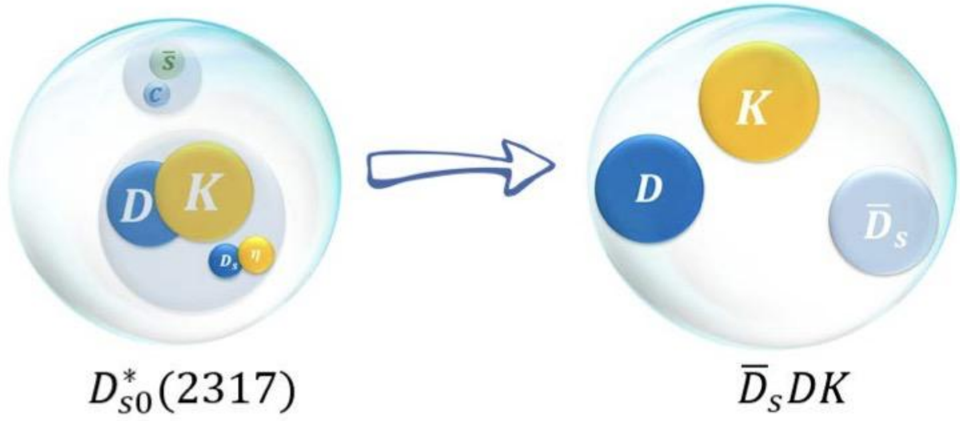

Existence of 0^{--} $\bar{D}_s D K$ on nature of $D_{s0}^*(2317)$

Accepted by PRL: 2501.11358

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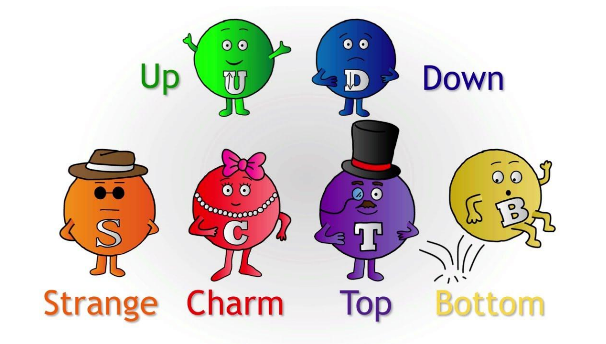
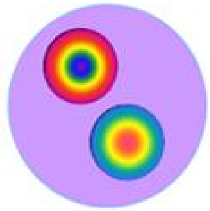
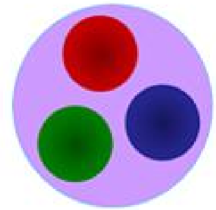
2025.0714@桂林

第八届强子谱和强子结构研讨会

Quark model



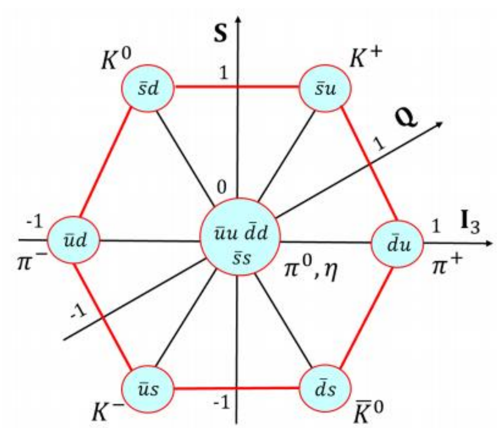


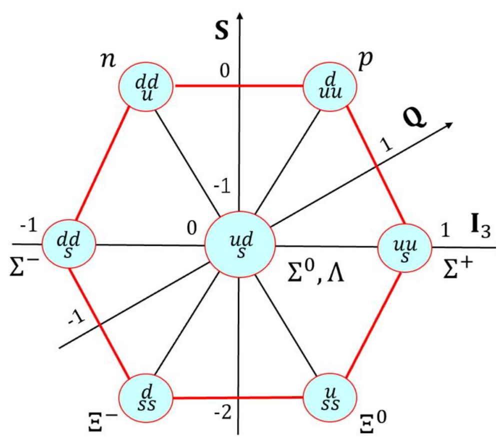


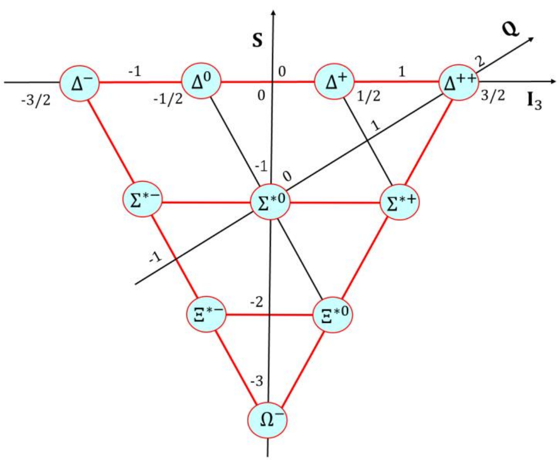
Gell-Mann

Meson： $3\otimes\bar{3} = 1\oplus 8$

Baryon： $3\otimes 3\otimes 3 = 1\oplus 8\oplus 8' \oplus 10$

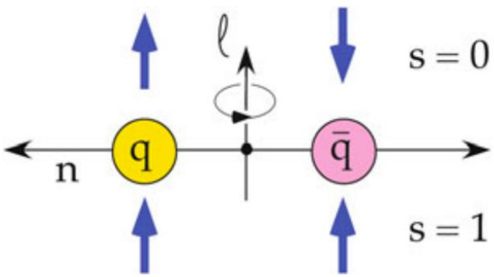


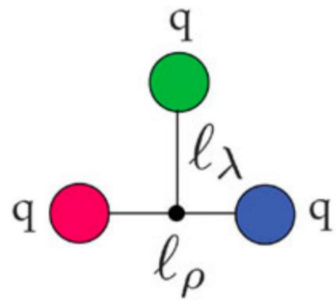




1964年，Gell-Mann等人提出夸克模型，将当时发现的强子分类为介子和重子，并成功预言了$\boldsymbol{\Omega}^-$粒子。

Exotic quantum numbers in Hadrons





$$I^G(J^{PC})$$

	d	u	s	c	b	t
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0
S – strangeness	0	0	−1	0	0	0
C – charm	0	0	0	+1	0	0
B – bottomness	0	0	0	0	−1	0
T – topness	0	0	0	0	0	+1

$$\mathrm{Q} = \mathrm{I}_z + \frac{\mathcal{B} + \mathrm{S} + \mathrm{C} + \mathrm{B} + \mathrm{T}}{2}$$

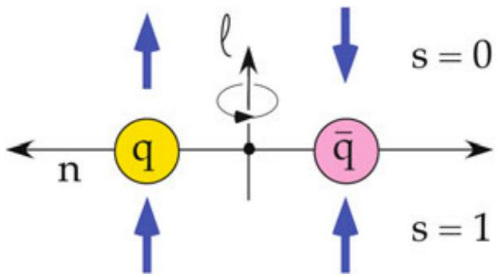
Spin and Parity of two-body systems

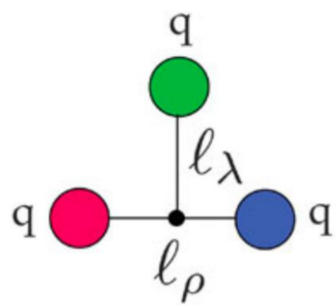
$$|\ell - s| \leq J \leq |\ell + s| \qquad P = (-1)^{\ell} P_a P_b$$

C and G parity of $\Phi\bar{\Phi}$ ($F\bar{F}$ and $B\bar{B}$)

$$\boxed{C(\Phi\overline{\Phi}) = (-1)^{\ell+s}} \qquad G = (-1)^{I+\ell+s}$$

Exotic quantum numbers in Hadrons





$$I^G(J^{PC})$$

		d	u	s	c	b	t
Q	– electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
I	– isospin	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
I_z	– isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0
S	– strangeness	0	0	-1	0	0	0
C	– charm	0	0	0	$+1$	0	0
B	– bottomness	0	0	0	0	-1	0
T	– topness	0	0	0	0	0	$+1$

$$\mathrm{Q} = \mathrm{I}_z + \frac{\mathcal{B} + \mathrm{S} + \mathrm{C} + \mathrm{B} + \mathrm{T}}{2}$$

