On the three-body state $K^*D^*K^*$

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OUTLINE

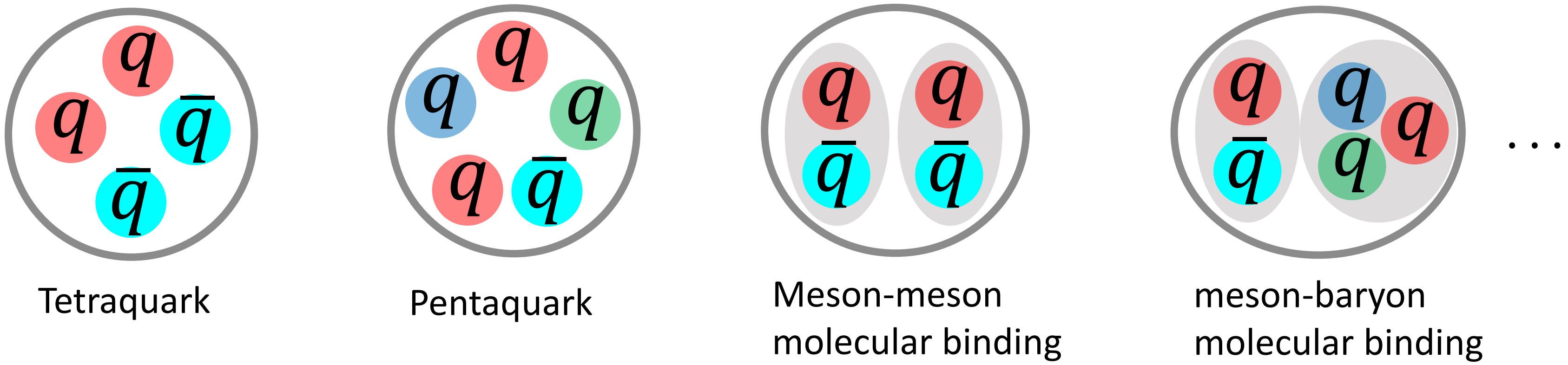
- Motivation
- Chiral unitary approach
- Meson-meson interaction
- Fixed Center Approach
- $K^*D^*K^*$ three body systems
- Summary and Outlook

1. Motivation

1.1 Hadronic molecular state

- The exotic hadronic states may be tetraquarks, pentaquarks, glueballs, hybrids, or hadronic molecular states.

- Hadronic molecular states can be composed of a pair of hadrons, such as meson-meson or meson-baryon molecular states. In principle, molecular states formed by three hadrons are also theoretically allowed.



1.2 Few-body systems composed of mesons

- Similar to the forces generated between nucleons, there is also and interaction between mesons, that sometimes is stronger than the known nuclear forces.

  Is it possible that many body systems made only from mesons exist, but we failed to identify them?

- Unlike systems with baryons, mesons is no meson conservation number and systems with many mesons can decay into systems with fewer mesons. Its decay width tends to increase, making it more difficult to identify.

  Flavor is conserved in strong interactions, so multimeson states with different flavors certainly more stable.
  High spin leads to decay modes with high angular momentum which are not favored.

  T.-W. Wu et al. Sci. Bull. 67 (2022) 1735

- The three-body bound states are receiving more and more attention. A. Martinez Torres et al. Few-Body Syst. 61 (2020) 35

multi $-\rho$	L. Roca, E. Oset, PRD 82 (2010) 054013	$D^*D^*K^{*-}$	N. Ikeno, M. Bayar, E. Oset, PRD 107 (2023) 034006
$NDK, ND\bar{K}, ND\bar{D}$	C. W. Xiao, M. Bayar, E. Oset, PRD 84 (2011) 034037	$K^*B^{(*)}B^*$	M. Bayar, N. Ikeno, L. Roca, PRD 107 (2023) 054042
$D^{(*)}B^{(*)}B^{(*)}$	J. M. Dias, L. Roca, S. Sakai, PRD 97 (2018) 056019	DND^*	V. Montesinos et al. PRD 110 (2024) 054043
$D^*D^*D^*$	S.-Q. Luo et al. PRD 105 (2022) 074033	$\cdots$	

1.2 Few-body systems composed of mesons

$T_{c\bar{s}}(2900)$

- The D^*K^* system was investigated and two states were found corresponding to $J^P = 0^+$, $J^P = 2^+$ in the case of $I = 1$.

