Isospin violating decays via intermediate meson loops

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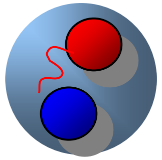
Outline

1. General remarks on the non-pQCD and hadron spectroscopy
2. Isospin-violating decay of $B_s^* \to B_s \pi^0$
3. U-spin-violating decay of $\chi_{c2} \to VP$
4. A brief summary

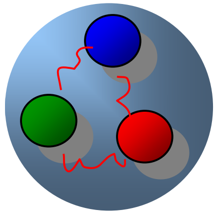
Our comprehension of the non-perturbative QCD seems to be much less than we thought!

- **Conventional quark model prescription**
- **Exotic states are expected by QCD**

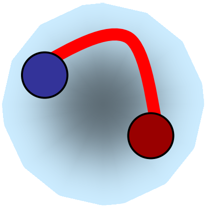
meson



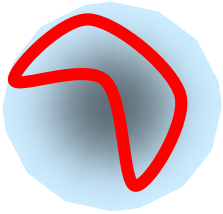
baryon



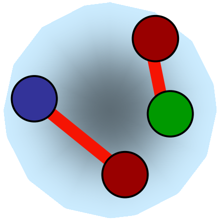
Hybrid



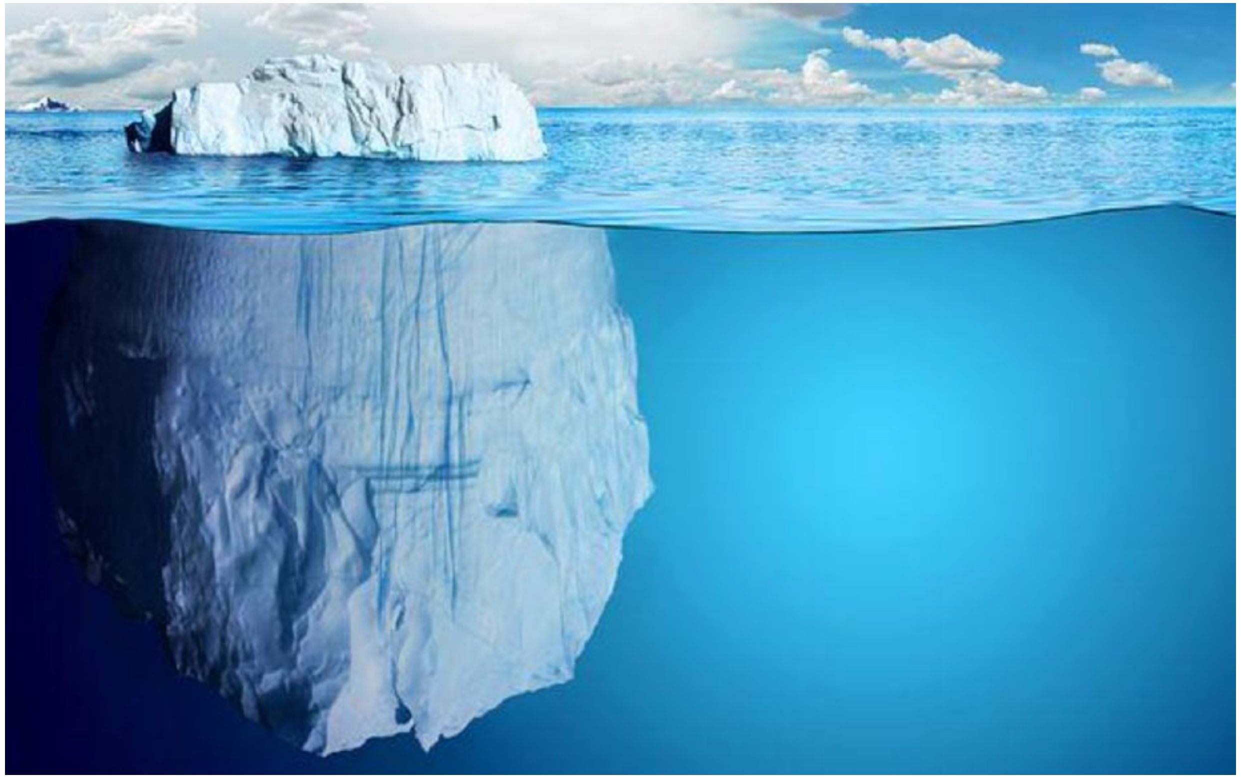
Glueball



Tetraquark



- **How the non-pQCD manifest itself in the quark-gluon interactions?**
- **What is the proper effective degrees of freedom in the description of hadron structures?**



Hadrons beyond the conventional quark model

Exotics of Type-I:

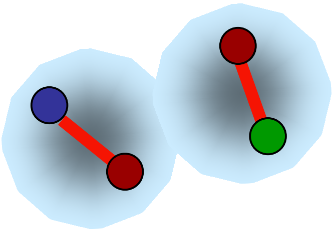
J^{PC} are not allowed by $Q\bar{Q}$ configurations, e.g. $0^{--}, 1^{-+}$ …

- Direct observation

Exotics of Type-II:

J^{PC} are the same as $Q\bar{Q}$ configurations

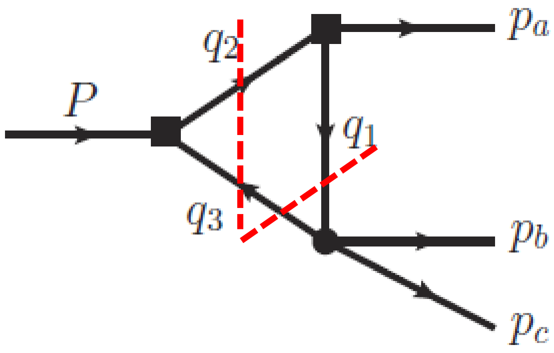
- Outnumbering of conventional QM states?
- Peculiar properties?



"Exotics" of Type-III:

Leading kinematic singularity can cause measurable effects, e.g. the triangle singularity.

- What's the impact?
- How to distinguish a genuine state from kinematic effects?



Success of Quark Model:
Hadrons are made of quarks (antiquarks) as QCD color singlet

Hamiltonian in a non-relativistic quark model :

$$\boxed{H = \left(\sum_{i=1}^{4} m_i + T_i\right) - T_G + \sum_{i<j} V_{ij}(r_{ij})}$$

$$T_i = \frac{p_i^2}{2m_i}, \qquad V_{ij}(r_{ij}) = V_{ij}^{\rm OGE}(r_{ij}) + V_{ij}^{\rm Conf}(r_{ij}),$$

$$V_{ij}^{\rm Conf}(r_{ij}) = -\frac{3}{16}(\lambda_i \cdot \lambda_j) \cdot b r_{ij},$$

Potential smearing factor

$$V_{ij}^{\rm OGE} = \frac{\alpha_{ij}}{4}(\lambda_i \cdot \lambda_j)\left\{\boxed{\frac{1}{r_{ij}}}\boxed{-\frac{\pi}{2} \cdot \frac{\sigma_{ij}^3 e^{-\sigma_{ij}^2 r_{ij}^2}}{\pi^{3/2}} \cdot \frac{4}{3m_i m_j}(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)}\right\}$$

Coulomb

Spin-spin correl.

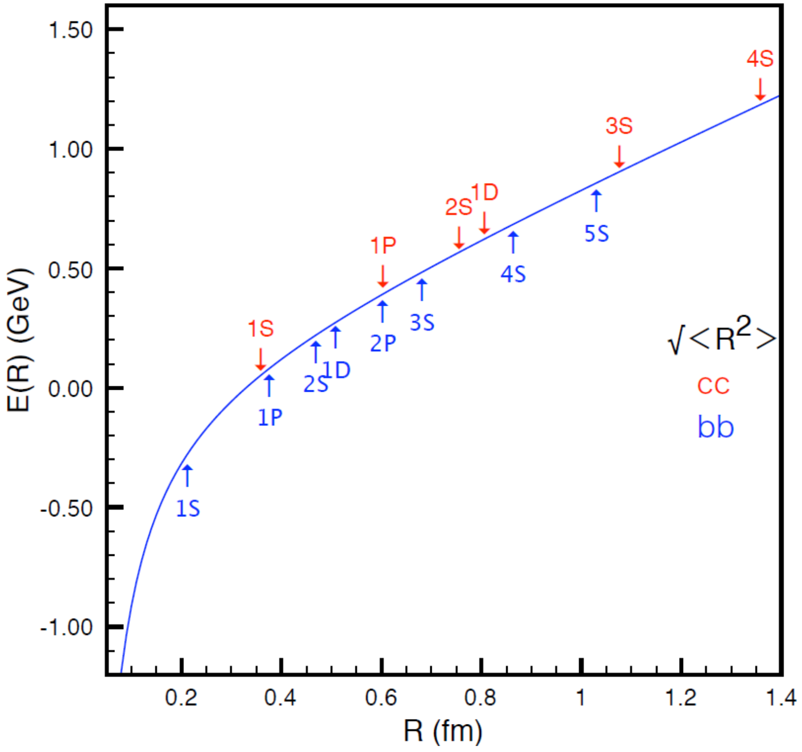
$$V_{ij}^{LS} = -\frac{\alpha_{ij}}{16}\frac{\lambda_i \cdot \lambda_j}{r_{ij}^3}\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4}{m_i m_j}\right)\left\{\mathbf{L}_{ij} \cdot (\mathbf{S}_i + \mathbf{S}_j)\right\}$$

$$-\frac{\alpha_{ij}}{16}\frac{\lambda_i \cdot \lambda_j}{r_{ij}^3}\left(\frac{1}{m_i^2} - \frac{1}{m_j^2}\right)\left\{\mathbf{L}_{ij} \cdot (\mathbf{S}_i - \mathbf{S}_j)\right\},$$

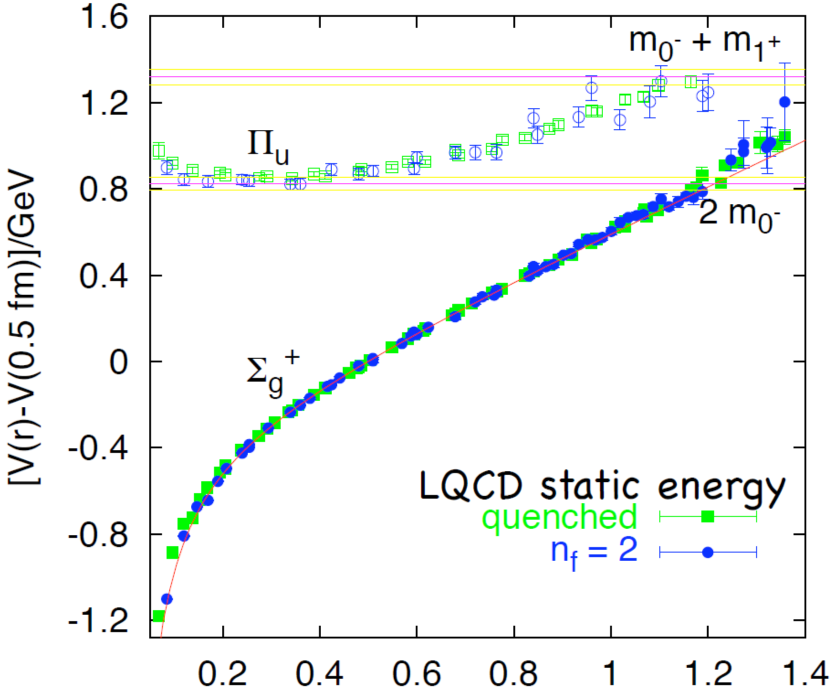
$$V_{ij}^{T} = -\frac{\alpha_{ij}}{4}(\lambda_i \cdot \lambda_j)\frac{1}{m_i m_j r_{ij}^3}\left\{\frac{3(\mathbf{S}_i \cdot \mathbf{r}_{ij})(\mathbf{S}_j \cdot \mathbf{r}_{ij})}{r_{ij}^2} - \mathbf{S}_i \cdot \mathbf{S}_j\right\}$$

- **Cornell potential model**
- **Godfrey-Isgur model**
- **A lot of recent development …**

The connection between the quark model and QCD **ONLY** becomes clear in certain circumstances: in the heavy quark limit the soft QCD for quark-antiquark or quark-quark interactions can become much simpler.



