A novel and self-consistency analysis for the $\eta_c \to \gamma\gamma$ process

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王声权

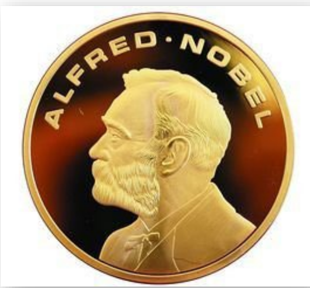
2025.07.13

Outline

- Introduction
- Renormalization scale setting
- Results and discussions
- Summary

Introduction

Quark model
1964(1969)

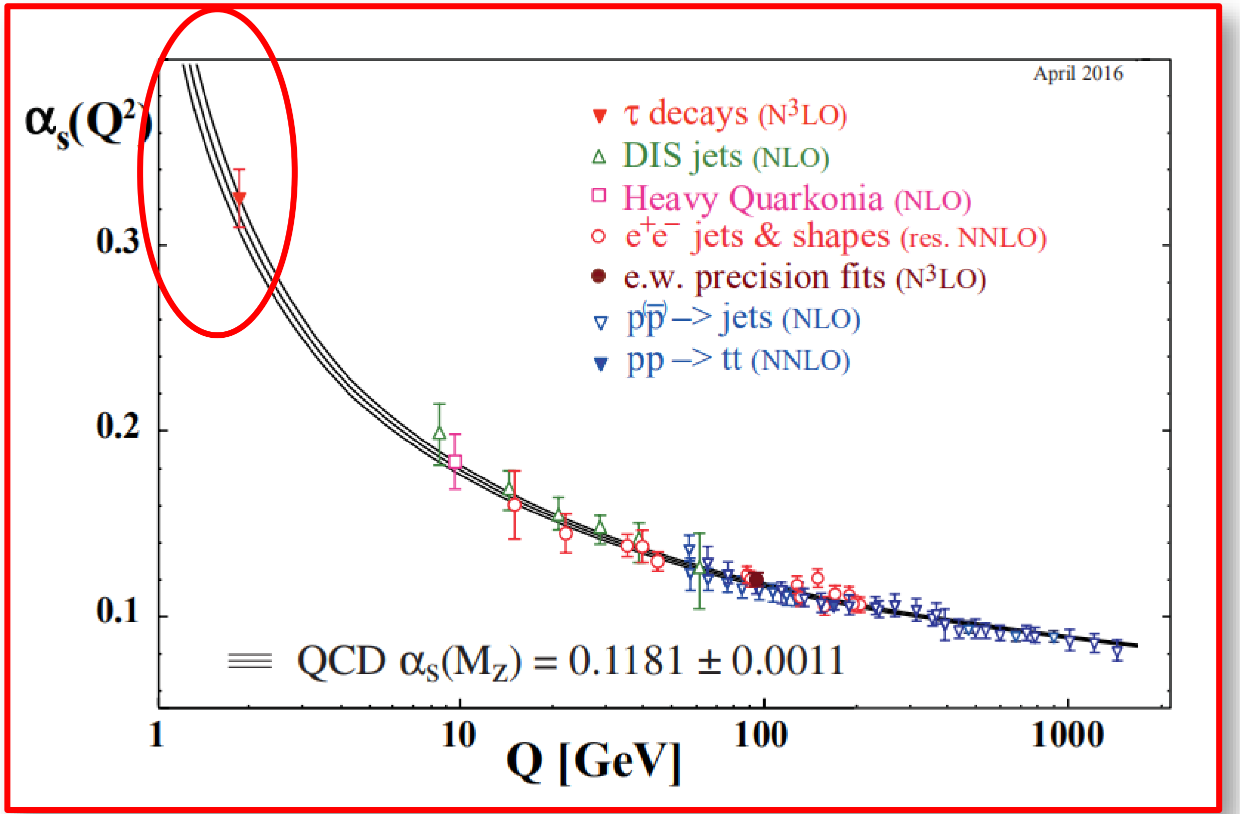


QCD
1973(2004)

默里·盖尔曼(Murray Gell-Mann)

戴维·格罗斯(David J. Gross)
戴维·普利策(H. David Politzer)
弗兰克·维尔泽克(Frank Wilczek)

Introduction



Introduction

The simplest and cleanest charmonium decay process

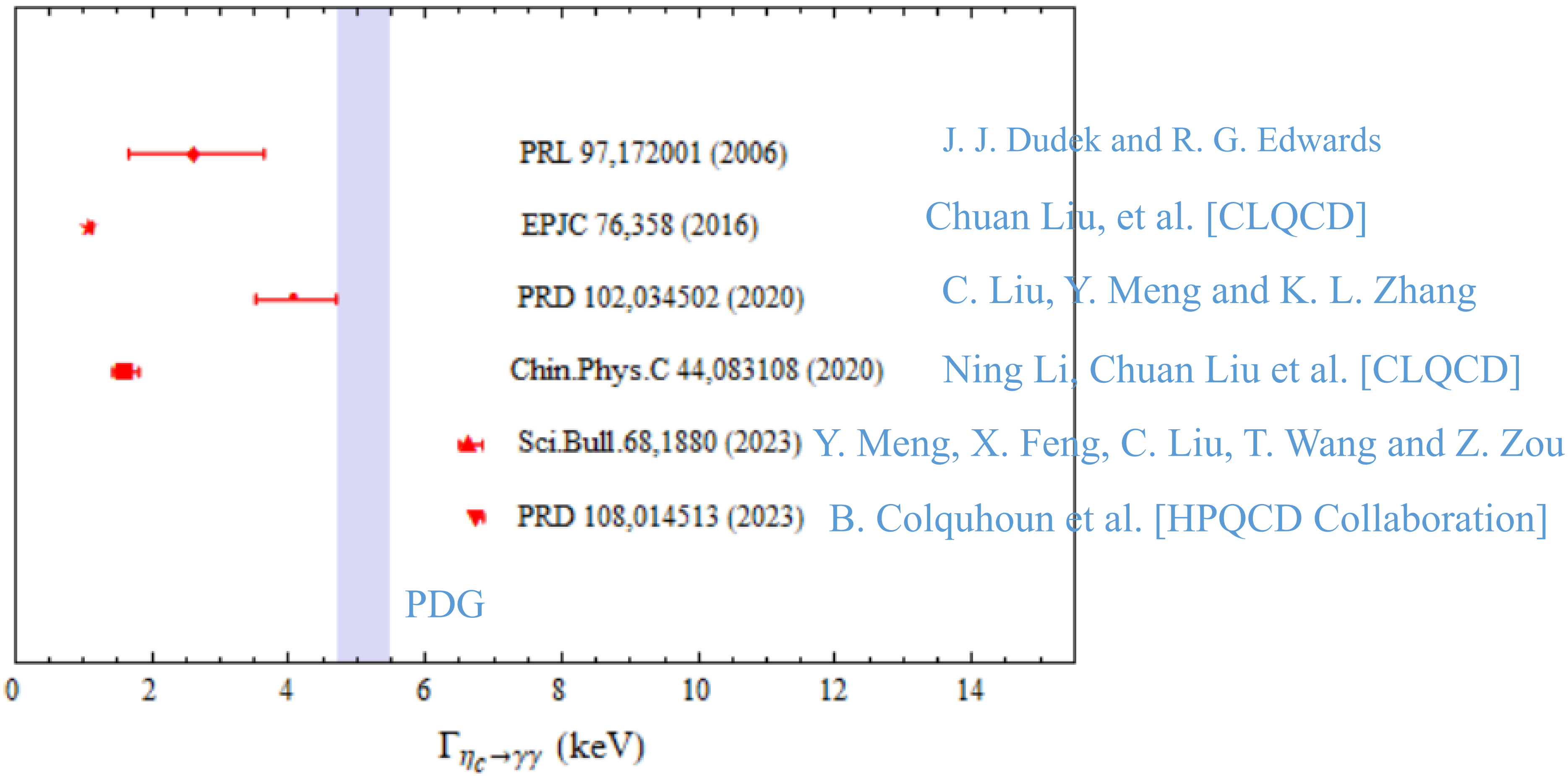
$$\Gamma_{\eta_c \to \gamma\gamma} = \frac{\pi}{4} \alpha^2 m_{\eta_c}^3 |F(0)|^2,$$

