Theoretical study on the recently observed $T_{c\bar{s}}$ state

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Hadron spectra and hadron structures：Exotic states

Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

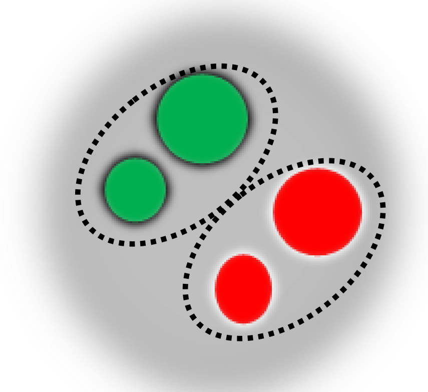
M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way" [1-3], we are tempted to look for some fundamental explanation of the situation. A highly promised approach is the purely dynamical "bootstrap" model for all the strongly interacting particles within which one may try to derive isotopic spin and strangeness conservation and broken eightfold symmetry from self-consistency alone [4]. Of course, with only strong interactions, the orientation of the asymmetry in the unitary space cannot be specified; one hopes that in some way the selection of specific components of the F-spin by electromagnetism and the weak interactions determines the choice of isotopic spin and hypercharge directions.

ber $n_t - n_{\bar{t}}$ would be zero for all known baryons and mesons. The most interesting example of such a model is one in which the triplet has spin $\frac{1}{2}$ and $z = -1$, so that the four particles d^-, s^-, u^0 and b^0 exhibit a parallel with the leptons.

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" [6] q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc. It is assuming that the lowest



Compact multiquark

8419/TH.412
21 February 1964

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

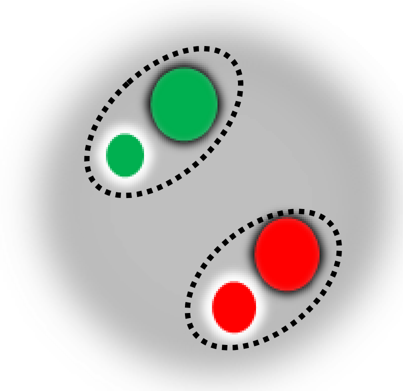
II *)

G. Zweig
CERN---Geneva

*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.

...

6) In general, we would expect that baryons are built not only from the product of three aces, AAA, but also from $AAAA\bar{A}$, $AAAAAA\bar{A}\bar{A}$, etc., where $\bar{A}$ denotes an anti-ace. Similarly, mesons could be formed from $A\bar{A}$, $AA\bar{A}\bar{A}$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $A\bar{A}$ and AAA, that is, "deuces and treys".

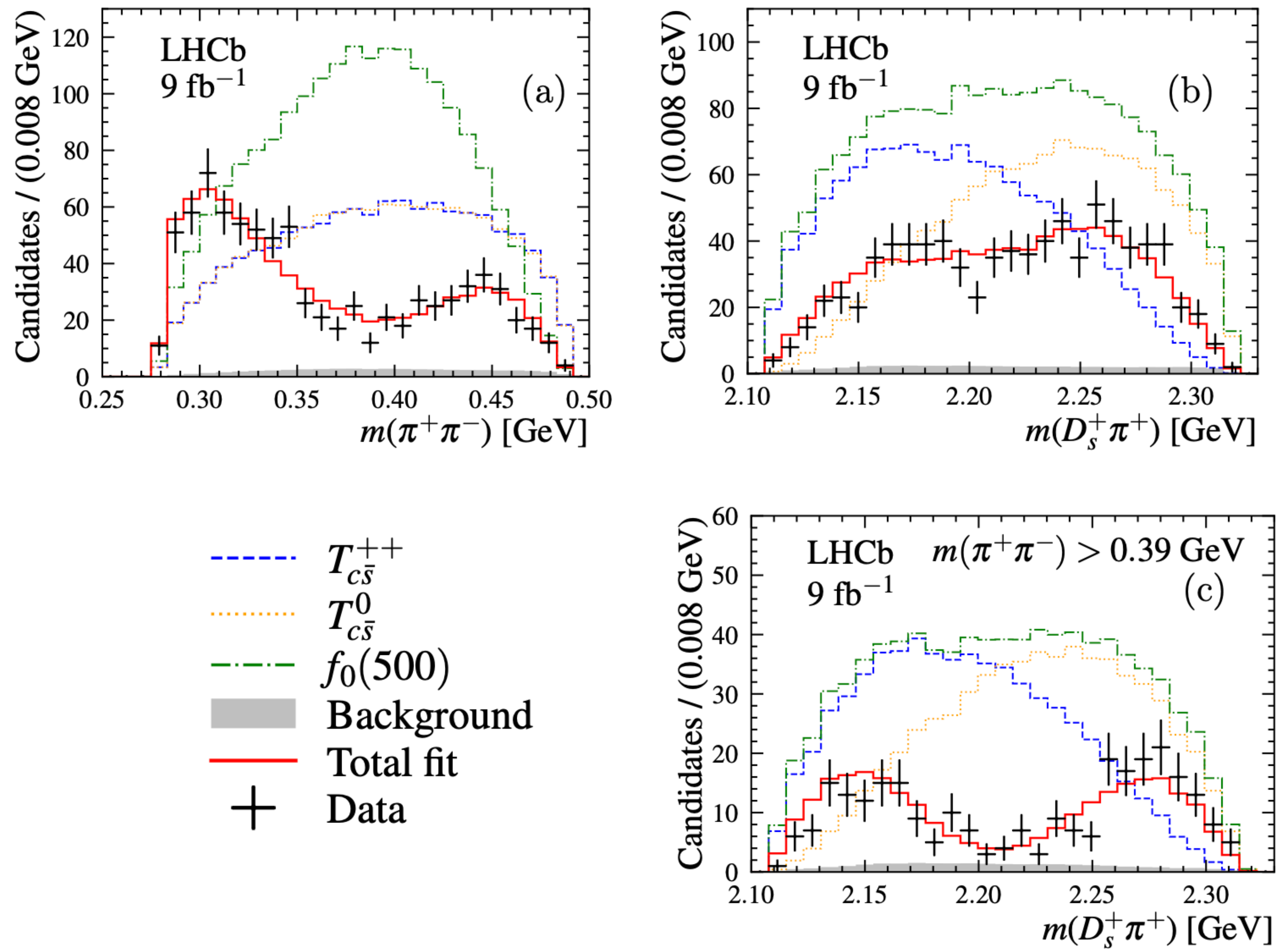


Hadronic molecule

Exotic structure in $B \to D^{(*)} D_s^+ \pi^+ \pi^-$ decays

LHCb, arXiv:2411.03399

$$B \to D^{(*)} D_{s1}(2460)$$
$$\downarrow$$
$$D_s^+ \pi^+ \pi^-$$



Model	Resonance	Mass (MeV)	Width (MeV)
$f_0(500)$ + RBW $T_{c\bar{s}}(0^+)$	$f_0(500)$	$464 \pm 23 \pm 14$	$214 \pm 28 \pm 8$
	$T_{c\bar{s}}^{++}/T_{c\bar{s}}^{0}$	$2312 \pm 27 \pm 11$	$264 \pm 46 \pm 21$
$f_0(500)$ + K-matrix $T_{c\bar{s}}(0^+)$	$f_0(500)$	$472 \pm 32 \pm 19$	$226 \pm 24 \pm 18$
	$T_{c\bar{s}}^{++}/T_{c\bar{s}}^{0}$	$2328 \pm 12 \pm 12$	$96 \pm 16 \pm 23$