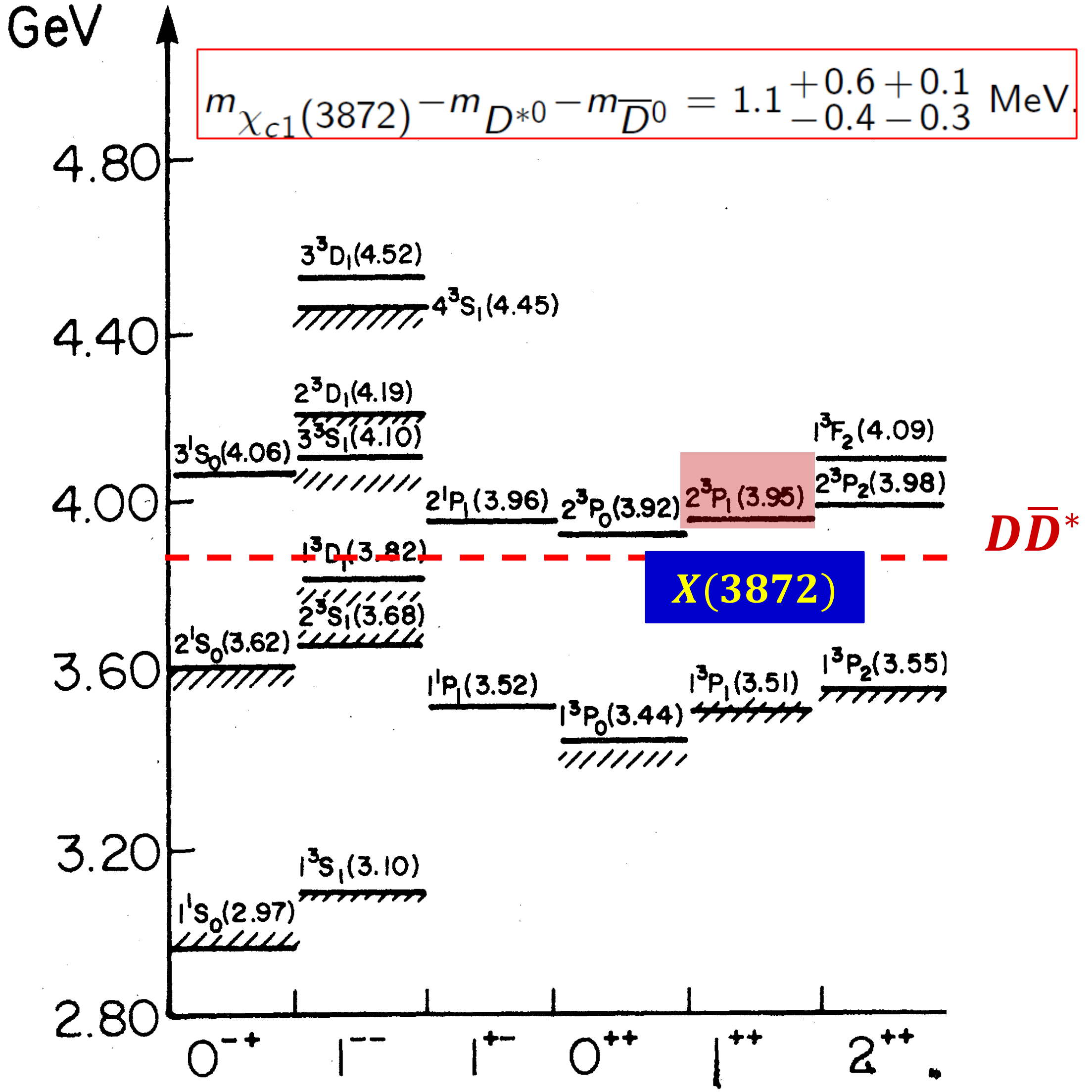
Quenched LQCD simulation



G. S. Bali, et al., Phys. Rev. D62, 054503 (2000)
M. Foster and C. Michael (UKQCD), Phys. Rev. D59, 094509 (1999)

The QM state $\chi_{c1}(2P)$ is about 60 MeV higher than the physical state $X(3872)$.

$n^{2S+1}L_J$	Name	J^{PC}	Exp. [6]	[8]	[11]	LP	SP
1^3S_1	J/ψ	1^{--}	3097^a	3090	3097	3097	3097
1^1S_0	$\eta_c(1S)$	0^{-+}	2984^a	2982	2979	2983	2984
2^3S_1	$\psi(2S)$	1^{--}	3686^a	3672	3673	3679	3679
2^1S_0	$\eta_c(2S)$	0^{-+}	3639^a	3630	3623	3635	3637
3^3S_1	$\psi(3S)$	1^{--}	4040^a	4072	4022	4078	4030
3^1S_0	$\eta_c(3S)$	0^{-+}	$\cdots$	4043	3991	4048	4004
4^3S_1	$\psi(4S)$	1^{--}	4415?	4406	4273	4412	4281
4^1S_0	$\eta_c(4S)$	0^{-+}	$\cdots$	4384	4250	4388	4264
5^3S_1	$\psi(5S)$	1^{--}	$\cdots$	$\cdots$	4463	4711	4472
5^1S_0	$\eta_c(5S)$	0^{-+}	$\cdots$	$\cdots$	4446	4690	4459
1^3P_2	$\chi_{c2}(1P)$	2^{++}	3556^a	3556	3554	3552	3553
1^3P_1	$\chi_{c1}(1P)$	1^{++}	3511^a	3505	3510	3516	3521
1^3P_0	$\chi_{c0}(1P)$	0^{++}	3415^a	3424	3433	3415	3415
1^1P_1	$h_c(1P)$	1^{+-}	3525^a	3516	3519	3522	3526
2^3P_2	$\chi_{c2}(2P)$	2^{++}	3927^a	3972	3937	3967	3937
2^3P_1	$\chi_{c1}(2P)$	1^{++}	$\cdots$	3925	3901	3937	3914
2^3P_0	$\chi_{c0}(2P)$	0^{++}	3918?	3852	3842	3869	3848
2^1P_1	$h_c(2P)$	1^{+-}	$\cdots$	3934	3908	3940	3916



W.J. Deng et al., PRD95, 034026 (2017)

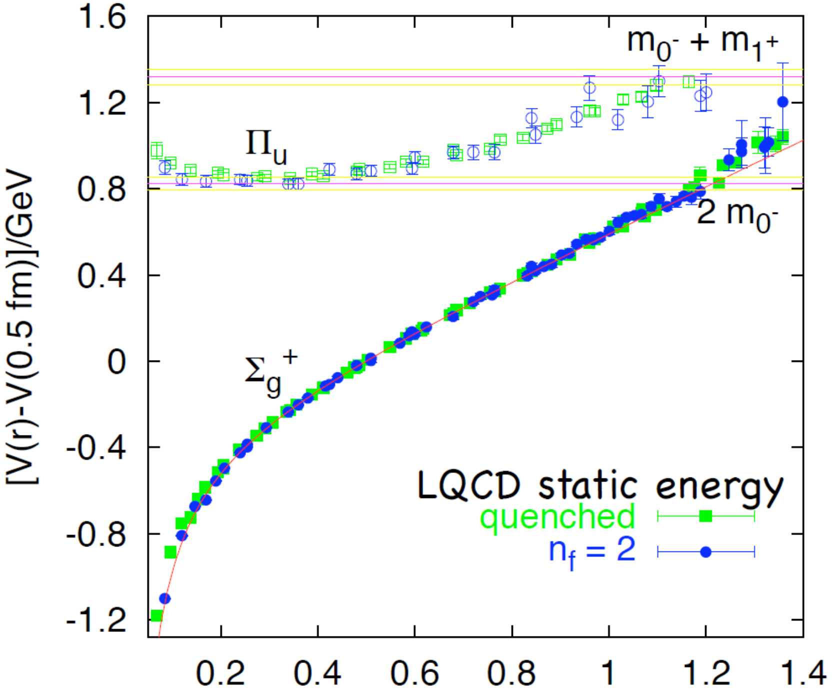
Godfrey and Isgur, PRD32, 189 (1985)

[8] T. Barnes, S. Godfrey, and E. S. Swanson, PRD 72, 054026 (2005).

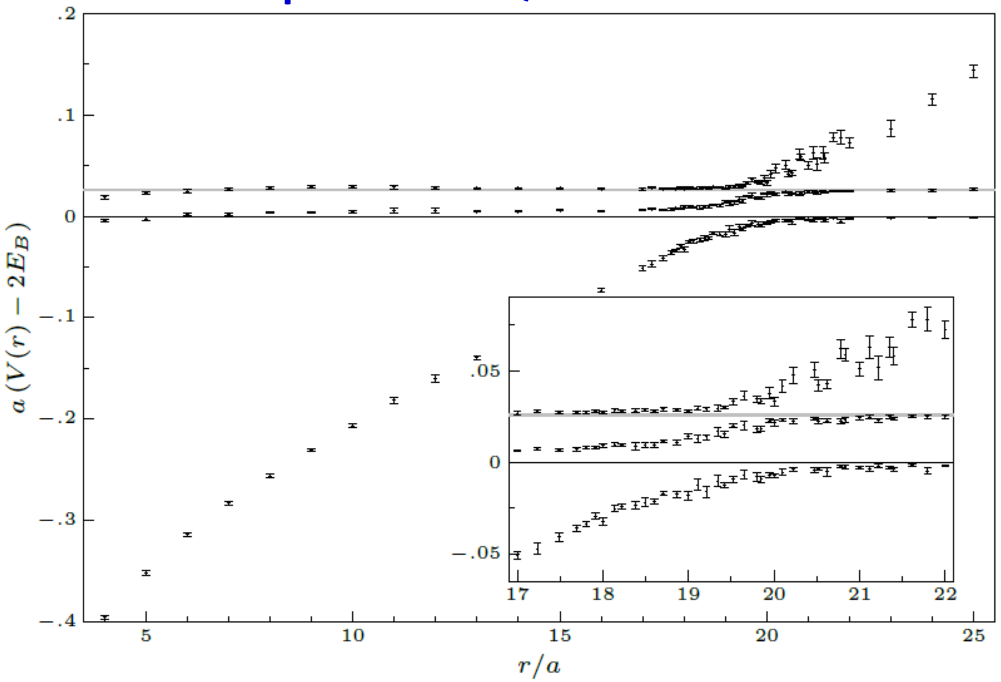
[11] B. Q. Li and K. T. Chao, PRD 79, 094004 (2009).

Open threshold effects: A missing piece of dynamics in the potential QM

Quenched LQCD simulation



Unquenched LQCD simulation



The creation energy for a quark pair with $J^{PC} = 0^{++}$: $E \simeq 2m_\pi \simeq 280$ MeV.

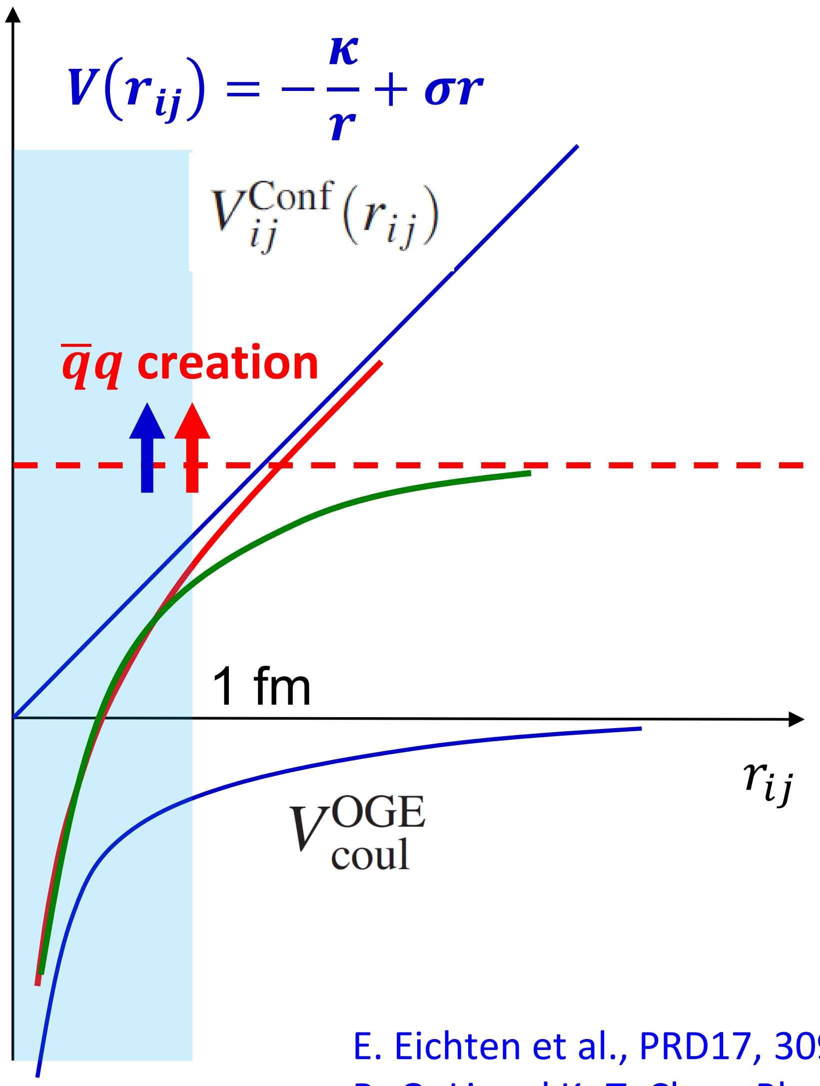
The radial excitation energy for nucleon: $m_{N(1440)} - m_{N(980)} \simeq 460$ MeV.