R. Molina, T. Branz, E. Oset, PRD 82 (2010) 014010

R. Molina, E. Oset, PRD 107 (2023) 056015

$$m \approx 2834 \text{ MeV}, \qquad \Gamma \approx 19 \text{ MeV}$$

$B^+ \to D^{*-}D^+K^+$ R. Aaij et al. (LHCb) PRL 133 (2024) 131902

- Adding an extra K^{*+} to the $D^{*+}K^{*+}$.
  Creating a three body state with $J^P = 3^-$, where all the couples will be in $J = 2$.

$$Q = 3, \qquad S = 2, \qquad J = 3, \qquad I = \frac{3}{2}.$$

The quark content of this state is $u\bar{s}c\bar{d}u\bar{s}$, and no quark-antiquark annihilation.

- Using the fixed center approximation (FCA) to treat the three body interaction system of $K^*D^*K^*$.

2. Chiral unitary approach

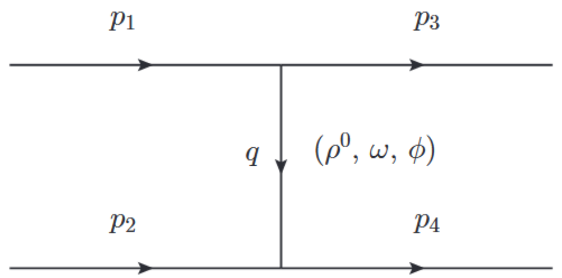
- The chiral unitary approach takes into account both chiral symmetry and unitarity.
- The hidden-gauge Lagrangian is

$$\mathcal{L} = -\frac{1}{4}\langle \bar{V}_{\mu\nu}\bar{V}^{\mu\nu}\rangle + \frac{1}{2}M_v^2\left\langle \left[V_\mu - \langle i/g\rangle\Gamma_\mu\right]^2\right\rangle$$

The Lagrangian above provides four-vector and three-vector contact interaction terms.

$$\mathcal{L}_{VVVV} = \frac{1}{2}g^2\langle [V_\mu - V_\nu]V^\mu V^\nu\rangle$$

$$\mathcal{L}_{VVV} = ig\langle (\partial_\mu V_\nu - \partial_\nu V_\mu)V^\mu V^\nu\rangle$$



The three-vector contact terms lead to the vector-exchange diagrams.

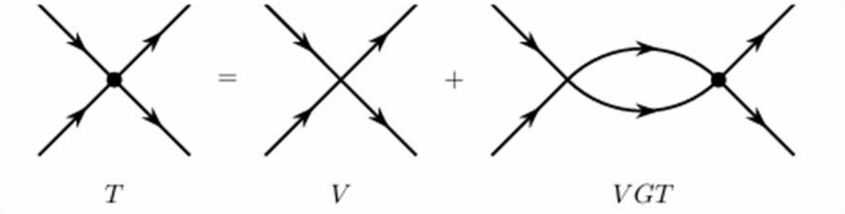
The interaction (potential) is given by: $V_{ij} = \langle i|-\mathcal{L}|j\rangle$

$$V_\mu = \begin{pmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & \frac{\omega-\rho^0}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi \end{pmatrix}$$

The coupling constant $g = \frac{M_V}{2f}$, $M_V = 800$ MeV, with the pion decay constant $f_\pi = 93$ MeV.

- Bethe-Salpeter equation:

$$T = V + VGT = (1 - VG)^{-1}V$$



- The meson-meson loop functions G:

using a cutoff

$$G(s) = \int_0^{q_{\max}} \frac{q^2 \mathrm{d}|q|}{(2\pi)^2} \frac{\omega_1 + \omega_2}{\omega_1\omega_2[s - (\omega_1 + \omega_2)^2 + i\epsilon]}$$

using dimensional regularization:

$$G(s) = \frac{1}{16\pi^2}\left[\alpha + \log\frac{M_1^2}{\mu^2} + \frac{M_2^2 - M_1^2 + s}{2s}\log\frac{M_2^2}{M_1^2} + \frac{p}{\sqrt{s}}\left(\log\frac{s - M_2^2 + M_1^2 + 2p\sqrt{s}}{-s + M_2^2 - M_1^2 + 2p\sqrt{s}} + \log\frac{s + M_2^2 - M_1^2 + 2p\sqrt{s}}{-s - M_2^2 + M_1^2 + 2p\sqrt{s}}\right)\right]$$

- Searching for resonance poles $M_R + i\frac{\Gamma_R}{2}$。

$$\det(1 - VG) = 0$$

Close to a pole, the meson-meson amplitudes can be approximated by Breit-Wigner form.

$$T_{ij} \backsimeq \frac{g_i g_j}{s - s_R}$$

Calculating the couplings to each channel from the residue of the amplitudes

$$g_i g_j = \lim_{s \to s_R}(s - s_R)T_{ij}$$

$$g_i g_j = \frac{1}{2\pi i}\oint_C \mathrm{d}s T_{ij}(s)$$

3. Meson-meson interaction

- Studying the two body interactions between vector mesons.
  - To dynamically generate a D^*K^* molecular state with $I(J^P) = 1(2^+)$ using a cutoff.
  - To obtain the mass, decay width, and coupling constants to each coupled channel of the molecular state.
  - To calculate the $J = 2$ $K^*K^* \to K^*K^*$ transition potential and obtain information on the interaction.

