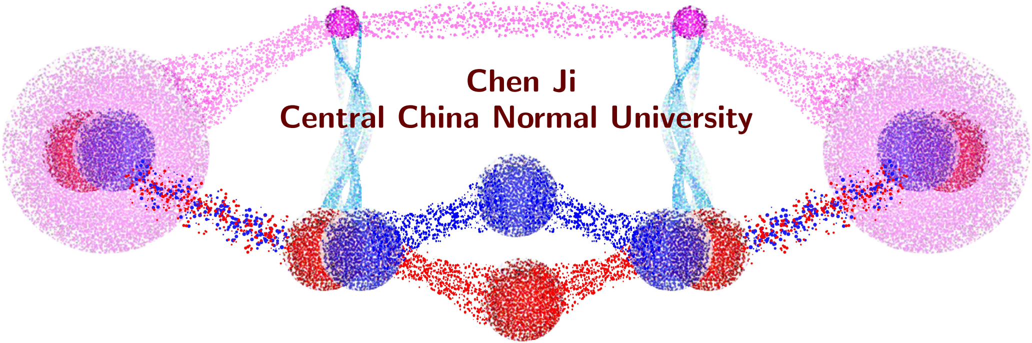
華中師範大學
CENTRAL CHINA NORMAL UNIVERSITY

Nuclear Structure Effects to High-Precision Spectroscopy in Hydrogen-Like Atoms



Chen Ji
Central China Normal University

MIP 2025, Hunan University 2025.05.17-19

Nuclear structures from spectroscopy

- Precision spectroscopy provides abundant information on nuclear structures.

Nuclear structure observables

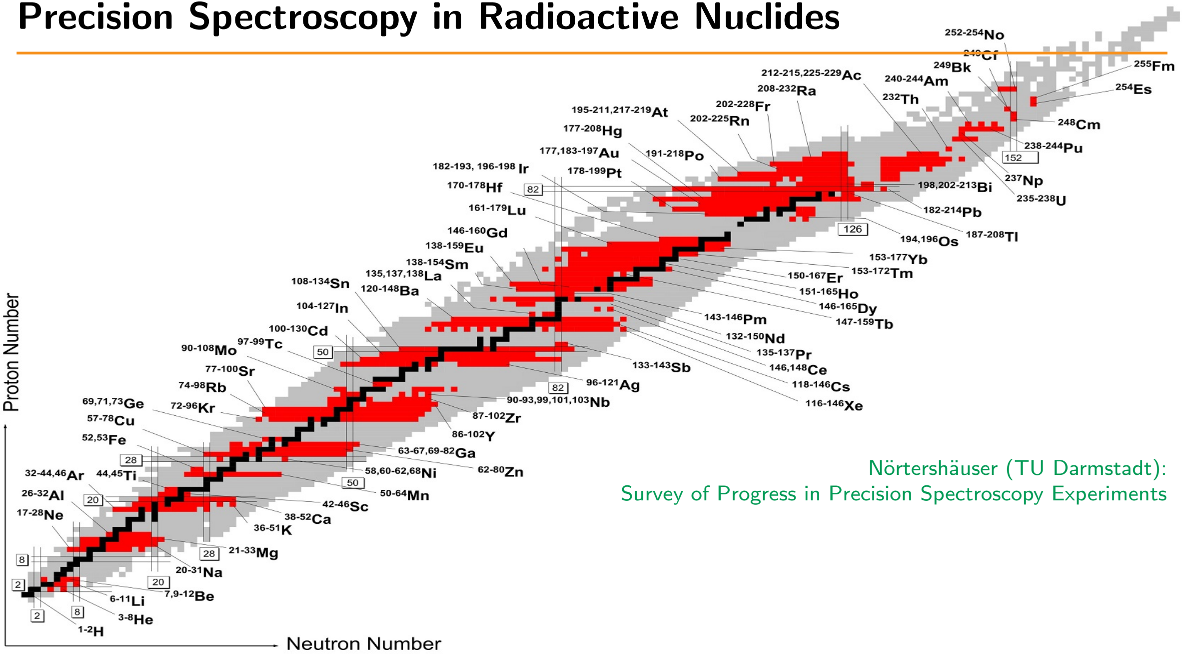
Nuclear spin
Charge radius
Magnetic dipole moment
electric quadrupole moment
magnetic radius

Nuclear structure physics

Nuclear shell evolution
New β stability line, neutron-rich drip line
Halo structure of radioactive nuclei
Internal nucleon distribution
Nuclear deformation

- Precision measurements on nuclear structures provides crucial guidance to building nuclear Hamiltonian models and nuclear many-body theories
  - Deuteron quadrupole moment $\rightarrow$ nuclear tensor force
  - Magnetic moment/radius $\rightarrow$ meson-exchange current
  - Charge radii for $A \geq 3$ systems $\rightarrow$ three-nucleon force

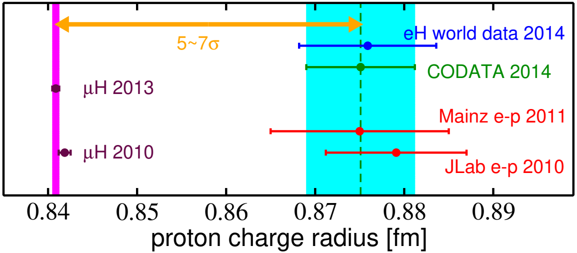
Precision Spectroscopy in Radioactive Nuclides



Nörtershäuser (TU Darmstadt):
Survey of Progress in Precision Spectroscopy Experiments

Proton radius puzzle

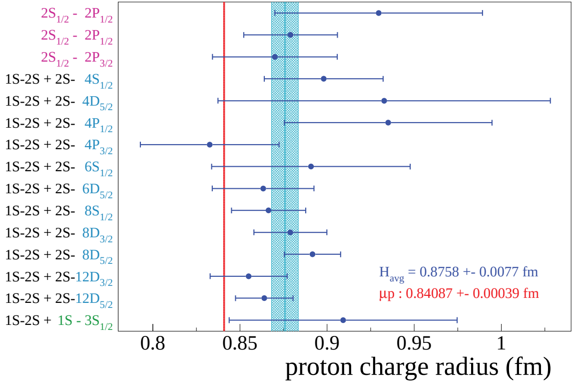
- electron-proton interaction experiments: $r_p = 0.8770(45)$ fm
  - eH spectroscopy
  - e–p scattering
- μ-p interaction experiments: $r_p = 0.8409(4)$ fm
  - μH Lamb shift (ΔE_{2S-2P}) [PSI-CREMA]
    Pohl *et al.*, Nature (2010); Antognini *et al.*, Science (2013)

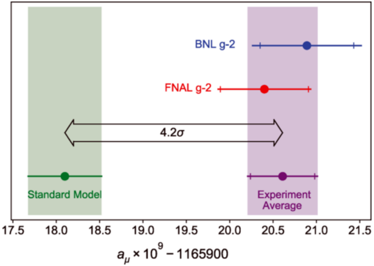


Solve the radius puzzle

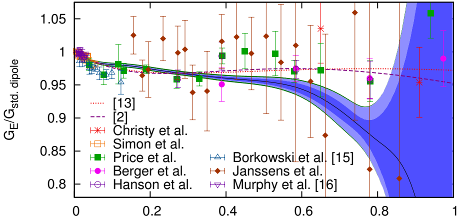
- Possible explanation:
  - Lepton-universality violation? $(g_\mu - 2)$
  - exotic hadron structure?
  - Neglected systematic uncertainty?

