Resonance parameters of the vector charmonium-like state G(3900)

Quanxing Ye (叶全兴)

第八届强子谱和强子结构研讨会

桂林, 广西师范大学 2025.7.14

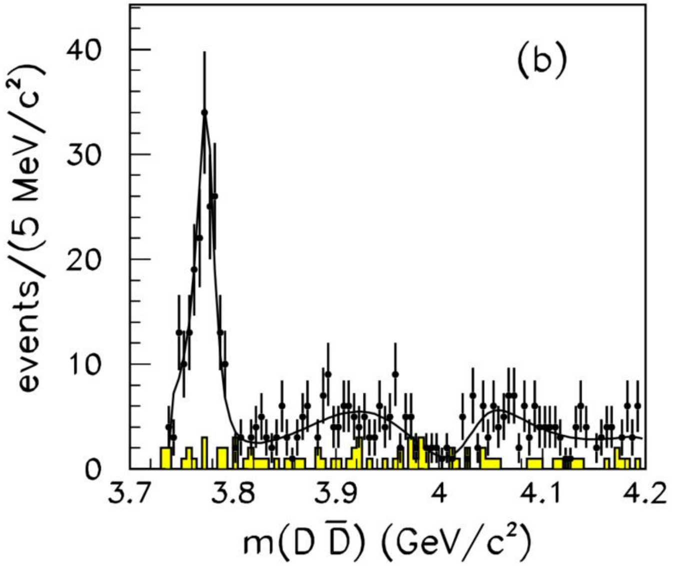
Quanxing Ye, Zhenyu Zhang, Meng-Lin Du, Ulf-G. Meiβner, Peng-Yu Niu, Qian Wang,

arXiv:2504.17431 [hep-ph] (accepted by PRD)

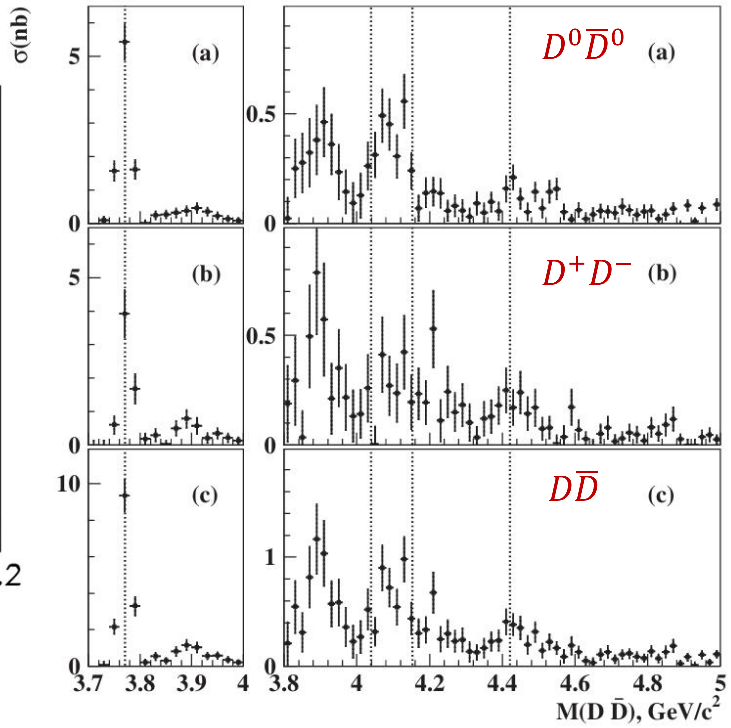
Background

- The observation of G(3900)

[*BABAR*, PRD 76 (2007), 111105(R)]

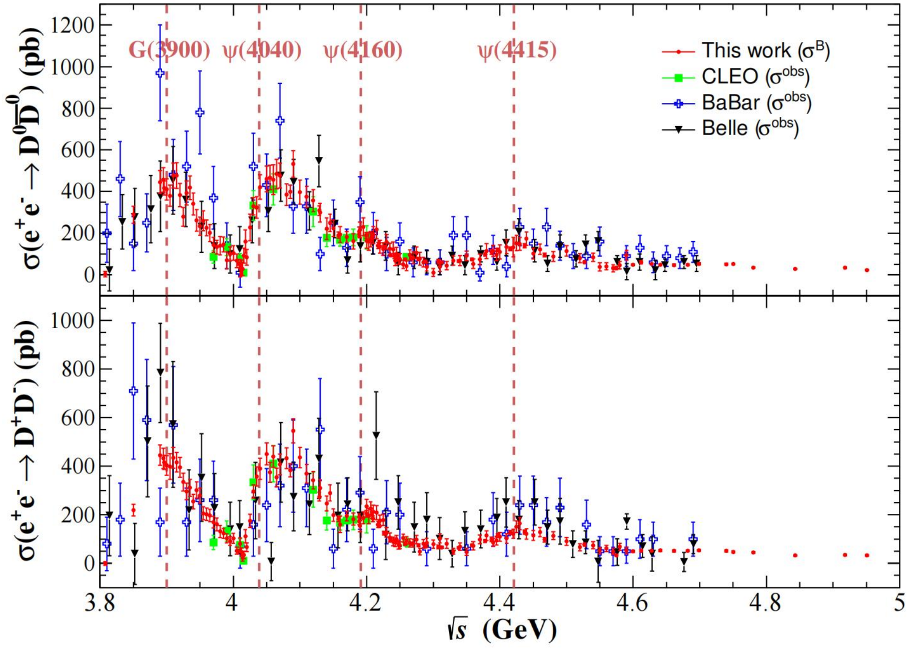


[Belle, PRD 77 (2008), 111103(R)]



- ◆ $M(G(3900)) = 3943 \pm 17_{stat} \pm 12_{syst}$ MeV, $\sigma = 52 \pm 8_{stat} \pm 7_{syst}$ MeV

[BESIII, PRL 133 (2024), 081901]



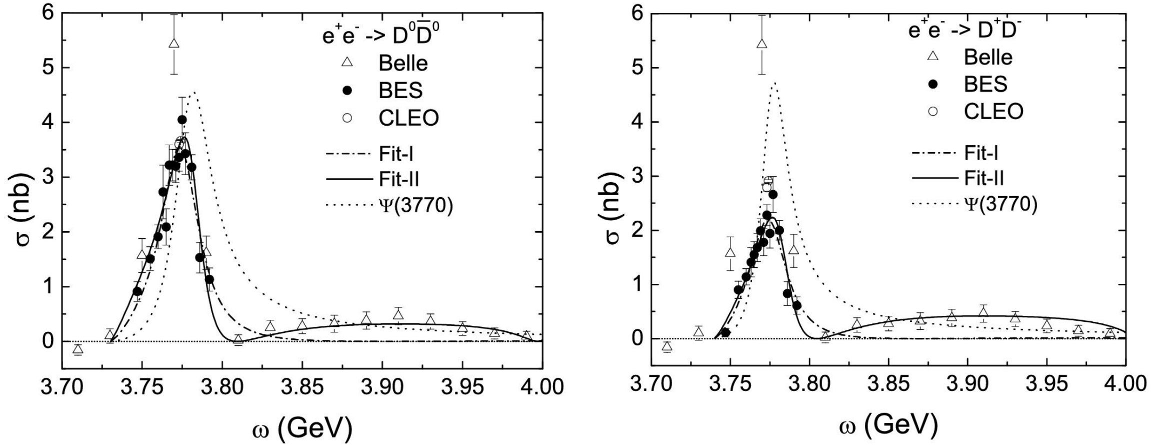
- ◆ Discovered in $e^+e^- \to D\bar{D}$
- ◆ Close to $D\bar{D}^*$threshold
- ◆ $M(G(3900)) = 3872.5 \pm 14.2_{stat} \pm 3.0_{syst}$ MeV, $\Gamma = 179.7 \pm 14.1_{stat} \pm 7.0_{syst}$ MeV
- ◆Statistical significance > 20 σ

