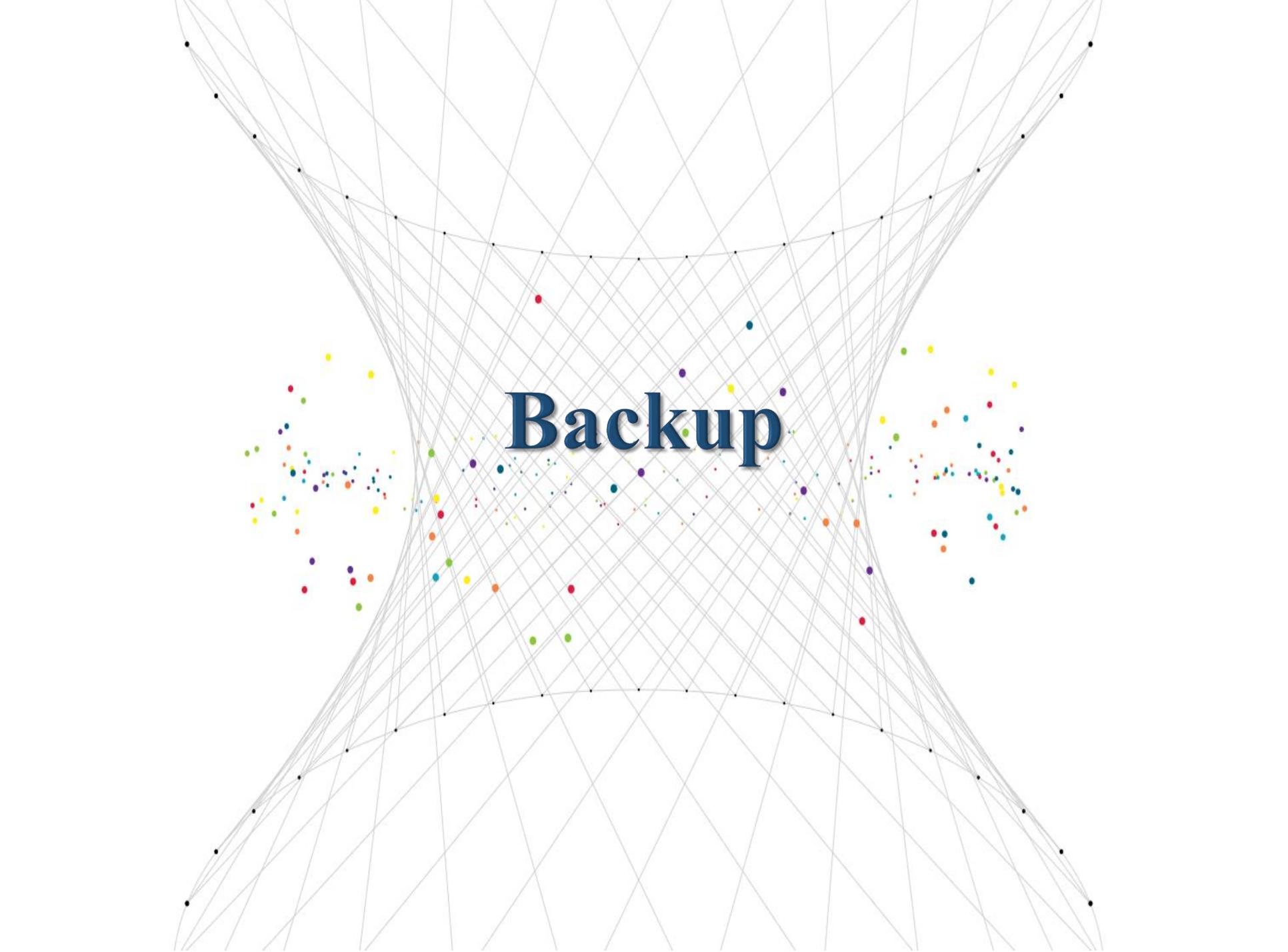


Summary

- **The partial widths and branching ratios for different two-pseudoscalar-meson decay channels of states dynamically generated by two vector meson are calculated by triangular diagrams, and interference between different channels and neglected real part of previous box diagrams are considered.**
- **Our results are similar with previous ones, which are obtain by box diagram and real axis method.**
- **Data from experiment is limited and old, and some of them are not consistent with each other. We expect new measurements from BES III, Belle II and LHCb collaborations can check old and limited data in RPP to obtain more reliable data to test our results.**

THANKS



The image features a complex network diagram with a central text element. The network is composed of numerous nodes, represented by small black dots, arranged in a roughly circular pattern. These nodes are interconnected by a dense web of thin, light gray lines, creating a complex, multi-layered structure. The overall shape of the network is somewhat hourglass-like, with a narrower central section and wider sections at the top and bottom. In the center of this network, the word "Backup" is written in a large, bold, blue serif font. The text is slightly shadowed, giving it a three-dimensional appearance as if it's floating within or attached to the network. The background is plain white, which makes the network and the text stand out clearly.

Backup

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- Coupling

- Data from RPP

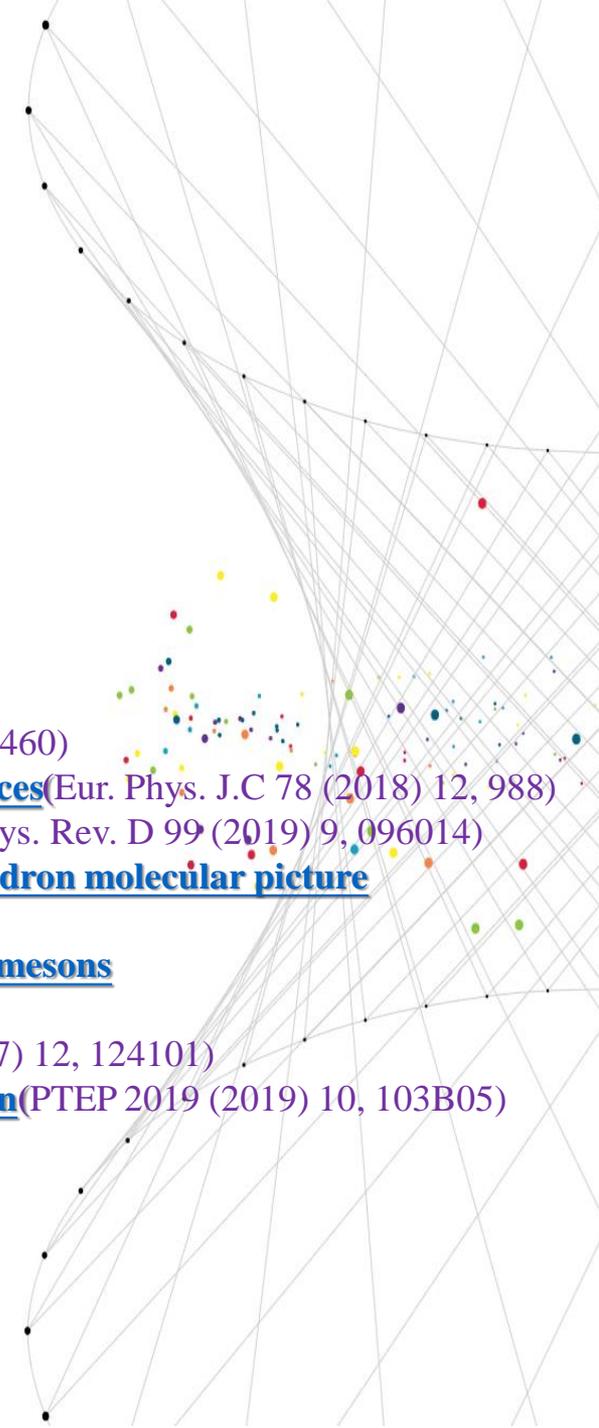
- $f_2(1270)$
- $f_2'(1525)$
- $K_2^*(1430)$
- $f_0(1710)$
- $f_0(1500)$
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- Previous results of box diagram