The Particle Data Group's reported value:

$$\Gamma_{\eta_c \to \gamma\gamma} = 5.1 \pm 0.4 \text{ keV}$$

Phys. Rev. D 110, 030001 (2024)

Lattice


PRL 97,172001 (2006)
J. J. Dudek and R. G. Edwards
EPJC 76,358 (2016)
Chuan Liu, et al. [CLQCD]
PRD 102,034502 (2020)
C. Liu, Y. Meng and K. L. Zhang
Chin.Phys.C 44,083108 (2020)
Ning Li, Chuan Liu et al. [CLQCD]
Sci.Bull.68,1880 (2023)
Y. Meng, X. Feng, C. Liu, T. Wang and Z. Zou
PRD 108,014513 (2023)
B. Colquhoun et al. [HPQCD Collaboration]
PDG
0
2
4
6
8
10
12
14
$\Gamma_{\eta_c \to \gamma\gamma}$ (keV)

Introduction

BLFQ	**3.7(6)**

Yang Li, Meijian Li, and James P. Vary, Phys. Rev. D 105.L071901 (2022)

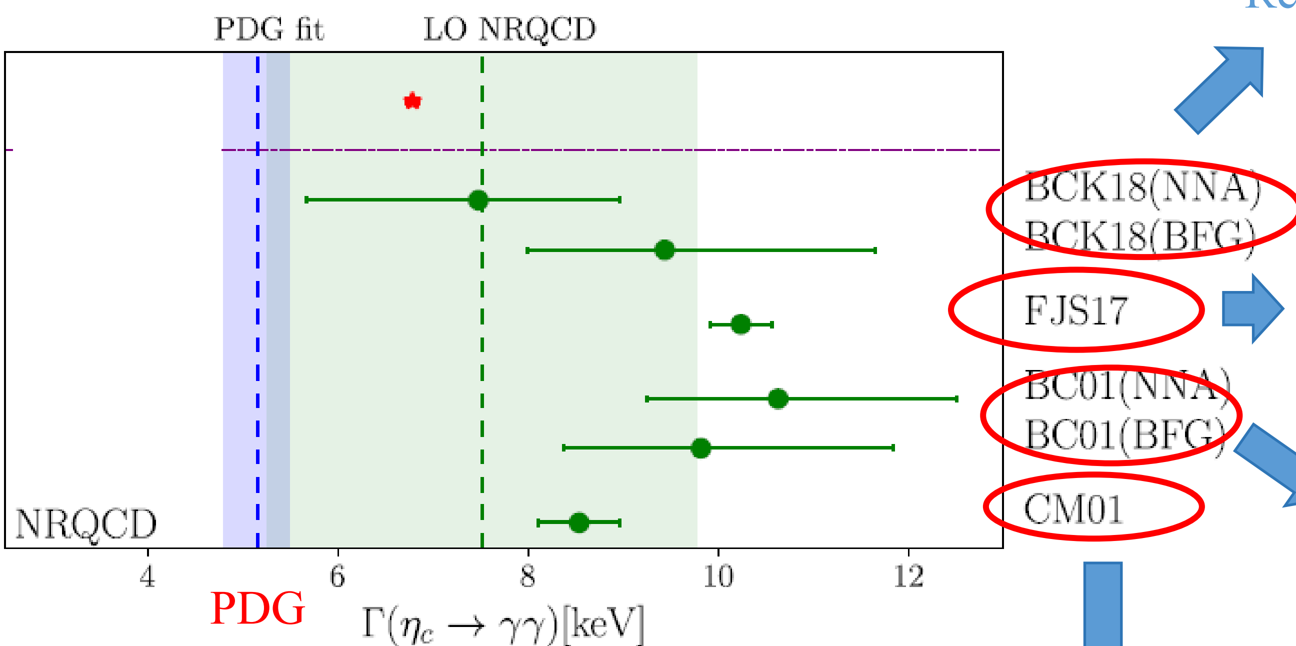
J. Chen, M. Ding, L. Chang, and Y.-X. Liu, Phys. Rev. D 95, 016010 (2017).