Background

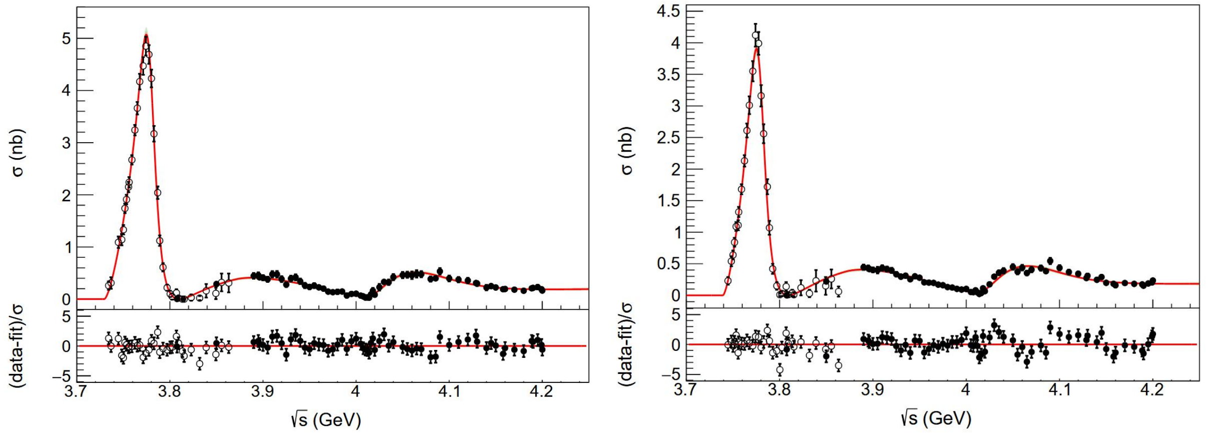
- Studies of G(3900)

◆ Threshold enhancement

[Xu Cao at al, arXiv (2014), 1410.1375]



[N. Hüsken et al, PRD 109 (2024), 114010]



◆ Couple-channel T-matrix

◆ Distortion of $\psi(3770)$ tail induced by the $D\bar{D}^*$ threshold

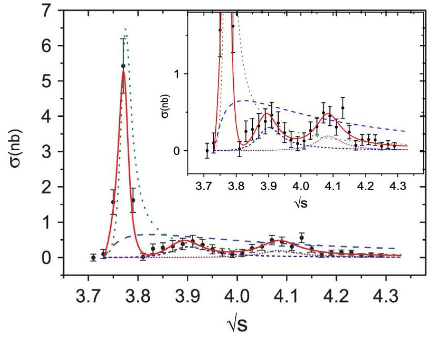
◆ Couple-channel K-matrix

◆ No additional bare pole near 3900 MeV is needed

Background

◆ P-wave $D\bar{D}^*$ molecular resonance

[Yuan-Jiang Zhang et al, PRD 81 (2010), 034011]

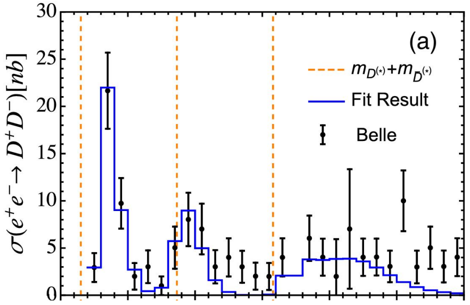


◆ Effective Lagrangian approach

◆ Extract $\gamma D\bar{D}^*$ transition form factor

◆ $M(G(3900)) = 3894^{+11}_{-11}$ MeV
$\Gamma(G(3900)) = 89.8^{+12.6}_{-12.6}$ MeV

[Meng-Lin Du et al, PRD 94 (2016), 096006]

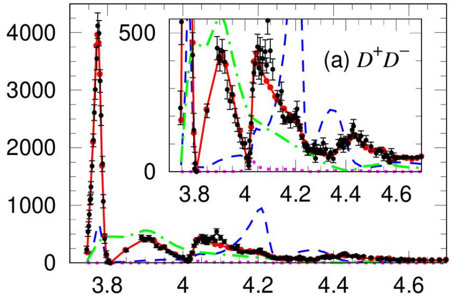


$e^+e^- \to D^{(*)}\bar{D}^{(*)}$ SU(2)

◆ Under HQSS, LSE approch

◆ $M(G(3900)) = 3879$ MeV
$\Gamma(G(3900)) = 64$ MeV

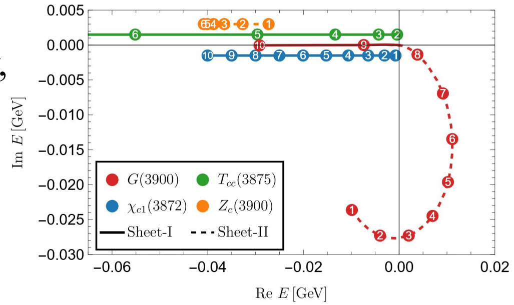
[S. X. Nakamura et al, arXiv (2023), 2312.17658]



◆ Global fitting (10 two-body, 9 three-body, 1 four-body final states)

◆ $M(G(3900)) = 3896.0^{+1.4}_{-1.4}$ MeV
$\Gamma(G(3900)) = 70.0^{+3.9}_{-3.9}$ MeV

[Zi-Yang Lin et al, PRL 133 (2024), 241903]



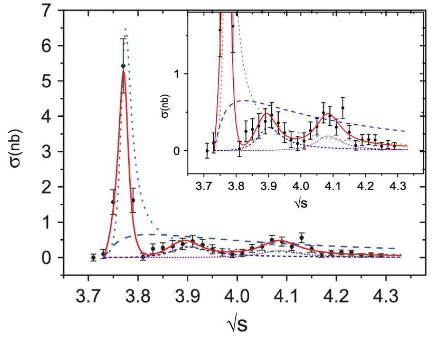
◆ OBE: $\rho, \omega, \sigma, \pi, \eta$
HM: $X(3872), Z_c(3900), T_{cc}(3875)$