- Argument on V-V interaction

- A chiral covariant approach to $\rho\rho$ scattering (Eur. Phys. J. C 77 (2017) 7, 460)
- Interactions between vector mesons and dynamically generated resonances (Eur. Phys. J.C 78 (2018) 12, 988)
- $\rho\rho$ scattering revisited with coupled channels of pseudoscalar mesons (Phys. Rev. D 99 (2019) 9, 096014)
- Further study of $f_0(1710)$ with the coupled-channel approach and the hadron molecular picture (Phys. Rev. D 104 (2021) 11, 114001)
- Two dynamical generated a_0 resonances by interactions between vector mesons (Eur.Phys.J.C 82 (2022) 6, 509)
- On the chiral covariant approach to $\rho\rho$ scattering (Chin. Phys. C 41 (2017) 12, 124101)
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- My view
- Summary

- On shell factorization





Backup Coupling

strangeness=0 and isospin=0						
	pole position	$K^* \bar{K}^*$	$\rho\rho$	$\omega\omega$	$\omega\phi$	$\phi\phi$
$f_2(1270)$	(1275, $-i$)	(4733, $-53i$)	(10889, $-99i$)	($-440, 7i$)	(777, $-13i$)	($-675, 11i$)
$f_2'(1525)$	(1525, $-3i$)	(10121, $101i$)	($-2443, 649i$)	($-2709, 8i$)	(5016, $-17i$)	($-4615, 17i$)
$f_0(1710)$	(1726, $-14i$)	(7124, $96i$)	($-1030, 1086i$)	($-1763, 108i$)	(3010, $-210i$)	($-2493, -204i$)
$f_0(1500)$	(1512, $-26i$)	(1208, $-419i$)	(7906, $-1084i$)	($-40, i30$)	(34, $-42i$)	(12, $24i$)
strangeness=0 and isospin=1						
	pole position	$K^* \bar{K}^*$	$\rho\omega$	$\rho\phi$		
$a_0(1710)$	(1780, $-66i$)	(7526, $-1525i$)	($-4042, 1389i$)	(4998, $-1869i$)		
strangeness=1 and isospin=1/2						
	pole position	ρK^*	$K^* \omega$	$K^* \phi$		
$K_2^*(1430)$	(1431, $-i$)	(10901, $-71i$)	(2267, $-13i$)	($-2898, 17i$)		

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Backup Data from RPP

$f_2(1270)$

$\Gamma(f_2(1270) \rightarrow \pi\pi)$ Γ_1 —				
VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
157.2^{+5.0}_{-1.0}		OUR FIT		
157.0 ^{+6.0} _{-1.0}		¹ LONGACRE	1986 MPS	22 $\pi^- p \rightarrow n 2 K_S^0$
• • We do not use the following data for averages, fits, limits, etc. • •				
152 ± 8	870	² SCHEGELSKY	2006A RVUE	$\gamma\gamma \rightarrow K_S^0 K_S^0$

$\Gamma(f_2(1270) \rightarrow K\bar{K})$ Γ_3 —				
VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
8.5 ± 0.8		OUR FIT		Error includes scale factor of 2.8.
9.0 ^{+0.7} _{-0.3}		¹ LONGACRE	1986 MPS	22 $\pi^- p \rightarrow n 2 K_S^0$
• • We do not use the following data for averages, fits, limits, etc. • •				
7.5 ± 2.0	870	² SCHEGELSKY	2006A RVUE	$\gamma\gamma \rightarrow K_S^0 K_S^0$

$\Gamma(f_2(1270) \rightarrow \eta\eta)$ Γ_5 —				
VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
0.75 ± 0.14		OUR FIT		Error includes scale factor of 2.1.
1.0 ± 0.1		¹ LONGACRE	1986 MPS	22 $\pi^- p \rightarrow n 2 K_S^0$
• • We do not use the following data for averages, fits, limits, etc. • •				
1.8 ± 0.4	870	² SCHEGELSKY	2006A RVUE	$\gamma\gamma \rightarrow K_S^0 K_S^0$



Backup Data from RPP

$\Gamma(f_2(1270) \rightarrow \pi\pi)/\Gamma_{\text{total}}$

Γ_1/Γ

VALUE	EVTS	DOCUMENT ID	TECN	COMMENT
$0.843^{+0.028}_{-0.010}$		OUR FIT		Error includes scale factor of 1.2.
0.837 ± 0.020		OUR AVERAGE		
0.849 ± 0.025		CHABAUD	1983	ASPK 17 $\pi^- p$ polarized
0.85 ± 0.05	250	BEAUPRE	1971	HBC 8 $\pi^+ p \rightarrow \Delta^{++} f_2$
0.8 ± 0.04	600	OH	1970	HBC 1.26 $\pi^- p \rightarrow \pi^+ \pi^- n$
• • We do not use the following data for averages, fits, limits, etc. • •				
$0.856 \pm 0.001 \pm 0.05$		¹ ALBRECHT	2020	RVUE 0.9 $\bar{p} p \rightarrow \pi^0 \pi^0 \eta, \pi^0 \eta \eta, \pi^0 K^+ K^-$



Backup Data from RPP

$\Gamma(f_2(1270) \rightarrow K\bar{K})/\Gamma_{\text{total}}$

Γ_3/Γ

VALUE	DOCUMENT ID	TECN	COMMENT
• • We do not use the following data for averages, fits, limits, etc. • •			
$0.033 \pm 0.001 \pm 0.005$	¹ ALBRECHT	2020	RVUE 0.9 $\bar{p} p \rightarrow \pi^0 \pi^0 \eta, \pi^0 \eta \eta, \pi^0 K^+ K^-$

$\Gamma(f_2(1270) \rightarrow \eta\eta)/\Gamma_{\text{total}}$

Γ_5/Γ

VALUE (10 ⁻³)	DOCUMENT ID	TECN	COMMENT
4.0 ± 0.8	OUR FIT	Error includes scale factor of 2.1.	
2.9 ± 0.5	OUR AVERAGE		
2.7 ± 0.7	BINON	2005	GAMS 33 $\pi^- p \rightarrow \eta \eta n$
2.8 ± 0.7	ALDE	1986D	GAM4 100 $\pi^- p \rightarrow 2 \eta n$
5.2 ± 1.7	BINON	1983	GAM2 38 $\pi^- p \rightarrow 2 \eta n$
• • We do not use the following data for averages, fits, limits, etc. • •			
$4.0 \pm 1.0 \pm 2.0$	¹ ALBRECHT	2020	RVUE 0.9 $\bar{p} p \rightarrow \pi^0 \pi^0 \eta, \pi^0 \eta \eta, \pi^0 K^+ K^-$



Backup Data from RPP

$$\Gamma(f_2(1270) \rightarrow K\bar{K})/\Gamma(f_2(1270) \rightarrow \pi\pi)$$

 Γ_3/Γ_1

We average only experiments which either take into account $f_2(1270) - a_2(1320)$ interference explicitly or demonstrate that $a_2(1320)$ production is negligible.

VALUE	EVTS	DOCUMENT ID	TECN	COMMENT	
0.054^{+0.005}_{-0.006}	OUR FIT	Error includes scale factor of 2.7.			
0.041^{+0.004}_{-0.005}	OUR AVERAGE				
0.045 ± 0.01		¹ BARGIOTTI	2003	OBLX	$\bar{p}p$
0.037 ^{+0.008} _{-0.021}		ETKIN	1982B	MPS	23 $\pi^- p \rightarrow n 2 K_S^0$
0.045 ± 0.009		CHABAUD	1981	ASPK	17 $\pi^- p$ polarized
0.039 ± 0.008		LOVERRE	1980	HBC	4 $\pi^- p \rightarrow K\bar{K}N$
• • We do not use the following data for averages, fits, limits, etc. • •					
0.052 ± 0.025		ABLIKIM	2004E	BES2	$J/\psi \rightarrow \omega K^+ K^-$
0.036 ± 0.005		² COSTA	1980	OMEG	1-2.2 $\pi^- p \rightarrow K^+ K^- n$
0.030 ± 0.005		³ MARTIN	1979	RVUE	
0.027 ± 0.009		⁴ POLYCHRONAKOS	1979	STRC	7 $\pi^- p \rightarrow n 2 K_S^0$
0.025 ± 0.015		EMMS	1975D	DBC	4 $\pi^+ n \rightarrow p f_2$
0.031 ± 0.012	20	ADERHOLZ	1969	HBC	8 $\pi^+ p \rightarrow K^+ K^- \pi^+ p$

