Coulomb interaction and isospin breaking effects in $D^{(*)}\Sigma_c^{(*)}$ systems

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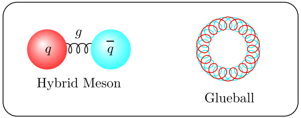
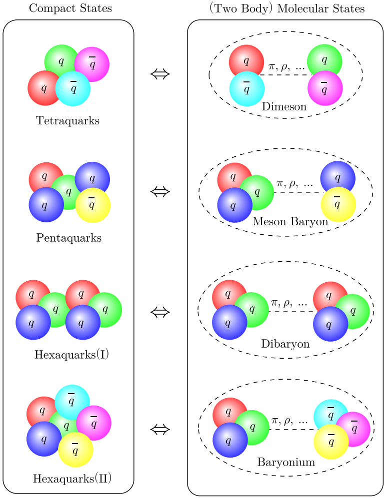
Outline

Backgrounds

- Exotic states discoveries in past decades, beyond the traditional quark model
- Several theoretical structures proposed:
  - Molecular states
  - Compact states
  - Hybrid states and glueballs
- Experimental discoveries:
  - $\chi_{c1}(3872), \mathrm{T}_{cc}(3875), \ldots$
  - $\mathrm{P}_{\psi}(4380), \mathrm{P}_{\psi s}(4459), \ldots$

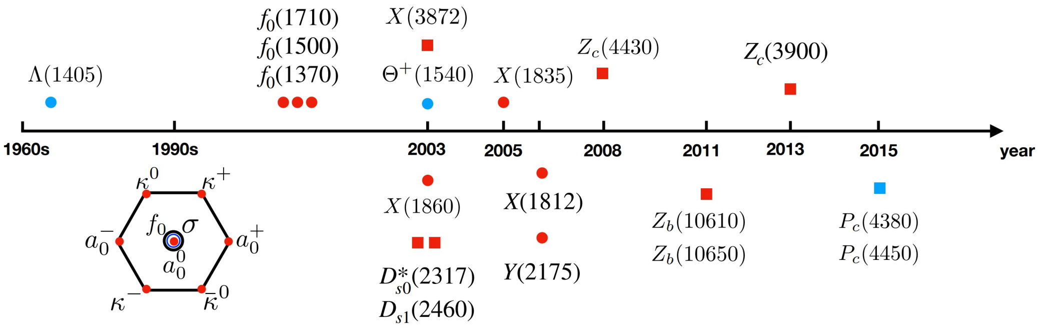


Gluon involved states



Experimental discovery timeline

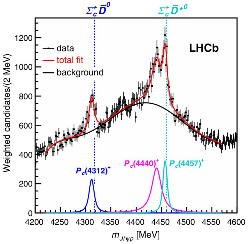
An early timeline of observations of some typical exotic hadronic states.



Yan-Rui Liu, Hua-Xing Chen, Wei Chen, Xiang Liu, Shi-Lin Zhu. (arXiv:1903.11976)

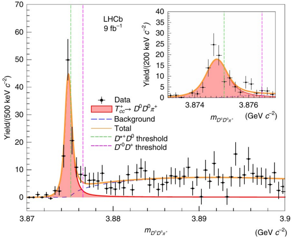
Recent experimental discoveries of typical multiquarks

P_ψ: Hidden charmed pentaquarks



$\Lambda_b^0 \to J/\psi\, p\, K^-$ **(LHCb, 2019)**

T_{cc}: Double charmed tetraquarks



$T_{cc}^+ \to D^0\, D^0\, \pi^+$ **(LHCb, 2021)**

- Masses lying nearl under the thresholds of two-body hadronic systems
- What about doubly charmed pentaquarks P_{cc}?

Previous theoretical studies

- **About P_{cc} states: Well bound in several states**
  - Lattice QCD: PhysRevD.101.074030, 2020; ...
  - OBE model: PhysLetB.2021.136693, 2021; ...
  - Quark model: PhysRevD.103.116017, 2021; ...
  - QCD Sum rules: PhysRevD.109.094018, 2024; ...
- **About Coulomb potential in hadronic systems**
  - In dibaryon: May be very strong or have no infuence (arXiv:2107.04957, 2021)
  - In $\chi_{c1}(3872)$: Lesser contributions (PhysRevD.109.094002, 2024)

The one-boson-exchange (OBE) potential model

Effective Lagrangians:

(Yan-Rui Liu, 2012)

$$\mathcal{L}_{\mathrm{M}} = g_{\mathrm{S}}\langle H_m \sigma \bar{H}_m\rangle + \mathrm{i}\beta\langle H_n v_\rho (\hat{V}^\rho - \mathbb{V}'^\rho)_{nm}\bar{H}_m\rangle + \mathrm{i}g\langle H_n \hat{X}^\rho_{nm}\gamma_\rho\gamma_5\bar{H}_m\rangle + i\lambda\langle H_n \hat{F}^{\alpha\beta}_{nm}\sigma_{\alpha\beta}\bar{H}_m\rangle \quad (1)$$

$$\mathcal{L}_{\mathrm{B}_6} = l_{\mathrm{S}} g_{\mu\nu}\mathrm{Tr}[\bar{S}^\mu \sigma S^\nu] + \mathrm{i}\beta_{\mathrm{S}} g_{\mu\nu}\mathrm{Tr}[\bar{S}^\mu(\hat{V}^\rho - \mathbb{V}'^\rho)v_\rho S^\nu] + \frac{3}{2}g_1\varepsilon_{\mu\nu\rho\sigma}v^\sigma\mathrm{Tr}[\bar{S}^\mu \hat{X}^\rho S^\nu] + \lambda_{\mathrm{S}}\mathrm{Tr}[\bar{S}^\mu \hat{F}_{\mu\nu}S^\nu] \quad (2)$$

Meson exchange: Scalar, **P**seudoscalar, **V**ector

$$\mathbb{P}_{mn} \equiv \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta^0 \end{bmatrix} \quad (3) \qquad \mathbb{V}^\mu_{mn} \equiv \begin{bmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi^0 \end{bmatrix}^\mu \quad (4)$$

Feynmann diagram → potential → Schördinger equation → binding energy

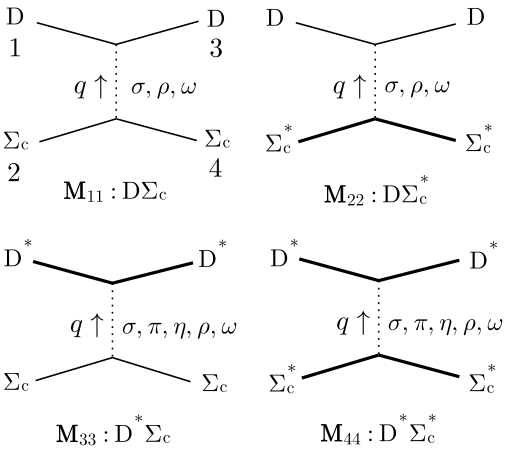
P_{cc} as $D^{(*)}\Sigma_c^{(*)}$ molecular states

- Feynmann diagrams at tree level:
- Momentum potential:
  Breit's approximation

$$V \propto -\mathcal{M}_{1,2\to 3,4} \tag{5}$$

- Monopole form factor added in the Fourier transformation to avoid ultraviolet divergence

$$F(q^2) \equiv \frac{\Lambda^2 - m_{ex}^2}{\Lambda^2 - q^2} \tag{6}$$



Isospin: transformation and coupled channel

- Two kinds of equivalent representations in isospin coupled OBE potential under isospin symmetry
- i.e. for $\mathrm{D}^*\Sigma_\mathrm{c}$, $|I I_3\rangle = |\frac{1}{2}\frac{1}{2}\rangle, |\frac{3}{2}\frac{1}{2}\rangle$:

$$V_{pq}^{\otimes} = \begin{bmatrix} \langle \mathrm{D}^{*+}\Sigma_\mathrm{c}^{+} | \mathrm{D}^{*+}\Sigma_\mathrm{c}^{+}\rangle & \langle \mathrm{D}^{*+}\Sigma_\mathrm{c}^{+} | \mathrm{D}^{*0}\Sigma_\mathrm{c}^{++}\rangle \\ \langle \mathrm{D}^{*0}\Sigma_\mathrm{c}^{++} | \mathrm{D}^{*+}\Sigma_\mathrm{c}^{+}\rangle & \langle \mathrm{D}^{*0}\Sigma_\mathrm{c}^{++} | \mathrm{D}^{*0}\Sigma_\mathrm{c}^{++}\rangle \end{bmatrix}, \tag{7}$$

$$V_{pq}^{\oplus} = \begin{bmatrix} \langle \mathrm{D}^*\Sigma_\mathrm{c}, I I_3 = \frac{1}{2}\frac{1}{2}\rangle & 0 \\ 0 & \langle \mathrm{D}^*\Sigma_\mathrm{c}, I I_3 = \frac{3}{2}\frac{1}{2}\rangle \end{bmatrix}. \tag{8}$$

$$V_{pq}^{\oplus} = U V_{pq}^{\otimes} U^{-1}, \tag{9}$$

$$U = \begin{pmatrix} \sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} \\ \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \end{pmatrix}. \tag{10}$$

- No external perturbations: $I = \frac{3}{2}$ and $I = \frac{1}{2}$ are totally separated
  - Potential matrices in coupling representation are diagonal