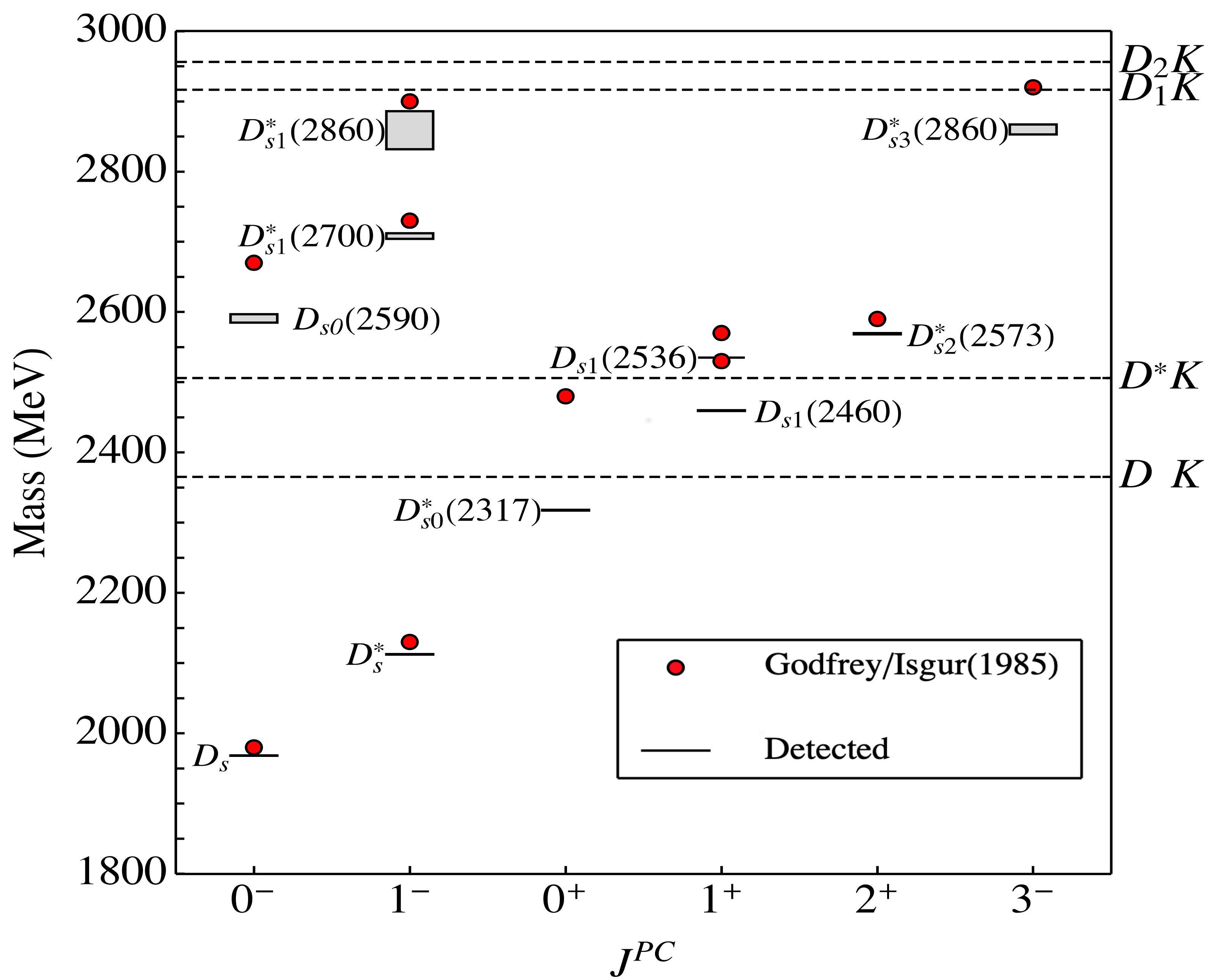
$D_{s1}(2460)$ exotic structure

Mesons in a Relativized Quark Model with Chromodynamics

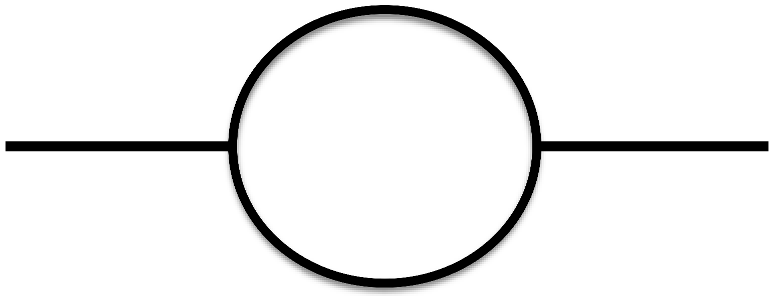
#1

S. Godfrey (Toronto U.), Nathan Isgur (Toronto U.) (1985)

Published in: *Phys.Rev.D* 32 (1985) 189-231



Coupled-channel effect



1. Yu. S. Kalashnikova, **Phys.Rev.D 72, 034010 (2005)**

   ☞ Charmonium

2. F.-K. Guo, S. Krewald, and U.-G. Meißner, **Phys.Lett.B 665,157 (2008)**
   Z.-Y. Zhou and Z. Xiao, **Phys. Rev. D 84, 034023 (2011)**

   ☞ Charmed and charmed-strange spectra

3. Y. Lu, M. N. Anwar, B. S. Zou, **Phys.Rev.D 94, 034021 (2016)**

   ☞ Bottomonium

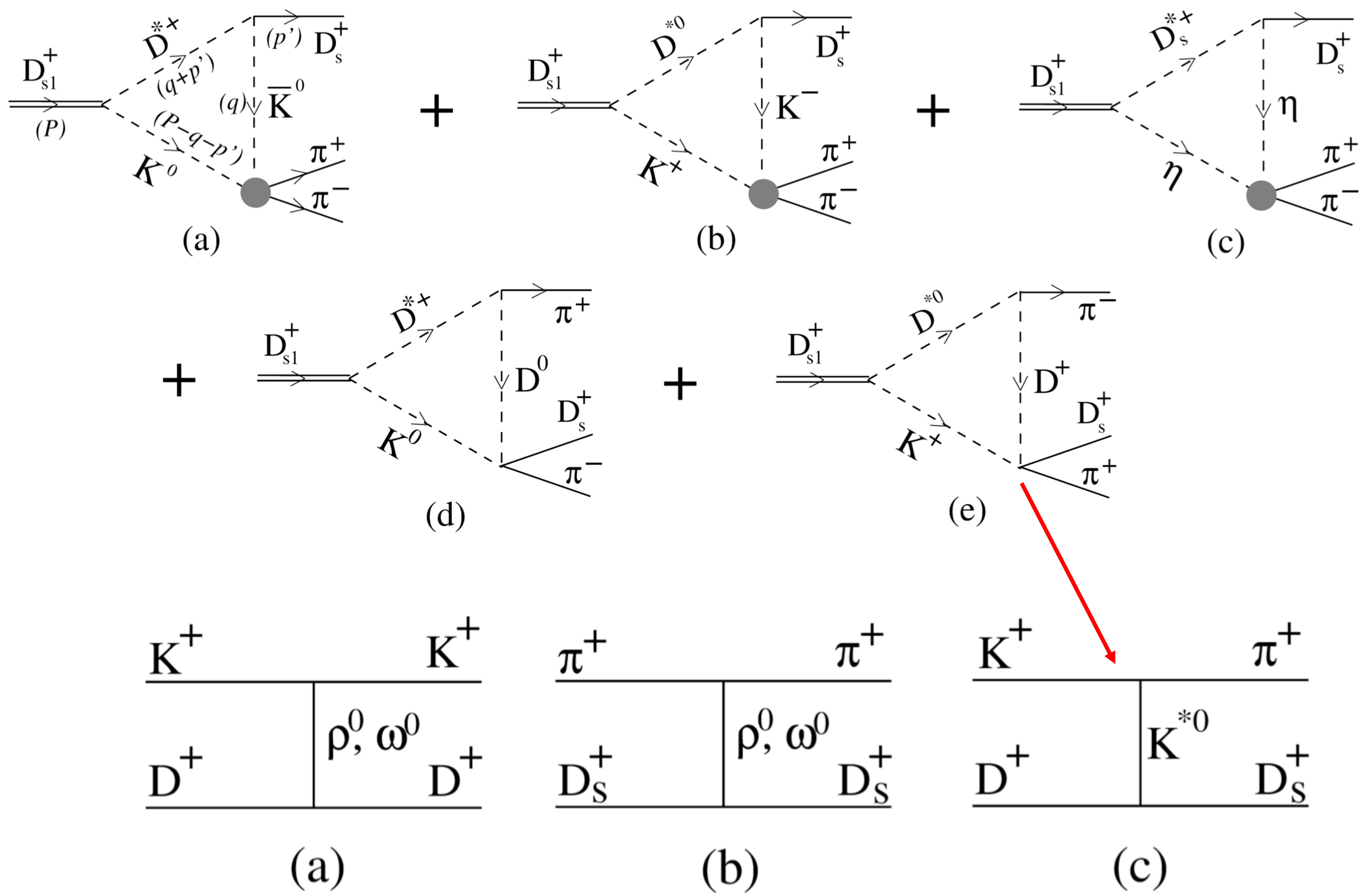
……

- **Coupled-channel effect due to hadron loop could cause sizable mass shift on the state in quark model.**