No explanation has been completed accepted



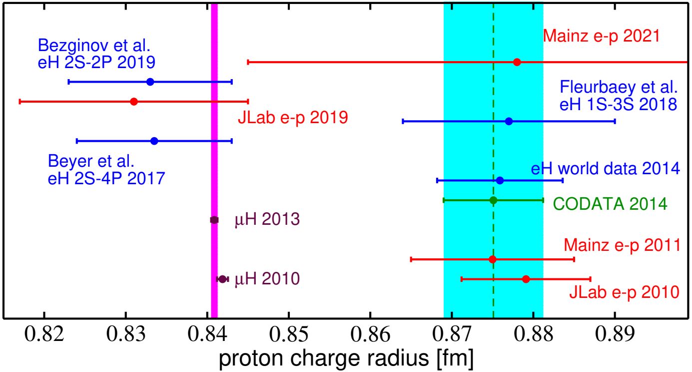


$(g-2)_\mu$ collaboration, PRL 126 (2021) 141801



Solve the radius puzzle

- New experiment to measure the proton radius
  - $e-p$ scattering (JLab, Mainz, Tohoku U.)
  - $\mu-p$ scattering (PSI-MUSE)
  - hydrogen spectroscopy (MPQ, LKB, York U.)



We seem to better (not fully) understand the proton radius now.

Spectroscopy measurement of nuclear radii in other atoms

- Lamb shift in muonic atoms/ions (PSI-CREMA)
  - $\mu^2\mathrm{H}$ [Pohl *et al.*, Science 2016]
  - $\mu^4\mathrm{He}^+$ [Krauth *et al.*, Nature 2021]
  - $\mu^3\mathrm{He}^+$ [K. Schuhmann *et al.*, arXiv:2305.11679]
  - $\mu^3\mathrm{H}$, $\mu\mathrm{Li}$, $\mu\mathrm{Be}$ [planned]

Nuclear charge radii

- $e^{3,4}\mathrm{He}$ spectroscopy

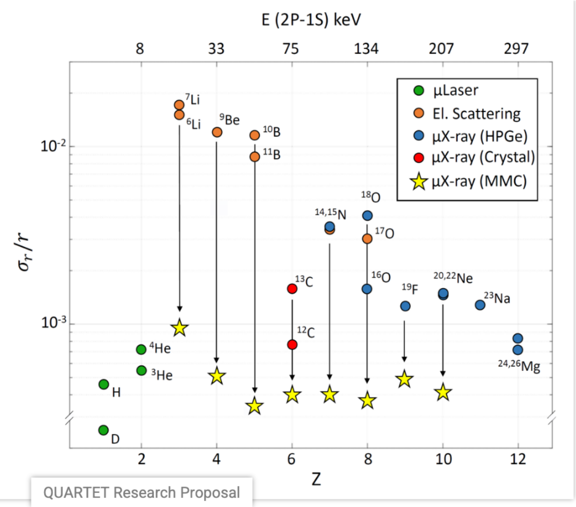
$^3\mathrm{He}$-$^4\mathrm{He}$ charge-radius isotope-shift

- hyperfine splitting in muonic measurements (PSI-CREMA)
  - $\mu^2\mathrm{H}$, $\mu^3\mathrm{He}^+$ [In plan]

Nuclear magnetic Zemach radius

QUARTET: Spectroscopy for nuclear charge radii

- Uses metallic magnetic microcalorimeters to measure X-rays
- High energy bandwidth, high resolution, high detection efficiency
- Improves precision in charge radius measurements of light nuclei from Li to Ne



pic from D. Unger's seminar talk at U Heidelberg (2024)

Nuclear structure effects to Lamb shift

- Extract nuclear charge radius from Lamb shift in muonic atoms

$$\delta E_{\rm LS} = \delta_{\rm QED} + \mathcal{A}_{\rm OPE}\, R_E^2 + \delta_{\rm TPE}$$

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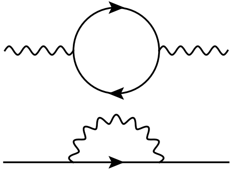
$$\delta E_{\mathrm{LS}} = \delta_{\mathrm{QED}} + \mathcal{A}_{\mathrm{OPE}}\, R_E^2 + \delta_{\mathrm{TPE}}$$

- **QED effects**
  - Vacuum polarization (Uehling effect)
  - Lepton self energy
  - relativistic recoil



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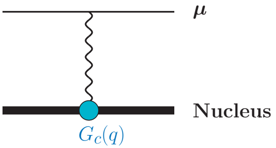
- **Nuclear structure effects**
  - $\propto R_E^2 \Longrightarrow$ **one-photon exchange (OPE)**
    $\mathcal{A}_{\mathrm{OPE}} \approx m_\mu^3 (Z\alpha)^4/12$



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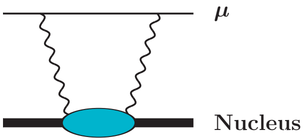
- **Nuclear structure effects**
  - $\boldsymbol{\delta_{\rm TPE}} \Longrightarrow$ **two-photon exchange (TPE)**
    - elastic part: Zemach moment $\delta_{\rm Zem}$
    - inelastic part: nuclear polarizability $\delta_{\rm pol}$



Nuclear structure effects to Lamb shift

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$$\delta E_{\rm LS} = \delta_{\rm QED} + \mathcal{A}_{\rm OPE}\, R_E^2 + \boxed{\delta_{\rm TPE}}$$

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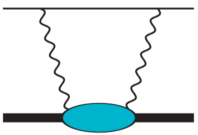


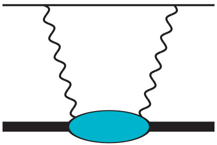
- **The accuracy of extracting R_E relies on the theoretical input of $\delta_{\rm TPE}$**

  $\mu^2{\rm H}$ experiment: $\delta_{\rm pol}$ requires 1% accruacy

  $\mu^{3,4}{\rm He}^+$ experiment: $\delta_{\rm pol}$ requires 5% accuracy

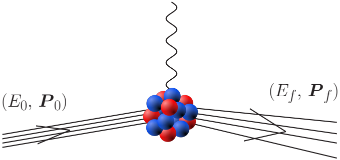
Nuclear polarizability from sum rules for photo-nuclear reactions

$$\delta_{\rm pol} = \sum_{g,\, S_{\hat{O}}} \int_{\omega_{th}}^{\infty} d\omega\, \underbrace{g(\omega)}_{\text{weight}}\, \underbrace{S_{\hat{O}}(\omega)}_{\text{response function}}$$



