The consistent interactions between the axion and vector mesons

Jia Liu (刘佳)
Peking University

ArXiv: 2406.11948

Yang Bai, Ting-Kuo Chen, Jia Liu, Xiaolin Ma

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Outlines

- Axion general introduction
- Axion-scalar meson interactions
- Axion-vector meson interactions
- Phenomenology at BESIII and STCF
- Summary

The QCD axion and the Strong CP problem

$$\mathscr{L} \supset -\frac{\theta g_s^2}{32\pi^2} G\tilde{G} - \left(\bar{u}_L M_u u_R + \bar{d}_L M_d d_R + \mathrm{h.c.}\right)$$

- The CKM matrix from $M_{u,d}$
  - CP violating phase $\theta_{\rm CP}$ ~ 1.2 radian

- QCD induced CP violating phase, $\bar{\theta}$

$$\bar{\theta} = \theta + \arg\left[\det\left[M_u M_d\right]\right]$$

- $\bar{\theta}$ is invariant under quark chiral rotation

$$d^n_{\rm EDM} \sim \bar{\theta} \times 10^{-16}\ \mathrm{e\ cm}$$

- According to neutron EDM experiment

$$d^n_{\rm exp} < 10^{-26}\ \mathrm{e\ cm}$$

$$\bar{\theta} \lesssim 1.3 \times 10^{-10}\ \mathrm{radian}$$

The Peccei-Quinn solution to Strong CP problem

- Experiment requires $\bar{\theta} = \theta + \arg\left[\det\left[M_u M_d\right]\right] \lesssim 10^{-1}\mathrm{rad}$
- PQ: promote the constant $\bar{\theta}$ to a dynamical field, a
- Vafa-Witten theorem: vector-like theory (QCD) has ground state $\langle\theta\rangle = 0$
- Introduce a *global* PQ-symmetry $U(1)_{\mathrm{PQ}}$, *anomalous* under the QCD
  - The massless Goldstone boson a is called *axion*
    - $a \to a + \kappa f_a \;\; \Rightarrow \mathscr{S} \to \mathscr{S} + \frac{\kappa}{32\pi^2}\int d^4x G\tilde{G}$, cancels $\bar{\theta}$
    - Low energy: $\mathscr{L} = \sum_q \bar{q}\left(iD_\mu\gamma^\mu - m_q\right)q - \frac{1}{4}GG + \frac{g_s^2}{32\pi^2}\frac{a}{f_a}G\tilde{G} + \frac{1}{2}\left(\partial_\mu a\right)^2 + \mathscr{L}_{\mathrm{int}}[\partial_\mu a]$

The invisible axion models

- SM particles does not directly charge under $U(1)_{\mathrm{PQ}}$
  - KSVZ model:
    - Heavy vector-like quark: $Q_{L,R}$
    - Q_L and Q_R has different charge under $U(1)_{\mathrm{PQ}}$
    - A heavy complex scalar $\Phi = re^{ia}$ charge under $U(1)_{\mathrm{PQ}}$
    - Yukawa: $y\Phi\bar{Q}_L Q_R \supset \frac{yf_a}{\sqrt{2}} e^{ia/f_a}\bar{Q}_L Q_R$
    - $\mathscr{L} \supset \frac{g_s^2}{32\pi^2}\frac{a}{f_a}G\tilde{G}$

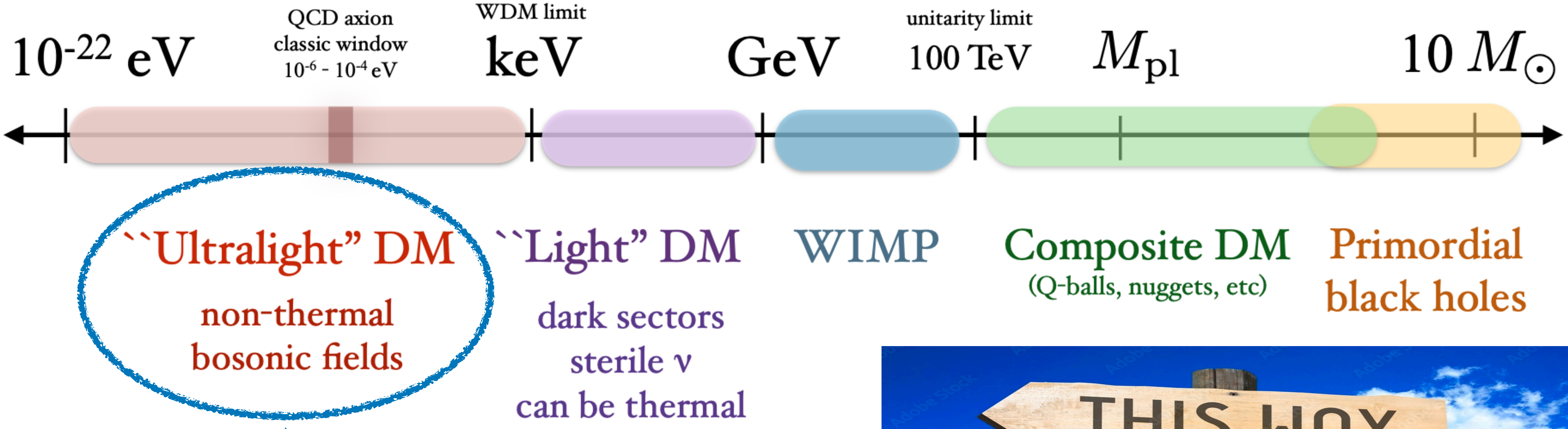
The invisible axion models

- DFSZ model:

- Two Higgs doublet $H_{u,d}$ and a complex singlet Φ charged under $U(1)_{\rm PQ}$, with phase factor $e^{i\phi_{u,d,0}}$
- Similar to previous UV model, but $\langle\Phi\rangle \gg v_h$
- Yukawa: $\left(\bar{Q}Y_u H_u u_R + \bar{Q}Y_d H_d d_R + \bar{L}Y_e H_d e_R\right) + h.c.$
- Potential term: e.g. $H_u H_d \Phi^2$, Axion mode: $a = \frac{1}{f_a}\sum_{i=u,d,0} Q_i v_i \phi_i$
- Axion have direct quark and lepton couplings
- Low energy: $\mathscr{L} \supset \frac{\alpha_s}{8\pi^2}\frac{a}{f_a}G\tilde{G} + \frac{\alpha_{em}}{8\pi}\frac{E}{N}\frac{a}{f_a}F\tilde{F} - \bar{f}_L M_f f_R + \frac{\partial_\mu a}{2f_a}\bar{f}c_f\gamma^\mu\gamma_5 f$

The dark matter candidate models

1904.07915， TASI lecture



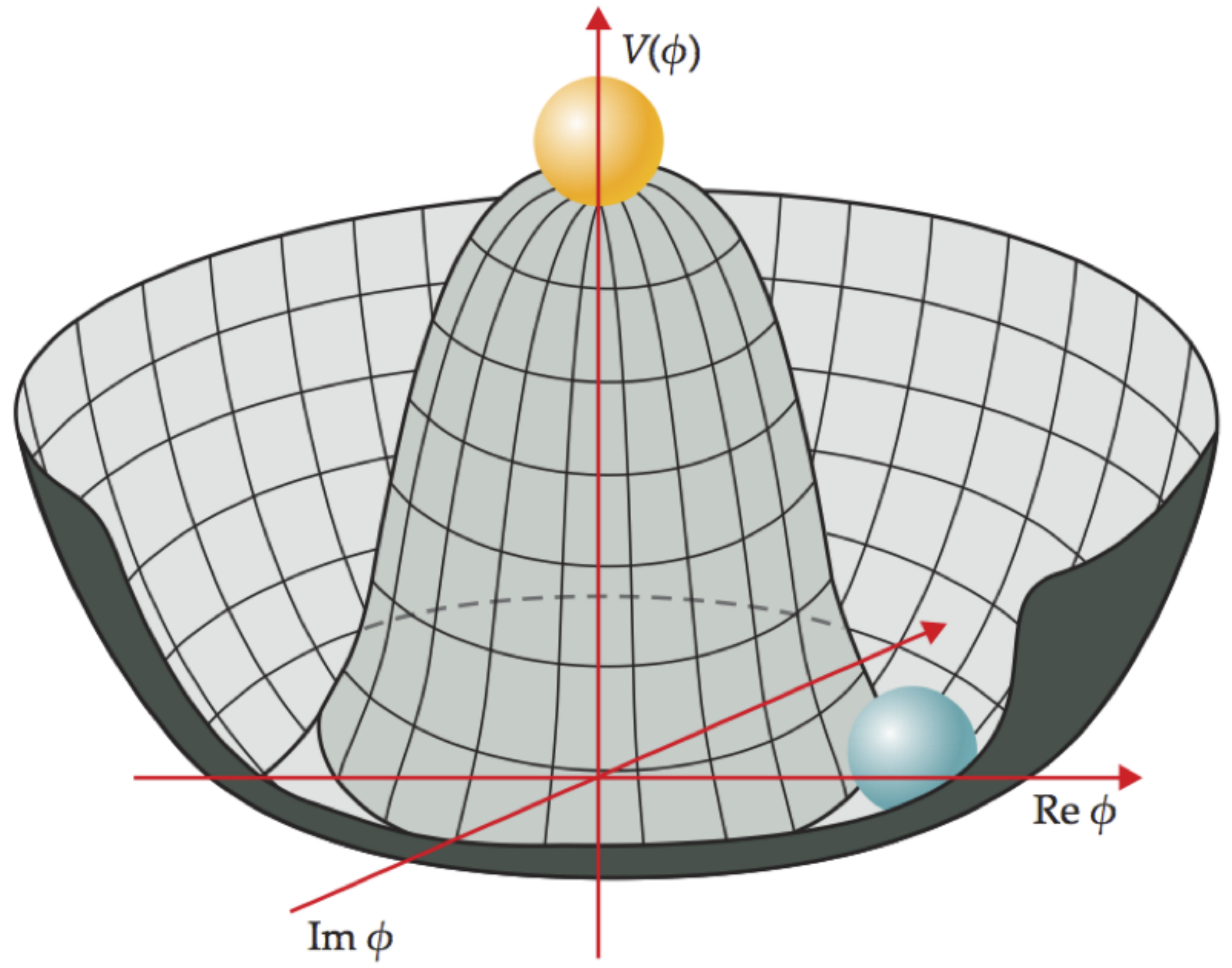
Axion and ALP dark matter

HEP at a cross-road: explore all directions!

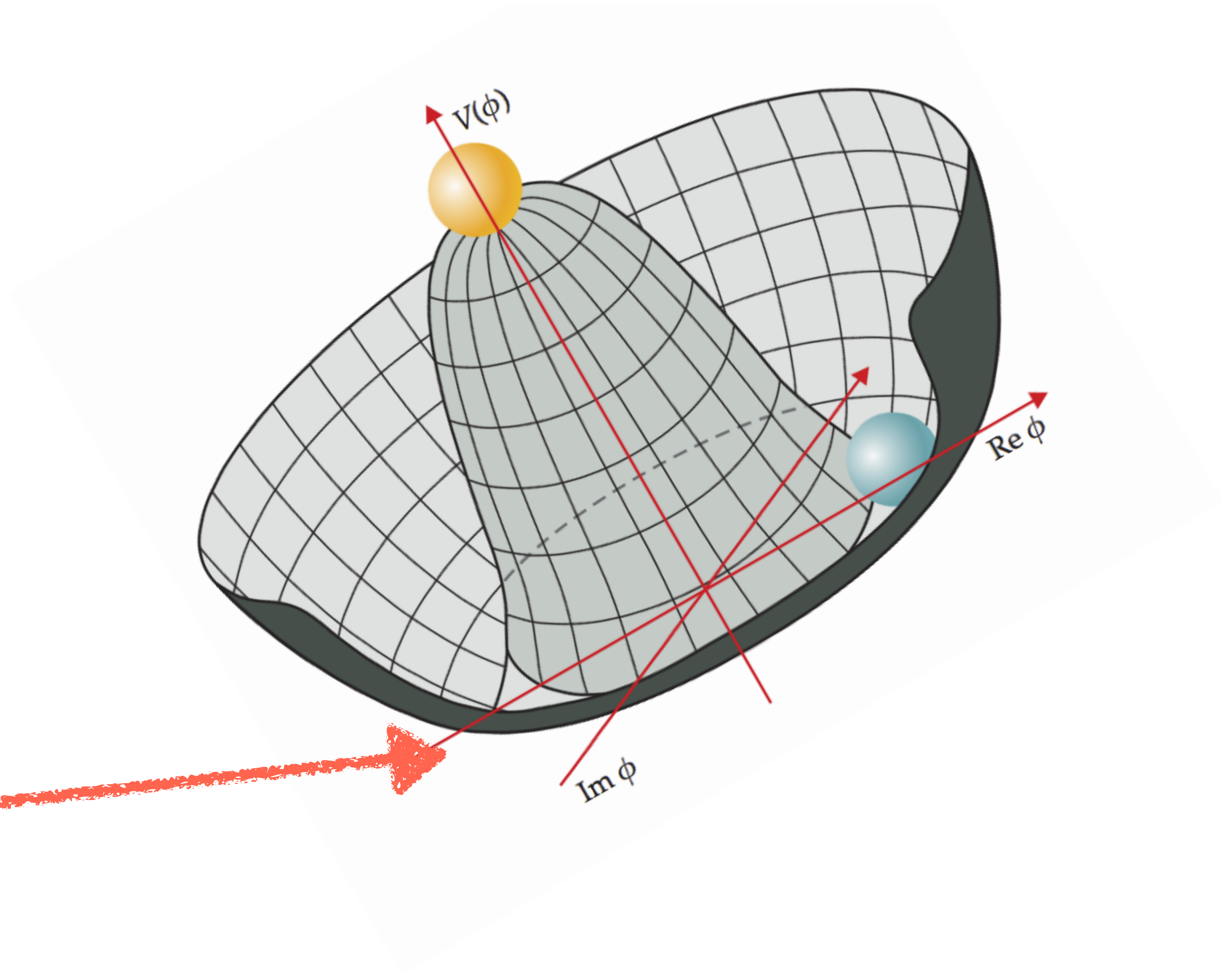
Misalignment and Axion Dark Matter

- Global U(1)$_{PQ}$ symmetry
  - Spontaneous broken leads to massless goldstone (Axion)

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$



- At QCD scale ~ O(1) GeV,
  - Potential from Chiral Lagrangian explicitly breaks the symmetry leads to massive axion
  - Energy stored in coherent oscillation of axion field
  - When $m_\phi \sim \frac{\Lambda^2}{f_\phi} \sim H$, misalignment happens and the fields turns into particles: cold dark matter
  - QCD vacuum picks $\Theta = \theta_{\text{QCD}} + \xi\langle a\rangle/f_a = 0$

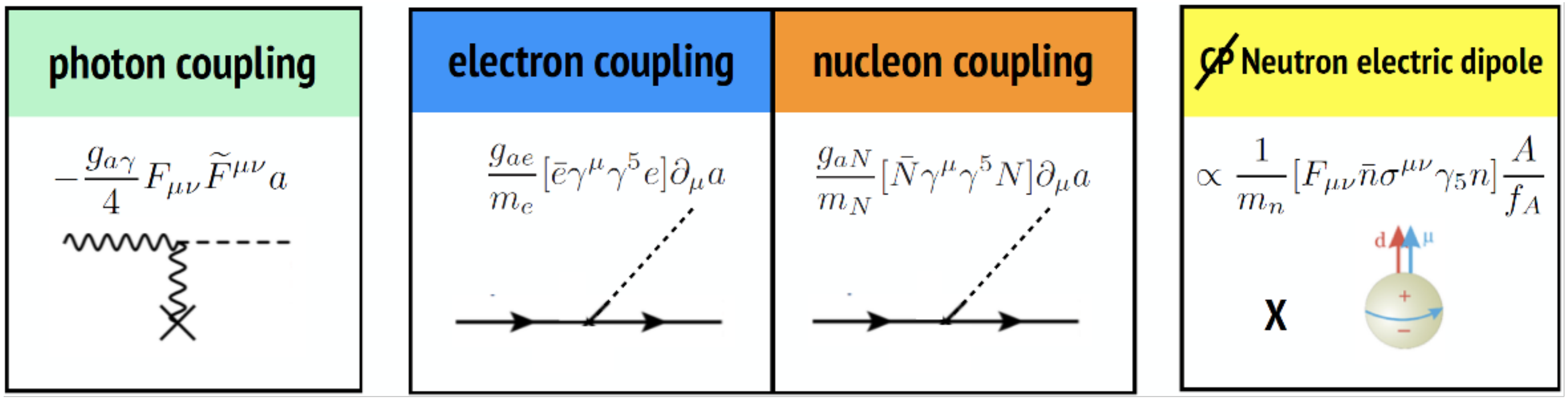


The axion effective Lagrangian at quark-level

- Axion can couple to SM gauge bosons and fermions

$$\mathscr{L}_{\mathrm{ALP}} = g_{ag}\frac{a}{f_a}G\tilde{G} + g_{a\gamma}\frac{a}{f_a}F\tilde{F} + g_{af}\frac{\partial_\mu a}{2f_a}\bar{f}\gamma^\mu\gamma_5 f$$

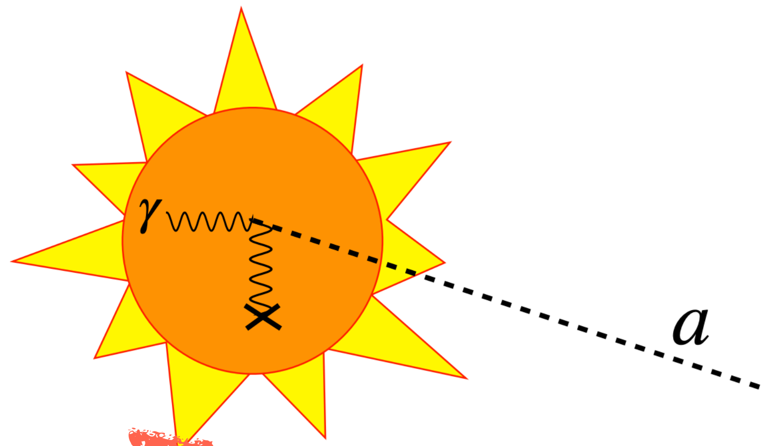
- Detection of axion through various couplings

