

含双奇异夸克分子态类型四夸克态

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第八届强子谱和强子结构研讨会 2025年7月11日至15日 广西桂林

Fu-Lai Wang, Si-Qiang Luo, Ri-Qing Qian, and Xiang Liu, Phys. Rev. D 110, 114041 (2024)

提 纲

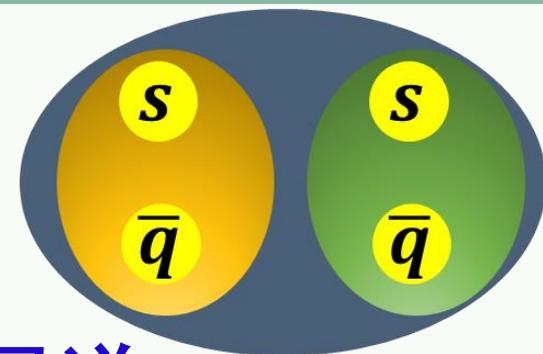
1 研究背景

2 含双奇异夸克分子态类型四夸克态候选者的质量谱

3 含双奇异夸克分子态类型四夸克态候选者的强衰变行为

4 含双奇异夸克分子态类型四夸克态候选者的电磁性质

5 总结



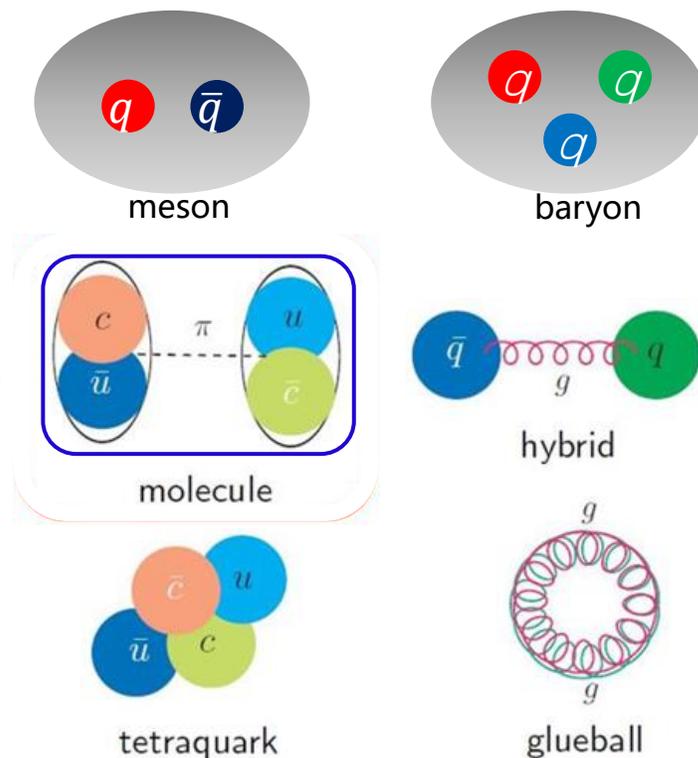
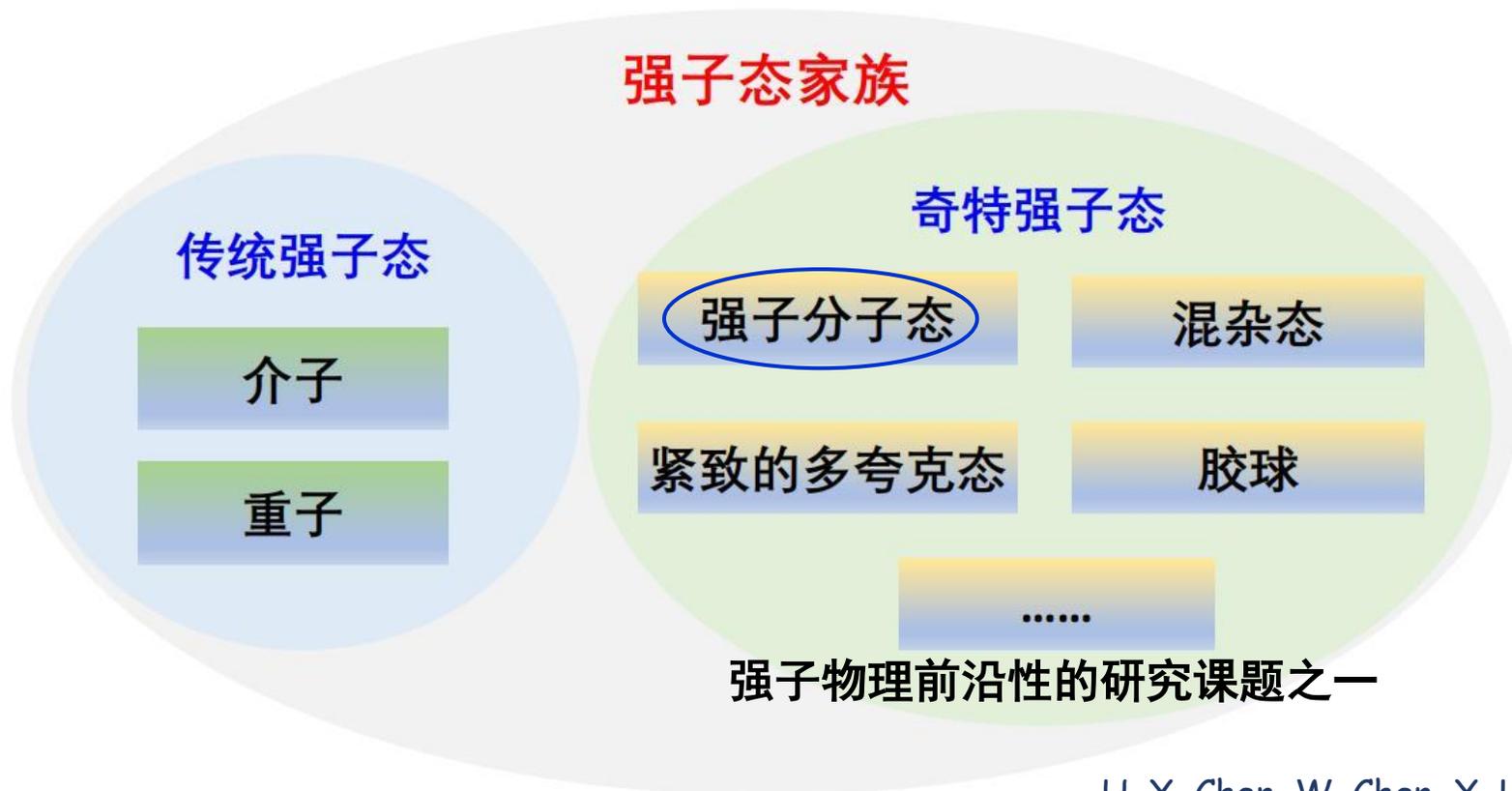


1

研究背景

强子物理的研究对象——强子态（夸克与胶子）

- 强子物理主要研究强子态的谱学和结构等问题。



H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, Phys. Rept. 639, 1 (2016).

- 为理解强子的内部结构与强相互作用的非微扰问题提供重要的信息；
- 在奇特强子态的研究中，过去二十多年强子分子态引起了大家的广泛关注。

强子分子态的定义和典型特征

- 强子分子态是由两个(多个)色单态的强子形成的松散束缚态。

$P_{\psi}^N(4312)$

$P_{\psi}^N(4440)$ $\Sigma_c \bar{D}^{(*)}$ 分子态

$P_{\psi}^N(4457)$

[LHCb], PRL 122, 222001 (2019)

$T_{cc}^+(3875)$ DD^* 分子态

[LHCb], Nature Phys. 18, 751-754(2022).

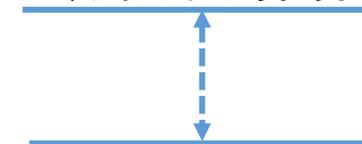
- 强子分子态的典型特征:

➤ 强子分子态是**束缚态**, 组分强子的阈值大于强子分子态的质量;

➤ 强子分子态的**束缚较松散** (较小的束缚能和较大的尺寸)。

H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, Phys. Rept. 639, 1 (2016).

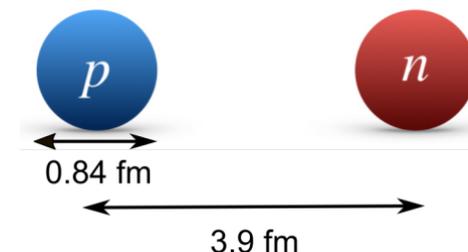
组分强子的阈值



强子分子态

- 氘核是由质子与中子形成的分子态类型六夸克态, 有较小的束缚能(~ 2.225 MeV)和较大的尺寸(~ 3.9 fm)。

R. Machleidt, PRC 63, 024001 (2001).



R. Pohl et al., Nature 466, 213 (2010).

2019年关于 P_c 态的高精度实验数据支持隐粲分子态类型五夸克态的解释

PHYSICAL REVIEW LETTERS 122, 222001 (2019)

Editors' Suggestion

Featured in Physics

Observation of a Narrow Pentaquark State, $P_c(4312)^+$, and of the Two-Peak Structure of the $P_c(4450)^+$

第一轮+第二轮

$\Lambda_b^0 \rightarrow J/\psi p K^-$

R. Aaij *et al.**
(LHCb Collaboration)

(Received 6 April 2019; published 5 June 2019)