The orbital excitation energy for nucleon: $m_{N(1535)} - m_{N(980)} \simeq 550$ MeV.

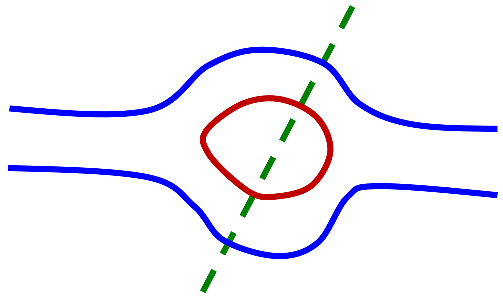
However, the effects of the open channels on the soft QCD potential is also evident!

G. S. Bali, et al., Phys. Rev. D62, 054503 (2000)
M. Foster and C. Michael (UKQCD), Phys. Rev. D59, 094509 (1999)
J. Bulava, et al., Phys. Lett. B793, 493 (2019)

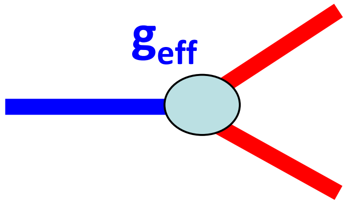
Open threshold effects: A missing piece of dynamics in the potential QM



- Color screening effects? String breaking effects?



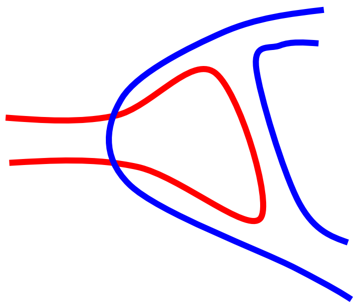
- The effect of vacuum polarization due to dynamical quark pair creation may be manifested by the strong coupling to open thresholds and compensated by that of the hadron loops, i.e. coupled-channel effects.



E. Eichten et al., PRD17, 3090 (1987); E. J. Eichten, K. Lane, and C. Quigg, Phys. Rev. D 69, 094019 (2004)
B.-Q. Li and K.-T. Chao, Phys. Rev. D79, 094004 (2009);
T. Barnes and E. Swanson, Phys.Rev. C77, 055206 (2008)

Typical processes where the open threshold coupled channels can play a role

The open channel couplings introduce NOT ONLY additional dynamics (add. effective DOF) into the hadron structures, BUT ALSO novel kinematic effects, i.e. triangle singularity …

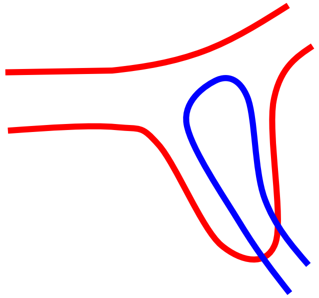


$\psi(3770) \to non D\bar{D}$ Y.J. Zhang et al, PRL(2009); X. Liu, B. Zhang, X.Q. Li, PLB(2009)

"ρπ puzzle" Q. Wang et al. PRD(2012), PLB(2012)

$\chi_{c1} \to VV,\ \chi_{c2} \to VP$ X.-H. Liu et al, PRD81, 014017(2010); X. Liu et al, PRD81, 074006(2010)

$\eta_c(\eta_c') \to VV$ Q. Wang et al, PRD2012



$\psi' \to J/\psi\pi^0, \psi' \to J/\psi\eta$

$\psi' \to \gamma\eta_c, J/\psi \to \gamma\eta_c$

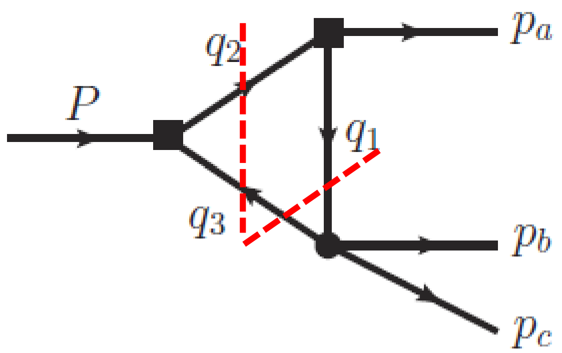
$\boxed{D_s^* \to D_s\pi^0}$

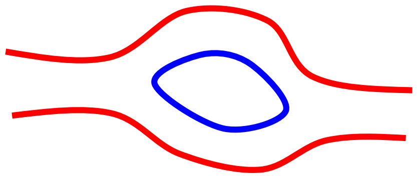
G. Li and Q. Zhao, PRD(2011)074005

F.K. Guo, C. Hanhart, G. Li, U.-G. Meiβner and Q. Zhao, PRD82, 034025 (2010); PRD83, 034013 (2011)

F.K. Guo and Ulf-G Meiβner, PRL108(2012)112002

J. Wang and Q. Zhao, PRD111, 096007 (2025)





$D_{s1}(2460) - D_{s1}(2536)$

The mass shift in charmonia and charmed mesons, E.Eichten et al., PRD17(1987)3090

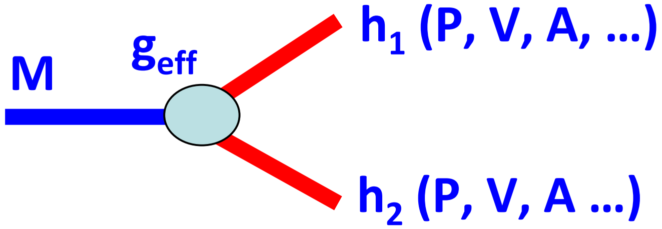
X.-G. Wu and Q. Zhao, PRD85, 034040 (2012)

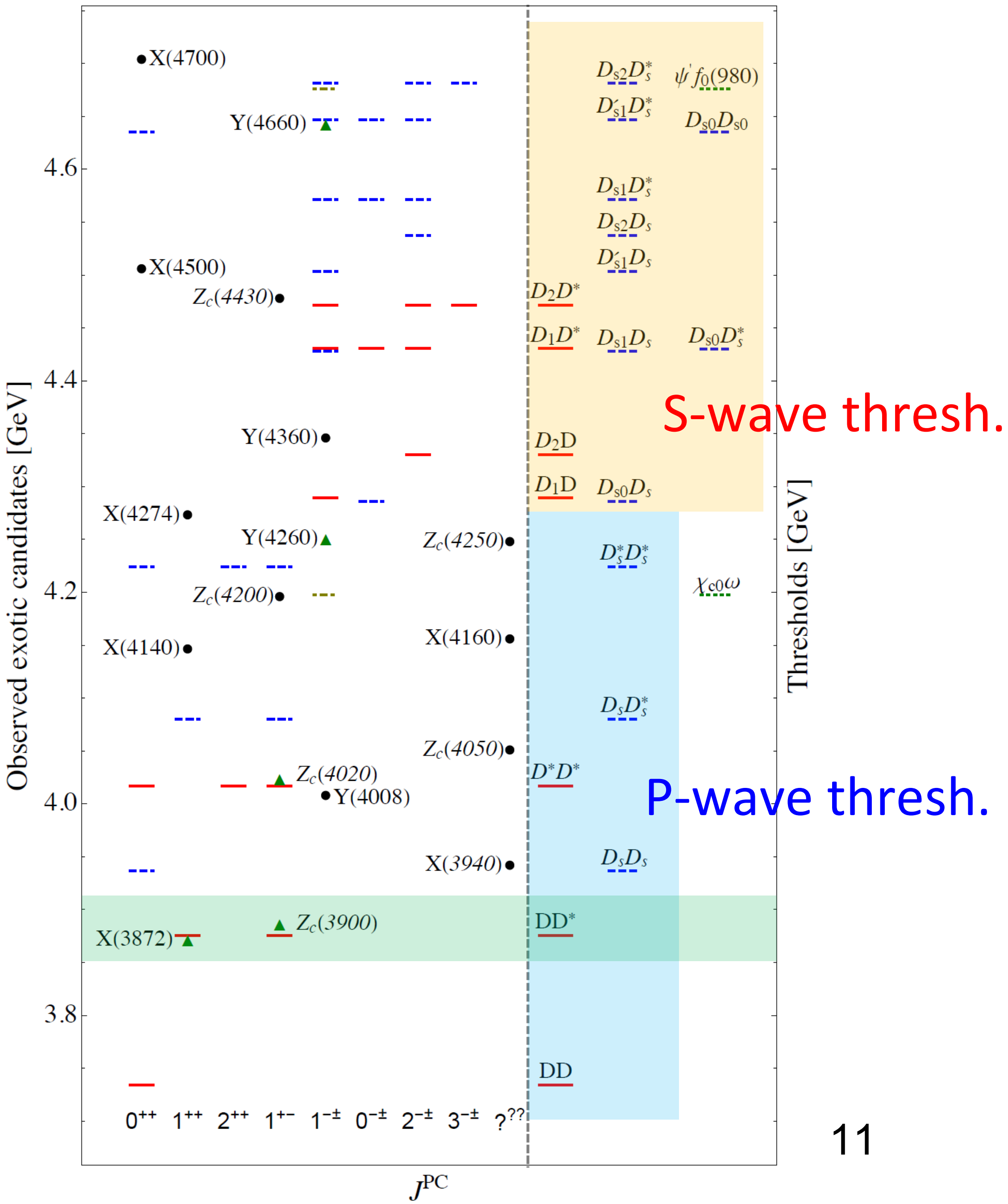
Hadronic molecules

The narrow two-body open thresholds:

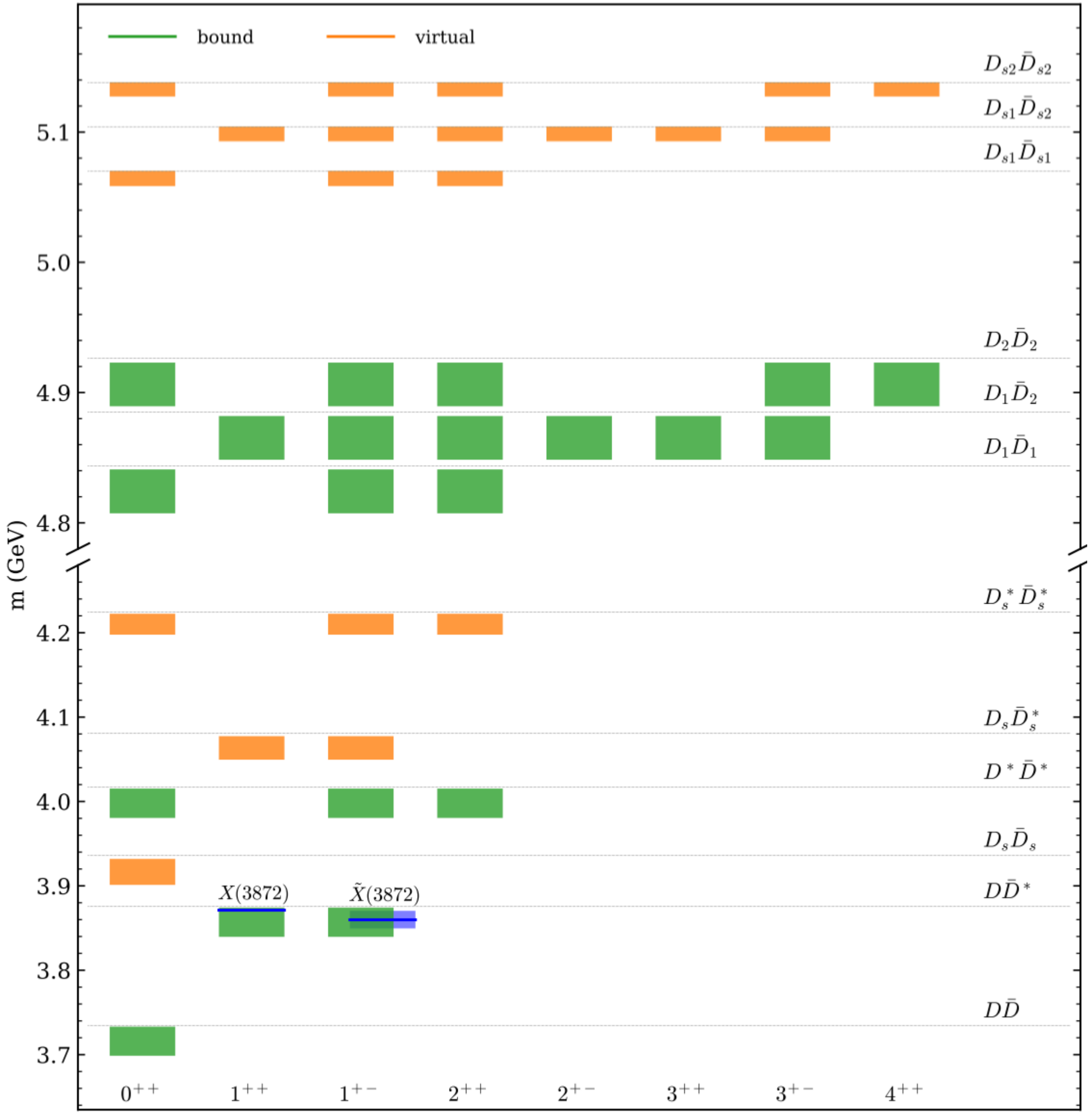
Their possible impact on the spectrum should be systematically investigated.