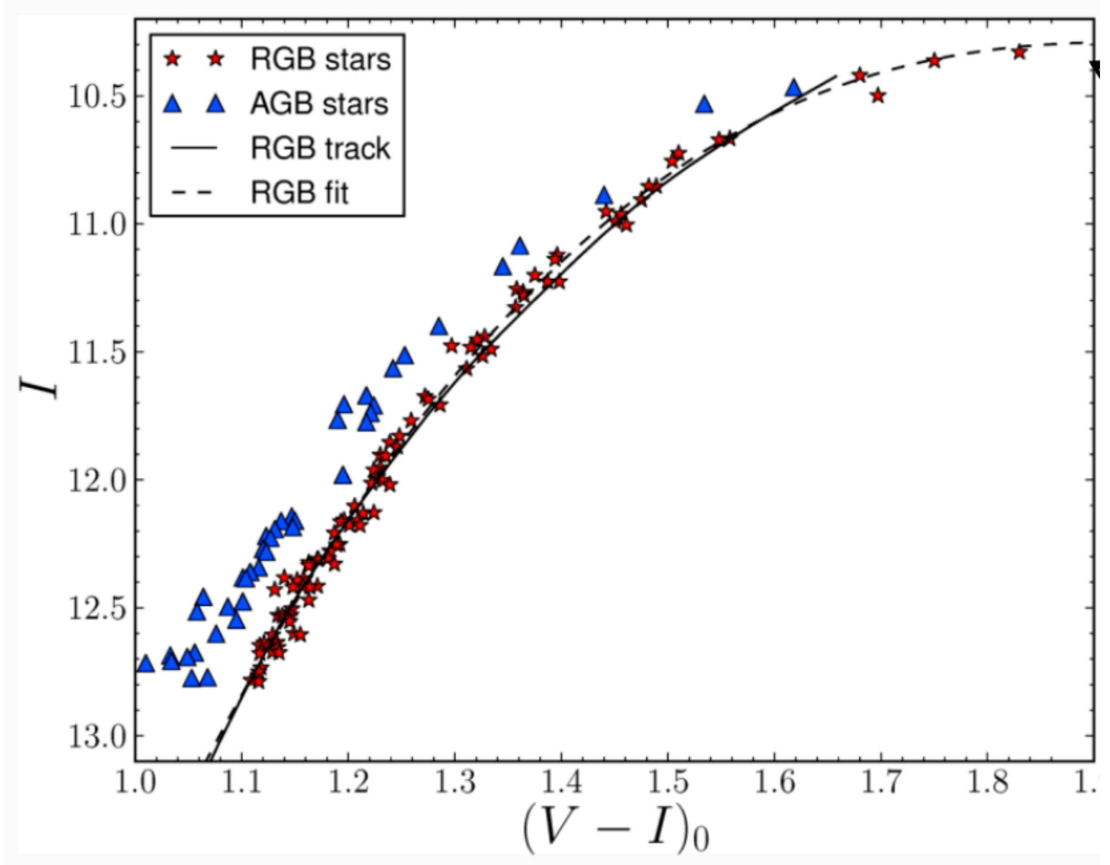


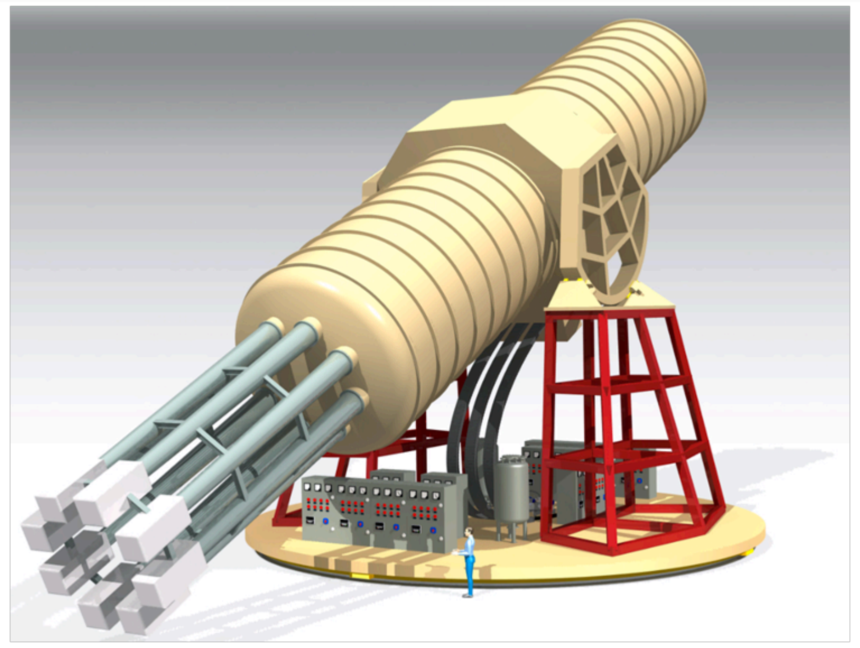
Experimental searches for Axion-Like Particles axion

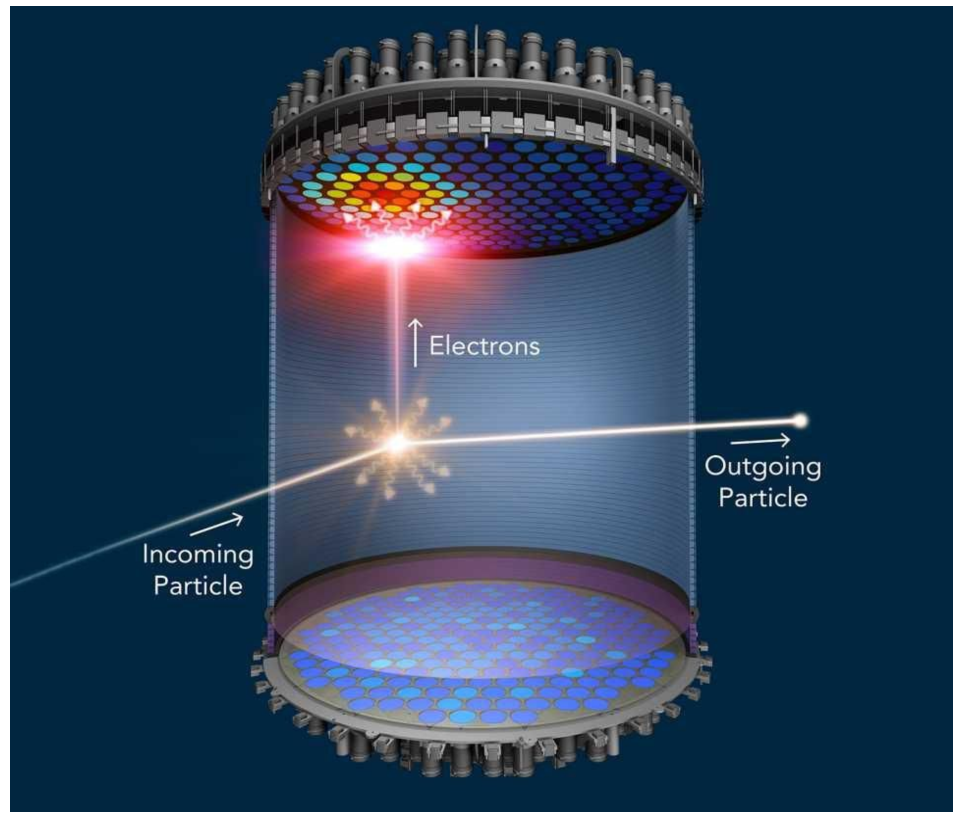
Methodology:

- Dark Matter Axion: haloscopes ..
- Axion independent searches:
  - Rare meson decays
  - Stellar cooling
  - Supernova
  - Helioscopes: solar axion (CAST, IAXO, or DM direct detection searches)
  - Light shining through walls
  - Polarization
  - Fifth force
  - Radio wave detection



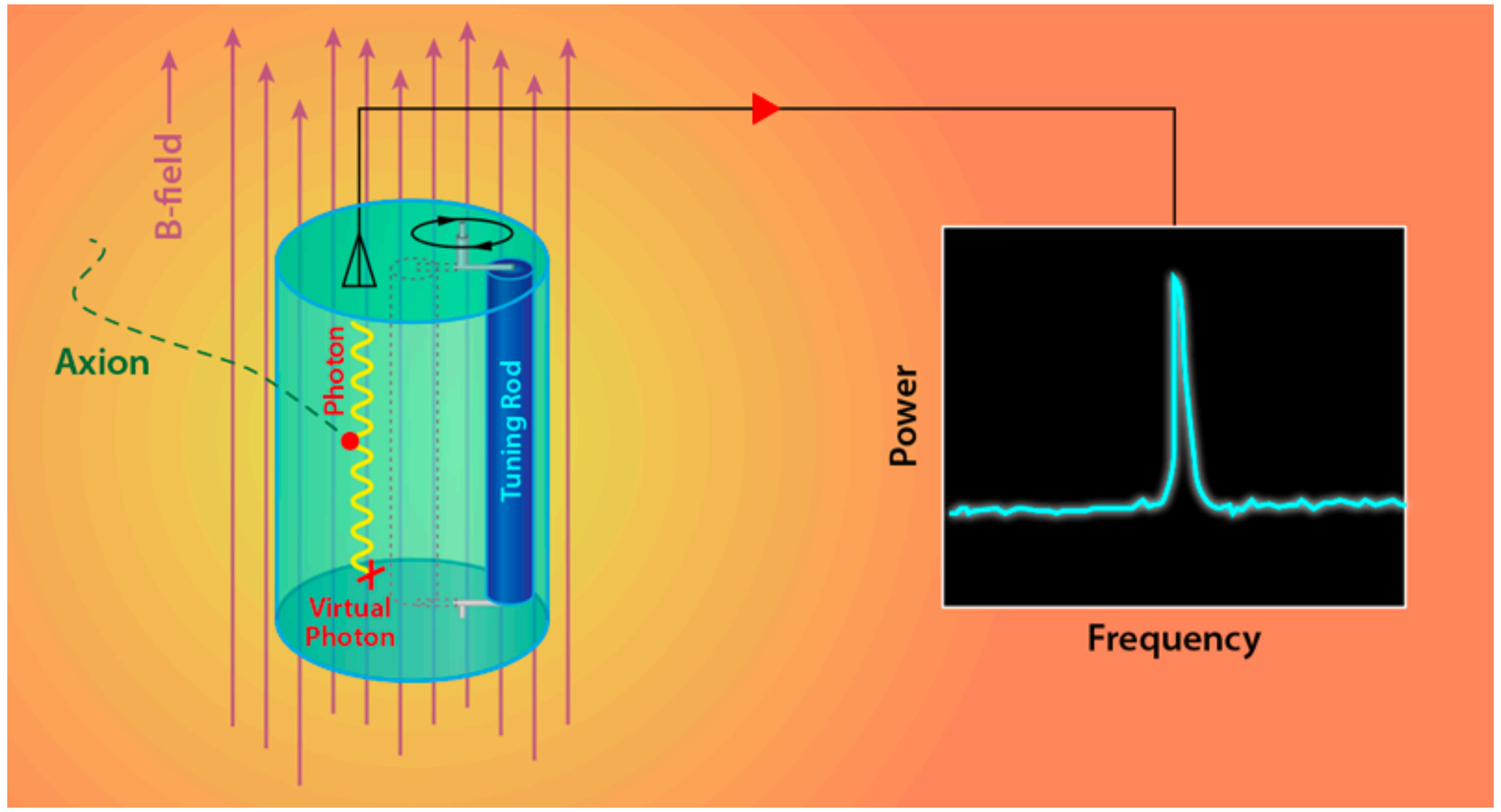






The detection of ultralight bosonic dark matter

- Mass ranges from $[10^{-22},\ 10^{3}]\ \mathrm{eV}$， DM exist as classical fields
  - Interacting feebly with SM sector, interdisciplinary collaboration with Atomic Molecular Optics, Astrophysics, Astronomy and Cosmology
  - Various detection methods:
    - Star as Laboratory：exotic energy loss （A', ALP, S)
    - Early universe CMB, Gamma ray propagation, Black Hole picture and polarization (ALP、A')
    - Lab resonant cavity searches:（ADMX, HAYSTAC …）(ALP, A')
    - Lab broad-band searches: (WISPDMX, Dark E-field) (ALP, A')
    - 5th force, Equivalent Principle test （S, A')
    - DM direct detection experiments (XENONnT, PANDAX-4T, CDEX) (ALP, A')
    - Radio astronomy (ALP, A')



Experimental searches is related to model and couplings

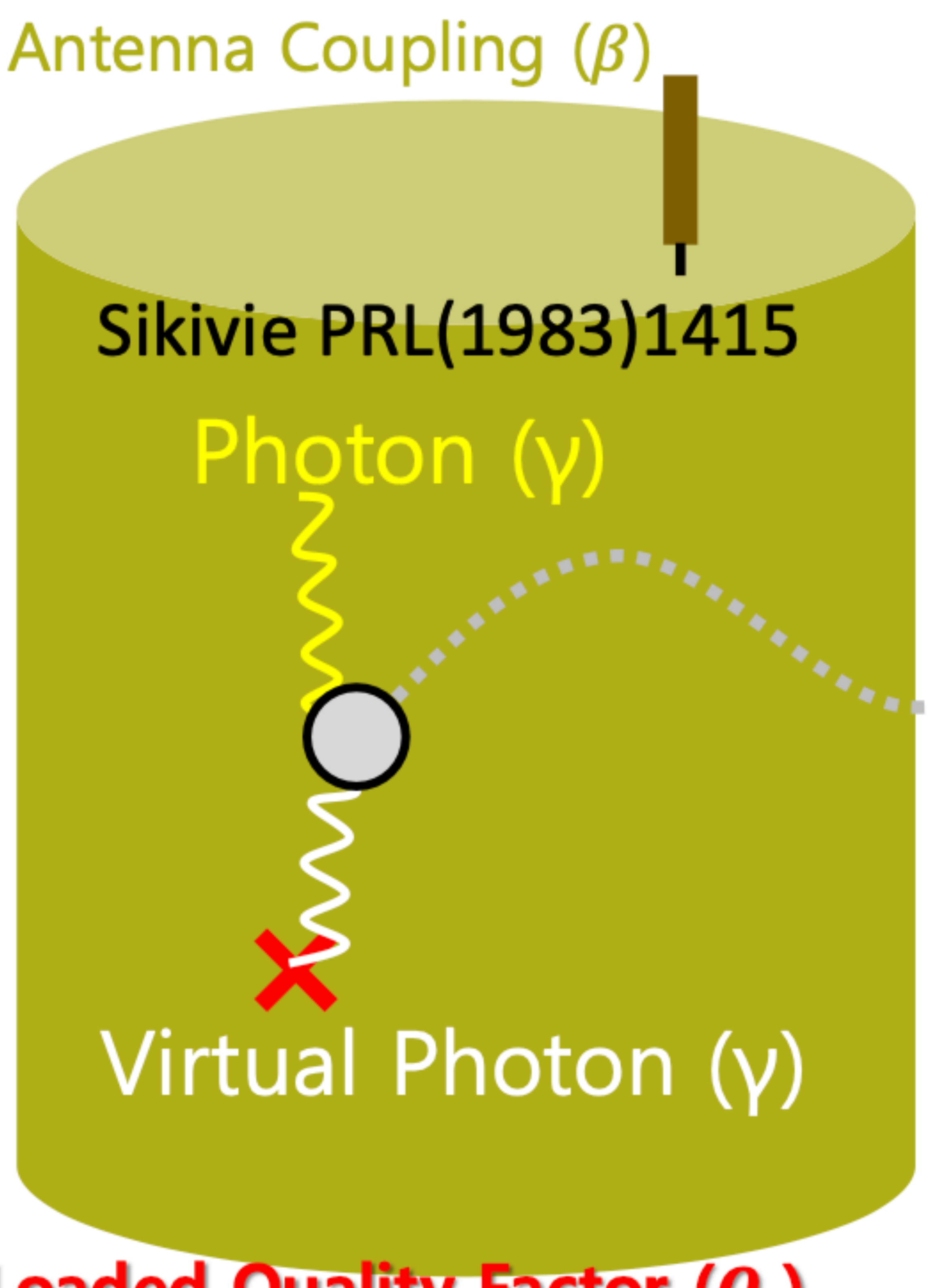
$$g_{a\gamma\gamma} a F_{\mu\nu} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \sim g_{a\gamma\gamma} a \vec{E} \cdot \vec{B}$$

The resonant searches for ALP via photon coupling

- Tuning cavity resonant frequency to match axion mass

$$g_{a\gamma\gamma} a F_{\mu\nu} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \sim g_{a\gamma\gamma} a \vec{E} \cdot \vec{B}$$

From Danho Ahn@Patras2023



Loaded Quality Factor (Q_l)
Volume (V)
Resonant Angular Frequency (ω_0)
Form Factor (C)
System Temperature (T_{sys})

Signal Power (Coupling Constant, Dark Matter Axion Density, Axion Mass, Axion Quality Factor)

$$P_{sig} = \frac{\beta}{1+\beta} g_{a\gamma\gamma}^2 \frac{\rho_a}{m_a^2} B^2 V \omega_0 C \frac{Q_a Q_l}{Q_a + Q_l}$$

Kim *et al.* JCAP03(2020)066

Scan Rate

$$\frac{df}{dt} \propto \frac{B^4 V^2 C^2}{k_B^2 T_{sys}^2} Q_l Q_a$$

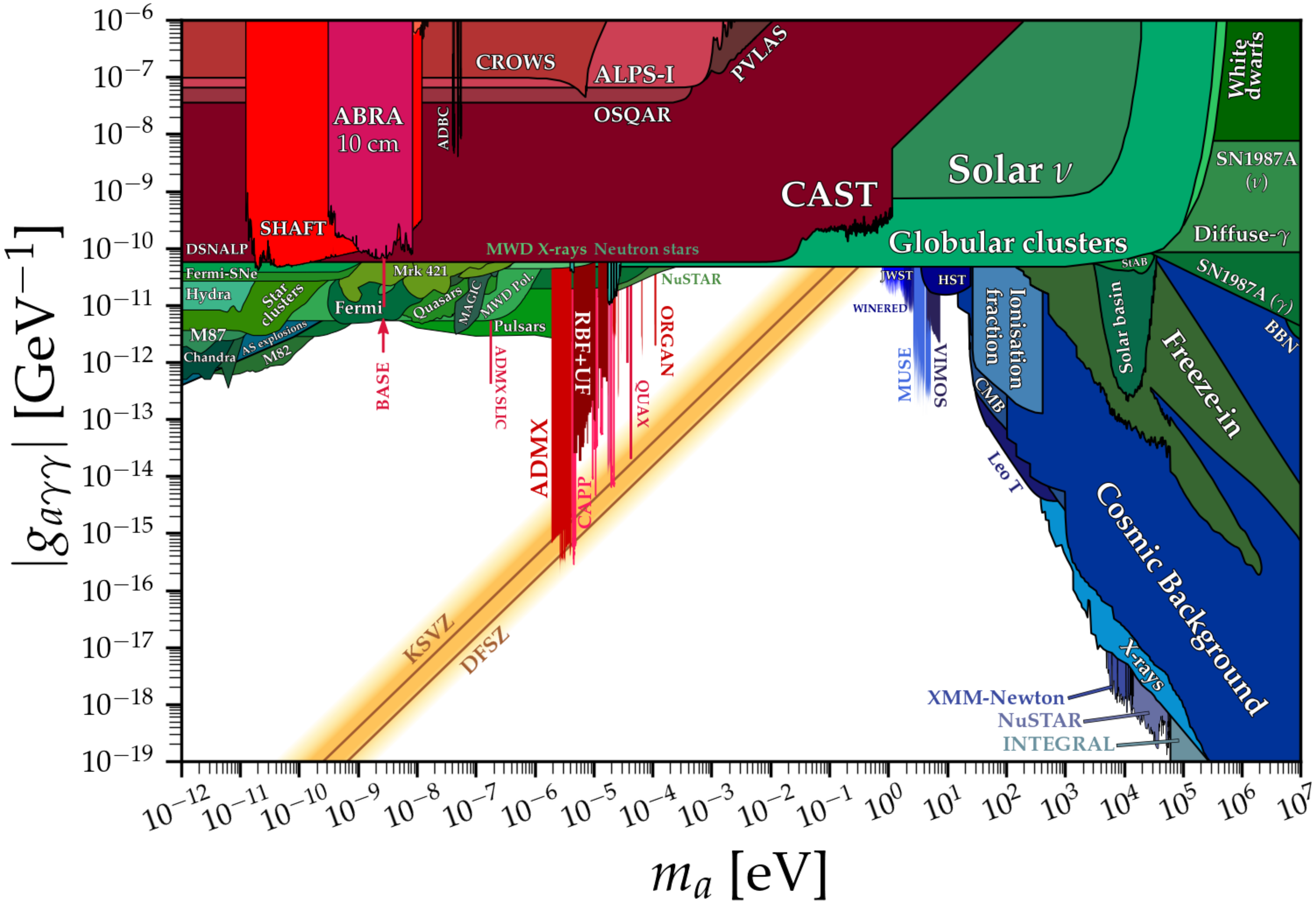
System Noise Temperature ~ 200 mK

$Q_l \gg Q_a \sim 10^6$

Refer to Session 02, Thu, Dr. Jinsu Kim

The resonant searches for ALP via photon coupling

- The overview of ALP-photon coupling searches
- Very competitive research field



https://cajohare.github.io/AxionLimits/

The resonant searches for ALP via photon coupling

https://cajohare.github.io/AxionLimits/

The resonant searches of nucleon couplings

- The ALP DM field

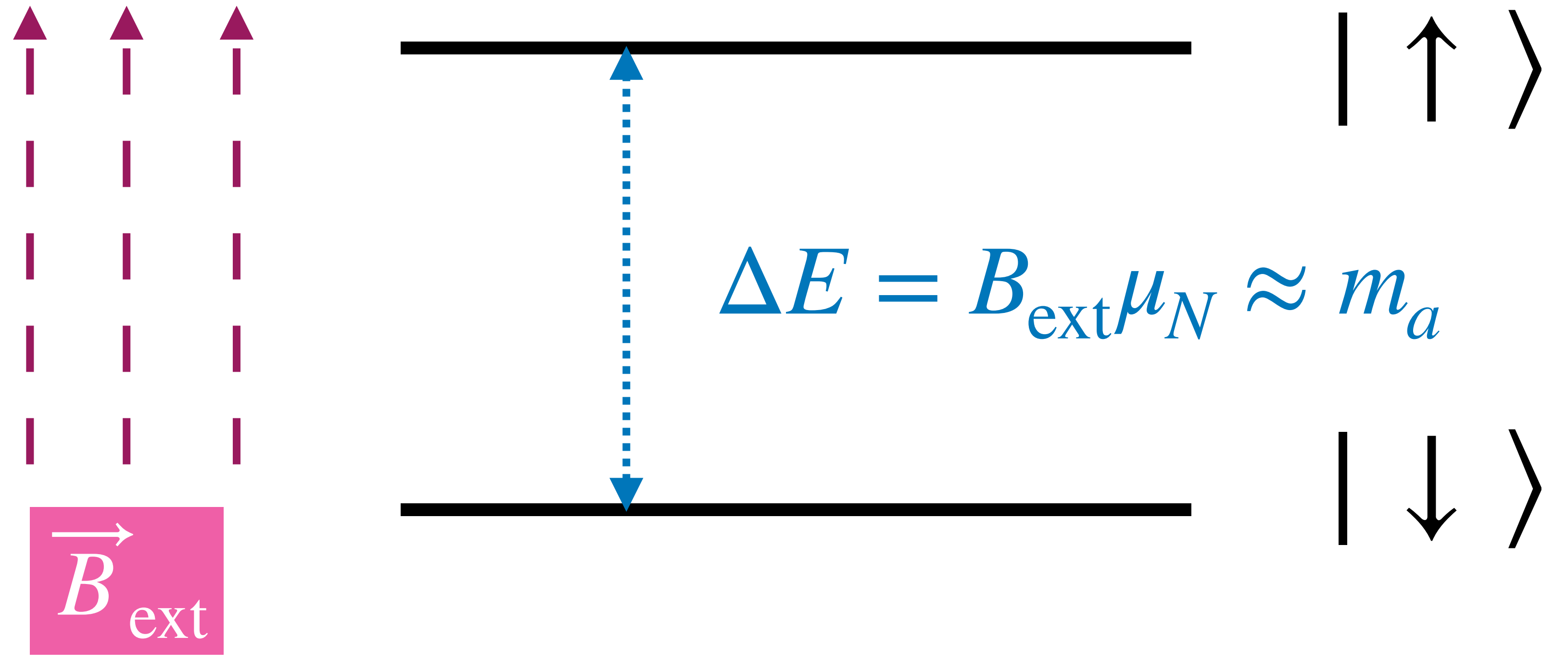
$$a(x,t) \approx a_0 \cos(\omega t - \vec{p}\cdot\vec{x} + \theta_0)$$