$$P_c^+(4312) : M = 4311.9 \pm 0.7_{-0.6}^{+6.8} \text{ MeV},$$

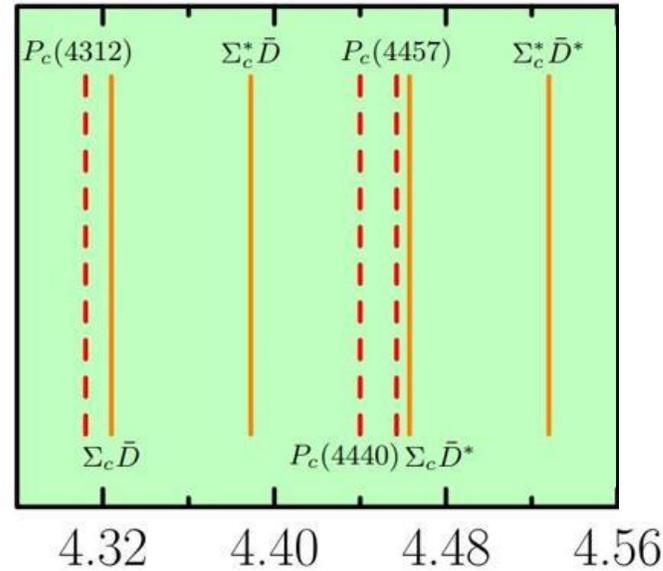
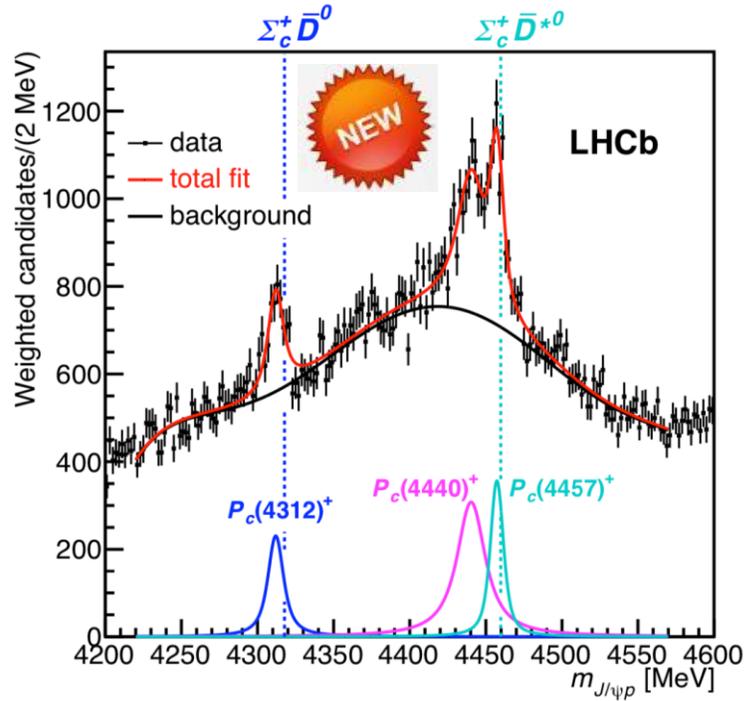
$$\Gamma = 9.8 \pm 2.7_{-4.5}^{+3.7} \text{ MeV},$$

$$P_c^+(4440) : M = 4440.3 \pm 1.3_{-4.7}^{+4.1} \text{ MeV},$$

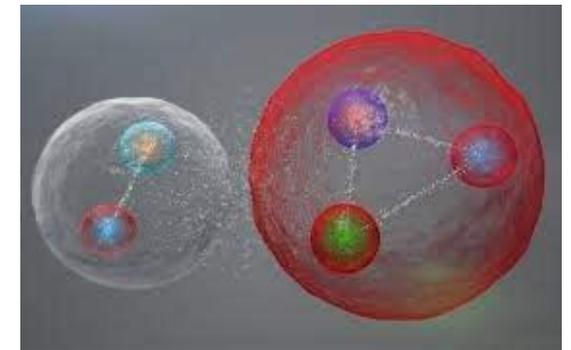
$$\Gamma = 20.6 \pm 4.9_{-10.1}^{+8.7} \text{ MeV},$$

$$P_c^+(4457) : M = 4457.3 \pm 0.6_{-1.7}^{+4.1} \text{ MeV},$$

$$\Gamma = 6.4 \pm 2.0_{-1.9}^{+5.7} \text{ MeV}.$$



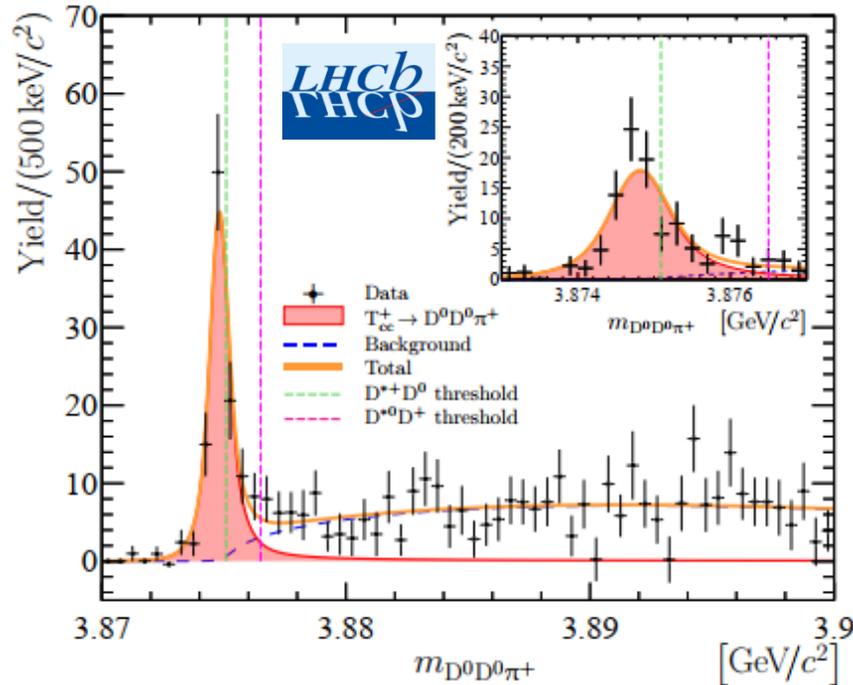
S波 $\Sigma_c \bar{D}^{(*)}$ 分子态的特征能谱



隐粲分子态类型五夸克态

$T_{cc}^+(3875)$ 的实验发现@2021年

nature physics LETTERS
<https://doi.org/10.1038/s41567-022-01614-y>
 OPEN
Observation of an exotic narrow doubly charmed tetraquark
 绝对奇特态 (味道)
 LHCb Collaboration*



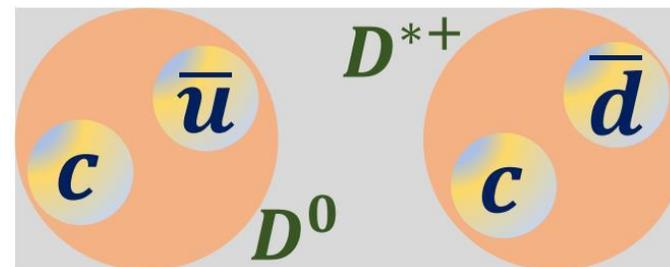
➤ 质量与宽度:

$$\delta m = -273 \pm 61 \pm 5_{-14}^{+11} \text{ keV} \quad \delta m^U = -361 \pm 40 \text{ keV}$$

$$\Gamma = 410 \pm 165 \pm 43_{-38}^{+18} \text{ keV} \quad \Gamma^U = 47.8 \pm 1.9 \text{ keV}$$

➤ $J^P = 1^+$ 同位旋标量的双粲四夸克态。

同位旋标量且 $J^P = 1^+$ 的 DD^* 分子态



$T_{cc}^+(3875)$ 的实验发现强烈的支持同位旋标量 $J^P = 1^+$ 的 DD^* 分子态的解释, 为在粲能区存在分子态类型四夸克态提供了强有力的实验证据支持。

$T_{cc}^+(3875)$ 伙伴态的理论研究

$$T_{ss} \leftarrow T_{cc}^+(3875) \rightarrow T_{bc} \rightarrow T_{bb}$$

质量 

PHYSICAL REVIEW LETTERS **132**, 201902 (2024)

Bound Isoscalar Axial-Vector $bc\bar{u}\bar{d}$ Tetraquark T_{bc} from Lattice QCD Using Two-Meson and Diquark-Antidiquark Variational Basis

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²Department of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha R

 (Received 11 September 2023; revised 1 April 2024; accepted 19 April 2024; pub

PHYSICAL REVIEW D **108**, 054502 (2023)

Lattice study on a tetraquark state T_{bb} in the HAL QCD method

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¹Yukawa Institute for Theoretical Physics Kyoto University, Kyoto 606-8502, Japan

²Center for Gravitational Physics and Quantum Information, Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan

³RIKEN Nishina Center (RNC), Saitama 351-0198, Japan

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 (Received 19 June 2023; accepted 9 August 2023; published 11 September 2023)

$T_{cc}^+(3875)$ 的实验发现激发了理论上研究 T_{cc} 伙伴态的热潮，目前主要关注 T_{bc}/T_{bb} 系统。
然而， T_{ss} 系统没有得到理论上的关注。

绝对奇特态（味道）对于强子分子态的鉴别具有重要意义

通过研究 XYZ 态和 $T_{cc}^+(3875)$ 态确定粲能区存在四夸克态的启示：

绝对奇特态（味道） 对于奇特强子态的鉴别具有重要意义。

Hidden charm tetraquarks			Double charm tetraquark		粲能区存在分子态类型 四夸克态被广泛接受
$X(3872)$	$Z_c(3900)$	$X(3915)$	$T_{cc}^+(3875)$ 绝对奇特态（味道）		
$Z(3930)$	$Y(3940)$	$Y(4008)$			
$Z_c(4020)$	$Z_c(4025)$	$Z^+(4051)$			
...					
Hidden strangeness tetraquarks			Double strangeness tetraquark		轻夸克能区存在分子 态类型四夸克态？
$f_0(980)$ 1982年 Isgur, ...	$f_1(1285)$	$f_0(1710)$?		
...					

F. K. Guo, C. Hanhart, U. Meissner, Q. Wang, Q. Zhao, B. S. Zou, Rev. Mod. Phys. 90, 015004 (2018).

- $\bar{K}^{(*)}\bar{K}^{(*)}$ 系统是 $D^{(*)}D^{(*)}$ 系统的奇异夸克伙伴态。
- $\bar{K}^{(*)}\bar{K}^{(*)}$ 系统的优势——味道绝对奇特态。

含双奇异夸克分子态类型四夸克态与含双粲分子态类型四夸克态 $T_{cc}^+(3875)$ 的研究同等重要，含双奇异夸克分子态类型四夸克态的研究为轻夸克能区存在分子态类型多夸克态提供重要信息。

含双奇异夸克分子态类型多夸克态的理论与实验研究

理论上含双奇异夸克分子态类型多夸克态的研究比较早，可以追溯到上世纪七十年代，其中最著名的是 H 双重子态的研究。

Perhaps a Stable Dihyperon*

R. L. Jaffe†

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305, and Department of Physics and Laboratory of Nuclear Science, ‡ Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 1 November 1976)

In the quark bag model, the same gluon-exchange forces which make the proton lighter than the $\Delta(1236)$ bind six quarks to form a stable, flavor-singlet (with strangeness of -2) $J^P = 0^+$ dihyperon (H) at 2150 MeV. Another isosinglet dihyperon (H^*) with $J^P = 1^+$ at 2335 MeV should appear as a bump in $\Lambda\Lambda$ invariant-mass plots. Production and decay systematics of the H are discussed.

Robert L. Jaffe, PRL 38, 195-198(1977).