¹ Coupled channel analysis of $\pi^+\pi^-\pi^0$, $K^+K^-\pi^0$ and $K^+K^0\pi^+$



Backup Data from RPP

$\Gamma(f_2(1270) \rightarrow \eta\eta)/\Gamma(f_2(1270) \rightarrow \pi\pi)$

Γ_5/Γ_1

VALUE	CL%	DOCUMENT ID	TECN	COMMENT
0.003 ± 0.001		BARBERIS	2000E	450 $p p \rightarrow p_f \eta \eta p_s$
• • We do not use the following data for averages, fits, limits, etc. • •				
< 0.05	95	EDWARDS	1982F	CBAL $e^+ e^- \rightarrow e^+ e^- 2 \eta$
< 0.016	95	EMMS	1975D	DBC $4 \pi^+ n \rightarrow p f_2$
< 0.09	95	EISENBERG	1974	HBC $4.9 \pi^+ p \rightarrow \Delta^{++} f_2$

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Backup Data from RPP

$f_2'(1525)$

$\Gamma(f_2'(1525) \rightarrow K\bar{K})$					Γ_1
VALUE (MeV)		DOCUMENT ID	TECN	COMMENT	
64⁺⁶₋₅	OUR FIT				
63 ⁺⁶ ₋₅		¹ LONGACRE	1986 MPS	22 $\pi^- p \rightarrow K_S^0 K_S^0 n$	

$\Gamma(f_2'(1525) \rightarrow \pi\pi)$					Γ_3
VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT	
0.59 ± 0.12					
1.4 ^{+1.0} _{-0.5}		¹ LONGACRE	1986 MPS	22 $\pi^- p \rightarrow K_S^0 K_S^0 n$	
• • We do not use the following data for averages, fits, limits, etc. • •					
0.2 ^{+1.0} _{-0.2}	870	² SCHEGELSKY	2006A RVUE	$\gamma \gamma \rightarrow K_S^0 K_S^0$	

$\Gamma(f_2'(1525) \rightarrow \eta\eta)$					Γ_2
VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT	
7.4 ± 1.9					
• • We do not use the following data for averages, fits, limits, etc. • •					
5.0 ± 0.8	870	¹ SCHEGELSKY	2006A RVUE	$\gamma \gamma \rightarrow K_S^0 K_S^0$	
24 ⁺³ ₋₁		² LONGACRE	1986 MPS	22 $\pi^- p \rightarrow K_S^0 K_S^0 n$	



Backup Data from RPP

$\Gamma(f_2'(1525) \rightarrow K\bar{K})/\Gamma_{\text{total}}$

Γ_1/Γ

VALUE

DOCUMENT ID

TECN

COMMENT

• • We do not use the following data for averages, fits, limits, etc. • •

$0.746 \pm 0.002^{+0.166}_{-0.162}$

¹ ALBRECHT

2020

RVUE

$0.9 \bar{p} p \rightarrow \pi^0 \pi^0 \eta, \pi^0 \eta \eta, \pi^0 K^+ K^-$

$\Gamma(f_2'(1525) \rightarrow \eta\eta)/\Gamma_{\text{total}}$

Γ_2/Γ

VALUE

DOCUMENT ID

TECN

COMMENT

• • We do not use the following data for averages, fits, limits, etc. • •

$0.059 \pm 0.003 \pm 0.026$

¹ ALBRECHT

2020

RVUE

$0.9 \bar{p} p \rightarrow \pi^0 \pi^0 \eta, \pi^0 \eta \eta, \pi^0 K^+ K^-$

seen

UEHARA

2010A

BELL

$10.6 e^+ e^- \rightarrow e^+ e^- \eta \eta$

0.10 ± 0.03

² PROKOSHKIN

1991

GAM4

$300 \pi^- p \rightarrow \pi^- p \eta \eta$



Backup Data from RPP

$\Gamma(f_2'(1525) \rightarrow \pi\pi)/\Gamma_{\text{total}}$					Γ_3/Γ
VALUE (10^{-2})	CL%	DOCUMENT ID	TECN	COMMENT	
0.82 ± 0.16		OUR FIT			
0.75 ± 0.16		OUR AVERAGE			
0.7 ± 0.2		COSTA	1980	OMEG	10 $\pi^- p \rightarrow K^+ K^- n$
2.7 $^{+7.1}_{-1.3}$		¹ GORLICH	1980	ASPK	17,18 $\pi^- p$
0.75 ± 0.25		^{1,2} MARTIN	1979	RVUE	
• • We do not use the following data for averages, fits, limits, etc. • •					
3.4 ± 1.5 ± 1.0		³ ALBRECHT	2020	RVUE	0.9 $\bar{p} p \rightarrow \pi^0 \pi^0 \eta, \pi^0 \eta \eta, \pi^0 K^+ K^-$
< 6	95	AGUILAR-BENIT..	1981B	HBC	4.2 $K^- p \rightarrow \Lambda K^+ K^-$
19 ± 3		CORDEN	1979	OMEG	12–15 $\pi^- p \rightarrow \pi^+ \pi^- n$
< 4.5	95	BARREIRO	1977	HBC	4.15 $K^- p \rightarrow \Lambda K_S^0 K_S^0$
1.2 ± 0.4		¹ PAWLICKI	1977	SPEC	6 $\pi N \rightarrow K^+ K^- N$
< 6.3	90	BRANDENBURG	1976C	ASPK	13 $K^- p \rightarrow K^+ K^- (\Lambda, \Sigma)$
< 0.86		¹ BEUSCH	1975B	OSPK	8.9 $\pi^- p \rightarrow K^0 \bar{K}^0 n$



Backup Data from RPP

$$\Gamma(f_2'(1525) \rightarrow \pi\pi) / \Gamma(f_2'(1525) \rightarrow K\bar{K})$$

 Γ_3/Γ_1

VALUE	DOCUMENT ID	TECN	COMMENT
0.0092 ± 0.0018	OUR FIT		
0.075 ± 0.035	AUGUSTIN	1987 DM2	$J/\psi \rightarrow \gamma\pi^+\pi^-$

$$\Gamma(f_2'(1525) \rightarrow \eta\eta) / \Gamma(f_2'(1525) \rightarrow K\bar{K})$$

 Γ_2/Γ_1

VALUE	CL%	EVTS	DOCUMENT ID	TECN	COMMENT
0.116 ± 0.028			OUR FIT		
0.115 ± 0.028			OUR AVERAGE		
0.119 ± 0.015 ± 0.036		61	¹ BINON	2007 GAMS	32.5 $K^- p \rightarrow \eta\eta(\Lambda / \Sigma^0)$
0.11 ± 0.04			² PROKOSHKIN	1991 GAM4	300 $\pi^- p \rightarrow \pi^- p\eta\eta$
• • We do not use the following data for averages, fits, limits, etc. • •					
< 0.14	90		BARBERIS	2000E	450 $p p \rightarrow p f\eta\eta p_s$
< 0.50			BARNES	1967 HBC	4.6,5.0 $K^- p$

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Backup Data from RPP

$K_2^*(1430)$

$\Gamma(K_2^*(1430) \rightarrow K\pi)/\Gamma_{\text{total}}$

Γ_1/Γ

	DOCUMENT ID	TECN	CHG	COMMENT
0.499 ± 0.012	OUR FIT			
0.488 ± 0.014	OUR AVERAGE			
0.485 ± 0.006 ± 0.020		¹ ASTON	1988	LASS 0 11 $K^- p \rightarrow K^- \pi^+ n$
0.49 ± 0.02		¹ ESTABROOKS	1978	ASPK ± 13 $K^\pm p \rightarrow pK\pi$

$\Gamma(K_2^*(1430) \rightarrow K\eta)/\Gamma(K_2^*(1430) \rightarrow K^*(892)\pi)$

Γ_7/Γ_2

	DOCUMENT ID	TECN	CHG	COMMENT
0.006^{+0.014}_{-0.004}	OUR FIT Error includes scale factor of 1.2.			
0.07 ± 0.04		FIELD	1967	HBC - 3.8 $K^- p$