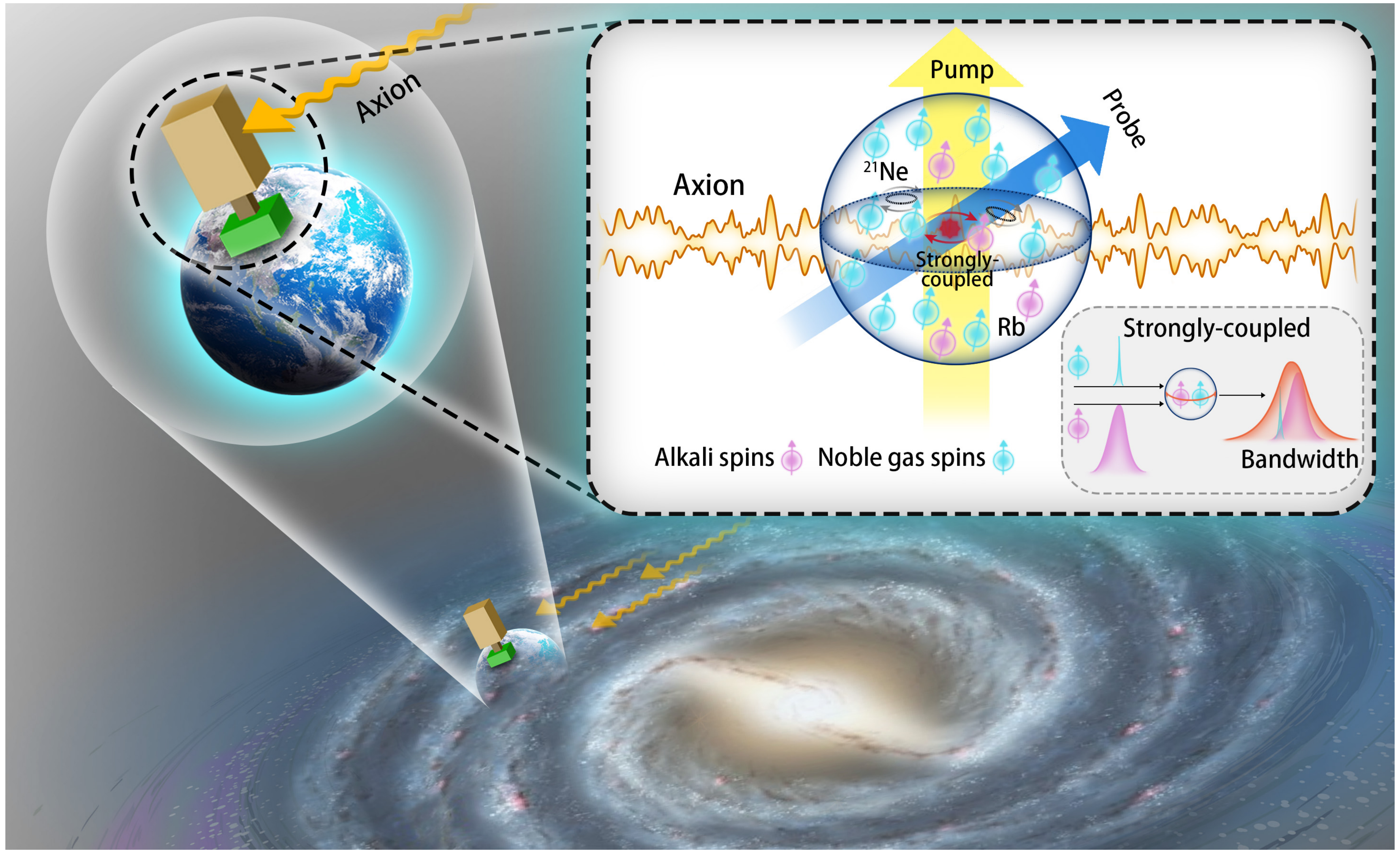
- The axion-wind Hamiltonian

$$H = g_{aNN}\frac{\partial_\mu a}{2f_a}\bar{N}\gamma^\mu\gamma_5 N = g_{aNN}\vec{\nabla} a\cdot\vec{\sigma}_N$$

$$\approx g_{aNN}\vec{v}_a\cdot\vec{\sigma}_N \times \sqrt{2\rho_a}\sin(p\cdot x)$$

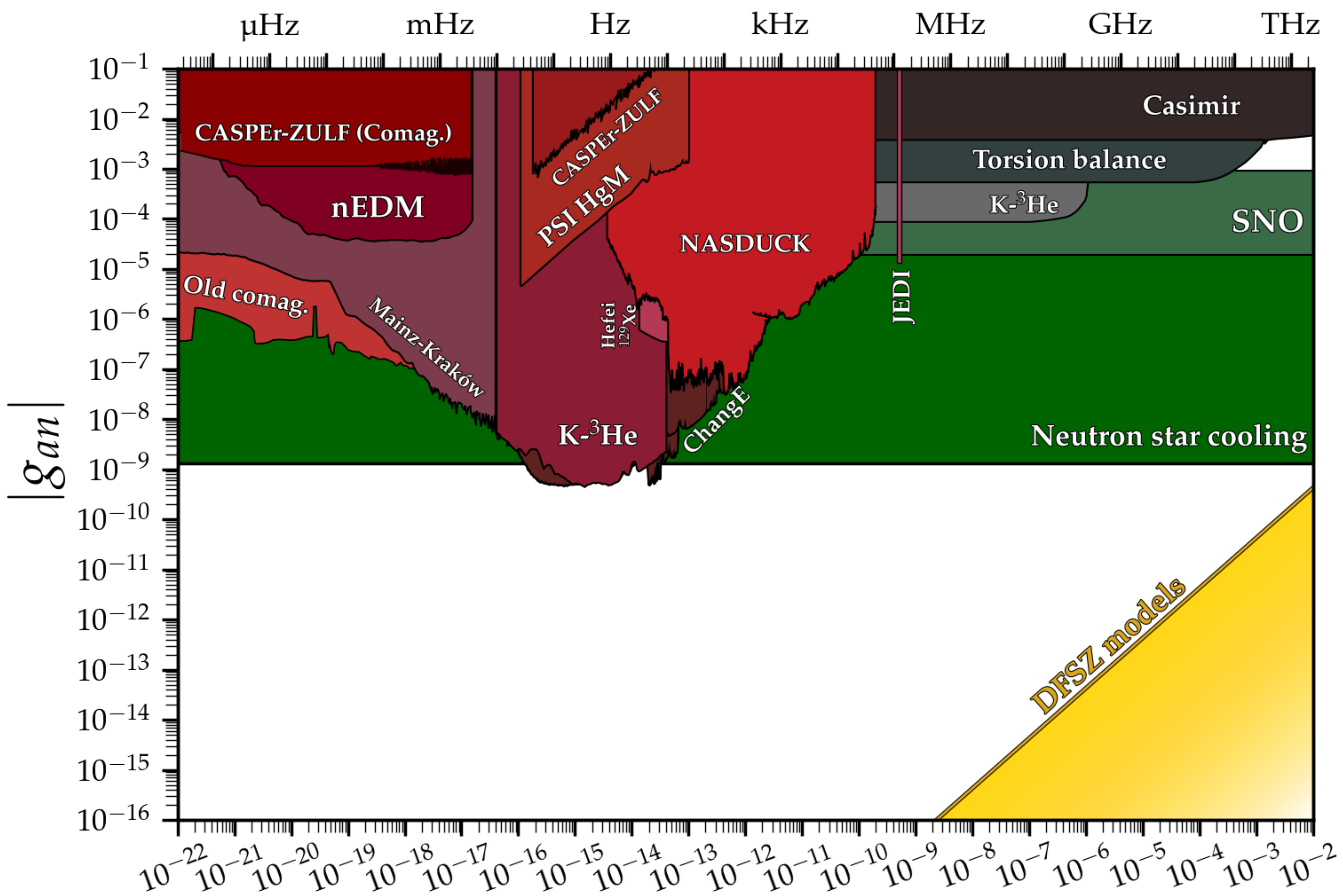
- A Zeeman split in B field

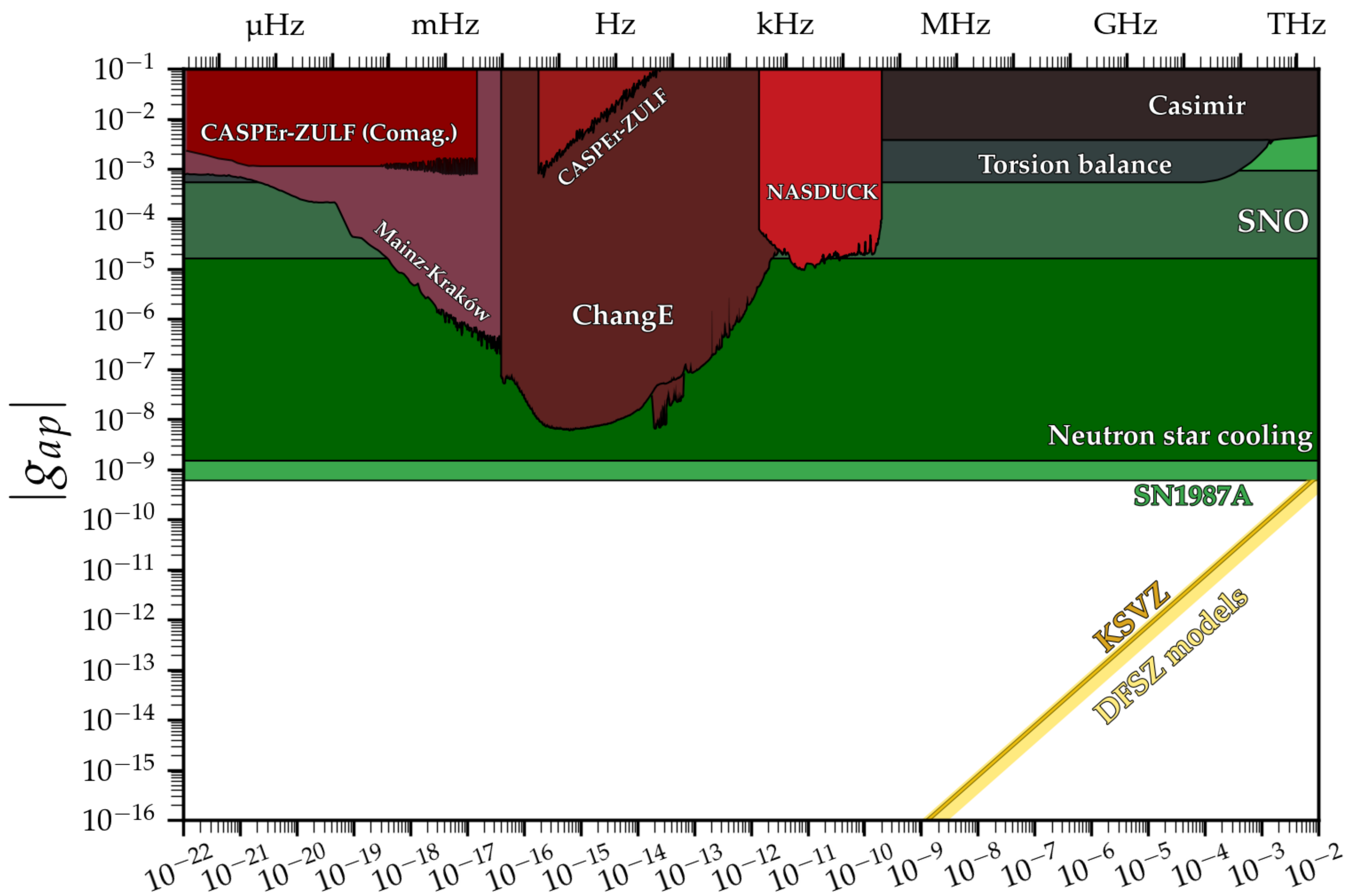




ChangE results

- ChangE experiments set competitive limits on ALP-nucleon couplings (AxionLimits version)
- Improving ALP-proton coupling limits by 10^5 - 10^6
- Provideing best limits on ALP-neutron couplings at ~[0.02, 0.2] Hz and [10, 200] Hz





Kai et al, 2306.08039; Xu et al, Commun.Phys. 7 (2024) 1, 226
Cong et al, JAIS 2024 (2024) 505

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The axion effective Lagrangian at quark-level

- A more detailed effective Lagrangian

$$\mathcal{L}_{\text{eff},0} = \bar{q}_0(iD_\mu\gamma^\mu - \mathbf{m}_{q,0})q_0 + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$
$$+ g_{ag,0}\frac{a}{f_a}G\tilde{G} + g_{a\gamma,0}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L\mathbf{k}_{L,0}\gamma^\mu q_L + \bar{q}_R\mathbf{k}_{R,0}\gamma^\mu q_R + \ldots\right)$$

Bauer et al, PRL 127 (2021), 081803

- Quark mass $\mathbf{m}_{q,0}$ diagonal and real

- Coupling to both left/right fermions $\mathbf{k}_{L,0}$ and $\mathbf{k}_{R,0}$

The axion-dependent chiral rotation

- Use an axion-dependent chiral rotation to eliminate $aG\tilde{G}$ term

Bauer et al, PRL 127 (2021), 081803

$$q_0(x) = \exp\left[-i(\boldsymbol{\delta}_{q,0} + \boldsymbol{\kappa}_{q,0}\gamma_5)c_{gg}\frac{a(x)}{f_a}\right] q(x) \qquad \mathrm{Tr}(\kappa_{q,0}) = 1$$

- New effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \bar{q}(iD_\mu\gamma^\mu - \mathbf{m}_q(a))q + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$

$$+ g_{a\gamma}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L\mathbf{k}_L(a)\gamma^\mu q_L + \bar{q}_R\mathbf{k}_R(a)\gamma^\mu q_R + \ldots\right)$$

The axion-dependent chiral rotation

- Define the chiral rotations (2-flavor for simplicity)

$$\boldsymbol{\theta}_L \equiv \boldsymbol{\delta}_{q,0} - \boldsymbol{\kappa}_{q,0} \qquad U_L \equiv \exp\left[-i\boldsymbol{\theta}_L a/f_a\right]$$

$$\boldsymbol{\theta}_R \equiv \boldsymbol{\delta}_{q,0} + \boldsymbol{\kappa}_{q,0} \qquad U_R \equiv \exp\left[-i\boldsymbol{\theta}_R a/f_a\right]$$

- The relations between parameters

$$\mathbf{m}_q(a) = U_L^\dagger \mathbf{m}_0 U_R \to \begin{pmatrix} m_{u,0}e^{-2i\kappa_{u,0}c_{gg}} & 0 \\ 0 & m_{d,0}e^{-2i\kappa_{d,0}c_{gg}} \end{pmatrix}$$

$$\mathbf{k}_L(a) = U_L^\dagger[\mathbf{k}_{L,0} + c_{gg}\boldsymbol{\theta}_{L,0}]U_L \to \mathbf{k}_{L,0} + c_{gg}\boldsymbol{\theta}_{L,0}$$

$$\mathbf{k}_R(a) = U_R^\dagger[\mathbf{k}_{R,0} + c_{gg}\boldsymbol{\theta}_{R,0}]U_R \to \mathbf{k}_{R,0} + c_{gg}\boldsymbol{\theta}_{R,0}$$

Anomalous axion contribution

$$g_{a\gamma} = g_{a\gamma_0} - 2N_c c_{gg}\mathrm{Tr}\left[\mathbf{Q}^2\boldsymbol{\kappa}_{q,0}\right]$$

The consistent ChPT axion Lagrangian

$$\mathscr{L}_{\text{eff}} = \bar{q}(iD_\mu\gamma^\mu - \mathbf{m}_q(a))q + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$

$$+ g_{a\gamma}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L\mathbf{k}_L(a)\gamma^\mu q_L + \bar{q}_R\mathbf{k}_R(a)\gamma^\mu q_R + \ldots\right)$$

- ChPT Lagrangian matching

$$U = \exp[(\sqrt{2}i/f_\pi)\pi^a\boldsymbol{\tau}^a]$$

$$\mathscr{L}_{\chi\text{PT}} = \frac{f_\pi^2}{8}\left[(D^\mu U)(D_\mu U)^\dagger\right] + \frac{f_\pi^2}{4}B_0\text{Tr}\left[\mathbf{m}_q(a)U^\dagger + h.c.\right] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + g_{a\gamma}\frac{a}{f_a}F\tilde{F}$$

- The axion derivative coupling

Bauer et al, PRL 127 (2021), 081803

$$D^\mu U \to D^\mu U - i\frac{\partial^\mu a}{f_a}\left(\mathbf{k}_L U - U\mathbf{k}_R\right)$$

The importance of consistency

- The physical results should be independent of auxiliary parameters

$$q_0(x) = \exp\left[-i(\boldsymbol{\delta}_{q,0} + \boldsymbol{\kappa}_{q,0}\gamma_5)c_{gg}\frac{a(x)}{f_a}\right] q(x)$$

- The most important channel BR($K \to \pi a$) is off by a factor of 37 for 35 years

H. Georgi, D. B. Kaplan and L. Randall, Phys. Lett. B 169, 73-78 (1986)

- Model-independent expression for $K \to \pi a$ and $\pi^- \to e^-\bar{\nu}_e a$ have been obtained for all axion couplings

Bauer et al, PRL 127 (2021), 081803

Axion couplings to other mesons/baryons/EFT

- Axion couplings to other mesons , e.g. η, η' etc

  Gao, Guo, Oller, Zhou JHEP04(2023)022; Wang, Guo, Zhou PRD 109(2024)075030; Wang, Guo Lu, Zhou 2403.16064; Cao, Guo, 2408.15825

- Axion couplings to baryons

  Vonk, Guo, Meissner JHEP03(2020)138, Lu, Du, Guo, Meissner, Vonk JHEP05(2020)001, Vonk Guo, Meissner JHEP08(2021)024 …