3. Meson-meson interaction

3.1 D^*K^* interaction

Considering the isospin coupled channels: D^*K^*, $D_s^*\rho$

- Tree-level amplitudes in the case of $I=1, J=2$.

	K^*D^*	$D_s^*\rho$
K^*D^*	$\frac{g^2}{2}\left(\frac{1}{m_\rho^2}-\frac{1}{m_\omega^2}\right) \times (p_1+p_3)(p_2+p_4)$	$-2g^2-\frac{g^2(p_1+p_4)(p_2+p_3)}{m_{D^*}^2}-\frac{g^2(p_1+p_3)(p_2+p_4)}{m_{K^*}^2}$
$D_s^*\rho$	$-2g^2-\frac{g^2(p_1+p_4)(p_2+p_3)}{m_{D^*}^2}-\frac{g^2(p_1+p_3)(p_2+p_4)}{m_{K^*}^2}$	0

R. Molina, T. Branz, E. Oset, PRD 82 (2010) 014010

- Calculating the couplings to each channel

couplings g_i in units of MeV .

	$\mathrm{I}(\mathrm{J}^\mathrm{P})$	$\sqrt{s_{\mathrm{pole}}}$	$\lvert g_{D^*K^*}\rvert$	$\lvert g_{D_s^*\rho}\rvert$
$\mu=1500$, $\alpha=-1.474$	$1(2^+)$	2786	11041	11092
$q_{\max}=965$ MeV	$1(2^+)$	**2834**	**12130**	**12061**

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- considering the decay width of the ρ and K^* mesons

Position and width of the state.

	$\mathrm{I}(\mathrm{J}^\mathrm{P})$	$\sqrt{s_{\mathrm{pole}}}$ (MeV)	Γ_{pole} (MeV)
$\mu=1500$, $\alpha=-1.474$	$1(2^+)$	2834	19
$q_{\max}=965$ MeV	$1(2^+)$	**2834**	**29**

R. Molina, E. Oset, PRD 107 (2023) 056015

- The free parameters $q_{\max}$ is fitted assuming that the interaction generates the $M(2834)$ resonance. It is taken to be **965 MeV .**

3.1 D^*K^* interaction

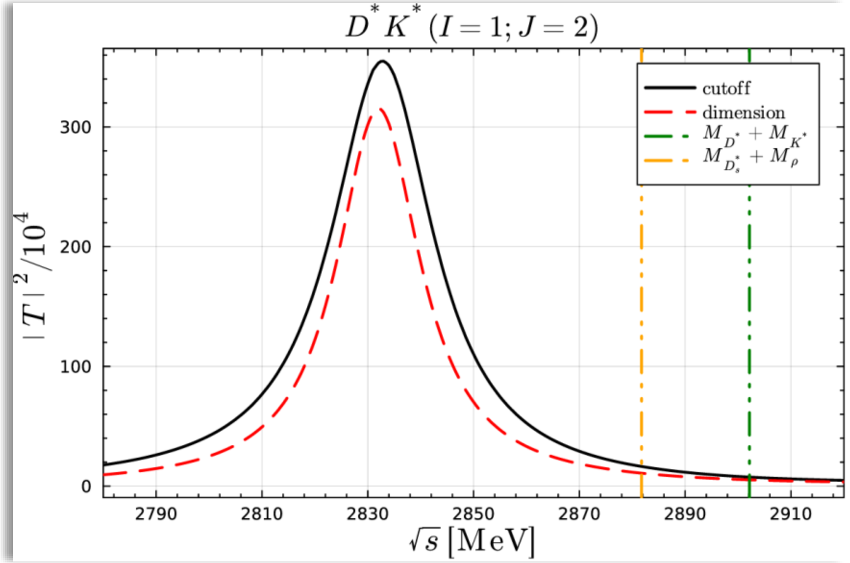
- Due to $|D^*K^*, I=1\rangle = |D^{*+}K^{*+}\rangle$,