◆ $M(G(3900)) = 3869.2^{+6.7}_{-6.7}$ MeV
$\Gamma(G(3900)) = 29.0^{+5.2}_{-5.2}$ MeV

Background

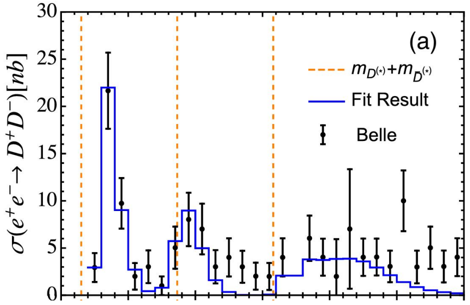
◆ P-wave $D\bar{D}^*$ molecular resonance

[Yuan-Jiang Zhang et al, PRD 81 (2010), 034011]



- Effective Lagrangian approach
- Extract $\gamma D\bar{D}^*$ transition form factor
- $M(G(3900)) = 3894^{+11}_{-11}$ MeV
  $\Gamma(G(3900)) = 89.8^{+12.6}_{-12.6}$ MeV

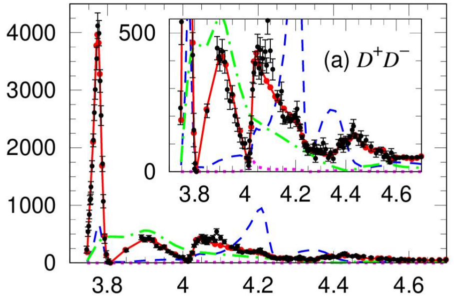
[Meng-Lin Du et al, PRD 94 (2016), 096006]



$e^+e^- \to D^{(*)}\bar{D}^{(*)}$ SU(2)

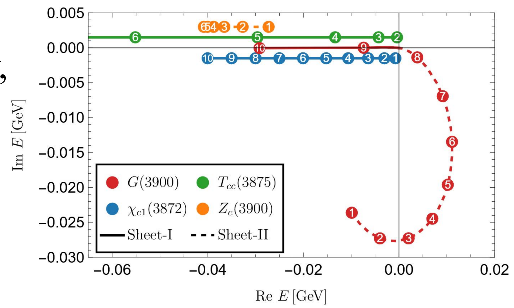
- Under HQSS, LSE approch
- $M(G(3900)) = 3879$ MeV
  $\Gamma(G(3900)) = 64$ MeV

[S. X. Nakamura et al, arXiv (2023), 2312.17658]



- Global fitting (10 two-body, 9 three-body, 1 four-body final states)
- $M(G(3900)) = 3896.0^{+1.4}_{-1.4}$ MeV
  $\Gamma(G(3900)) = 70.0^{+3.9}_{-3.9}$ MeV

[Zi-Yang Lin et al, PRL 133 (2024), 241903]



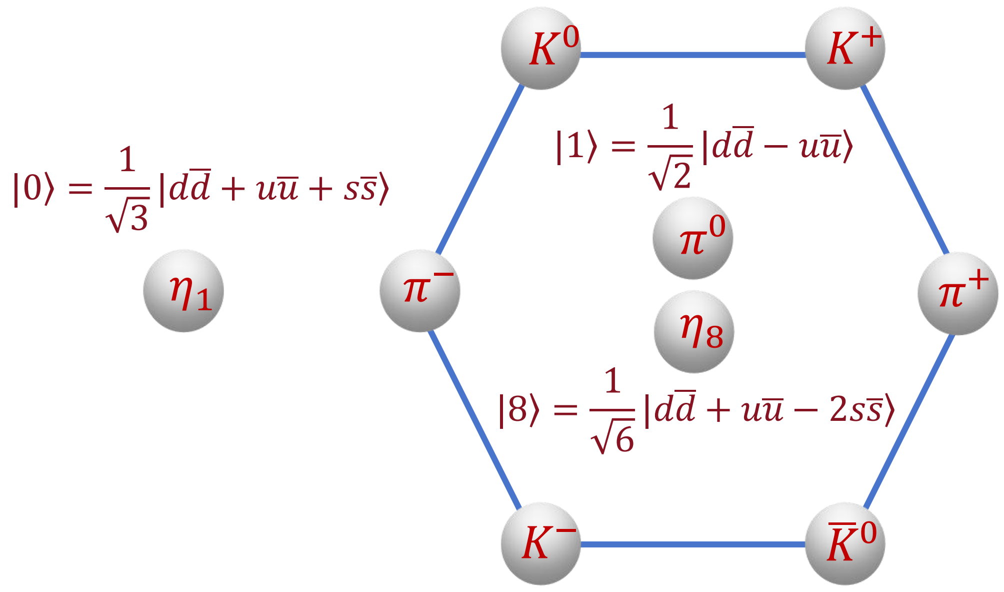
- OBE: $\rho, \omega, \sigma, \pi, \eta$
  HM: $X(3872), Z_c(3900), T_{cc}(3875)$
- $M(G(3900)) = 3869.2^{+6.7}_{-6.7}$ MeV
  $\Gamma(G(3900)) = 29.0^{+5.2}_{-5.2}$ MeV

◆ Motivation

- Perform a global analysis under HQS and SU(3) symmetry
- Exploring whether G(3900) is dynamically generated or a renormalized bare state

Transformation from hadron basis to SU(3) basis

- The transformation

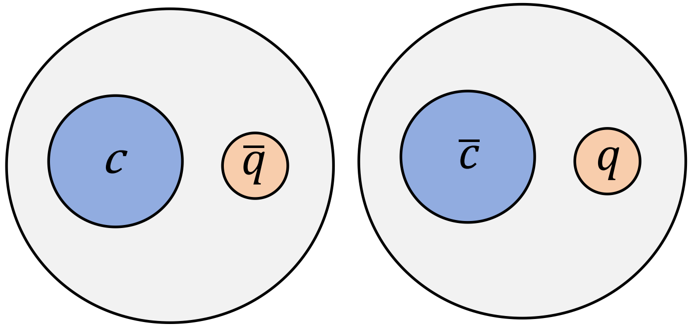


$$\begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{6}} & 0 \end{pmatrix} \otimes 1_{4\times4}$$

hadron basis $\xrightarrow{\text{Transformation}}$ SU(3) basis

$$\begin{pmatrix} |D^{(*)+}D^{(*)-}\rangle \\ |D^{(*)0}\bar{D}^{(*)0}\rangle \\ |D_s^{(*)+}D_s^{(*)-}\rangle \end{pmatrix} \qquad \begin{pmatrix} |D^{(*)}\bar{D}^{(*)}\rangle^0 \\ |D^{(*)}\bar{D}^{(*)}\rangle^8 \\ |D^{(*)}\bar{D}^{(*)}\rangle^1 \end{pmatrix}$$

- The conventions
  - a=d, u, s the light quark in the charmed meson pairs
  - i=0, 8, 1 the three different SU(3) basis