$uuddss(\Lambda\Lambda)$

$J^P = 0^+, I = 0$

$m_H \approx 2150$ MeV, 低于 $\Lambda\Lambda$ 道阈值80 MeV

口袋模型

自从R. L. Jaffe预言存在 $\Lambda\Lambda$ 强子分子态，理论与实验上引起了研究 $\Lambda\Lambda$ 强子分子态的热潮，然而到目前为止实验上没有给出存在 $\Lambda\Lambda$ 强子分子态的明确证据。

H. Clement, Prog. Part. Nucl. Phys. 93, 195 (2017).

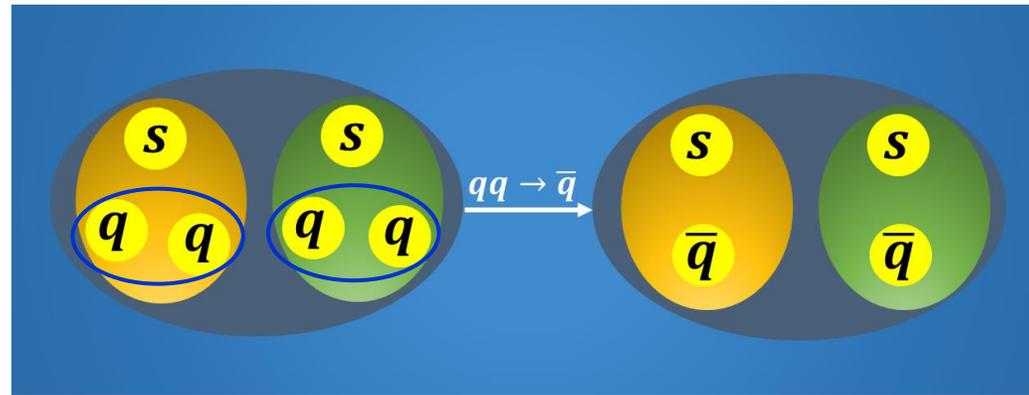
$\bar{K}^{(*)}\bar{K}^{(*)}$ 系统是 $\Lambda\Lambda$ 系统的伙伴态

到目前为止，高能物理实验上观测到的六夸克态候选者远远少于四夸克态候选者。

理论与实验上应该优先关注含双奇异夸克分子态类型四夸克态候选者。



$$3 \otimes 3 = \bar{3} \oplus 6 \quad \bar{3} \otimes 3 = 1 \text{ (颜色)}$$



$\bar{K}^{(*)}\bar{K}^{(*)}$ 系统是 $\Lambda\Lambda$ 系统的伙伴态

含双奇异夸克分子态类型四夸克态的研究为轻夸克能区存在分子态类型多夸克态提供重要信息。



2

含双奇异夸克分子态类型四夸克态候选者 的质量谱

强子间相互作用是研究强子分子态质量谱的重要输入

- 强子间相互作用是研究强子分子态质量谱的重要输入，然而强相互作用在低能情况下不能微扰求解。

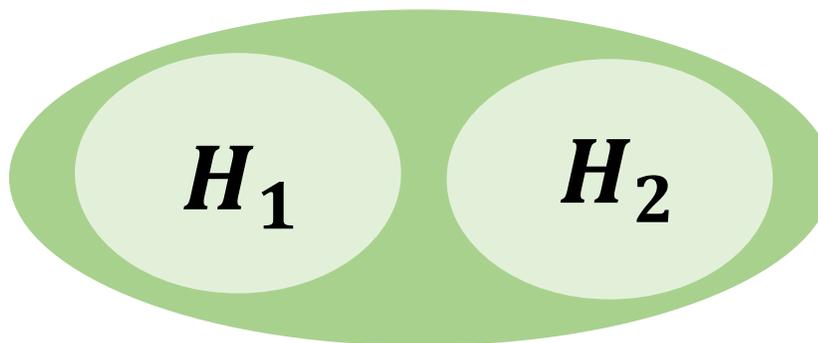
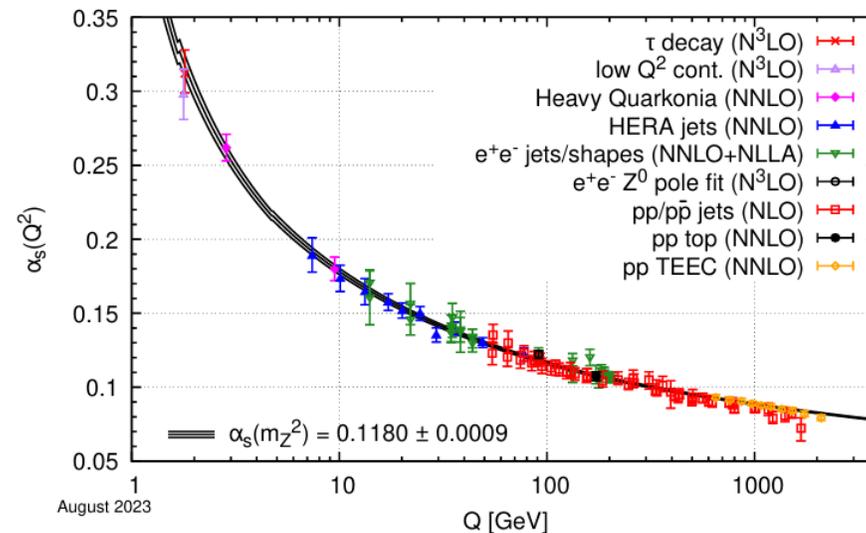
$$-\frac{1}{2\mu}\left(\nabla^2 - \frac{l(l+1)}{r^2}\right)\psi(r) + \boxed{V(r)}\psi(r) = E\psi(r)$$

- 束缚能 E
- 数值空间波函数 $\psi(r)$

- 讨论强子间相互作用的模型与方法：

- ① 单玻色子交换模型；
- ② 格点QCD模拟；
- ③ QCD求和规则；
- ④ 手征微扰理论；
- ⑤ 组分夸克模型；

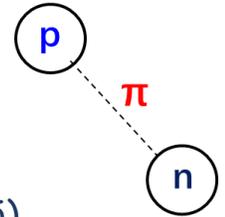
.....



H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, Phys. Rept. 639, 1 (2016).

单玻色子交换模型

- **介子交换模型**:核力是两个核子间交换 π 产生的, 质子和中子间通过交换 π 形成氘核。 π 交换只提供核子间相互作用的长程力。



H. Yukawa, Proc. Phys. Math. Soc. Jap. 17 (1935).

- 在强子间相互作用的研究中通过考虑**不同的轻味介子**描述**长程、中程和短程力**:

$$\sigma, \quad \mathbb{P} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}, \quad \mathbb{V} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}.$$

中程

$0.8 \text{ fm} \leq r \leq 2.0 \text{ fm}$

长程

$r \geq 2.0 \text{ fm}$

短程

$r \leq 0.8 \text{ fm}$

- 在强子间相互作用的研究中考虑各种**效应（修正）**——更精细的结构:
 - ① S - D 波混合效应;
 - ② 耦合道效应;
 - ③ 同位旋破缺效应;

H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, Phys. Rept. 639, 1 (2016).

单玻色子交换模型对高能物理实验发现的强子分子态候选者的成功预言

CPC(HEP & NP), 2012, 36(1): 6-13

Chinese Physics C

Vol. 36, No. 1, Jan., 2012

2011

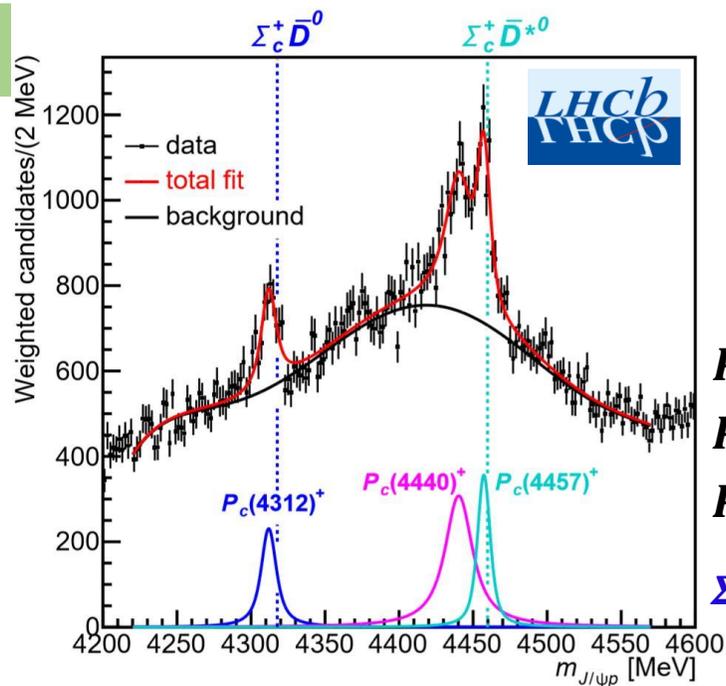
Possible hidden-charm molecular baryons composed of an anti-charmed meson and a charmed baryon*

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LIU Xiang(刘翔)^{2,4,2)} ZHU Shi-Lin(朱世琳)^{1,3)}

¹ Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

Abstract: Using the one-boson-exchange model, we studied the possible existence of very loosely bound hidden-charm molecular baryons composed of an anti-charmed meson and a charmed baryon. Our numerical results indicate that the $\Sigma_c \bar{D}^*$ and $\Sigma_c \bar{D}$ states exist, but that the $\Lambda_c \bar{D}$ and $\Lambda_c \bar{D}^*$ molecular states do not.

2019



$P_\psi^N(4312)$
 $P_\psi^N(4440)$
 $P_\psi^N(4457)$
 $\Sigma_c \bar{D}^{(*)}$ 分子态

[LHCb] PRL 122, 222001 (2019).