Theoretical interpretation

- Molecular $D_{s1}(2460)[D^*K, D_s^*\eta]$ state decay

Roca, Dias and Oset, arXiv:2502.18401



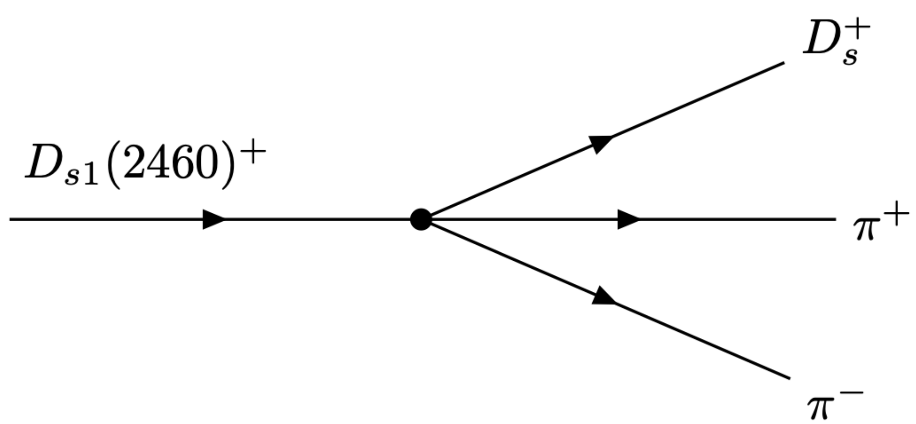
$$V_{K^+D^+,K^+D^+} = 0; \quad V_{\pi^+D_s^+,\pi^+D_s^+} = 0;$$

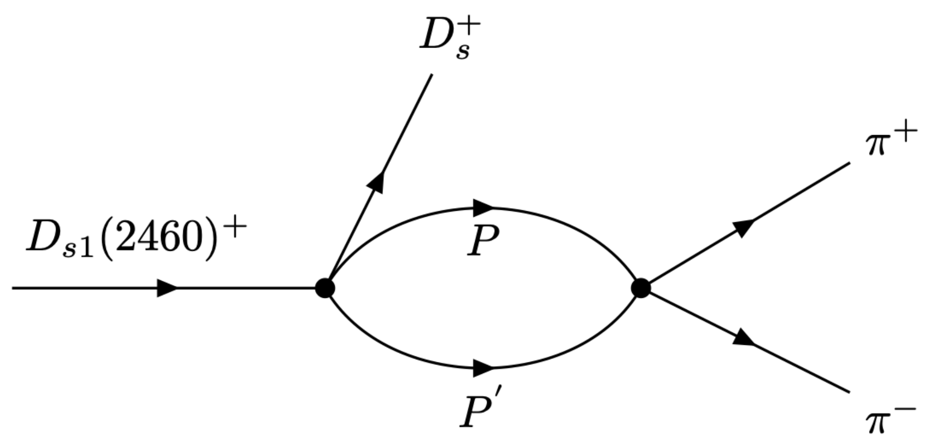
$$V_{K^+D^+,\pi^+D_s^+} = \frac{g^2}{m_{K^*}^2}(p_1+p_3)_\mu(p_2+p_4)^\mu.$$

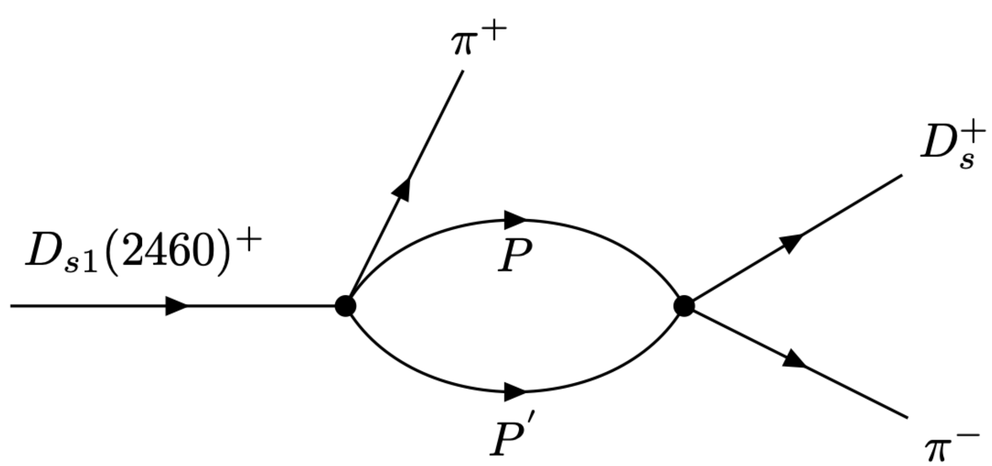
Theoretical interpretation

- $D_s\pi - DK$ coupled channel scattering: $T_{c\bar{s}}$ resonances is dynamically generated from the pseudoscalar-pseudoscalar meson interaction within the chiral unitary approach.

ZY Wang, YS Li and SQ Luo, Phys.Rev.D 111 (2025) 7, 076009







$$V^{I=1}_{D_s\pi\to D_s\pi} = 0,$$

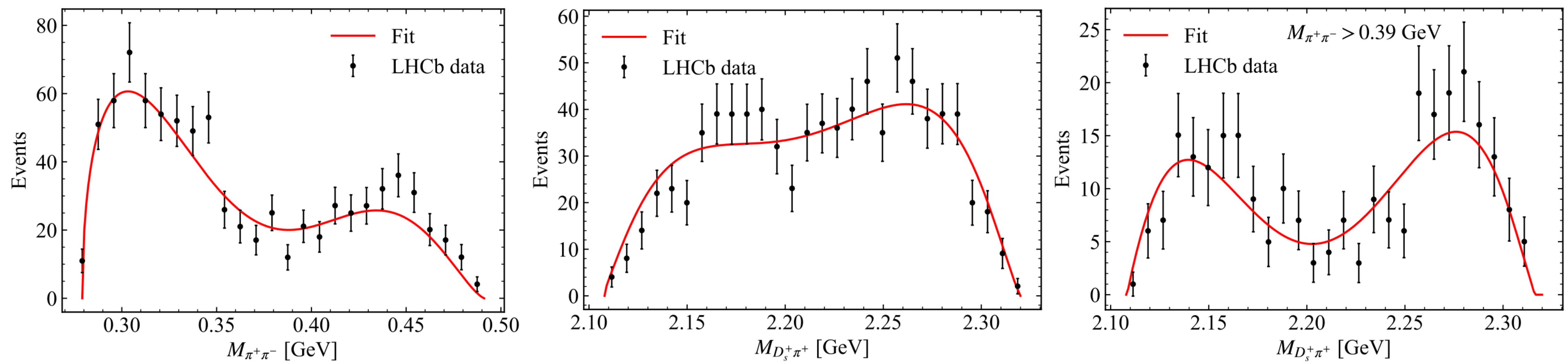
$$V^{I=1}_{D_s\pi\to DK} = -\frac{1}{8f^2}\left(3s - \left(M_1^2 + M_2^2 + m_1^2 + m_2^2\right) - \frac{\Delta_1\Delta_2}{s}\right),$$

$$V^{I=1}_{DK\to DK} = 0,$$

Theoretical interpretation

- $D_s\pi - DK$ coupled channel scattering: $T_{c\bar{s}}$ resonances is dynamically generated from the pseudoscalar-pseudoscalar meson interaction within the chiral unitary approach.

