



中国科学院大学

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The Inflation Trilogy and Primordial Black Holes

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Based on arXiv: 2410.10996 (Paulo B. Ferraz, João G. Rosa)

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Outline

Overview of cosmological inflation

Motivation

The Model

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Conclusion

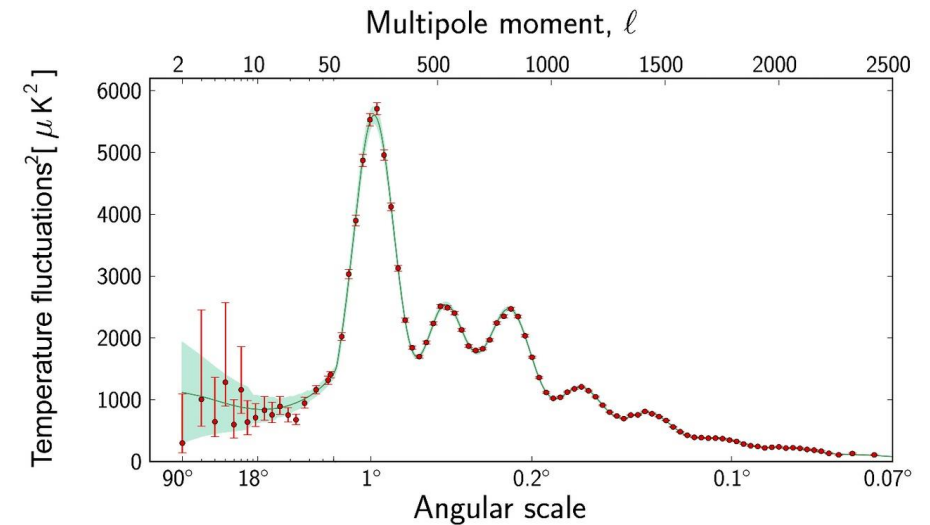
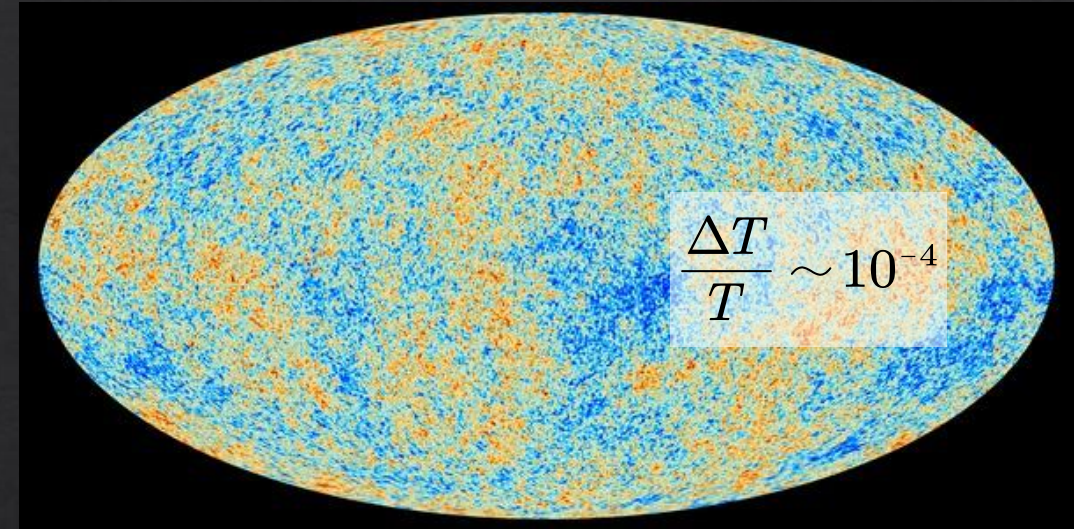
Overview of Cosmological Inflation

Why Inflation?

- Flatness problem
- Horizon problem
- Origin of primordial density fluctuation

(Primordial) density fluctuations are:

- small
- scale-invariant
- Gaussian
- ...