2013

One-channel analysis of the possible $D^{(*)}D^{(*)}$, $\bar{B}^{(*)}\bar{B}^{(*)}$ and $D^{(*)}\bar{B}^{(*)}$ molecular states

PHYSICAL REVIEW D 88, 114008 (2013)

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¹Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

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⁴Research Center for Hadron and CSR Physics,

⁵Center of High Energy Physics

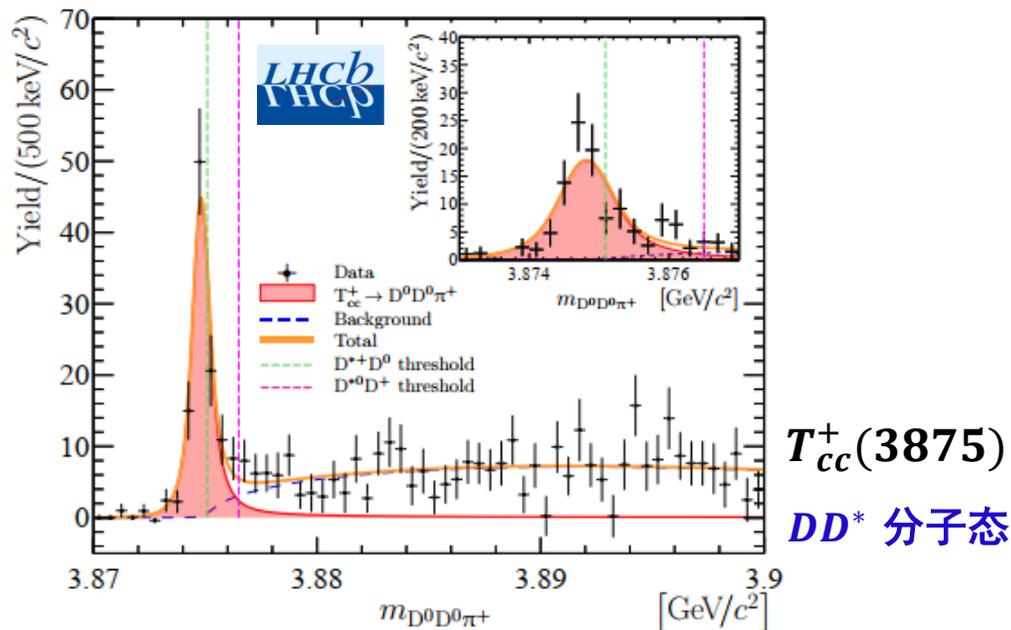
⁶Collaborative Innovation Center of Extreme Matter Structure

(Received 22 November 2012; revised 12 December 2012)

J^P	$D^{(*)}D^{(*)}$	$\bar{B}^{(*)}\bar{B}^{(*)}$	$D^{(*)}\bar{B}^{(*)}$	OBE	China	
0^+			***			
	Λ (GeV)	0.95	1.00	1.05	1.10	
	B.E. (MeV)	0.47	5.44	18.72	42.82	
	M (MeV)	3875.38	3870.41	3857.13	3833.03	
	r_{rms} (fm)	4.46	1.58	0.91	0.64	
0	1^+	P_1 (%)	97.97	92.94	85.64	77.88
		P_2 (%)	0.58	0.55	0.32	0.15
		P_3 (%)	1.41	6.42	13.97	21.91
		P_4 (%)	0.04	0.09	0.08	0.05

9年

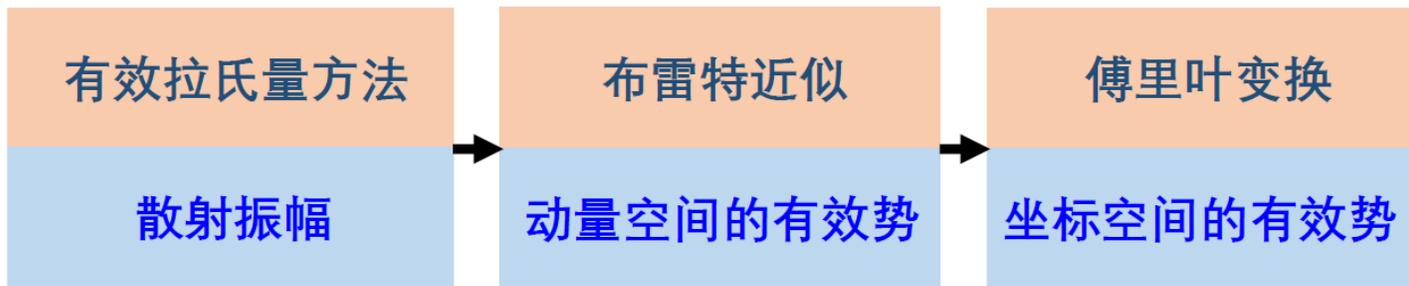
2021



T_{cc}^+ (3875)
 DD^* 分子态

[LHCb] Nature Phys. 18 (2022) 7, 751-754.

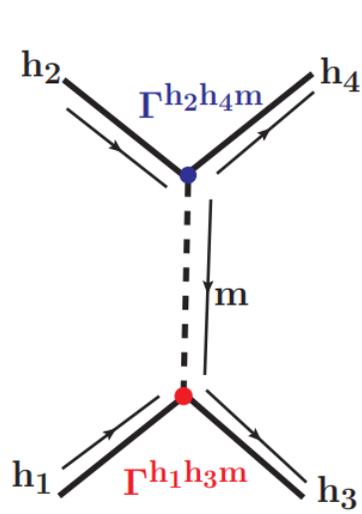
在单玻色子交换模型中讨论强子分子态候选者质量谱的步骤



E. M. Lifshitz, Quantum electrodynamics, 1982.

求解耦合道薛定谔方程

束缚态解: (i) 束缚能, (ii) 方均根半径, (iii) 每个道的贡献



$$i\mathcal{M}^{h_1 h_2 \rightarrow h_3 h_4}(q) = \sum_{m=\sigma, P, V} i\Gamma_{(\mu)}^{h_1 h_3 m} P_m^{(\mu\nu)} i\Gamma_{(\nu)}^{h_2 h_4 m}$$

$$\mathcal{V}_E^{h_1 h_2 \rightarrow h_3 h_4}(q) = -\frac{\mathcal{M}^{h_1 h_2 \rightarrow h_3 h_4}(q)}{\sqrt{\prod_i 2m_i \prod_f 2m_f}}$$

$$\mathcal{V}_E^{h_1 h_2 \rightarrow h_3 h_4}(\mathbf{r}) = \int \frac{d^3 q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \mathcal{V}_E^{h_1 h_2 \rightarrow h_3 h_4}(q) \mathcal{F}^2(q^2, m_E^2)$$

形状因子: $\mathcal{F}(q^2, m_E^2) = \frac{\Lambda^2 - m_E^2}{\Lambda^2 - q^2}$ N. A. Tornqvist, Z. Phys. C 61, 525 (1994).
 N. A. Tornqvist, Nuovo Cim. A 107, 2471 (1994).¹⁷

F. L. Wang, R. Chen, Z. W. Liu, and X. Liu, PRC 101, 025201 (2020).

Models	$\Lambda(\text{MeV})$	$E(\text{MeV})$	$r_{\text{RMS}}(\text{fm})$
OPE(IN)	/	/	/
OPE(Y)	1064	-2.23	3.74
OBE(IN)	1174	-2.25	3.67
OBE(Y)	864	-2.26	3.75

$\Lambda \sim 1.0 \text{ GeV}$

研究 $\bar{K}^{(*)}\bar{K}^{(*)}$ 系统束缚性质的有效拉式量——有效拉氏量近似

- 根据对称性要求，研究 $\bar{K}^{(*)}\bar{K}^{(*)}$ 系统束缚性质的有效拉式量构造为：

$$\begin{aligned} \mathcal{L}_{\bar{K}^{(*)}\bar{K}^{(*)}\mathbb{P}} &= -\frac{2ig'}{f_\pi} v^\alpha \varepsilon_{\alpha\mu\nu\lambda} \bar{K}_b^{*\mu} \bar{K}_a^{*\lambda\dagger} \partial^\nu \mathbb{P}_{ba} \\ &\quad - \frac{2g'}{f_\pi} (\bar{K}_b^{*\mu} \bar{K}_a^\dagger + \bar{K}_b \bar{K}_a^{*\mu\dagger}) \partial_\mu \mathbb{P}_{ba}, \\ \mathcal{L}_{\bar{K}^{(*)}\bar{K}^{(*)}\sigma} &= -2g'_s \bar{K}_a \sigma \bar{K}_a^\dagger + 2g'_s \bar{K}_{a\mu}^* \sigma \bar{K}_a^{*\mu\dagger}, \\ \mathcal{L}_{\bar{K}^{(*)}\bar{K}^{(*)}\mathbb{V}} &= -\sqrt{2}\beta' g_V \bar{K}_b \bar{K}_a^\dagger v \cdot \mathbb{V}_{ba} + \sqrt{2}\beta' g_V \bar{K}_{b\mu}^* \bar{K}_a^{*\mu\dagger} v \cdot \mathbb{V}_{ba} \\ &\quad - 2\sqrt{2}i\lambda' g_V \bar{K}_b^{*\mu} \bar{K}_a^{*\nu\dagger} (\partial_\mu \mathbb{V}_\nu - \partial_\nu \mathbb{V}_\mu)_{ba} \\ &\quad - 2\sqrt{2}\lambda' g_V v^\lambda \varepsilon_{\lambda\mu\alpha\beta} (\bar{K}_b \bar{K}_a^{*\mu\dagger} + \bar{K}_b^{*\mu} \bar{K}_a^\dagger) \partial^\alpha \mathbb{V}_{ba}^\beta. \end{aligned}$$

$$\begin{aligned} \bar{K}^{(*)} &= (K^{(*)-}, \bar{K}^{(*)0}) \\ \mathbb{P} &= \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} \\ \mathbb{V} &= \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix} \end{aligned}$$

M. Z. Liu, J. J. Xie, and L. S. Geng, PRD 102, 091502 (2020).

J. Z. Wang, Z. Y. Lin, B. Wang, L. Meng, and S. L. Zhu, PRD 110, 114003(2024).

- 描述相互作用顶点的耦合常数通过拟合实验数据或采用唯象模型获得。

➤ $g' = 1.12$ ：拟合 $K^* \rightarrow K\pi$ 宽度的实验数据 J. Z. Wang, Z. Y. Lin, B. Wang, L. Meng, and S. L. Zhu, PRD 110, 114003(2024).

➤ $g'_s = 0.76$, $\lambda' = 0.56 \text{ GeV}^{-1}$ ：夸克模型 D. O. Riska and G. E. Brown, Nucl. Phys. A 679, 577 (2001).

➤ $\beta' = 0.835$ ：矢量介子的隐局域对称性 R. Molina, T. Branz, and E. Oset, PRD 82, 014010 (2010).

