On the quantum numbers of the $X(1880)$

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Contents

J/ψ decay

- There is structure near $p\bar{p}$ threshold on the $\eta'\pi^+\pi^-$ invariant spectrum in $J/\psi \to \gamma\eta'\pi^+\pi^-$, the $X(1835)$ was firstly discovered by BES. [Phys.Rev.Lett.95:262001(2005)]

- The anomalous structures near $p\bar{p}$ threshold were found in the processes $J/\psi \to \gamma 3(\pi^+\pi^-)$, $J/\psi \to \gamma K_S^0 K_S^0 \eta$ and $J/\psi \to \gamma\phi$ by BESIII. [Phys.Rev.D 88(9):091502(2013), Phys.Rev.Lett.95:262001(2005), Phys.Rev.D 97(5):051101(2018)]

- The latest measurement for $J/\psi \to \gamma 3(\pi^+\pi^-)$ was performed by BESIII. [Phys.Rev.Lett.132(15):151901(2024)]
  - Higher statistics and more precision.
  - $X(1880)$ and $X(1835)$ are reported.

Threshold effect

We previous work about J/ψ decay

- The combined analysis are perform for $J/\psi \to \gamma\eta'\pi^+\pi^-$, $J/\psi \to \gamma p\bar{p}$ decay and $p\bar{p} \to 3(\pi^+\pi^-)$ according $N\bar{N}$ scattering and DWBA method.
  [L.Y.Dai Phys.Rev.D98(1):014005(2018)]
- The more processes are included ($J/\psi \to \gamma K_S^0 K_S^0 \eta$, and $J/\psi \to \gamma 3(\pi^+\pi^-)$) in the combined analysis.
  [Q.H.-Yang Phys.Rev.D107(3):034030(2023)]

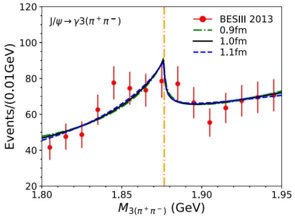
The conclusion in previous work

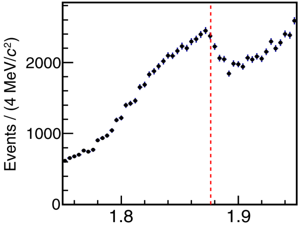
- The data can described very well according the $N\bar{N}$ scattering at intermediate state.
- No states are found in the origin $N\bar{N}$ scattering amplitude.
- The structures around $p\bar{p}$ threshold in these processes are caused by the $N\bar{N}$ threshold effect.

$X(1880)$ in $3(\pi^+\pi^-)$ invariant mass spectrum

- What is the quantum number of $X(1880)$?(It is an arduous task for experimentalists performing a partial wave analysis)

- Can the more precise data for the $3(\pi^+\pi^-)$ invariant mass spectrum be described by the $N\bar{N}$ scattering at intermediate state?(Can it be more precisely determined that anomalous behaviors around the $p\bar{p}$ threshold for $J/\psi \to \gamma 3(\pi^+\pi^-)$ be described by threshold effect?)

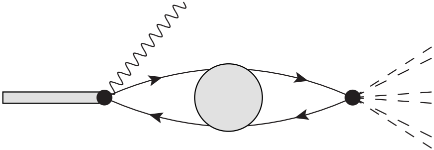




[Q.H.-Yang Phys.Rev.D107(3):034030(2023), Phys.Rev.Lett.132(15):151901(2024)]

$N\bar{N}$ scattering

The Feynman diagram for $J/\psi \to \gamma 3(\pi^+\pi^-)$



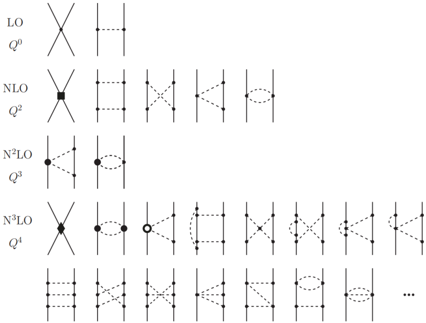
Lippmann-Schwinger equation

$$T_{L''L'}(p'', p'; E_k) = V_{L''L'}(p'', p') + \sum_L \int \frac{dpp^2}{(2\pi)^3} V_{L''L}(p'', p) \frac{1}{2E_k - 2E_p + i0^+} T_{LL'}(p, p'; E_k)$$

$V_{L''L'}(p'', p')$ is $SU(2)$ interaction potential.