- energy-weighted sum rules $g(\omega)$
- nuclear response function $S_{\hat{O}}(\omega)$

$$S_O(\omega) = \sum\!\!\!\!\!\!\int_f |\langle\psi_f|\hat{O}|\psi_0\rangle|^2 \delta(E_f - E_0 - \omega)$$



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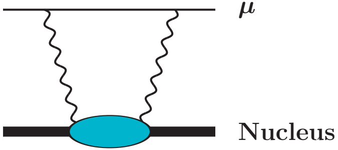


Contributing terms in nuclear polarizability δ_{pol}:

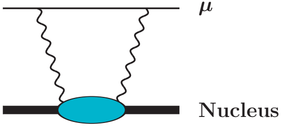
- multipole expansion to EM operators
  - E0, E1, E2, M1 response sum rules
- relativistic and Coulomb-distortion corrections
- intrinsic nucleon structure corrections

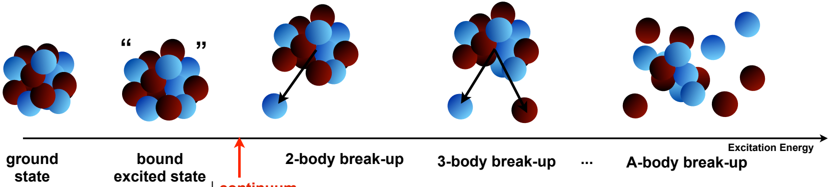
CJ, Bacca, Barnea, Hernandez, Nevo-Dinur, JPG 45 (2018) 093002

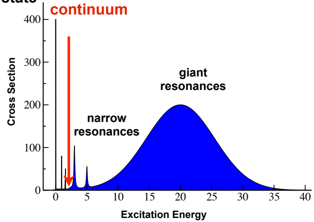
Nuclear response function: continuum spectrum

- The nucleus is virtually excited during the TPE process

$$S_O(\omega) = \sum_f |\langle \psi_f | \hat{O} | \psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$$





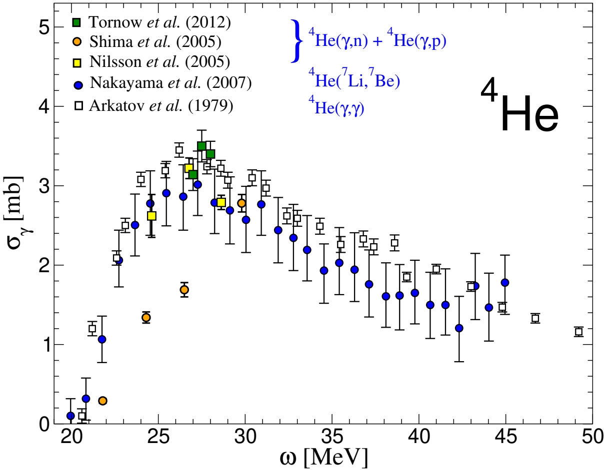


$$\sigma \propto \left| \langle \Psi_f | J^\mu | \Psi_0 \rangle \right|^2$$

Exact knowledge limited in energy and mass number

Determine $S_{\hat{O}}$ from photo-nuclear reaction experiments

$$\sigma_\gamma(\omega) = 4\pi^2\alpha\omega S_{E1}(\omega)$$



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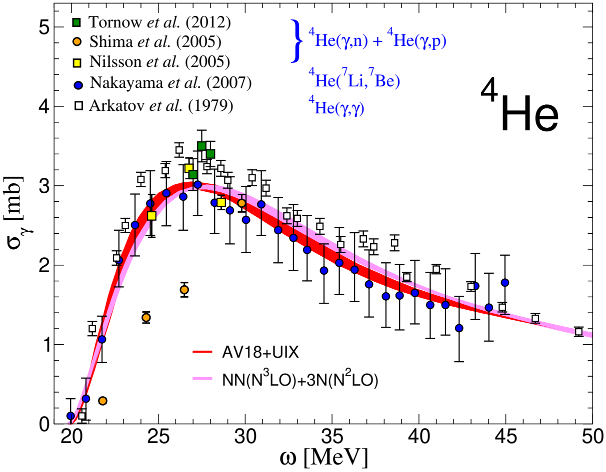


Ab initio nuclear-structure calculations

Recent developments in nuclear structure theories:

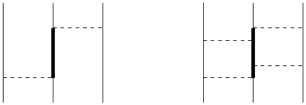
- Microscopic theories of nucleon-nucleon interactions
- Theory for electroweak interactions involving nucleons
- Ab initio calculations of nuclear structures and reactions
- Uncertainty quantification for nuclear theory

Realistic nuclear potential

- **Argonne v_{18} NN interaction**
  - fit to 1787 pp & 2514 np scattering data ($E_{lab} \leq 350$ MeV, $\chi^2/\mathrm{datum} = 1.1$)
  - nn scattering length & deuteron binding energy

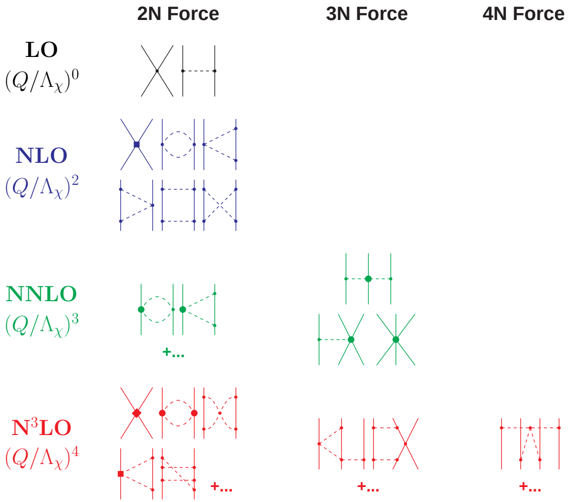
- **Urbana IX NNN force**

$$V_{ijk} = V_{ijk}^{2\pi P} + V_{ijk}^{R}$$



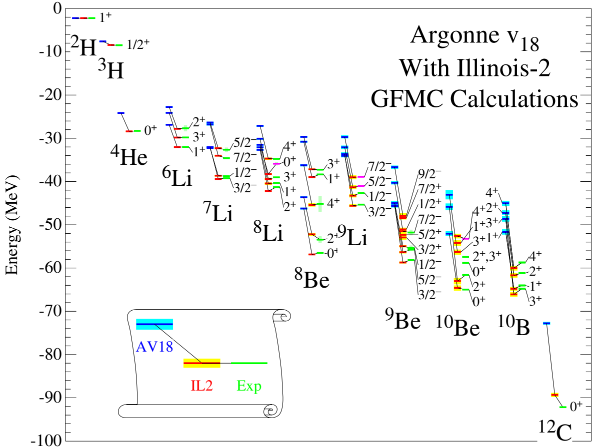
Chiral effective field theory (χEFT)