$$t_{D^{*+}K^{*+}\to D^{*+}K^{*+}} = t^{I=1}_{D^*K^*\to D^*K^*}$$

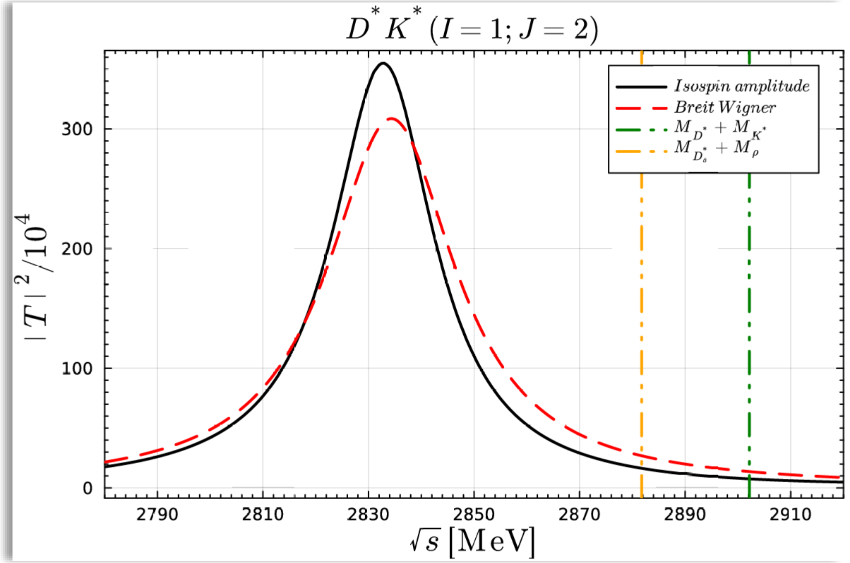
To consider the width Γ_R of the dynamically generated state, we approximate amplitude by Breit-Wigner form:

$$t^{I=1}_{D^*K^*\to D^*K^*} = \frac{\left(g^{I=1}_{D^*K^*}\right)^2}{s_{13} - M_R^2 + iM_R\Gamma_R}$$

$$\sqrt{s_{13}} = M_{\text{inv}}(D^*K^*)$$



The $|T|^2$ as a function of center-of-mass energy, using a cutoff (solid line) and using dimensional regularization (dashed line).



The $|T|^2$ as a function of center-of-mass energy for the isospin amplitude (solid line) and the Breit-Wigner form (dashed line).

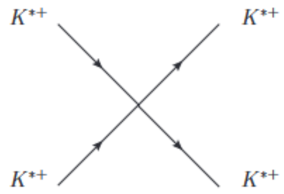
3.2 K^*K^* interaction

- The lowest order transition potential describing $K^{*+}K^{*+} \to K^{*+}K^{*+}$ of $J=2$ includes

$$V = V_{\text{ex}}^{(J=2)} + V_{\text{c}}^{(J=2)}$$

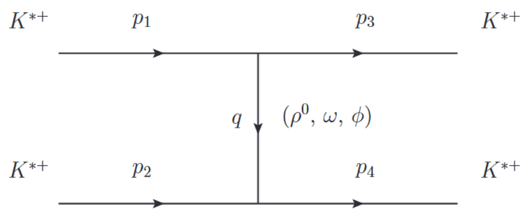
  - the contact term from direct contact of four-vector mesons:

$$V_{\text{c}}^{(J=2)} = 4g^2$$



  - the term from the exchange of light vectors:

$$V_{\text{ex}}^{(J=2)} = \frac{2g^2}{M_V^2}[(p_1+p_3)\cdot(p_2+p_4)+(p_1+p_4)\cdot(p_2+p_3)]$$
$$= \frac{2g^2}{M_V^2}\left(3s_{23} - 4m_{K^{*+}}^2\right)$$



- To regularize the divergence of G integral using a cutoff, taking into account the decay width of the K^* meson