The Heavy-Light decomposition

- Spin rearrangement

$$|l([s_{l_1}s_{Q_1}]_{j_1}[s_{l_2}s_{Q_2}]_{j_2})_s\rangle_J$$

$$= \sum_{s_l,\, s_Q,\, s_q} (-1)^{l+s_Q+s_q+J}\, \widehat{s_q}\widehat{s_Q}\widehat{j_1}\widehat{j_2}\hat{s}\widehat{s_l} \begin{Bmatrix} s_{l_1} & s_{Q_1} & j_1 \\ s_{l_2} & s_{Q_2} & j_2 \\ s_q & s_Q & s \end{Bmatrix} \times \begin{Bmatrix} l & s_q & s_l \\ s_Q & J & s \end{Bmatrix} |(l[s_{l_1}s_{l_2}]_{s_q})_{s_l}[s_{Q_1}s_{Q_2}]_{s_Q}\rangle_J$$

- The decomposition of $1^{-\mp}$ $D^{(*)}\bar{D}^{(*)}$ pair

$j^{PC} = 1^{--}$

$$|D\bar{D}\rangle^i_{1^{--}} = \frac{1}{2}|\boxed{0}\otimes\boxed{1}\rangle^i + \frac{1}{2\sqrt{3}}|1\otimes 0\rangle^i - \frac{1}{2}|1\otimes 1\rangle^i + \frac{\sqrt{5}}{2\sqrt{3}}|1\otimes 2\rangle^i$$

$$|D\bar{D}^* + c.c.\rangle^i_{1^{--}} = -\frac{1}{2\sqrt{3}}|1\otimes 0\rangle^i + \frac{1}{2}|1\otimes 1\rangle^i + \frac{\sqrt{5}}{2\sqrt{3}}|1\otimes 2\rangle^i$$

$$|D^*\bar{D}^*\rangle^{i\,s=0}_{1^{--}} = \frac{\sqrt{3}}{2}|0\otimes 1\rangle^i - \frac{1}{6}|1\otimes 0\rangle^i + \frac{1}{2\sqrt{3}}|1\otimes 1\rangle^i - \frac{\sqrt{5}}{6}|1\otimes 2\rangle^i$$

$$|D^*\bar{D}^*\rangle^{i\,s=2}_{1^{--}} = \frac{\sqrt{5}}{3}|1\otimes 0\rangle^i + \frac{\sqrt{5}}{2\sqrt{3}}|1\otimes 1\rangle^i + \frac{1}{6}|1\otimes 2\rangle^i$$

$j^{PC} = 1^{-+}$

$$|D\bar{D}^* + c.c.\rangle^i_{1^{-+}} = -\frac{1}{\sqrt{2}}|0\otimes 1\rangle^i + \frac{1}{\sqrt{2}}|1\otimes 1\rangle^i$$

$$|D^*\bar{D}^*\rangle^{i\,s=1}_{1^{-+}} = \frac{1}{\sqrt{2}}|0\otimes 1\rangle^i + \frac{1}{\sqrt{2}}|1\otimes 1\rangle^i$$

- 12 couple channels

Potentials

- Potentials between charm meson pairs

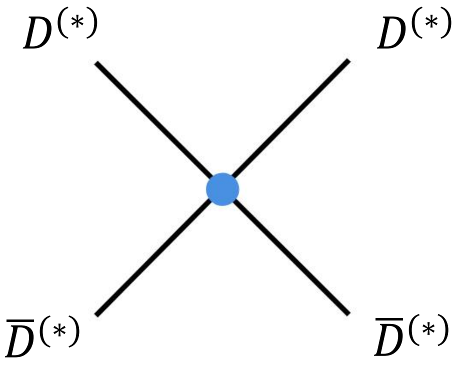
◆ Low-energy constants

$$C_1^i \equiv {}^i\langle 0\otimes 1|\mathcal{H}_{CT}|0\otimes 1\rangle^j\delta_{ij}$$
$$C_2^i \equiv {}^i\langle 1\otimes 0|\mathcal{H}_{CT}|1\otimes 0\rangle^j\delta_{ij}$$
$$C_3^i \equiv {}^i\langle 1\otimes 1|\mathcal{H}_{CT}|1\otimes 1\rangle^j\delta_{ij}$$
$$C_4^i \equiv {}^i\langle 1\otimes 2|\mathcal{H}_{CT}|1\otimes 2\rangle^j\delta_{ij}$$

◆ Contact potentials

$$V_{nn'}^i = {}_n^i\langle D^{(*)}\bar{D}^{(*)}|\mathcal{H}_{CT}|D^{(*)}\bar{D}^{(*)}\rangle_{n'}^j\delta_{ij}$$

$$V_{CT} = \begin{pmatrix} V^0 & & \\ & V^8 & \\ & & V^1 \end{pmatrix}$$



- Potentials between charm meson pairs

◆ Bare states $\psi(3770)$, $\psi(4040)$ and $\psi(4160)$ lie within 3.7-4.25 GeV

◆ Treat them as $\psi(1D)$, $\psi(3S)$ and $\psi(2D)$, respectively

◆ Couplings

Model II

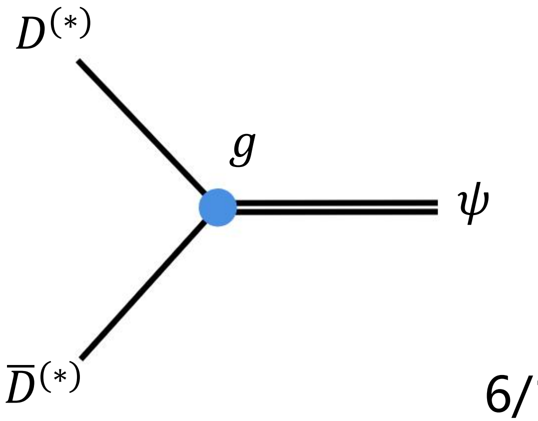
$$g_{1D}^0 \equiv {}^0\langle 1\otimes 2|\mathcal{H}_{bare}|1\otimes 2\rangle_{1D}^0$$
$$g_{3S}^0 \equiv {}^0\langle 1\otimes 0|\mathcal{H}_{bare}|1\otimes 0\rangle_{3S}^0$$
$$g_{2D}^0 \equiv {}^0\langle 1\otimes 2|\mathcal{H}_{bare}|1\otimes 2\rangle_{2D}^0$$