- The effective theory of QCD at low energies
- Nuclear potential based on power-counting (Q/Λ_χ expansion)
- low-energy constants determined from scattering data

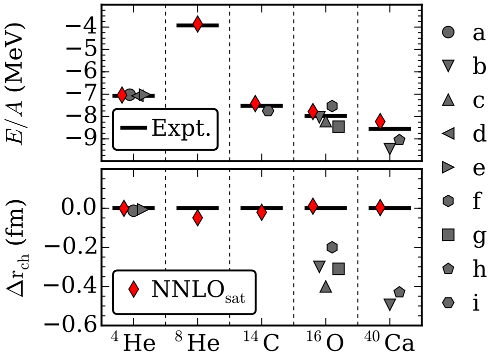


Nuclear ab inito calculations

Combined with quantum many-body calculations, state-of-the-art nuclear forces can accurately describe a wide range of nuclear structures



AV18+UIX Carlson al., RMP 87, 1067 (2015)



χEFT
Ekström et al., PRC 91, 051301 (2015)

Ab-initio calculations of nuclear polarizability $\delta_{\rm pol}$

- $\mu^{2,3}$H, $\mu^{3,4}$He$^+$:
  - Numerical ab-initio methods

    Effective Interaction Hyperspherical Harmonics Expansion
    Lorentz Integral Transform (response function)
    Lanczos Algorithm (sum rules)

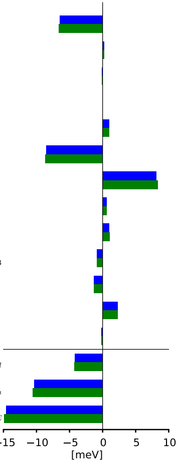
    bound state $\rightarrow$ resonance/continuum
  - Nuclear potentials

    AV18+UIX
    χEFT NN(N^{3}LO)+NNN(N^{2}LO)
    Analyze nuclear-theory uncertainty by comparing $\delta_{\rm pol}$ from different potential models

CJ, Nevo-Dinur, Bacca, Barnea, PRL 111 (2013) 143402
Hernandez, CJ, Bacca, Nevo-Dinur, Barnea, PLB 736 (2014) 344
Nevo Dinur, CJ, Bacca, Barnea, PLB 755 (2016) 380
Hernandez, Ekström, Nevo Dinur, CJ, Bacca, Barnea, PLB 788 (2018) 377
CJ, Bacca, Barnea, Hernandez, Nevo-Dinur, JPG 45 (2018) 093002
Emmons, CJ, Platter, JPG 48 (2021) 035101
CJ, Zhang, Platter, PRL 133 (2024) 042502

TPE & nuclear polarizability: nuclear-model uncertainty

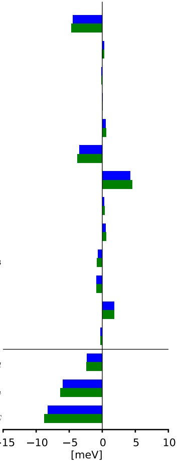
δ_{TPE} in $\mu\,^3$He$^+$



	AV18/UIX	χEFT
$\delta_{D1}^{(0)}$	-6.479	-6.633
$\delta_{L}^{(0)}$	0.232	0.240
$\delta_{T}^{(0)}$	-0.103	-0.107
$\delta_{M}^{(0)}$	0.006	0.006
$\delta_{C}^{(0)}$	1.000	1.020
$\delta_{R3}^{(1)}$	-8.539	-8.711
$\delta_{Z3}^{(1)}$	8.100	8.327
$\delta_{R^2}^{(2)}$	0.632	0.654
$\delta_{Q}^{(2)}$	1.015	1.038
$\delta_{D1D3}^{(2)}$	-0.841	-0.862
$\delta_{R1}^{(1)}$	-1.294	-1.314
$\delta_{Z1}^{(1)}$	2.256	2.291
$\delta_{NS}^{(2)}$	-0.193	-0.199
δ_{pol}^{A}	-4.208	-4.250
δ_{Zem}^{A}	-10.356	-10.618
δ_{TPE}^{A}	-14.564	-14.868

$\delta_{\mathrm{TPE}} = -14.72$ meV $\pm 1.5\%(1\sigma)$

δ_{TPE} in $\mu\,^4$He$^+$



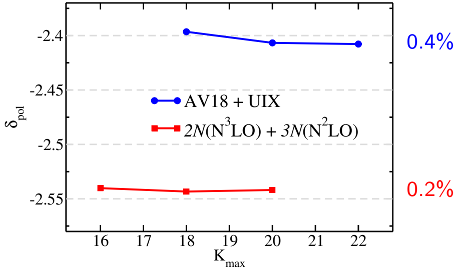
	AV18/UIX	χEFT
$\delta_{D1}^{(0)}$	-4.418	-4.701
$\delta_{L}^{(0)}$	0.289	0.308
$\delta_{T}^{(0)}$	-0.126	-0.134
$\delta_{M}^{(0)}$	0.008	0.006
$\delta_{C}^{(0)}$	0.519	0.553
$\delta_{R3}^{(1)}$	-3.442	-3.717
$\delta_{Z3}^{(1)}$	4.183	4.526
$\delta_{R^2}^{(2)}$	0.259	0.324
$\delta_{Q}^{(2)}$	0.484	0.561
$\delta_{D1D3}^{(2)}$	-0.666	-0.784
$\delta_{R1}^{(1)}$	-0.848	-0.874
$\delta_{Z1}^{(1)}$	1.753	1.811
$\delta_{NS}^{(2)}$	-0.279	-0.293
δ_{pol}^{A}	-2.284	-2.414
δ_{Zem}^{A}	-5.936	-6.337
δ_{TPE}^{A}	-8.220	-8.751

$\delta_{\mathrm{TPE}}^{A} = -8.49$ meV $\pm 4.4\%(1\sigma)$

TPE & nuclear polarizability: other uncertainty

Numerical uncertainty

- convergence of EIHH model space (μ^4He$^+$)



Atomic-physics uncertainty

- $(Z\alpha)^6$ correction three-photon exchange
- relativistic and Coulomb distortion effects to sum rules beyond E1
- higher-order nucleonic-structure corrections
- Overall atomic-physics uncertainty
  - 1.5% in μ^3He$^+$
  - 1.3% in μ^4He$^+$

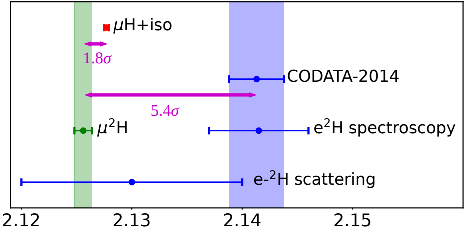
- Combine all uncertainties:

  $\delta_{\rm TPE}(\mu^3{\rm He}^+) = -14.72$ meV $\pm 2.1\%$

  $\delta_{\rm TPE}(\mu^4{\rm He}^+) = -8.49$ meV $\pm 4.6\%$
- The TPE prediction fulfills the 5% accuracy requirements from $\mu^{3,4}$He$^+$ experiments

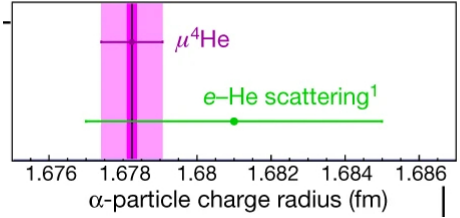
Nuclear charge radii from Lamb shifts in μ^2H and $\mu^{3,4}$He

- Our predictions of nuclear TPE effects have been used by CREMA to extract nuclear charge radii from Lamb shift measurements
- Theoretical uncertainties in TPE effects dominate the error in the extracted nuclear charge radii



$r_d = 2.12562(13)_{\rm exp}(77)_{\rm theo}$ fm

Pohl, *et al.*, Science (2016)



$r_\alpha = 1.67824(13)_{\rm exp}(82)_{\rm theo}$ fm

Krauth et al., Nature (2021)

TPE theory:

Hernandez, CJ, Bacca, Nevo-Dinur, Barnea, PLB 736 (2014) 344; PRC 100 (2019) 064315 (μ^2H)

Hernandez, Ekström, Nevo Dinur, CJ, Bacca, Barnea, PLB 788 (2018) 377 (μ^2H)

CJ, Nevo-Dinur, Bacca, Barnea, PRL 111 (2013) 143402 (μ^4H)

CJ, Bacca, Barnea, Hernandez, Nevo-Dinur, JPG 45 (2018) 093002 ($\mu^{2,3}$H, $\mu^{3,4}$He$^+$)

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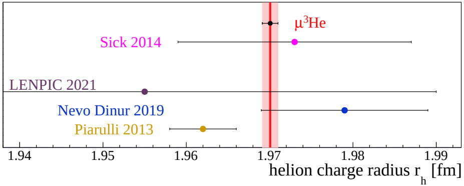
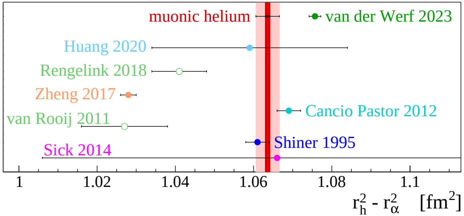


$$r_h = 1.97007(12)_{\rm exp}(93)_{\rm theo}\ \text{fm}$$

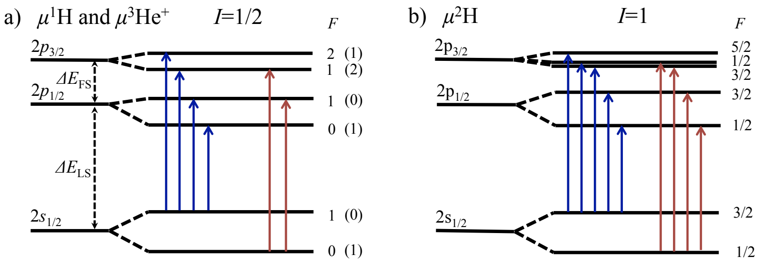


$$r_h^2 - r_\alpha^2 = 1.0636(6)_{\rm exp}(30)_{\rm theo}\ \text{fm}^2$$

Schuhmann et al. (CREMA) arXiv:2305.11679

TPE theory:
Nevo Dinur, CJ, Bacca, Barnea, PLB 755 (2016) 380 (μ^3H, μ^3He$^+$)
CJ, Bacca, Barnea, Hernandez, Nevo-Dinur, JPG 45 (2018) 093002 ($\mu^{2,3}$H, $\mu^{3,4}$He$^+$)

Nuclear Zemach Radius from μ-atom Hyperfine Splitting



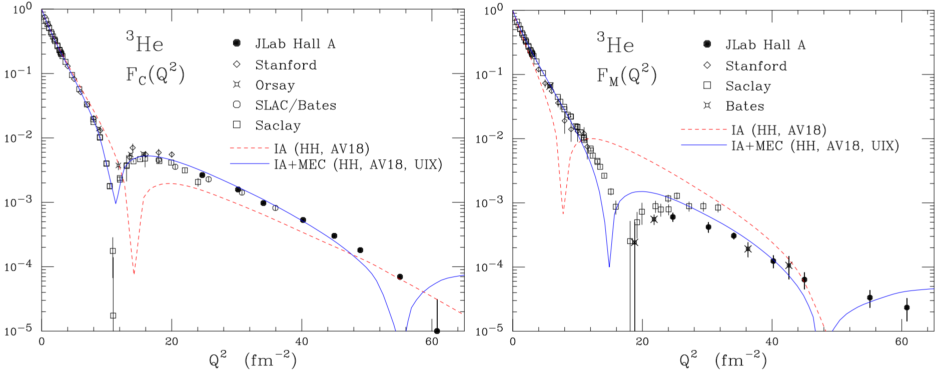
- Zemach radius R_Z depends on both nuclear charge and magnetic distributions

$$R_Z = \iint d\boldsymbol{r} d\boldsymbol{r}' \rho_E(\boldsymbol{r}) \rho_M(\boldsymbol{r}') |\boldsymbol{r} - \boldsymbol{r}'|$$

- μH hyperfine structure: CREMA, J-PARC, RIKEN-RAL
- μ^3He$^+$ hyperfine structure: CREMA

Discrepancies in Nuclear Magnetic Distribution Studies

- ^{3}**He** charge form factor F_C: agreement between experiment and theory
- ^{3}**He** magnetic form factor F_M: discrepancies between experiment and theory at high Q
- Muonic atom spectroscopy provides more precise measurements for nuclear magnetic distributions



Nuclear Structure Effects in Hyperfine Splitting

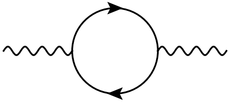
- Extract nuclear Zemach radius from e/μ atom hyperfine splitting

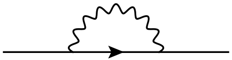
$$E_{\text{HFS}}(nS) = E_F(1 + \delta_{\text{QED}} + \delta_{\text{TPE}})$$

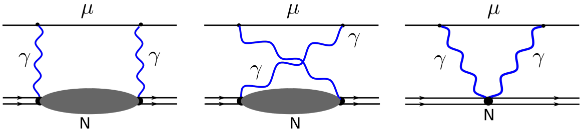
- Fermi contact term
  - Nuclear-lepton spin point coupling

$$E_F = \frac{2\pi\alpha g_m}{3m_\ell m_N}\phi_n^2(0)\langle\vec{\sigma}^{(\ell)}\cdot\vec{I}\rangle$$