Spin and Parity of two-body systems

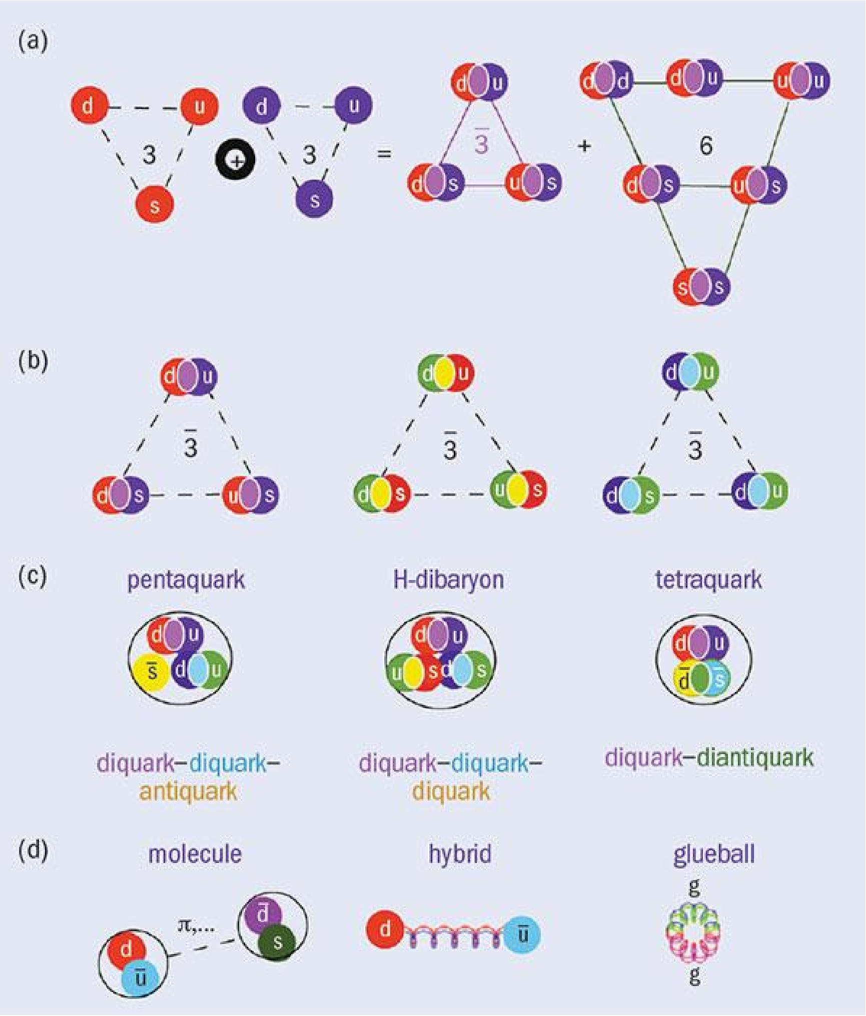
$$|\ell - s| \le J \le |\ell + s| \qquad P = (-1)^\ell P_a P_b$$

C and G parity of $\Phi\bar{\Phi}$ ($\mathrm{F}\bar{\mathrm{F}}$ and $\mathrm{B}\bar{\mathrm{B}}$)

$$\boxed{C(\Phi\overline{\Phi}) = (-1)^{\ell+s}} \qquad G = (-1)^{I+\ell+s}$$

Quantum numbers $\boldsymbol{J^{PC}} = \mathbf{0^{--}}, \mathbf{0^{+-}}, \mathbf{1^{-+}}, \mathbf{2^{+-}}, \mathbf{3^{-+}} \ldots$ are forbidden in $q\bar{q}$ mesons.

Exotic hadronic states



Normal hadron：

meson: quark-antiquark pair

baryon: 3 quarks

Exotic hadron state:

1. Glueball

Composed of gluons

2. **Hybrid**

Composed of quarks and gluons

3. **Multiquark state**

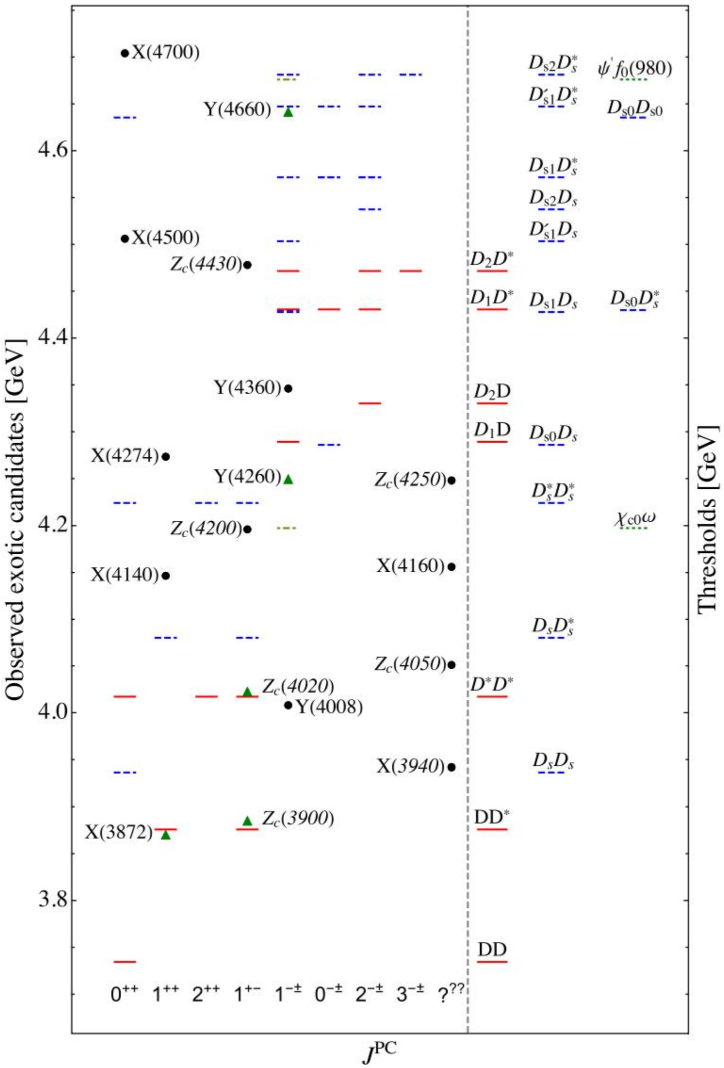
Composed of multi quarks(>3)

4. **Hadronic molecule**

Composed of 2 or more hadrons

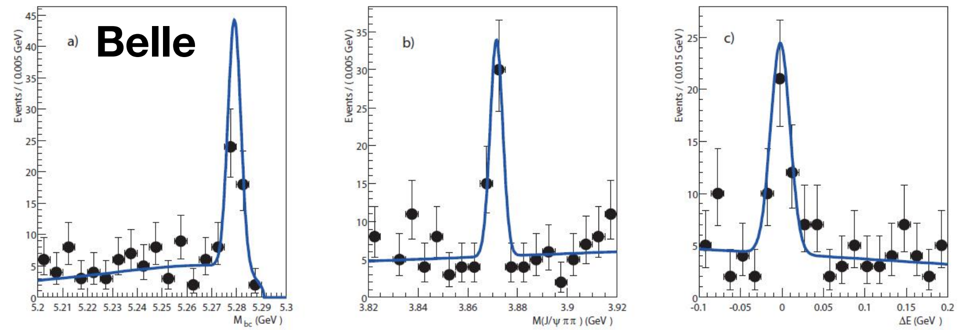
Hadronic molecules

F.K. Guo, et al. Rev.Mod.Phys.90.015004

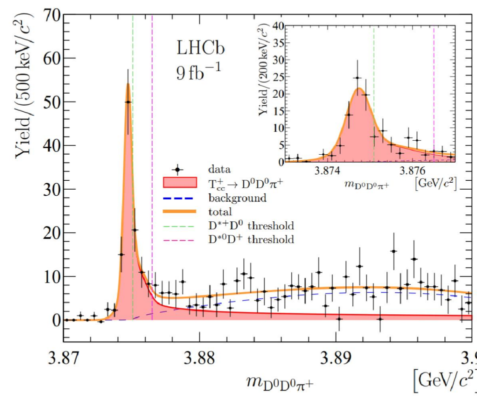


Many XYZ particles are close to two hadron mass thresholds.

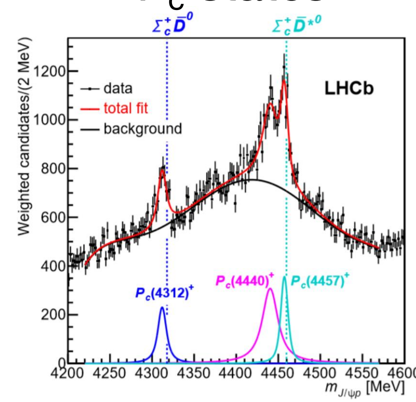
X(3872)