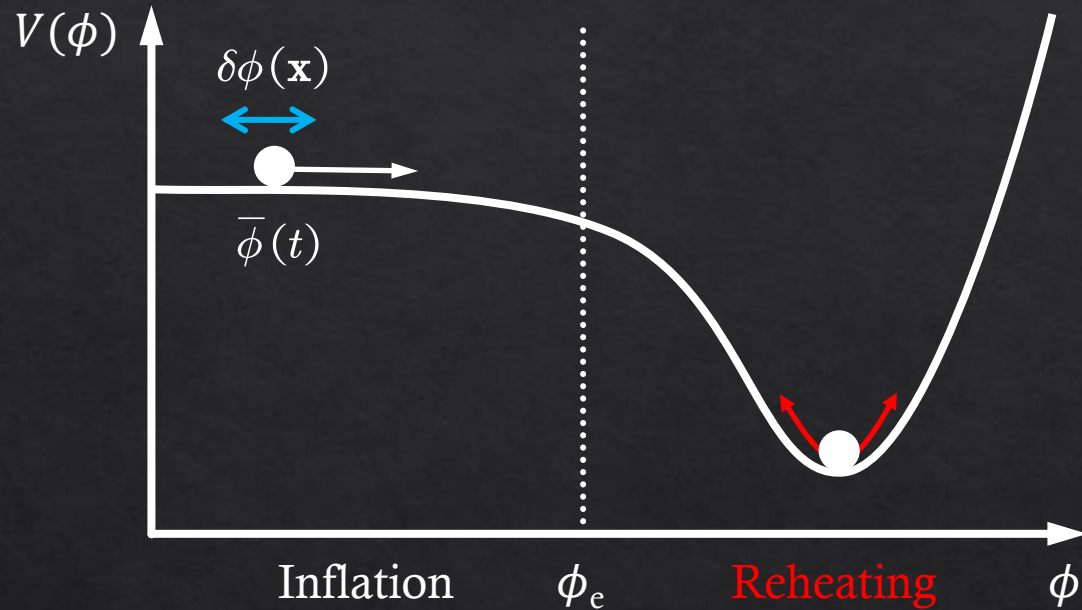


[Planck 2013]

Cold Inflation

$$H^2 \approx \frac{V}{3M_{\text{pl}}^2}$$

Slow-roll Inflation



EOM:

$$\cancel{\ddot{\phi}} + 3H\dot{\phi} + V'(\phi) = 0$$

Slow-roll conditions:

$$\epsilon_V = \frac{M_{\text{pl}}^2}{2} \left(\frac{V'}{V} \right)^2 \ll 1, \quad \eta_V = M_{\text{pl}}^2 \frac{V''}{V} \ll 1$$

- In cold inflation, interactions of the inflaton with other fields are considered negligible:

$$\dot{\rho}_r + 4H\rho_r = 0 \Rightarrow \rho_r \sim 1/a^4 \Rightarrow \text{Universe supercools}$$

↪ “Radiation density redshifts away”

Cold Inflation

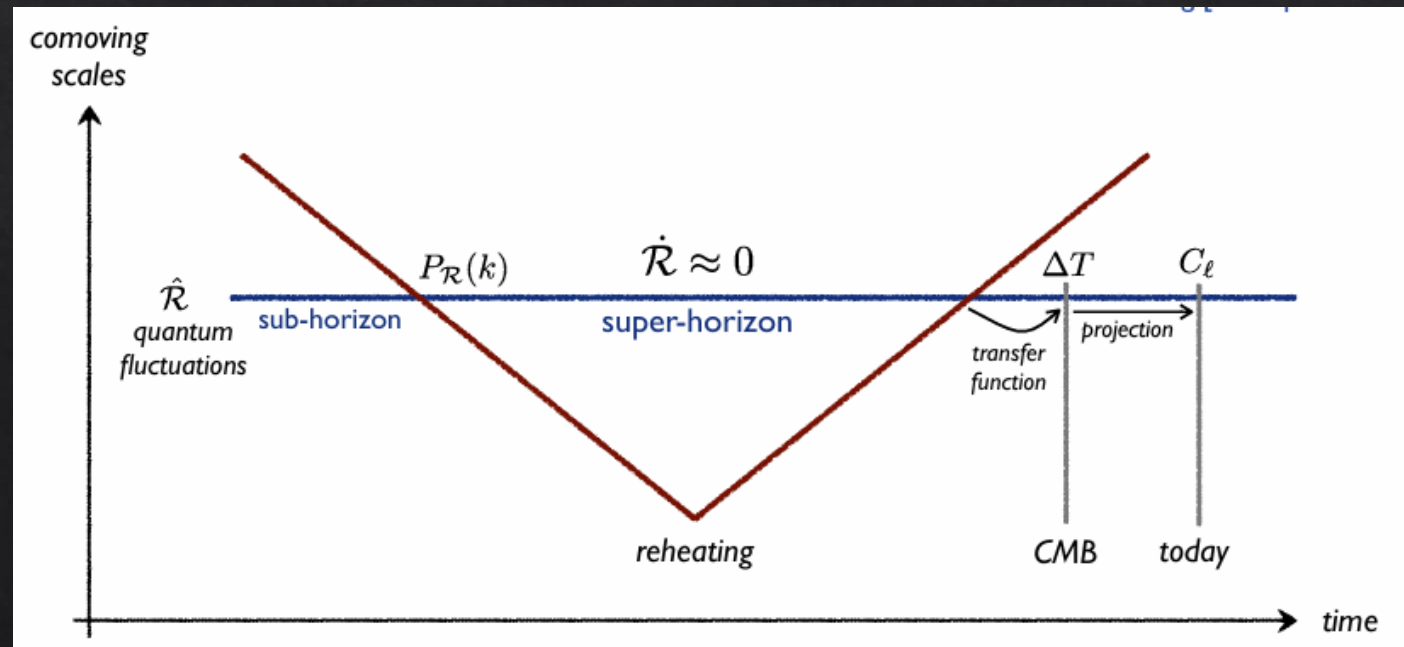
- Quantum fluctuations:

$$\delta\phi(\mathbf{x}) \sim H$$

- Primordial curvature power spectrum:

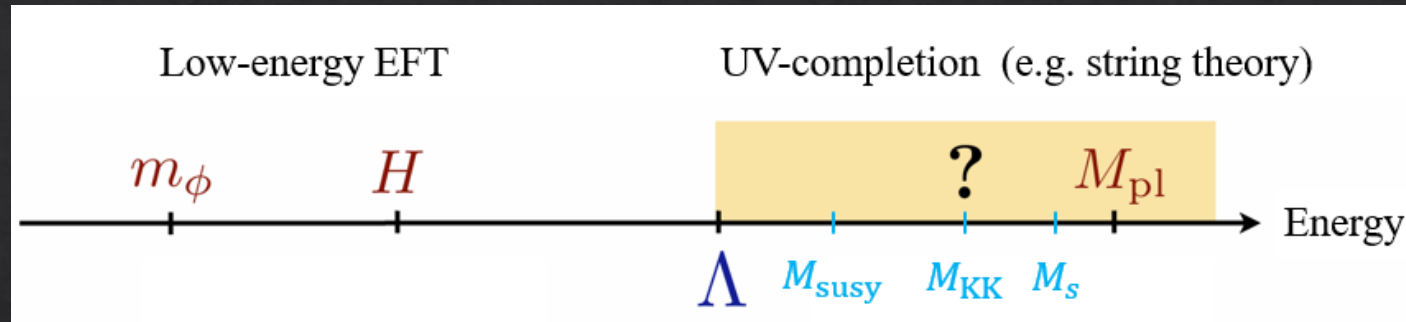
$$\Delta_{\mathcal{R}}^2(k) = \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k) = A_s(k_*) \left(\frac{k}{k_*} \right)^{n_s-1}, \quad A_s(k_*) \simeq 10^{-9}, n_s \simeq 0.968$$

- nearly scale-invariant
- slightly red-tilted spectrum



Cold Inflation

- Shortcoming: eta-problem



Successful inflation requires that the inflaton mass is smaller than the Hubble scale

$$\eta = \frac{m_\phi^2}{3H^2} \sim \mathcal{O}(0.01) .$$

Integrating out the massive particles yields

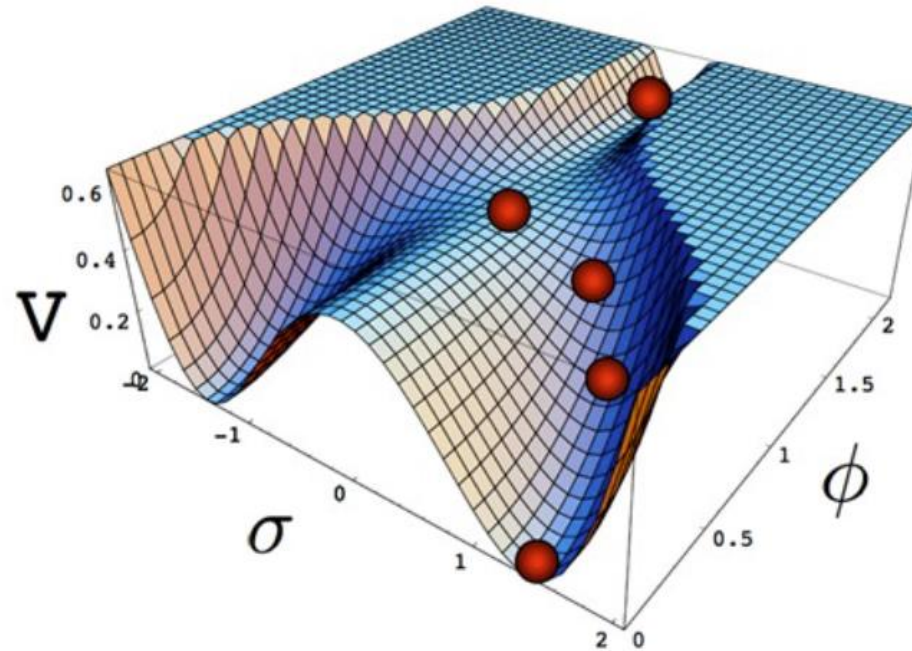
$$\Delta V = c_1 V(\phi) \frac{\phi^2}{\Lambda^2} ,$$

$$\Rightarrow \Delta\eta_V = \frac{M_{\text{pl}}^2}{V} (\Delta V)'' \approx 2c_1 \left(\frac{M_{\text{pl}}}{\Lambda} \right)^2 > 1 .$$