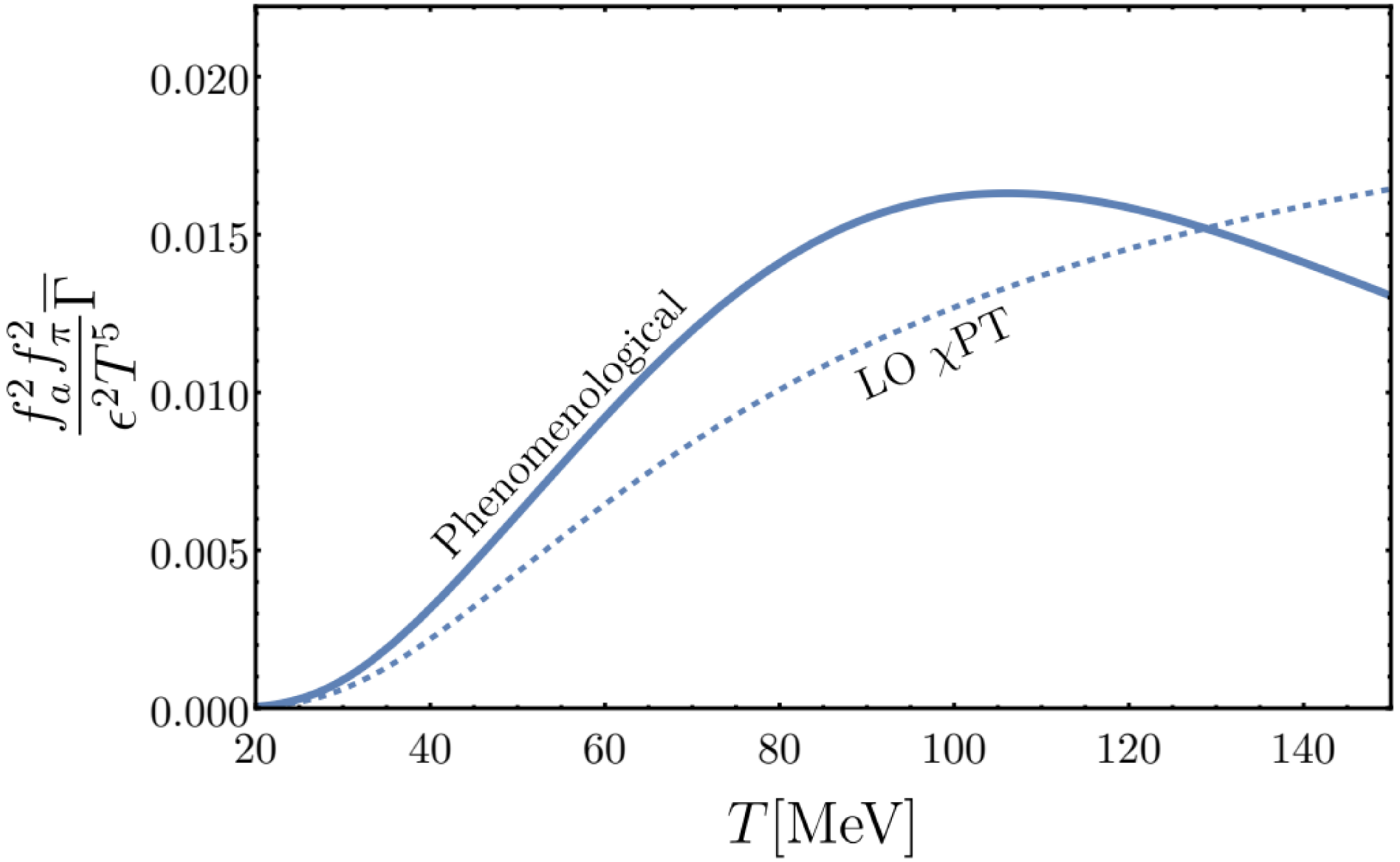
- Axion EFT

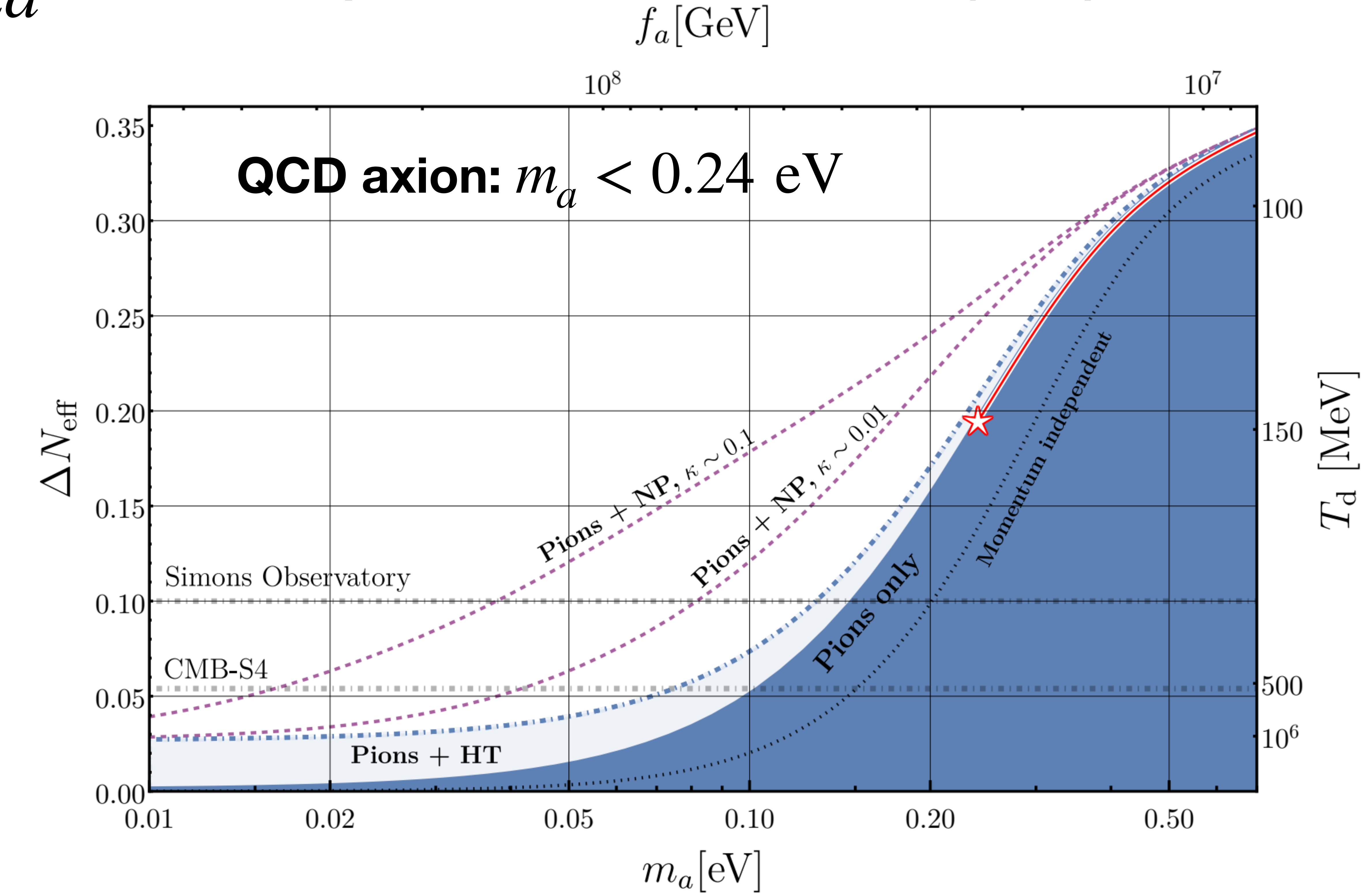
  Hu, Jiang, Li, Xiao, Yu, PRD 103(2021)095025; Song, Sun, Yu JHEP01(2024)161

- …

Why accurate interactions are important?

- 1. Prediction for thermal axion and its near future test by CMB observation
  - Thermal axion production (high T): $q\bar{q} \to ga, qg \to qa$ *Ferreira, Notari PRL 120 (2018)191301*
  - QCD phase transition: *D'Eramo, Hajkarim, Yun PRL 128 (2022)152001*
  - Improved axion-pion scattering production: $\pi\pi \leftrightarrow \pi a$ *Notari, Rompineve, Villladoro PRL 131 (2023)011004*

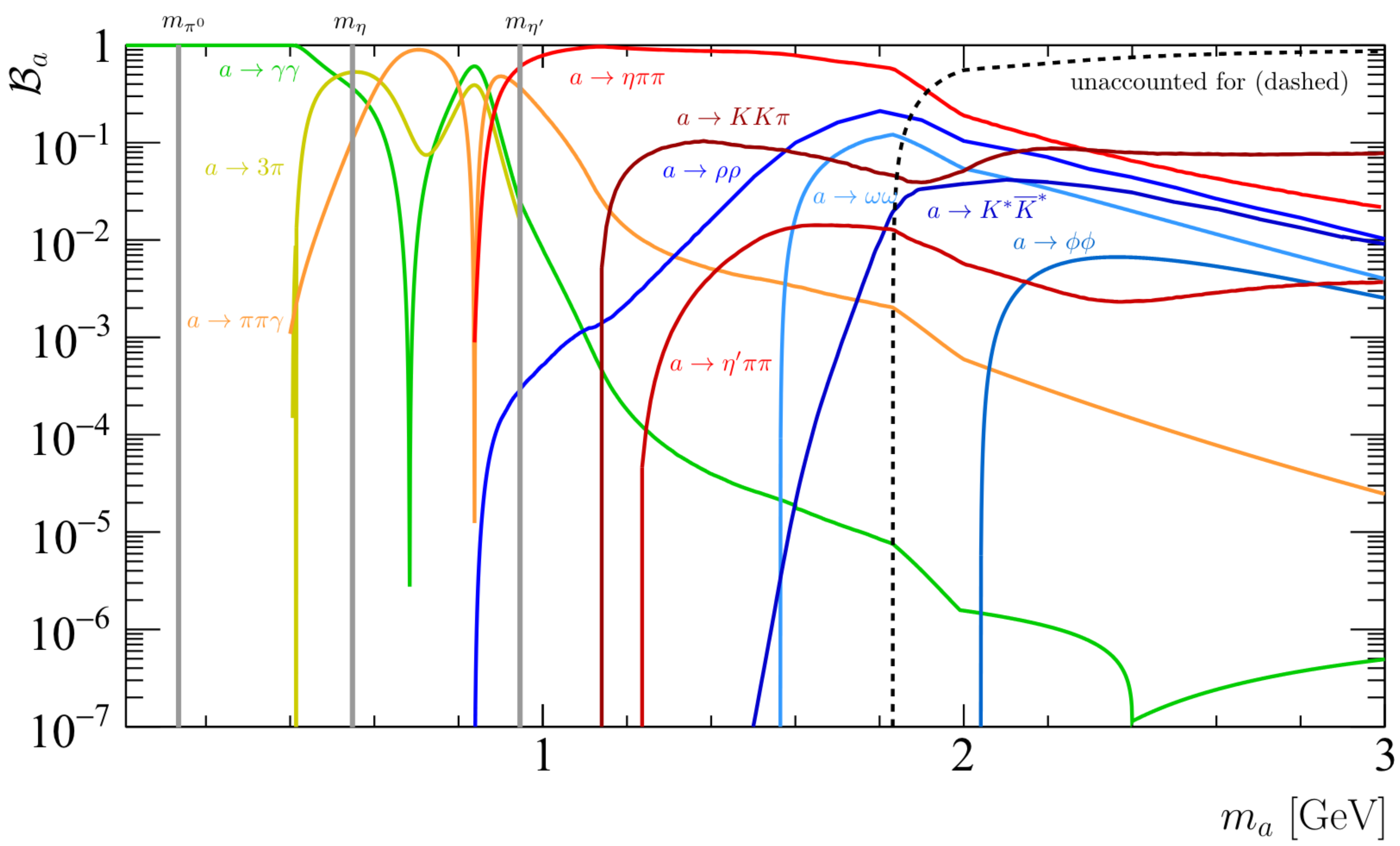


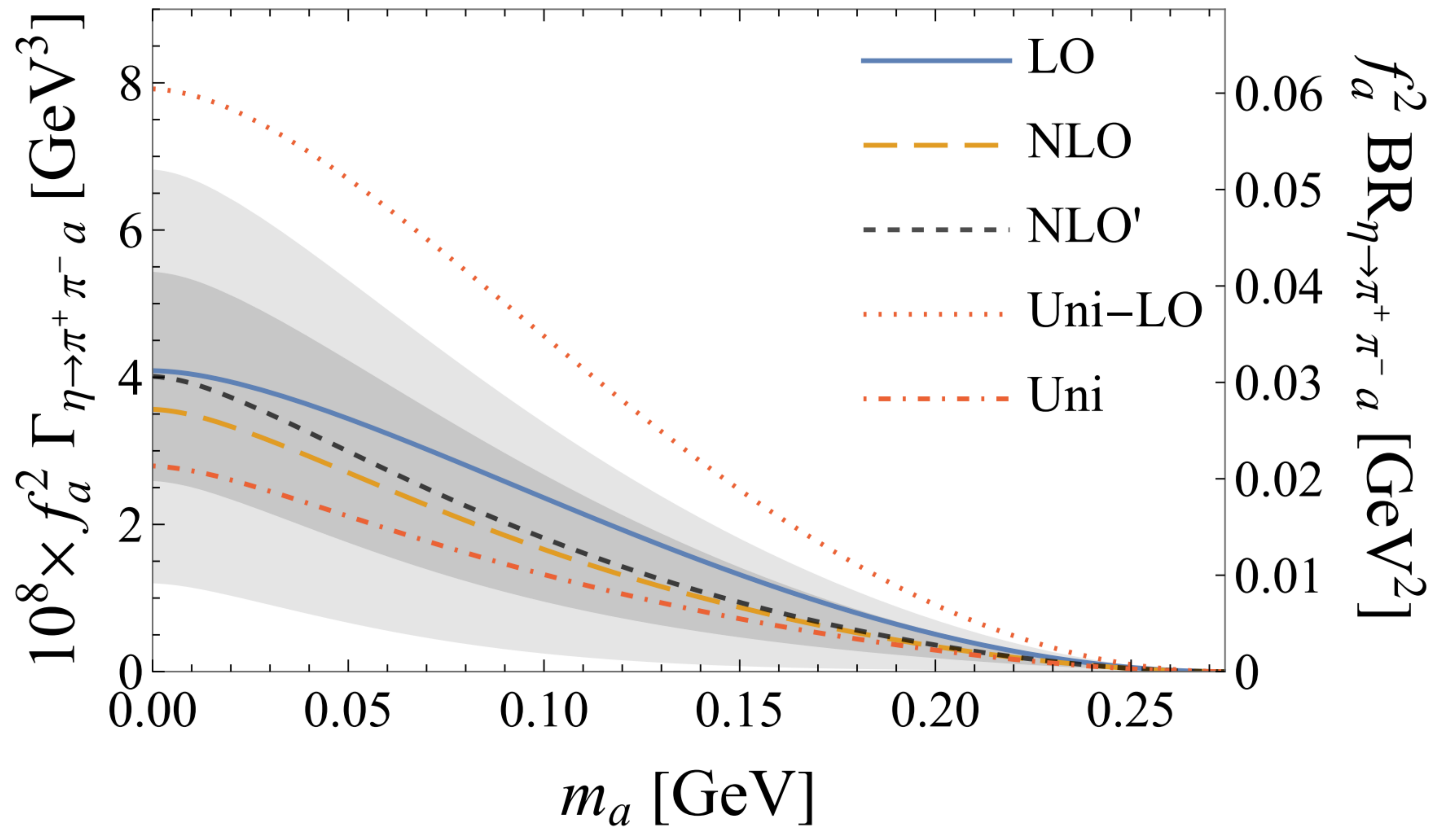


Why accurate interactions are important?

- 2. Axion related exotic decay width and BR
  - Axion decay BR: *Aloni, Soreq, Williams PRL 123 (2019) 031803*
  - Other meson decays to Axion: $K \to \pi a, \eta \to \pi\pi a$ etc…