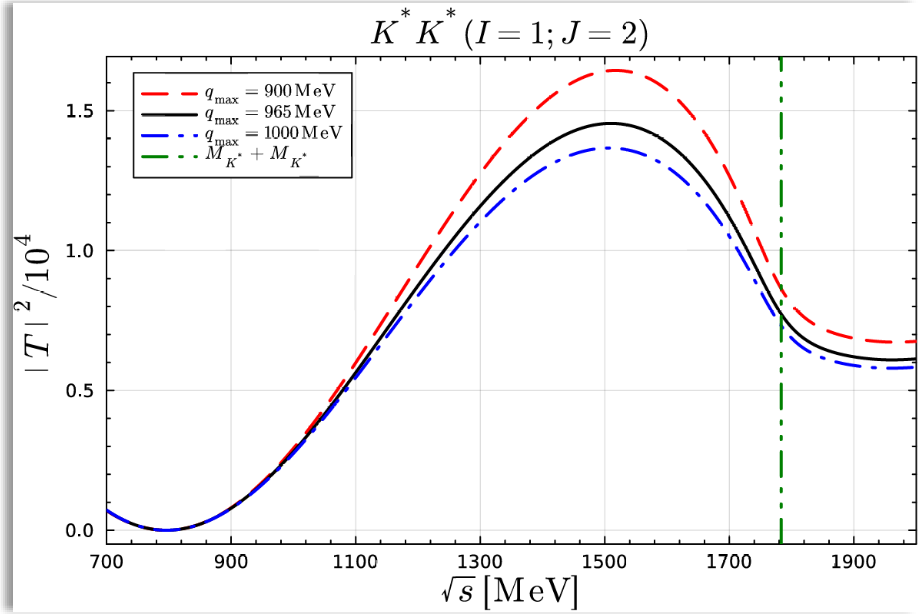
$$\tilde{G}(s) = \frac{1}{\mathcal{N}^2}\int_{(M_1-3.5\Gamma_1)^2}^{(M_1+3.5\Gamma_1)^2} d\tilde{m}_1^2\left(\frac{-1}{\pi}\right)\text{Im}\frac{1}{\tilde{m}_1^2 - M_1^2 + i\tilde{\Gamma}_1\tilde{m}_1}$$
$$\times \int_{(M_2-3.5\Gamma_2)^2}^{(M_2+3.5\Gamma_2)^2} d\tilde{m}_2^2\left(\frac{-1}{\pi}\right)\text{Im}\frac{1}{\tilde{m}_2^2 - M_2^2 + i\tilde{\Gamma}_2\tilde{m}_2}$$
$$\times G(s, \tilde{m}_1^2, \tilde{m}_2^2)$$

3.2 K^*K^* interaction

- The total $K^{*+}K^{*+}$ two body scattering amplitudes T can be obtained from the BS equation

$$T(K^{*+}K^{*+}) = (1 - VG)^{-1}V$$

- $K^{*+}K^{*+}$ is a repulsive potential, but its strength is weaker than the strength of the $D^{*+}K^{*+}$ attractive potential.
  K^*K^* : at threshold $V \propto (M_{K^*} + M_{K^*})^2$,
  D^*K^* : at threshold $V \propto (M_{D^*} + M_{K^*})^2$,
  Therefore, the formation of a three-body bound state is possible.



$|T|^2$ as a function of center-of-mass energy, with $q_{\max}$ = 900 MeV(red dashed line), 965 MeV (black solid line), 1000 MeV (blue dash-dotted line).

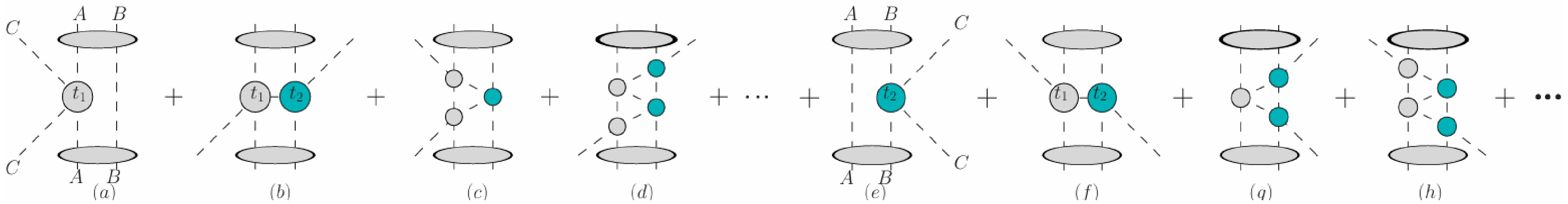
4. Fixed Center Approach

- Faddeev equations are the most common tools to solve the three-body equations.

$$T^i = t^i \delta^3(k_i' - k_i) + \sum_{j\neq i=1}^{3} T_R^{ij}, i = 1, 2, 3. \qquad T_R^{ij} = t^i g^{ij} t^j + t^i [G^{iji} T_R^{ji} + G^{ijk} T_R^{jk}]$$

- The FCA is employed as an approximate method to the Faddeev equations.

$$T = T_1 + T_2, \quad T_1 = t_1 + t_1 G_0 T_2, \quad T_2 = t_2 + t_2 G_0 T_1.$$



Diagrammatic representation of the FCA

4. Fixed Center Approach

- G_0 is the propagator of particle 3 modulated by the wave function of the two-body cluster:

$$G_0 = \int \frac{\mathrm{d}^3 q}{(2\pi)^3} F(q) \frac{1}{{q^0}^2 - \mathbf{q}^2 - m_3^2 + i\epsilon}$$