Model I

◆ Potentials

$$V_{c\bar{c}\ nj}^0 \equiv {}_n^0\langle D^{(*)}\bar{D}^{(*)}|\mathcal{H}_{bare}|j\rangle^0$$

only couple to SU(3) singlet



The reduction of LSE

- The (12+n)×(12+n) T-matrix is reduced to 12×12 T-matrix

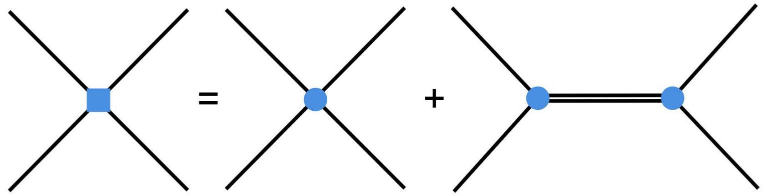
◆ $T_{oo}(E)=\boxed{f_\Lambda(p)}\left[\left[\hat{V}_{oo}^{eff}(E)\right]^{-1}-G_{CT}(E)\right]^{-1}\boxed{f_\Lambda(p')}$

$$G_{CT}^{ii}(E)=\int\frac{d^3q}{(2\pi)^3}\frac{q^2 f_\Lambda^2(q)}{E-m_{i1}-m_{i2}-q^2/(2\mu)+i\varepsilon^+}$$
$$=-\frac{\mu\Lambda}{(2\pi)^{3/2}}\left(k^2+\frac{\Lambda^2}{4}\right)+\frac{\mu k^3}{2\pi}e^{-2k^2/\Lambda^2}\left[erfi\left(\frac{\sqrt{2}k}{\Lambda}\right)-i\right]$$

$$f_\Lambda(p)=diag[exp(-p_i^2/\Lambda^2)]_{12\times 12}$$

ensure the unitarity of T-matrix

$$\hat{V}_{oo}^{eff}=R\begin{pmatrix} V^0+V_{c\bar{c}}^0 G_{c\bar{c}} V_{c\bar{c}}^{0\,T} & & \\ & V^8 & \\ & & V^1 \end{pmatrix}R^{-1}$$



$\hat{V}_{oo}^{eff}$

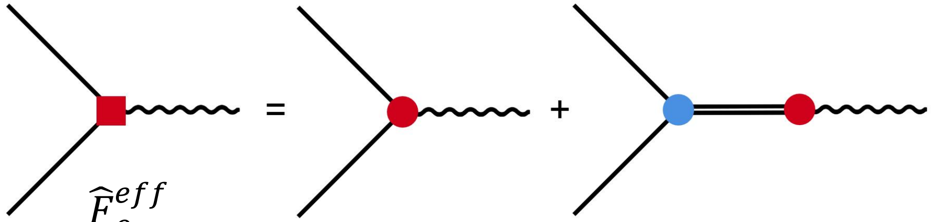
- The physical production amplitude

$$\mathcal{U}_0(E)=f_\Lambda(p)\left(1_{12\times 12}-\hat{V}_{oo}^{eff}G_{CT}(E)\right)^{-1}\hat{F}_o^{eff}$$

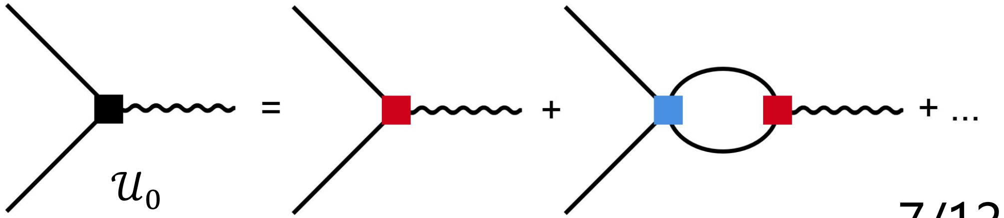
production for bare state

$$\hat{F}_o^{eff}=R\begin{pmatrix}\left[\boxed{F^0}+V_{c\bar{c}}^0 G_{c\bar{c}}\boxed{f_b}\right]_{4\times 1}\\ \boxed{\begin{matrix}[F^8]\\ [F^1]\end{matrix}}\end{pmatrix}$$

production for $(D^{(*)}\bar{D}^{(*)})^i$ pair



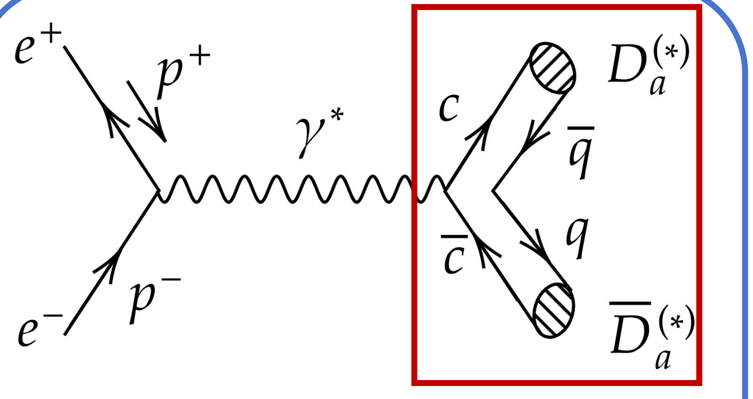
$\hat{F}_o^{eff}$



$\mathcal{U}_0$

The cross section of $e^+e^- \to D^{(*)}\overline{D}^{(*)}$

- The scattering amplitude for $e^+e^- \to D^{(*)}\overline{D}^{(*)}$



$$\mathcal{M}_n^a = \bar{v}(p_+)(-ie\gamma_\mu)u(p_-)\frac{-ig^{\mu\nu}}{s+i\varepsilon^+}A_{n\nu}^a$$

- The covariant amplitue

$$A_1^{ai} = \mathcal{U}_1^a(p_{\overline{D}} - p_D)^i$$
$$A_2^{ai} = \mathcal{U}_2^a \epsilon^{ijk}(p_{\overline{D}} - p_D)_j \varepsilon_{\lambda k}^*$$
$$A_3^{ai} = \frac{1}{\sqrt{3}}\mathcal{U}_3^a(p_{\overline{D}^*} - p_{D^*})^i \varepsilon_{D^*}^* \cdot \varepsilon_{\overline{D}^*}^*$$
$$A_4^{ai} = \frac{\sqrt{3}}{\sqrt{5}}\mathcal{U}_4^a P_2^{ij,mn}(p_{\overline{D}^*} - p_{D^*})_j \varepsilon_{D^*m}^* \varepsilon_{\overline{D}^*n}^*$$

[S. U. Chung, PRD 48 (1993), 1225] [B. S. Zou et al, EPJA 16 (2003), 537-547]