DSE/BSE [16]	6.39
LFQM [36]	4.88
LFQM [37,38]	5.7–9.7
NRQM/LF [8,9]	1.7–3.9
NRQM [8,9]	5.2–21

Light front: PRD 98, 034018; JPG 34, 687; PRD 82, 034021

kT -factorization approach: Phys. Rev. D 100, 054018 (2019); J. High Energy Phys. 06 (2020) 101

NRQCD



PDG

N. Brambilla, et al. Phys. Rev. D 98, 114020 (2018)

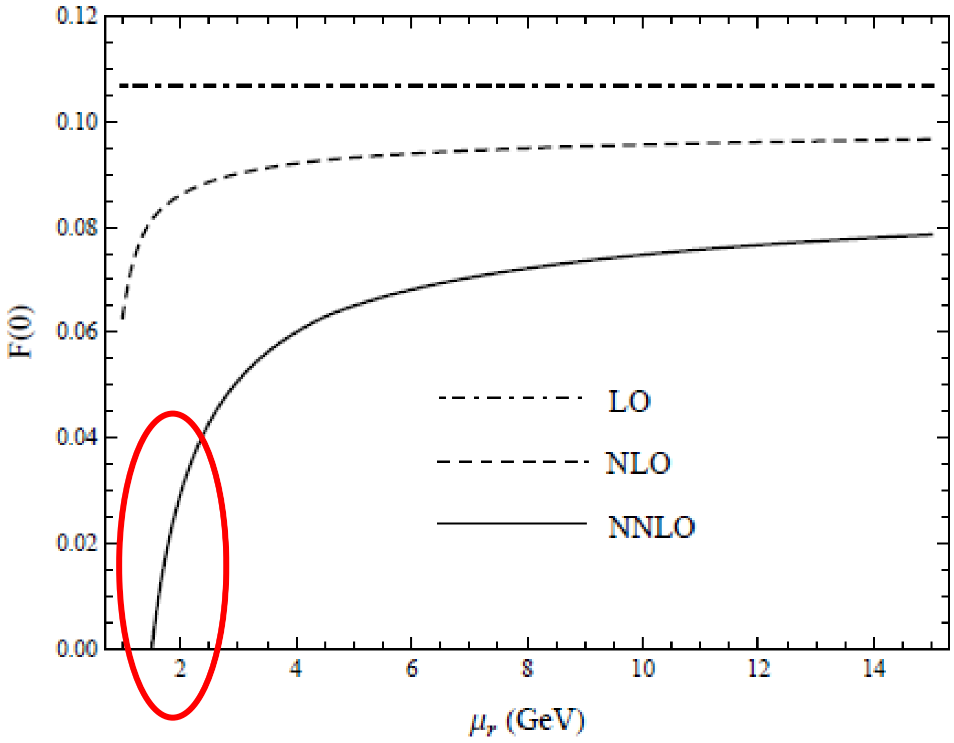
Feng, Jia, and Sang, Phys. Rev. Lett. 119, 252001 (2017), 10σ deviation

G. T. Bodwin and Y.-Q. Chen, Phys. Rev. D 64, 114008 (2001)

A. Czarnecki and K. Melnikov, Phys. Lett. B 519, 212 (2001)

Introduction

NRQCD



$$\Gamma_{\eta_c \to \gamma\gamma} = \frac{\pi}{4} \alpha^2 m_{\eta_c}^3 |F(0)|^2,$$

Renormalization scale setting

$$\rho(\mu_R) = r_0 \alpha_s(\mu_R)[1 + \sum_{k=1}^{\infty} r_k (\frac{Q}{\mu_R}) \frac{\alpha_s^k(\mu_R)}{\pi^k}]$$

为消除红外发散或紫外发散
引入重整化理论

$$g_0 = Z_g \mu^{\varepsilon/2} g \quad (\varepsilon\text{=4-d})$$

正规化、重整化、能标设定
↓ ↓ ↓
准确预言具同等重要性

成为当前理论中重要系统误差之一，
极大地影响微扰论计算精度及预言能力

计算到无穷阶的微扰论预言需与人为引入的参数无关
－－ 重整化群不变性

物理量

$$\frac{\partial \rho}{\partial \mu_R} \equiv 0; \frac{\partial \rho}{\partial \mathrm{R}} \equiv 0$$

Renormalization scale setting

如何解决能标问题

Brodsky–Lepage–Mackenzie method (BLM)

引用1214次

PHYSICAL REVIEW D VOLUME 28, NUMBER 1 1 JULY 1983

On the elimination of scale ambiguities in perturbative quantum chromodynamics

Stanley J. Brodsky
Institute for Advanced Study, Princeton, New Jersey 08540
*and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305**

G. Peter Lepage
Institute for Advanced Study, Princeton, New Jersey 08540
*and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853**

Paul B. Mackenzie
Fermilab, Batavia, Illinois 60510
(Received 23 November 1982)

源自QED观察
引入轻子圈
问题来自高阶如何处理

Principle of Minimum Sensitivity (PMS)

引用1200次

PHYSICAL REVIEW D VOLUME 23, NUMBER 12 15 JUNE 1981

Optimized perturbation theory

P. M. Stevenson
Physics Department, University of Wisconsin-Madison, Madison, Wisconsin 53706
(Received 21 July 1980; revised manuscript received 17 February 1981)

源自数学处理
引入驻点
问题来自物理

RG-improved effective coupling method (FAC)

引用553次

Volume 95B, number 1 PHYSICS LETTERS 8 September 1980

RENORMALIZATION GROUP IMPROVED PERTURBATIVE QCD

G. GRUNBERG [1]
Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, NY 14853, USA

源自实验 - 理论一致性
引入有效耦合常数
问题来自与微扰论理念冲突

Renormalization scale setting

Contents lists available at SciVerse ScienceDirect
Progress in Particle and Nuclear Physics
journal homepage: www.elsevier.com/locate/ppnp

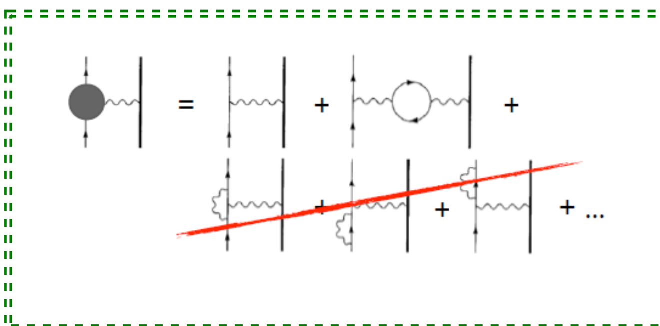
Review

The renormalization scale-setting problem in QCD

Xing-Gang Wu [a,*], Stanley J. Brodsky [b], Matin Mojaza [b,c]
[a] Department of Physics, Chongqing University, Chongqing 401331, PR China
[b] SLAC National Accelerator Laboratory, Stanford University, CA 94039, USA
[c] CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230, Denmark

BLM/FAC/PMS

In the case of QED, the renormalization scale can be set unambiguously by using the Gell-Mann-Low method, which automatically sums all vacuum polarization contributions to the photon propagators to all orders.