$T_{cc}(3875)$



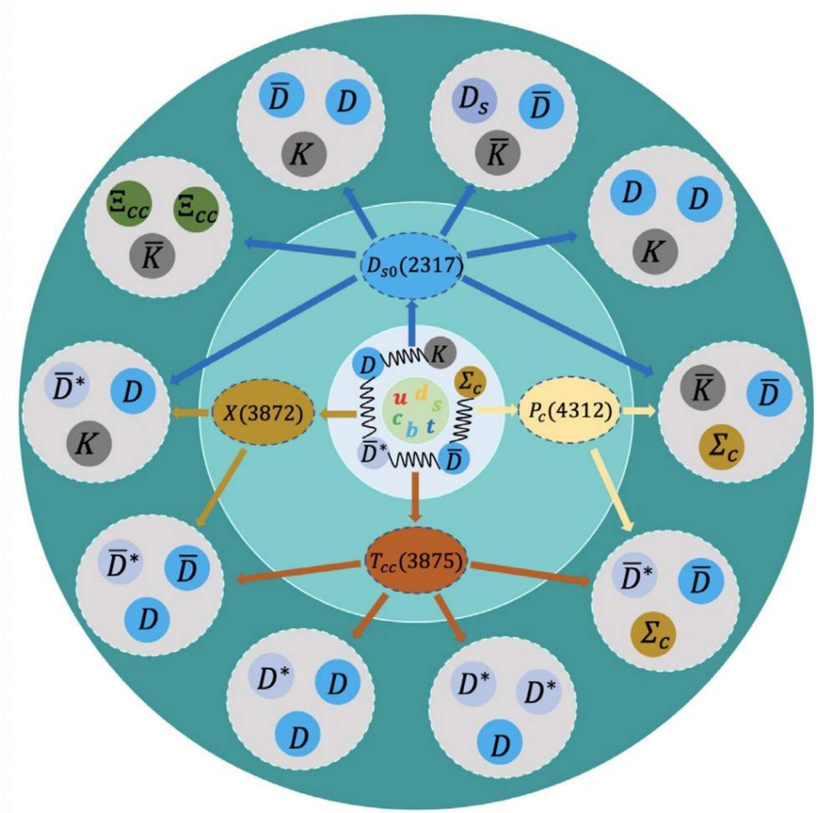
P_c states

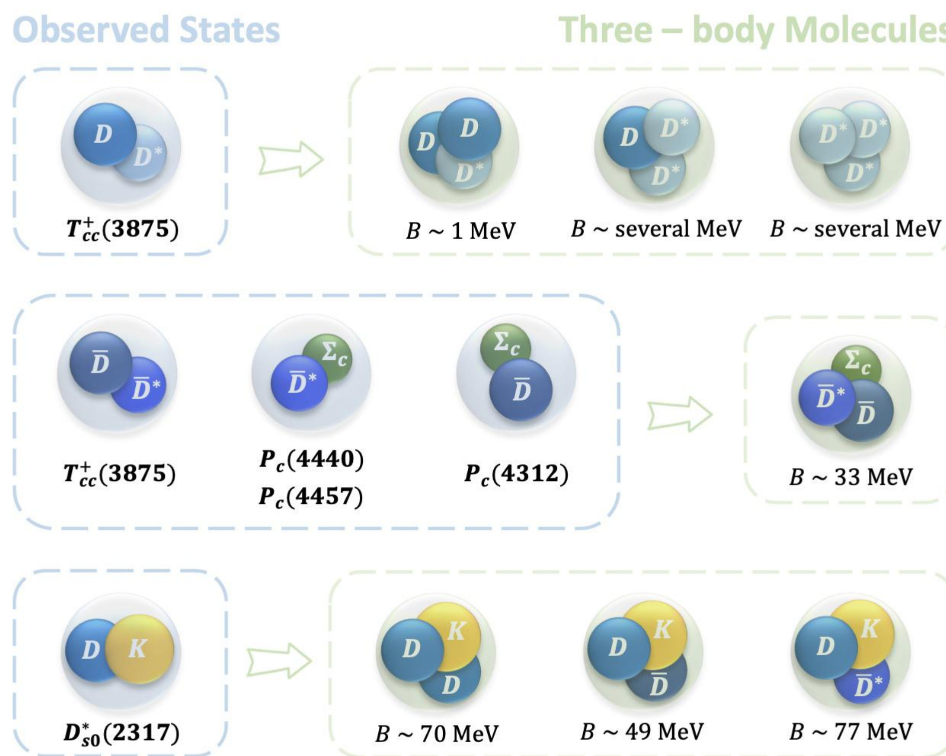


X(3872)、T_{cc}(3875)、P_c pentaquarks、D_{s0}^*(2317) are interpreted as hadronic molecules.

Three-body molecules

T.W. Wu, et al. Sci. Bull. 67 (2022) 1735-1738   M.Z. Liu, et al. Phys.Rept. 1108 (2025) 1-108

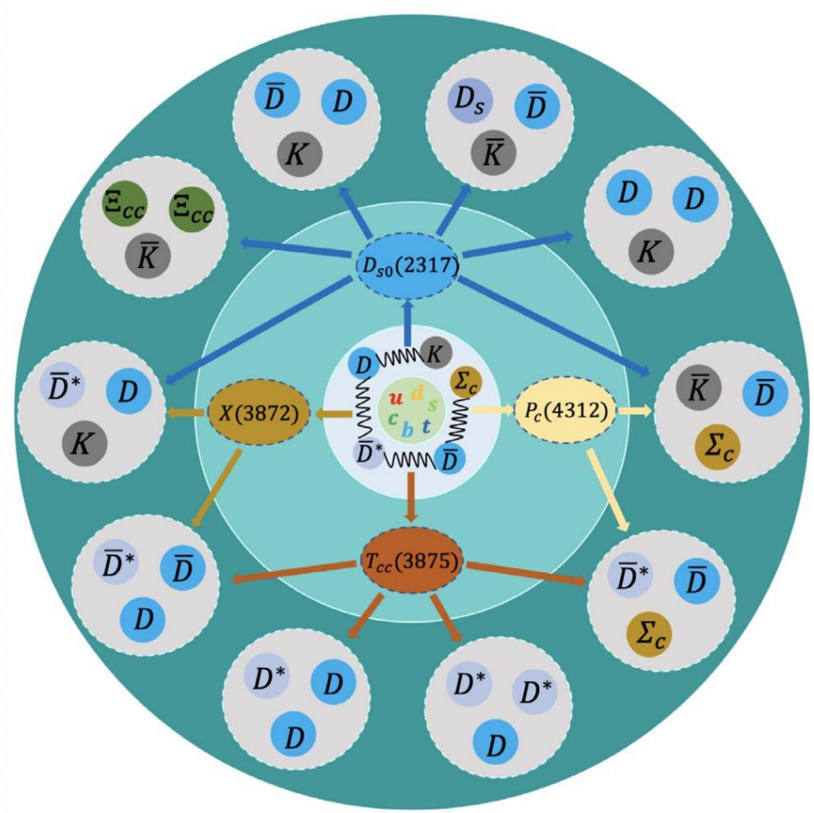




Three-body molecules based on two-body candidates

Three-body molecules

T.W. Wu, et al. Science Bulletin 67 (2022) 1735-1738



Study mainly focused on:

Existence (mass\structure)

Decay (channel\partial width)

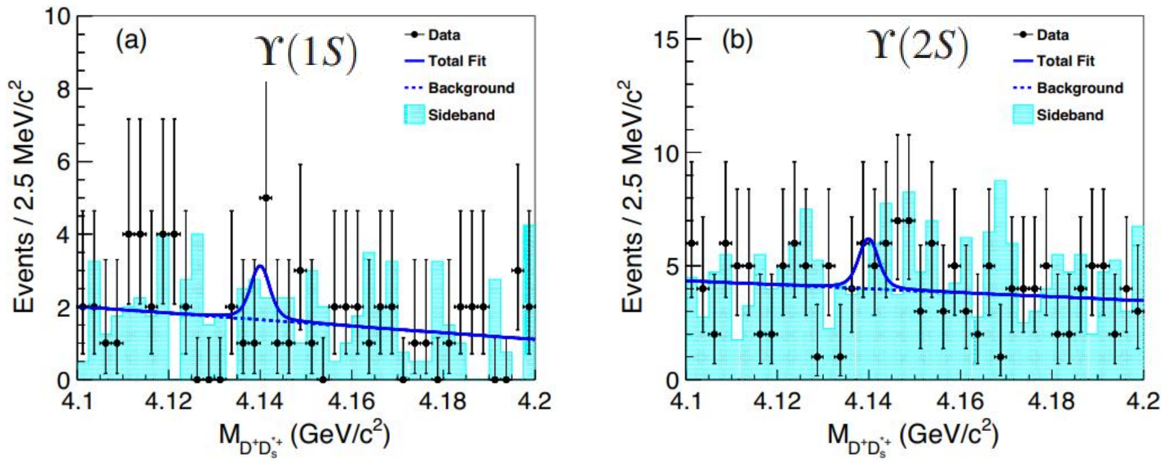
Production (B.F.\events)

How to find and verify three-body molecules?

Seraching for DDK

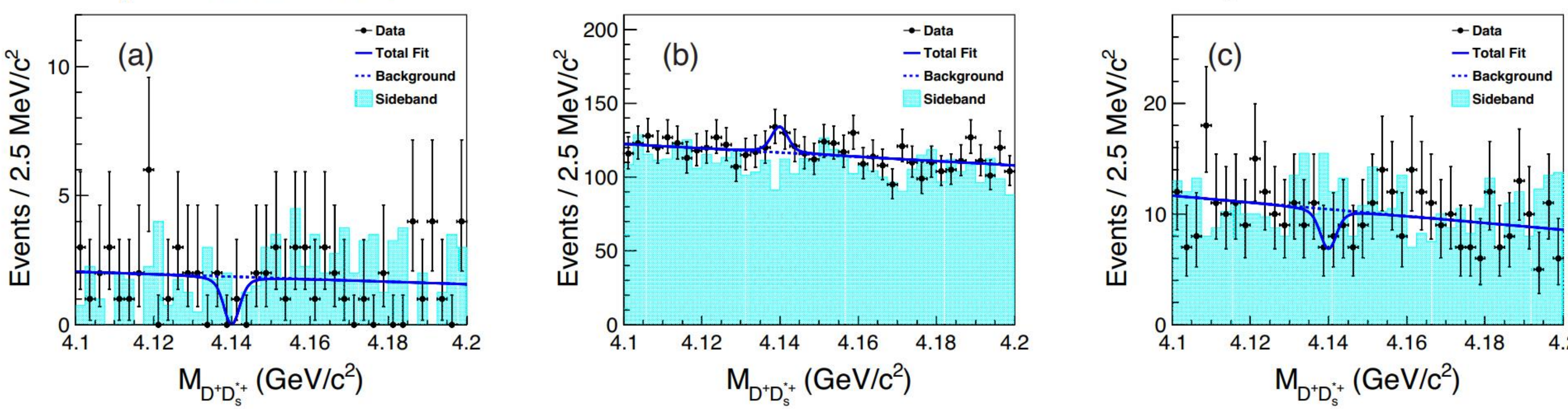
Phys.Rev. D102 (2020) 112001@Belle

$$\Upsilon(1S)/\Upsilon(2S) \to R^{++} + \text{anything},\ R^{++} \to D^+ D_s^{*+}$$



$$e^+e^- \to R^{++} + \text{anything at } \sqrt{s} = 10.520/10.580/10.867\ \text{GeV},\ R^{++} \to D^+ D_s^{*+}$$

$\sqrt{s} = 10.520$ GeV　$\sqrt{s} = 10.580$ GeV　$\sqrt{s} = 10.867$ GeV



Doubly charmed more difficult to produce.