- $F(q)$ is the form factor of the two-body cluster:

$$F(q) = \frac{1}{N} \int_{\substack{|\vec{p}| \le q_{\max}, \\ |\vec{p}-\vec{q}| \le q_{\max}}} \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{M_R - \omega_1(\vec{p}) - \omega_2(\vec{p})} \frac{1}{M_R - \omega_1(\vec{p}-\vec{q}) - \omega_2(\vec{p}-\vec{q})}$$

- N is the normalization factor of the form factor:

$$N = \int_{|\vec{p}| \le q_{\max}} \frac{\mathrm{d}^3 p}{(2\pi)^3} \left(\frac{1}{M_R - \omega_1(\vec{p}) - \omega_2(\vec{p})} \right)^2$$

q^0 is the energy of particle 3 in the rest frame of the cluster, $q^0 = \frac{s - m_3^2 - M_R^2}{2M_R}$, $\omega_i(\vec{p}) = \sqrt{m_i^2 + \vec{p}^2}$.

5. $K^*D^*K^*$ three body systems

- Studying the three body interactions $K^*D^*K^*$ using FCA.
  - To treat the $J = 2$ D^*K^* system as a cluster and study the interaction of a third K^* particle with the two particles inside the cluster.
  - To discuss the spin-isospin structure of these amplitudes and their normalization, and obtain information on the three body interaction.

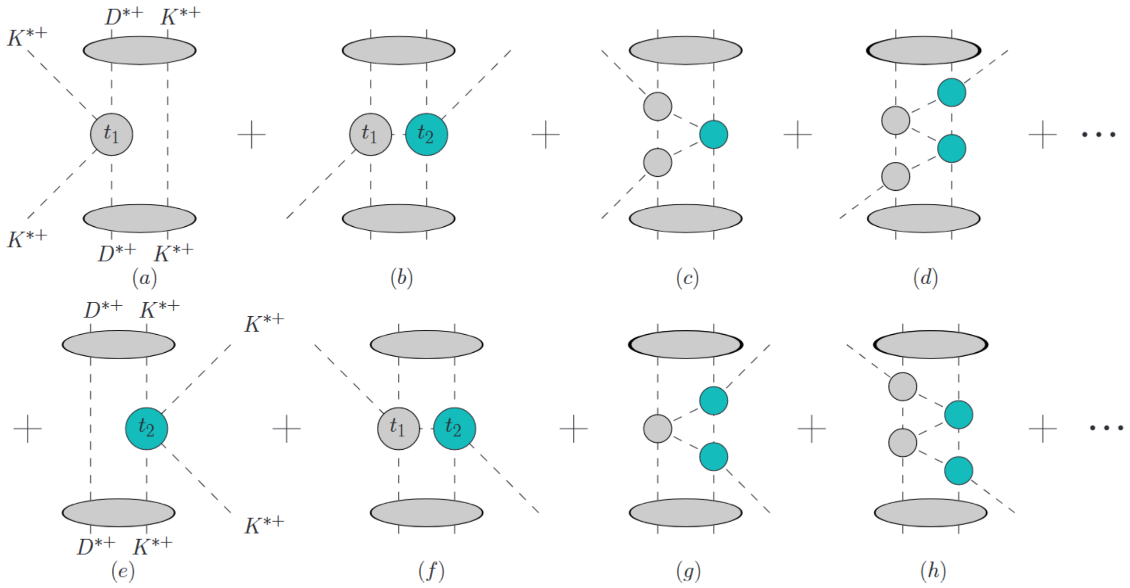
3.3 $K^*D^*K^*$ three body interaction

- Discuss the isospin structure of the amplitudes t_1 and t_2

$$t_1 = t_1(K^{*+}D^{*+}, I = 1)$$

$$t_2 = t_2(K^{*+}K^{*+}, I = 1)$$

- The normalization factor is $\frac{1}{\sqrt{2\omega_i}}$ in the Mandl and Shaw convention, where ω_i is the energy of particle i.



- the S matrix of single scattering

$$S_i^{(1)} = (-it_i)\frac{1}{\mathcal{V}^2}\frac{1}{\sqrt{2\omega_3}}\frac{1}{\sqrt{2\omega_3'}}\frac{1}{\sqrt{2\omega_i}}\frac{1}{\sqrt{2\omega_i'}}(2\pi)^4\delta^4(k+k_c-k'-k_c'), \qquad i = 1, 2.$$

- the S matrix of double scattering

$$S^{(2)} = (-it_1t_2)(2\pi)^4\delta^4(k+k_c-k'-k_c')\frac{1}{\mathcal{V}^2}\frac{1}{\sqrt{2\omega_3}}\frac{1}{\sqrt{2\omega_3'}}\frac{1}{\sqrt{2\omega_1}}\frac{1}{\sqrt{2\omega_1'}}\frac{1}{\sqrt{2\omega_2}}\frac{1}{\sqrt{2\omega_2'}}G_0$$

k, k' refer to the initial and final four-momenta of the particle 3, k_c, k_c' refer to initial and final fourmomenta of the cluster.