BLM=> nf-term
BLM method reduces in the Abelian limit to the Gell-Mann-Low method

Quantum Electrodynamics at Small Distances
M. Gell-Mann and F. E. Low
Phys. Rev. **95**, 1300 – Published 1 September 1954

Renormalization scale setting

PMC首篇正式论文

最初想法是将BLM推到无穷阶

后期发现两者在低阶等价，但PMC理念更基础

PHYSICAL REVIEW D **85**, 034038 (2012)

Scale setting using the extended renormalization group and the principle of maximum conformality: The QCD coupling constant at four loops

Stanley J. Brodsky[1,*] and Xing-Gang Wu[1,2,†]

[1]*SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, California 94025, USA*
[2]*Department of Physics, Chongqing University, Chongqing 401331, China*
(Received 30 November 2011; published 22 February 2012)

PRL **109**, 042002 (2012) PHYSICAL REVIEW LETTERS week ending 27 JULY 2012

Eliminating the Renormalization Scale Ambiguity for Top-Pair Production Using the Principle of Maximum Conformality

Stanley J. Brodsky[1,*] and Xing-Gang Wu[1,2,†]

[1]*SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, California 94025, USA*
[2]*Department of Physics, Chongqing University, Chongqing 401331, People's Republic of China*
(Received 29 March 2012; published 23 July 2012)

PRL **110**, 192001 (2013) PHYSICAL REVIEW LETTERS week ending 10 MAY 2013

Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

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CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230 Odense, Denmark and SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA

Stanley J. Brodsky[†]
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(Received 13 January 2013; published 10 May 2013)

PMC基本思想

$$\beta^{\mathcal{R}} = \mu_r^2 \frac{\partial}{\partial \mu_r^2}\left(\frac{\alpha_s^{\mathcal{R}}(\mu_r)}{4\pi}\right) = -\sum_{i=0}^{\infty} \beta_i^{\mathcal{R}} \left(\frac{\alpha_s^{\mathcal{R}}(\mu_r)}{4\pi}\right)^{i+2}$$

基于重整化群方程，利用微扰序列中的非共形β项确定高能物理过程的有效强耦合常数数值，获得与重整化能标选择无关的理论预言。通过最大程度的逼近共形微扰序列，可同时获得与重整化方案无关的理论预言，符合重整化群不变性要求。

附产品：由于消除具有发散性质的重整化子项，PMC序列将自然地具有更好的微扰收敛性。该收敛性与重整化能标选择无关，因此可以将之认为是高能物理过程的内禀属性。在阿贝尔极限下，将回归QED理论中的GM-L方案。

$$n!\beta_i^n \alpha_s^n$$

Renormalization scale setting

Scale Setting Using the Extended Renormalization Group and the Principle of Maximum Conformality: the QCD Coupling Constant at Four Loops.
Phys.Rev. D85 (2012) 034038.

Eliminating the Renormalization Scale Ambiguity for Top-Pair Production Using the Principle of Maximum Conformality
Phys.Rev.Lett. 109 (2012) 042002.

Self-Consistency Requirements of the Renormalization Group for Setting the Renormalization Scale
Phys.Rev. D86 (2012) 054018.

Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD
Phys.Rev.Lett. 110 (2013) 192001.

The Renormalization Scale-Setting Problem in QCD
Prog.Part.Nucl.Phys. 72 (2013) 44-98.

Reanalysis of the BFKL Pomeron at the next-to-leading logarithmic accuracy
JHEP 1310 (2013) 117

Systematic Scale-Setting to All Orders: The Principle of Maximum Conformality and Commensurate Scale Relations
Phys.Rev. D89 (2014) 014027.

Renormalization Group Invariance and Optimal QCD Renormalization Scale-Setting
Rept.Prog.Phys. 78 (2015) 126201.

General Properties on Applying the Principle of Minimum Sensitivity to High-order Perturbative QCD Predictions
Phys.Rev. D91 (2015) , 034006.

Setting the renormalization scale in perturbative QCD: Comparisons of the principle of maximum conformality with the sequential extended Brodsky-Lepage-Mackenzie approach.
Phys.Rev. D91 (2015), 094028.

Degeneracy Relations in QCD and the Equivalence of Two Systematic All-Orders Methods for Setting the Renormalization Scale
Phys.Lett. B748 (2015) 13-18.

The Generalized Scheme-Independent Crewther Relation in QCD
Phys.Lett. B770 (2017) 494-499

Novel All-Orders Single-Scale Approach to QCD Renormalization Scale-Setting
Phys.Rev. D95 (2017) , 094006.

Renormalization scheme dependence of high-order perturbative QCD predictions
Phys.Rev. D97 (2018), 036024.

Novel demonstration of the renormalization group invariance of the fixed-order predictions using the principle of maximum conformality and the C -scheme coupling
Phys.Rev. D97 (2018), 094030.