Isospin: transformation and coupled channel

- If perturbations exist:

$$V_{\mathrm{CL}}^{\otimes} = \begin{bmatrix} V_{\mathrm{CL}} & 0 \\ 0 & 0 \end{bmatrix} \quad \rightarrow \quad V_{\mathrm{CL}}^{\oplus} = U V_{\mathrm{CL}}^{\otimes} U^{-1} = \begin{pmatrix} \frac{1}{3} & \frac{\sqrt{2}}{3} \\ \frac{\sqrt{2}}{3} & \frac{2}{3} \end{pmatrix} V_{\mathrm{CL}}, \tag{11}$$

$$\Delta M^{\otimes} = \begin{bmatrix} \Delta M & 0 \\ 0 & 0 \end{bmatrix} \quad \rightarrow \quad \Delta M^{\oplus} = U \Delta M^{\otimes} U^{-1} = \begin{pmatrix} \frac{1}{3} & \frac{\sqrt{2}}{3} \\ \frac{\sqrt{2}}{3} & \frac{2}{3} \end{pmatrix} \Delta M, \tag{12}$$

- Assuming the perturbations are small enough:

$$V_{\mathrm{CL}}^{\oplus} = \begin{pmatrix} \frac{1}{3} & \\ & \frac{2}{3} \end{pmatrix} V_{\mathrm{CL}}, \quad \Delta M^{\oplus} = \begin{pmatrix} \frac{1}{3} & \\ & \frac{2}{3} \end{pmatrix} \Delta M. \tag{13}$$

- Coupled isospin degenerates to the uncoupled situation

Smeared Coulomb potential

- Exponential charge distribution

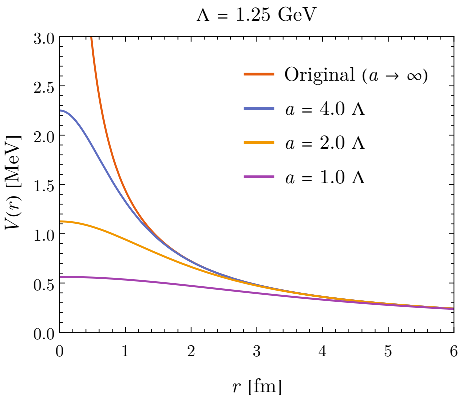
$$\rho(r) = \frac{a^3}{8\pi} e^{-ar}, \qquad (14)$$

- Non-point Coulomb potential

$$V_{\rm CL}(r) = Nk\frac{1}{r}\left[1 - G_4(ar)e^{-ar}\right] \qquad (15)$$

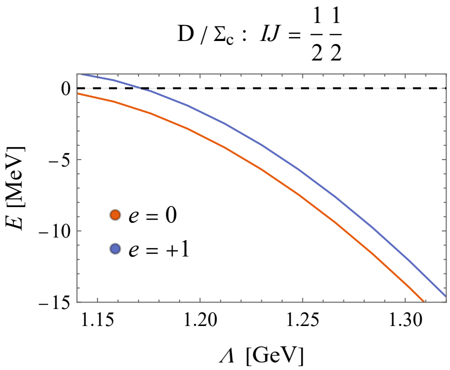
$$G_4(x) = 1 + \frac{11}{16}x + \frac{3}{16}x^2 + \frac{1}{48}x^3 \qquad (16)$$

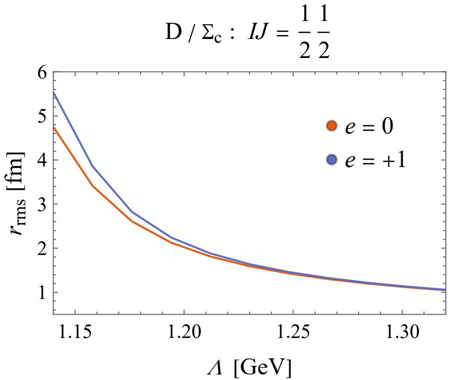
where a is taken to 2Λ.



Smeared Coulomb potentials, taking at $\Lambda = 1.25$ GeV.

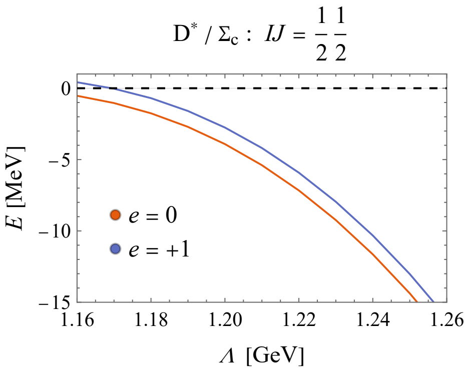
Results of $D\Sigma_c^{(*)}$ system, $I = 1/2$