- The cross section for $e^+e^- \to D^{(*)}\overline{D}^{(*)}$

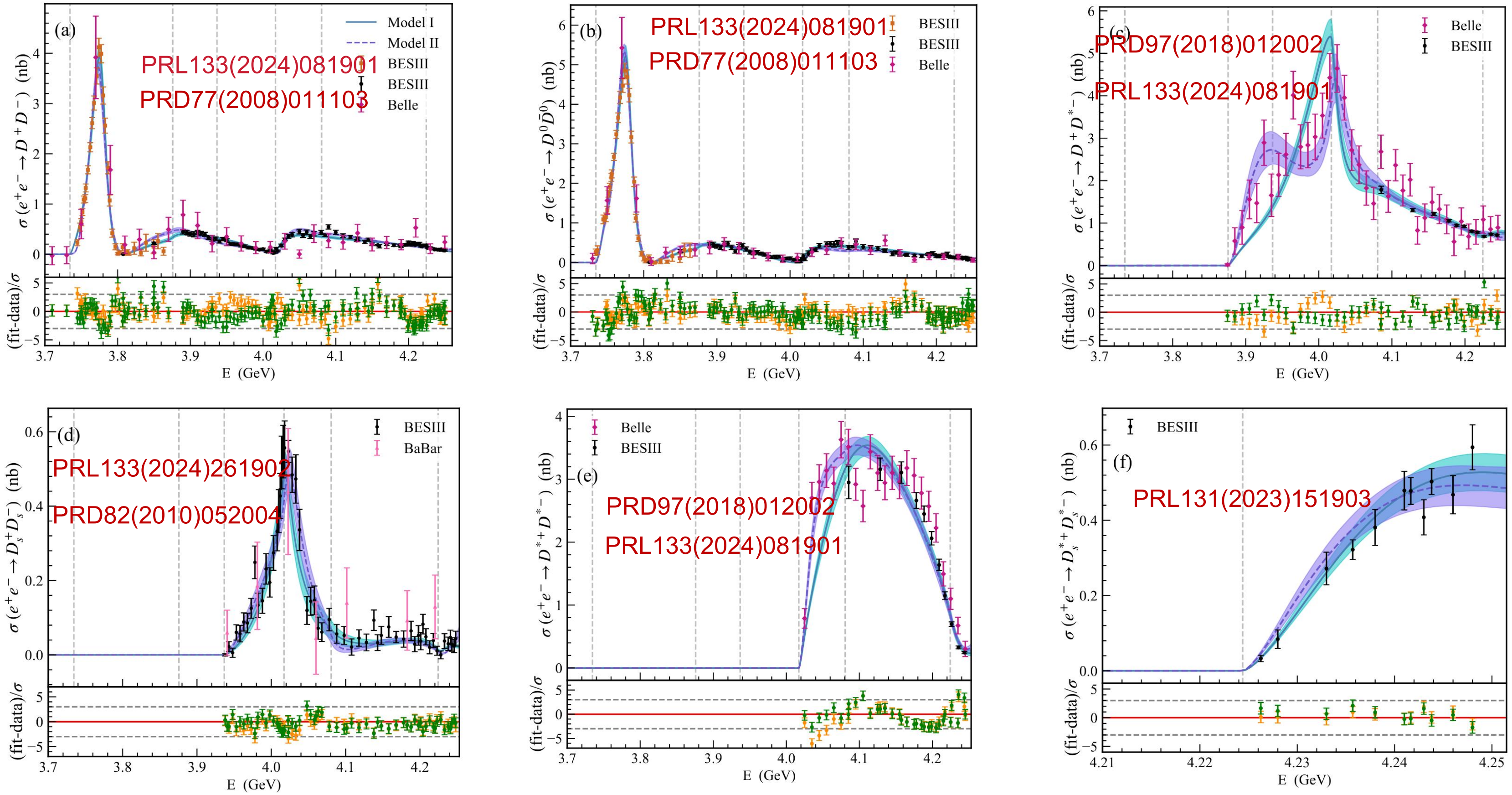
$$|\overline{\mathcal{M}}_n^a|^2 = \frac{1}{2}\sum_r \frac{1}{2}\sum_s \sum_\lambda \sum_{\lambda'} |\mathcal{M}_n^a|^2$$

$$|\overline{\mathcal{M}}_1^a|^2 = \frac{8\pi\alpha}{s}|p_D|^2|\mathcal{U}_1^a|^2(1-cos^2\theta)$$
$$|\overline{\mathcal{M}}_2^a|^2 = \frac{8\pi\alpha}{s}|p_D|^2|\mathcal{U}_2^a|^2(1+cos^2\theta)$$
$$|\overline{\mathcal{M}}_3^a|^2 = \frac{8\pi\alpha}{s}|p_{D^*}|^2|\mathcal{U}_3^a|^2(1-cos^2\theta)$$
$$|\overline{\mathcal{M}}_4^a|^2 = \frac{28\pi\alpha}{5s}|p_{D^*}|^2|\mathcal{U}_4^a|^2(1-\frac{1}{7}cos^2\theta)$$

$$\frac{d\sigma_n^a}{dcos\theta} = \frac{|p_{D^{(*)}}|}{16\pi s^{3/2}}|\overline{\mathcal{M}}_n^a|^2$$

Results

● The line shapes in comparison with the experimental data



- Blue line (Model I), purple line (Model II)
- Residuals: Orange (Model I), in Green (Model II)
- $\chi^2/d.o.f = 2.17$ for Model I, $\chi^2/d.o.f = 2.66$ for Model II

Results

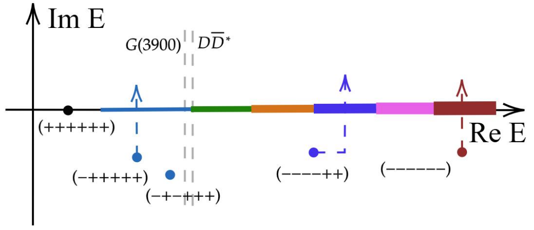
The dynamical parameters

Parameters	Model I	Model II
g^0_{1D} [GeV^{-1}]	0.66 ± 0.04	-12.93 ± 0.26
g^0_{3S} [GeV^{-1}]	-14.66 ± 0.37	-14.11 ± 0.96
g^0_{2D} [GeV^{-1}]	-17.09 ± 0.23	–
m^0_{1D} [GeV]	3.807 ± 0.001	3.804 ± 0.001
m^0_{3S} [GeV]	4.229 ± 0.002	4.253 ± 0.005
m^0_{2D} [GeV]	3.692 ± 0.003	–

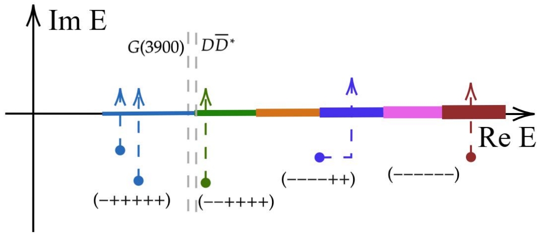
The pole positions

Riemann sheets	Model I	Model II
$(+,+,+,+,+,+)$	3.691.60	–
$(-,+,+,+,+,+)$	–	$3743.07 \pm 7.36i$ [7]
	$3778.42 \pm 11.81i$ [12]	$3775.29 \pm 14.31i$ [14]
$(-,+,-,+,+,+)$	$3832.52 \pm 74.53i$	–
$(-,-,+,+,+,+)$	–	$3883.91 \pm 46.53i$ [47]
$(-,-,-,-,+,+)$	$4011.05 \pm 10.13i$ [16]	$4019.42 \pm 17.40i$ [17]
$(-,-,-,-,-,-)$	$4232.78 \pm 23.96i$ [24]	$4278.21 \pm 21.59i$ [22]