Cold Inflation

A sample of hybrid inflation

$$V(\sigma, \phi) = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\sigma^2$$



ϕ : inflaton

σ : waterfall field

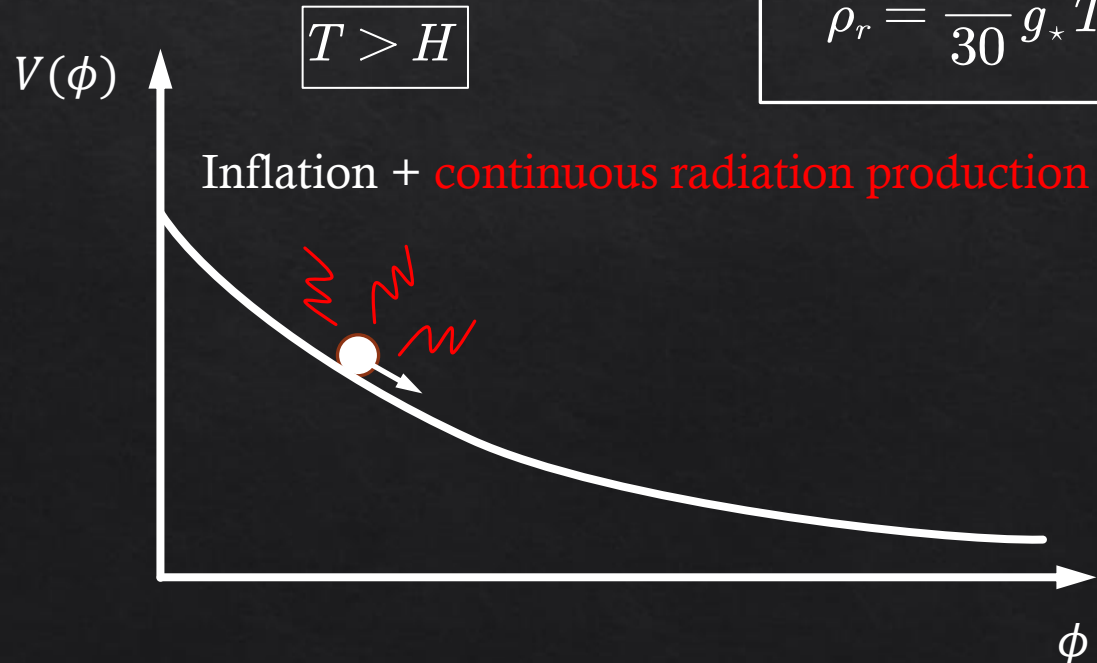
Warm Inflation

- Interactions of the inflaton with other d.o.f. are important during inflation, (generate dissipation terms) small fraction of vacuum energy density can be converted to radiation

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V'(\phi) = 0$$

$$\dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2$$

$$\rho_r = \frac{\pi^2}{30} g_* T^4$$



$$\frac{\Upsilon}{3H} \equiv Q > 1$$

$$\epsilon_H = -\frac{\dot{H}}{H^2} \simeq \frac{1+Q}{2M_{\text{pl}}^2} \frac{\dot{\phi}^2}{H^2} \simeq \frac{\epsilon_V}{1+Q} < 1 \Rightarrow \epsilon_V < 1+Q$$

$$\eta_H = \frac{\dot{\epsilon}_H}{H\epsilon_H} < 1 \Rightarrow \eta_V < 1+Q$$

Warm Inflation

- Eta-problem can be alleviated in warm inflation due to the additional dissipative friction.
- Primordial density fluctuations are sourced by thermal fluctuations:

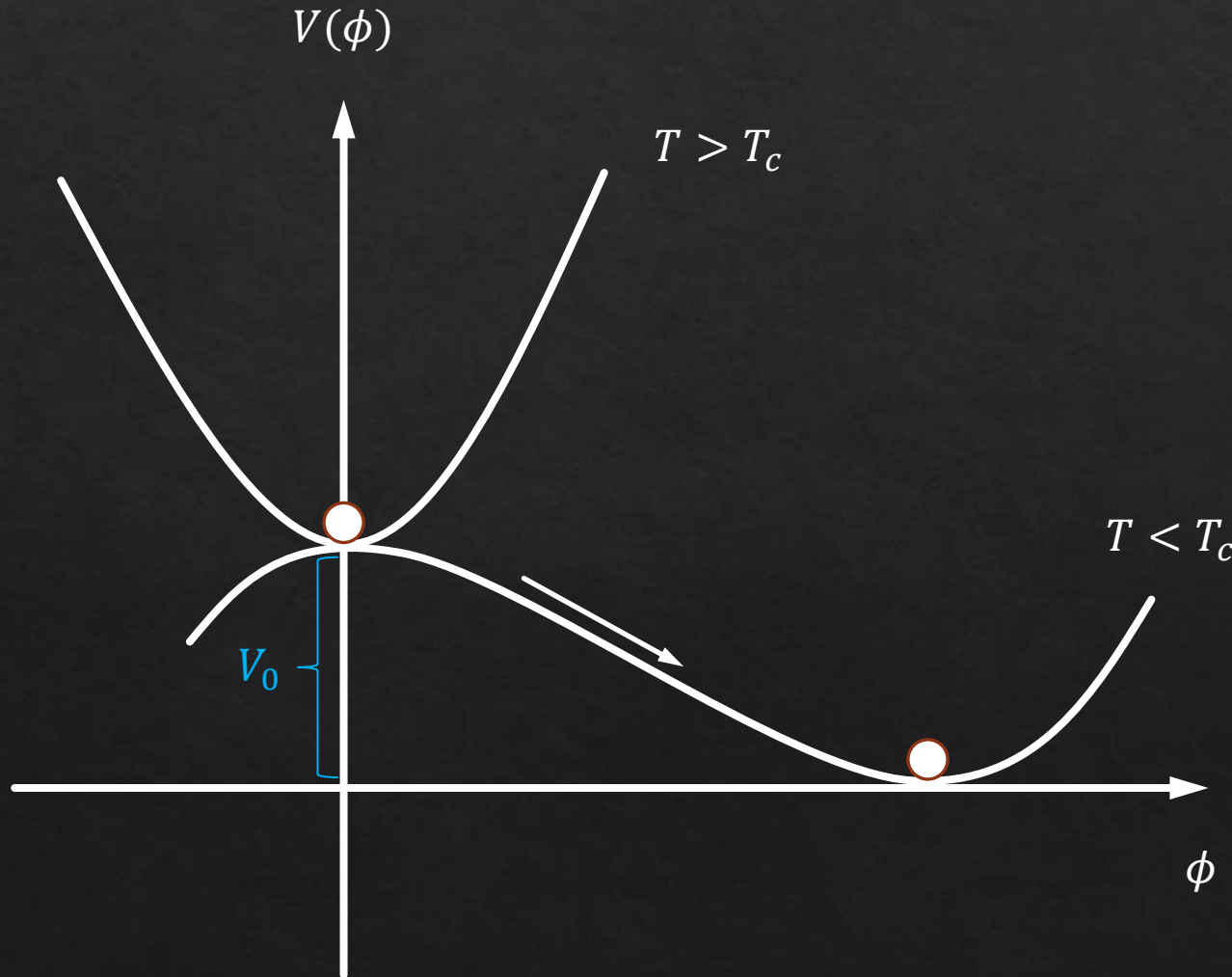
$$\delta\phi \sim H + \sqrt{HT} + \sqrt{\Upsilon T}$$

- Strong dissipative effects lead to a blue-tilted primordial spectrum

$$\Delta_R^2(k) = A_s \left(\frac{k}{k_\star} \right)^{n_s - 1}, \quad n_s > 1$$

Thermal inflation [Lyth and Stewart, 1995]

- Occurs after reheating.
- Dilute the unwanted cosmological relics.



$$V(\phi, T) = V_0 + \frac{1}{2} (g^2 T^2 - m^2) \phi^2 + \frac{\lambda^2 \phi^6}{M_p^2}$$