Features of $\bar{D}_s D K$ system

√ **Study on three-body systems with C-parity**

C−parity interaction included

√ **No mixture of conventional hadrons**

$\mathbf{0}^{--}$ exotic quantum numbers

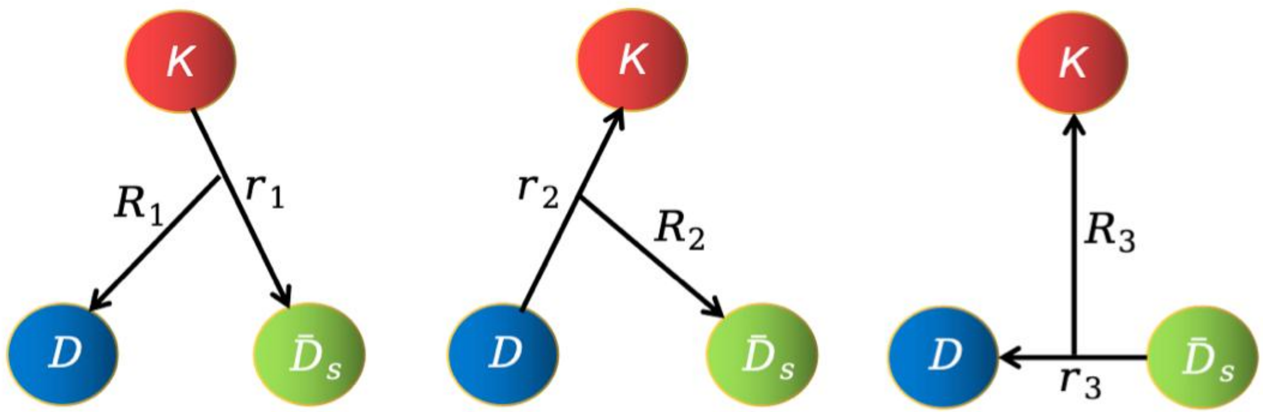
√ **Two-body system can not bind**

Suppressed by the OBE model or OZI rule

√ **Produced in both e^+e^- and pp collisions**

Hidden charm/strange state easier to observe

Wave function of $\bar{D}_s D K$ with C parity



$$\bar{D}_s D K$$

$$\Psi^C = \frac{1}{\sqrt{2}}(\Psi_{\bar{D}_s D K} + C\Psi'_{D_s \bar{D}\bar{K}}), \qquad \langle \Psi^C | (H - E) | \Psi^C \rangle = 0$$

$$C = \pm 1$$

$$\Psi'_{D_s \bar{D}\bar{K}} = \hat{C}\Psi_{\bar{D}_s D K} = \sum_{c=1,3} \Phi(r'_c, R'_c).$$

$$H = T + V + V_C$$

Two-body interaction

C-parity interaction

Solve three-body problem with GEM

Two-body interactions

DK interaction parameterized by a constant C_a

with **Contact-range EFT approach**

$DK - D_s\eta$ coupled channel interaction in matrix form

$$V_{DK-D_s\eta}^{J^P=0^+} = \begin{pmatrix} C_a & -\frac{\sqrt{3}}{2}C_a \\ -\frac{\sqrt{3}}{2}C_a & 0 \end{pmatrix}$$

V_{DK} in a Gaussian form in coordinate space

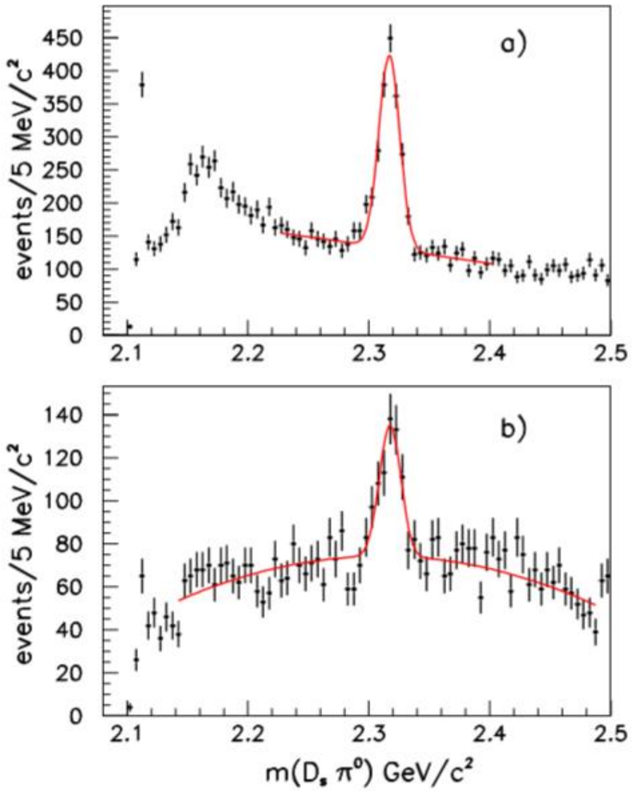
$$V(r) = C_a \frac{e^{(r/R_c)2}}{\pi^{3/2} R_c^3},$$

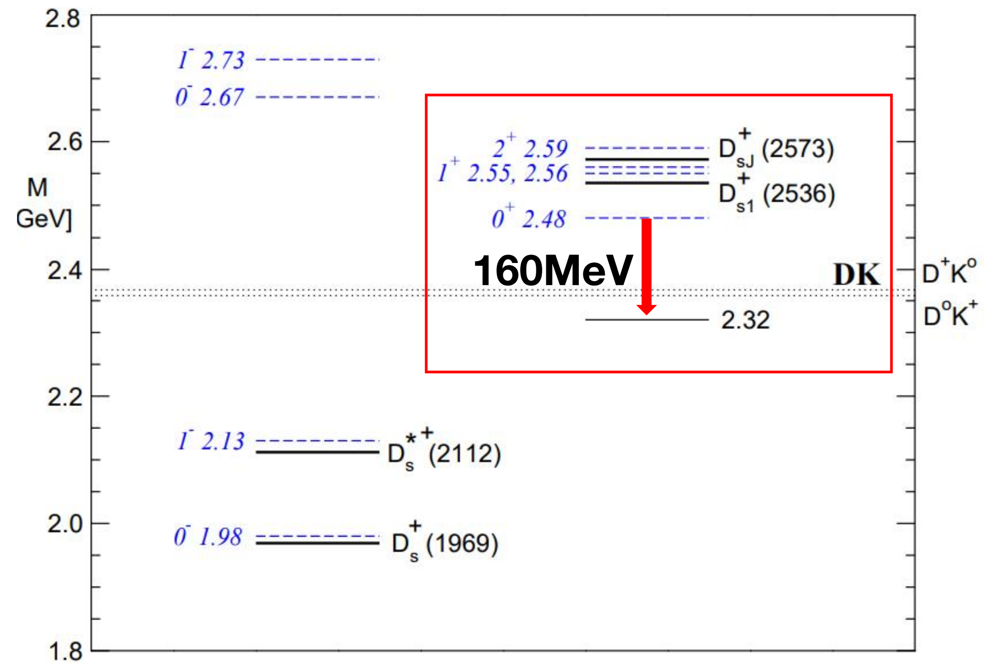
With SU(3) flavor symmtry and experimental fitting

$$C_a^{DK} : C_a^{\bar{D}_s K} : C_a^{\bar{D}_s D} \approx 1 : 0.5 : 0.1$$

$D_{s0}^{*}(2317)$

Ds0(2317), discovered by BABAR, CLEO and D0。





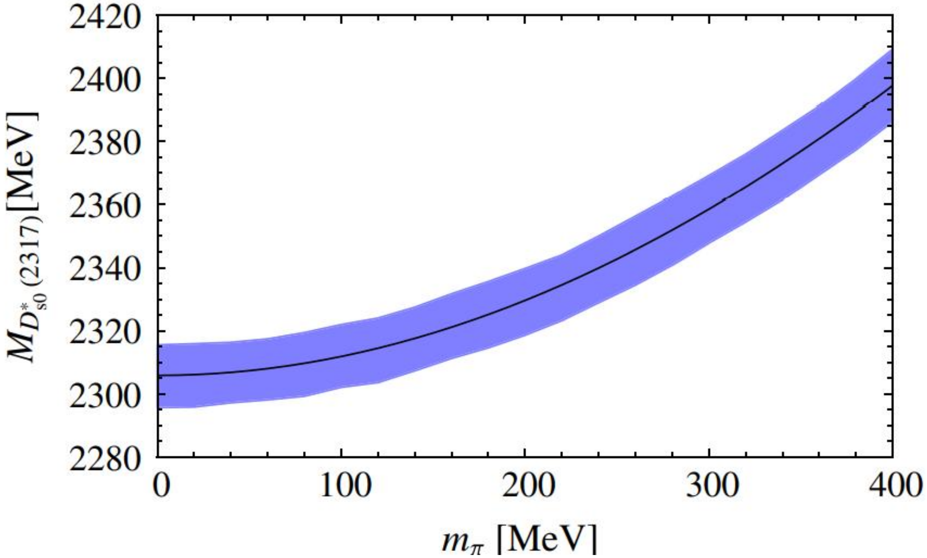
T. Barnes, et al. Phys. Rev. D 68, 054006

@PDG (Expe.)
Mass： 2317.7 MeV
Width： < 3.8MeV
Partner D_{s1}: 2460 MeV
$M(D_{s1})-M(D_{s0})$=140 MeV

@Quark model (Theo.)
Mass： 2480 MeV
Width： 270-990 MeV
Partner D_{s1}: 2560 MeV
$M(D_{s1})-M(D_{s0})$=80 MeV

Explanations of Ds0*(2317)