Backup Data from RPP

$\Gamma(K_2^*(1430) \rightarrow K\eta)/\Gamma(K_2^*(1430) \rightarrow K\pi)$ Γ_7/Γ_1 —

CL%	DOCUMENT ID	TECN	CHG	COMMENT
$0.0030^{+0.0070}_{-0.0020}$	OUR FIT	Error includes scale factor of 1.3.		
0 ± 0.0056		¹ ASTON	1988B LASS	- 11 $K^- p \rightarrow K^- \eta p$
• • We do not use the following data for averages, fits, limits, etc. • •				
< 0.04	95	AGUILAR-BENIT..	1971B HBC	3.9,4.6 $K^- p$
< 0.065		² BASSOMPIERRE	1969 HBC	5.0 $K^+ p$
< 0.02		BISHOP	1969 HBC	3.5 $K^+ p$

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Backup Data from RPP

$f_0(1710)$

$\Gamma(f_0(1710) \rightarrow K\bar{K})/\Gamma_{\text{total}}$ Γ_1/Γ —

VALUE	EVTS	DOCUMENT ID	TECN	COMMENT
• • We do not use the following data for averages, fits, limits, etc. • •				
seen	1004	¹ DOBBS	2015	$J/\psi \rightarrow \gamma K^+ K^-$
seen	349	¹ DOBBS	2015	$\psi(2S) \rightarrow \gamma K^+ K^-$
0.36 ± 0.12		ALBALADEJO	2008	RVUE
$0.38^{+0.09}_{-0.19}$		² LONGACRE	1986	MPS $22 \pi^- p \rightarrow n 2 K_S^0$

$\Gamma(f_0(1710) \rightarrow \eta\eta)/\Gamma_{\text{total}}$ Γ_2/Γ —

VALUE	DOCUMENT ID	TECN
• • We do not use the following data for averages, fits, limits, etc. • •		
0.22 ± 0.12	ALBALADEJO	2008
$0.18^{+0.03}_{-0.13}$	¹ LONGACRE	1986

$\Gamma(f_0(1710) \rightarrow \pi\pi)/\Gamma_{\text{total}}$ Γ_4/Γ —

VALUE	EVTS	DOCUMENT ID	TECN	COMMENT
• • We do not use the following data for averages, fits, limits, etc. • •				
seen	381	¹ DOBBS	2015	$J/\psi \rightarrow \gamma \pi^+ \pi^-$
seen	237	¹ DOBBS	2015	$\psi(2S) \rightarrow \gamma \pi^+ \pi^-$
not seen		AMSLER	2002	CBAR $0.9 \bar{p} p \rightarrow \pi^0 \eta \eta, \pi^0 \pi^0 \pi^0$
$0.039^{+0.002}_{-0.024}$		² LONGACRE	1986	RVUE



Backup Data from RPP

$\Gamma(f_0(1710) \rightarrow \pi\pi)/\Gamma(f_0(1710) \rightarrow K\bar{K})$

Γ_4/Γ_1

VALUE	CL%	DOCUMENT ID	TECN	COMMENT
0.23 ± 0.05		OUR AVERAGE		Error includes scale factor of 1.2.
0.64 ± 0.27 ± 0.18		LEES	2018A	BABR $\Upsilon(1S) \rightarrow \gamma\pi^+\pi^-, \gamma K^+K^-$
0.41 $^{+0.11}_{-0.17}$		ABLIKIM	2006V	BES2 $e^+e^- \rightarrow J/\psi \rightarrow \gamma\pi^+\pi^-$
0.2 ± 0.024 ± 0.036		BARBERIS	1999D	OMEG $450 p p \rightarrow K^+K^-, \pi^+\pi^-$
0.39 ± 0.14		ARMSTRONG	1991	OMEG $300 p p \rightarrow pp\pi\pi, ppK\bar{K}$

• • We do not use the following data for averages, fits, limits, etc. • •

0.32 ± 0.14		ALBALADEJO	2008	RVUE	
< 0.11	95	¹ ABLIKIM	2004E	BES2	$J/\psi \rightarrow \omega K^+K^-$
5.8 $^{+9.1}_{-5.5}$		² ANISOVICH	2002D	SPEC	Combined fit

¹ Using data from ABLIKIM 2004A.



Backup Data from RPP

$\Gamma(f_0(1710) \rightarrow \eta\eta)/\Gamma(f_0(1710) \rightarrow K\bar{K})$

Γ_2/Γ_1

VALUE	CL%	DOCUMENT ID	TECN	COMMENT
0.48 ±0.15		BARBERIS	2000E	450 $p p \rightarrow p_f \eta \eta p_s$
• • We do not use the following data for averages, fits, limits, etc. • •				
0.46 ^{+0.70} _{-0.38}		¹ ANISOVICH	2002D SPEC	Combined fit
< 0.02	90	² PROKOSHKIN	1991 GA24	300 $\pi^- p \rightarrow \pi^- p \eta \eta$

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Backup Data from RPP

$f_0(1500)$

<i>Mode</i>	<i>Fraction (Γ_i / Γ)</i>	<i>Scale Factor/ Conf. Level</i>	<i>P(MeV/c)</i>
$\Gamma_1 \quad f_0(1500) \rightarrow \pi\pi$	$(34.5 \pm 2.2) \times 10^{-2}$	S=1.2	749

$\Gamma(f_0(1500) \rightarrow \pi\pi) / \Gamma_{total}$ Γ_1 / Γ —

<i>VALUE</i>	<i>DOCUMENT ID</i>	<i>TECN</i>
• • We do not use the following data for averages, fits, limits, etc. • •		
0.454 ± 0.104	BUGG	1996 RVUE

References ▾



Backup Data from RPP

<i>Mode</i>	<i>Fraction (Γ_i / Γ)</i>	<i>Scale Factor/ Conf. Level</i>	<i>P(MeV/c)</i>
$\Gamma_{13} \quad f_0(1500) \rightarrow K\bar{K}$	$(8.5 \pm 1.0) \times 10^{-2}$	S=1.1	579

$\Gamma(f_0(1500) \rightarrow K\bar{K}) / \Gamma_{\text{total}}$			Γ_{13} / Γ
<i>VALUE</i>	<i>DOCUMENT ID</i>	<i>TECN</i>	
• • We do not use the following data for averages, fits, limits, etc. • •			
0.044 ± 0.021	BUGG	1996	RVUE



Backup Data from RPP

$f_0(1500)$

<i>Mode</i>	<i>Fraction (Γ_i / Γ)</i>	<i>Scale Factor/ Conf. Level</i>	<i>P(MeV/c)</i>
$\Gamma_{11} \quad f_0(1500) \rightarrow \eta\eta$	$(6.0 \pm 0.9) \times 10^{-2}$	S=1.1	528

$\Gamma(f_0(1500) \rightarrow \eta\eta) / \Gamma_{\text{total}}$ Γ_{11} / Γ —

<i>VALUE</i>	<i>DOCUMENT ID</i>	<i>TECN</i>	<i>COMMENT</i>
• • We do not use the following data for averages, fits, limits, etc. • •			
large	ALDE	1988 GAM4	300 $\pi^- N \rightarrow \eta\eta\pi^- N$
large	BINON	1983 GAM2	38 $\pi^- p \rightarrow 2 \eta n$



Backup Data from RPP

$\Gamma(f_0(1500) \rightarrow K\bar{K})/\Gamma(f_0(1500) \rightarrow \pi\pi)$

Γ_{13}/Γ_1

VALUE	DOCUMENT ID	TECN	COMMENT
0.246 ± 0.025	OUR FIT		
0.236 ± 0.026	OUR AVERAGE		
0.25 ± 0.03	¹ BARGIOTTI	2003 OBLX	$\bar{p}p$
0.19 ± 0.07	² ABELE	1998 CBAR	0.0 $\bar{p}p \rightarrow K_L^0 K^\pm \pi^\mp$
0.20 ± 0.08	³ ABELE	1996B CBAR	0.0 $\bar{p}p \rightarrow \pi^0 K_L^0 K_L^0$
• • We do not use the following data for averages, fits, limits, etc. • •			
0.16 ± 0.05	⁴ ANISOVICH	2002D SPEC	Combined fit
0.33 ± 0.03 ± 0.07	BARBERIS	1999D OMEG	450 $pp \rightarrow K^+ K^-, \pi^+ \pi^-$