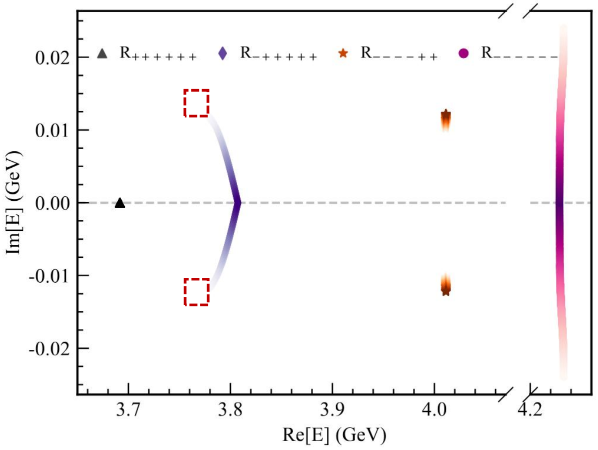
Model I



Model II



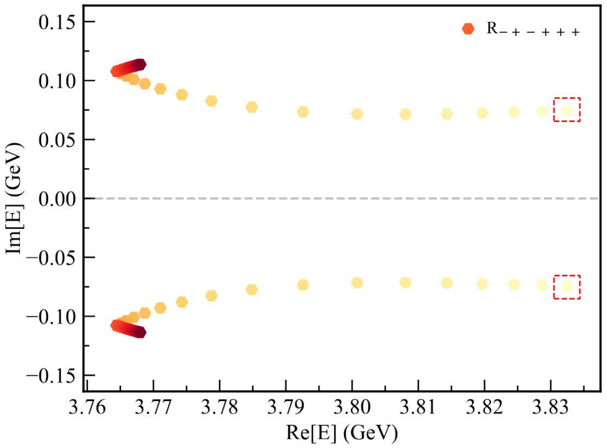
The trajectory of poles in Model I



$g^0_{1D} \to 0$

$g^0_{3S} \to 0$

$g^0_{2D} \to 0$



Results

The dynamical parameters

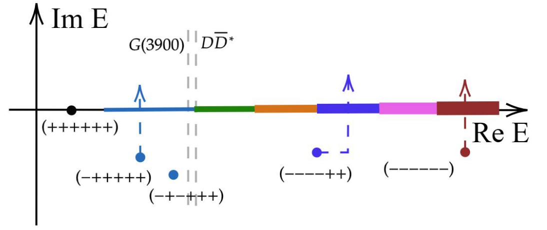
Parameters	Model I	Model II
g^0_{1D} [GeV^{-1}]	0.66 ± 0.04	-12.93 ± 0.26
g^0_{3S} [GeV^{-1}]	-14.66 ± 0.37	-14.11 ± 0.96
g^0_{2D} [GeV^{-1}]	-17.09 ± 0.23	–
m^0_{1D} [GeV]	3.807 ± 0.001	3.804 ± 0.001
m^0_{3S} [GeV]	4.229 ± 0.002	4.253 ± 0.005
m^0_{2D} [GeV]	3.692 ± 0.003	–

The pole positions

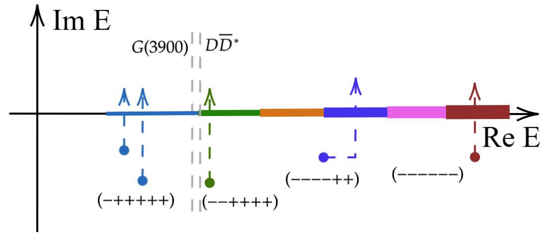
Riemann sheets	Model I	Model II
$(+,+,+,+,+,+)$	$3.691.60$	–
$(-,+,+,+,+,+)$	–	$3743.07 \pm 7.36i$ [7]
	$3778.42 \pm 11.81i$ [12]	$3775.29 \pm 14.31i$ [14]
$(-,+,-,+,+,+)$	$3832.52 \pm 74.53i$	–
$(-,-,+,+,+,+)$	–	$3883.91 \pm 46.53i$ [47]
$(-,-,-,-,+,+)$	$4011.05 \pm 10.13i$ [16]	$4019.42 \pm 17.40i$ [17]
$(-,-,-,-,-,-)$	$4232.78 \pm 23.96i$ [24]	$4278.21 \pm 21.59i$ [22]

● bare states ● dynamically generated states

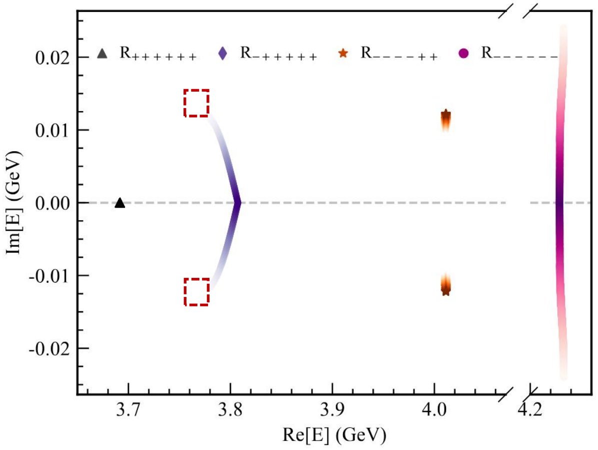
Model I



Model II



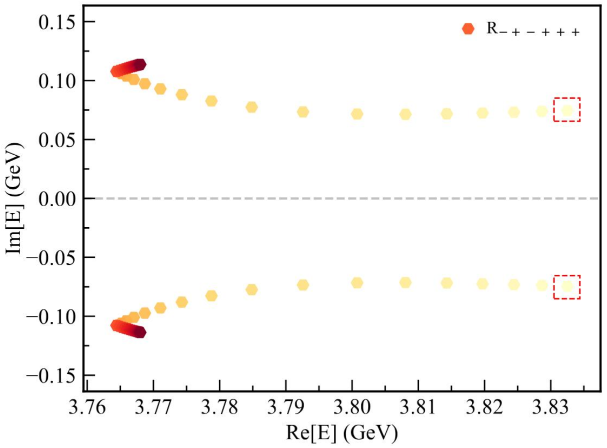
The trajectory of poles in Model I



$g^0_{1D} \to 0$

$g^0_{3S} \to 0$

$g^0_{2D} \to 0$



Results

● The dynamical parameters

Parameters	Model I	Model II
g^0_{1D} [GeV^{-1}]	0.66 ± 0.04	-12.93 ± 0.26
g^0_{3S} [GeV^{-1}]	-14.66 ± 0.37	-14.11 ± 0.96
g^0_{2D} [GeV^{-1}]	-17.09 ± 0.23	–
m^0_{1D} [GeV]	3.807 ± 0.001	3.804 ± 0.001
m^0_{3S} [GeV]	4.229 ± 0.002	4.253 ± 0.005
m^0_{2D} [GeV]	$\boxed{3.692 \pm 0.003}$	–