- 根据玻色-爱因斯坦统计， $\bar{K}^{(*)}\bar{K}^{(*)}$ 系统允许的量子数：

$$[\bar{K}\bar{K}] : 1(0^+) \quad [\bar{K}^*\bar{K}^*] : 1(0^+), 0(1^+), 1(2^+).$$

$\bar{K}^{(*)}\bar{K}^{(*)}$ 系统的OBE相互作用势

$$\begin{aligned}
 V_{I,J}^{\bar{K}\bar{K}\rightarrow\bar{K}\bar{K}} &= -g_s'^2 Y_\sigma + \frac{1}{2}\beta'^2 g_V^2 \mathcal{G}_V(I) Y_V, \\
 V_{I,J}^{\bar{K}\bar{K}^*\rightarrow\bar{K}\bar{K}^*} &= -g_s'^2 \mathcal{O}_1[J] Y_\sigma + \frac{1}{2}\beta'^2 g_V^2 \mathcal{O}_1[J] \mathcal{G}_V(I) Y_V, \\
 V_{I,J}^{\bar{K}\bar{K}^*\rightarrow\bar{K}^*\bar{K}} &= \frac{g'^2}{3f_\pi^2} (\mathcal{O}_2[J] \mathcal{Z}_r + \mathcal{O}_3[J] \mathcal{T}_r) \mathcal{H}'_P(I) Y_{P0} \\
 &+ \frac{2}{3}\lambda'^2 g_V^2 (2\mathcal{O}_2[J] \mathcal{Z}_r - \mathcal{O}_3[J] \mathcal{T}_r) \mathcal{G}'_V(I) Y_{V0}, \\
 V_{I,J}^{\bar{K}^*\bar{K}^*\rightarrow\bar{K}^*\bar{K}^*} &= -g_s'^2 \mathcal{O}_4[J] Y_\sigma \\
 &- \frac{g'^2}{3f_\pi^2} (\mathcal{O}_5[J] \mathcal{Z}_r + \mathcal{O}_6[J] \mathcal{T}_r) \mathcal{H}_P(I) Y_P \\
 &+ \frac{1}{2}\beta'^2 g_V^2 \mathcal{O}_4[J] \mathcal{G}_V(I) Y_V \\
 &- \frac{2}{3}\lambda'^2 g_V^2 (2\mathcal{O}_5[J] \mathcal{Z}_r - \mathcal{O}_6[J] \mathcal{T}_r) \mathcal{G}_V(I) Y_V,
 \end{aligned}$$

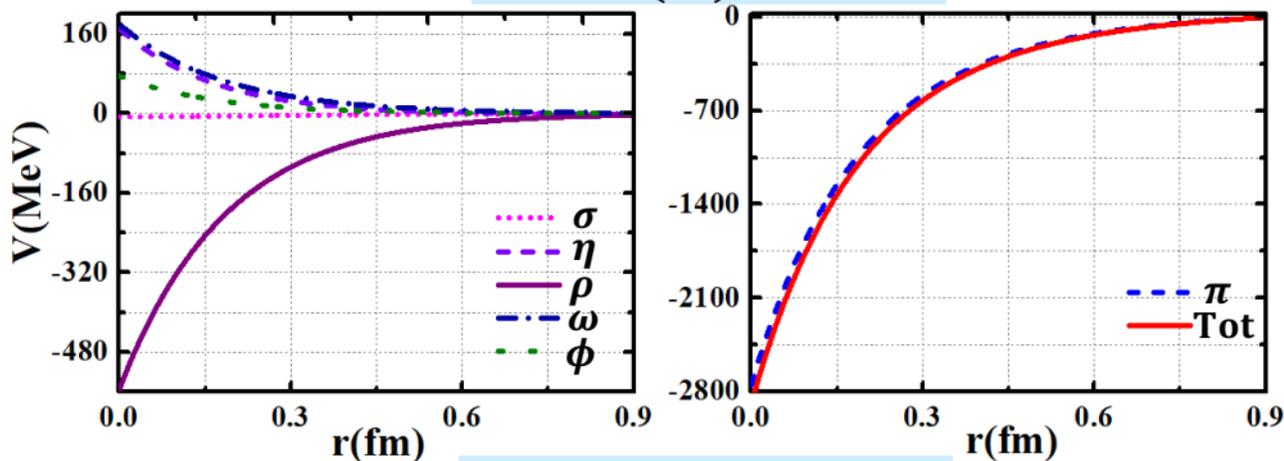
$$\begin{aligned}
 V_{I,J}^{\bar{K}\bar{K}\rightarrow\bar{K}^*\bar{K}^*} &= \frac{g'^2}{3f_\pi^2} (\mathcal{O}_7[J] \mathcal{Z}_r + \mathcal{O}_8[J] \mathcal{T}_r) \mathcal{H}_P(I) Y_P \\
 &+ \frac{2}{3}\lambda'^2 g_V^2 (2\mathcal{O}_7[J] \mathcal{Z}_r - \mathcal{O}_8[J] \mathcal{T}_r) \mathcal{G}_V(I) Y_V, \\
 V_{I,J}^{\bar{K}\bar{K}^*\rightarrow\bar{K}^*\bar{K}^*} &= \frac{g'^2}{3f_\pi^2} (\mathcal{O}_9[J] \mathcal{Z}_r + \mathcal{O}_{10}[J] \mathcal{T}_r) \mathcal{H}''_P(I) Y_{P1} \\
 &+ \frac{2}{3}\lambda'^2 g_V^2 (2\mathcal{O}_9[J] \mathcal{Z}_r - \mathcal{O}_{10}[J] \mathcal{T}_r) \mathcal{G}''_V(I) Y_{V1}, \\
 V_{I,J}^{\bar{K}^*\bar{K}\rightarrow\bar{K}^*\bar{K}^*} &= -\frac{g'^2}{3f_\pi^2} (\mathcal{O}_{11}[J] \mathcal{Z}_r + \mathcal{O}_{12}[J] \mathcal{T}_r) \mathcal{H}''_P(I) Y_{P1} \\
 &- \frac{2}{3}\lambda'^2 g_V^2 (2\mathcal{O}_{11}[J] \mathcal{Z}_r - \mathcal{O}_{12}[J] \mathcal{T}_r) \mathcal{G}''_V(I) Y_{V1}.
 \end{aligned}$$

$$Y_i = \begin{cases} |q_i| \leq m, & \frac{e^{-m_i r} - e^{-\Lambda_i^2 r}}{4\pi r} - \frac{\Lambda_i^2 - m_i^2}{8\pi\Lambda_i} e^{-\Lambda_i r}, \\ |q_i| > m, & \frac{\cos(m'_i r) - e^{-\Lambda_i r}}{4\pi r} - \frac{\Lambda_i^2 + m_i'^2}{8\pi\Lambda_i} e^{-\Lambda_i r}, \end{cases} \quad m_i = \sqrt{m^2 - q_i^2}, \quad m'_i = \sqrt{q_i^2 - m^2}, \quad \text{and} \quad \Lambda_i = \sqrt{\Lambda^2 - q_i^2}$$

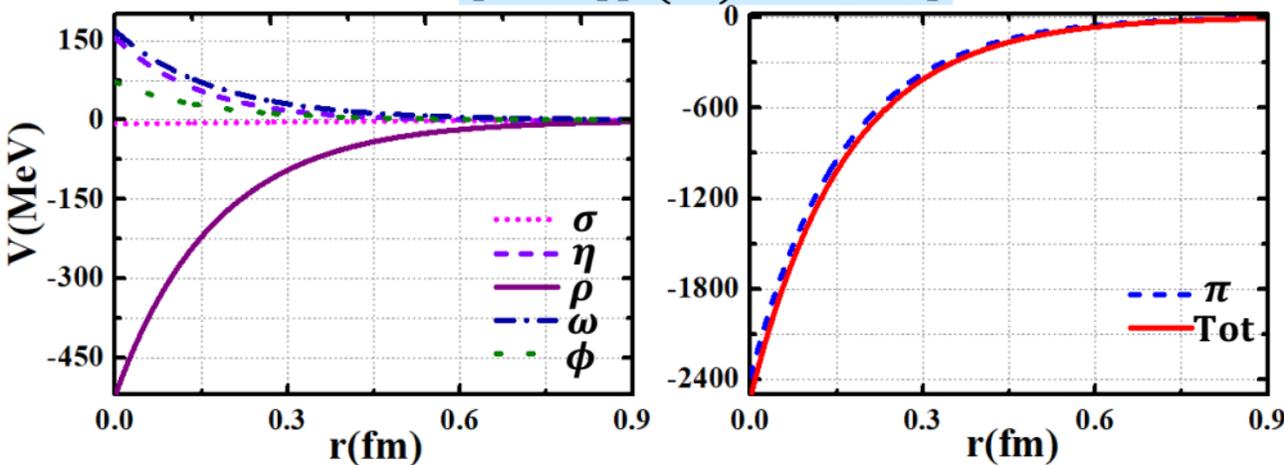
$$\mathcal{Z}_r = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} \quad \text{and} \quad \mathcal{T}_r = r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r}$$

$\bar{K}^{(*)}\bar{K}^{(*)}$ 系统的OBE相互作用势—— π 交换相互作用主导

$[\bar{K}\bar{K}^*][I(J^P) = 0(1^+)]$



$[\bar{K}^*\bar{K}^*][I(J^P) = 0(1^+)]$



- π 交换相互作用主导 $I(J^P) = 0(1^+)\bar{K}\bar{K}^*$ 和 $I(J^P) = 0(1^+)\bar{K}^*\bar{K}^*$ 系统的相互作用, 这与在单玻色子交换模型框架下研究强子分子态得到的普适结论一致。
H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, Phys. Rep. 639, 1 (2016).

- 与 π 交换相互作用相关的耦合常数通过拟合 $K^* \rightarrow K\pi$ 衰变宽度确定, 这为 π 交换相互作用提供可靠的估计。
J. Z. Wang, Z. Y. Lin, B. Wang, L. Meng, and S. L. Zhu, PRD 110, 114003(2024).

$I(J^P) = 1(0^+) \bar{K} \bar{K}$ 系统的相互作用：OBE模型与格点QCD模拟结果比较

PHYSICAL REVIEW D 77, 094507 (2008)

$K^+ K^+$ scattering length from lattice QCD

Silas R. Beane,¹ Thomas C. Luu,² Kostas Orginos,^{3,4} Assumpta Parreño,⁵ Martin J. Savage,⁶
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(Received 29 October 2007; published 21 May 2008)

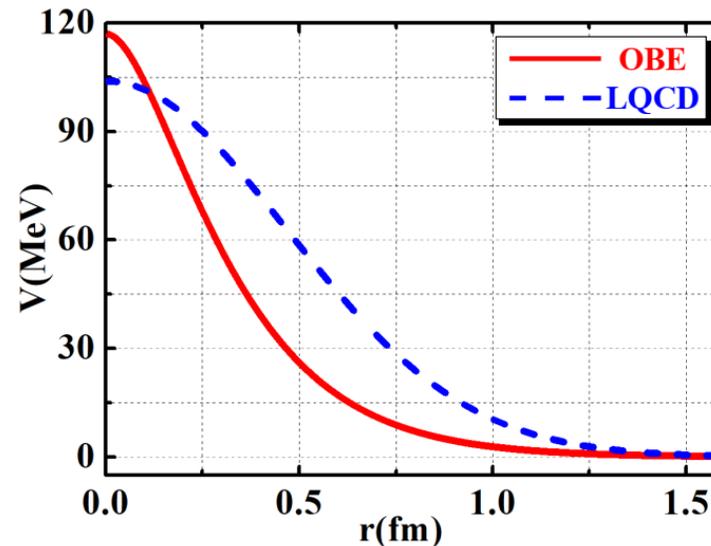
$I(J^P) = 1(0^+) \bar{K} \bar{K}$ 系统的相互作用用一个高斯函数描述：

$$V_{[\bar{K}\bar{K}][1(0^+)]} = U_{[\bar{K}\bar{K}]}^{I=1} e^{-(r/b)^2}$$

相互作用强度 104 MeV
相互作用力程 0.66 fm

Y. Kanada-En'yo and D. Jido, PRC 78, 025212 (2008).

散射长度： $a_{K^+ K^+[1(0^+)]} = -0.141 \pm 0.006$ fm



通过OBE模型与格点QCD模拟得到的 $I(J^P) = 1(0^+) \bar{K} \bar{K}$ 系统的相互作用结果一致。

$I(J^P) = 0(1^+) \bar{K} \bar{K}^*$ 系统的束缚性质

Scenarios	Λ (GeV)	E (MeV)	r_{RMS} (fm)	P_1 (%)	P_2 (%)	P_3 (%)	P_4 (%)
I 单道分析	1.21	-4.27	2.83	100.00			
	1.27	-14.83	1.47	100.00			
	1.33	-30.02	1.05	100.00			
II <i>S-D</i> 波混合分析	1.03	-3.88	3.86	95.98	4.02		
	1.11	-13.01	1.90	96.76	3.24		
	1.19	-30.37	1.19	96.60	3.40		
III 耦合道分析	0.79	-3.86	3.88	94.08	3.85	2.01	0.06
	0.83	-12.50	1.91	91.73	1.93	6.24	0.10
	0.87	-30.49	1.14	88.13	1.32	10.44	0.11