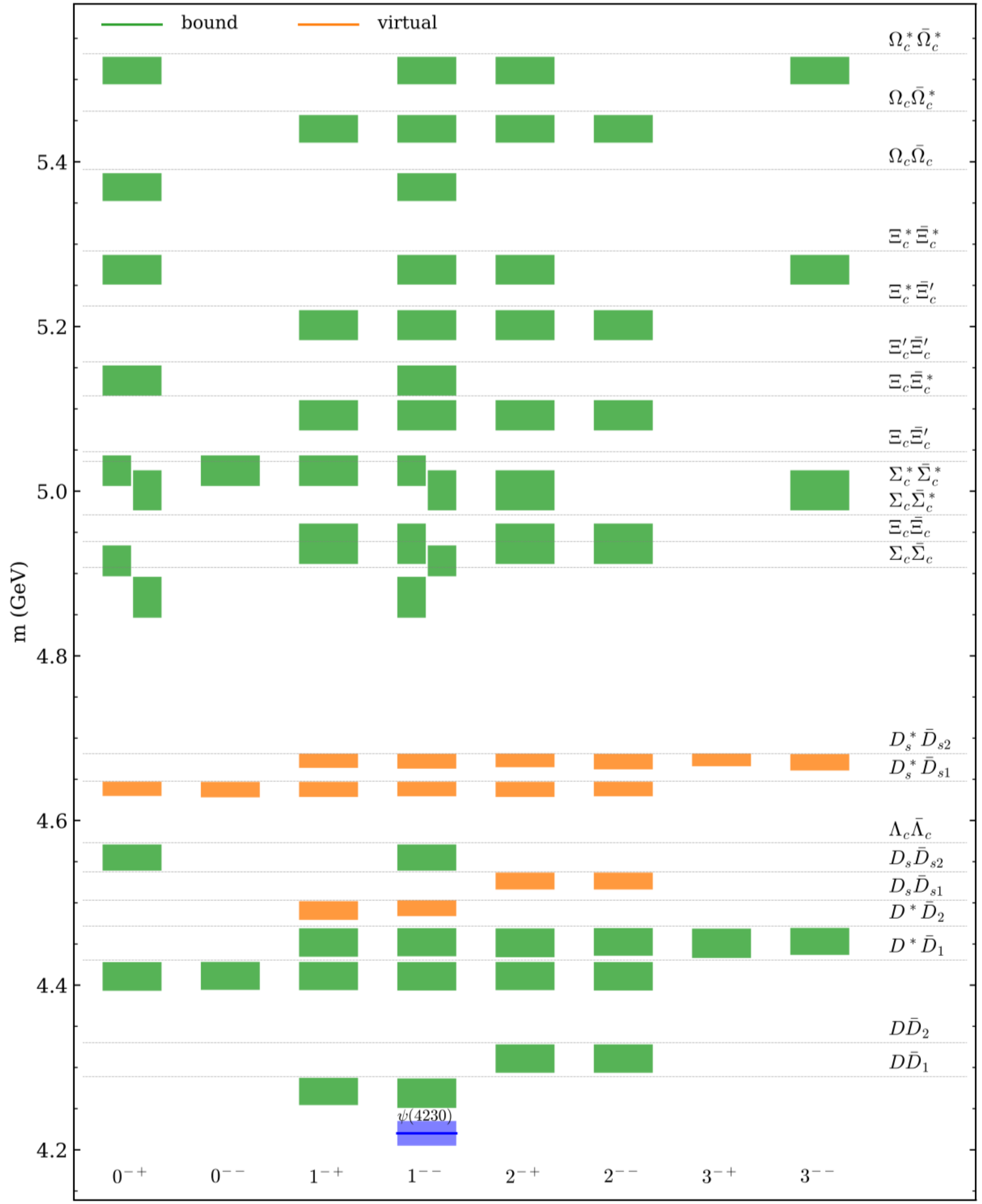
	$S-wave(L=0)$	$P-wave(L=1)$
PP	$0^{+(\pm)}$	$1^{-(\pm)}$
PV	$1^{+(\pm)}$	$0^{-(\pm)},\ 1^{-(\mp)},\ 2^{-(\pm)}$
VV	$0^{+(+)},\ 1^{+(-)},\ 2^{+(+)}$	$1^{-(+)};$ $0^{-(-)},\ 1^{-(-)},\ 2^{-(-)};$ $1^{-(+)},\ 2^{-(+)},\ 3^{-(+)}$
PA	$1^{-(-)}$	$\cdots\cdots$
VA	$0^{-(\pm)},\ 1^{-(\mp)},\ 2^{-(\pm)}$	$\cdots\cdots$





X.-K. Dong, F.-K. Guo, B.-S. Zou, Progr. Phys. 41 (2021) 65 [arXiv:2101.01021]





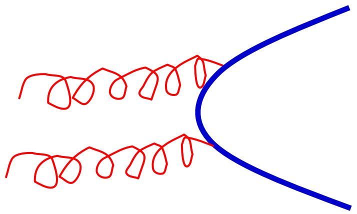
2. Isospin-violating decay of $B_s^* \to B_s \pi^0$

J. Wang and Q. Zhao, PRD 111 (2025) 096007, 2503.13138 [hep-ph]

Isospin symmetry and SU(2) flavor doublets

三类轻味夸克：

量子数\夸克	down	up	strange
电荷	$-1/3$	$+2/3$	$-1/3$
同位旋	$1/2$	$1/2$	0
同位旋$\hat{z}$分量	$-1/2$	$+1/2$	0
奇异数	0	0	-1



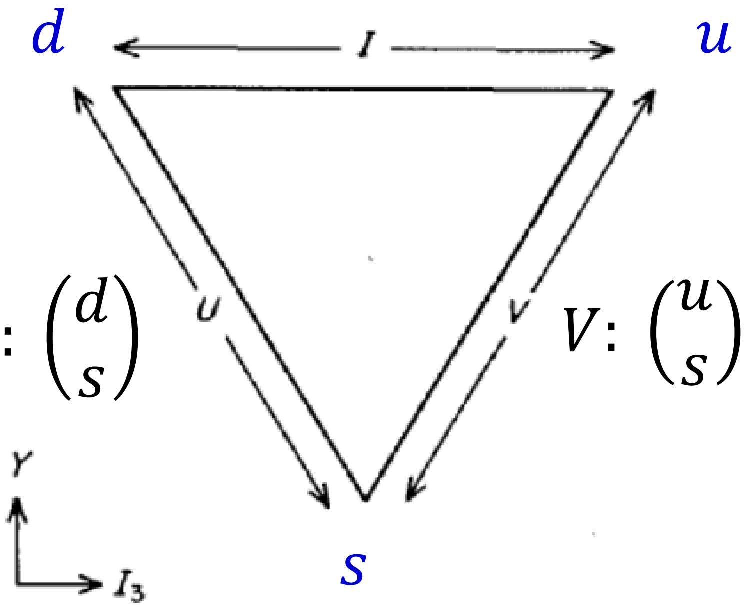
$$q\bar{q} = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$$

$$u\bar{u} = \underbrace{\frac{1}{2}(u\bar{u} + d\bar{d})}_{I=0} + \underbrace{\frac{1}{2}(u\bar{u} - d\bar{d})}_{I=1}$$

SU(3) flavor triplet with I, U, V SU(2) doublets

$$m_u \simeq m_d < m_s$$

$$I: \begin{pmatrix} u \\ d \end{pmatrix}$$



$$U: \begin{pmatrix} d \\ s \end{pmatrix} \qquad V: \begin{pmatrix} u \\ s \end{pmatrix}$$

Measurement of the quantum numbers for the first time in 2023!

BESIII, PL B846 (2023)138245

$\boxed{D_s^{*\pm}}$ $\qquad I(J^P) = 0(1^-)$

$J^P = 1^-$ established by ABLIKIM 23AZ.

$D_s^{*\pm}$ MASS

The fit includes $D^\pm$, D^0, $D_s^\pm$, $D^{*\pm}$, D^{*0}, $D_s^{*\pm}$, $D_1(2420)^0$, $D_2^*(2460)^0$, and $D_{s1}(2536)^\pm$ mass and mass difference measurements.

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
2112.2±0.4 OUR FIT				
2106.6±2.1±2.7	[1] BLAYLOCK	87	MRK3	$e^+e^- \rightarrow D_s^\pm \gamma X$

[1] Assuming $D_s^\pm$ mass $= 1968.7 \pm 0.9$ MeV.

$m_{D_s^{*\pm}} - m_{D_s^{\pm}}$

The fit includes $D^{\pm}$, D^0, $D_s^{\pm}$, $D^{*\pm}$, D^{*0}, $D_s^{*\pm}$, $D_1(2420)^0$, $D_2^*(2460)^0$, and $D_{s1}(2536)^{\pm}$ mass and mass difference measurements.

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
143.8 ± 0.4 OUR FIT					
143.9 ± 0.4 OUR AVERAGE					
143.76 ± 0.39 ± 0.40		GRONBERG	95	CLE2	$e^+ e^-$
144.22 ± 0.47 ± 0.37		BROWN	94	CLE2	$e^+ e^-$
142.5 ± 0.8 ± 1.5		[2] ALBRECHT	88	ARG	$e^+ e^- \rightarrow D_s^{\pm} \gamma X$
139.5 ± 8.3 ± 9.7	60	AIHARA	84D	TPC	$e^+ e^- \rightarrow$ hadrons
• • • We do not use the following data for averages, fits, limits, etc. • • •					
143.0 ± 18.0	8	ASRATYAN	85	HLBC	FNAL 15-ft, ν-^{2}H
110 ± 46		BRANDELIK	79	DASP	$e^+ e^- \rightarrow D_s^{\pm} \gamma X$

[2] Result includes data of ALBRECHT 84B.

$D_s^{*\pm}$ WIDTH

VALUE (MeV)	*CL%*	*DOCUMENT ID*		*TECN*	*COMMENT*
< 1.9	90	GRONBERG	95	CLE2	e^+e^-
$<$ 4.5	90	ALBRECHT	88	ARG	$E_{\rm cm}^{ee} = 10.2$ GeV
• • • We do not use the following data for averages, fits, limits, etc. • • •					
$<$ 4.9	90	BROWN	94	CLE2	e^+e^-
$<$22	90	BLAYLOCK	87	MRK3	$e^+e^- \rightarrow D_s^{\pm}\gamma X$

D_s^{*+} DECAY MODES

D_s^{*-} modes are charge conjugates of the modes below.

	Mode	Fraction (Γ_i/Γ)
Γ_1	$D_s^+\gamma$	(93.6 ± 0.4) %
Γ_2	$D_s^+\pi^0$	(5.77 ± 0.35) %
Γ_3	$D_s^+e^+e^-$	$(6.7 \pm 1.6) \times 10^{-3}$
Γ_4	$e^+\nu_e$	$(2.1\ ^{+1.2}_{-0.9}) \times 10^{-5}$

Isospin-violating through $\eta - \pi^0$ mixing

For $D_s^{*+} \rightarrow D_s^+ \eta$, the corresponding effective Lagrangian is

$$\mathcal{L}_{D_s^* D_s \eta} = i g_{D^* DP} \sin \alpha_P D_s (D_s^*)_\mu \partial^\mu \eta,$$

where the mixing angle $\alpha_P = 40.6°$ is adopted for the $\eta - \eta'$ mixing.

The $\eta - \eta'$ mixing angle is given by the leading order chiral expansion

$$\tan(2\theta_{\eta\pi^0}) = \frac{\sqrt{3}}{2} \frac{m_d - m_u}{m_s - \hat{m}}, \quad \text{with} \quad \hat{m} = (m_u + m_d)/2.$$

Since $\theta_{\eta\pi^0}$ is small, we can make the following approximation: $\theta_{\eta\pi^0} \simeq \frac{\sqrt{3}}{4} \frac{m_d - m_u}{m_s - \hat{m}}$,

[13] Chi-Yee Cheung and Chien-Wen Hwang, Three symmetry breakings in strong and radiative decays of strange heavy mesons, Eur. Phys. J. C **76**, 19 (2016).

[15] Peter L. Cho and Mark B. Wise, Comment on $D_s^* \to D_s \pi^0$ decay, Phys. Rev. D **49**, 6228 (1994).

[16] A. N. Ivanov, On the $D_s^{*+} \to D_s^+ + \pi_0$ decay in the effective quark model with chiral $U(3) \times U(3)$ symmetry, p. 5, 1998.

[17] Kunihiko Terasaki, Decays of Charmed Vector Mesons—$\eta\pi^0$ mixing as an origin of isospin non-conservation, p. 11, 2015.