Bauer et al, PRL 127 (2021), 081803
Wang, Guo Lu, Zhou 2403.16064



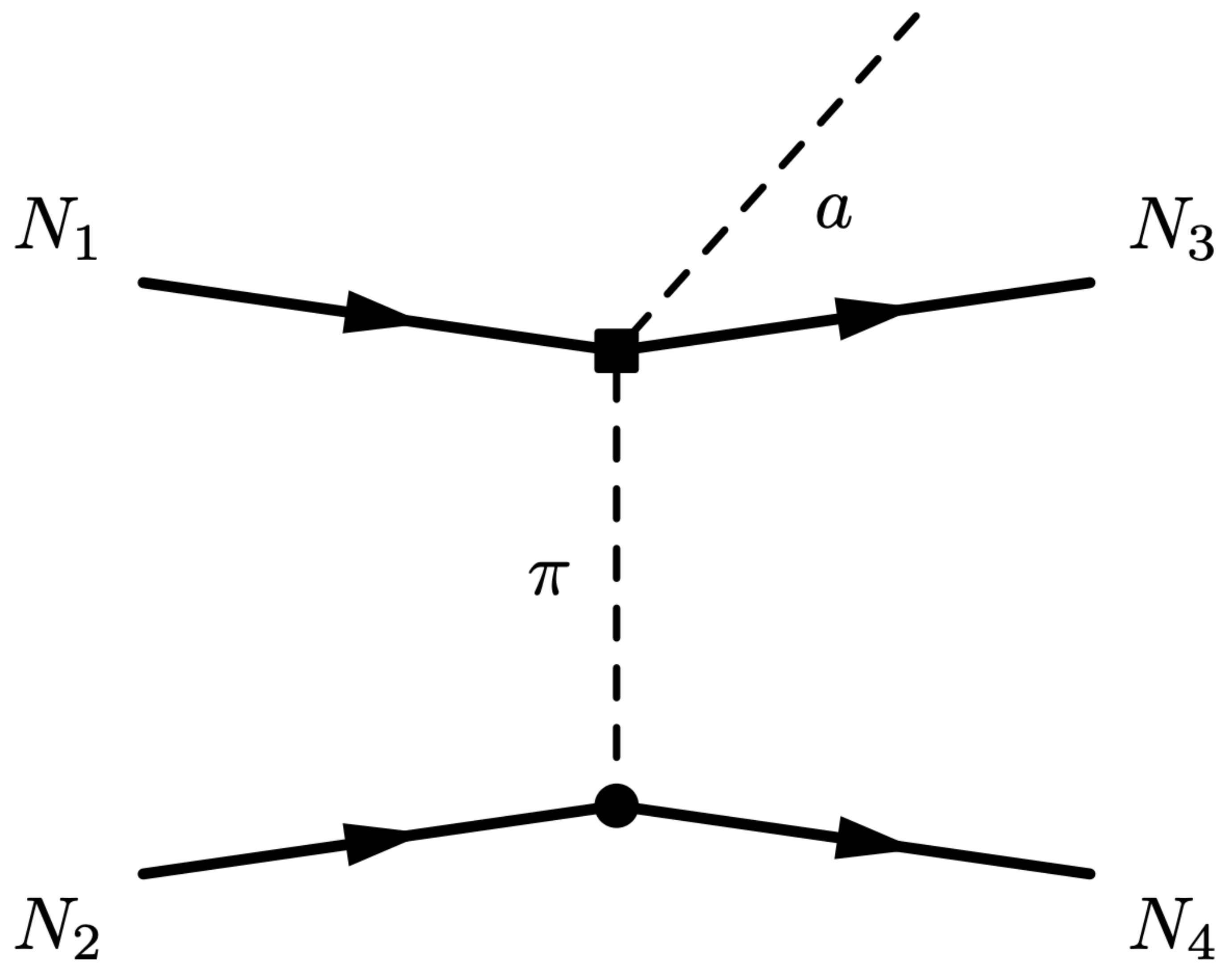


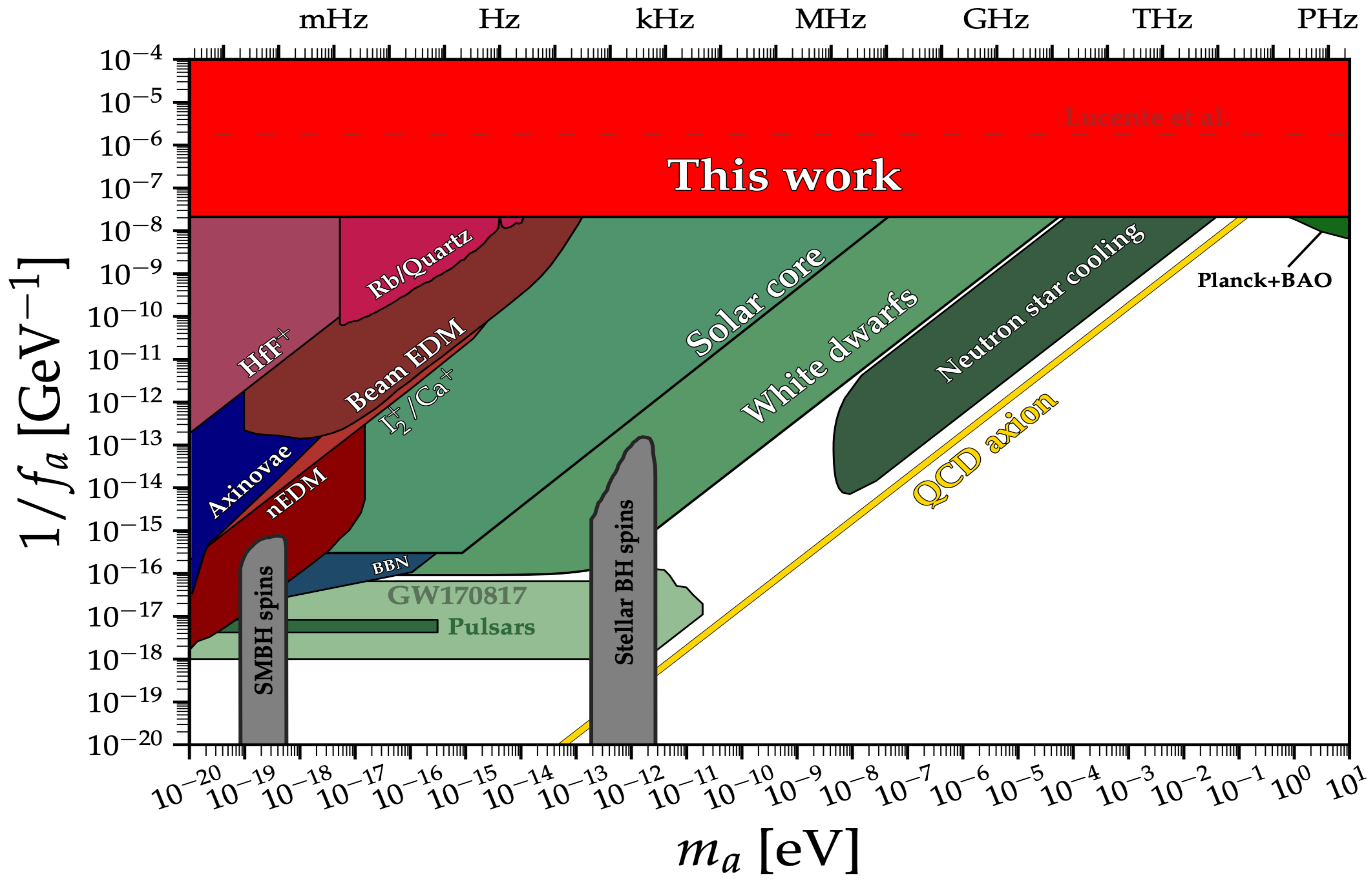
Why accurate interactions are important?

- 3. Astrophysical bounds for axions
  - Supernova constraints on axion coupling f_a:
  - Isospin breaking operator at NLO improves bound by two orders?

Springmann, Stadlbauer, Stelzl, Weiler 2410.19902

$$\mathcal{L}_{\pi N}^{(2)} \supset -\hat{c}_5 m_\pi^2 \frac{4z}{(1+z)^2} \bar{N} \left(\frac{\pi^a a}{f_\pi f_a} \right) \tau^a N,$$





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Wess-Zumino-Witten Interactions in QCD

- Describing anomalies in QCD
- Ensuring gauge invariance and completing chiral L
- Low-energy dynamics of mesons
  e.g. multiple mesons and photons interactions, $\pi_0 \to \gamma\gamma$

$$\Gamma_{WZW}(U,\mathcal{A}_L,\mathcal{A}_R) = \Gamma_0(U) \; + \mathcal{C}\int \mathrm{Tr}\Big\{(\mathcal{A}_L\alpha^3 + \mathcal{A}_R\beta^3) - \frac{i}{2}[(\mathcal{A}_L\alpha)^2 - (\mathcal{A}_R\beta)^2]$$
$$+ i(\mathcal{A}_L U\mathcal{A}_R U^\dagger \alpha^2 - \mathcal{A}_R U^\dagger \mathcal{A}_L U\beta^2) + i(d\mathcal{A}_R dU^\dagger \mathcal{A}_L U - d\mathcal{A}_L dU\mathcal{A}_R U^\dagger)$$
$$+ i\big[(d\mathcal{A}_L\mathcal{A}_L + \mathcal{A}_L d\mathcal{A}_L)\alpha + (d\mathcal{A}_R\mathcal{A}_R + \mathcal{A}_R d\mathcal{A}_R)\beta\big] + (\mathcal{A}_L^3\alpha + \mathcal{A}_R^3\beta)$$
$$- (d\mathcal{A}_L\mathcal{A}_L + \mathcal{A}_L d\mathcal{A}_L)U\mathcal{A}_R U^\dagger + (d\mathcal{A}_R\mathcal{A}_R + \mathcal{A}_R d\mathcal{A}_R)U^\dagger \mathcal{A}_L U$$
$$+ (\mathcal{A}_L U\mathcal{A}_R U^\dagger \mathcal{A}_L\alpha + \mathcal{A}_R U^\dagger \mathcal{A}_L U\mathcal{A}_R\beta) + i\left[\mathcal{A}_L^3 U\mathcal{A}_R U^\dagger - \mathcal{A}_R^3 U^\dagger \mathcal{A}_L U - \frac{1}{2}(U\mathcal{A}_R U^\dagger \mathcal{A}_L)^2\right]\Big\}.$$

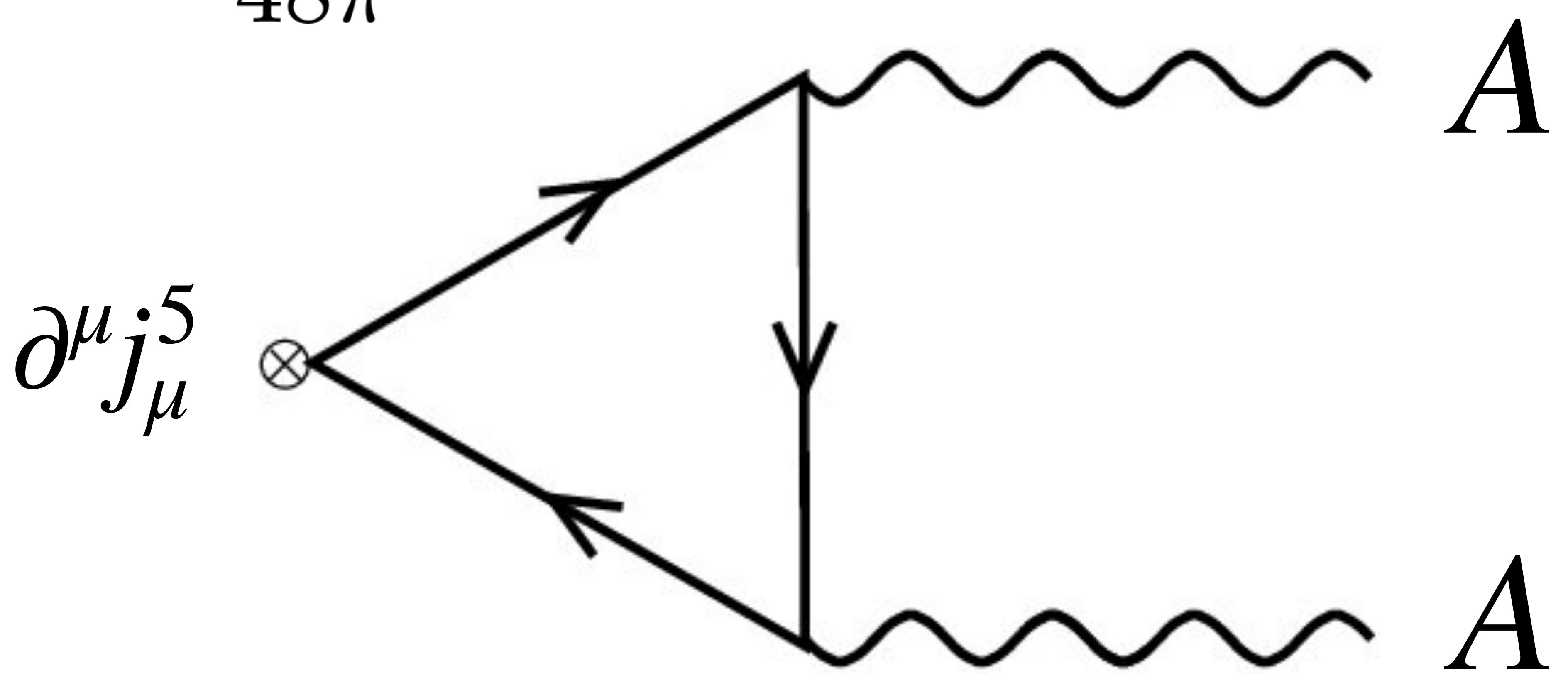
$$\alpha = dUU^\dagger$$
$$\beta = U^\dagger dU$$

$$\Gamma_0(U) = -\frac{i\mathcal{C}}{5}\int_{M^5} \mathrm{Tr}\left(\alpha^5\right) = \frac{iN_c}{240\pi^2}\int d^5x\, \epsilon^{ABCDE}\; \mathrm{Tr}\left(\alpha_A\alpha_B\alpha_C\alpha_D\alpha_E\right),$$

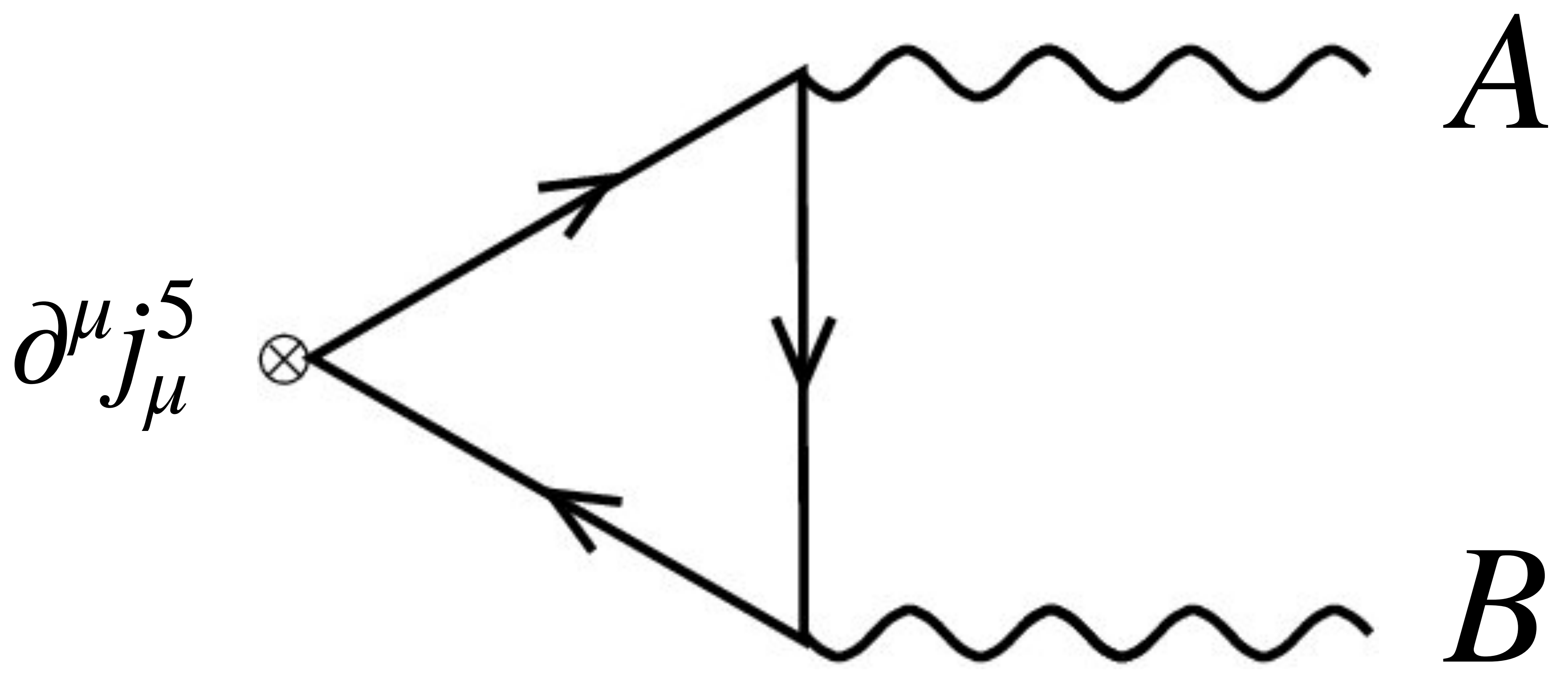
Global currents and background vector fields

- Background fields can couple to currents of $\mathscr{L}_{\chi\mathrm{PT}}$
  - Baryon currents $U(1)_B$ in neutron star, ω meson
  - Z boson vector in neutrino dense environment
- SM gauge invariance needs counter terms

$$\partial^\mu j^5_\mu = \frac{1}{48\pi^2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$



$$\delta\partial^\mu j^5_\mu \propto \epsilon_{\mu\nu\rho\sigma}A^{\mu\nu}B^{\rho\sigma}$$



WZW counter terms for global symmetry

J. A. Harvey, C. T. Hill, and R. J. Hill,
PRL 99 (2007) 261601,
PRD 77(2008) 085017

- Generic WZW interactions with counter terms
  - Vector fields in 1-form: $\mathscr{A}_{L/R} \equiv \mathbb{A}_{L/R} + \mathbb{B}_{L/R}$
    Similar to Hidden Local Symmetry

$$\mathscr{L}^{\text{full}}_{\text{WZW}}(U, \mathscr{A}_{L/R}) = \mathscr{L}_{\text{WZW}}(U, \mathscr{A}_L, \mathscr{A}_R) + \mathscr{L}_c(\mathbb{A}_{L/R}, \mathbb{B}_{L/R})$$

- Counter terms ensures SM invariance

$$\Gamma_c = -2\mathscr{C} \int Tr\Big[(\mathbb{A}_L d\mathbb{A}_L + d\mathbb{A}_L \mathbb{A}_L)\mathbb{B}_L + \frac{1}{2}\mathbb{A}_L(\mathbb{B}_L d\mathbb{B}_L + d\mathbb{B}_L \mathbb{B}_L) - \frac{3}{2} i \mathbb{A}_L^3 \mathbb{B}_L - \frac{3}{4} i \mathbb{A}_L \mathbb{B}_L \mathbb{A}_L \mathbb{B}_L - \frac{i}{2} \mathbb{A}_L \mathbb{B}_L^3\Big] - (L \leftrightarrow R)$$

- Suitable for chiral gauge fields and background fields

Axion treatment as a fictitious background field

$$\mathscr{L}_{\text{eff}} = \bar{q}(iD_\mu\gamma^\mu - \mathbf{m}_q(a))q + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$

$$+g_{a\gamma}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L\mathbf{k}_L(a)\gamma^\mu q_L + \bar{q}_R\mathbf{k}_R(a)\gamma^\mu q_R + \ldots\right)$$

Yang Bai, Ting-Kuo Chen, JL, Xiaolin Ma
2406.11948

- $D_\mu = \partial_\mu - ig(A_L P_L + A_R P_R)$
- Hints from quark-level L: $D_\mu \to D_\mu + i\dfrac{\partial_\mu a}{f_a}\left(\mathbf{k}_L P_L + \mathbf{k}_R P_R\right)$
- Hints from ChPT L: $D^\mu U \to D^\mu U - i\dfrac{\partial^\mu a}{f_a}\left(\mathbf{k}_L U - U\mathbf{k}_R\right)$

$$\mathscr{L}_{\chi\text{PT}} = \frac{f_\pi^2}{8}\left[(D^\mu U)(D_\mu U)^\dagger\right] + \frac{f_\pi^2}{4}B_0\text{Tr}\left[\mathbf{m}_q(a)U^\dagger + h.c.\right] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + g_{a\gamma}\frac{a}{f_a}F\tilde{F}$$

Axion treatment as a fictitious background field

- Vector fields in 1-form: $\mathscr{A}_{L/R} \equiv \mathbb{A}_{L/R} + \mathbb{B}_{L/R}$
  Similar to Hidden Local Symmetry

- Axion 1-form field can be added into background fields:

$$\mathbb{B}_{L/R} \to \mathbb{B}_{L/R} + \mathbf{k}_{L/R,0} \frac{da}{f_a}$$

- 2-flavor ChPT with SM gauge bosons and background fields

$$\mathbb{A}_L = \frac{e}{s_\mathrm{w}} W^a \frac{\boldsymbol{\tau}^a}{2} + \frac{e}{c_\mathrm{w}} W^0 \mathbf{Y}_Q, \; \mathbb{A}_R = \frac{e}{c_\mathrm{w}} W^0 \mathbf{Y}_q$$

$$\mathbb{B}_V \equiv \mathbb{B}_L + \mathbb{B}_R = g \begin{pmatrix} \rho_0 & \sqrt{2}\rho^+ \\ \sqrt{2}\rho^- & -\rho_0 \end{pmatrix} + g' \begin{pmatrix} \omega & \\ & \omega \end{pmatrix} + (\mathbf{k}_{L,0} + \mathbf{k}_{R,0}) \frac{da}{f}$$

$$\mathbb{B}_A \equiv \mathbb{B}_L - \mathbb{B}_R = g \begin{pmatrix} a_1 & \sqrt{2}a^+ \\ \sqrt{2}a^- & -a_1 \end{pmatrix} + g' \begin{pmatrix} f_1 & \\ & f_1 \end{pmatrix} + (\mathbf{k}_{L,0} - \mathbf{k}_{R,0}) \frac{da}{f}$$

The consistent axion Lagrangian at low energy

- ChPT:

$$\mathscr{L}_{\chi\mathrm{PT}} = \frac{f_\pi^2}{8}\mathrm{Tr}\left[(D^\mu U)(D_\mu U)^\dagger\right] + \frac{f_\pi^2}{4}B_0\,\mathrm{Tr}\left[\mathbf{m}_q(a)U^\dagger + h.c.\right] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + \frac{a}{f}\sum_{\mathscr{A}_{1,2}} c_{\mathscr{A}_1\mathscr{A}_2} F_{\mathscr{A}_1\,\mu\nu}\widetilde{F}^{\mu\nu}_{\mathscr{A}_2}$$

- Full WZW: $\mathscr{L}^{\mathrm{full}}_{\mathrm{WZW}}(U, \mathscr{A}_{L/R}) = \mathscr{L}_{\mathrm{WZW}}(U, \mathscr{A}_L, \mathscr{A}_R) + \mathscr{L}_c(\mathbb{A}_{L/R}, \mathbb{B}_{L/R})$

- Full $\mathscr{L}$: $\mathscr{L}^{\mathrm{full}}_{\mathrm{axion}} \equiv \left[\mathscr{L}_{\chi\mathrm{PT}} + \mathscr{L}^{\mathrm{full}}_{\mathrm{WZW}}\right]\left(U, \mathbf{m}_q(a), \mathscr{A}_{L/R} + \mathbf{k}_{L/R}(a)da/f_a\right)$