The QCD Renormalization Group Equation and the Elimination of Fixed-Order Scheme-and-Scale Ambiguities Using the Principle of Maximum Conformality
Prog.Part.Nucl.Phys. 108 (2019) 103706

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Renormalization scale setting

Infinite-order scale-setting using the principle of maximum conformality: A remarkably efficient method for eliminating renormalization scale ambiguities for perturbative QCD

Phys.Rev.D 102 (2020) 1, 014015

Renormalization scale setting for heavy quark pair production in e+e− annihilation near the threshold region

Phys.Rev.D 102 (2020) 1, 014005

New analyses of event shape observables in electron-positron annihilation and the determination of α_s running behavior in perturbative domain
JHEP 09 (2022) 137

Extending the predictive power of perturbative QCD using the principle of maximum conformality and the Bayesian analysis
Eur.Phys.J.C 83 (2023) 4, 326

The Principle of Maximum Conformality Correctly Resolves the Renormalization-Scheme-Dependence Problem
e-Print: 2311.17360 [hep-ph]

High precision tests of QCD without scale or scheme ambiguities : The 40thanniversary of the Brodsky–Lepage–Mackenzie method
Prog.Part.Nucl.Phys. 135 (2024) 104092

……

Renormalization scale setting

$$\Gamma_{\eta_c\to\gamma\gamma} = \frac{\pi}{4}\,\alpha^2\, m_{\eta_c}^3\, |F(0)|^2,$$

$$F(0) = c^{(0)}\left[1 + \delta^{(1)}\, a_s(\mu_r) + \delta^{(2)}(\mu_r)\, a_s^2(\mu_r)\right].$$

$$c^{(0)} = \frac{e_c^2\langle\eta_c|\psi^\dagger\chi(\mu_\Lambda)|0\rangle}{m_c^{5/2}},$$

$$\delta^{(1)} = C_F\left(\frac{\pi^2}{8} - \frac{5}{2}\right),$$

$$\delta^{(2)}(\mu_r) = \delta^{(1)}\frac{\beta_0}{4}\ln\frac{\mu_r^2}{m_c^2} - \frac{\pi^2}{2}C_F\left(C_F + \frac{C_A}{2}\right)\ln\frac{\mu_\Lambda^2}{m_c^2} + f_{lbl}^{(2)} + f_{reg}^{(2)}. \quad (5)$$

Renormalization scale setting

$$f^{(2)}_{reg} = f^{(2)}_{reg,in} + f^{(2)}_{reg,n_f}\, n_f,$$

$$\delta^{(2)}(\mu_r) = \delta^{(2)}_{in}(\mu_r) + \delta^{(2)}_{n_f}(\mu_r)\, n_f,$$

where

$$\begin{aligned}
\delta^{(2)}_{in}(\mu_r) &= \frac{11}{4}\delta^{(1)} \ln\frac{\mu_r^2}{m_c^2} - \frac{17}{9}\pi^2 \ln\frac{\mu_\Lambda^2}{m_c^2} \\
&\quad + f^{(2)}_{lbl} + f^{(2)}_{reg,in}, \\
\delta^{(2)}_{n_f}(\mu_r) &= \delta^{(2)}_{reg,n_f} - \frac{1}{6}\delta^{(1)} \ln\frac{\mu_r^2}{m_c^2}.
\end{aligned}$$

The bound state process in the physical V scheme, a reliable prediction is achieved

J. Yan, X. G. Wu, Z. F. Wu, J. H. Shan and H. Zhou,

Phys. Lett. B **853**, 138664 (2024).

S. Q. Wang, S. J. Brodsky, X. G. Wu, L. Di Giustino and J. M. Shen,

Phys. Rev. D **102**, 014005 (2020).

Renormalization scale setting

$$V(Q^2) = -\frac{4\,\pi^2\,C_F\,a_s^V(Q)}{Q^2}$$

$$\begin{aligned} F(0) &= c^{(0)}\left[1 + \delta_V^{(1)}\,a_s^V(\mu_r) + \left(\delta_{in,V}^{(2)}(\mu_r) \right.\right. \\ &\qquad \left.\left. + \delta_{n_f,V}^{(2)}(\mu_r)\,n_f\right)\left(a_s^V(\mu_r)\right)^2\right], \end{aligned}$$

$$F(0) = c^{(0)}\left[1 + \delta_V^{(1)}\,a_s^V(Q_\star) + \delta_{con,V}^{(2)}(\mu_r)\,\left(a_s^V(Q_\star)\right)^2\right].$$

$$Q_\star = \mu_r \exp\left[\frac{3\,\delta_{n_f,V}^{(2)}(\mu_r)}{2\,T_R\,\delta_V^{(1)}}\right],$$

$$\delta_{con,V}^{(2)}(\mu_r) = \frac{11\,C_A\,\delta_{n_f,V}^{(2)}(\mu_r)}{4\,T_R} + \delta_{in,V}^{(2)}(\mu_r).$$

Results and discussions

MS\bar scheme

Conv.:

NLO is moderate

NNLO gives large negative contributions, encounters large scale uncertainty

$$F^{\rm Conv}(0)|_{\mu_r=1{\rm GeV}} = c^{(0)}(1-0.25-1.11).$$
$$F^{\rm Conv}(0)|_{\mu_r=m_c} = c^{(0)}(1-0.18-0.60).$$
$$F^{\rm Conv}(0)|_{\mu_r=2m_c} = c^{(0)}(1-0.13-0.36).$$

In fact, when μr < 1.3 GeV, the F(0) becomes negative

PMC:

$$F^{\rm PMC}(0) \equiv c^{(0)}(1-0.13-0.37)$$

Scale uncertainty is eliminated and PMC scale = 2mc

Optimal scale under conventional method is 2mc

Results and discussions

V scheme

Conv.:

μ_r	LO	NLO	NNLO	$F(0)$
1 GeV	0.1066	-0.0438	-0.2276	-0.1648
m_c	0.1066	-0.0253	-0.0815	-0.0001
$2m_c$	0.1066	-0.0165	-0.0392	0.0509

TABLE I: The QCD corrections for $F(0)$ using the conventional scale setting for three typical renormalization scales $\mu_r = 1$ GeV, m_c and $2m_c$. The factorization scale is set to: $\mu_\Lambda = 1$ GeV.

$$\text{LO:NLO:NNLO} \sim 1 : -0.41 : -2.13$$

$$\text{LO:NLO:NNLO} \sim 1 : -0.24 : -0.76$$

$$\text{LO:NLO:NNLO} \sim 1 : -0.15 : -0.37$$

PMC:

$$Q_\star = 4.49\, m_c = 6.74 \text{ GeV}$$

$$F(0) = 0.1066 - 0.0123 - 0.0245 = 0.0698$$

1： -0.12： -0.23

Results and discussions

Factorization scale uncertainty:

Conv.:

$$F^{\text{Conv}}(0)|_{\mu_r=m_c} = 0.43c^{(0)},\ \ 0.22c^{(0)},\ \ -0.06c^{(0)}$$

PMC:

$$F^{\text{PMC}}(0) = 0.62c^{(0)},\ \ 0.50c^{(0)},\ \ 0.34c^{(0)}.$$

for μf= 1 GeV, mc and 2mc

Mass mc uncertainty:

Conv.:

$$F(0) = 0.22c^{(0)},\ 0.14c^{(0)},\ 0.07c^{(0)}$$

PMC:

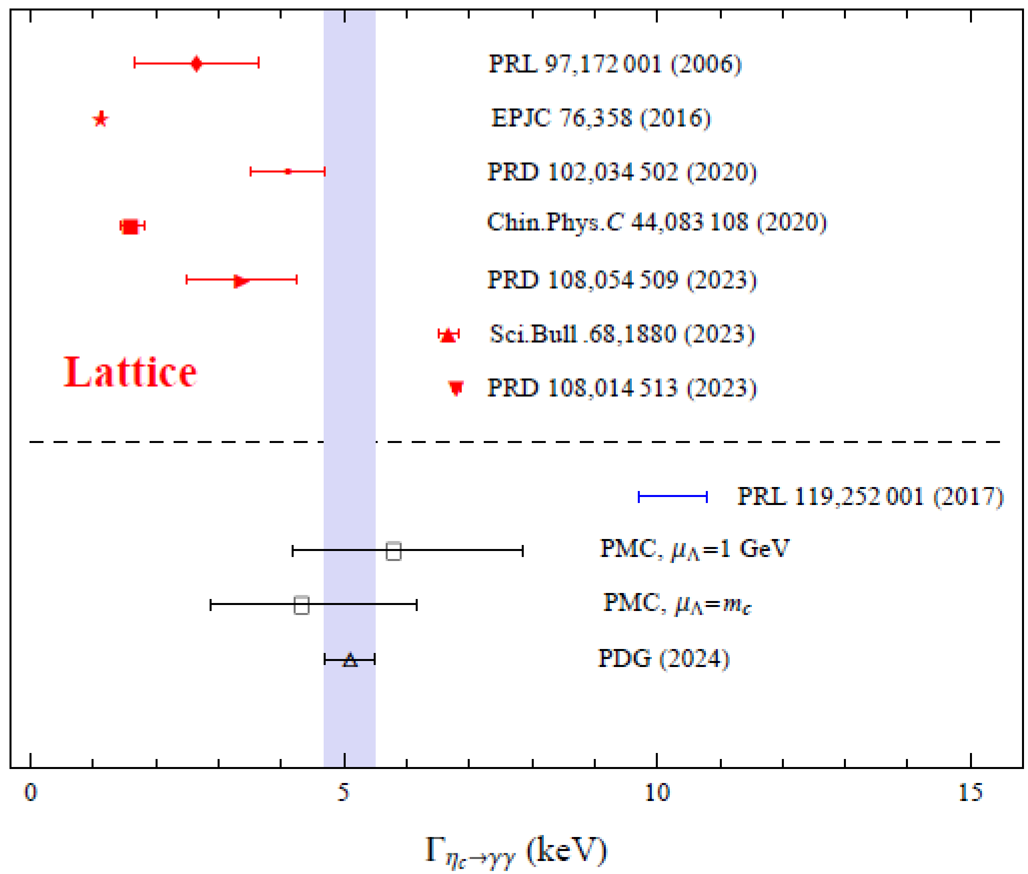
$$F(0) = 0.50c^{(0)},\ 0.46c^{(0)},\ 0.43c^{(0)}$$

for mc= 1.68, 1.5 and 1.4 GeV

Results and discussions

$$\Gamma_{\eta_c\to\gamma\gamma} = 5.79^{+1.79+1.00+0.15}_{-1.32-0.92-0.15}\ \text{keV for } \mu_\Lambda = 1\ \text{GeV},$$

$$\Gamma_{\eta_c\to\gamma\gamma} = 4.32^{+1.48+1.11+0.15}_{-1.05-0.98-0.16}\ \text{keV for } \mu_\Lambda = \text{m}_c,$$



Results and discussions

$$\gamma^*\gamma \to \eta_c$$

In 2010, the BaBar collaboration measured the transition form factor $F(Q^2)$ in a wide range of 2 GeV^2< Q^2< 50 GeV^2.

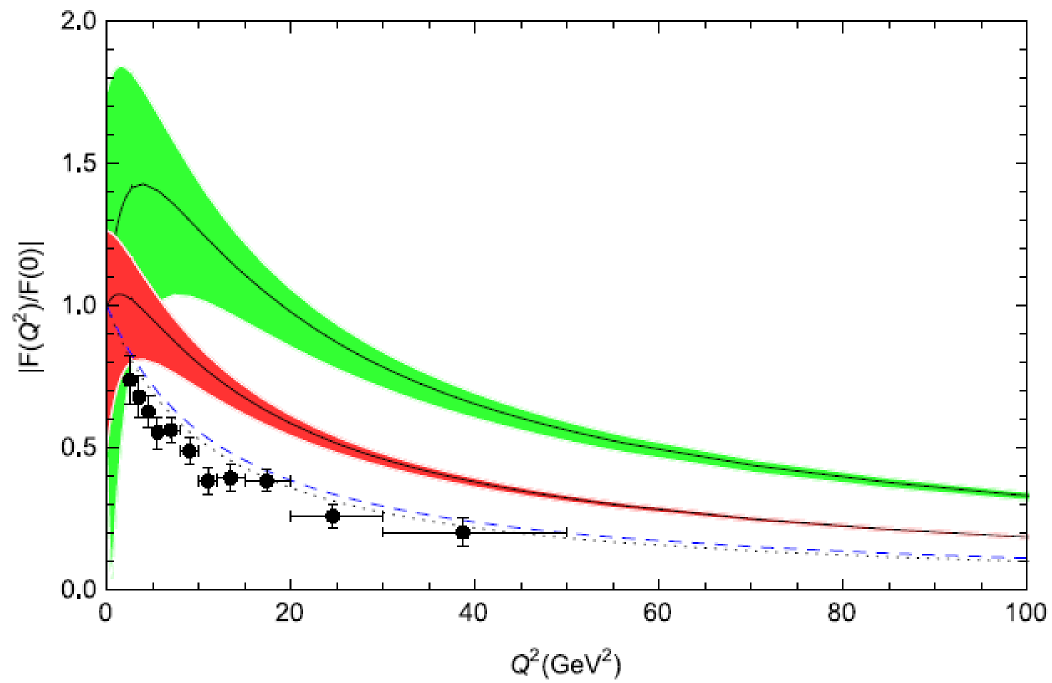
$$|F(Q^2)/F(0)| = \frac{1}{1 + Q^2/\Lambda},$$

J. P. Lees *et al.* [BaBar Collaboration], Phys. Rev. D **81**, 052010 (2010).

Results and discussions

Phys. Rev. Lett. **115**, 222001 (2015).