- Quantum electrodynamics corrections
  - Vacuum polarization effects
  - Lepton self-energy corrections
  - Relativistic recoil corrections
- Two-photon exchange
  - Nuclear polarizability
  - Elastic terms (Zemach radius)







Nuclear Structure Effects in $e/\mu - D$ Hyperfine Splitting

- TPE terms dominate the discrepancy between measurements and QED predictions
- TPE effects in e^2H HFS: theory and experiment agree (controversial)
- TPE effects in μ^2H HFS: significant discrepancy between theory and experiment

e^2H 1S $E_{HFS}(2\gamma)$ [kHz]

$\nu_{\rm exp} - \nu_{\rm qed}$	45 [1]
Khriplovich, Milstein 2004	43 (model dependent)
Friar 2005	46 $(+18)$
	$(1N$ pol/recoil)

μ^2H 2S $E_{HFS}(2\gamma)$ [meV]

$\nu_{\rm exp} - \nu_{\rm qed}$	0.0966(73) [2]
Kalinowski, Pachucki 2018	0.0383

[1] Wineland, Ramsey, PRA (1972)
[2] Pohl et al., Science (2016)

Nuclear Structure Effects in $e/\mu - D$ Hyperfine Splitting

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- Previous theories did not rigorously treat nuclear excitations (polarizability terms)

PHYSICAL REVIEW LETTERS **133**, 042502 (2024)

Nuclear Structure Effects on Hyperfine Splittings in Ordinary and Muonic Deuterium

Chen Ji,[1,2,*] Xiang Zhang,[1] and Lucas Platter[3,4]
[1]Key Laboratory of Quark and Lepton Physics, Institute of Particle Physics, Central China Normal University, Wuhan 430079, China
[2]Southern Center for Nuclear-Science Theory, Institute of Modern Physics, Chinese Academy of Sciences, Huizhou 516000, China
[3]Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA
[4]Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA

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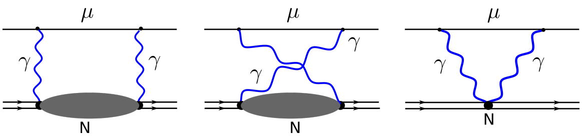
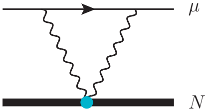
TPE Corrections in ^{2}H and μ^2H Hyperfine Splitting





- TPE corrections

$$E_{\mathrm{TPE}} = E_{\mathrm{el}} + E_{\mathrm{pol}} + E_{\mathrm{1N}}$$

- Elastic part $F_c(q)$, $F_m(q)$, $F_Q(q)$: $\sim r_Z$
- Inelastic vector polarization
- E_{1N}: single-nucleon TPE

$$\delta^{(0,1)}_{\mathrm{pol}} \propto \int d\omega \int dq h^{(0,1)}(\omega, q) S^{(0,1)}(\omega, q)$$

Charge-magnetic current interference: $S^{(0)}(\omega, q) = -\frac{1}{q^2}\mathrm{Im} \sum\!\!\!\!\!\!\int_{N \neq N_0} \int \frac{d\hat{q}}{4\pi} \langle N_0 II| \left[\vec{q} \times \vec{J}^{\dagger}_m(\vec{q})\right]_3 |N\rangle\langle N|\rho(\vec{q})|N_0 II\rangle\delta(\omega -$

Convection-magnetic current interference: $S^{(1)}(\omega, q) = -\mathrm{Im} \sum\!\!\!\!\!\!\int_{N \neq N_0} \int \frac{d\hat{q}}{4\pi} \epsilon^{3jk} \langle N_0 II|\vec{J}^{\dagger}_{m,j}(\vec{q})|N\rangle\langle N|\vec{J}_{c,k}(\vec{q})|N_0 II\rangle|N_0 I$

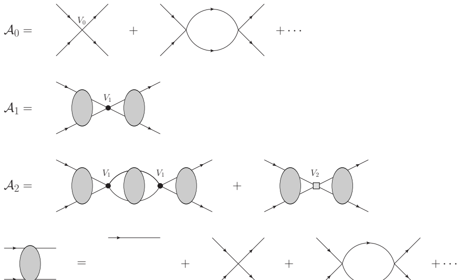
- Pionless effective field theory $\not{\pi}$EFT calculation   CJ*, Zhang, Platter, Phys. Rev. Lett. 133, 042502 (2024)

Nuclear Polarizability Effects with $\not\pi$EFT

- $\not\pi$EFT constructs low-energy NN and NNN interactions through contact potentials
- Predictions require only a few inputs: a_t, r_t, Q_d at NNLO (5% accuracy)

$$
\begin{aligned}
\mathcal{L} \quad = \quad & N^\dagger \left[i\partial_0 + \frac{\nabla^2}{2M} \right] N - C_0 \left(N^T P_i N\right)^\dagger \left(N^T P_i N\right) \\
& + \frac{1}{8} C_2 \left[\left(N^T P_i N\right)^\dagger \left(N^T \overleftrightarrow{\nabla}^2 P_i N\right) + h.c. \right] - \frac{1}{16} C_4 \left(N^T \overleftrightarrow{\nabla}^2 P_i N\right)^\dagger \left(N^T \overleftrightarrow{\nabla}^2 P_i N\right) \\
& + \frac{1}{4} C_0^{(sd)} \left\{ \left(N^T P^i N\right)^\dagger \left[N^T P^j \left(\overleftrightarrow{\nabla}_i \overleftrightarrow{\nabla}_j - \frac{1}{3}\delta_{ij} \overleftrightarrow{\nabla}^2 \right) N \right] + \text{h.c.} \right\}
\end{aligned}
$$

- np scattering t-matrix $\mathcal{A}_n$ order-by-order expansion:



- on-shell t-matrix

$$
\mathcal{A}_t(p,p;E) = -\frac{4\pi}{m_N}\frac{1}{\gamma + ip}\left[1 + \rho(\gamma - ip)/2 + \rho^2(\gamma - ip)^2/4\right]
$$

- off-shell t-matrix

$$
\mathcal{A}_t^{(0)}(k,p;E) = -\frac{4\pi}{m_N}\frac{1}{\gamma + ip}
$$

$$
\mathcal{A}_t^{(1)}(k,p;E) = -\frac{2\pi}{m_N}\frac{\rho}{\gamma + ip}\left[\gamma - ip + \frac{1}{2(\gamma - \mu)}\left(k^2 - p^2\right)\right]
$$