Matching between $\mathscr{L}_{\rm eff}$ and $\mathscr{L}_{\rm axion}^{\rm full}$

$$\mathscr{L}_{\rm eff,\,0}(q_0, \mathbf{m}_{q,0}, \mathbf{k}_{L,0}, \mathbf{k}_{R,0})$$

$$q_0 = \exp\left(-i\, c_{gg}\, \boldsymbol{\kappa}_{q,0}\, \gamma_5 \frac{a}{f}\right) q$$

$$\mathscr{L}_{\rm eff}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R)$$

$$q' = \exp\left[i\left(\boldsymbol{\delta}_q + \boldsymbol{\kappa}_q \gamma_5\right)\frac{a}{f}\right] q$$

$$\mathscr{L}_{\rm eff}(q', \mathbf{m}'_q, \mathbf{k}'_L, \mathbf{k}'_R) + \delta\mathscr{L}_a^{\rm ano}$$

matching

matching

$$\mathscr{L}_{\rm axion}^{\rm full} \equiv \mathscr{L}_{\chi{\rm PT}}(U, \mathbf{m}_q, \mathscr{A}_{L/R} + \mathbf{k}_{L/R}\, da) + \mathscr{L}_{\rm WZW}^{\rm full}(U, \mathbf{m}_q, \mathscr{A}_{L/R} + \mathbf{k}_{L/R}\, da)$$

$$U' = U_L^\dagger U U_R$$

$$\mathscr{L}_{\chi{\rm PT}}(U', \mathbf{m}'_q, \mathscr{A}_{L/R} + \mathbf{k}'_{L/R}\, da) + \mathscr{L}_{\rm WZW}^{\rm full}(U', \mathbf{m}'_q, \mathscr{A}_{L/R} + \mathbf{k}'_{L/R}\, da) + \delta\mathscr{L}_{\rm WZW}^{\rm ano}$$

Effective Lagrangian for axions

- Initial effective Lagrangian:

$$\mathscr{L}_{\text{eff},0} = \mathscr{L}_{\text{SM}} + \bar{q}_0(i\not{D} - \mathbf{m}_{q,0})q_0 + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + c_{gg}\frac{\alpha_s}{4\pi}\frac{a}{f}G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{a}{f}\sum_{\mathscr{A}_{1,2}} c^0_{\mathscr{A}_1\mathscr{A}_2}F_{\mathscr{A}_1\,\mu\nu}\tilde{F}^{\mu\nu}_{\mathscr{A}_2} + \mathscr{L}_c$$

- Eliminating aGG term:

$$q_0(x) = \exp\left[-i(\boldsymbol{\delta}_{q,0} + \boldsymbol{\kappa}_{q,0}\gamma_5)c_{gg}\frac{a(x)}{f}\right]q(x), \quad \text{with } \mathrm{Tr}\left(\boldsymbol{\kappa}_{q,0}\right) = 1$$

- New effective Lagrangian:

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_{\text{SM}} + \bar{q}\,i\not{D}\,q - [\bar{q}_L\,\mathbf{m}_q(a)\,q_R + h.c.] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + \frac{a}{f}\sum_{\mathscr{A}_{1,2}} c_{\mathscr{A}_1\mathscr{A}_2}F_{\mathscr{A}_1\,\mu\nu}\tilde{F}^{\mu\nu}_{\mathscr{A}_2} + \mathscr{L}_c$$

Auxiliary chiral rotation for effective Lagrangian

- Chiral rotation without regenerating aGG term $\mathrm{Tr}(\boldsymbol{\kappa}_q) = 0$

$$q' = \exp\left[i\left(\boldsymbol{\delta}_q + \boldsymbol{\kappa}_q \gamma_5\right) a/f\right] q$$

- Left/right rotation matrices

$$\boldsymbol{\theta}_{L/R} \equiv \boldsymbol{\delta}_q \mp \boldsymbol{\kappa}_q \qquad U_{L/R} \equiv \exp\left[-i\boldsymbol{\theta}_{L/R} a/f\right]$$

- Mass and coupling shifts

$$\mathbf{m}_q' = U_L^\dagger \mathbf{m}_q U_R, \quad \mathbf{k}_{L/R}' = U_{L/R}^\dagger(\mathbf{k}_{L/R} + \boldsymbol{\theta}_{L/R}) U_{L/R} = \mathbf{k}_{L/R} + \boldsymbol{\theta}_{L/R}$$

- Chiral basis change for effective Lagrangian

$$\mathscr{L}_{\text{eff}}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R) \to \mathscr{L}_{\text{eff}}(q', \mathbf{m}_q', \mathbf{k}_L', \mathbf{k}_R') + \delta\mathscr{L}_a^{\text{ano}}$$

The axion anomalous interactions

$$\delta \mathscr{L}_a^{\text{ano}} = -\,\delta \left[\mathscr{L}_{\text{WZW}} + \mathscr{L}_c\right](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R)$$

- The exact expressions

$$\delta\left[\Gamma_{\text{WZW}} + \Gamma_c\right](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R) = -2\mathscr{C}\frac{a}{f}\int \text{Tr}\Big\{\boldsymbol{\theta}_L\Big[3(d\mathbb{A}_L - i\mathbb{A}_L^2)^2 + 3(d\mathbb{A}_L - i\mathbb{A}_L^2)(D\mathbb{B}_L) + D\mathbb{B}_L D\mathbb{B}_L - \frac{i}{2}D(\mathbb{B}_L^3)$$
$$+ i\mathbb{B}_L(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L - i(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L^2\Big]\Big\} - (L \leftrightarrow R)\,,$$

- Covariant derivative $D\mathbb{B}_{L,R} = d\mathbb{B}_{L,R} - i\mathbb{A}_{L,R}\mathbb{B}_{L,R} - i\mathbb{B}_{L,R}\mathbb{A}_{L,R}$

- Covariant field strength: $F = d\mathbb{A}_L - i\mathbb{A}_L^2$

The axion anomalous interactions

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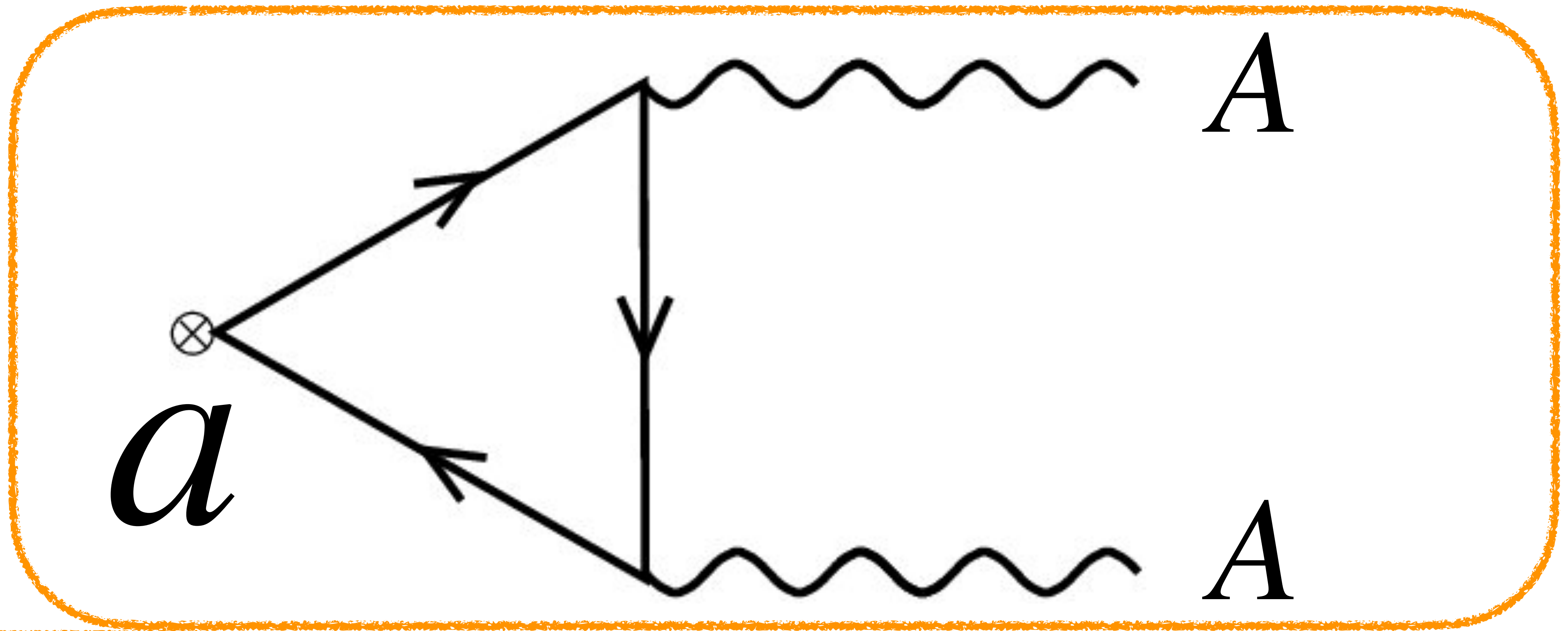

- The exact expressions

$$\delta\left[\Gamma_{\text{WZW}} + \Gamma_c\right](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R) = -2\mathscr{C}\frac{a}{f}\int \text{Tr}\Big\{\boldsymbol{\theta}_L\Big[3(d\mathbb{A}_L - i\mathbb{A}_L^2)^2 + 3(d\mathbb{A}_L - i\mathbb{A}_L^2)(D\mathbb{B}_L) + D\mathbb{B}_L D\mathbb{B}_L - \frac{i}{2}D(\mathbb{B}_L^3)$$
$$+ i\mathbb{B}_L(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L - i(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L^2\Big]\Big\} - (L \leftrightarrow R)\,,$$

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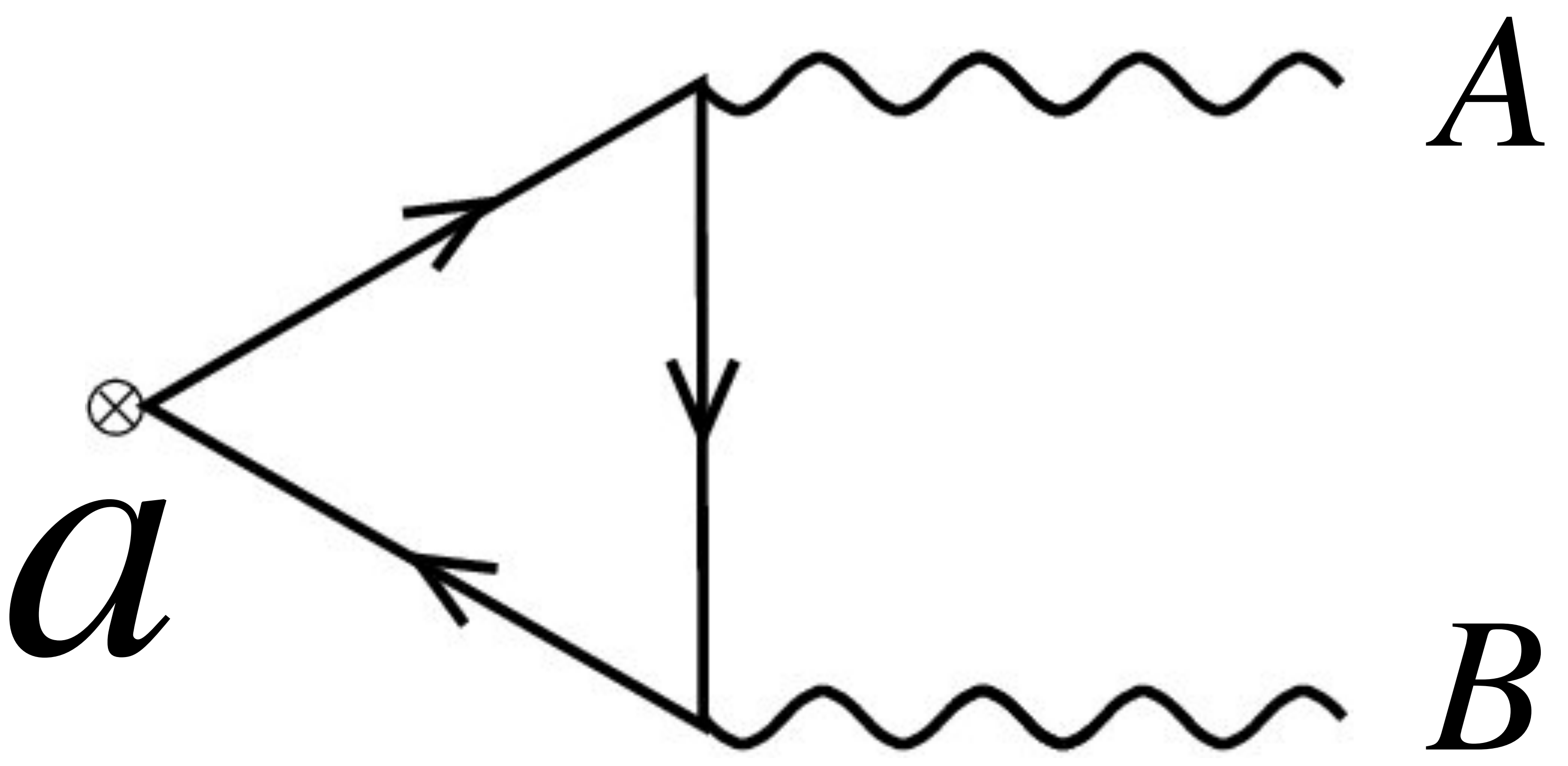
- Covariant field strength: $F = d\mathbb{A}_L - i\mathbb{A}_L^2$

The axion anomalous interactions

$$\delta \mathscr{L}_a^{\text{ano}} = -\delta\left[\mathscr{L}_{\text{WZW}} + \mathscr{L}_c\right](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R)$$



- The exact expressions

$$\delta\left[\Gamma_{\text{WZW}} + \Gamma_c\right](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R) = -2\mathscr{C}\frac{a}{f}\int \text{Tr}\Big\{\boldsymbol{\theta}_L\Big[3(d\mathbb{A}_L - i\mathbb{A}_L^2)^2 + 3(d\mathbb{A}_L - i\mathbb{A}_L^2)(D\mathbb{B}_L) + D\mathbb{B}_L D\mathbb{B}_L - \frac{i}{2}D(\mathbb{B}_L^3)$$
$$+ i\mathbb{B}_L(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L - i(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L^2\Big]\Big\} - (L \leftrightarrow R)\,,$$

- Covariant derivative $D\mathbb{B}_{L,R} = d\mathbb{B}_{L,R} - i\mathbb{A}_{L,R}\mathbb{B}_{L,R} - i\mathbb{B}_{L,R}\mathbb{A}_{L,R}$

- Covariant field strength: $F = d\mathbb{A}_L - i\mathbb{A}_L^2$

Effective and Chiral Lagrangian matching

$$\mathscr{L}_{\rm eff}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R) \to \mathscr{L}_{\chi\rm PT}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R)$$

- The correspondence

$$\mathscr{L}_{\rm eff} = \mathscr{L}_{\rm SM} + \bar{q} i \not{D} q - [\bar{q}_L \mathbf{m}_q(a) q_R + h.c.] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + \frac{a}{f} \sum_{\mathscr{A}_{1,2}} c_{\mathscr{A}_1 \mathscr{A}_2} F_{\mathscr{A}_1 \mu\nu} \widetilde{F}^{\mu\nu}_{\mathscr{A}_2} + \mathscr{L}_c$$

$$\mathscr{L}_{\chi\rm PT} = \frac{f_\pi^2}{8} \mathrm{Tr}\left[(D^\mu U)(D_\mu U)^\dagger\right] + \frac{f_\pi^2}{4} B_0 \mathrm{Tr}\left[\mathbf{m}_q(a) U^\dagger + h.c.\right] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + \frac{a}{f} \sum_{\mathscr{A}_{1,2}} c_{\mathscr{A}_1 \mathscr{A}_2} F_{\mathscr{A}_1 \mu\nu} \widetilde{F}^{\mu\nu}_{\mathscr{A}_2}$$

- The anomalous matching condition between UV and IR

$$\mathscr{L}_{\chi\rm PT}^{\rm ano} \equiv \frac{a}{f_a} \sum_{\mathscr{A}_{1,2}} c_{\mathscr{A}_1 \mathscr{A}_2} F_{\mathscr{A}_1 \mu\nu} \widetilde{F}^{\mu\nu}_{\mathscr{A}_2}$$

Effective and Chiral Lagrangian matching

$$\mathcal{L}_{\text{eff}}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R) \to \mathcal{L}^{\text{full}}_{\text{axion}} = \mathcal{L}_{\chi\text{PT}}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R) + \mathcal{L}^{\text{full}}_{\text{WZW}}(U, \mathscr{A}_{L/R})$$

- The correspondence

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \bar{q} i \not{D} q - [\bar{q}_L \mathbf{m}_q(a) q_R + h.c.] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m^2_{a,0}}{2} a^2 + \frac{a}{f} \sum_{\mathscr{A}_{1,2}} c_{\mathscr{A}_1 \mathscr{A}_2} F_{\mathscr{A}_1 \mu\nu} \tilde{F}^{\mu\nu}_{\mathscr{A}_2} + \mathcal{L}_c$$

$$\mathcal{L}_{\chi\text{PT}} = \frac{f_\pi^2}{8} \text{Tr}\left[(D^\mu U)(D_\mu U)^\dagger\right] + \frac{f_\pi^2}{4} B_0 \text{Tr}\left[\mathbf{m}_q(a) U^\dagger + h.c.\right] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m^2_{a,0}}{2} a^2 + \frac{a}{f} \sum_{\mathscr{A}_{1,2}} c_{\mathscr{A}_1 \mathscr{A}_2} F_{\mathscr{A}_1 \mu\nu} \tilde{F}^{\mu\nu}_{\mathscr{A}_2}$$

- WZW term and counter terms

$$\mathcal{L}^{\text{full}}_{\text{WZW}}(U, \mathscr{A}_{L/R}) = \mathcal{L}_{\text{WZW}}(U, \mathscr{A}_L, \mathscr{A}_R) + \mathcal{L}_c(\mathbb{A}_{L/R}, \mathbb{B}_{L/R})$$

Consistent matching between $\mathscr{L}_{\text{eff}}$ and $\mathscr{L}^{\text{full}}_{\text{axion}}$

$$\mathscr{L}_{\text{eff},0}(q_0, \mathbf{m}_{q,0}, \mathbf{k}_{L,0}, \mathbf{k}_{R,0})$$

$$q_0 = \exp\left(-i\, c_{gg}\, \boldsymbol{\kappa}_{q,0}\, \gamma_5 \frac{a}{f}\right) q$$

$$\delta \mathscr{L}^{\text{ano}}_{a} = -\delta\left[\mathscr{L}_{\text{WZW}} + \mathscr{L}_{c}\right](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R) = \delta \mathscr{L}^{\text{ano}}_{\text{WZW}}$$

$$\mathscr{L}_{\text{eff}}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R) \longrightarrow \mathscr{L}_{\text{eff}}(q', \mathbf{m}'_q, \mathbf{k}'_L, \mathbf{k}'_R) + \delta \mathscr{L}^{\text{ano}}_{a}$$

$$q' = \exp\left[i\left(\boldsymbol{\delta}_q + \boldsymbol{\kappa}_q \gamma_5\right)\frac{a}{f}\right] q$$

matching

matching

$$\mathscr{L}^{\text{full}}_{\text{axion}} \equiv \mathscr{L}_{\chi\text{PT}}(U, \mathbf{m}_q, \mathscr{A}_{L/R} + \mathbf{k}_{L/R} da) + \mathscr{L}^{\text{full}}_{\text{WZW}}(U, \mathbf{m}_q, \mathscr{A}_{L/R} + \mathbf{k}_{L/R} da)$$

$$U' = U_L^{\dagger} U U_R$$

$$\mathscr{L}_{\chi\text{PT}}(U', \mathbf{m}'_q, \mathscr{A}_{L/R} + \mathbf{k}'_{L/R} da) + \mathscr{L}^{\text{full}}_{\text{WZW}}(U', \mathbf{m}'_q, \mathscr{A}_{L/R} + \mathbf{k}'_{L/R} da) + \delta \mathscr{L}^{\text{ano}}_{\text{WZW}}$$