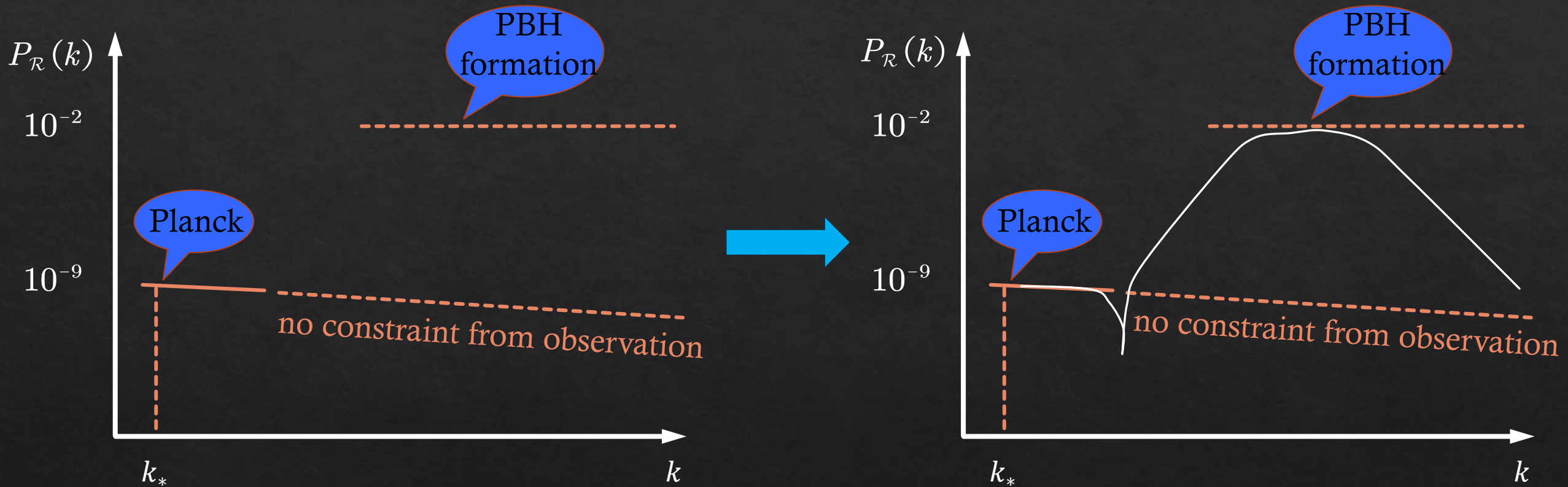
$$T_t = \left(\frac{30}{\pi^2 g_*} \right)^{1/4} V_0^{1/4}, \quad T_c \equiv m/g$$

$$N = \log \frac{T_t}{T_c}$$

Motivation

- Requiring a single field to drive the full 50-60 e-folds of inflationary expansion may be too much to ask for in a theory
- Avoid eta-problem
- Explain all of the dark matter by PBHs

Motivation



Inflationary Trilogy

- Inflationary trilogy: undergoes three distinct stages of inflation – cold, warm and thermal – driven by three different scalar fields



- Cold inflation contributes most of the e-folds
- Warm inflation avoids eta-problem and produces PBHs
- Thermal inflation dilutes the unwanted relics