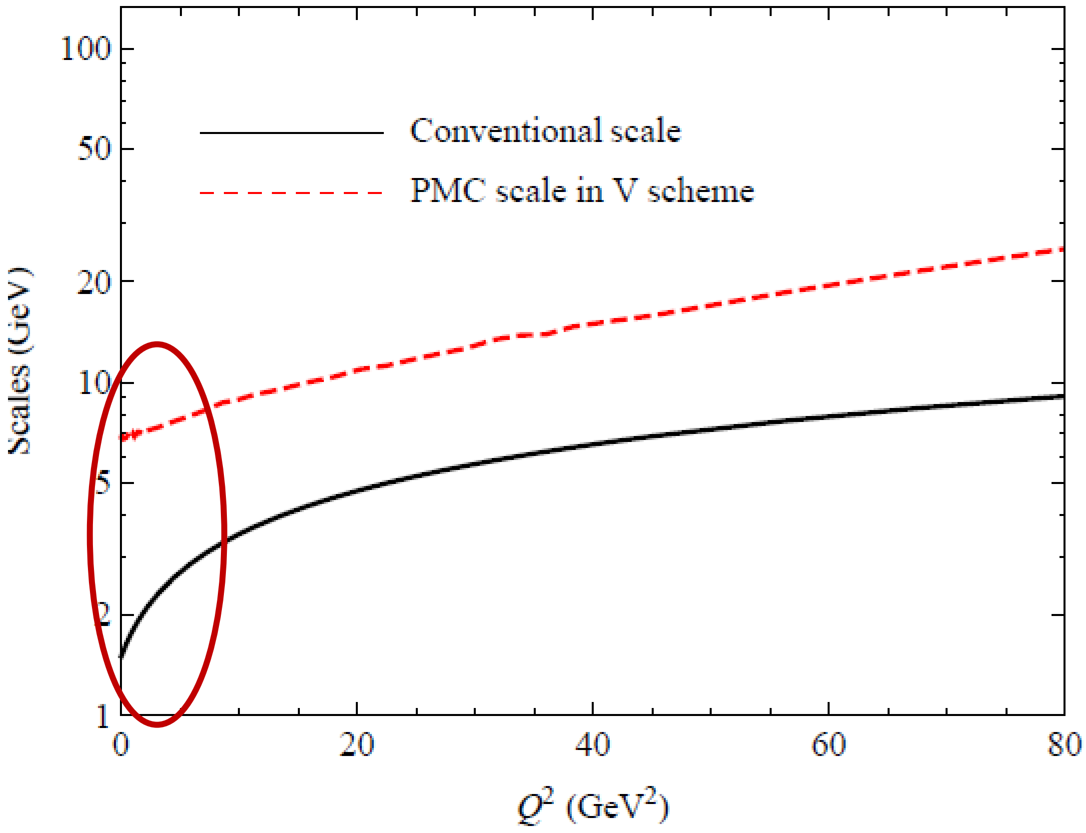
The NNLO NRQCD prediction fails to explain the BaBar measurements



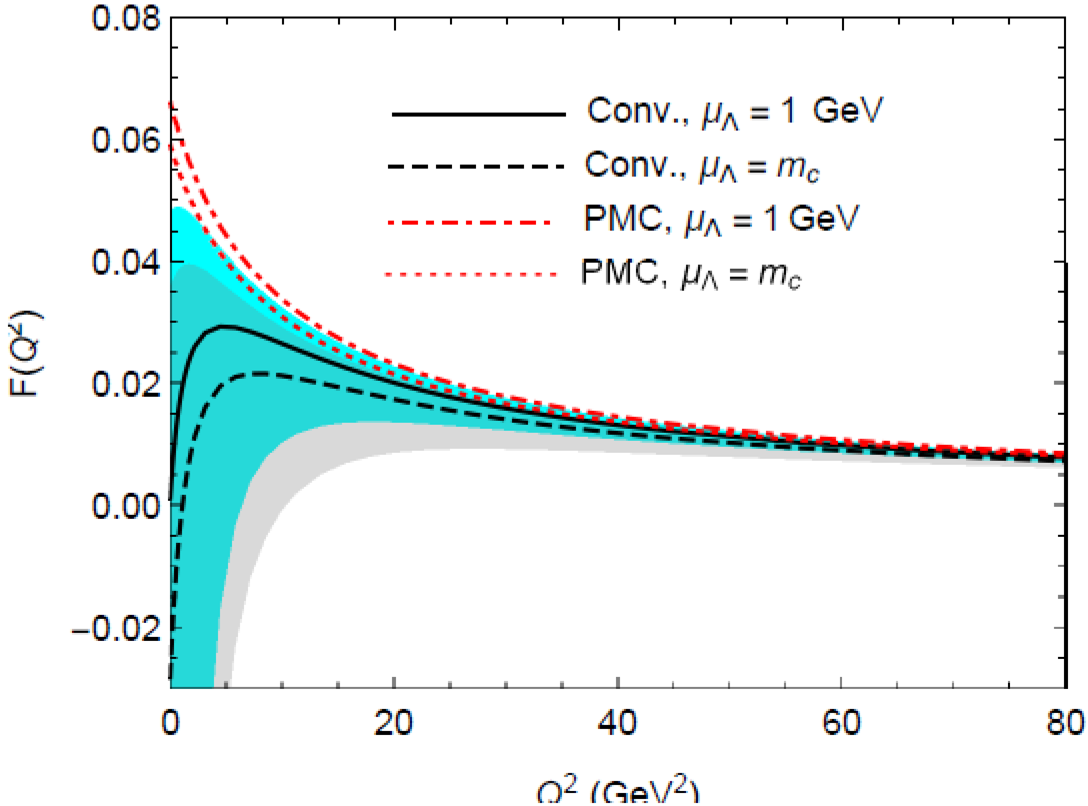
applicability of NRQCD ?

New physics ?