The scattering potential of $N\bar{N}$

The Feynman diagram of $N\bar{N}$ scattering up to N^{3}LO



- Meson exchange potential: one pion, two pion and three pion exchange;
- Contact potential;
- Annihilation potential.

Regulation

Meson exchange potential

- Transform to position space by Fourier transformation
- Regulating in position space by following function

$$f(r) = \left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^6$$

where the cut-off are took R=0.9,1.0,1.1,1.2 fm

- Transform to momentum space by inverse Fourier transformation

Contact and annihilation potential

- Regulating in momentum space by following function

$$f(p', p) = \exp\left(-\frac{p'^2 + p^2}{\Lambda^2}\right)$$

where the cut off $\Lambda = 2R^{-1}$

Two step DWBA

The amplitude of $J/\psi \to \gamma 3(\pi^+\pi^-)$ can be obtained through solving the following set of couple equations

$$F_1(Q) = A_1^0(p') + \int_0^\infty \frac{dk k^2}{(2\pi)^3} A_1^0(k) \frac{1}{Q - 2E_k + i\epsilon} T(k, p'; E_{p'}),$$

$$F_2(Q) = A_2^0(p) + \int_0^\infty \frac{dk k^2}{(2\pi)^3} T(p, k; E_k) \frac{1}{2E_k - Q + i\epsilon} A_2^0(k)\,.$$

$$F_3(Q) = A_3^0(Q) + \int_0^\infty \frac{dk k^2}{(2\pi)^3} F_1(E_k) \frac{1}{Q - 2E_k + i\epsilon} A_2^0(k)\,,$$

where the subscript $1, 2, 3$ represent the processes of $J/\psi \to \gamma N\bar{N}$, $N\bar{N} \to \gamma(\pi^+\pi^-)$, and $J/\psi \to \gamma 3(\pi^+\pi^-)$, respectively. The transition Born amplitude and annihilation potential A_i^0 for different partial wave

$$A_{1,2}^{0,S}(p) = \tilde{C}_{1,2}^S + C_{1,2}^S p^2 + D_{1,2}^S p^4\,,$$

$$A_{1,2}^{0,P}(p) = C_{1,2}^P p + D_{1,2}^P p^3\,,$$

$$A_{1,2}^{0,D}(p) = D_{1,2}^D p^2\,,$$

$$A_3^0(Q) = \tilde{C}_3 + C_3 Q\,.$$

The decay rate and cross section

The Lorentz invariant amplitudes $\mathcal{M}_{J/\psi\to\gamma 3(\pi^+\pi^-)}$ and $\mathcal{M}_{p\bar{p}\to 3(\pi^+\pi^-)}$

$$\begin{aligned}\mathcal{M}_{J/\psi\to\gamma 3(\pi^+\pi^-)} &= -32\pi^{\frac{7}{2}}\sqrt{E_\gamma E_{J/\psi} E_1 E_2 E_3} F_{J/\psi},\\ \mathcal{M}_{N\bar{N}\to 3(\pi^+\pi^-)} &= -32\pi^{\frac{7}{2}} E_N \sqrt{E_1 E_2 E_3} F_{N\bar{N}}.\end{aligned}$$