M. Altenbuchinger et al. Phys.Rev.D 89, 014026

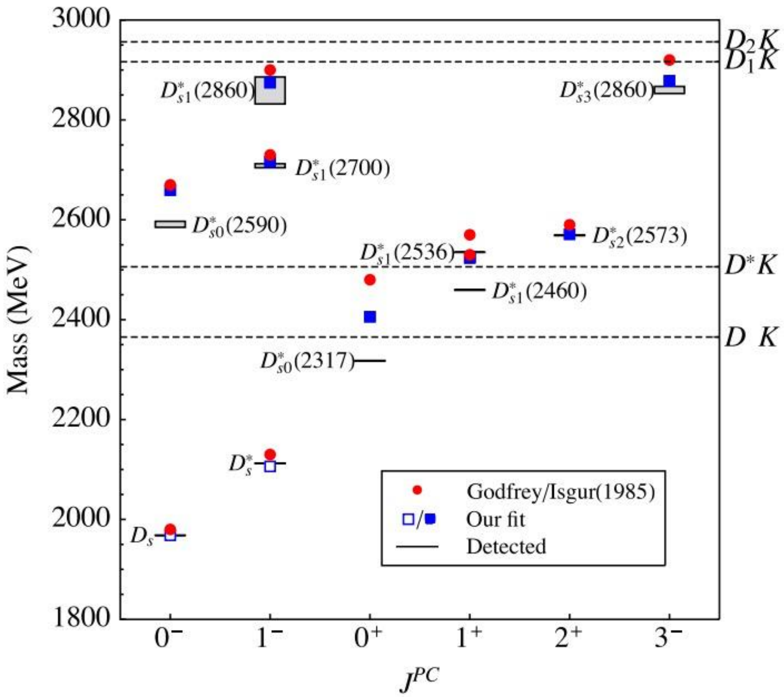


DK pole position@ChPT

TABLE V. Pole positions $\sqrt{s} = M - i\frac{\Gamma}{2}$ (in units of MeV) of charm mesons dynamically generated in the HQS UChPT.

(S, I)	$J^P = 0^+$	$J^P = 1^+$
(1, 0)	2317 ± 10	2457 ± 17
(0, 1/2)	$(2105 \pm 4) - i(103 \pm 7)$	$(2248 \pm 6) - i(106 \pm 13)$

Z. Yang et al. PhysRevLett.128.112001



68 % DK+32 % $c\bar{s}$@LQCD

ChPT, Lattice and Exp. all surport that the D_{s0}(2317) is a DK molecule or at least has a large DK component.

DK interaction fitting $D_{s0}(2317)$

Considering $D_{s0}(2317)$ as a $DK - D_s\eta$ molecule $+c\bar{s}$ state

Momentum space

Couplings	$\Lambda = 0.50$	$\Lambda = 1.00$	$\Lambda = 1.50$	$\Lambda = 2.00$	$\Lambda = 0.50$	$\Lambda = 1.00$	$\Lambda = 1.50$	$\Lambda = 2.00$
$g_{D^*_{s0}DK}$ (GeV)	19.37	14.72	13.32	12.66	16.20	12.28	11.16	10.63
$g_{D^*_{s0}D_s\eta}$ (GeV)	13.23	9.54	8.40	7.86	10.42	7.70	6.89	6.50
$C_a(\mathrm{fm}^2)$	-5.78	-1.84	-1.03	-0.71	-6.96	-2.06	-1.12	-0.75
Compositeness	$\Lambda = 0.50$	$\Lambda = 1.00$	$\Lambda = 1.50$	$\Lambda = 2.00$	$\Lambda = 0.50$	$\Lambda = 1.00$	$\Lambda = 1.50$	$\Lambda = 2.00$
P_{DK}	0.92	0.90	0.89	0.88	0.65	0.63	0.62	0.62
$P_{D_s\eta}$	0.08	0.10	0.11	0.12	0.05	0.07	0.08	0.08

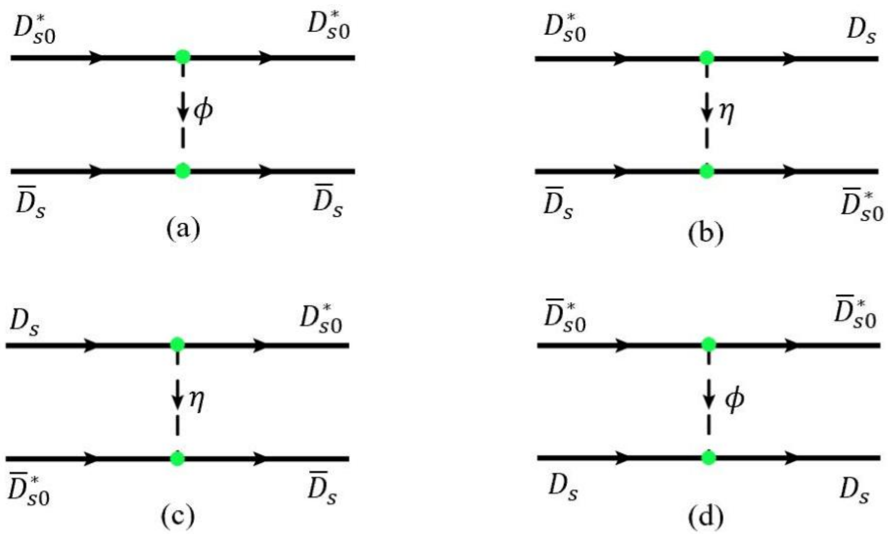
100% molecule | 70 % molecule+30 % $c\bar{s}$

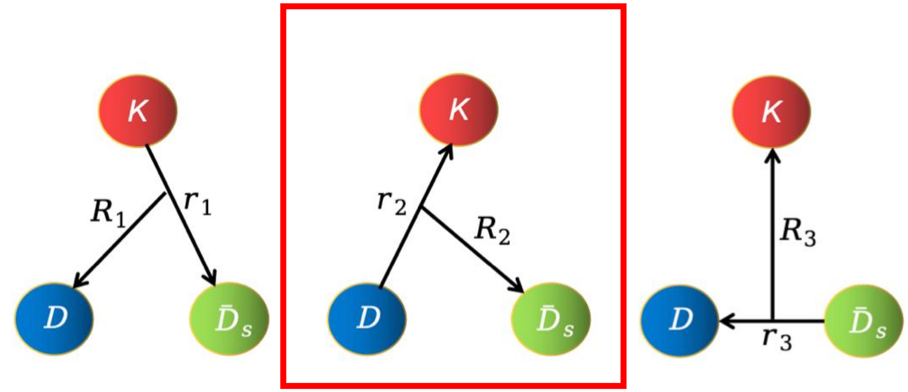
Coordinate space

Components of $D^*_{s0}(2317)$	$M(DK-D_s\eta)$	$M(DK)$	$M(c\bar{s})$ [4]	$P(c\bar{s})$	$P(DK)$	$P(D_s\eta)$
70% molecule+30% $c\bar{s}$	2280	2349	2406	30%	60%	10%
100% molecule	2318	2358	2406	0%	90%	10%
50% molecule+50% $c\bar{s}$	2230	2336	2406	50%	42%	8%

$DK - D_s\eta > c\bar{s}$ $B_{DK} < 45$ MeV

C-parity interaction $\bar{D}_s D_{s0}^* - D_s \bar{D}_{s0}^*$





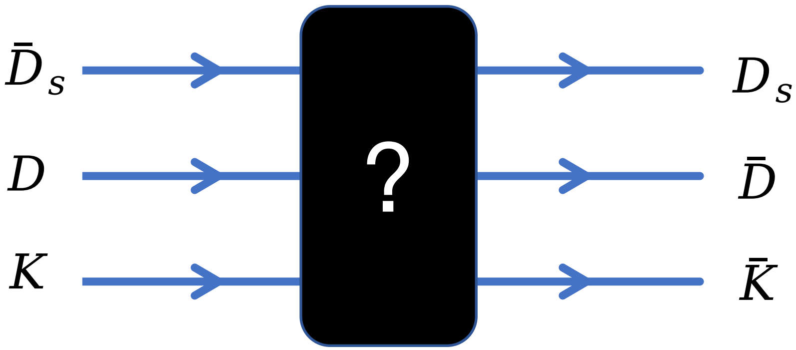
η exchange potential

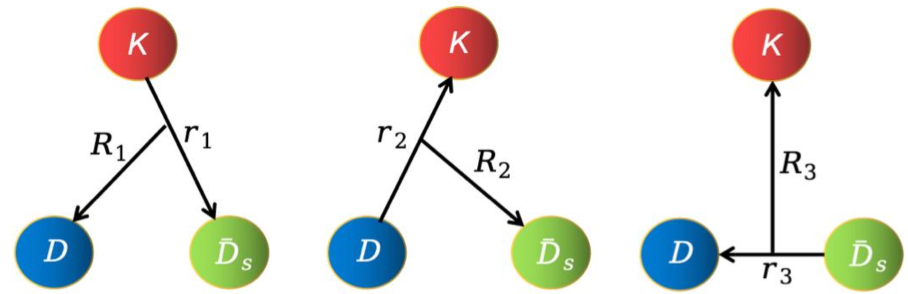
V_C simplified as

$$V^{C=\pm} = \mp\frac{2}{3}\frac{k^2}{f_\pi^2}q_0^2\left(\frac{e^{-mr}-e^{-\Lambda r}}{4\pi r} - \frac{\Lambda^2-m^2}{8\pi\Lambda}e^{-\Lambda r}\right)$$

$q_0 = m_{D_{s0}^*} - m_{D_s}, k = 0.56,$ $\Lambda = \alpha\Lambda_{QCD} + m_{eff}, m_{eff} = \sqrt{m_\eta^2 - (m_{D_{s0}^*} - m_{D_s})^2}.$