$I(J^P) = 0(1^+) \bar{K} \bar{K}^*$ 系统是含双奇异夸克分子态类型四夸克态候选者，这与基于手征夸克模型和夸克蜕定域色屏蔽模型得到的结论一致。

J. Z. Ji, Y. H. Xing, X. X. Wu, N. Xu, and Y. Tan, *CPC* 49, 013101(2025).

X. J. Liu, H. M. Zheng, D. Y. Chen, H. X. Huang, and J. L. Ping, arXiv:2505.12828.

$I(J^P) = 0(1^+) \bar{K}^* \bar{K}^*$ 系统的束缚性质

Scenarios	Bound-state properties				
	Λ (GeV)	E (MeV)	r_{RMS} (fm)		
I 单道分析	1.23	-4.55	2.31		
	1.30	-15.28	1.34		
	1.36	-29.95	1.00		
II S - D 波混合分析	Λ (GeV)	E (MeV)	r_{RMS} (fm)	$P_{ {}^3S_1\rangle}$ (%)	$P_{ {}^3D_1\rangle}$ (%)
	1.11	-3.99	2.56	98.02	1.98
	1.19	-14.45	1.46	97.43	2.57
	1.27	-32.64	1.04	97.18	2.82

Mode	Rate (MeV)	Mode	Rate (MeV)
$(K\pi)^\pm$	51.4 ± 0.8	$(K\pi)^0$	47.2 ± 0.5
$K^\pm \gamma$	0.050 ± 0.005	$K^0 \gamma$	0.117 ± 0.010

R. L. Workman et al. [Particle Data Group], PTEP 2022, 083C01 (2022).

当考虑 K^* 介子的质量分布时:

$2\mu_{[K^*K^*]}$	841.67 MeV		891.67 MeV		941.67 MeV	
Λ	E	r_{RMS}	E	r_{RMS}	E	r_{RMS}
1.23	-1.10	4.22	-4.55	2.31	-9.60	1.61
1.30	-7.96	1.83	-15.28	1.34	-24.05	1.08
1.36	-18.92	1.25	-29.95	1.00	-42.32	0.85

$I(J^P) = 0(1^+) \bar{K}^* \bar{K}^*$ 系统是含双奇异夸克分子态类型四夸克态候选者。

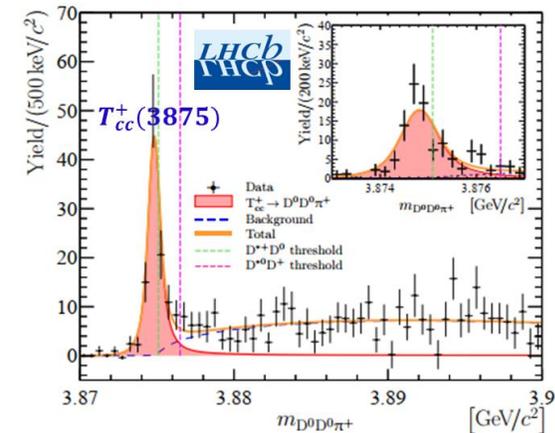
X. J. Liu, H. M. Zheng, D. Y. Chen, H. X. Huang, and J. L. Ping, arXiv:2505.12828.



3

含双奇异夸克分子态类型四夸克态候选者 的强衰变行为

$T_{cc}^+(3875)$ 作为 DD^* 分子态的强衰变行为的理论研究



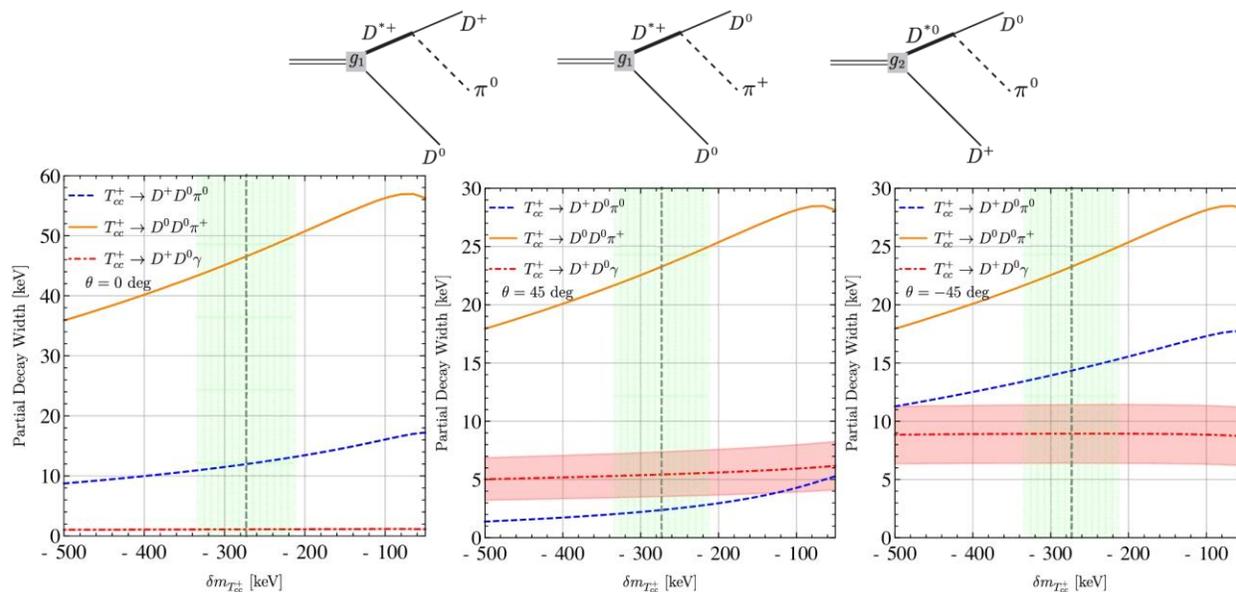
$$\delta m = -273 \pm 61 \pm 5^{+11}_{-14} \text{ keV}$$

$$\Gamma = 410 \pm 165 \pm 43^{+18}_{-38} \text{ keV}$$

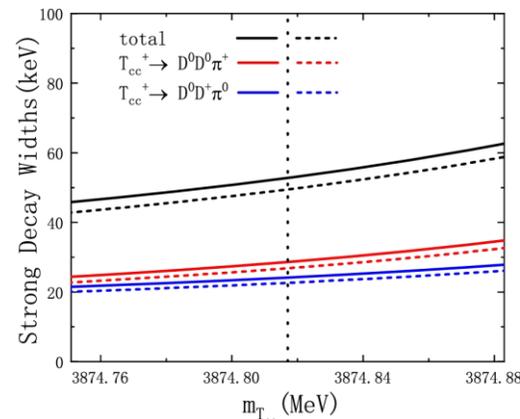
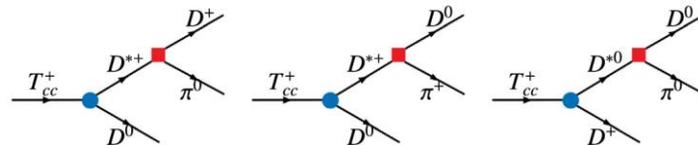
$$\delta m^U = -361 \pm 40 \text{ keV}$$

$$\Gamma^U = 47.8 \pm 1.9 \text{ keV}$$

DD^* 分子态



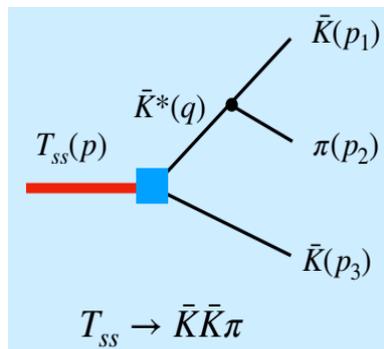
L. Meng, G. J. Wang, B. Wang, and S. L. Zhu, PRD 104, 051502 (2021).



X. Z. Ling, M. Z. Liu, L. S. Geng, E. Wang, and J. J. Xie, PLB 826, 136897 (2022).

有效拉氏量近似
树图水平的散射过程

$I(J^P) = 0(1^+) \bar{K} \bar{K}^*$ 分子态候选者的三体强衰变行为



$$\mathcal{L}_{T_{ss} \bar{K} \bar{K}^*} = g_{T_{ss} \bar{K} \bar{K}^*} T_{ss}^\mu \bar{K}_\mu^* \bar{K}.$$

温伯格组分条件

$$g_{T_{ss} \bar{K} \bar{K}^*}^2 = \frac{16\pi(m_{\bar{K}} + m_{\bar{K}^*})^{5/2} \sqrt{2|E_B|}}{(m_{\bar{K}} m_{\bar{K}^*})^{1/2}}$$

S. Weinberg, Phys. Rev. 130, 776-783 (1963).

$$i\mathcal{M}_{[\bar{K} \bar{K}^*][0(1^+)] \rightarrow \bar{K} \bar{K} \pi} = ig_{T_{ss} \bar{K} \bar{K}^*} \epsilon_{T_{ss}}^\lambda \frac{i(-g_{\lambda\rho} + q_\lambda q_\rho / m_{\bar{K}^*}^2)}{q^2 - m_{\bar{K}^*}^2 + im_{\bar{K}^*} \Gamma_{\bar{K}^*}} \sqrt{m_{\bar{K}^*} m_{\bar{K}}} \left(\frac{-2g'}{f_\pi} \right) (ip_2^\rho).$$

$$\Gamma = 2 \frac{1}{S} \frac{1}{2m_{T_{ss}}} \sum_{\text{pol.}} \frac{1}{3} \int |\mathcal{M}|^2 d\Phi_3 = 2 \frac{1}{S} \frac{1}{(2\pi)^3} \frac{1}{32m_{T_{ss}}^3} \sum_{\text{pol.}} \frac{1}{3} \int |\mathcal{M}|^2 dm_{12}^2 dm_{23}^2$$

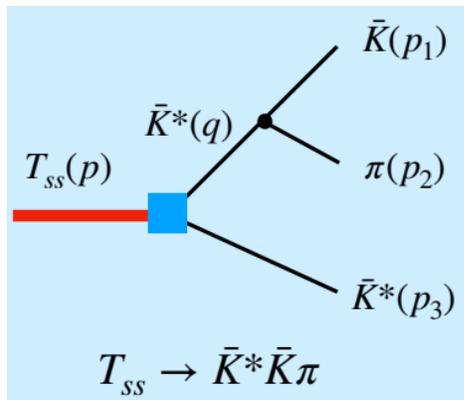
R. L. Workman et al. [Particle Data Group], PTEP 2022, 083C01 (2022).