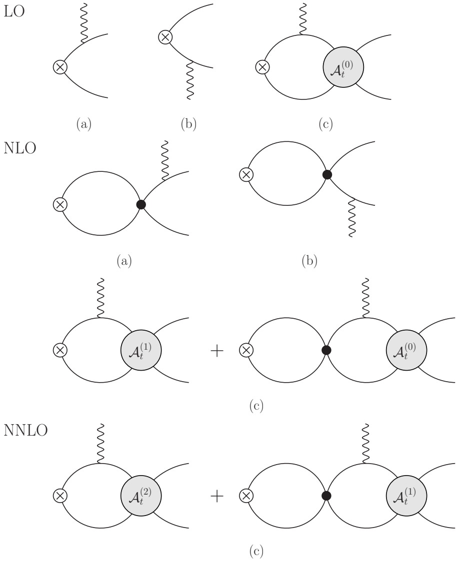
$$
\mathcal{A}_t^{(2)}(k,p;E) = -\frac{\pi}{m_N}\frac{\rho^2}{\gamma + ip}\left[(\gamma - ip)^2 + \frac{\gamma - ip}{\gamma - \mu}\left(1 + \frac{\gamma + ip}{\gamma - \mu}\right)\frac{k^2 - p^2}{2}\right]
$$

TPE Corrections in ^{2}H and μ^2H Hyperfine Splitting

- 1N electromagnetic current operators:

  Nuclear charge density ρ_E, convection current $\vec{J}_c$, and magnetic current $\vec{J}_m$ all contribute to HFS

$$\mathcal{L}_{\text{EM},1b} = -eN^\dagger \frac{1+\tau_3}{2} N A_0 - \frac{ie}{2m_N}\left[N^\dagger \overleftrightarrow{\nabla} \frac{1+\tau_3}{2} N\right] \cdot \vec{A} + \frac{e}{2m_N} N^\dagger \left(\kappa_0 + \kappa_1 \tau_3\right) \vec{\sigma} \cdot \vec{B} N$$

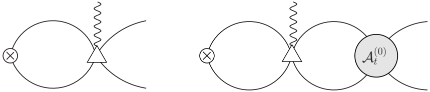


TPE Corrections in ^{2}H and μ^2H HFS

- $2N$ convection and magnetic currents (meson exchange current):

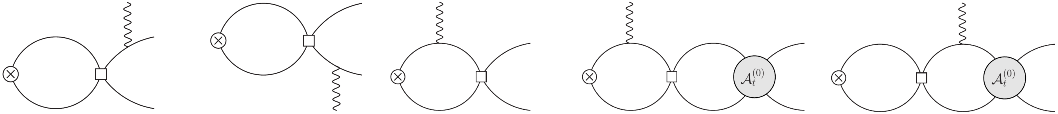
$$\mathcal{L}_{2,C} = ie\frac{C_2}{4}\left[(N^T P_i N)^\dagger (N^T \overleftrightarrow{\nabla} P_i \tau_3 N) + \text{h.c.}\right] \cdot \vec{A}$$

$$\mathcal{L}_{2,B} = -ieL_2\epsilon_{ijk}\left(N^T P_i N\right)^\dagger \left(N^T P_j N\right) B_k + \text{h.c.}$$



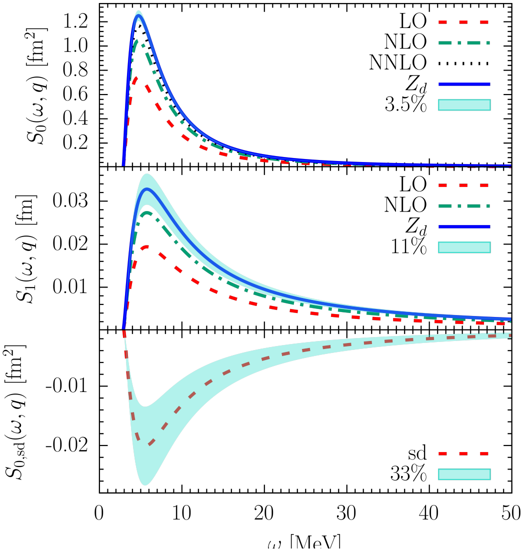
- SD mixing coupling:

$$\mathcal{L}_{2,Q} = -eL_Q\left(N^T P_i N\right)^\dagger \left(N^T P_j N\right)\left(\nabla^i\nabla^j - \frac{1}{3}\nabla^2\delta_{ij}\right) A_0$$



Nuclear Response Functions in TPE effects to HFS

- $S^{(0)}(\omega, q)$: charge density-magnetic current transitions (LO)
- $S^{(1)}(\omega, q)$: convection current-magnetic current transitions (NLO)
- $S^{(0)}_{\rm sd}(\omega, q)$: SD coupling corrections (NNLO)
- Nuclear response functions converge order by order



CJ*, Zhang, Platter, Phys. Rev. Lett. 133, 042502 (2024)

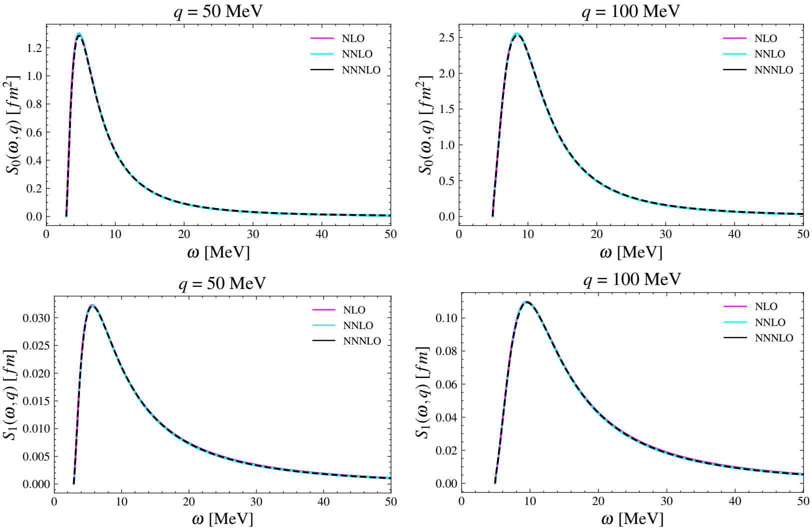
TPE Corrections in ^{2}H and μ^2H HFS ($\not{\pi}$EFT)

	^{2}H (1S)	μ^2H (1S)	μ^2H (2S)
E_{1p} (Antognini 2022)	$-35.54(8)$	$-1.018(2)$	$-0.1272(2)$
E_{1n} (Tomalak 2019)	$9.6(1.0)$	$0.08(3)$	$0.010(4)$
E_{el}	$-42.1(2.1)$	$-0.984(46)$	$-0.123(6)$
E_{pol}	$109.8(4.5)$	$2.86(12)$	$0.358(14)$
E_{TPE}	kHz	meV	meV
This work	$41.7(4.4)$	$0.94(11)$	$0.117(13)$
Khriplovich, Milstein 2004	43		
Friar, Payne 2005 $_{mod}$	64.5		
Kalinowskim, Pauckci 2018		$0.304(68)$	$0.0383(86)$
$\nu_{exp} - \nu_{qed}$	45.2		$0.0966(73)$