Inflationary Trilogy

Cold Inflation

Warm Inflation

Thermal Inflation

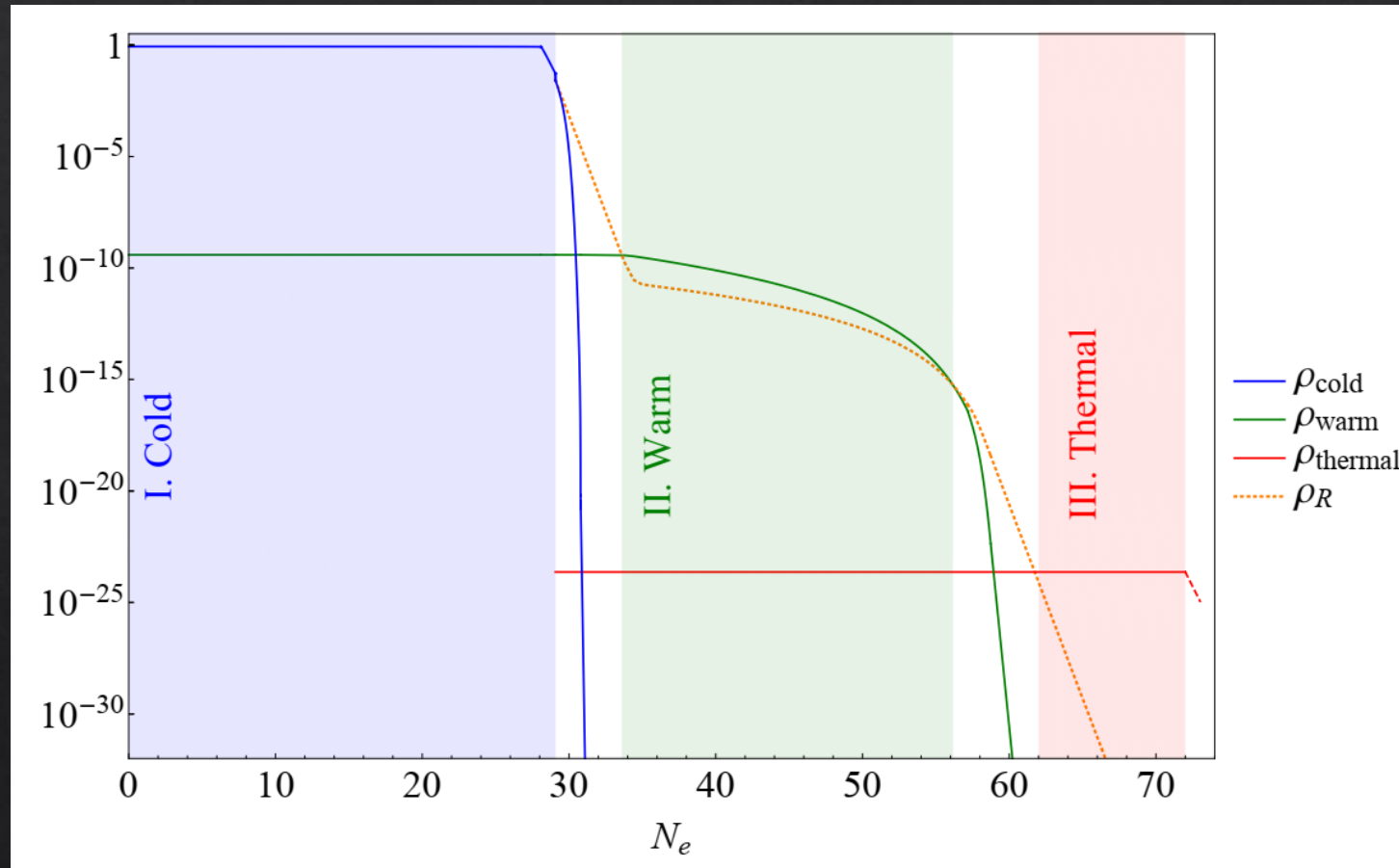


Fig. Evolution of the energy density of the relevant scalar fields and radiation fluid

The Model - Cold inflation

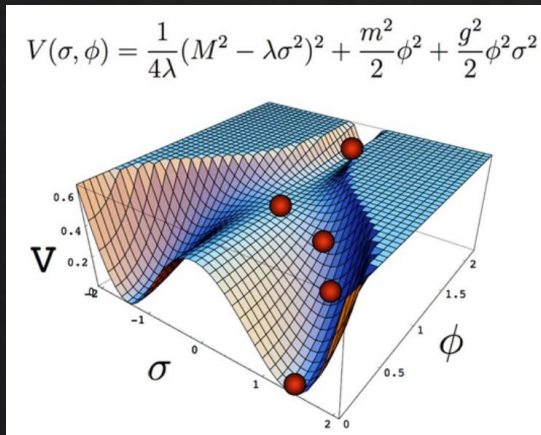
- The tree-level scalar potential in a SUSY hybrid inflation model is given by

$$V_0 = 2\kappa^2 |\phi_c|^2 |\sigma|^2 + \kappa^2 (|\sigma|^2 - M_c^2)^2.$$

ϕ_c : singlet chiral superfield – cold inflaton

$\sigma, \bar{\sigma}$: a pair of conjugate superfields – waterfall field

- After taking radiative corrections into account, and at 1-loop order these are given by



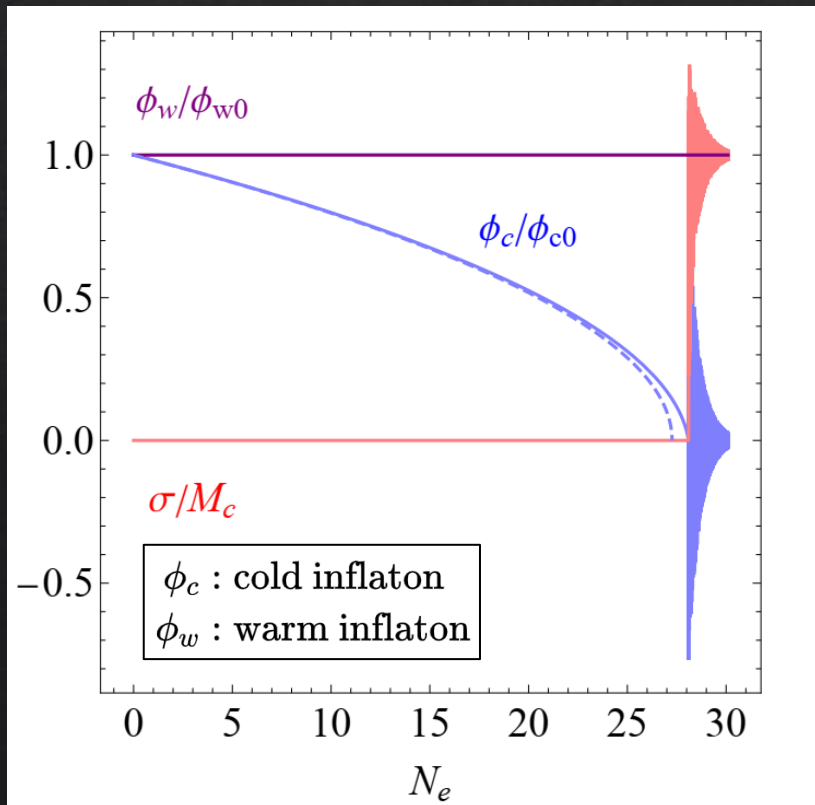
$$\Delta V_1 \simeq \frac{(\kappa M_c)^4}{8\pi^2} \mathcal{N}f[x], \quad x = \frac{|\phi_c|}{M_c},$$