C-parity interaction $\bar{D}_s D_{s0}^* - D_s \bar{D}_{s0}^*$





η exchange potential

V_C simplified as

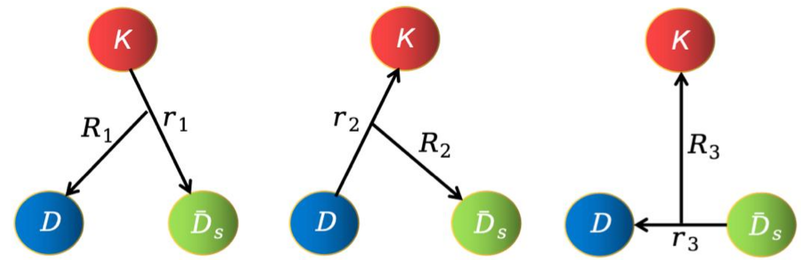
$$V^{C=\pm} = \mp\frac{2}{3}\frac{k^2}{f_\pi^2}q_0^2\left(\frac{e^{-mr}-e^{-\Lambda r}}{4\pi r} - \frac{\Lambda^2-m^2}{8\pi\Lambda}e^{-\Lambda r}\right)$$

$q_0 = m_{D_{s0}^*} - m_{D_s}, k = 0.56,\quad \Lambda = \alpha\Lambda_{QCD} + m_{eff}, m_{eff} = \sqrt{m_\eta^2 - (m_{D_{s0}^*} - m_{D_s})^2}.$

Binding, weights and spatial struture

0^{--} $\bar{D}_s D K$ molecule: X(4310)

Scenarios	B.E.(0^{--})	$P_{\bar{D}_s K-D}$	$P_{DK-\bar{D}_s}$	$P_{\bar{D}_s D-K}$
$\alpha=1$	22^{+23}_{-14}	11^{+1}_{-1} %	78^{-1}_{+2} %	11^{+0}_{-1} %
$\alpha=2$	20^{+22}_{-13}	10^{+1}_{-1}%	80^{-1}_{+2} %	10^{+0}_{-1} %
Scenarios	$r_{\bar{D}_s K}$	r_{DK}	$r_{\bar{D}_s D}$	$\langle T\rangle$
$\alpha=1$	$1.6^{-0.4}_{+0.9}$	$1.2^{-0.3}_{+0.5}$	$1.4^{-0.3}_{+0.8}$	177^{+81}_{-74}
$\alpha=2$	$1.7^{-0.4}_{+1.4}$	$1.2^{-0.3}_{+0.7}$	$1.6^{-0.5}_{+1.3}$	169^{+82}_{-78}
Scenarios	$\langle V_{D_s\bar{K}}\rangle$	$\langle V_{DK}\rangle$	$\langle V_{D_s\bar{D}}\rangle$	$\langle V^{C=-}_{D_s\bar{D}^*_{s0}}\rangle$
$\alpha=1$	-40^{-25}_{+20}	-147^{-73}_{+62}	-14^{-7}_{+6}	2^{+0}_{-1}
$\alpha=2$	-37^{-25}_{+22}	-143^{-73}_{+64}	-13^{-6}_{+7}	3^{+2}_{-1}



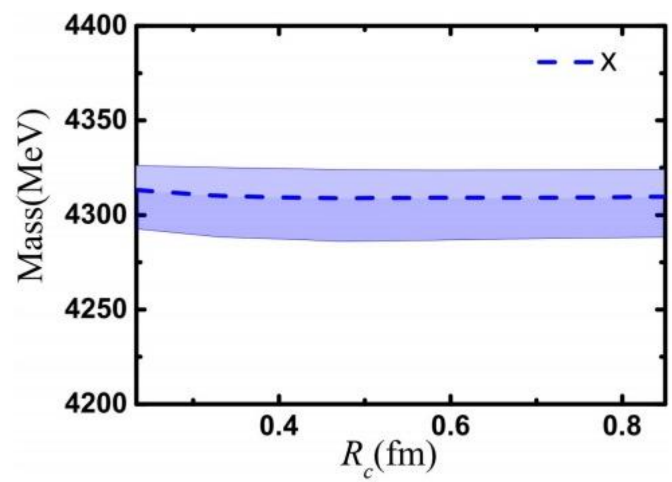
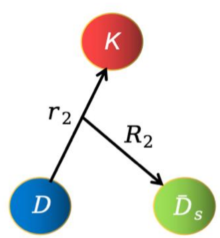


FIG. 5. Mass of X as a function of the cutoff R_c.

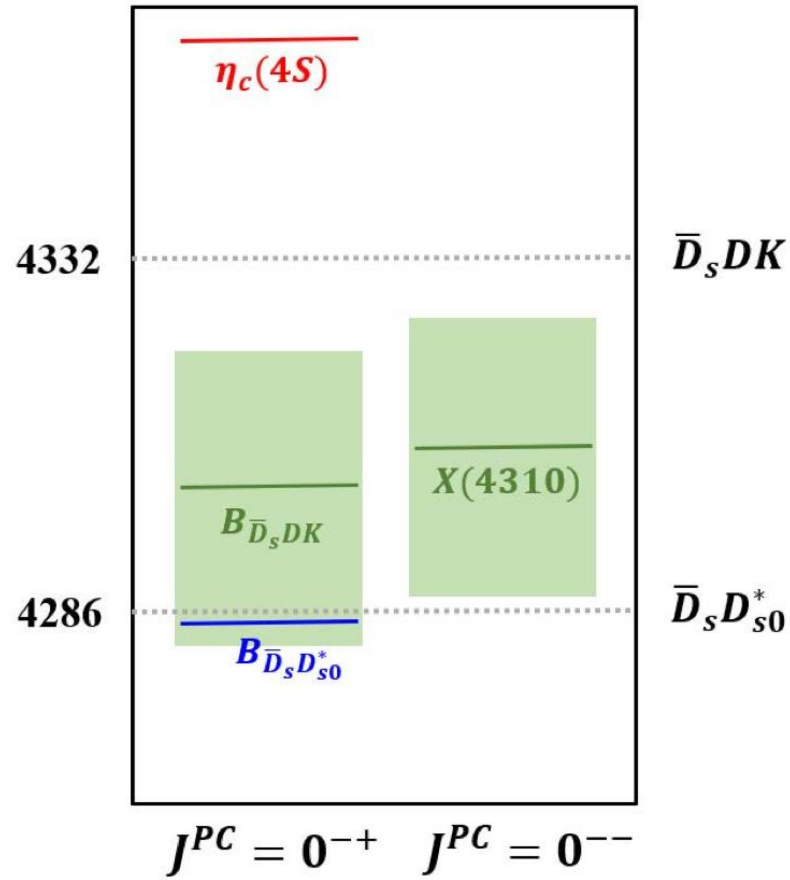
Mainly contributed by



Comparion with 0^{-+}

0^{-+} $\bar{D}_s D K$ molecule: 4304 MeV

Sets	B.E.(0^{-+})	$P_{\bar{D}_s K-D}$	$P_{DK-\bar{D}_s}$	$P_{\bar{D}_s D-K}$
$\alpha=1$	26^{+22}_{-16}	13^{+0}_{-1} %	76^{-0}_{+0} %	11^{-0}_{+1} %
$\alpha=2$	28^{+23}_{-17}	14^{+0}_{-1}%	74^{+1}_{+1} %	12^{-1}_{+0} %
Scenarios	$r_{\bar{D}_s K}$	r_{DK}	$r_{\bar{D}_s D}$	$\langle T \rangle$
$\alpha=1$	$1.5^{-0.3}_{+0.7}$	$1.1^{-0.2}_{+0.5}$	$1.3^{-0.3}_{+0.6}$	187^{+78}_{-73}
$\alpha=2$	$1.4^{-0.3}_{+0.6}$	$1.1^{-0.2}_{+0.4}$	$1.2^{-0.2}_{+0.5}$	195^{+77}_{-73}
Scenarios	$\langle V_{D_s \bar{K}} \rangle$	$\langle V_{DK} \rangle$	$\langle V_{D_s \bar{D}} \rangle$	$\langle V^{C=+}_{D_s \bar{D}^*_{s0}} \rangle$
$\alpha=1$	-44^{-24}_{+20}	-151^{-71}_{+61}	-16^{-6}_{+6}	-2^{-0}_{+1}
$\alpha=2$	-47^{-24}_{+21}	-154^{-70}_{+61}	-17^{-6}_{+6}	-4^{-1}_{+2}

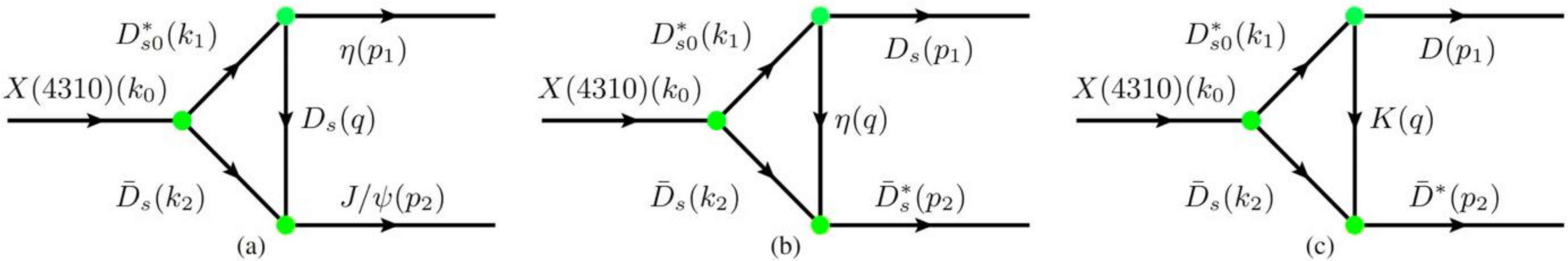


0^{-+} $\bar{D}_s D K$: coupling with $c\bar{c}$ and $\bar{D}_s D^*_{s0}$

0^{--} $\bar{D}_s D K$: no coupling with $c\bar{c}$ and $\bar{D}_s D^*_{s0}$

Decays of 0^{--} $\bar{D}_s D K$ molecule X(4310)