ZY Wang, YS Li and SQ Luo, Phys.Rev.D 111 (2025) 7, 076009



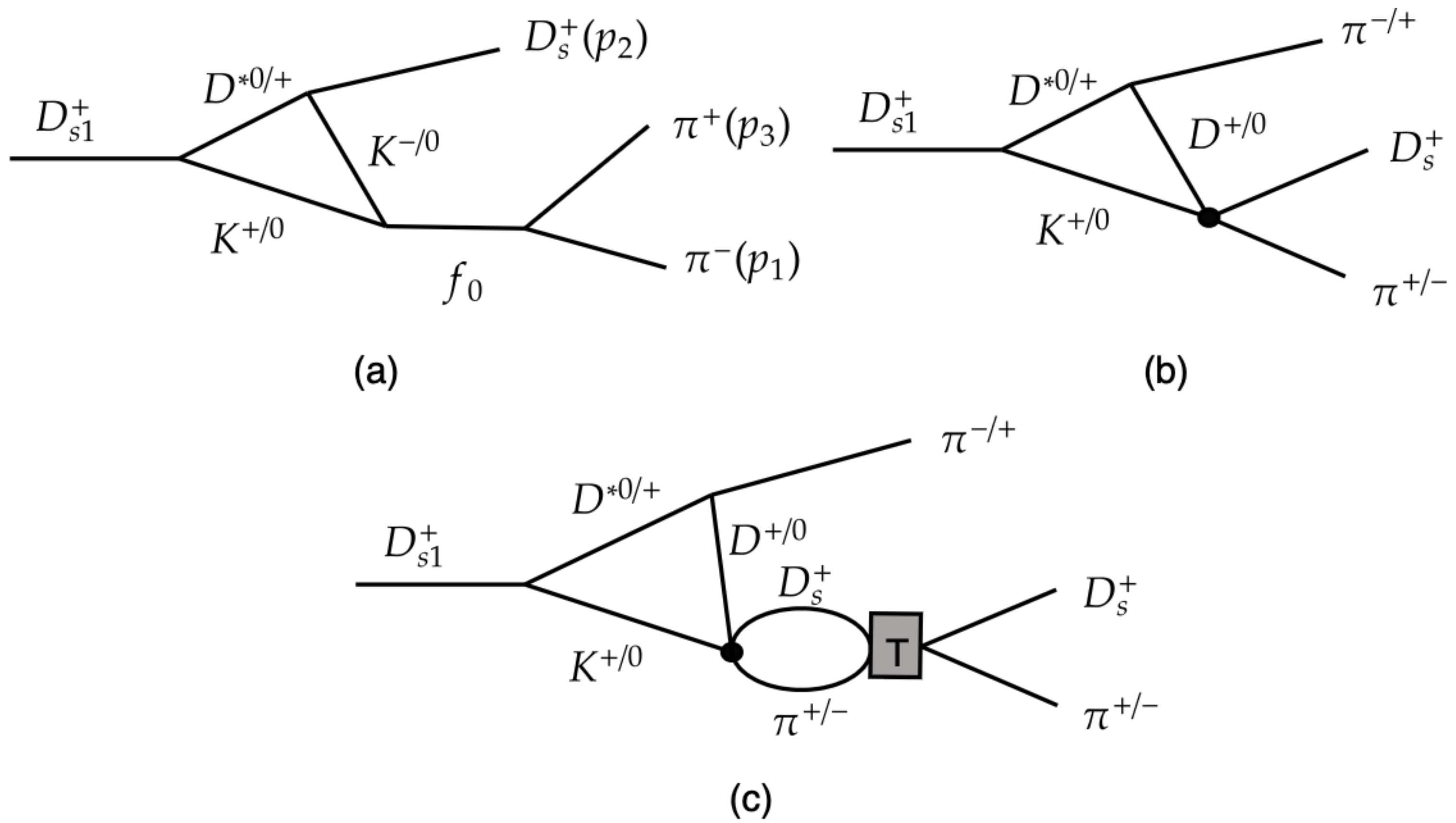
- A pole at $\sqrt{s_p} = (2.394 - 0.057i)$GeV on the second Riemann sheet; a pole at $\sqrt{s_p} = (2.246 - 0.093i)$GeV on the third Riemann sheet.

(S,I)	$h'_5 = +1$			$h'_5 = -1$		
	Re	Im	RS	Re	Im	RS
$(0, \frac{1}{2})$	2107	±123	II	2107	±105	II
	2452	±17	III	2519	±69	III
$(0, \frac{1}{2})$ $(V_{ii} = 0)$	2466	±24	III	2388	±49	III
$(1, 0)$	2318	0	I	2318	0	I
$(1, 1)$	2309	±111	III	2283	±196	III

F.F. Guo, C. Hanhart and U.G. Meissner, Eur. Phys. J. A40, 171-179(2009)

Our model

- The feynman diagrams



- The amplitude

$$
\begin{aligned}
i\mathcal{M}_a &= \frac{r_1}{m_{13}^2 - m_{f_0} + i m_{f_0}\Gamma_{f_0}} \int \frac{d^4 q}{(2\pi)^4} \frac{\epsilon^\mu L_\mu}{[q^2 - m_D^2][(q+p_2)^2 - m_{D^*}^2][(p_0 - p_2 - q)^2 - m_K^2]}, \\
i\mathcal{M}_b &= r_2 \int \sqrt{\pi}\sin\theta d\theta \int \frac{d^4 q}{(2\pi)^4} \frac{\epsilon^\mu N_\mu}{[q^2 - m_D^2][(q+p_1)^2 - m_{D^*}^2][(p_0 - p_1 - q)^2 - m_K^2]} + (p_1 \leftrightarrow p_3), \\
i\mathcal{M}_c &= r_2 \int \frac{\sin\theta d\theta k^2 dk}{2\pi^{3/2}} \int \frac{d^4 q}{(2\pi)^4} \frac{\epsilon^\mu N_\mu G_{D_s\pi}(k, m_{23}) T_{D_s\pi\to D_s\pi}(k, p_{\rm on}, m_{23})}{[q^2 - m_D^2][(q+p_1)^2 - m_{D^*}^2][(p_0 - p_1 - q)^2 - m_K^2]} + (p_1 \leftrightarrow p_3),
\end{aligned}
$$

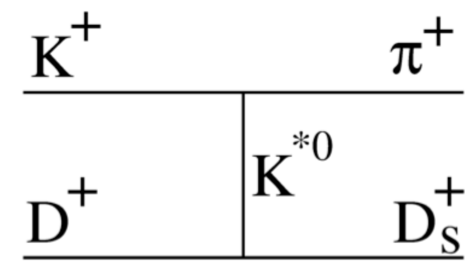
$e^{i\phi}$

Our model

- For the T-matrix,

$$T(\vec{k}_{D^*},\vec{k}'_{D^*};E)=\mathcal{V}(\vec{k}_{D^*},\vec{k}'_{D^*};E)+\int d\,\vec{q}\,\frac{\mathcal{V}(\vec{k}_{D^*},\vec{q};E)T(\vec{q},\vec{k}'_{D^*};E)}{E-\sqrt{m_D^2+q^2}-\sqrt{m_{D^*}^2+q^2}+i\epsilon}$$

$$\mathcal{V}_{DK\to D_s\pi}^{K^*}=\frac{g_{K^*}}{m_{K^*}^2}(p_1+p_3)_\mu(p_2+p_4)^\mu$$



Similar for $V_{DK\to D_s\pi}^{D^*}$. The resulting effective potential reads

$$\mathcal{V}_{\alpha\beta}=\left(V_{\alpha\beta}^{K^*}+V_{\alpha\beta}^{D^*}\right)\left(\frac{\Lambda_\alpha^2}{\Lambda_\alpha^2+p_\alpha^2}\right)^2\left(\frac{\Lambda_\beta^2}{\Lambda_\beta^2+p_\beta'^2}\right)^2$$

$\alpha,\beta=1,2$ for $D_s\pi$ and DK channel, respectively. The cutoff for these two channels can be different.