5. $K^*D^*K^*$ three body systems

- The scattering process between particle 3 and the cluster as a whole:

$$S = (-i\tilde{t})(2\pi)^4\delta^4(k + k_c - k' - k'_c)\frac{1}{\mathcal{V}^2}\frac{1}{\sqrt{2\omega_3}}\frac{1}{\sqrt{2\omega'_3}}\frac{1}{\sqrt{2\omega_c}}\frac{1}{\sqrt{2\omega'_c}}$$

- For single scattering, compare the macroscopic and microscopic representations

$$\tilde{t}_1 = \frac{2\omega_c}{\sqrt{2\omega_1}\sqrt{2\omega'_1}}t_1 \simeq \frac{M_c}{m_1}t_1$$

$$\tilde{t}_2 = \frac{2\omega_c}{\sqrt{2\omega_2}\sqrt{2\omega'_2}}t_2 \simeq \frac{M_c}{m_2}t_2$$

$$\omega_i \approx \omega'_i, \sqrt{2\omega_i} = \sqrt{2m_i}$$

- Compare the macroscopic and microscopic representations of the secondary scattering S matrix

$$\tilde{G}_0 = \frac{1}{2M_c}\int_{|\vec{q}|\le q_{\max}}\frac{\mathrm{d}^3q}{(2\pi)^3}F(q)\frac{1}{{q^0}^2 - \mathbf{q}^2 - m_3^2 + i\epsilon}$$

- The total three body scattering amplitudes is

$$\tilde{T} = \frac{\tilde{t}_1 + \tilde{t}_2 + 2\tilde{t}_1\tilde{t}_2\tilde{G}_0}{1 - \tilde{t}_1\tilde{t}_2\tilde{G}_0^2}$$

$$s_1 = m_3^2 + (\xi m_1)^2 + 2\xi m_1 q^0\,, \quad s_2 = m_3^2 + (\xi m_2)^2 + 2\xi m_2 q^0, \quad \xi = \frac{M_c}{m_1+m_2}$$

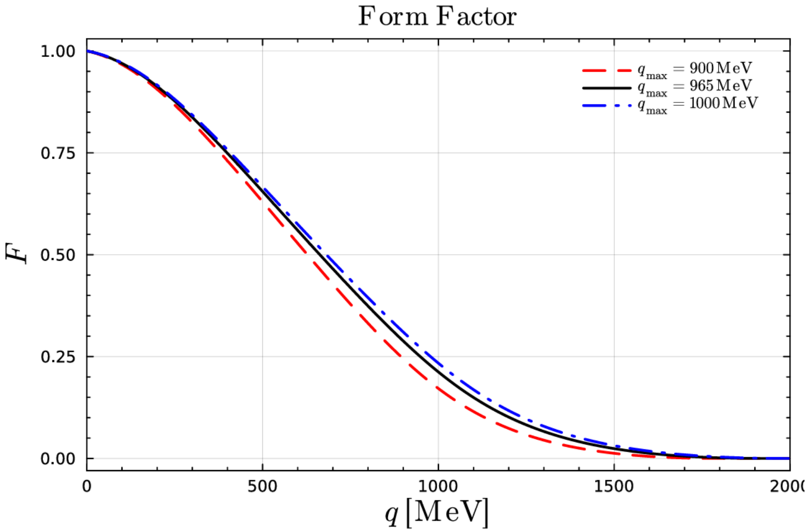
5. $K^*D^*K^*$ three body systems

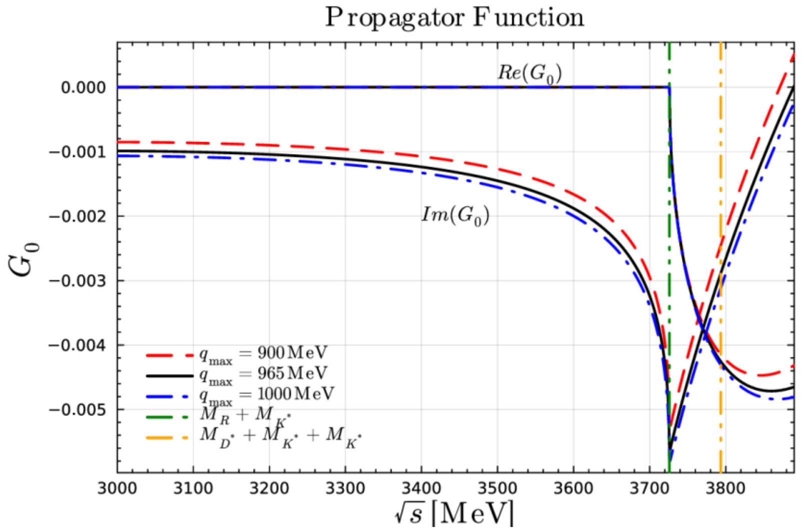
Calculating the form factors and loop functions using different cutoff values.