[18] Bin Yang, Bo Wang, Lu Meng, and Shi-Lin Zhu, Isospin violating decay $D_s^* \to D_s \pi^0$ in chiral perturbation theory, Phys. Rev. D **101**, 054019 (2020).

J. Gasser and H. Leutwyler, Chiral perturbation theory: Expansions in the mass of the strange quark, Nucl. Phys.B250, 465 (1985).

Effective Lagrangians

$$\mathcal{L} = -ig_{\mathcal{D}^*\mathcal{D}\mathcal{P}}(\mathcal{D}^i\partial^\mu\mathcal{P}_{ij}\mathcal{D}_\mu^{*j\dagger} - \mathcal{D}_\mu^{*i}\partial^\mu\mathcal{P}_{ij}\mathcal{D}^{j\dagger}) + \frac{1}{2}g_{\mathcal{D}^*\mathcal{D}^*\mathcal{P}}\varepsilon_{\mu\nu\alpha\beta}\mathcal{D}_i^{*\mu}\partial^\nu\mathcal{P}^{ij}\overleftrightarrow{\partial}^\alpha\mathcal{D}_j^{*\beta\dagger}$$
$$- ig_{\mathcal{D}\mathcal{D}\mathcal{V}}\mathcal{D}_i^\dagger\overleftrightarrow{\partial}_\mu\mathcal{D}^j(\mathcal{V}^\mu)^i_j - 2f_{\mathcal{D}^*\mathcal{D}\mathcal{V}}\epsilon_{\mu\nu\alpha\beta}(\partial^\mu\mathcal{V}^\nu)^i_j(\mathcal{D}_i^\dagger\overleftrightarrow{\partial}^\alpha\mathcal{D}^{*\beta j} - \mathcal{D}_i^{*\beta\dagger}\overleftrightarrow{\partial}^\alpha\mathcal{D}^j)$$
$$+ ig_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}\mathcal{D}_i^{*\nu\dagger}\overleftrightarrow{\partial}_\mu\mathcal{D}_\nu^{*j}(\mathcal{V}^\mu)^i_j + 4if_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}\mathcal{D}_{i\mu}^{*\dagger}(\partial^\mu\mathcal{V}^\nu - \partial^\nu\mathcal{V}^\mu)^i_j\mathcal{D}_\nu^{*j},$$

where $\mathcal{D}$ and $\mathcal{D}^*$ represent the pseudoscalar and vector charm meson fields, respectively, i.e.,

$$\mathcal{D} = (D^0, D^+, D_s^+), \qquad \mathcal{D}^* = (D^{*0}, D^{*+}, D_s^{*+}),$$

and $\mathcal{P}$ and $\mathcal{V}$ are 3×3 matrices representing the pseudoscalar nonet and vector nonet meson fields

$$\mathcal{P} = \begin{pmatrix} \frac{\sin\alpha_P\eta'+\cos\alpha_P\eta+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\sin\alpha_P\eta'+\cos\alpha_P\eta-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \cos\alpha_P\eta' - \sin\alpha_P\eta \end{pmatrix} \mathcal{V} = \begin{pmatrix} \frac{\rho^0+\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega-\rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}$$

For the light hadron vertices, we adopt the following effective Lagrangians:

$$\mathcal{L}_{VPP} = ig_{VPP}\mathrm{Tr}[(\mathcal{P}\partial_\mu\mathcal{P} - \partial_\mu\mathcal{P}\mathcal{P})\mathcal{V}^\mu],$$
$$\mathcal{L}_{VVP} = g_{VVP}\varepsilon_{\alpha\beta\mu\nu}\mathrm{Tr}[\partial^\alpha\mathcal{V}^\mu\partial^\beta\mathcal{V}^\nu\mathcal{P}],$$
$$\mathcal{L}_{VVV} = ig_{VVV}\mathrm{Tr}[(\partial_\mu V_\nu - \partial_\nu V_\mu)V^\mu V^\nu].$$

The following coupling constants are adopted:

$$g_{D^*D^*\pi} = \frac{g_{D^*D\pi}}{\sqrt{m_D m_{D^*}}} = \frac{2g}{f_\pi}, \qquad g_{DDV} = g_{D^*D^*V} = \frac{\beta g_V}{\sqrt{2}}, \qquad f_{D^*DV} = \frac{f_{D^*D^*V}}{m_{D^*}} = \frac{\lambda g_V}{\sqrt{2}}, \qquad g_{D^*D_sK} = \sqrt{\frac{m_{D_s}}{m_D}}g_{D^*D\pi},$$

$$g_{D_s^*DK} = \sqrt{\frac{m_{D_s^*}}{m_{D^*}}}g_{D^*D\pi}, \qquad g_V = \frac{m_\rho}{f_\pi}, \qquad g_{D^*D^*K} = \frac{g_{D^*DK}}{\sqrt{m_D m_{D^*}}} = \frac{2g}{f_K} \qquad g_{D_sDV} = \sqrt{\frac{m_{Ds}}{m_D}}g_{DDV},$$

$$g_{D^*D_s^*K^*} = \sqrt{\frac{m_{D_s^*}}{m_{D^*}}}g_{D^*D^*V}, \qquad f_{D^*D_s^*K^*} = \sqrt{\frac{m_{D_s^*}}{m_{D^*}}}f_{D^*D^*V}, \qquad \frac{g_{D_s^*D_s\eta}}{\sqrt{m_{D_s^*}m_{D_s}}} = \frac{g_{D^*D\pi}\sin\alpha_P}{\sqrt{m_{D^*}m_D}},$$

with $g = 0.59, \beta = 0.9, f_\pi = 132\ \mathrm{MeV}, f_K = 155\ \mathrm{MeV},\ \lambda = 0.56\ \mathrm{GeV}^{-1}$

The relative strengths and phases of the coupling constants for vector and scalar mesons can be determined by $SU(3)$ flavor symmetry relations and expressed by overall coupling constants g_{VVP} and g_{VPP},

$$g_{K^{*+}K^{*-}\pi^0} = -g_{K^{*0}\bar{K}^{*0}\pi^0} = \frac{1}{\sqrt{2}} g_{VVP},$$

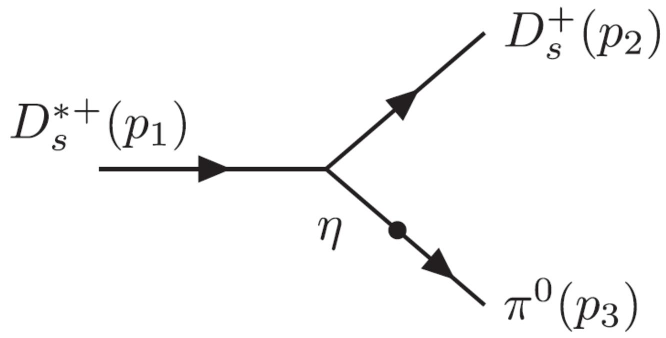
$$g_{K^{*0}\bar{K}^0\pi^0} = -g_{K^{*0}\pi^0\bar{K}^0} = g_{\bar{K}^{*0}\pi^0 K^0} = -g_{\bar{K}^{*0}K^0\pi^0} = \frac{1}{\sqrt{2}} g_{VPP},$$

$$g_{K^{*+}\pi^0 K^-} = -g_{K^{*+}K^-\pi^0} = g_{K^{*-}K^+\pi^0} = -g_{K^{*-}\pi^0 K^+} = \frac{1}{\sqrt{2}} g_{VPP}.$$

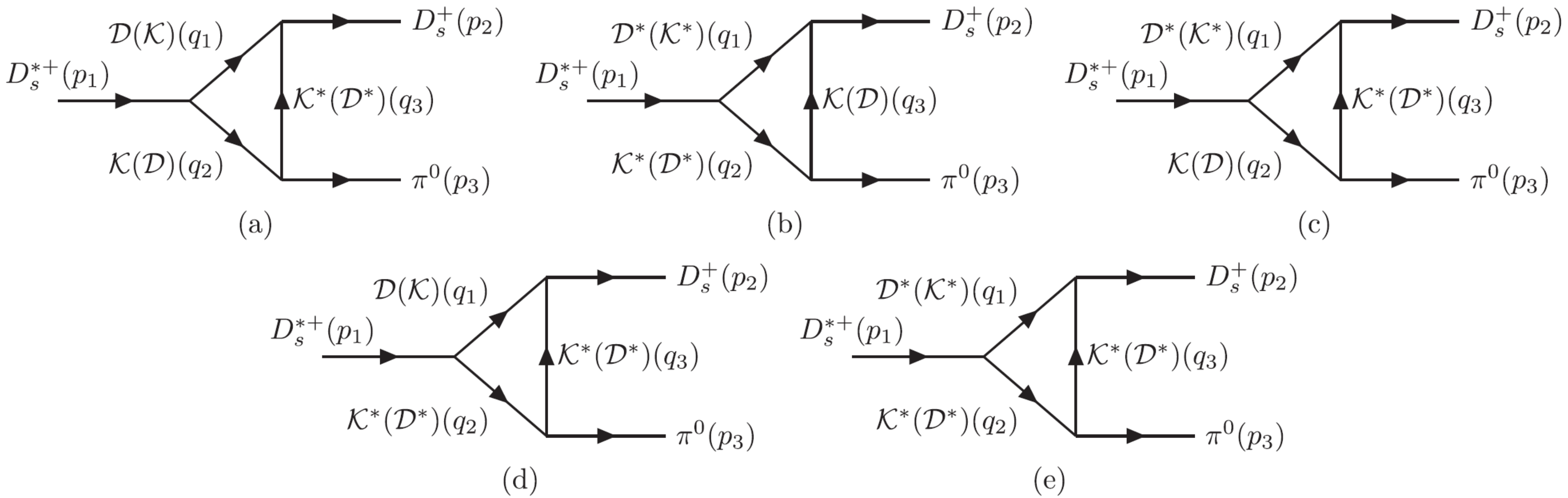
The tree-level amplitude:

$$i\mathcal{M}_{\text{tree}} = i g_{D_s^* D_s \eta} \varepsilon_{D_s^*} \cdot p_3 \theta_{\eta\pi^0} = i g_{\text{tree}} \varepsilon_{D_s^*} \cdot (p_2 - p_3),$$

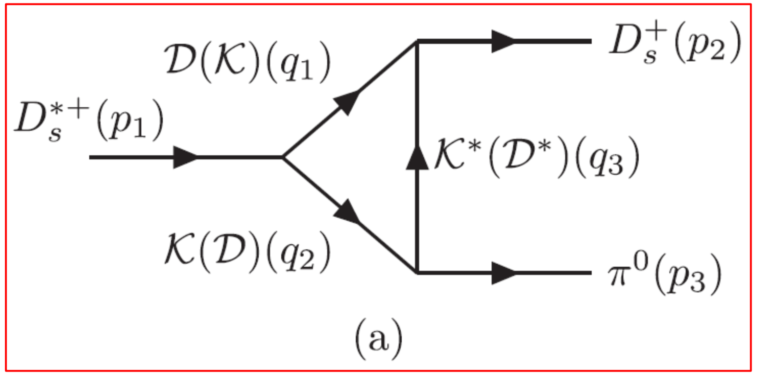
with $g_{\text{tree}} \equiv g_{D_s^* D_s \eta} \theta_{\eta\pi^0}/2$



The loop amplitude at one-loop level without $\eta - \pi^0$ mixing:



Examples:

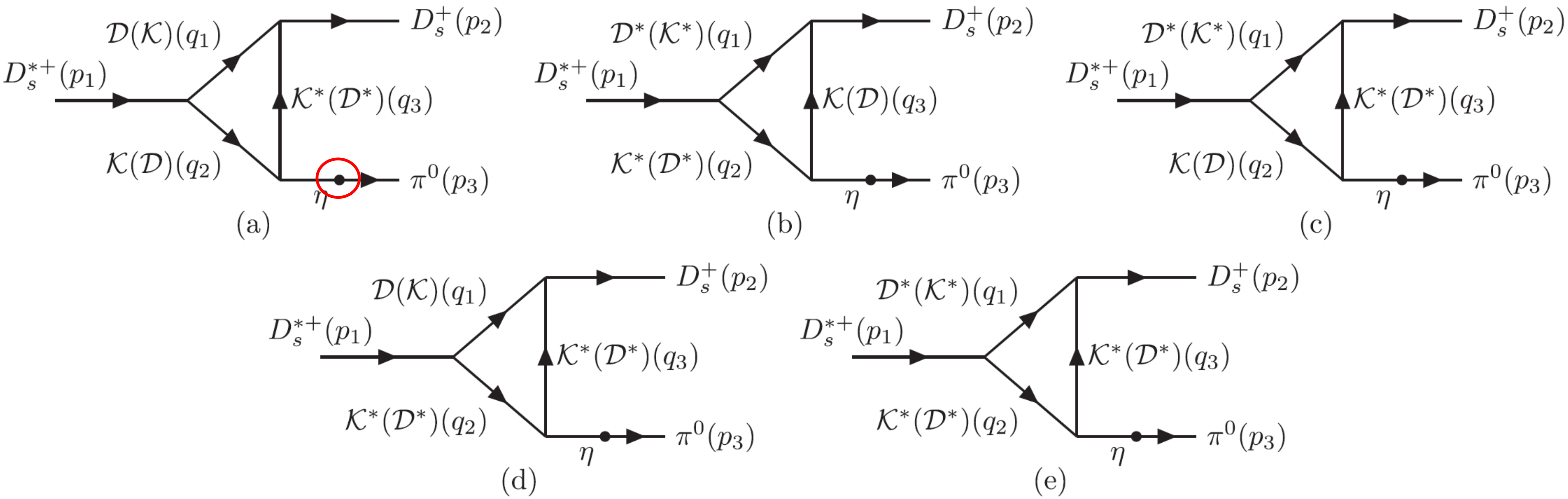


$$i\mathcal{M}_a(\mathcal{D},\mathcal{K},\mathcal{K}^*) = \int \frac{d^4 q_3}{(2\pi)^4} \frac{g_{\mathcal{D}_s^*\mathcal{D}\mathcal{K}} q_2 \cdot \varepsilon^s g_{\mathcal{D}_s\mathcal{D}\mathcal{K}^*}(q_1+p_2)_\alpha \left(g^{\alpha\beta} - \frac{q_3^\alpha q_3^\beta}{m_3^2}\right) g_{\mathcal{K}^*\mathcal{K}\pi}(p_3+q_2)_\beta}{(q_1^2-m_1^2)(q_2^2-m_2^2)(q_3^2-m_3^2)} \mathcal{F}(q_i^2),$$

$$i\mathcal{M}_a(\mathcal{K},\mathcal{D},\mathcal{D}^*) = \int \frac{d^4 q_3}{(2\pi)^4} \frac{g_{\mathcal{D}_s^*\mathcal{D}\mathcal{K}} g_{\mathcal{D}^*\mathcal{D}_s\mathcal{K}} g_{\mathcal{D}^*\mathcal{D}\pi} q_1 \cdot \varepsilon^s q_1^\mu \left(g_{\mu\nu} - \frac{q_{3\mu} q_{3\nu}}{m_3^2}\right) p_3^\nu}{(q_1^2-m_1^2)(q_2^2-m_2^2)(q_3^2-m_3^2)} \mathcal{F}(q_i^2),$$

with a UV cutoff $\mathcal{F}(p_i^2) = \prod_i \left(\frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 - p_i^2}\right)$ and $\Lambda_i \equiv m_i + \alpha\Lambda_{\mathrm{QCD}}$. $\Lambda_{QCD} = 220$ MeV

The loop amplitude at one-loop level with $\eta - \pi^0$ mixing:



$$
\left\{
\begin{aligned}
i\mathcal{M}_a(\mathcal{D},\mathcal{K},\mathcal{K}^*,\eta) &= i\mathcal{M}_a(\mathcal{D},\mathcal{K},\mathcal{K}^*)\cdot\frac{g_{\mathcal{K}\mathcal{K}^*\eta}\theta_{\eta\pi^0}}{g_{\mathcal{K}\mathcal{K}^*\pi}},\\
i\mathcal{M}_a(\mathcal{K},\mathcal{D},\mathcal{D}^*,\eta) &= i\mathcal{M}_a(\mathcal{K},\mathcal{D},\mathcal{D}^*)\cdot\frac{g_{\mathcal{D}^*\mathcal{D}\eta}\theta_{\eta\pi^0}}{g_{\mathcal{D}^*\mathcal{D}\pi}},
\end{aligned}
\right.
$$

Power counting of the loop amplitude:

Assuming the intermediate mesons are close to being on-shell, the propagator reads

$$1/(t-m_3^2) \simeq -1/m_3^2$$

The integrand counts: $(v^5/v^4) \times p_\pi \cdot p_{D_s}/m_3^2 \simeq vE_{D_s}E_\pi/m_3^2$

For the two loop amplitudes which cancel each other, we have

$$vE_{D_s}E_\pi(1/m_{K^{*\pm}}^2 - 1/m_{K^{*0}}^2) \simeq vE_\pi\delta_{K^*}/m_{K^*}^2 \simeq v\underbrace{(m_\pi/m_{K^*})}_{\text{Loop power suppression}}\underbrace{(\delta_{K^*}/m_{K^*})}_{\text{Isospin breaking term}}$$

$$\boxed{\delta_{K^*} \equiv m_{K^{*0}} - m_{K^{*\pm}}}$$

The loop amplitude can be expressed as

$$i\mathcal{M}_{\text{loop}} = ig_{\text{loop}}\varepsilon_{D_s^{*+}} \cdot (p_2 - p_3)$$

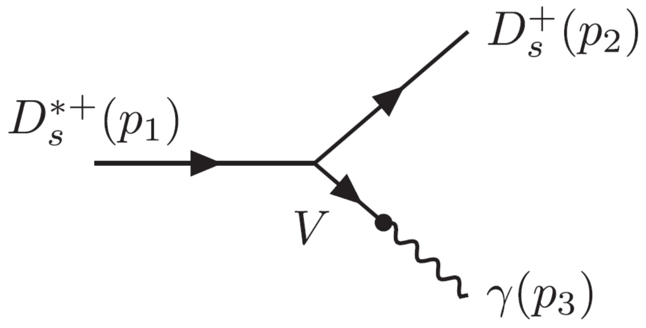
The total amplitude can be written as

$$\begin{aligned} i\mathcal{M}_{D_s^{*+}\to D_s^+\pi^0} &= i(g_{\text{tree}} + g_{\text{loop}})\varepsilon_{D_s^{*+}} \cdot (p_{D_s^+} - p_{\pi^0}) \\ &\equiv ig_{\text{total}}\varepsilon_{D_s^{*+}} \cdot (p_{D_s^+} - p_{\pi^0}), \end{aligned}$$

Tree and loop amplitudes of $D_s^{*+} \to D_s^+ \gamma$ in the VMD model

The radiative decay of D_s^* and its loop correction can be evaluated in the same framework

$$i\mathcal{M}^{\gamma}_{\rm tree} = i g^{\gamma}_{\rm tree}(\gamma)\varepsilon_{\mu\nu\alpha\beta}p_1^{\mu}p_3^{\nu}\varepsilon_1^{\alpha}\varepsilon_3^{\beta}$$



where g^{γ}_{tree} can be calculated using the VMD model

$$g^{\gamma}_{\rm tree} = i g_{D_s^* D_s V}\frac{e m_V^2}{f_V}G_V$$

$$G_V \equiv \frac{-i}{p_\gamma^2 - m_V^2 + i m_V \Gamma_V} = \frac{-i}{-m_V^2 + i m_V \Gamma_V}$$

where $V = \rho, \omega, \phi, J/\psi$ and e/f_V can be determined by $V \to e^+e^-$.

$$\frac{e}{f_V} = \left[\frac{3\Gamma_{V\to e^+e^-}}{2\alpha_e|\mathbf{p}_e|}\right]^{\frac{1}{2}}$$

To match the coupling constants in heavy quark effective field theory, we have

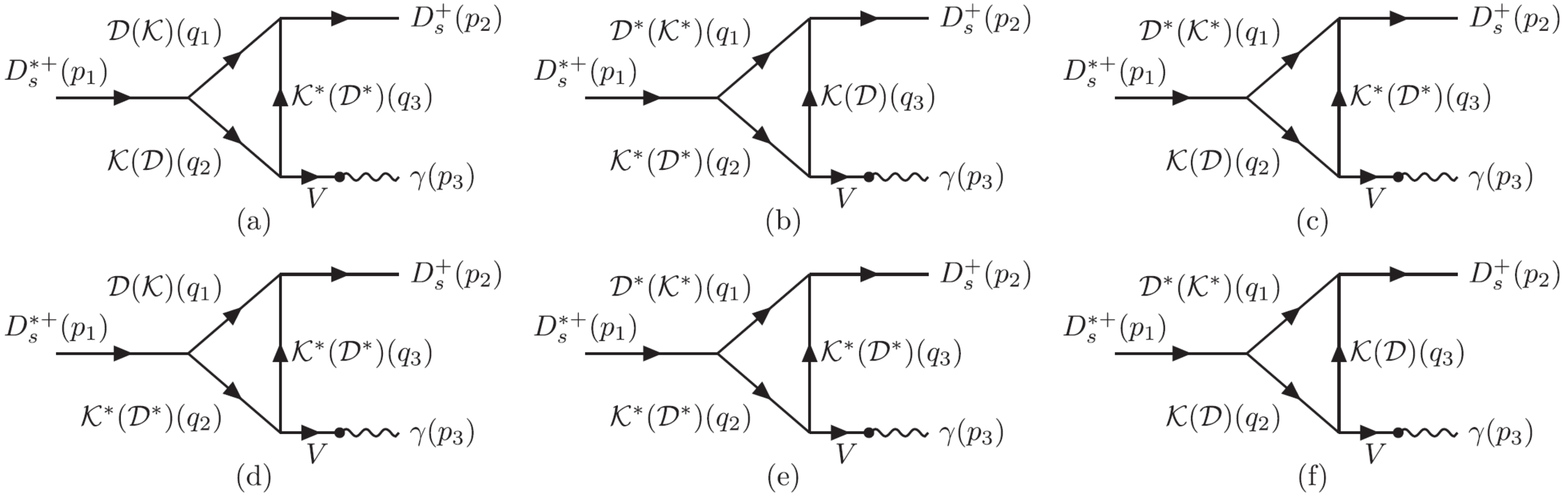
$$g_{D_s^* D_s V} = 4 f_{D_s^* D_s V}$$

For the tree-level $V = \phi, J/\psi$, the coupling constant can be extracted:

$$g_{D_s^* D_s \gamma} = i\left(g_{D_s^* D_s \phi} \frac{e m_\phi^2}{f_\phi} G_\phi + g_{D_s^* D_s \psi} \frac{e m_\psi^2}{f_\psi} G_\psi \right)$$

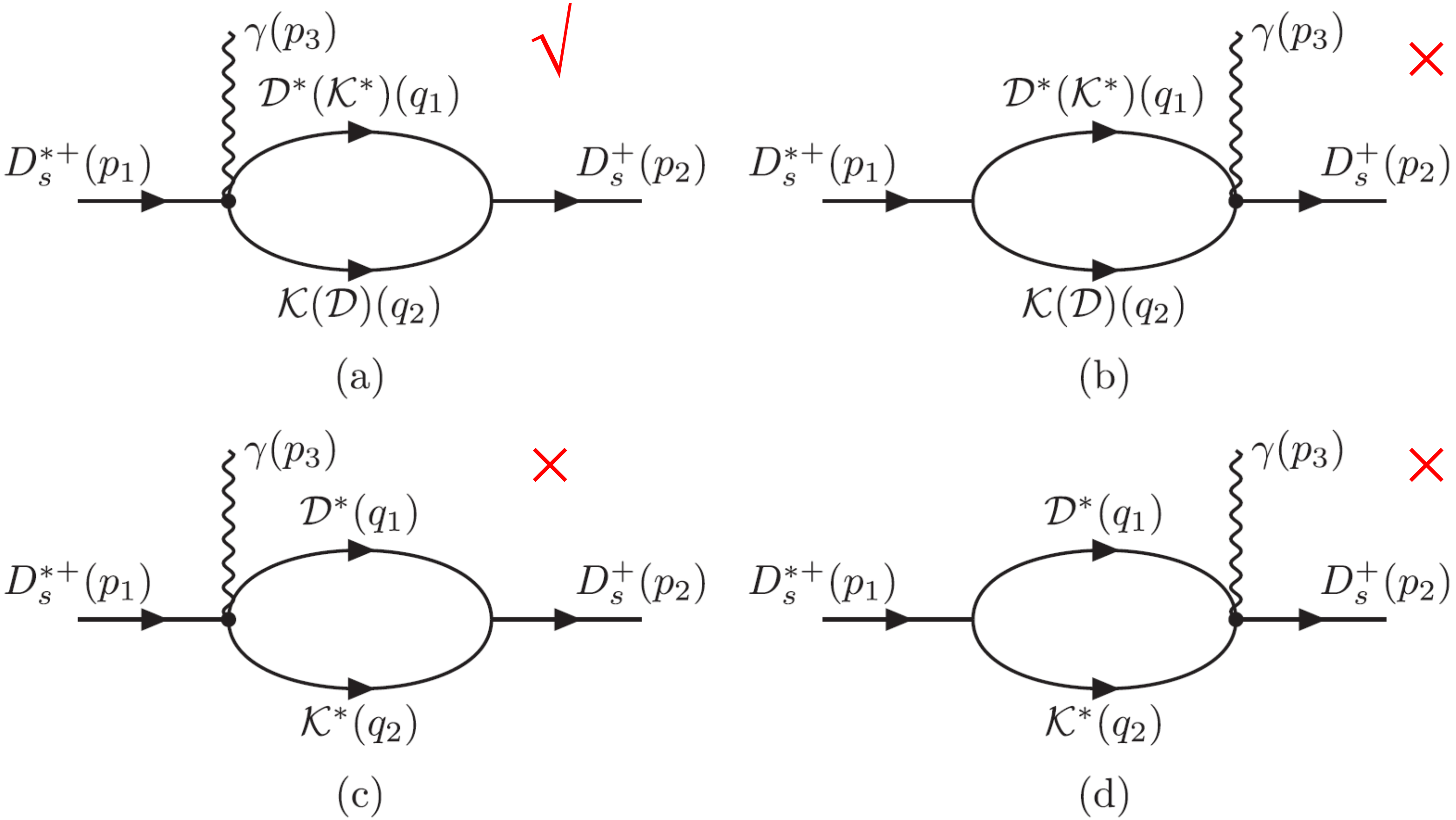
with $g_{D_s^* D_s \psi} = 2 g_{J/\psi D \bar{D}} / \tilde{M}, \tilde{M} = \sqrt{M_{D_s^*} M_{D_s}}$