Results and discussions

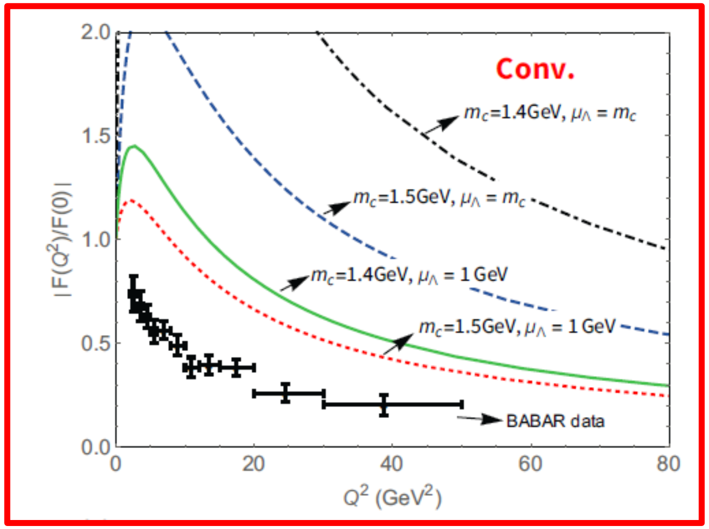


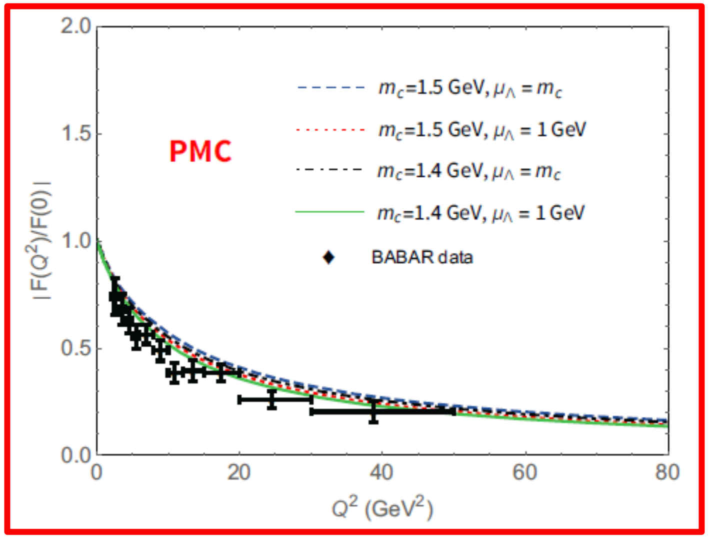
Results and discussions



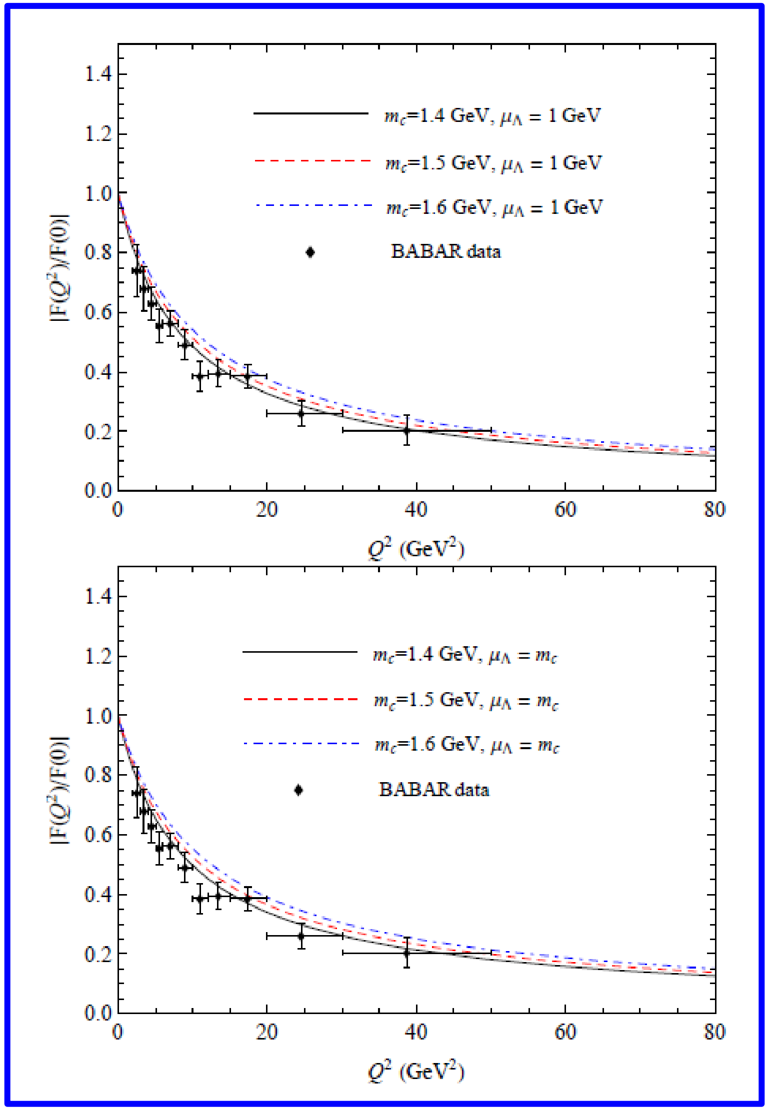
Results and discussions

MS\bar-scheme





V-scheme



Summary

$$\beta^{\mathcal{R}} = \mu_r^2 \frac{\partial}{\partial \mu_r^2}\left(\frac{\alpha_s^{\mathcal{R}}(\mu_r)}{4\pi}\right) = -\sum_{i=0}^{\infty} \beta_i^{\mathcal{R}} \left(\frac{\alpha_s^{\mathcal{R}}(\mu_r)}{4\pi}\right)^{i+2}$$

- ✓ To eliminate the renormalization scheme-and-scale ambiguities.
- ✓ There is no renormalon divergence in the pQCD series
- ✓ The more convergent perturbative series is in general achieved
- ✓ A novel and self-consistency analysis for the ηc → γγ process is achieved.
- ✓ e^+e^- →J/\psi+\eta_c, e^+e^- →J/\psi+J/\psi, J/\psi→e^+e^- process are being prepared.

Thanks for your attention!

Join us!