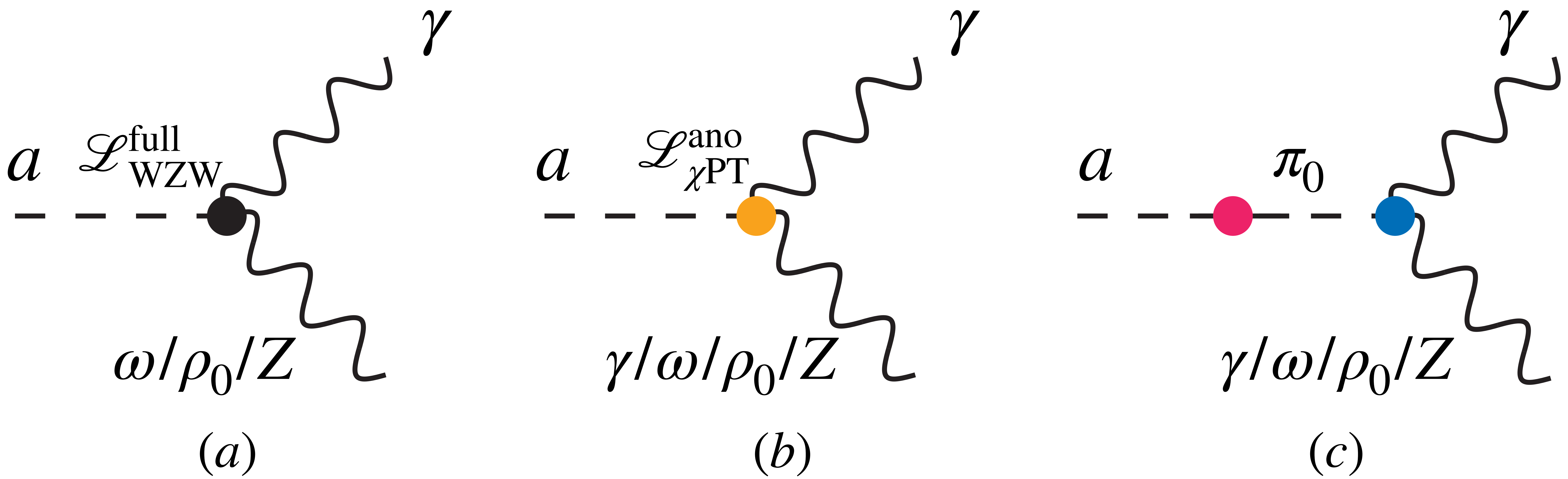
Outlines

- Axion general introduction
- Axion-scalar meson interactions
- Axion-vector meson interactions
- Phenomenology at BESIII and STCF
- Summary

Consistent physical amplitudes

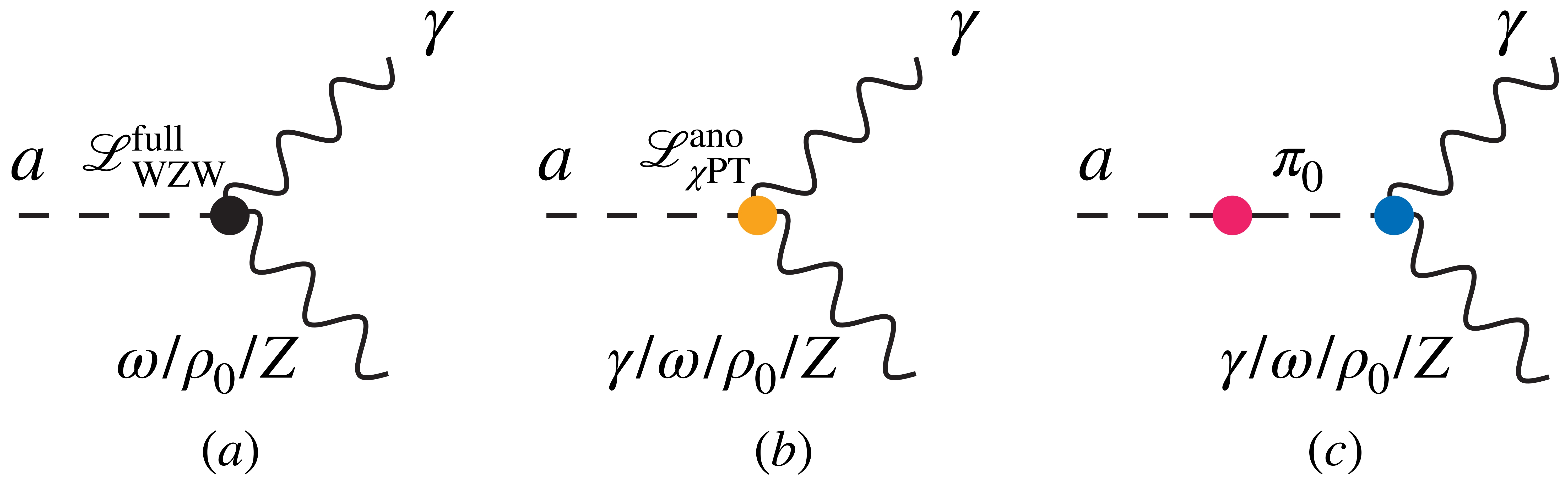
- A consistent Lagrangian will give physical amplitudes independent of auxiliary rotations

- Full WZW interactions are important for a-A-B amplitudes



Consistent physical amplitudes for $a - \gamma - \gamma$

- Auxiliary rotations are cancelled



$ad\gamma d\gamma: \quad c_{\rm WZW} \equiv 0$ $\qquad ad\gamma d\gamma: \quad c_{\rm ano} \equiv -\frac{e^2 N_c}{48\pi^2 f} 12(Q_u^2\kappa_u + Q_d^2\kappa_d)$ $\qquad \pi_0 d\gamma d\gamma: \quad c_{\pi_0} \equiv \frac{e^2 N_c}{48\pi^2 f_\pi} 6\sqrt{2}(Q_d^2 - Q_u^2)$

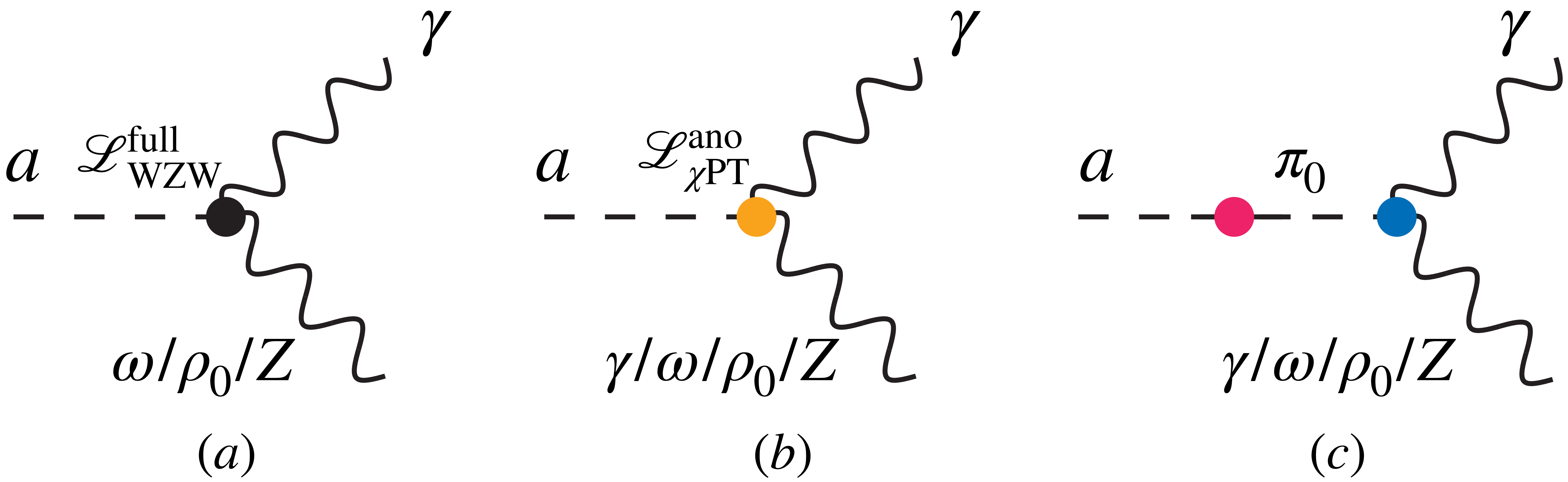
$$\mathscr{M}(a \to \gamma\gamma)\textbf{(auxiliary)} = CF \times \left(c_{\rm ano} + \theta'_{a-\pi_0} c_{\pi_0} + c_{\rm WZW}\right)$$

$$= CF \times e^2 \left\{ \frac{-N_c}{48\pi^2 f_a} 12(Q_u^2\kappa_u + Q_d^2\kappa_d) + i\frac{f_\pi}{\sqrt{2}f}\left[(\kappa_u - \kappa_d)p_a^2 - 2\frac{m_u\kappa_u - m_d\kappa_d}{m_u + m_d} m_\pi^2\right] \frac{i}{p_a^2 - m_\pi^2} \times \frac{N_c}{48\pi^2 f_\pi} 6\sqrt{2}(Q_d^2 - Q_u^2) \right\}$$

$\kappa_u + \kappa_d = 0$
$p_a^2 = m_a^2$ $\quad \to 0$

Consistent physical amplitudes for $a-\gamma-\omega$

- Auxiliary rotations are cancelled



$a d\omega d\gamma:\ c_{\text{wzw}} = \frac{-eg' N_c}{48\pi^2 f} 6(Q_u \kappa_u + Q_d \kappa_d)$ $\quad a d\omega d\gamma:\ c_{\text{ano}} = \frac{-eg' N_c}{48\pi^2 f} 6(Q_u \kappa_u + Q_d \kappa_d)$ $\quad \pi_0 d\omega d\gamma:\ c_{\pi_0} = \frac{eg' N_c}{48\pi^2 f_\pi} 6\sqrt{2}(Q_d - Q_u)$

$\mathscr{M}(a \to \omega\gamma)$**(auxiliary)** $= CF \times (c_{\text{ano}} + \theta'_{a-\pi_0} c_{\pi_0} + c_{\text{wzw}})$

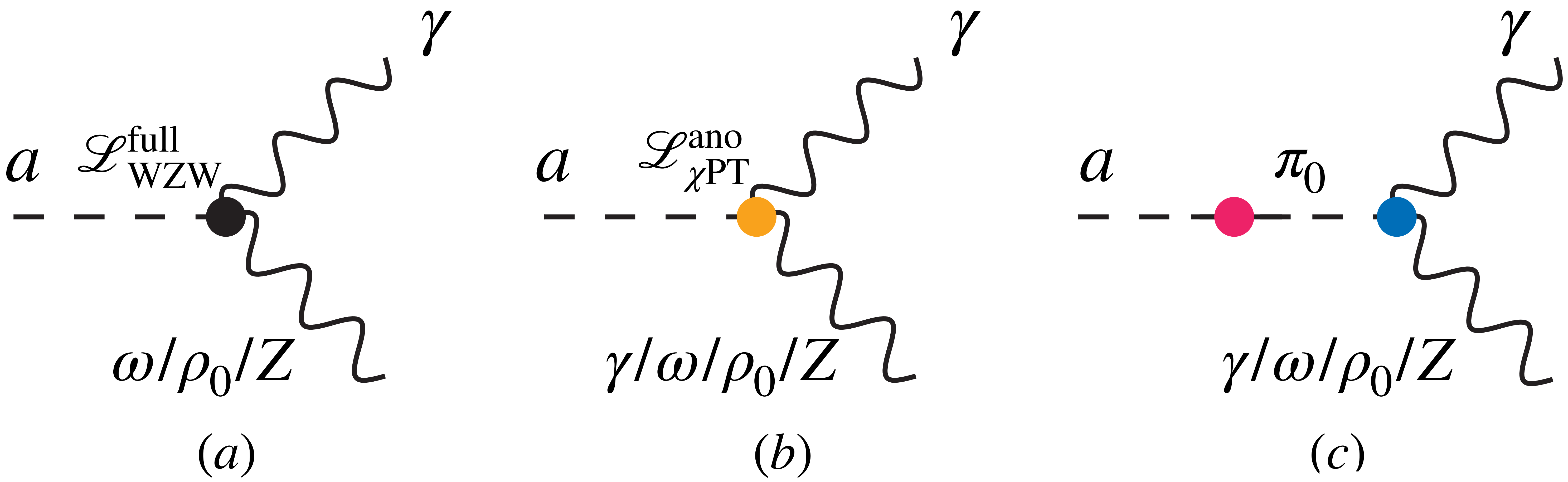
$$= CF \times eg' \left[\frac{-N_c}{48\pi^2 f} 12(Q_u\kappa_u + Q_d\kappa_d) + i\frac{f_\pi}{\sqrt{2}f}((\kappa_u - \kappa_d)p_a^2 - 2\frac{m_u\kappa_u - m_d\kappa_d}{m_u + m_d} m_\pi^2)\frac{i}{p_a^2 - m_\pi^2} \times \frac{N_c}{48\pi^2 f_\pi} 6\sqrt{2}(Q_d - Q_u) \right]$$

$\to 0$

Consistent physical amplitudes for $a - \gamma - Z$

- Auxiliary rotations are cancelled

$$a d\gamma d Z: \quad c_{\rm ano} = \frac{N_c}{48\pi^2 f}\frac{2e^2}{3c_w s_w}\left[3\delta_d + 6\delta_u - 3\kappa_d - 6\kappa_u + 4s_w^2(\kappa_d + 4\kappa_u)\right]$$



$$a d\gamma d Z: \quad c_{\rm WZW} = \frac{-2e^2 N_c}{48\pi^2 f s_w c_w}(\delta_d + 2\delta_u)$$

$$\pi_0 d\gamma d Z: \quad c_{\pi_0} = \frac{-e^2 N_c}{48\pi^2 f_\pi s_w c_w}\sqrt{2}(c_w^2 - 3s_w^2)$$

$$\mathscr{M}(a \to Z^*\gamma)\textbf{(auxiliary)} = CF \times (c_{\rm ano} + \theta'_{a-\pi_0} c_{\pi_0} + c_{\rm WZW})$$

$$= CF \times \left[c_{\rm WZW} + c_{\rm ano} + i\frac{f_\pi}{\sqrt{2}f}\left((\kappa_u - \kappa_d)p_a^2 - 2\frac{m_u\kappa_u - m_d\kappa_d}{m_u + m_d}m_\pi^2\right)\frac{i}{p_a^2 - m_\pi^2} \times c_{\pi_0}\right]$$

$$\to 0$$

Consistent amplitudes for three point vertex

$$c^{\rm eff}_{\gamma\gamma} = c^0_{\gamma\gamma} + \frac{e^2 c_{gg}}{16\pi^2 f}\left(-\frac{10}{3} - 2\frac{m_u - m_d}{m_u + m_d}\frac{m_\pi^2}{m_a^2 - m_\pi^2}\right) - \frac{e^2}{16\pi^2 f}\frac{m_a^2}{m_\pi^2 - m_a^2}(c_u - c_d)$$

$$c^{\rm eff}_{\omega\gamma} = eg'\left\{\frac{-c_{gg}}{8\pi^2 f} - \frac{3}{8\pi^2 f}\left[\frac{m_a^2}{m_\pi^2 - m_a^2}\left(\frac{c_u - c_d}{2}\right) + c_{gg}\frac{m_u - m_d}{m_u + m_d}\frac{m_\pi^2}{m_a^2 - m_\pi^2}\right] + \frac{1}{16\pi^2 f}(c_d + c_Q - 2c_u)\right\}$$

$$c^{\rm eff}_{\rho\gamma} = eg\left\{\frac{-3c_{gg}}{8\pi^2 f} - \frac{1}{8\pi^2 f}\left[\frac{m_a^2}{m_\pi^2 - m_a^2}\left(\frac{c_u - c_d}{2}\right) + c_{gg}\frac{m_u - m_d}{m_u + m_d}\frac{m_\pi^2}{m_a^2 - m_\pi^2}\right] + \frac{1}{16\pi^2 f}\left(3c_Q - 2c_u - c_d\right)\right\}$$

$$c^{\rm eff}_{\gamma Z} = c^0_{\gamma Z} + \frac{N_c c_{gg}}{48\pi^2 f}\frac{e^2}{s_w c_w}(-9 + 20 s_w^2) - c_{\pi_0}\frac{f_\pi}{\sqrt{2}f}\left(\frac{m_a^2}{m_\pi^2 - m_a^2}\frac{c_d - c_u}{2} - c_{gg}\frac{m_u - m_d}{m_u + m_d}\frac{m_\pi^2}{m_a^2 - m_\pi^2}\right) - \frac{N_c}{48\pi^2 f}\frac{2e^2}{s_{2w}}(c_d + 2c_u + 3c_Q)$$

- Vertex $\omega \to \gamma a$ benefit from large $g' \approx 5.7 \gg e$

$\mathbf{k}_{L,0} = \{c_Q, c_Q\}$ $\quad \mathbf{k}_{R,0} = \{c_u, c_d\}$

Phenomenology at BESIII and STCF

- New channel $e^+e^- \to \gamma^*(J/\Psi) \to \omega a$

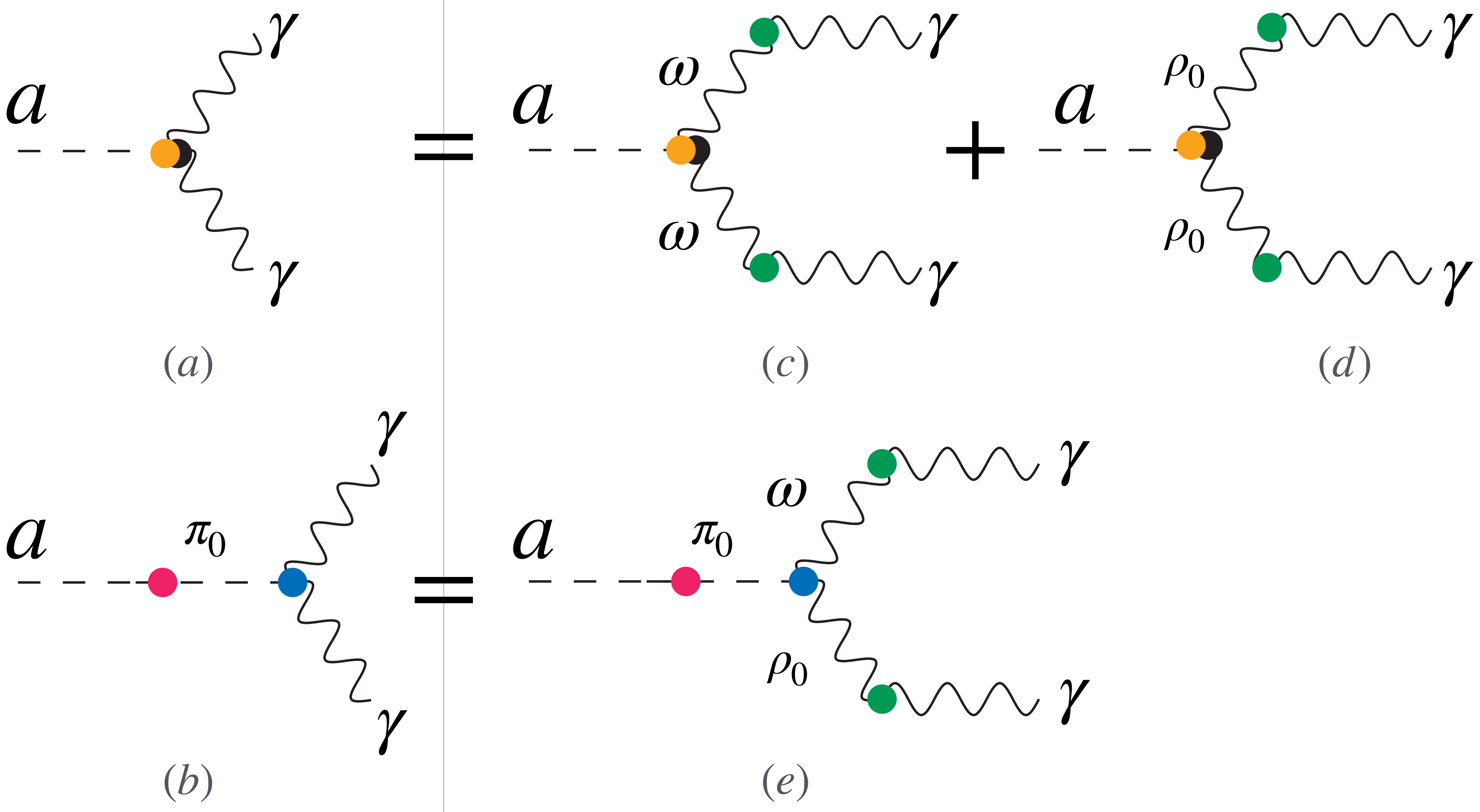
$$c_{\omega\gamma}^{\rm eff}(q^2) = -\frac{eg'c_{gg}}{8\pi^2 f}\frac{m_\omega^2}{m_\omega^2 - q^2 - i\sqrt{q^2}\Gamma_\omega} - \frac{3eg'c_{gg}}{8\pi^2 f}\frac{m_u - m_d}{m_u + m_d}\frac{m_\pi^2}{m_a^2 - m_\pi^2}\sum_{i=0}^{3}\frac{A_i M_i^2 e^{i\phi_i}}{M_i^2 - q^2 - i\sqrt{q^2}\Gamma_i(\sqrt{q^2})}$$