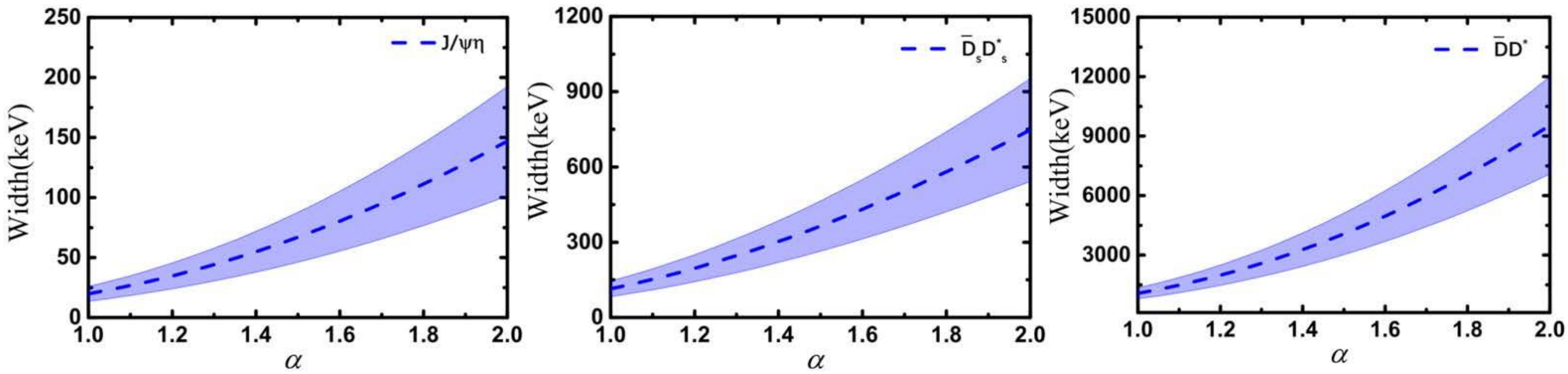
Triangle diagrams of the strong decays



$$
\begin{aligned}
i\mathcal{M}_a &= g_{XD^*_{s0}\bar{D}_s} g_{D^*_{s0}D_s\eta} g_{\psi\bar{D}_s D_s} \int \frac{d^4q}{(2\pi)^4} (k_2^\mu - q^\mu) \frac{1}{k_1^2 - m^2_{D^*_{s0}}} \frac{1}{k_2^2 - m^2_{\bar{D}_s}} \frac{1}{q^2 - m^2_{D_s}} \varepsilon_\mu(p_2) F(q^2), \\
i\mathcal{M}_b &= g_{XD^*_{s0}\bar{D}_s} g_{D^*_{s0}D_s\eta} g_{\bar{D}_s D^*_s\eta} \int \frac{d^4q}{(2\pi)^4} q^\mu \frac{1}{k_1^2 - m^2_{D^*_{s0}}} \frac{1}{k_2^2 - m^2_{\bar{D}_s}} \frac{1}{q^2 - m^2_{\eta}} \varepsilon_\mu(p_2) F(q^2), \\
i\mathcal{M}_c &= g_{XD^*_{s0}\bar{D}_s} g_{D^*_{s0}DK} g_{\bar{D}_s D^* K} \int \frac{d^4q}{(2\pi)^4} q^\mu \frac{1}{k_1^2 - m^2_{D^*_{s0}}} \frac{1}{k_2^2 - m^2_{\bar{D}_s}} \frac{1}{q^2 - m^2_{K}} \varepsilon_\mu(p_2) F(q^2),
\end{aligned}
$$

$$\Gamma = \frac{1}{2J+1} \frac{1}{8\pi} \frac{|\vec{p}|}{M^2} |\bar{\mathcal{M}}|^2 \qquad F(q,\Lambda,m) = \left(\frac{\Lambda^2 - m_E^2}{\Lambda^2 - q^2}\right)^2$$

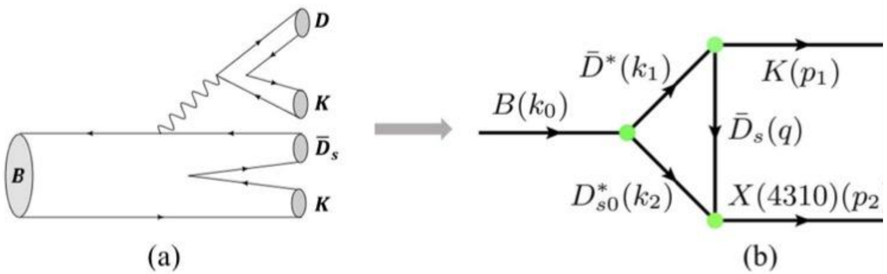
Partial decay widths



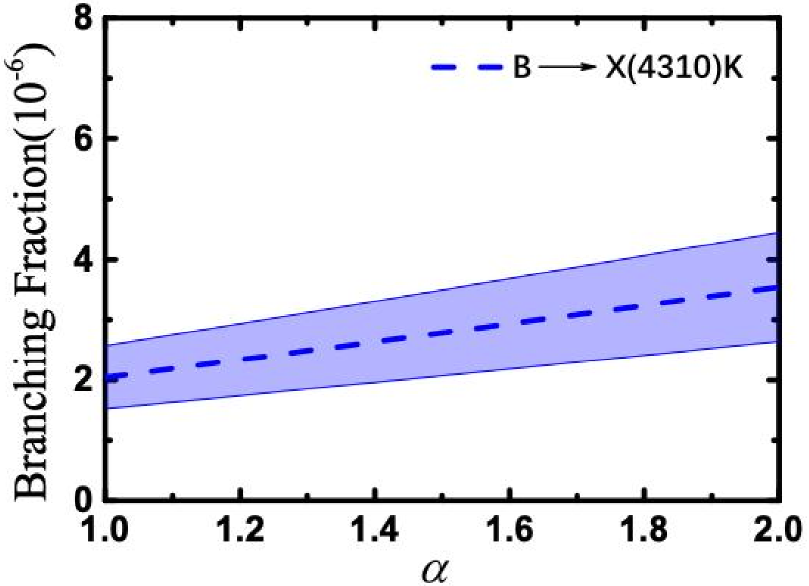
$X \to J/\psi\eta \sim 10^1$ $\quad$ $X \to \bar{D}_sD_s^* \sim 10^2$ $\quad$ $X \to \bar{D}^*D \sim 10^3$

Dominent decay: $X \to \bar{D}^*D$

Productions of 0^{--} $\bar{D}_s D K$ molecule



0^{--} $\bar{D}_s D K$ molecule:



$$\mathcal{M} = g_{\bar{D}^*\bar{D}_s K} g_{D^*_{s0}\bar{D}_s X} \mathcal{A}(B \to D^*_{s0}\bar{D}^*)^\mu \frac{-g^{\mu\nu} + \frac{k_1^\mu k_1^\nu}{k_1^2}}{(k_1^2 - m_{\bar{D}^*}^2)(k_2^2 - m_{D^*_{s0}}^2)(q^2 - m_{\bar{D}_s}^2)} p_1^\nu F(q^2),$$

$$\mathcal{A}(B \to D^*_{s0}\bar{D}^*) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs} a_1 f_{D^*_{s0}} \{ -q_1 \cdot \varepsilon(q_2)(m_{D^*} + m_B) A_1(q_1^2) + (k_0 + q_2) \cdot \varepsilon(q_2) q_1 \cdot (k_0 + q_2) \frac{A_2(q_1^2)}{m_{D^*} + m_B} + (k_0 + q_2) \cdot \varepsilon(q_2) [(m_{D^*} + m_B) A_1(q_1^2) - (m_B - m_{D^*}) A_2(q_1^2) - 2m_{D^*} A_0(q_1^2)] \},$$

Production rate: $B \to [X(4310) \to \bar{D}^* D] K \sim (3 \pm 1) \times 10^{-6}$