Backup

Data from RPP

$$\Gamma(f_0(1500) \rightarrow \eta\eta) / \Gamma(f_0(1500) \rightarrow \pi\pi)$$

 Γ_{11}/Γ_1

VALUE	DOCUMENT ID	TECN	COMMENT
0.173 ± 0.024	OUR FIT		Error includes scale factor of 1.1.
0.175 ± 0.027	OUR AVERAGE		
0.18 ± 0.03	BARBERIS	2000E	450 $p p \rightarrow p_f \eta \eta p_s$
0.157 ± 0.060	¹ AMSLER	1995D	CBAR 0.0 $\bar{p} p \rightarrow \pi^0 \pi^0 \pi^0, \pi^0 \eta \eta, \pi^0 \pi^0 \eta$
• • We do not use the following data for averages, fits, limits, etc. • •			
0.080 ± 0.033	AMSLER	2002	CBAR 0.9 $\bar{p} p \rightarrow \pi^0 \eta \eta, \pi^0 \pi^0 \pi^0$
0.11 ± 0.03	² ANISOVICH	2002D	SPEC Combined fit
0.078 ± 0.013	³ ABELE	1996C	RVUE Compilation
0.230 ± 0.097	⁴ AMSLER	1995C	CBAR 0.0 $\bar{p} p \rightarrow \eta \eta \pi^0$



Backup Data from RPP

$\Gamma(f_0(1500) \rightarrow K\bar{K})/\Gamma(f_0(1500) \rightarrow \eta\eta)$

Γ_{13}/Γ_{11}

VALUE	CL%	DOCUMENT ID	TECN	COMMENT
1.43 ± 0.24	OUR FIT	Error includes scale factor of 1.1.		
1.85 ± 0.41		BARBERIS	2000E	450 $p p \rightarrow p_f \eta \eta p_s$
• • We do not use the following data for averages, fits, limits, etc. • •				
1.5 ± 0.6		¹ ANISOVICH	2002D	SPEC Combined fit
< 0.4	90	² PROKOSHKIN	1991	GAM4 300 $\pi^- p \rightarrow \pi^- p \eta \eta$
< 0.6		³ BINON	1983	GAM2 38 $\pi^- p \rightarrow 2 \eta n$

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Backup Data from RPP

$f_0(1370)$

$\Gamma(f_0(1370) \rightarrow \pi\pi)/\Gamma_{\text{total}}$ Γ_1/Γ —

VALUE	CL%	DOCUMENT ID	TECN	COMMENT
• • We do not use the following data for averages, fits, limits, etc. • •				
< 0.10	95	OCHS	2013	RVUE
0.26 ± 0.09		BUGG	1996	RVUE
< 0.15		¹ AMSLER	1994	CBAR $\bar{p} p \rightarrow \pi^+ \pi^- 3 \pi^0$
< 0.06		GASPERO	1993	DBC $0.0 \bar{p} n \rightarrow \text{hadrons}$

$\Gamma(f_0(1370) \rightarrow K\bar{K})/\Gamma_{\text{total}}$ Γ_{11}/Γ —

VALUE	DOCUMENT ID	TECN
• • We do not use the following data for averages, fits, limits, etc. • •		
0.35 ± 0.13	BUGG	1996 RVUE



Backup Data from RPP

$\Gamma(f_0(1370) \rightarrow K\bar{K})/\Gamma(f_0(1370) \rightarrow \pi\pi)$

Γ_{11}/Γ_1

VALUE

DOCUMENT ID

TECN

COMMENT

• • We do not use the following data for averages, fits, limits, etc. • •

0.08 ±0.08

[ABLIKIM](#)

[2005](#)

BES2

$J/\psi \rightarrow \phi\pi^+\pi^-, \phi K^+K^-$

0.91 ±0.20

¹ [BARGIOTTI](#)

[2003](#)

OBLX

$\bar{p}p$

0.12 ±0.06

² [ANISOVICH](#)

[2002D](#)

SPEC

Combined fit

0.46 ±0.15 ±0.11

[BARBERIS](#)

[1999D](#)

OMEG

450 $p p \rightarrow K^+K^-, \pi^+\pi^-$

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Backup

Previous results of box diagram

PoS EFT09 (2009) 040

Our treatment of the box amplitudes enables us to obtain the decay branching ratios of the generated states into two pseudoscalar mesons using the real-axis method in the following way:

- First, we calculate the width of the selected state with and without the contributions of the box diagrams, $\Gamma(\text{total}) = \Gamma(\text{VV}) + \Gamma(\text{PP})$ and $\Gamma(\text{VV})$.
- Second, we estimate the partial decay width into a particular two-pseudoscalar channel of the state by including only the contribution of the particular channel. Taking the $\pi\pi$ channel as an example, this way we obtain $\Gamma(\text{w}\pi\pi)$. The contribution of the $\pi\pi$ channel is then determined as $\Gamma(\pi\pi) = \Gamma(\text{w}\pi\pi) - \Gamma(\text{VV})$ and its branching ratio is calculated as $\Gamma(\pi\pi)/\Gamma(\text{total})$. The partial decay branching ratios into other two-pseudoscalar channels are calculated similarly. It should be noted that we have assumed that interference between contributions of different channels is small, which seems to be justified since the sum of the calculated partial decay widths agrees well with the total decay width.



Backup

Previous results of box diagram

$f_2(1270)$

PoS EFT09 (2009) 040	
$\Gamma(\pi\pi)/\Gamma(total)$	$\sim 88\%$
$\Gamma(K\bar{K})/\Gamma(total)$	$\sim 10\%$
$\Gamma(\eta\eta)/\Gamma(total)$	$< 10\%$
$\Gamma(VV)/\Gamma(total)$	$< 1\%$

$f_2'(1525)$

PoS EFT09 (2009) 040	
$\Gamma(\pi\pi)/\Gamma(total)$	$< 1\%$
$\Gamma(K\bar{K})/\Gamma(total)$	$\sim 66\%$
$\Gamma(\eta\eta)/\Gamma(total)$	$\sim 21\%$
$\Gamma(VV)/\Gamma(total)$	$\sim 13\%$

$K_2^*(1430)$

PoS EFT09 (2009) 040	
$\Gamma(K\pi)/\Gamma(total)$	$\sim 93\%$
$\Gamma(K\eta)/\Gamma(total)$	$\sim 5\%$
$\Gamma(VV)/\Gamma(total)$	$\sim 2\%$



Backup

Previous results of box diagram

$f_0(1710)$

PoS EFT09 (2009) 040	
$\Gamma(\pi\pi)/\Gamma(total)$	< 1%
$\Gamma(K\bar{K})/\Gamma(total)$	~ 55%
$\Gamma(\eta\eta)/\Gamma(total)$	~ 27%
$\Gamma(VV)/\Gamma(total)$	~ 18%

$a_0(1710)$

PoS EFT09 (2009) 040	
$\Gamma(K\bar{K})/\Gamma(total)$	~ 27%
$\Gamma(\pi\eta)/\Gamma(total)$	~ 23%
$\Gamma(VV)/\Gamma(total)$	~ 50%

f_0

PoS EFT09 (2009) 040	
$\Gamma(\pi\pi)/\Gamma(total)$	~ 72%
$\Gamma(K\bar{K})/\Gamma(total)$	~ 10%
$\Gamma(\eta\eta)/\Gamma(total)$	< 1%
$\Gamma(VV)/\Gamma(total)$	~ 18%