UV divergence in the triangle loop

- S- and D-wave coupling

$$\mathcal{M}_S = g_S \epsilon_i^{\mu} \epsilon_{j,\mu}^{\dagger}, \quad \mathcal{M}_D = \frac{g_D}{M^2} \epsilon_i^{\mu} \epsilon_j^{\dagger\nu} \left(q_\mu q_\nu - \frac{g_{\mu\nu} q^2}{4} \right)$$

- The amplitude

$$\begin{aligned}
i\mathcal{M}_a &= \frac{r_1}{m_{13}^2 - m_{f_0} + i m_{f_0}\Gamma_{f_0}} \int \frac{d^4 q}{(2\pi)^4} \frac{\epsilon^{\mu} L_\mu}{[q^2 - m_D^2][(q+p_2)^2 - m_{D^*}^2][(p_0 - p_2 - q)^2 - m_K^2]}, \\
i\mathcal{M}_b &= r_2 \int \sqrt{\pi} \sin\theta d\theta \int \frac{d^4 q}{(2\pi)^4} \frac{\epsilon^{\mu} N_\mu}{[q^2 - m_D^2][(q+p_1)^2 - m_{D^*}^2][(p_0 - p_1 - q)^2 - m_K^2]} + (p_1 \leftrightarrow p_3), \\
i\mathcal{M}_c &= r_2 \int \frac{\sin\theta d\theta k^2 dk}{2\pi^{3/2}} \int \frac{d^4 q}{(2\pi)^4} \frac{\epsilon^{\mu} N_\mu G_{D_s\pi}(k, m_{23}) T_{D_s\pi \to D_s\pi}(k, p_{\text{on}}, m_{23})}{[q^2 - m_D^2][(q+p_1)^2 - m_{D^*}^2][(p_0 - p_1 - q)^2 - m_K^2]} + (p_1 \leftrightarrow p_3),
\end{aligned}$$

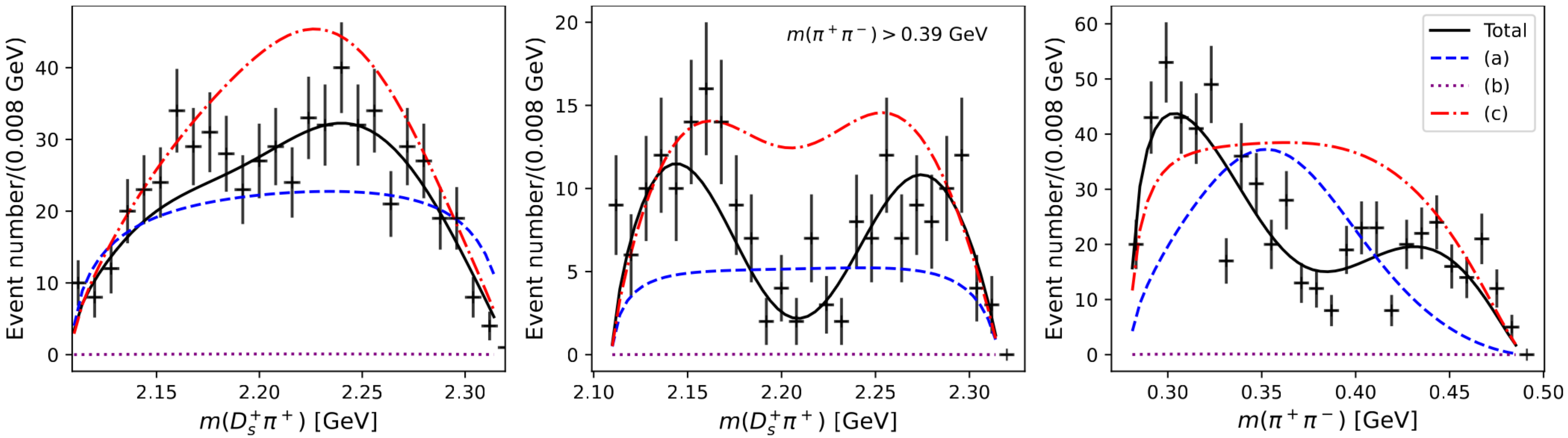
$$\begin{aligned}
L_\mu &= P_{\mu\nu}(p_2 + q, m_{D^*})(p_2 - q)^{\nu} \\
N_\mu &= P_{\mu\nu}(p_1 + q, m_{D^*})(p_1 - q)^{\nu}(p_2 - q + 2p_3)^{\alpha} \\
&\quad \times P_{\alpha\beta}(p_2 - q, m_{K^*})(p_2 + q)^{\beta}
\end{aligned}$$

$$P_{\mu\nu}(p, m) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}$$

- Regularize the loop integral using dimensional regularization, and use the Msbar renormalization scheme.

Fit to the experimental lineshapes

- $\chi^2/dof = 1.5$



- The parameters of the model determined from the fit :

$$\Lambda_1 = 1.9^{+0.4}_{-0.4}\ \text{GeV}, \quad \Lambda_2 = 0.5\ \text{GeV (fixed)},$$
$$g_{K^*} = 60.0^{+5}_{-4}\ \text{GeV}^2, \quad \phi = 10.27^{+0.18}_{-0.17},$$
$$m_{f_0} = 363^{+22}_{-23}\ \text{MeV}, \quad \Gamma_{f_0} = 178^{+42}_{-34}\ \text{MeV}.$$

- The pole position (in second Riemann sheet) is :

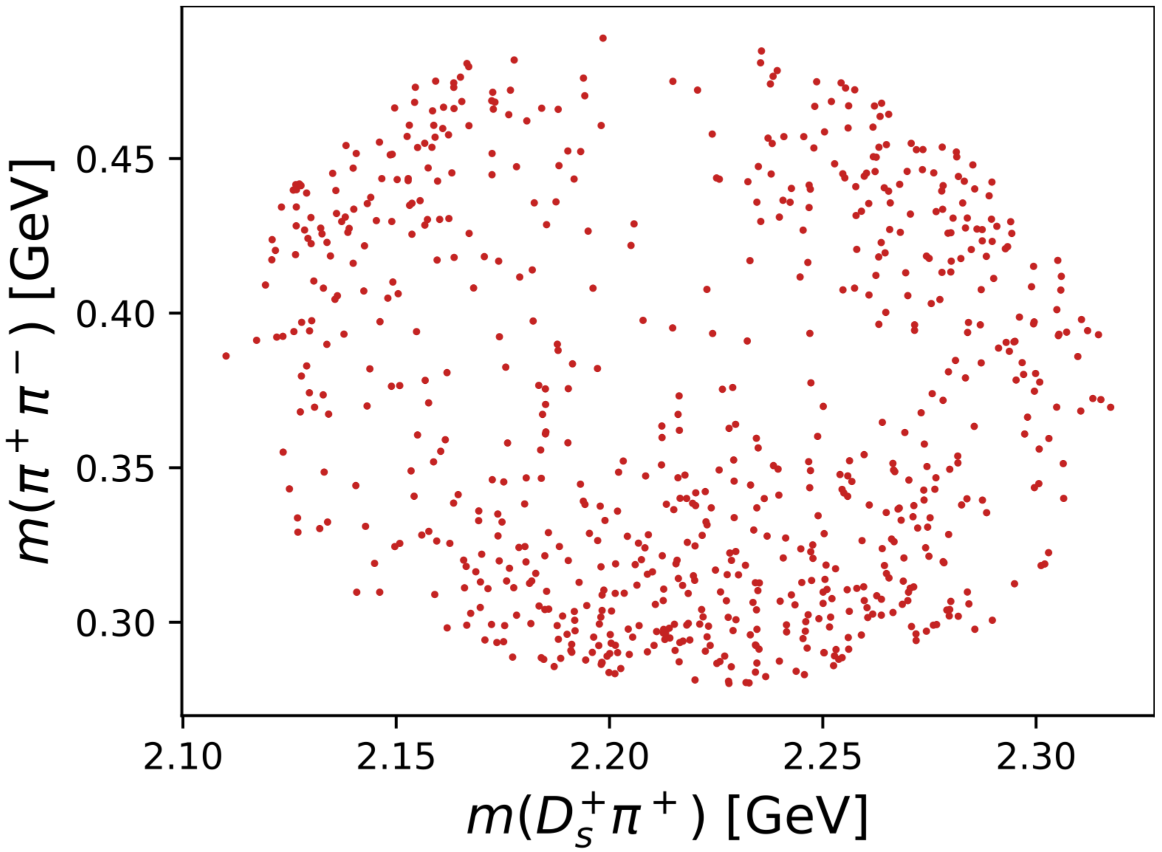
$$E_p = M - \Gamma/2i = 2270^{+33}_{-14} - 110^{+47}_{-26} i$$