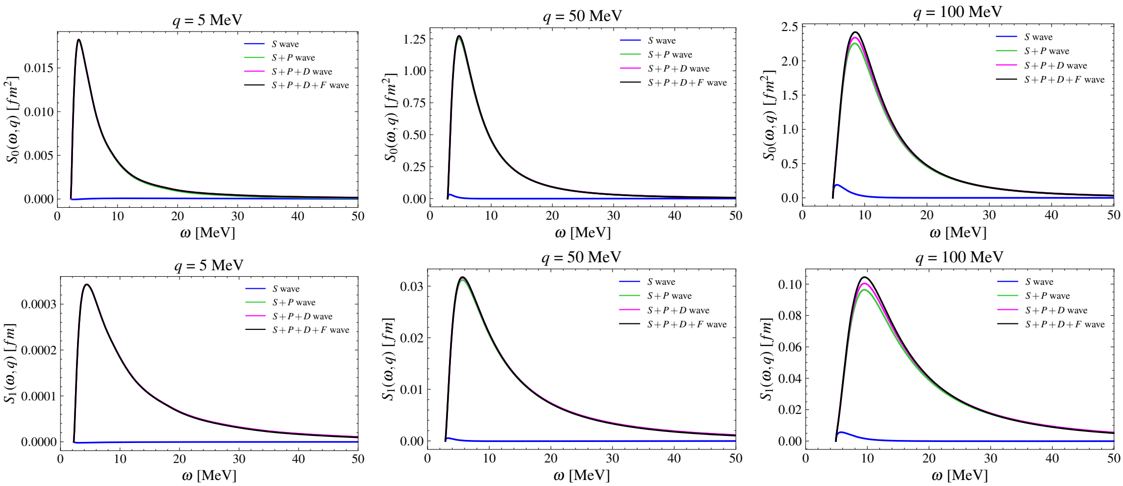
- Our work well explains the experiment-QED theory discrepancy $(0.8 - 1.3\sigma)$
- Theoretical uncertainties in single-nucleon TPE might be underestimated (chiral perturbation and dispersion relation calculations differ by 1 order)

CJ, Zhang, Platter, Phys. Rev. Lett. 133, 042502 (2024)

Nuclear Response Functions from χEFT (Convergence)



Nuclear Response Functions from χEFT (Partial Waves)



Nuclear Polarizability Contribution to ^{2}H and μ^2H HFS (χEFT)

Hyperfine Energy Shift E_{pol}			
	$^2\mathrm{H}(1S)$ kHz	$\mu^2\mathrm{H}(1S)$ meV	$\mu^2\mathrm{H}(2S)$ meV
$\mathrm{NLO}_{\mathrm{RS450}}$	109.5	2.819	0.352
$\mathrm{N2LO}_{\mathrm{RS450}}$	109.4	2.816	0.352
$\mathrm{N3LO}_{\mathrm{Idaho}}$	110.2	2.834	0.354
Pionless EFT	109.8(4.5)	2.86(12)	0.358(14)

- Preliminary results of nuclear polarizability contributions to HFS in χEFT
- Consistent with $\not\pi$EFT prediction

TPE effects to HFS in other atoms

- TPE effects to HFS in other atoms lack systmatic studies
  - How does nuclear polarizability change with Z and A?
  - contributions from two-nucleon currents?
- Without direct calculation, extract nuclear polarizability through:
  - HFS spectroscopy - QED - scattering data (Zemach radius)
  - Unclear mechanism of nuclear polarizability effects

PHYSICAL REVIEW A **107**, 052802 (2023)

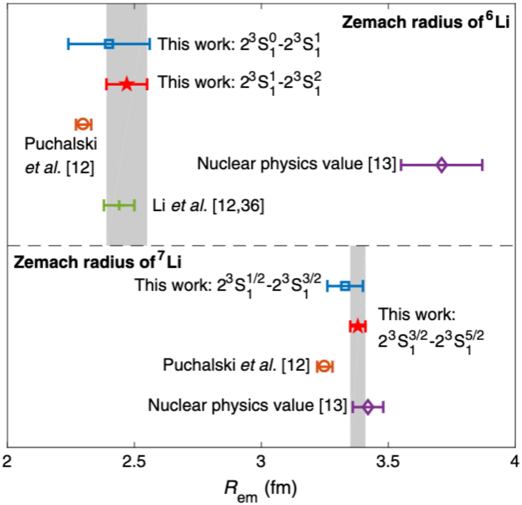
Nuclear polarizability effects in $^3He^+$ hyperfine splitting

Vojtěch Patkóš,[1] Vladimir A. Yerokhin,[2] and Krzysztof Pachucki[3]
Faculty of Mathematics and Physics, Charles University, Ke Karlovu 3, 121 16 Prague 2, Czech Republic
[2]*Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany*
[3]*Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland*

"We obtain a surprising result that the nuclear polarizability of the helion yields just 3% of the total nuclear correction, which is smaller than for the proton."

TPE effects to HFS in ^{6,7}Li

- $R_z(^7\text{Li})$ from HFS spectroscopy - QED agrees with R_z from e-^{7}Li scattering
- large discrepancy exists for $R_z(^6\text{Li})$
- imply that $\delta_{pol}(^7\text{Li})$ ll $\delta_{pol}(^6\text{Li})$



Puchalski, Pachucki, PRL 111, 243001 (2013)
Qi et al., PRL 125, 183002 (2020)
Li et al., PRL 124, 063002 (2020)
Guan et al., PRA 102, 030801(R) (2020)

Conclusion

- radius puzzle & spectroscopy in hydrogen-like atoms
  - Challenge higher-order QED theory
  - TPE effects connect atomic transition with photo-nuclear reaction
  - Use low-energy nuclear theory to probe precision physics
- TPE effects to Lamb shift
  - determine nuclear charge radii
  - Ab inito calculations improve theoretical accuracy to percentage
  - more accurate than extracting information from photonuclear reaction data
- TPE effects to hyperfine splitting
  - determine nuclear magnetic structure
  - Ab initio theory to determine TPE effects to HFS
  - further improve accuracy in nuclear theory (χEFT, or $\not\pi$EFT at N^{3}LO)
  - uncertainty in nucleonic TPE needs to be reanalyzed
  - Future extension to study TPE effects to HFS in μ^3He, $e^{6,7}$Li

Collaborators

O.J. Hernandez, S. Bacca, T. Richardson	Johannes Gutenberg-Universität Mainz
N. Nevo-Dinur	TRIUMF
N. Barnea	Hebrew University of Jerusalem
A. Ekström	Chalmers University of Technology
S. Pastore, M. Piarulli	Washington University
R.B. Wiringa	Argonne National Laboratory
J.L. Bonilla, L. Platter	University of Tennessee, Knoxville
S.B. Emmons	Carson-Newman University