E_B (MeV)	-4	-17	-30
$\Gamma_{[\bar{K} \bar{K}^*][0(1^+)] \rightarrow \bar{K} \bar{K} \pi}$ (MeV)	37.0	43.2	33.8
可观的衰变宽度			

- $I(J^P) = 0(1^+) \bar{K} \bar{K}^*$ 分子态的束缚能为60 MeV时, 该衰变宽度为16.7 MeV, 该结果与夸克对产生模型预言的结果特别接近(17 MeV)。 J. Z. Ji, Y. H. Xing, X. X. Wu, N. Xu, and Y. Tan, CPC 49, 013101(2025).

- 鉴于LHCb实验组在 $DD\pi$ 不变质量谱上发现了双粲分子态类型四夸克态候选者 $T_{cc}^+(3875)$, 建议未来实验优先在 $\bar{K} \bar{K} \pi$ 不变质量谱上寻找 $I(J^P) = 0(1^+) \bar{K} \bar{K}^*$ 分子态候选者。

$I(J^P) = \mathbf{0}(1^+) \bar{K}^* \bar{K}^*$ 分子态候选者的三体强衰变行为



$$\mathcal{L}_{T_{ss} \bar{K}^* \bar{K}^*} = g_{T_{ss} \bar{K}^* \bar{K}^*} \epsilon_{\mu\nu\lambda\rho} \partial^\mu T_{ss}^\nu \bar{K}^{*\lambda} \bar{K}^{*\rho}$$

温伯格组分条件

$$g_{T_{ss} \bar{K}^* \bar{K}^*}^2 = \frac{16\pi \sqrt{|E_B|}}{m_{\bar{K}^*}^{1/2}}$$

S. Weinberg, Phys. Rev. 130, 776-783 (1963).

$$i\mathcal{M}_{[\bar{K}^* \bar{K}^*][0(1^+)] \rightarrow \bar{K}^* \bar{K} \pi} = ig_{T_{ss} \bar{K}^* \bar{K}^*} \epsilon_{\alpha\beta\lambda\rho} (-ip^\alpha) \epsilon_{T_{ss}}^\beta \epsilon_{K^*}^{*\rho} g^{\lambda\theta} \frac{i(-g_{\theta\tau} + q_\theta q_\tau / m_{\bar{K}^*})}{q^2 - m_{\bar{K}^*}^2 + im_{\bar{K}^*} \Gamma_{\bar{K}^*}} \sqrt{m_{\bar{K}} m_{\bar{K}^*}} \left(\frac{-2g'}{f_\pi} \right) (ip_2^\tau)$$

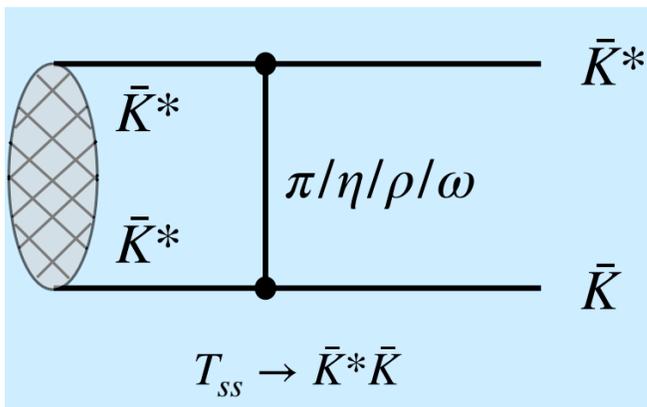
$$\Gamma = 2 \frac{1}{S} \frac{1}{2m_{T_{ss}}} \sum_{\text{pol.}} \frac{1}{3} \int |\mathcal{M}|^2 d\Phi_3 = 2 \frac{1}{S} \frac{1}{(2\pi)^3} \frac{1}{32m_{T_{ss}}^3} \sum_{\text{pol.}} \frac{1}{3} \int |\mathcal{M}|^2 dm_{12}^2 dm_{23}^2$$

R. L. Workman et al. [Particle Data Group], PTEP 2022, 083C01 (2022).

E_B (MeV)	- 4	- 17	- 30
$\Gamma_{[\bar{K}^* \bar{K}^*][0(1^+)] \rightarrow \bar{K}^* \bar{K} \pi}$ (MeV)	39.2	51.7	45.4

重要的三体强衰变道(P-波衰变)

$I(J^P) = 0(1^+) \bar{K}^* \bar{K}^*$ 分子态候选者的两体强衰变行为



$$\mathcal{M}_{[AB] \rightarrow CD} = \sqrt{\frac{m_{[AB]}}{2m_A m_B}} \int d^3\mathbf{k} \frac{d^3\mathbf{r}}{(2\pi)^3} \psi_{[AB]}(\mathbf{r}) \mathcal{M}_{AB \rightarrow CD}(\mathbf{k}, \mathbf{p})$$

衰变振幅 散射振幅

Y. J. Zhang, H. C. Chiang, P. N. Shen, and B. S. Zou, PRD 74, 014013 (2006).

$$\Gamma_{[AB] \rightarrow CD} = \frac{P}{32\pi^2 m_{[AB]}^2} \int d\Omega |\mathcal{M}_{[AB] \rightarrow CD}|^2$$

R. L. Workman et al. [Particle Data Group], PTEP 2022, 083C01 (2022).

E_B (MeV)	- 4	- 17	- 30
$\Gamma_{[\bar{K}^* \bar{K}^*][0(1^+)] \rightarrow \bar{K}^* \bar{K}}$ (MeV)	128.5	256.6	342.2

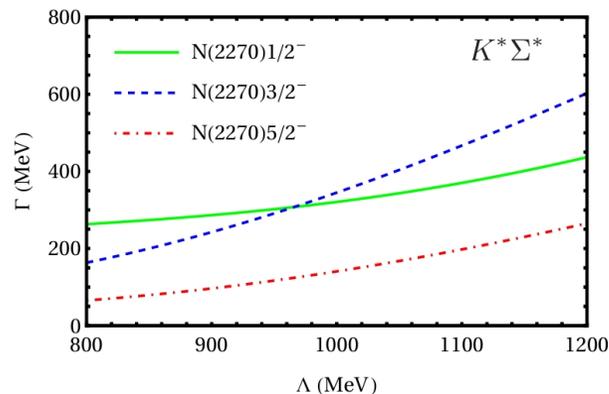
重要的两体强衰变道(S-波衰变)

$N(1875) K \Sigma^*$

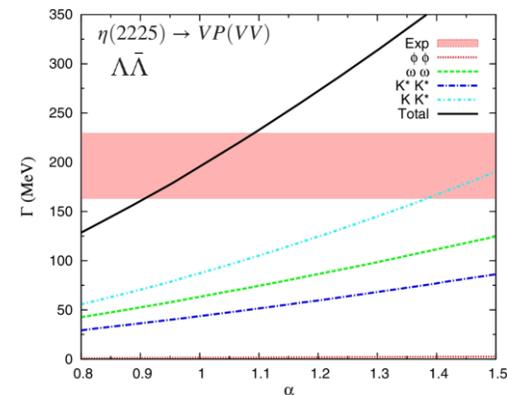
$N\sigma(500)$	1.5
πN	0.2
ρN	1.4
ωp	4.0
$K \Sigma$	0.02
$K \Lambda$	0.4
ηp	0.4
$\pi \Delta$	117.3

Y. H. Lin, C. W. Shen, and B. S. Zou, NPA 980, 21-31 (2018).

隐奇异分子态类型五夸克态候选者



D. Ben and S. M. Wu, arXiv:2409.18493.



Y. Dong, A. Faessler, T. Gutsche, Q. Lu, and V. E. Lyubovitskij, PRD 96, 074027 (2017).

隐奇异分子态类型六夸克态候选者

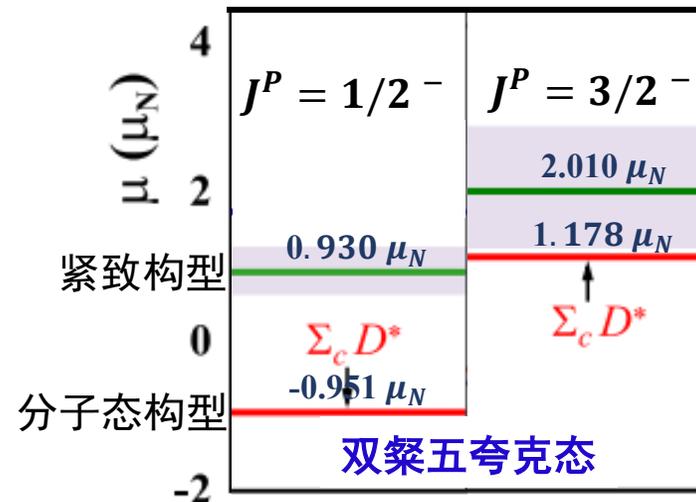


4

含双奇异夸克分子态类型四夸克态候选者 的电磁性质

强子的电磁性质——磁矩与辐射衰变宽度

- 磁矩与辐射衰变宽度是反映强子内部结构的重要电磁性质。
- 磁矩与辐射衰变宽度为确定强子的量子数与构型提供重要参考。
- 强子的磁矩与辐射衰变宽度是重要的实验可观测量。



H. Y. Zhou, F. L. Wang, Z. W. Liu, and X. Liu, PRD 106,034034 (2022).

- 强子电磁性质的研究模型与方法：

- ① 组分夸克模型；
- ② QCD求和规则；
- ③ 格点QCD模拟；
- ④ 手征微扰理论；

.....

p

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ Status: ****

p MAGNETIC MOMENT				
VALUE (μ_N)	DOCUMENT ID	TECN	COMMENT	
2.79284734463 ± 0.00000000082	TIESINGA	21	RVUE	2018 CODATA value
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●				
2.79284734462 ± 0.00000000082	SCHNEIDER	17	TRAP	Double Penning trap
2.7928473508 ± 0.00000000085	MOHR	16	RVUE	2014 CODATA value
2.792847356 ± 0.0000000023	MOHR	12	RVUE	2010 CODATA value
2.792847356 ± 0.0000000023	MOHR	08	RVUE	2006 CODATA value
2.792847351 ± 0.0000000028	MOHR	05	RVUE	2002 CODATA value
2.792847337 ± 0.0000000029	MOHR	99	RVUE	1998 CODATA value
2.792847386 ± 0.0000000063	COHEN	87	RVUE	1986 CODATA value