Experimental position

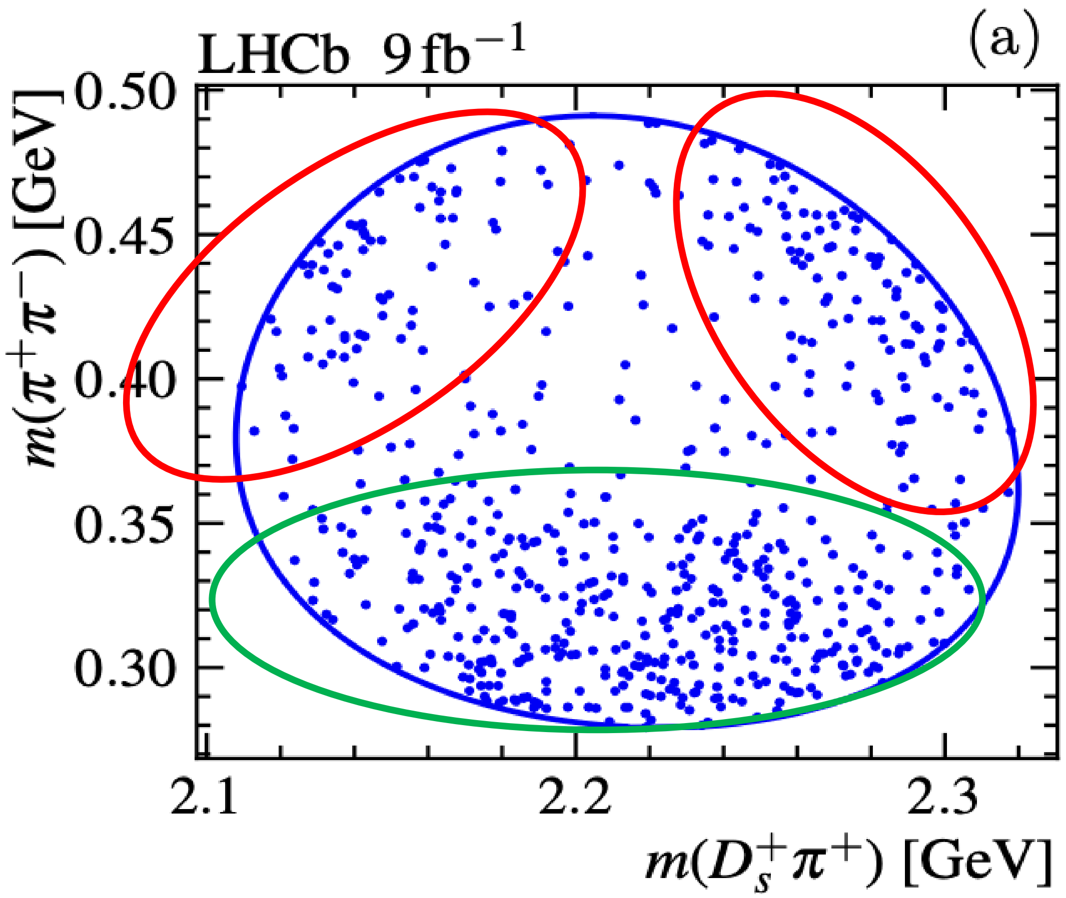
Mass (MeV)	Width (MeV)
$2328 \pm 12 \pm 12$	$96 \pm 16 \pm 23$

LHCb, arXiv:2411.03399

Dalitz plot



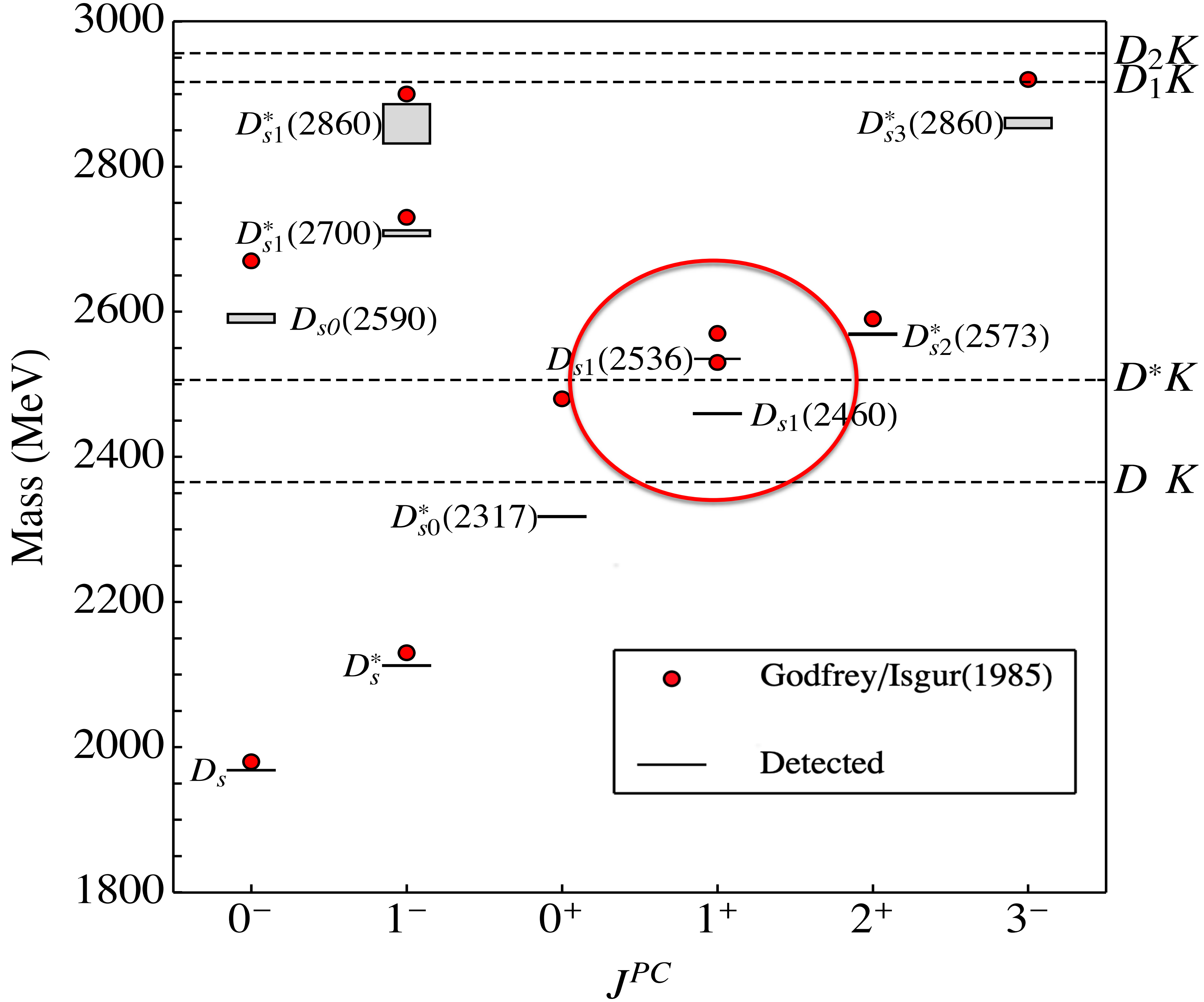
LHCb, arXiv:2411.03399



Is possible to investigate the structure of D_{s1} states?