In order to simplify phase integration, $\pi^+\pi^-$ regard as a whole.
$E_i (i = 1, 2, 3)$ denote the energy of three ($\pi^+\pi^-$) in the final state.
The decay rate and cross section

$$\begin{aligned}\frac{\mathrm{d}\Gamma}{\mathrm{d}Q} &= \int_{\beta(Q)} \mathrm{d}t_1 \mathrm{d}t_2 \frac{(m_{J/\psi}^2 - Q^2)|\mathcal{M}_{J/\psi\to\gamma 3(\pi^+\pi^-)}|^2}{6144\tilde{N}\pi^5 m_{J/\psi}^3 Q},\\ \sigma(Q) &= \int_{\beta(Q)} \mathrm{d}t_1 \mathrm{d}t_2 \frac{|\mathcal{M}_{p\bar{p}\to 3(\pi^+\pi^-)}|^2}{1024\tilde{N}\pi^3 Q^3\sqrt{Q^2 - 4m_p^2}}.\end{aligned}$$

where Q is both the invariant mass $M_{3(\pi^+\pi^-)}$ and the center-mass energy of $N\bar{N}$.

The model parameters

- The LECs $\tilde{C}_i$, C_i and D_i in contact potential, and $\tilde{C}_i^a$, C_i^a and D_i^a in annihilation potential (The $^1S_0$ partial wave as a example)

$$V(^1S_0) = \tilde{C}_{^1S_0} + C_{^1S_0}(p^2 + p'^2) + D^1_{^1S_0} p^2 p'^2 + D^2_{^1S_0}(p^4 + p'^4)$$
$$V_{\rm ann}(^1S_0) = -i(\tilde{C}^a_{^1S_0} + C^a_{^1S_0} p^2 + D^a_{^1S_0} p'^4)(\tilde{C}^a_{^1S_0} + C^a_{^1S_0} p'^2 + D^a_{^1S_0} p'^4)$$

  They are took the results from [L.-Y.Dai JHEP07(2017)]

- $\tilde{C}_i$, C_i in transition Born amplitude , and annihilation potential A_i^0

- Some normalization factor.

The partial wave

The partial wave and quantum number for $p\bar{p}$ system
($P = (-1)^{L+1}$,$C = (-1)^{L+S}$)

$J = 0$		$^1S_0(0^{-+})$	$^3P_0(1^{++})$	
$J = 1$	$^1P_1(1^{+-})$	$^3P_1(1^{++})$	$^3S_1(1^{--})$	$^3D_1(1^{--})$
$J = 2$	$^1D_2(2^{-+})$	$^3D_2(2^{--})$	$^3P_2(1^{++})$	$\cdots$

The quantum number of both J/ψ and γ: $J^{PC} = 1^{--}$.

The allowed partial wave of $3(\pi^+\pi^-)$ and $p\bar{p}$ system:

- $^1S_0$, $^3P_0$, $^3P_1$, $^1D_2$, $^3P_2$;
- Higher partial wave are ignored.

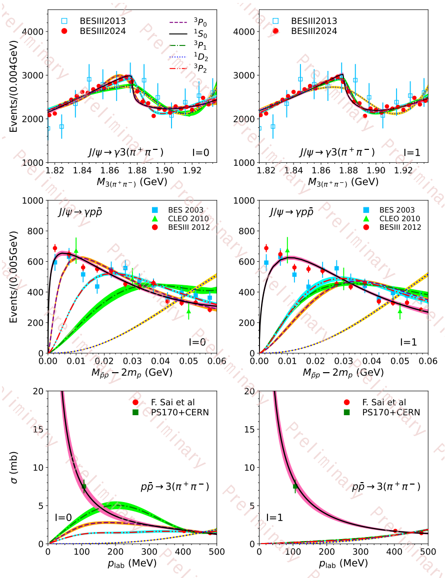
The fitting for isospin $I = 0, 1$ are considered.

The analysis strategy

The analytic strategy for the quantum number of $X(1880)$ as follows

- Fitting all dateset for the amplitude of differential partial wave and isospin .
- The partial wave with the correct quantum numbers should describe all the data well.