$q_{\max} = 900$ MeV (Red dashed line)

$q_{\max} = 965$ MeV (Black solid line)

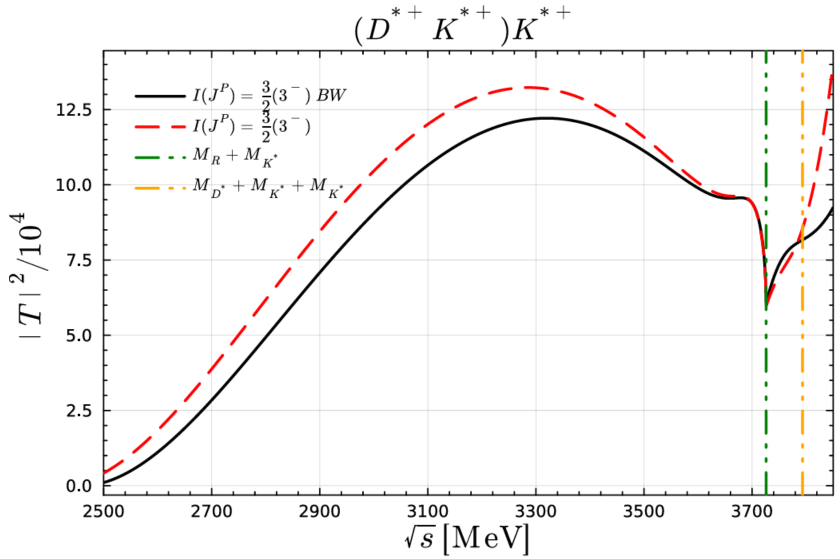
$q_{\max} = 1000$ MeV (Blue dash-dotted line)

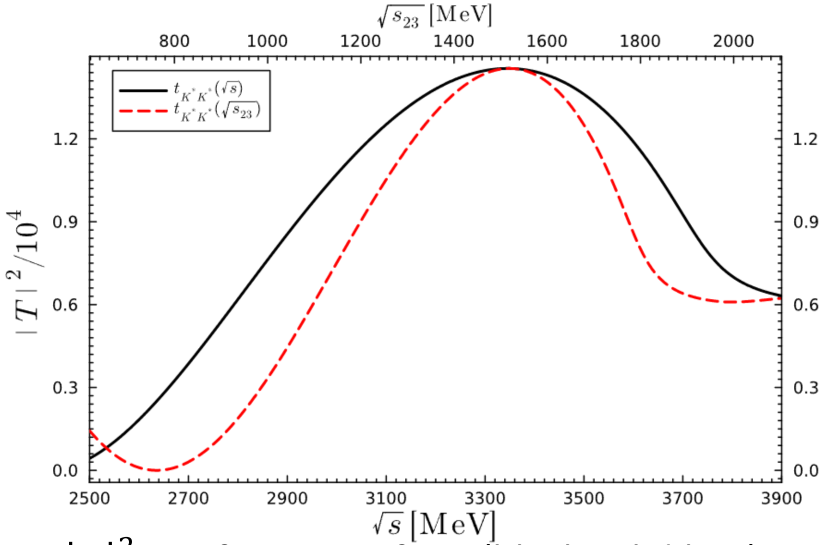




A peak is observed at 3320 MeV, with a height of 12.2×10^4, corresponding to a binding energy of approximately 406 MeV

A peak is observed at 3680 MeV, with a height of 9.57×10^4, corresponding to a binding energy of approximately 46 MeV.





$|T|^2$as a function of s_{23} (black solid line) and s(red dashed line), respectively.

6. Summary and Outlook

Summary

- Using a cutoff, a J=2 bound state is dynamically generated in the D^*K^* and $D_s^*\rho$ interactions, and the coupling constants to the various coupled channels are recalculated.

- The K^*K^* interaction is repulsive in that channel, but the interaction is weaker than the attraction of the D^*K^* .

- A peak is observed at 3320 MeV , with a height of 12.2×10^4, corresponding to a binding energy of approximately 406 MeV, indicating the possible existence of a certain structure at this energy.

Outlook

- No relevant experimental data are currently available; close attention is being paid to experimental developments to verify the internal structure of exotic hadronic states.

Thanks for your attention!