- The model satisfies partial Vector Meson Dominance, therefore we can use form factor for $\gamma^* - \omega - a$
- The differential cross-section

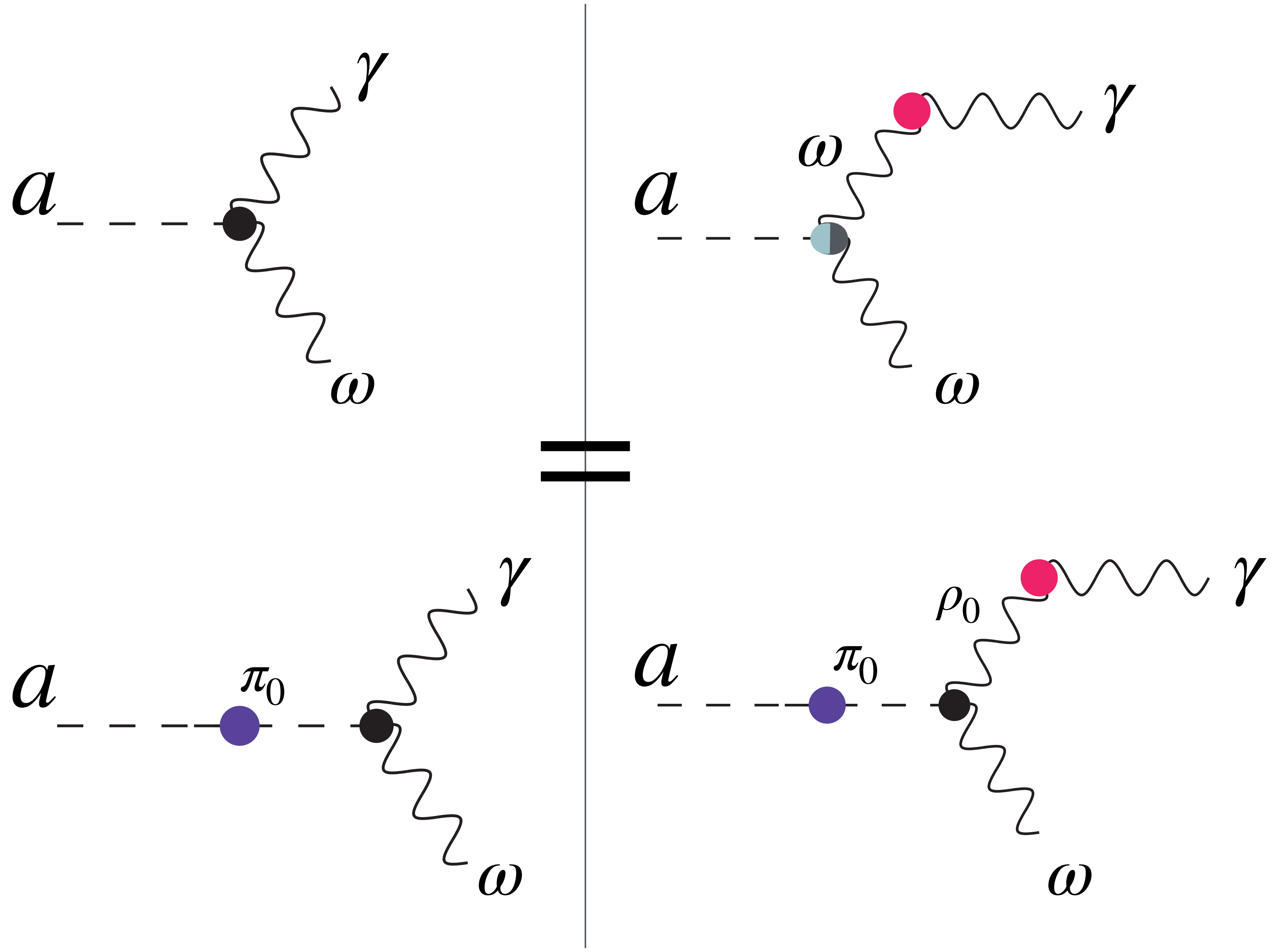
$$\frac{d\sigma(e^+e^- \to \omega\ a)}{d\cos\theta} = \frac{\alpha\,|c_{\omega\gamma}^{\rm eff}(q^2)|^2\left[m_a^4 + (m_\omega^2 - s)^2 - 2m_a^2(m_\omega^2 + s)\right]}{64 f^2 s^2}(1 + \cos\theta^2)$$

Vector Meson Dominance

$$a - \gamma - \gamma$$

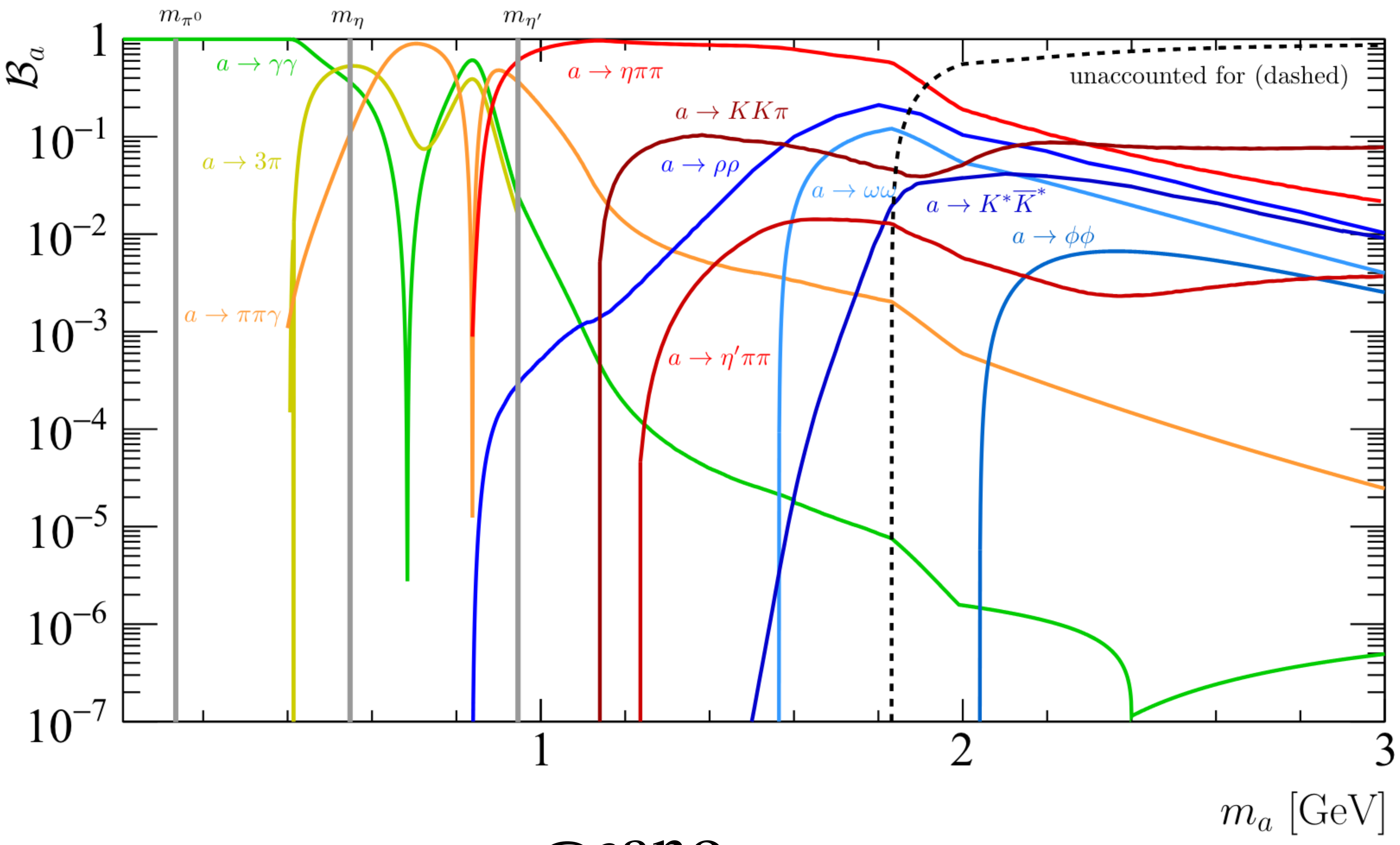


$$a - \gamma - \omega$$



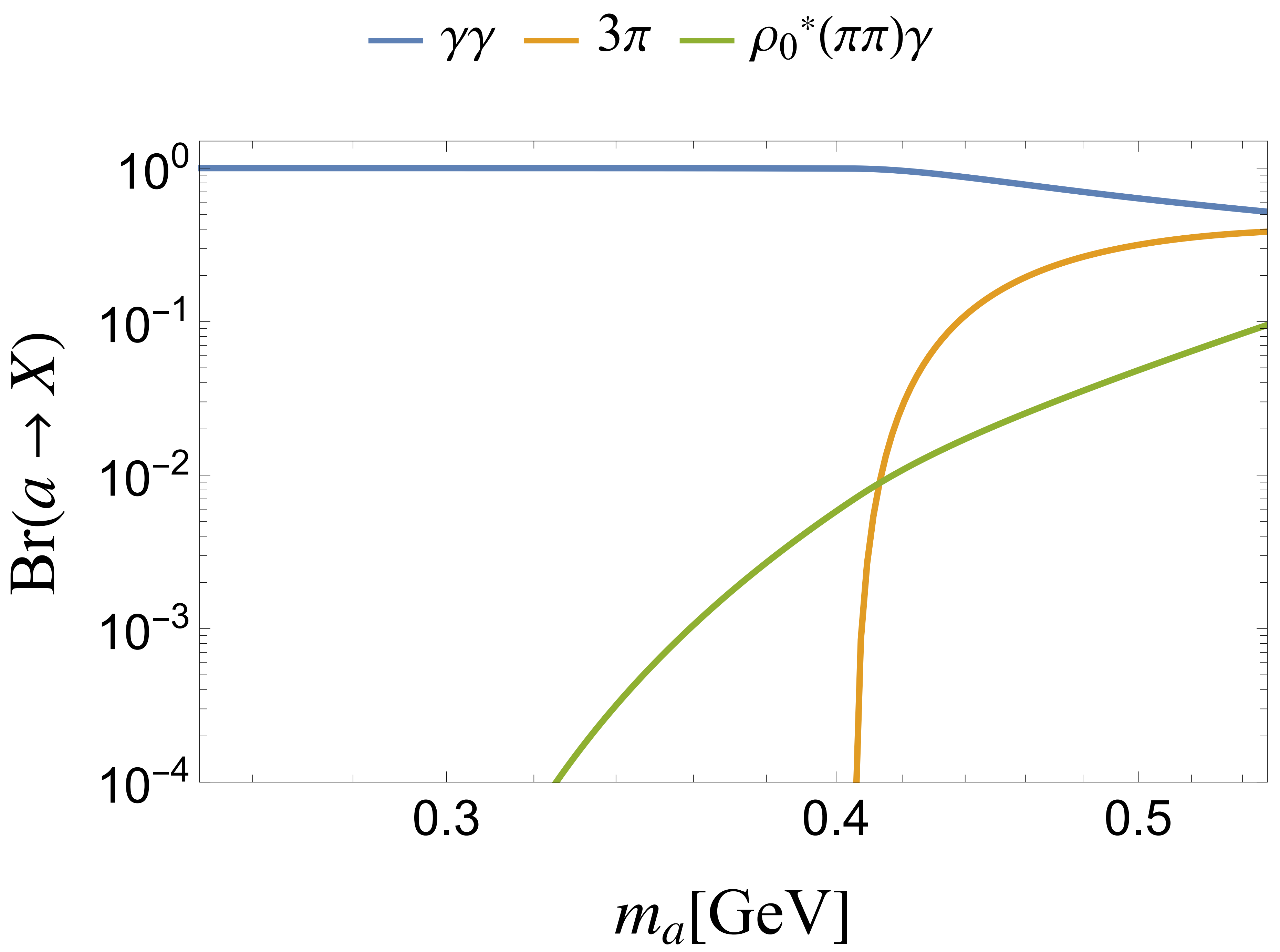
The decay of axion

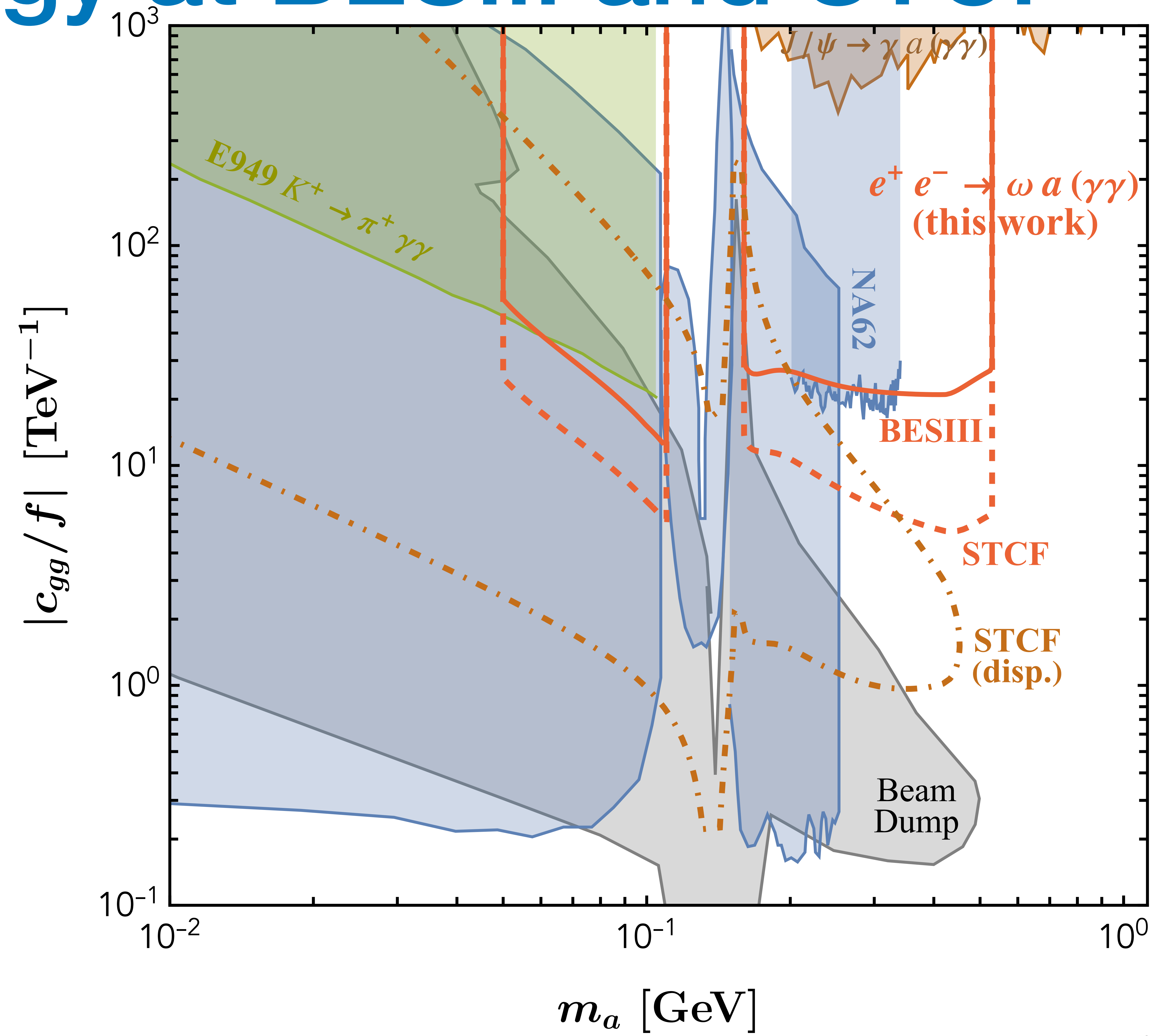
- Previous work (PRL 123 (2019) 031803) use Hidden Local Symmetry to describe pseudo scalar meson + vector meson interactions
- Assume axion mixes with π, η, η'
- Use data driven method to obtain form factor



- Lacks first chiral rotation contribution from $\mathscr{L}^{\text{ano}}_{\chi\text{PT}}$
- Lacks full WZW contribution from $\mathscr{L}^{\text{full}}_{\text{WZW}}$

Light axion phenomenology at BESIII and STCF





- Production $e^+e^- \to \gamma^*(J/\Psi) \to \omega a$
- Prompt decay: $a \to \gamma\gamma$
- Displaced decay of a

$$\frac{\mathrm{BR}(J/\psi \to \omega a)}{\mathrm{BR}(J/\psi \to ee)} = \frac{m_{J/\psi}^2}{32\pi\alpha}\left|c_{\omega\gamma}^{\mathrm{eff}}(q^2 = m_{J/\psi}^2)\right|^2 \left[\left(1 - \frac{(m_a + m_\omega)^2}{m_{J/\psi}^2}\right)\left(1 - \frac{(m_a - m_\omega)^2}{m_{J/\psi}^2}\right)\right]^{\frac{3}{2}}$$

Summary

- A full chiral axion Lagrangian for axion-pseudo-vector meson
  - Wess-Zumino-Witten counter term is necessary for gauge invariance
  - $\mathbb{B}_{L/R} \to \mathbb{B}_{L/R} + \mathbf{k}_{L/R} da/f_a$
  - UV-IR anomaly matching is necessary
- Consistent physical amplitudes without auxiliary rotation parameters
- New search channel involving $\omega \to \gamma a$ vertex at BESIII & STCF
- Future plan: extending to three light quarks scheme
  Needs to deal with η'; vector meson mediated processes in astrophysics

Backup: axion related WZW interactions

- Convention $\int d^4x \epsilon_{\mu\nu\rho\sigma} A^\mu B^\nu \partial^\rho C^\sigma \equiv \int ABdC$

$$\Gamma_{XdYda} = \frac{\mathscr{C}}{f}\int da \Bigg\{ \frac{2e^2}{s_{2\mathrm{w}}}(k_d + 2k_u + 3k_Q)\gamma dZ + eg(k_d + 2k_u + 3k_Q)\gamma da_1 - eg'(k_d - k_Q - 2k_u)\gamma df_1$$

$$+eg(k_d - 3k_Q + 2k_u)\gamma d\rho_0 - eg'(k_d + k_Q - 2k_u)\gamma d\omega + \frac{2e^2}{s_{2\mathrm{w}}^2}\left[(k_d + 4k_Q + k_u) - 2s_{\mathrm{w}}^2(k_d + 3k_Q + 2k_u)\right] ZdZ$$

$$+\frac{eg}{s_{2\mathrm{w}}}\left[(k_d + 4k_Q + k_u) - 2s_{\mathrm{w}}^2(k_d + 3k_Q + 2k_u)\right] Zda_1 - \frac{eg'}{s_{2\mathrm{w}}}\left[k_d - k_u + s_{\mathrm{w}}^2(-2k_d + 2k_Q + 4k_u)\right] Zdf_1$$

$$-\frac{eg}{s_{2\mathrm{w}}}\left[-3k_d - 3k_u + 2s_{\mathrm{w}}^2(k_d - 3k_Q + 2k_u)\right] Zd\rho_0 - \frac{eg'}{s_{2\mathrm{w}}}\left[3k_d - 3k_u - 2s_{\mathrm{w}}^2(k_d + k_Q - 2k_u)\right] Zd\omega$$

$$+g^2(k_d + 2k_Q + k_u)a_1 d\rho_0 + gg'(k_u - k_d)a_1 d\omega + gg'(k_u - k_d)f_1 d\rho_0 + g'^2(k_d + 2k_Q + k_u)f_1 d\omega$$

$$+g^2(k_d - 2k_Q + k_u)\rho_0 d\rho_0 + 2gg'(k_u - k_d)\rho_0 d\omega + g'^2(k_d - 2k_Q + k_u)\omega d\omega + \frac{3eg}{2s_{\mathrm{w}}}(k_u + k_d)W^{\pm} d\rho^{\mp}$$

$$+\frac{eg}{2s_{\mathrm{w}}}(k_d + 4k_Q + k_u)a^{\mp} dW^{\pm} + g^2(k_d + 2k_Q + k_u)a^{\mp} d\rho^{\pm} + \frac{e^2}{s_{\mathrm{w}}^2}(k_d + 4k_Q + k_u)W^- dW^+ \Bigg\}$$