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Backup

Argument on V-V interaction

A chiral covariant approach to $\rho\rho$ scattering

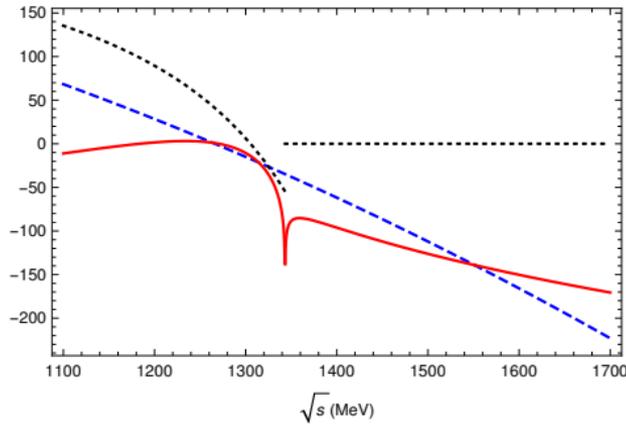
Eur. Phys. J. C 77 (2017) 7, 460

Left hand cut no $f_2(1270)$

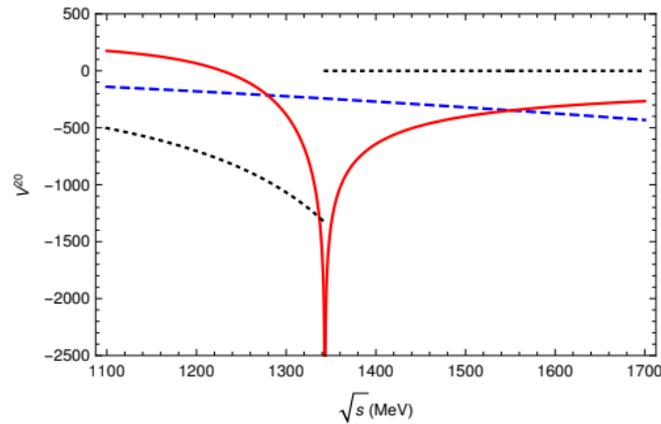
$$T_{\ell S; \bar{\ell} \bar{S}}^{(JI)}(s) = \frac{Y_{\ell}^0(\hat{\mathbf{z}})}{2(2J+1)} \sum_{\substack{\sigma_1, \sigma_2, \bar{\sigma}_1 \\ \bar{\sigma}_2, m}} \int d\mathbf{p}'' Y_{\ell}^m(\mathbf{p}'')^* (\sigma_1 \sigma_2 M | s_1 s_2 S) (m M \bar{M} | \ell S J) (\bar{\sigma}_1 \bar{\sigma}_2 \bar{M} | \bar{s}_1 \bar{s}_2 \bar{S}) (0 \bar{M} \bar{M} | \bar{\ell} \bar{S} J) \\ \times T^{(I)}(p_1, p_2, p_3, p_4; \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4), \quad (17)$$

with s the usual Mandelstam variable, $\mathbf{p}_1 = |\mathbf{p}| \hat{\mathbf{z}}$, $\mathbf{p}_2 = -|\mathbf{p}| \hat{\mathbf{z}}$, $\mathbf{p}_3 = \mathbf{p}''$ and $\mathbf{p}_4 = -\mathbf{p}''$, $M = \sigma_1 + \sigma_2$ and $\bar{M} =$

$$T^{(JI)}(s) = \left[I - V^{(JI)}(s) \cdot G(s) \right]^{-1} \cdot V^{(JI)}(s).$$



(a) $V^{(00)}(s)$



(b) $V^{(20)}(s)$



Backup

Argument on V-V interaction

According to the N/D method [54] a partial-wave amplitude can be written as

$$T = \frac{N(s)}{D(s)}, \quad (34)$$

where the function $D(s)$ has only the unitarity or right-hand cut (RHC) while $N(s)$ only has the left-hand cut (LHC). The secular equation for obtaining resonances and bound states corresponds to look for the zeros of $D(s)$,

$$D(s_i) = 0. \quad (35)$$

Below threshold along the real s axis this equation is purely real because $D(s)$ has a non-vanishing imaginary part only for $s > s_{\text{th}}$.

However, with our unitarization procedure from leading-order unitary chiral perturbation theory (UChPT) we have obtained the approximation

$$T^{(JI)}(s) = \frac{V^{(JI)}(s)}{1 - V^{(JI)}(s)g_c(s)}, \quad (36)$$

and the resulting equation to look for the bound states is

$$D_U(s) = 1 - V(s)g_c(s) = 0. \quad (37)$$

Notice that Eq. (36), contrary to the general Eq. (35), has an imaginary part below the branch-point singularity at $s = 3m^2$.



Backup

Argument on V-V interaction

$$N(s) = V^{(JI)}(s) ,$$

$$D(s) = \gamma_0 + \gamma_1(s - s_{th}) + \frac{1}{2}\gamma_2(s - s_{sth})^2 + \frac{(s - s_{th})s^2}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\rho(s')V^{(JI)}(s')}{(s' - s_{th})(s' - s)(s')^2} ,$$

$$T_{ND}(s) = \frac{N(s)}{D(s)} ,$$

$$\begin{aligned} \gamma_0 + \gamma_1(s - s_{th}) + \frac{1}{2}\gamma_2(s - s_{sth})^2 &= 1 - V(s)g_c(s) - \frac{(s - s_{th})s^2}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\rho(s')V(s')}{(s' - s_{th})(s' - s)(s')^2} \\ &\equiv \omega(s) . \end{aligned}$$

In this way,

$$\gamma_0 = 1 - V(s_{th})g_c(s_{th}) ,$$

$$\gamma_1 = \omega'(s_{th}) ,$$

$$\gamma_2 = \omega''(s_{th}) .$$



Backup

Argument on V-V interaction

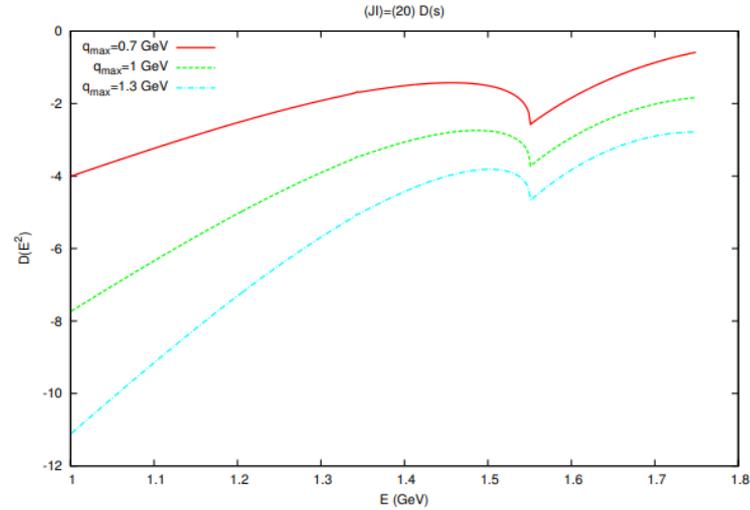


Figure 10: $D(s)$ function, Eq. (38), for $(J, I) = (2, 0)$. Above threshold only the real part is shown.

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Backup

Argument on V-V interaction

Interactions between vector mesons and dynamically generated resonances

Eur. Phys. J. C 78 (2018) 12, 988

S	$I^G(J^{PC})$	Pole [GeV]	PDG	Mass [GeV] [29]	Pole [GeV] [45]
0	$0^+(0^{++})$	[1.41 – 1.50]	$f_0(1370)$	[1.2 – 1.5]	1.512
			$f_0(1500)$	1.504 ± 0.006	
	$0^+(0^{++})$	[1.58 – 1.76]	$f_0(1710)$	$1.723^{+0.006}_{-0.005}$	1.726
	$0^-(1^{+-})$	[1.77 – 1.78]	–	–	1.802
	$1^-(0^{++})$	[1.75 – 1.79]	$a_0(1950)?$	1.931 ± 0.026	1.780
	$1^+(1^{+-})$	[1.44 – 1.50]	–	–	1.679
1	$1/2(0^+)$	[1.58 – 1.66]	–	–	1.643
2	$0(1^+)$	[1.68 – 1.74]	–	–	–

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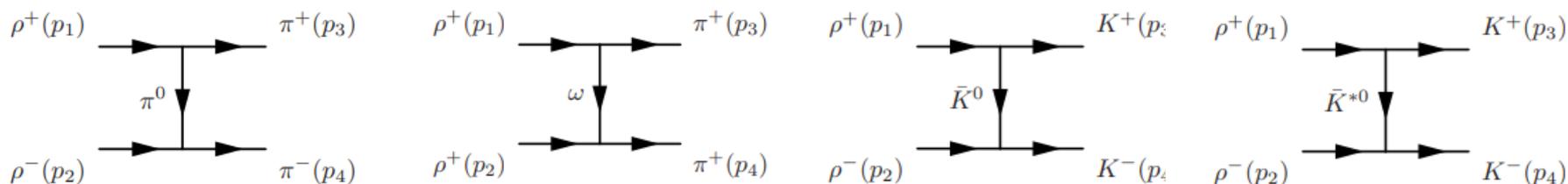
Backup