R. L. Workman et al. [Particle Data Group], PTEP 2022, 083C01 (2022).

组分夸克模型被广泛的用于讨论强子的电磁性质

- 组分夸克模型成功的描述了轻味重子的磁矩。

Baryons	Magnetic moment	Numerical	Experiment
p	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_d$	2.842	2.793
n	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_u$	-1.895	-1.913
Λ	μ_s	-0.625	-0.613 ± 0.006
Σ^+	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_s$	2.735	2.460 ± 0.006
Σ^-	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_s$	-1.055	-1.160 ± 0.025
Ξ^0	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_u$	-1.465	-1.250 ± 0.014
Ξ^-	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_d$	-0.518	-0.651 ± 0.0025
Ω^-	$3\mu_s$	-1.876	-2.020 ± 0.05

G. J. Wang, R. Chen, L. Ma, X. Liu, and S. L. Zhu, PRD 94, 094018 (2016).

- 在过去几年，组分夸克模型被广泛的用于讨论分子态类型多夸克态的磁矩与辐射衰变宽度。

- G. J. Wang, R. Chen, L. Ma, X. Liu, and S. L. Zhu, PRD 94, 094018 (2016)
- M. W. Li, Z. W. Liu, Z. F. Sun, and R. Chen, PRD 104, 054016 (2021)
- F. L. Wang, H. Y. Zhou, Z. W. Liu, and X. Liu, PRD 106, 054020 (2022)
- H. Y. Zhou, F. L. Wang, Z. W. Liu, and X. Liu, PRD 106, 034034 (2022)
- F. Gao and H. S. Li, CPC 46, 123111 (2022)
- F. L. Wang, S. Q. Luo, H. Y. Zhou, Z. W. Liu, and X. Liu, PRD 108, 034006 (2023)
- F. L. Wang and X. Liu, PRD 108, 074022 (2023)
- F. L. Wang and X. Liu, PRD 109, 014043 (2024)
- B. J. Lai, F. L. Wang, and X. Liu, PRD 109, 054036(2024)
-

强子磁矩的研究模型——组分夸克模型

- 在组分夸克模型中，强子的磁矩是其组分的自旋磁矩与轨道磁矩的矢量和。

强子的磁矩通过计算磁矩算符的期望值得到：

$$\hat{\mu}_{zj}^S = \mu_j \hat{\sigma}_{zj} \quad \text{with} \quad \mu_j = \frac{e_j}{2m_j}$$

$$\hat{\mu}_z^L = \mu_{\alpha\beta}^L \hat{L}_z \quad \text{with} \quad \mu_{\alpha\beta}^L = \frac{m_\alpha}{m_\alpha + m_\beta} \mu_\beta + \frac{m_\beta}{m_\alpha + m_\beta} \mu_\alpha$$

$$\mu_{T_{ss}} = \left\langle J_{T_{ss}}, J_{T_{ss}} \left| \sum_j \hat{\mu}_{zj}^S + \hat{\mu}_z^L \right| J_{T_{ss}}, J_{T_{ss}} \right\rangle$$

Y. R. Liu, P. Z. Huang, W. Z. Deng, X. L. Chen, and S. L. Zhu, PRC 69, 035205 (2004).
P. Z. Huang, Y. R. Liu, W. Z. Deng, X. L. Chen, and S. L. Zhu, PRD 70, 034003 (2004).

- 输入的参数——组分夸克质量：

$$m_u = 0.336 \text{ GeV}, \quad m_d = 0.336 \text{ GeV}, \quad m_s = 0.450 \text{ GeV}$$

Baryon	quark model	Experimental values
p	2.79	2.793
n	-1.86	-1.913
Λ	-0.61	-0.613 ± 0.004
Σ^+	2.69	2.458 ± 0.010
Σ^-	-1.038	-1.160 ± 0.025

S. Kumar, R. Dhir, and R. C. Verma, J. Phys. G 31, 141-147 (2005).

含双奇异夸克分子态类型四夸克态候选者的磁矩性质

$$\mu_{[\bar{K}\bar{K}^*][0(1^+)]} = \frac{1}{2}\mu_{\bar{K}^*0} + \frac{1}{2}\mu_{K^{*-}}$$

$$\mu_{[\bar{K}^*\bar{K}^*][0(1^+)]} = \frac{1}{2}\mu_{\bar{K}^*0} + \frac{1}{2}\mu_{K^{*-}}$$

T_{ss} molecules	Scenarios	Magnetic moments
$[\bar{K}\bar{K}^*][0(1^+)]$	I	$-1.16 \mu_N$
	II	$-1.14 \mu_N, -1.14 \mu_N, -1.14 \mu_N$
	III	$-1.16 \mu_N, -1.21 \mu_N, -1.23 \mu_N$
$[\bar{K}^*\bar{K}^*][0(1^+)]$	I	$-1.16 \mu_N$
	II	$-1.14 \mu_N, -1.14 \mu_N, -1.13 \mu_N$

含双奇异夸克分子态类型四夸克态候选者的磁矩性质和双粲分子态类型四夸克态候选者的磁矩性质类似，这与它们的内部结构相关。

Systems	J^P	Expressions	Magnetic moments
DD^*	1^+	$\frac{1}{2}(\mu_{D^*0} + \mu_{D^{*+}})$	-0.09
D^*D^*	1^+	$\frac{1}{2}(\mu_{D^*0} + \mu_{D^{*+}})$	-0.09

Y. D. Lei and H. S. Li, PRD 109, 076014 (2024).

含双奇异夸克分子态类型四夸克态候选者的辐射衰变行为

$$\mu_{T_{ss}} = \left\langle J_{T_{ss}}, J_{T_{ss}} \left| \sum_j \hat{\mu}_{zj}^S + \hat{\mu}_z^L \right| J_{T_{ss}}, J_{T_{ss}} \right\rangle$$

$$\Gamma_{T_{ss} \rightarrow T'_{ss} \gamma} = \alpha_{\text{EM}} \frac{2}{3} \frac{q^3}{m_p^2} \frac{|\mu_{T_{ss} \rightarrow T'_{ss}}|^2}{\mu_N^2} \quad q = \frac{m_{T_{ss}}^2 - m_{T'_{ss}}^2}{2m_{T_{ss}}}$$

F. L. Wang, S. Q. Luo, H. Y. Zhou, Z. W. Liu, and X. Liu, PRD 108, 034006 (2023).

$$\mu_{T_{ss} \rightarrow T'_{ss}} = \left\langle J_{T'_{ss}}, J_z \left| \sum_j \hat{\mu}_{zj}^S e^{-i\mathbf{q} \cdot \mathbf{r}_j} \right| J_{T_{ss}}, J_z \right\rangle^{J_z = \text{Min}\{J_{T_{ss}}, J_{T'_{ss}}\}}$$

Physical quantities	Scenarios	Values
$\mu_{[\bar{K}^* \bar{K}^*][0(1^+)] \rightarrow [\bar{K} \bar{K}^*][0(1^+)]}$	I	$-0.09 \mu_N, -0.13 \mu_N, -0.14 \mu_N$
	II	$-0.07 \mu_N, -0.12 \mu_N, -0.13 \mu_N$
	III	$-0.17 \mu_N, -0.36 \mu_N, -0.43 \mu_N$
$\Gamma_{[\bar{K}^* \bar{K}^*][0(1^+)] \rightarrow [\bar{K} \bar{K}^*][0(1^+)] \gamma}$	I	2.11 keV, 4.06 keV, 4.62 keV
	II	1.20 keV, 3.36 keV, 3.98 keV
	III	7.04 keV, 30.91 keV, 44.54 keV

Mode	Rate (MeV)
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$(K \pi)^\pm$	51.4 ± 0.8
$K^\pm \gamma$	0.050 ± 0.005

Mode	Rate (MeV)
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$(K \pi)^0$	47.2 ± 0.5
$K^0 \gamma$	0.117 ± 0.010

R. L. Workman et al. [Particle Data Group], PTEP 2022, 083C01 (2022).

含双奇异夸克分子态类型四夸克态候选者的辐射衰变宽度远小于它们的强衰变宽度，这与这些强子分子态组分强子的衰变行为相关。



5

总结

理论与实验紧密结合推动对轻夸克能区分子态类型四夸克态的研究

- 鉴于目前对 $K^{(*)}\bar{K}^{(*)}$ 隐奇异分子态类型四夸克态候选者和含双奇异夸克分子态类型六夸克态 $\Lambda\Lambda$ 的研究现状，我们建议未来实验和理论上应该关注含双奇异夸克分子态类型四夸克态候选者，这些候选者有奇特的味道量子数($ss\bar{q}\bar{q}$)。

- 质量谱:

- ✓ $I(J^P) = 0(1^+)\bar{K}\bar{K}^*$ 分子态

- ✓ $I(J^P) = 0(1^+)\bar{K}^*\bar{K}^*$ 分子态

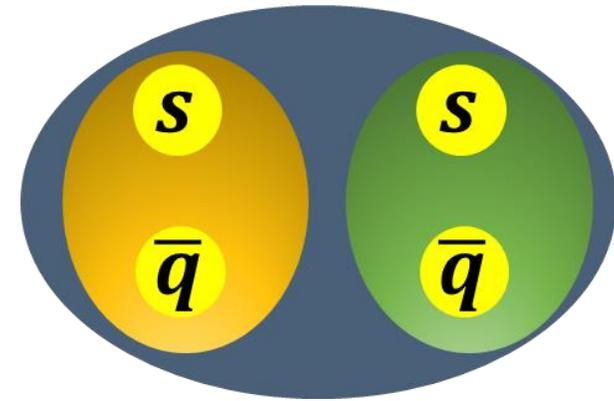
- 强衰变行为:

- 一些两体和三体强衰变道有可观的衰变宽度，建议在这些末态寻找 $\bar{K}^{(*)}\bar{K}^{(*)}$ 分子态候选者。

- 电磁性质:

- $\bar{K}^{(*)}\bar{K}^{(*)}$ 分子态候选者的内部结构与性质。

- 为寻找含双奇异夸克分子态类型四夸克态候选者提供重要信息，建议LHCb、Belle II和其它相关实验在B介子弱衰变过程研究 $I(J^P) = 0(1^+)\bar{K}\bar{K}^*$ 和 $\bar{K}^*\bar{K}^*$ 分子态候选者，这能够为在轻夸克能区存在分子态类型四夸克态提供重要信息。



谢谢大家！
敬请批评指正！