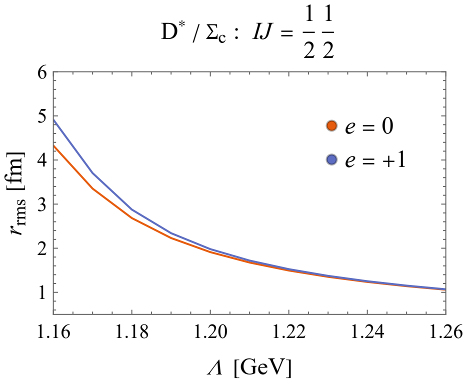




- $I = 1/2$: Lesser influence of Coulomb interaction
  - Due to the cancel of ρ, ω exchanges
- $I = 3/2$ states are unbound by only S-D mixing

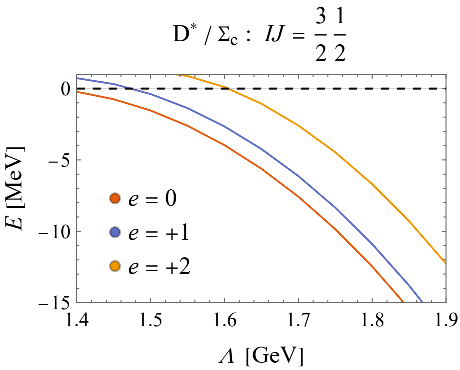
Results of $D^*\Sigma_c^{(*)}$ system, $I = 1/2$

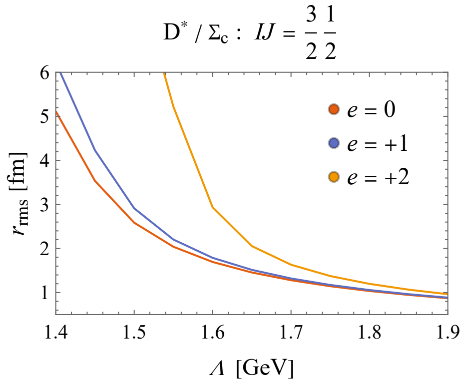




- $I = 1/2$: Lesser influence of Coulomb interaction
  - Relatively smaller than $D\Sigma_c^{(*)}$ systems
- Well bound in $J = 1/2,\ 3/2, (5/2)$

Results of $D^*\Sigma_c$ system, $I = 3/2$



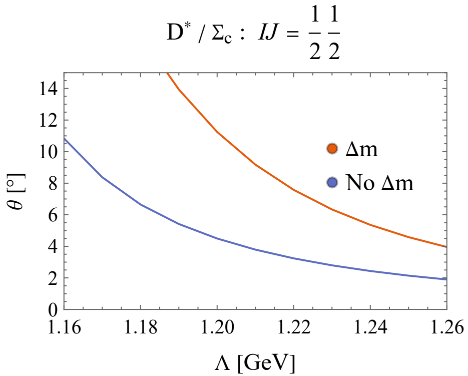


- $I = 3/2$: significant impact of Coulomb interaction
- Especially the absolute exotic state $D^{*+}\Sigma_c^{(*)++}(P_{cc}^{+++})$ with three positive charges

Isospin breaking angle

- Isospin mixing (breaking) angles: $\theta = \frac{180}{\pi} \arccos(\sqrt{P_{I=1/2}})$

	$D\Sigma_c$		$D\Sigma_c^*$		$D^*\Sigma_c$		$D^*\Sigma_c^*$	
J^P	Λ	θ	Λ	θ	Λ	θ	Λ	θ
	1.18	17.2°			1.18	17.4°	1.12	20.3°
$\frac{1}{2}^-$	1.25	7.2°	⋯	⋯	1.22	7.6°	1.16	9.3°
	1.30	4.7°			1.25	4.6°	1.19	5.5°
			1.18	17.1°	0.93	17.9°	1.08	19.5°
$\frac{3}{2}^-$	⋯	⋯	1.25	7.3°	1.00	4.8°	1.14	6.3°
			1.30	4.8°	1.04	3.0°	1.18	3.7°
							0.88	16.2°
$\frac{5}{2}^-$	⋯	⋯	⋯	⋯	⋯	⋯	0.95	4.9°
							1.00	2.9°



Summary

- **Repulsive Coulomb interactions may be important in $D^{(*)}\Sigma_c^{(*)}$**
  - $I = 1/2$: lesser influence
  - $I = 3/2$: huge impact, mass splitting up to 10^0 MeVs
  - Especially in absolute charge exotic state P_{cc}^{+++} $(D^{(*)+}\Sigma_c^{(*)++})$
- **Isospin breaking: relatively weak $(\theta : 3^\circ \sim 20^\circ)$**
  - Weak enough at high binding energy ($\sim>$ 10 MeV, $\sim< 5^\circ$)
  - Diagonal approximation: to simplify calculation for further studies, i.e. spin coupled channel

Thanks for listening !