Argument on V-V interaction

$\rho\rho$ scattering revisited with coupled-channels of pseudoscalar mesons

Phys. Rev. D 99 (2019) 9, 096014

$f_0(1500)$ rather than $f_0(1370)$



$q_{max}(GeV)$	0.875	1.0	1.2	1.4
$\rho\rho$ only	1494.8	1467.2	1427.3	1395.0
π	1530.0 - 4.9i	1519.5 - 8.4i	1501.5 - 12.3i	1488.6 - 14.6i
ω	1492.2 - 0.7i	1466.5 - 1.0i	1428.1 - 1.1i	1400.0 - 1.1i
K	1497.8 - 3.3i	1473.9 - 4.1i	1437.2 - 4.4i	1410.0 - 4.2i
K^*	1489.6 - 0.5i	1463.3 - 0.5i	1424.5 - 0.4i	1396.1 - 0.3i
3-channels	1529.8 - 4.9i	1519.0 - 8.6i	1500.9 - 13.5i	1488.4 - 16.7i

Table 1: Pole position for coupled-channels

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Argument on V-V interaction

Further study of $f_0(1710)$ with coupled-channel approach and hadron molecular picture

Phys. Rev. D 104 (2021) 11, 114001

$q_{max}(GeV)$	0.7	0.8	0.9	1.0	1.1
$Pole(GeV)$	$1.77 - 0.015i$	$1.75 - 0.028i$	$1.73 - 0.035i$	$1.72 - 0.045i$	$1.70 - 0.053i$

Table 3: The resonance pole for different cutoffs

$$\frac{\Gamma(f_0(1710) \rightarrow \pi\pi)}{\Gamma(f_0(1710) \rightarrow K\bar{K})} = 0.289 \pm 0.092 (0.23 \pm 0.05),$$
$$\frac{\Gamma(f_0(1710) \rightarrow \eta\eta)}{\Gamma(f_0(1710) \rightarrow K\bar{K})} = 0.294 \pm 0.048 (0.48 \pm 0.15).$$

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Backup

Argument on V-V interaction

Two dynamical generated a_0 resonances by interactions between vector mesons

Eur. Phys. J. C 82 (2022) 6, 509

$q_{max}(GeV)$	0.9	1.0	1.1
$Pole(GeV)$	$1.76 - 0.09i$	$1.72 - 0.10i$	$1.69 - 0.11i$

Table 3: The resonance pole for different cutoffs

$\Gamma(K^*K^* \rightarrow \rho\omega)$	$\Gamma(K^*K^* \rightarrow K\bar{K})$	$\Gamma(K^*K^* \rightarrow \pi\eta)$
$61.0MeV$	$74.4MeV$	$66.9MeV$
$\Gamma(\rho\phi \rightarrow \rho\omega)$	$\Gamma(\rho\phi \rightarrow K\bar{K})$	$\Gamma(\rho\phi \rightarrow \pi\eta)$
$60.8MeV$	$74.2MeV$	$66.6MeV$

Table 4: The partial decay widths

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lower to be nearly degenerate with its isoscalar partner $f_0(1710)$. The mass is consistent with the $a_0(1710)$ newly reported by Babar Collaboration [17] and BESII Collaboration [18]. The ratio of its decays to $\rho\omega, K\bar{K}, \pi\eta$ is predicted to be about 1 : 1 : 1. With pseudo-scalar

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Argument on V-V interaction

On the chiral covariant approach to $\rho\rho$ scattering

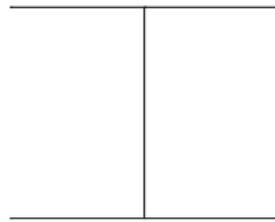
Chin. Phys. C 41 (2017) 12, 124101

$$\frac{1}{2} \int_{-1}^{+1} d \cos \theta \frac{1}{-2 \mathbf{p}^2 (1 - \cos \theta) - m_\rho^2 + i \varepsilon} = -\frac{1}{4 \mathbf{p}^2} \log \left(\frac{4 \mathbf{p}^2 + m_\rho^2}{m_\rho^2} + \frac{4 \mathbf{p}^2}{m_\rho^4} i \varepsilon \right)$$

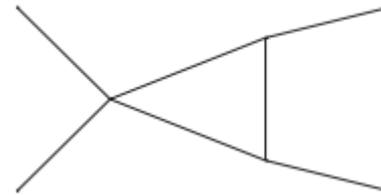
resonance $f_2(1270)$ does not appear with the potential of Eq. (8) is correct. The problem is that this is a clear situation where the on shell factorization cannot be done since the “on-shell” potential seats on top of a singularity of the extrapolated amplitude below threshold.



V_c



V_t

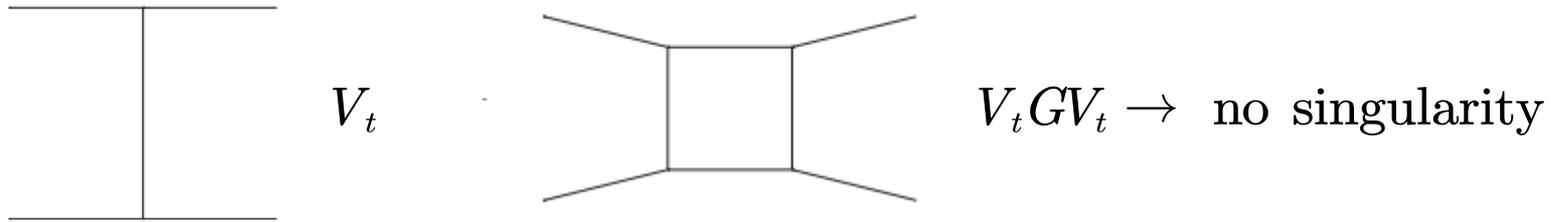


$V_c G V_t \rightarrow$ no singularity



Backup

Argument on V-V interaction



A method is developed, evaluating the loops with full ρ propagators, and we show that they do not develop singularities and do not have an imaginary part below threshold.

Chin. Phys. C 41 (2017) 12, 124101

PTEP 2019 (2019) 10, 103B05 Look back this paper.

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Comments on the dispersion relation method to vector-vector interaction

PTEP 2019 (2019) 10, 103B05

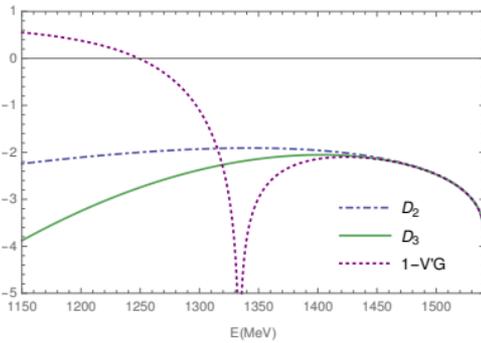


FIG. 7: The real part of the functions D_2 and D_3 from Eqs. (18) and (23) in comparison with $1 - V'G$.

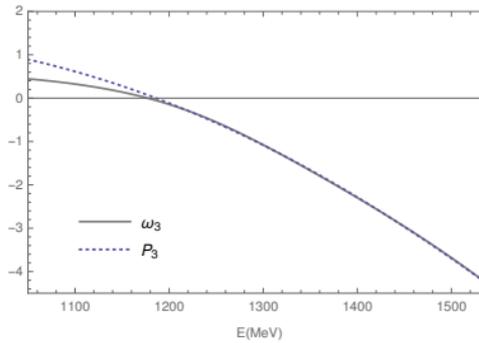


FIG. 9: The real part of the function ω_3 in comparison with P_3 from Eqs. (24), (25), and (26), with the convoluted potential V .