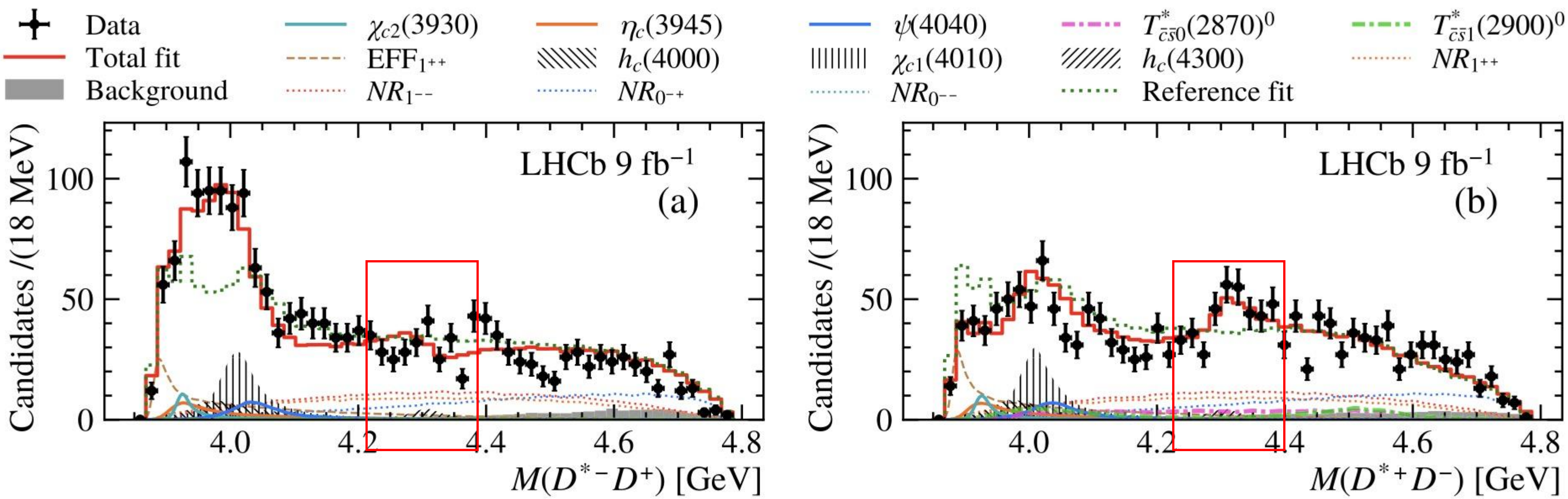
Hints in LHCb

TABLE I. Resonant and nonresonant components included in the baseline fit and their spin parities, fit fractions, and product branching fractions [$\mathcal{B}(B^+ \to RC) \times \mathcal{B}(R \to AB)$], where A, B, C are the three final-state particles. To obtain the branching fractions, including both $R \to D^{*+}D^-$ and $R \to D^{*-}D^+$, the values in the table should be multiplied by a factor of 2. The first uncertainties are statistical, estimated with a bootstrap method [32], the second are systematic, and the third are from the uncertainty of the $B^+ \to D^{*+}D^-K^+$ branching fraction. The masses and widths of the resonances marked with the † symbol are fixed to their known values [6].

Component	$J^{P(C)}$	Fit fraction [%] $B^+ \to D^{*+}D^-K^+$	Fit fraction [%] $B^+ \to D^{*-}D^+K^+$	Branching fraction $[10^{-4}]$
$\mathrm{EFF}_{1^{++}}$	1^{++}	$10.9^{+2.3}_{-1.2}{}^{+1.6}_{-2.1}$	$9.9^{+2.1}_{-1.0}{}^{+1.4}_{-1.9}$	$0.74^{+0.16}_{-0.08}{}^{+0.11}_{-0.14} \pm 0.07$
$\eta_c(3945)$	0^{-+}	$3.4^{+0.5}_{-1.0}{}^{+1.9}_{-0.7}$	$3.1^{+0.5}_{-0.9}{}^{+1.7}_{-0.6}$	$0.23^{+0.04}_{-0.07}{}^{+0.13}_{-0.05} \pm 0.02$
$\chi_{c2}(3930)^\dagger$	2^{++}	$1.8^{+0.5}_{-0.4}{}^{+0.6}_{-1.2}$	$1.7^{+0.5}_{-0.4}{}^{+0.6}_{-1.1}$	$0.12^{+0.03}_{-0.03}{}^{+0.04}_{-0.08} \pm 0.01$
$h_c(4000)$	1^{+-}	$5.1^{+1.0}_{-0.8}{}^{+1.5}_{-0.8}$	$4.6^{+0.9}_{-0.7}{}^{+1.4}_{-0.7}$	$0.35^{+0.07}_{-0.05}{}^{+0.10}_{-0.05} \pm 0.03$
$\chi_{c1}(4010)$	1^{++}	$10.1^{+1.6}_{-0.9}{}^{+1.3}_{-1.6}$	$9.1^{+1.4}_{-0.8}{}^{+1.2}_{-1.4}$	$0.69^{+0.11}_{-0.06}{}^{+0.09}_{-0.11} \pm 0.06$
$\psi(4040)^\dagger$	1^{--}	$2.8^{+0.5}_{-0.4}{}^{+0.5}_{-0.5}$	$2.6^{+0.5}_{-0.4}{}^{+0.4}_{-0.5}$	$0.19^{+0.04}_{-0.03}{}^{+0.03}_{-0.03} \pm 0.02$
$h_c(4300)$	1^{+-}	$1.2^{+0.2}_{-0.5}{}^{+0.2}_{-0.2}$	$1.1^{+0.2}_{-0.5}{}^{+0.2}_{-0.2}$	$0.08^{+0.01}_{-0.03}{}^{+0.02}_{-0.01} \pm 0.01$
$T^*_{\bar{c}\bar{s}0}(2870)^{0,\dagger}$	0^+	$6.5^{+0.9}_{-1.2}{}^{+1.3}_{-1.6}$	$\cdots$	$0.45^{+0.06}_{-0.08}{}^{+0.09}_{-0.10} \pm 0.04$
$T^*_{\bar{c}\bar{s}1}(2900)^{0\dagger}$	1^-	$5.5^{+1.1}_{-1.5}{}^{+2.4}_{-1.6}$	$\cdots$	$0.38^{+0.07}_{-0.10}{}^{+0.16}_{-0.11} \pm 0.03$
$\mathrm{NR}_{1^{--}}(D^{*\mp}D^\pm)$	1^{--}	$20.4^{+2.3}_{-0.6}{}^{+2.1}_{-2.6}$	$18.5^{+2.1}_{-0.5}{}^{+1.9}_{-2.3}$	$1.39^{+0.16}_{-0.04}{}^{+0.14}_{-0.17} \pm 0.12$
$\mathrm{NR}_{0^{--}}(D^{*\mp}D^\pm)$	0^{--}	$1.2^{+0.6}_{-0.1}{}^{+0.7}_{-0.6}$	$1.1^{+0.6}_{-0.1}{}^{+0.6}_{-0.5}$	$0.08^{+0.04}_{-0.01}{}^{+0.05}_{-0.04} \pm 0.01$
$\mathrm{NR}_{1^{++}}(D^{*\mp}D^\pm)$	1^{++}	$17.8^{+1.9}_{-1.4}{}^{+3.6}_{-2.6}$	$16.1^{+1.7}_{-1.3}{}^{+3.3}_{-2.3}$	$1.21^{+0.13}_{-0.10}{}^{+0.24}_{-0.17} \pm 0.11$
$\mathrm{NR}_{0^{-+}}(D^{*\mp}D^\pm)$	0^{-+}	$15.9^{+3.3}_{-1.2}{}^{+3.3}_{-3.3}$	$14.5^{+3.0}_{-1.1}{}^{+3.0}_{-3.0}$	$1.09^{+0.23}_{-0.08}{}^{+0.22}_{-0.23} \pm 0.09$

LHCb, Phys. Rev. Lett. 133,131902(2024)

Hints in LHCb



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LHCb, Phys. Rev. Lett. 133,131902(2024)

Estimation of events in LHC

BaBar, Phys. Rev. D 83, 032004(2011)

$$\mathcal{B}(B^+ \to D^{*\pm} D^{\mp} K^+]) \sim 6 \times 10^{-4}$$

LHCb, Phys. Rev. Lett. 133,131902(2024)

9fb^{-1}

$$B^+ \to D^{*\pm} D^{\mp} K^+ \sim 2 \times 10^3$$

LHCb, JHEP 12, 139 (2020)

$$\frac{\mathcal{B}(B^+ \to D^{*+} D^- K^+)}{\mathcal{B}(B^+ \to \bar{D}^0 D^0 K^+)} = 0.517 \pm 0.015 \pm 0.013 \pm 0.011\,,$$

$$\frac{\mathcal{B}(B^+ \to D^{*-} D^+ K^+)}{\mathcal{B}(B^+ \to \bar{D}^0 D^0 K^+)} = 0.577 \pm 0.016 \pm 0.013 \pm 0.013\,,$$

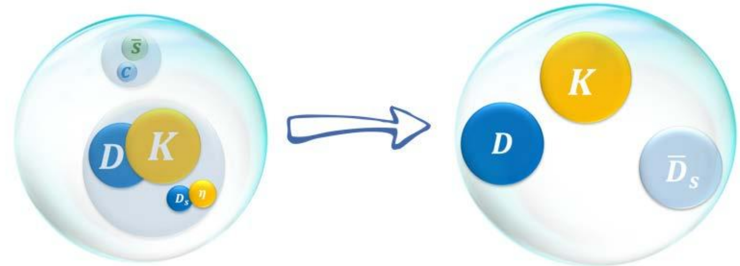
$$\frac{\mathcal{B}(B^0 \to D^{*-} D^0 K^+)}{\mathcal{B}(B^0 \to D^- D^0 K^+)} = 1.754 \pm 0.028 \pm 0.016 \pm 0.035\,,$$

$$\frac{\mathcal{B}(B^+ \to D^{*+} D^- K^+)}{\mathcal{B}(B^+ \to D^{*-} D^+ K^+)} = 0.907 \pm 0.033 \pm 0.014\,,$$

Process: $B^+ \to [X(4310) \to D^{*\pm} D^{\mp}] K^+ \sim 5 \times 10^{-7}$

LHC integrated luminosity:	50 fb^{-1}	Events:	10
	350 fb^{-1}		100

Summary:



$D^*_{s0}(2317)$ $\bar{D}_s DK$

- 4310 MeV $\mathbf{0^{--}}$ $\bar{D}_s DK$ exotic molecule is predicted on the nature of $D^*_{s0}(2317)$ as a molecule and $c\bar{s}$ mixture

- Main decay $X \to \bar{D}^* D$ with several MeV width

- Production process $B \to (X \to \bar{D}^* D)K$ with 10^{-6} branching fraction

Experiment searches are strongly recommended!