The fitting results for R=1.0 fm



- The best fitting: $I = 0\ {}^1S_0$;
- The sub-optimal fitting: $I = 1\ {}^1S_0$, $I = 0\ {}^3P_0$

Error band

- Bootstrap
- High order estimation

$$
\Delta X^{\mathrm{N^3LO}}(k) = \max\Big(Q^5\left|X^{\mathrm{LO}}(k)\right|,\ Q^3\left|X^{\mathrm{LO}}(k) - X^{\mathrm{NLO}}(k)\right|,\ Q^2\left|X^{\mathrm{NLO}}(k) - X^{\mathrm{N^2LO}}(k)\right|,\ Q\left|X^{\mathrm{N^2LO}}(k) - X^{\mathrm{N^3LO}}(k)\right|\Big),
$$

where

$$
Q = \max\left(\frac{k}{\Lambda_b},\ \frac{M_\pi}{\Lambda_b}\right)
$$

$\chi^2_{\mathrm{d.o.f}}$

R (fm)	Isospin	$^1S_0$	$^3P_0$	$^3P_1$	$^1D_2$	$^3P_2$
0.9	$I=0$	1.96	7.87	41.51	67.39	20.94
	$I=1$	4.53	35.09	30.72	69.05	29.97
1.0	$I=0$	1.99	6.74	33.81	66.18	18.58
	$I=1$	3.30	31.34	26.38	67.82	25.03
1.1	$I=0$	2.04	6.34	25.70	64.78	16.40
	$I=1$	3.32	28.00	21.20	66.45	20.85
1.2	$I=0$	2.07	6.42	19.80	63.24	14.90
	$I=1$	3.22	24.74	17.13	64.94	17.98

- The $\chi^2/\mathrm{d.o.f}$ of partial wave $I=0\ {}^1S_0$ is smallest for different Cut-off.
- The $\chi^2/\mathrm{d.o.f}$ of partial wave $I=1\ {}^1S_0$ and $I=0\ {}^3P_0$ are a little large than $I=0\ {}^1S_0$
- The $\chi^2/\mathrm{d.o.f}$ for others partial wave are significantly larger.

p-value

The definition of p value for a goodness-of-fit with χ^2

$$p = \int_{\chi^2_{\min}}^{\infty} f(t, n_d) dt \,,$$

$f(t, n_c)$ is χ^2 probability density function. n_d is the degrees of freedom. The statistic t is defined as

$$t(\boldsymbol{\theta}) = \sum_{i=1}^{N} \frac{(y_i - \mu_i(\boldsymbol{\theta}))}{\sigma^2_{\mathrm{tot},i}} \,,$$

where y_i is the i-th experimental data point and μ_i is the theoretical value. The variance are $\sigma_{\mathrm{tot},i} = \sqrt{\sigma^2_{\mathrm{exp},i} + \sigma^2_{\mathrm{theo},i}}$, with the experimental error $\sigma_{\mathrm{exp},i}$ and theory error $\sigma_{\mathrm{theo},i}$

p-value test

- The p-value larger than 0.05 indicated that the fitting results are significantly correlated with the experimental data

R (fm)	Isospin	$^1S_0$	$^3P_0$	$^3P_1$	$^1D_2$	$^3P_2$
0.9	$I=0$	0.261	0.000	0.000	0.000	0.000
	$I=1$	0.000	0.000	0.000	0.000	0.000
1.0	$I=0$	0.215	0.000	0.000	0.000	0.000
	$I=1$	0.000	0.000	0.000	0.000	0.000
1.1	$I=0$	0.168	0.000	0.000	0.000	0.000
	$I=1$	0.000	0.000	0.000	0.000	0.000
1.2	$I=0$	0.137	0.000	0.000	0.000	0.000
	$I=1$	0.000	0.000	0.000	0.000	0.000

- Only partial wave $I=0\ ^1S_0$ pass p-value test

Summary

- The combine analysis on the processes $J/\psi \to \gamma 3(\pi^+\pi^-)$, $J/\psi \to \gamma p\bar{p}$ and $p\bar{p} \to 3(\pi^+\pi^-)$ are performed for the difference partial wave and isospin.

- All datasets can be fitted well for partial wave $I = 0\ {}^1S_0$, and the p-value test is pass. We tend to think that the quantum number of the structure around $p\bar{p}$ threshold, the $X(1880)$, is $IJ^{PC} = 00^{-+}$.

- The $X(1880)$ is generated by the $N\bar{N}$ threshold effect.

Thank you for your patience!