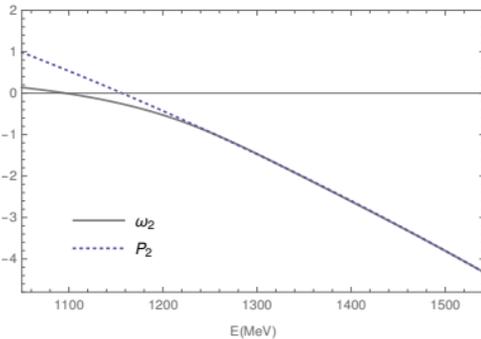


FIG. 8: The real part of the function ω_2 in comparison with P_2 , which appear in Eqs. (20), (21), and (22), with the convoluted potential V .

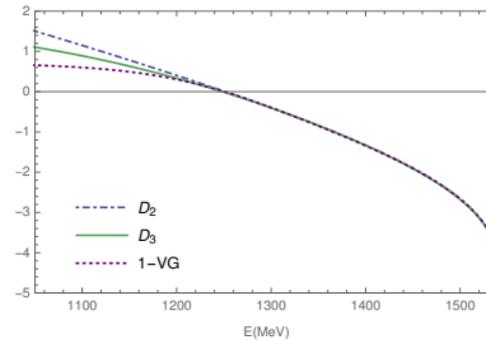


FIG. 10: The real part of the functions D_2 and D_3 from Eqs. (18) and (23) in comparison with the real part of $1 - VG$, with V the convoluted potential.

larity. Certainly, there is no convergence of the different orders in the region below the singularity and hence, neither D_2 nor D_3 , nor any higher order expansion, can be taken as a representation of a realistic D function below the singular peak. Thus, the claim that the $f_2(1270)$ does not appear from the $\rho\rho$ interaction based on the approach of Ref. [8] is not justified.

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Backup

Argument on V-V interaction

My view

- **Analytic continuation.**

In fact, these methods are different analytic continuation from above threshold (physical region) to low threshold (non-physical region).

- **On shell factorization.**

- **Width of interactive particle.**

They should be similar where near the threshold. However, argument appear on where far from threshold.

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Backup

Argument on V-V interaction

Summary

Some similar conclusion can be obtained by different methods in light meson sector:

$f_0(1710), a_0(1710)$ and a lower f_0 around 1500MeV

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Backup

On shell factorization

Nucl. Phys. A 620 (1997) 438-456

PhD thesis of Feng-Kun Guo

[34, 35] in the meson-baryon sector. Our channels are labelled 1 for the $K\bar{K}$ and 2 for the $\pi\pi$ states in $T = 0$ and 1 for $K\bar{K}$, 2 for $\pi\eta$ in $T = 1$. The coupled channel equations then become

$$\begin{aligned}t_{11} &= V_{11} + V_{11}G_{11}t_{11} + V_{12}G_{22}t_{21} \\t_{21} &= V_{21} + V_{21}G_{11}t_{11} + V_{22}G_{22}t_{21} \\t_{22} &= V_{22} + V_{21}G_{11}t_{12} + V_{22}G_{22}t_{22}\end{aligned}\tag{20}$$

$$G_{ii} = i \frac{1}{q^2 - m_{1i}^2 + i\epsilon} \frac{1}{(P - q)^2 - m_{2i}^2 + i\epsilon}\tag{21}$$

where P is the total fourmomentum of the meson-meson systems and q the fourmomentum of one intermediate meson. The terms VGT in eq. (20) actually mean

$$VGT = \int \frac{d^4q}{(2\pi)^4} V(k, p; q) G(P, q) t(q; k', p')\tag{22}$$

Note that as G_{ii} has no angular dependence and V_{ij} is purely S-wave then t_{ij} is S-wave.



Backup

On shell factorization

In principle one would have to solve the integrals in eqs. (20) by taking V and t off shell. However, this is not the case, as we show below, at least when dealing with S-wave, and we only need the on shell information. The argument goes as follows: As we can see from eqs. (18),(19), the on shell amplitudes are obtained by taking $p_i^2 = m_i^2$ and then we can write the off shell amplitudes as

$$V = V_{on} + \beta \sum_i (p_i^2 - m_i^2)$$

In order to illustrate the procedure let us simplify to one loop and one channel (the procedure is easily generalized to two channels). Hence we have

$$V^2 = V_{on}^2 + 2\beta V_{on} \sum_i (p_i^2 - m_i^2) + \beta^2 \sum_{ij} (p_i^2 - m_i^2)(p_j^2 - m_j^2) \quad (23)$$



Backup

On shell factorization

When performing the q^0 integration in the loop we have two poles, one for $q^0 = w_1(q)$ and the other one for $q^0 = P^0 + w_2(q)$, where the indices 1 and 2 indicate the two mesons inside the loop. Let us take the contribution from the first pole (the procedure follows analogously for the second pole). From the second term in eq. (23) we get the contribution

$$\frac{2\beta V_{on}}{(2\pi)^3} \int \frac{d^3q}{2w_1(q)} \frac{(P^0 - w_1)^2 - w_2^2}{(P^0 - w_1)^2 - w_2^2} = \frac{\beta V_{on}}{(2\pi)^3} \int dw_1 q \quad (24)$$

which for large Λ compared to the masses goes as $V_{on}\Lambda^2$ and has the same structure in the dynamical variables as the tree diagram. The third term in eq. (23) gives rise to the integral

$$\frac{\beta^2}{(2\pi)^3} \int \frac{d^3q}{2w_1(q)} [(P^0 - w_1)^2 - w_2^2] = \frac{\beta^2}{(2\pi)^3} \int \frac{d^3q}{2w_1(q)} [P^{02} - 2w_1 P^0 + (m_1^2 - m_2^2)] \quad (25)$$

coming from the first pole ($q^0 = w_1(q)$) and

$$\frac{\beta^2}{(2\pi)^3} \int \frac{d^3q}{2w_2(q)} [P^{02} + 2w_2 P^0 + (m_2^2 - m_1^2)]$$

coming from the second pole ($q^0 = P^0 + w_2(q)$). As we can see, the terms linear in P^0 cancel exactly, respecting the chiral structure which does not allow linear terms in P^0 .



Backup

On shell factorization

The term proportional to $P^{02} + m_1^2 - m_2^2$ in (25) leads again to a structure of the type $[P^{02} + (m_1^2 - m_2^2)]\Lambda^2$ and similarly happens with the quadratic contribution from the second pole which leads to $[P^{02} + (m_2^2 - m_1^2)]\Lambda^2$. These terms, together the $V_{on}\Lambda^2$ which we obtained before, combine with the tree level contribution, giving rise to an amplitude with the same structure as the tree level one but with renormalized parameters f and masses. However, since we are taking physical values for f ($f = f_\pi = 93 \text{ MeV}$) and the masses in the potential, these terms should be omitted. One can proceed like that to higher orders with the same conclusions.

Since we are taking V and t on shells they factorize outside the q integral. Thus the term VGT of eq. (22), after the q^0 integration is performed by choosing the contour in the lower half of the complex plane, is given by

$$V_{ij}G_{jj}t_{jk} = V_{ij}(s)t_{jk}(s)G_{jj}(s)$$
$$G_{jj}(s) = \int_0^{q_{max}} \frac{q^2 dq}{(2\pi)^2} \frac{\omega_1 + \omega_2}{\omega_1 \omega_2 [P^{02} - (\omega_1 + \omega_2)^2 + i\epsilon]}$$
(26)

where $\omega_i = (\vec{q}^2 + m_i^2)^{1/2}$ and $P^{02} = s$ and the subindex i stands for the two intermediate mesons of the j channel.

Thus the coupled channel Lippmann Schwinger equations get reduced to a set of algebraic equations:

$$At = V$$
(27)



Backup

On shell factorization

where

$$t = \begin{pmatrix} t_{11} \\ t_{21} \\ t_{22} \end{pmatrix} \quad V = \begin{pmatrix} V_{11} \\ V_{21} \\ V_{22} \end{pmatrix} \quad (28)$$

$$A = \begin{pmatrix} 1 - V_{11}t_{11} & -V_{12}G_{22} & 0 \\ -V_{21}G_{11} & 1 - V_{22}G_{22} & 0 \\ 0 & -V_{21}G_{11} & 1 - V_{22}G_{22} \end{pmatrix}$$



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