Loop amplitudes based on the VMD:



$$i\mathcal{M}_a(\mathcal{D},\mathcal{K},\mathcal{K}^*,\gamma) = \int \frac{d^4 q_3}{(2\pi)^4} \frac{g_{\mathcal{K}\mathcal{K}^*\gamma}\, g_{\mathcal{D}_s^*\mathcal{D}\mathcal{K}}\, g_{\mathcal{D}_s\mathcal{D}\mathcal{K}^*}\, q_2 \cdot \varepsilon^s (q_1 + p_2)_\alpha \left(g^{\alpha\beta} - \frac{q_3^\alpha q_3^\beta}{m_3^2} \right) \varepsilon_{\beta\sigma\mu\nu} \varepsilon_3^\sigma q_3^\mu p_3^\nu}{(q_1^2 - m_1^2)(q_2^2 - m_2^2)(q_3^2 - m_3^2)} \mathcal{F}(q_i^2)$$

Contact diagrams induced by the requirement of Lorentz gauge invariance:



Meson-photon coupling via the VMD model:

$$g_{K^+K^{*+}\gamma} = i\left(g_{\rho^0 K^{*+}K^-}\frac{em_{\rho^0}^2}{f_{\rho^0}}G_{\rho^0} + g_{\omega K^{*+}K^-}\frac{em_\omega^2}{f_\omega}G_\omega + g_{\phi K^{*+}K^-}\frac{em_\phi^2}{f_\phi}RG_\phi\right),$$

$$g_{K^0K^{*0}\gamma} = i\left(g_{\rho^0 K^{*0}\bar{K}^{*0}}\frac{em_{\rho^0}^2}{f_{\rho^0}}G_{\rho^0} + g_{\omega K^{*0}\bar{K}^{*0}}\frac{em_\omega^2}{f_\omega}G_\omega + g_{\phi K^{*0}\bar{K}^{*0}}\frac{em_\phi^2}{f_\phi}RG_\phi\right),$$

$$g_{D^{*0}D^0\gamma} = i\left(g_{\rho^0 D^{*0}\bar{D}^0}\frac{em_{\rho^0}^2}{f_{\rho^0}}G_{\rho^0} + g_{\omega D^{*0}\bar{D}^0}\frac{em_\omega^2}{f_\omega}G_\omega + g_{\phi D^{*0}\bar{D}^0}\frac{em_\phi^2}{f_\phi}RG_\phi\right),$$

$$g_{D^{*+}D^+\gamma} = i\left(g_{\rho^0 D^{*+}D^-}\frac{em_{\rho^0}^2}{f_{\rho^0}}G_{\rho^0} + g_{\omega D^{*+}D^-}\frac{em_\omega^2}{f_\omega}G_\omega + g_{\phi D^{*+}D^-}\frac{em_\phi^2}{f_\phi}RG_\phi\right),$$

where $R = 0.8$ is the SU(3) flavor-symmetry-breaking parameter for processes involving the strange quark pair.

TABLE I. Values of the VPP coupling constants.

Coupling constant	$g_{D_s^{*+}D^0K}$	$g_{D_s^+D^0K^*}$	$g_{D^{*0}D^0\pi}$	$g_{D^{*-}D^+\pi}$	$g_{D_s^{*+}D^+K}$	$g_{D^{*0}D_s^+K}$	$g_{D_s^+D^+K^*}$	$g_{D^{*+}D_s^+K}$	g_{VPP}
Numerical value	18.40	3.84	17.29	−17.33	18.42	17.77	3.84	17.78	4.18

TABLE II. Values of the VVP coupling constants.

Coupling constant	$g_{D_s^{*+}D^{*0}K}$	$f_{D^{*0}D_s^+K^*}$	$f_{D_s^*D^0K^*}$	$g_{D^{*0}\bar{D}^{*0}\pi^0}$	$g_{D^{*+}D^{*-}\pi^0}$	g_{VVP}
Numerical value (GeV^{-1})	7.81	2.38	2.47	8.94	−8.94	7.93

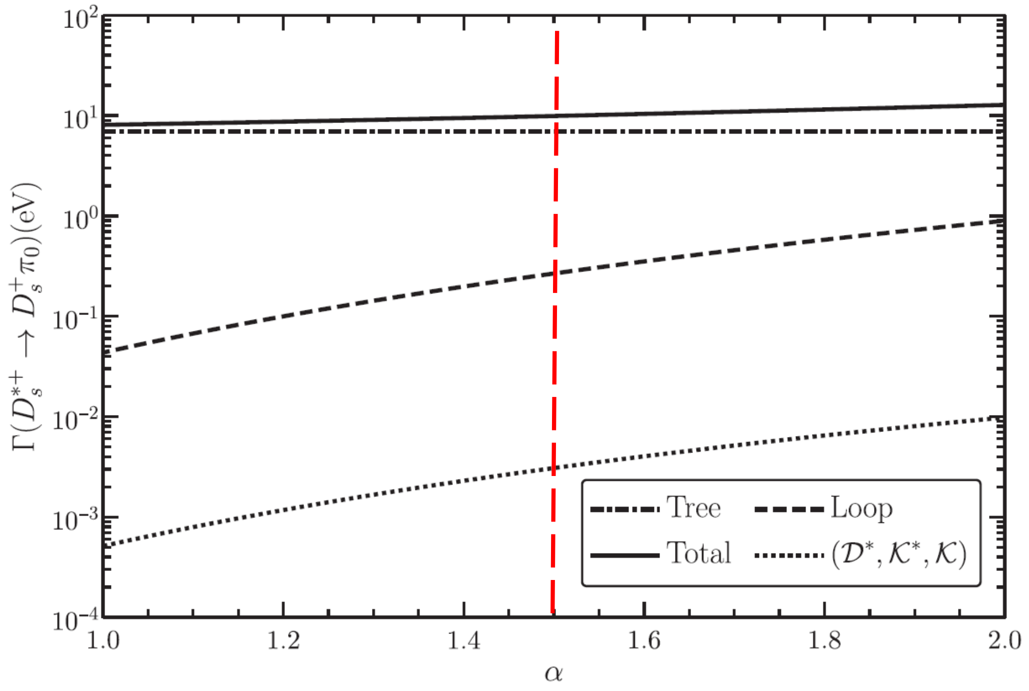
TABLE III. Values of the VVV coupling constants.

Coupling constant	g_{VVV}	$g_{D_s^{*+}D^{*0}K^*}$	$f_{D_s^*D^{*0}K^*}$
Numerical value	4.47	3.83	4.79

TABLE IV. Values of the electromagnetic coupling constants in the VMD model.

Coupling constant	$g_{K^{*+}K^+\gamma}$	$g_{K^{*0}K^0\gamma}$	$g_{D^{*0}D^0\gamma}$	$g_{D^{*+}D^+\gamma}$
Numerical value (GeV^{-1})	$-0.288+0.063i$	$0.369+0.062i$	$-0.383-0.082i$	$0.492+0.082i$
Coupling constant	$g_{K^{*+}K^{*+}\gamma}$	$g_{K^{*0}K^{*0}\gamma}$	$g_{D^{*0}D^{*0}\gamma}(f_{D^{*0}D^{*0}\gamma})$	$g_{D^{*+}D^{*+}\gamma}(f_{D^{*+}D^{*+}\gamma})$
Numerical value	$-0.162-0.035i$	$0.208+0.034i$	$-0.139-0.031i(-0.695-0.152i)$	$0.178+0.030i(0.892+0.150i)$
Coupling constant	$g_{K^+K^+\gamma}$	$g_{K^0K^0\gamma}$	$g_{D^0D^0\gamma}$	$g_{D^+D^+\gamma}$
Numerical value	$-0.288+0.063i$	$0.369+0.062i$	$-0.383-0.082i$	$0.492+0.082i$

Numerical results:



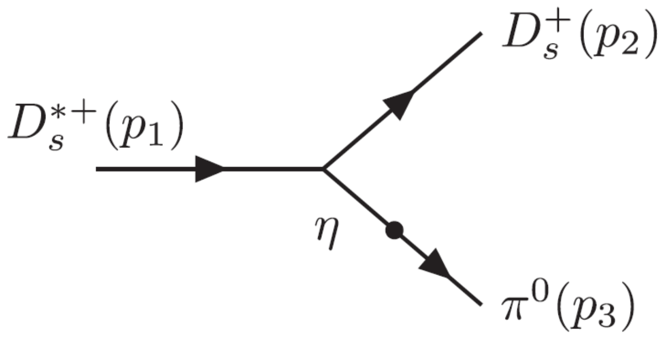
$\boxed{\alpha = 1.5 \pm 0.15}$

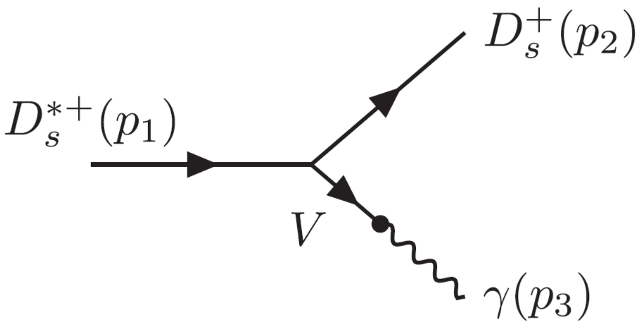
	CM [13]	χPT [18]	Our work
$\Gamma(D_s^{*+} \to D_s^+ \pi^0)$	277^{+28}_{-26}	$8.1^{+3.0}_{-2.6}$	$9.92^{+0.76}_{-0.66}$

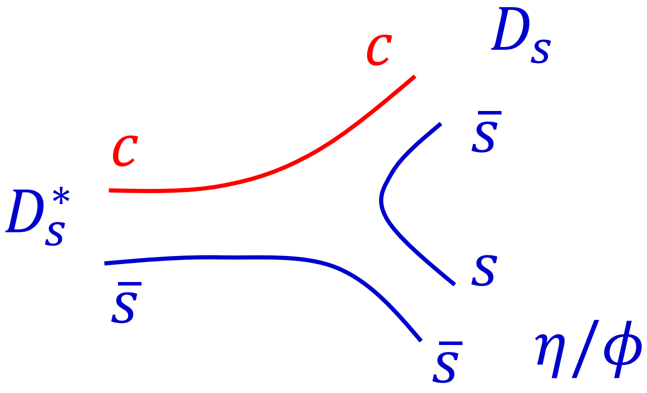
[13] Chi-Yee Cheung and Chien-Wen Hwang, Three symmetry breakings in strong and radiative decays of strange heavy mesons, Eur. Phys. J. C 76, 19 (2016).

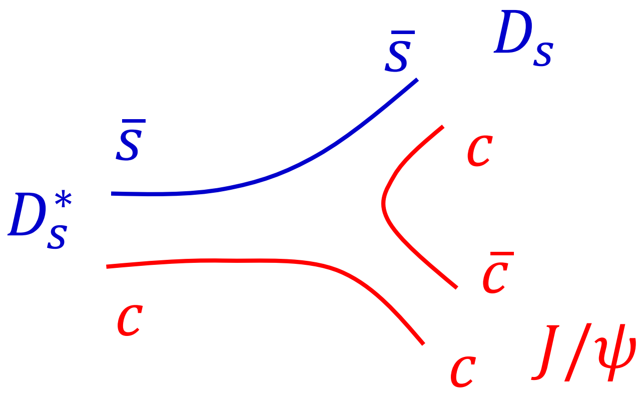
[18] Bin Yang, Bo Wang, Lu Meng, and Shi-Lin Zhu, Isospin violating decay $D_s^* \to D_s \pi^0$ in chiral perturbation theory, Phys. Rev. D 101, 054019 (2020).

Connection and difference between $D_s^* \to D_s \pi^0$ and $D_s^* \to D_s \gamma$









Cancellation occurs between the ϕ and J/ψ term.

The main uncertainties come from the $g_{J/\psi DD}$ coupling with $g_{J/\psi DD} = 7.0 \sim 7.5$.
Precise measurement of D_s^* width can provide a strong constraint on this coupling.