- $D_{s1}(2460) \rightarrow D_s^+ \pi^+ \pi^-$
- $D_{s1}(2536) \rightarrow D_s^+ \pi^+ \pi^-$

D_{s1} exotic structure

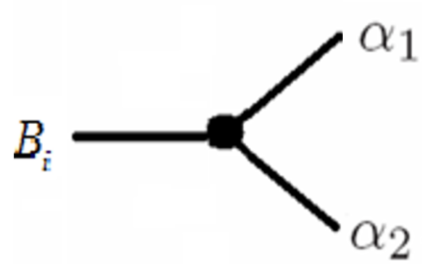


Coupled-channel framework

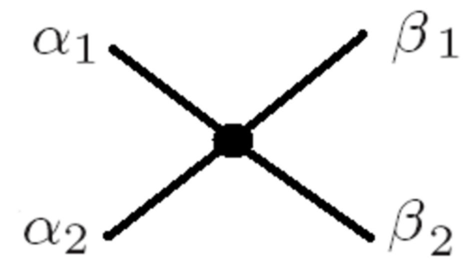
- Quark model bare state + hadronic molecule

Yang, Wang, Wu, Makoto, Zhu Phys.Rev.Lett. 128,112001(2022)

bare state core -> channel:



channel -> channel:



$$g = \sum_{\alpha,B} \int d^3\vec{k} \left\{ |\alpha(\vec{k})\rangle g_{\alpha B}(|\vec{k}|)\langle B| + h.c. \right\}$$

$$v = \sum_{\alpha,\beta} \int d^3\vec{k} d^3\vec{k}' |\alpha(\vec{k})\rangle V^L_{\alpha,\beta}(|\vec{k}|, |\vec{k}'|)\langle \beta(\vec{k}')|$$

Quark pair creation model (QPC):

$$g_{\alpha B}(|\vec{k}|) = \gamma I_{\alpha B}(|\vec{k}|) e^{-\frac{\vec{k}^2}{2\Lambda'^2}}$$

truncate the hard vertices given by usual QPC

P. G. Ortega, et al, Phys. Rev. D 94, 074037 (2016)

Effective Lagrangian: (exchanging mesons, e.g. ρ/ω)

Form factor: $\left(\frac{\Lambda^2}{\Lambda^2 + p_f^2}\right)^2 \left(\frac{\Lambda^2}{\Lambda^2 + p_i^2}\right)^2$

Component and pole mass of D_s states

Yang, Wang, Wu, Makoto, Zhu Phys.Rev.Lett. 128,112001(2022)

	$P(c\bar{s})[\%]$	ours	exp
$D_{s0}^*(2317)$	$32.0^{+5.2}_{-3.9}$	$2338.9^{+2.1}_{-2.7}$	2317.8 ± 0.5
$D_{s1}^*(2460)$	$52.4^{+5.1}_{-3.8}$	$2459.4^{+2.9}_{-3.0}$	2459.5 ± 0.6
$D_{s1}^*(2536)$	$98.2^{+0.1}_{-0.2}$	$2536.6^{+0.3}_{-0.5}$	2535.11 ± 0.06
$D_{s2}^*(2573)$	$95.9^{+1.0}_{-1.5}$	$2570.2^{+0.4}_{-0.8}$	2569.1 ± 0.8

$D_{s1}(2460)$

- Both the bare $c\bar{s}$ core and molecular components are significant and essential.

$D_{s1}(2536)$

- Mainly pure $c\bar{s}$.

Coupling of D_{s1} to D^*K (preliminary)

❖ S- and D-wave coupling

$$\mathcal{M}_S = g_S \epsilon_i^{\mu} \epsilon_{j,\mu}^{\dagger}, \quad \mathcal{M}_D = \frac{g_D}{M^2} \epsilon_i^{\mu} \epsilon_j^{\dagger\nu} \left(q_\mu q_\nu - \frac{g_{\mu\nu} q^2}{4} \right)$$

❖ $D_{s1}(2460)$

- The coupling of D_{s1} to D^*K can be obtained from the residue of the T-matrix:

$$g_i^2 = \frac{r}{2\pi} \int_0^{2\pi} T_{ii}(z(\theta)) e^{i\theta} d\theta \quad \longrightarrow \quad \left| \frac{g_D}{g_S} \right|_{theory}$$

- Fit to the experimental data $\longrightarrow$ $\left| \frac{g_D}{g_S} \right|_{\text{fit}} = 0.5$

❖ $D_{s1}(2536)$

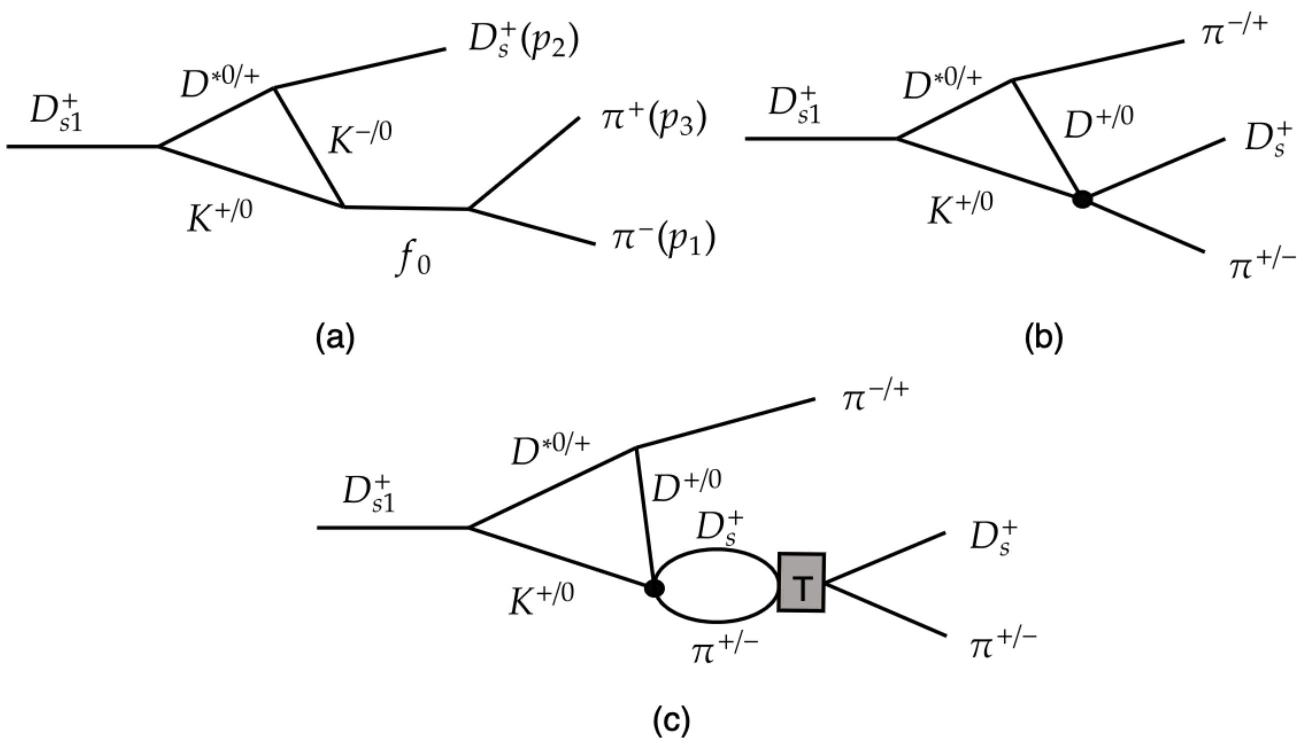
- The coupling can also obtained using the experimental branching fractions:

Particle Data Group

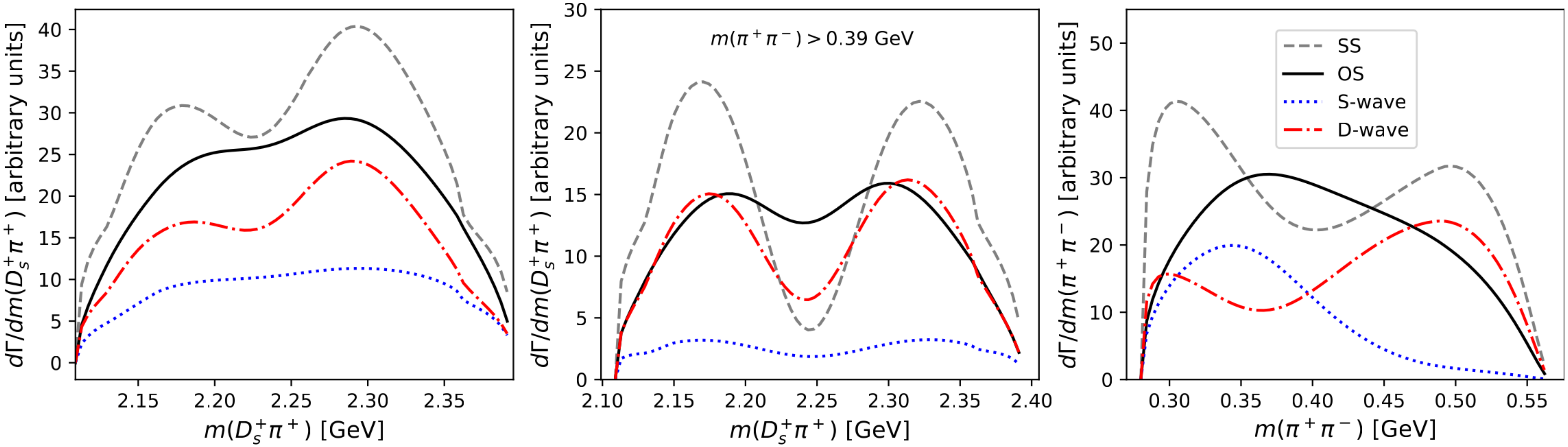
$$B(D^{*+}K^0)_{S-wave} = 0.61 \qquad B(D^{*+}K^0)_{D-wave} = 0.24$$

$$\longrightarrow \quad \left| \frac{g_D}{g_S} \right|_{\text{PDG}} = 7.24$$

Estimate the lineshape for $D_{s1}(2536)$ decay(preliminary)



- SS: same sign; OS: opposite sign for the S- and D-wave coupling
- SS and OS are for two choice of the phase for the S- and D-wave coupling.
- The two peak structure in the case of OS is not as obvious as that in Ds1(2460).



Summary

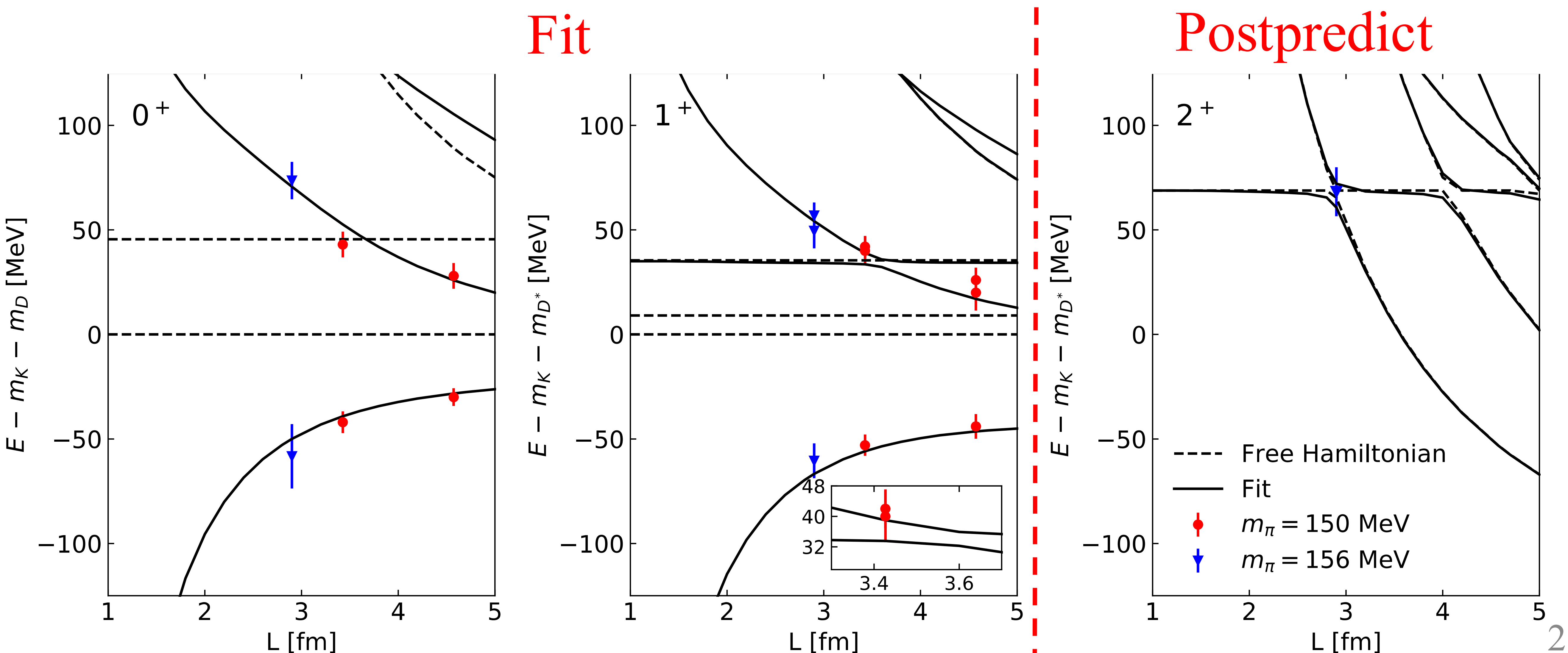
- The $T_{c\bar{s}}$ lineshape can be discribed through $D_s\pi - DK$ coupled-channel interactions, revealing how off-diagonal potential terms generate the observed resonance pole;
- Predictive calculations for $D_{s1}(2536)$ decays using structural parameters.

Backup slides：Fit the lattice data of $D_s(2317,2460,2536)$

$$(H_0+H_{\rm I})|\Psi\rangle = E|\Psi\rangle$$

Eigenvalues ⟺ Lattice levels

Lattice data from: C. B. Lang et al., **Phys. Rev. D 90, 034510 (2014);**
G. S. Bali et al., **Phys. Rev. D 96, 074501 (2017)**



Backup slides：Component and pole mass

- Component

$$(H_0+H_{\rm I})|\Psi\rangle = E|\Psi\rangle$$

$$|\Psi_E\rangle = C_0|B> + \sum_{\vec{k}_n=\frac{2\pi}{L}\vec{n}} C_E(\vec{k}_n)|\alpha(\vec{k}_n)\rangle$$

Eigenvector → Component

- Pole mass

In the infinite volume, the scattering T-matrix reads

$$T_{\alpha,\beta}(k,k';E) = \mathcal{V}_{\alpha,\beta}(k,k';E) + \sum_{\alpha'}\int q^2 dq \frac{\mathcal{V}_{\alpha,\alpha'}(k,q;E)T_{\alpha,\beta}(q,k';E)}{E-E_{\alpha'}(q)+i\epsilon}$$

where the effective potential reads

$$\mathcal{V}_{\alpha,\beta}(k,k';E) = \sum_B \frac{g_{\alpha B}(k)g^*_{\beta B}(k')}{E-m_B} + V^L_{\alpha,\beta}(k,k').$$

T-matrix → Pole mass