unphysical parameter

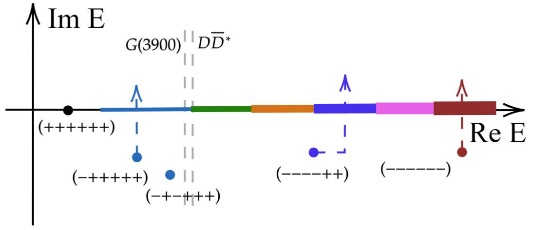
● The pole positions

Riemann sheets	Model I	Model II
$(+,+,+,+,+,+)$	3.691.60	–
$(-,+,+,+,+,+)$	–	$3743.07 \pm 7.36i$ [7]
	$3778.42 \pm 11.81i$ [12]	$3775.29 \pm 14.31i$ [14]
$(-,+,-,+,+,+)$	$\boxed{3832.52 \pm 74.53i}$	–
$(-,-,+,+,+,+)$	–	$\boxed{3883.91 \pm 46.53i\ [47]}$
$(-,-,-,-,+,+)$	$4011.05 \pm 10.13i$ [16]	$4019.42 \pm 17.40i$ [17]
$(-,-,-,-,-,-)$	$4232.78 \pm 23.96i$ [24]	$4278.21 \pm 21.59i$ [22]

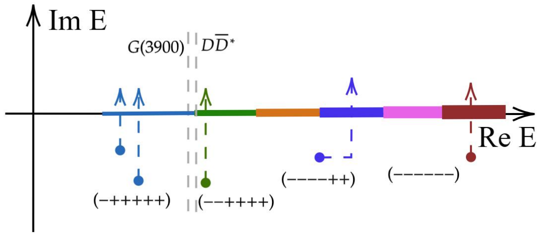
G(3900)

● bare states ● dynamically generated states

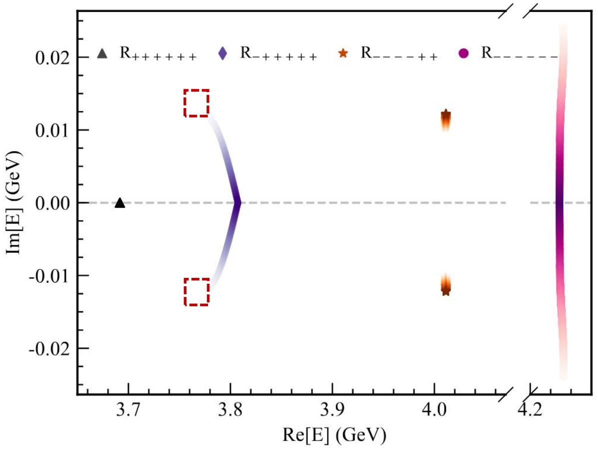
Model I



Model II



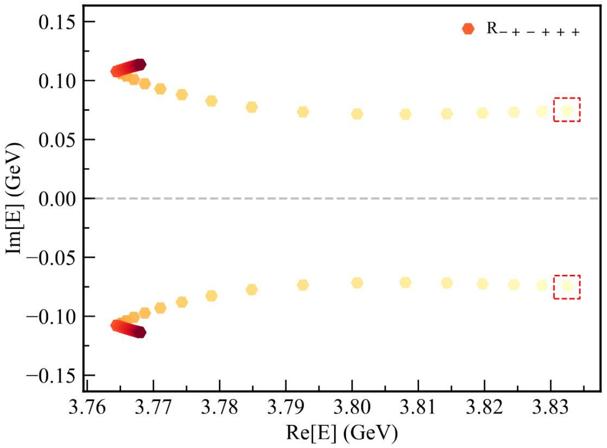
● The trajectory of poles in Model I

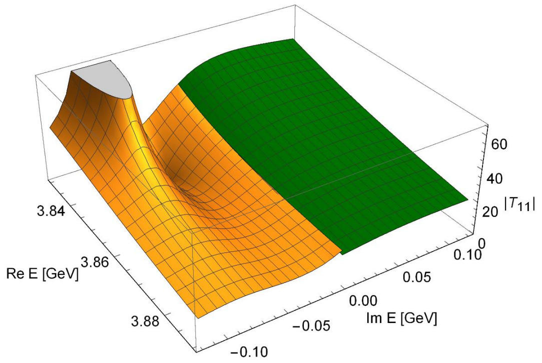


$g^0_{1D} \to 0$

$g^0_{3S} \to 0$

$g^0_{2D} \to 0$





Results

- The pole position of G(3900) in comparison with other works

This work	I $3832.6^{+0.9}_{-0.8} - 74.5^{+0.7}_{-2.2}i$	$\boxed{\text{II } 3883.9^{+0.4}_{-0.5} - 46.5^{+1.2}_{-1.2}i}$
PRD **81**, 034011	$3894^{+11}_{-11} - 44.9^{+6.3}_{-6.3}i$	
PRD **94**, 096006	$3879 - 32i$	
PRL **133**, 241903	$3869.2^{+6.7}_{-6.7} - 29.0^{+5.2}_{-5.2}i$	
2312.17658	$3896.0^{+1.4}_{-1.4} - 72.0^{+3.9}_{-3.9}i$	
Exp.	$3872.5^{+14.2+3.0}_{-14.2-3.0} - 89.9^{+7.0+2.5}_{-7.0-2.5}i$	

  - Pole position of G(3900): $3883.9^{+0.4}_{-0.5} - 46.5^{+1.2}_{-1.2}i$ MeV
  - G(3900) is a dynamically generated state

- The pole positions of 1^{-+} system

Riemann Sheets	Model I	Model II
$(+,+,+,+)$	3836.57	3869.57
$(-,+,+,+)$	$3885.42 \pm 9.48i$ [10]	$3891.73 \pm 26.19i$ [26]
$(-,-,+,+)$	$4001.56 \pm 3.94i$ [19]	$4017.93 \pm 2.71i$ [3]
$(-,-,-,+)$	$4085.70 \pm 27.08i$ [27]	$4087.76 \pm 21.92i$
$(-,-,-,-)$	$4224.18 \pm 31.26i$ [31]	$4213.85 \pm 9.63i$ [20]

  - Accessible through $e^+e^- \to \gamma + X$

Summary

● A couple channel analysis of the $e^+e^- \to D^{(*)}\overline{D}^{(*)}$ processes in HQSS and SU(3) within the energy region [3.7, 4.25] GeV.

● Pole position of G(3900), $3832.57^{+0.91}_{-0.79} - 74.53^{+0.68}_{-2.15}i$ MeV for Model I, $3883.9^{+0.4}_{-0.5} - 46.5^{+1.2}_{-1.2}i$ MeV for Model II.

● Pole trajectory analysis suggests G(3900) is a dynamically generated state, instead of a renormalized charmonium.

Thanks for your attention!