TABLE VII. Experimentally measured branching ratios and relative branching ratios of $D_s^{*+} \to D_s^+ \pi^0$ and $D_s^{*+} \to D_s^+ \gamma$.

	$\mathrm{BR}(D_s^{*+} \to D_s^+ \gamma)$	$\mathrm{BR}(D_s^{*+} \to D_s^+ \pi^0)$	$\mathrm{BR}(D_s^{*+} \to D_s^+ \pi^0)/\mathrm{BR}(D_s^{*+} \to D_s^+ \gamma)$
PDG [29]	$(93.6 \pm 0.4)\%$	$(5.77 \pm 0.35)\%$	$(6.2 \pm 0.4)\%$
BESIII [42]	$(93.54 \pm 0.38 \pm 0.22)\%$	$(5.76 \pm 0.38 \pm 0.16)\%$	$(6.16 \pm 0.43 \pm 0.18)\%$

TABLE IX. Contributions of the tree diagram, loop diagrams, and the combination of tree and loop diagrams to the partial decay width of $D_s^{*+} \to D_s^+ \gamma$, with $\alpha = 1.0$, 1.35, 1.5, 1.65, and 2.0 in units of eV and $g_{J/\psi D\bar{D}} = 7.23$.

α	1.0	1.35	1.5	1.65	2.0
Γ_{tree}	153.89	153.89	153.89	153.89	153.89
Γ_{loop}	0.54	2.22	3.59	5.55	13.14
Γ_{total}	155.30	157.84	159.69	162.16	171.15

Loop corrections to the radiative decays are small, but has a stringent constraint on the coupling $g_{J/\psi D\bar{D}}$.

The total width of D_s^* with uncertainty estimation.

	$\Gamma(D_s^{*+} \to D_s^+ \pi^0)$	$\Gamma(D_s^{*+} \to D_s^+ \gamma)$	$\Gamma_{\text{total}}(D_s^{*+})$
$\alpha = 1.5 \pm 0.15$, $g_{J/\psi D\bar{D}} = 7.23$	$9.92^{+0.76}_{-0.66}$	$159.7^{+2.5}_{-1.8}$	$169.6^{+2.6}_{-2.0}$
$\alpha = 1.5 \pm 0.15$, $g_{J/\psi D\bar{D}} = 7.23 \pm 0.06$	$9.92^{+0.76}_{-0.66}$	160^{+13}_{-12}	170^{+13}_{-12}

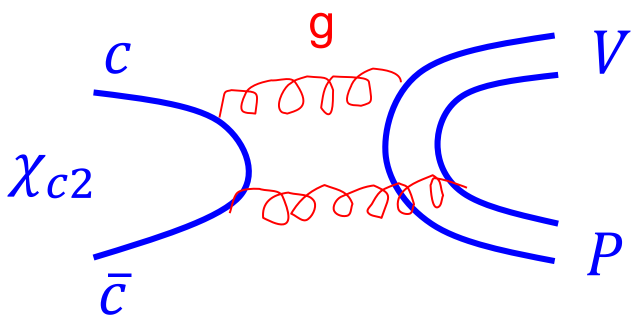
3. U-spin-violating decay of $\chi_{c2} \to VP$

X.H. Liu and Q. Zhao, *Evasion of helicity selection rule in $\chi_{c1} \to VV$ and $\chi_{c2} \to VP$ via intermediate charmed meson loops*, PRD 81, 014017 (2010)

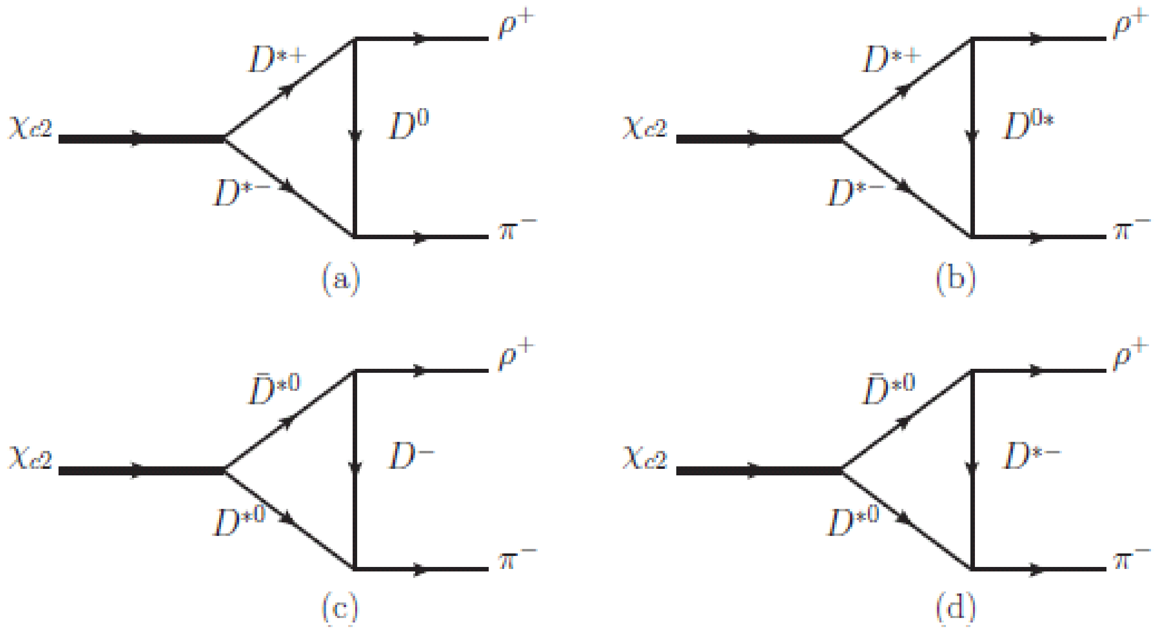
$\chi_{c2} \to VP$

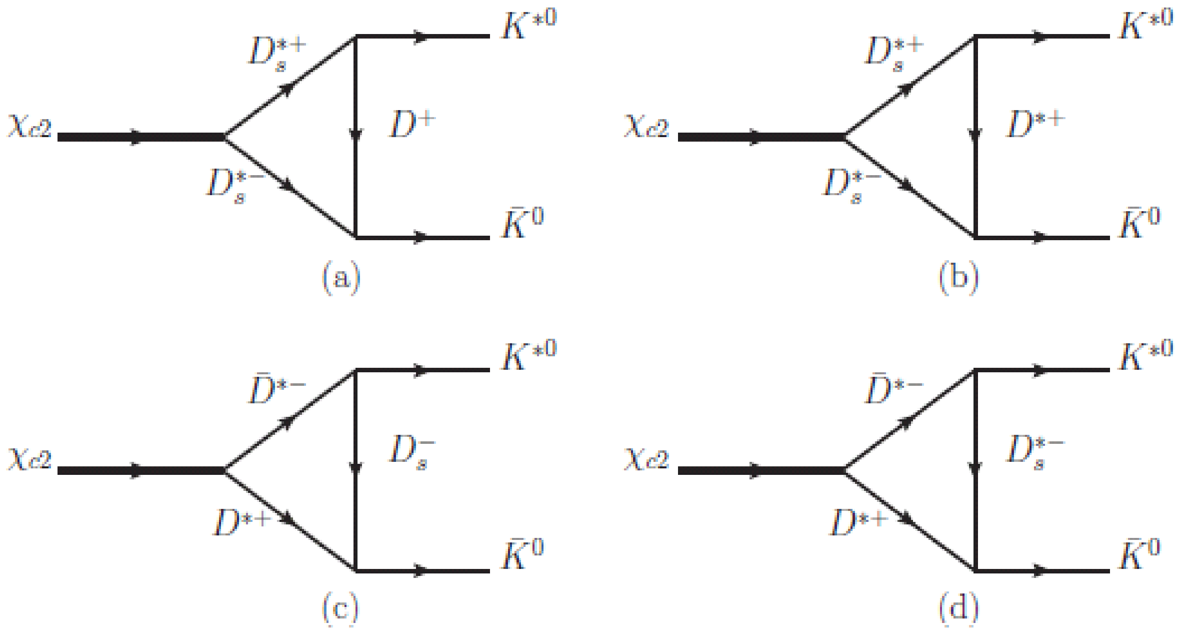
- **Further suppressed by approximate G-parity or isospin/U-spin conservation.**
- **Decay to neutral VP is forbidden by C-parity conservation.**

- **Short-ranged mechanism**

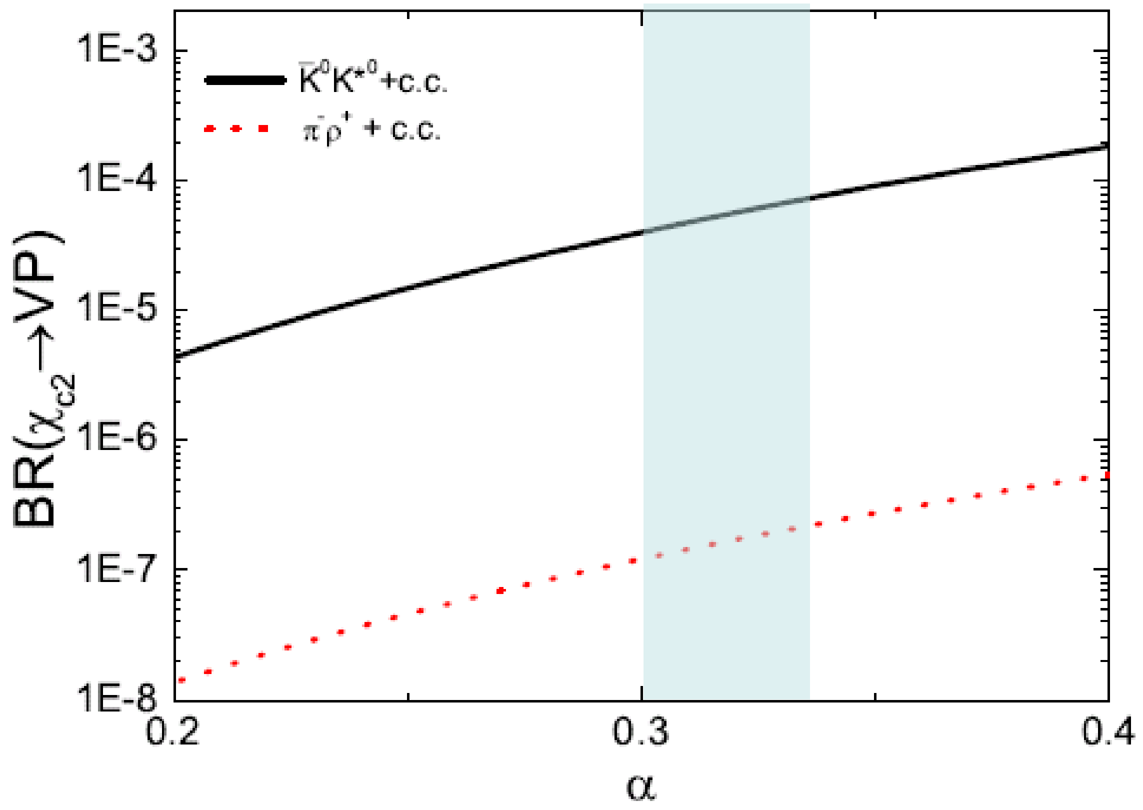


- **Long-ranged mechanism**





Predictions for the U-spin-violating decays of $\chi_{c2} \to VP$



Γ_{28}	$\rho(770)^{\pm}\pi^{\mp}$	$(6 \pm 4) \times 10^{-6}$
Γ_{34}	$K^*(892)^{\pm}K^{\mp}$	$(1.46 \pm 0.21) \times 10^{-4}$
Γ_{35}	$K^*(892)^0\overline{K}^0$ + c.c.	$(1.26 \pm 0.27) \times 10^{-4}$

BESIII, PR D96 (2017) 111102

$\mathrm{BR}(\times 10^{-5})$	$K^{*0}\bar{K}^0 + c.c.$	$K^{*+}K^- + c.c.$	$\rho^+\pi^- + c.c.$
Meson loop	$4.0 \sim 6.7$	$4.0 \sim 6.7$	$(1.2 \sim 2.0) \times 10^{-2}$
Exp. data	—	—	—

α=0.3 ~ 0.33

X.H. Liu and Q. Zhao, *Evasion of helicity selection rule in $\chi_{c1} \to VV$ and $\chi_{c2} \to VP$ via intermediate charmed meson loops*, PRD 81, 014017 (2010)

4. A brief summary

- What are the proper effective degrees of freedom for hadron internal structures?
- What are the possible color-singlet hadrons apart from the simplest conventional mesons ($q\bar{q}$) and baryons (qqq)? (e.g. multiquarks, hadronic molecules, hadroquarkonia …)
- What are the proper observables for determining the internal structures for hadrons ? Sometimes, symmetry breaking effects can help!
- How to distinguish genuine states from kinematic enhancements due to TS?
- What's happening in between "perturbative" and "non-perturbative"?
- … …

Thanks for your attention!