$$f[x] = \frac{1}{4} \left((x^4 + 1) \log \frac{x^4 - 1}{x^4} + 2x^2 \log \frac{x^2 + 1}{x^2 - 1} + 2 \log \frac{\kappa^2 M_c^2 x^2}{Q^2} - 3 \right)$$

The Model - Cold inflation

- The evolution of the cold, warm and waterfall scalar fields

$$\ddot{\phi}_i + 3H\dot{\phi}_i + \frac{\partial V}{\partial \phi_i} = 0, \quad H^2 = \frac{\sum_i \frac{1}{2} \dot{\phi}_i^2 + V(\phi_c, \phi_w, \sigma)}{3M_{\text{pl}}^2}$$



$$A_s(k_*) \simeq 2.1 \times 10^{-9}, \quad n_s(k_*) \simeq 0.965, \quad k_* \simeq 0.005 \text{ Mpc}^{-1}$$

$$\Downarrow$$

$$n_s \simeq 1 - \frac{1}{N_c} \Rightarrow N_c \simeq 28$$

Parameters setting:

$$M_c = 6.6 \times 10^{15} \text{ GeV}, \quad \lambda = 7 \times 10^{-16}, \quad \kappa = 1, \quad \mathcal{N} = 1$$

Initial conditions for the fields:

$$\phi_{c,0} = 266 M_c, \quad \phi_{w,0} \simeq 27 M_c$$

The Model - Warm inflation

- **Warm Little Inflaton:** [\[Phys.Rev.Lett. 117 \(2016\) 15, 151301\]](#)

The warm inflaton ϕ_w is the resultant singlet scalar field from the collective spontaneous breaking of a $U(1)$ gauge symmetry:

$$\Phi_1 = \frac{M_w}{\sqrt{2}} e^{i(\alpha + \phi_w)/M_w}, \quad \Phi_2 = \frac{M_w}{\sqrt{2}} e^{i(\alpha - \phi_w)/M_w} \quad \langle \Phi_1 \rangle = \langle \Phi_2 \rangle = M_w / \sqrt{2},$$

M_w is the symmetry breaking scale.

- Consider interactions between the warm inflaton and other d.o.f. consistent with the gauge symmetry, we may build the interaction Lagrangian:

$$\begin{aligned} -\mathcal{L}_{\phi\psi} &= \frac{g}{\sqrt{2}} (\Phi_1 + \Phi_2) \bar{\psi}_{1L} \psi_{1R} - i \frac{g}{\sqrt{2}} (\Phi_1 - \Phi_2) \bar{\psi}_{2L} \psi_{2R} \\ &= gM_w \cos(\phi_w/M_w) \bar{\psi}_{1L} \psi_{1R} + gM_w \sin(\phi_w/M_w) \bar{\psi}_{2L} \psi_{2R} \end{aligned}$$

The Model - Warm inflation

$$Q \equiv \frac{\Upsilon}{3H}$$

- Warm inflation dynamics:

$$\begin{aligned}\dot{\rho}_R + 4H\rho_R &= \Upsilon \dot{\phi}_w^2, \\ \ddot{\phi}_w + (3H + \Upsilon)\dot{\phi}_w + V'(\phi_w) &= 0, \\ \rho_R &= \frac{\pi^2}{30} g_* T^4, \quad H^2 = \frac{\rho_w + \rho_R}{3M_{\text{pl}}^2}.\end{aligned}$$

Slow-roll approx.

$$\epsilon_w < 1 + Q$$

$$\begin{aligned}4H\rho_R &\simeq \Upsilon \dot{\phi}_w^2, \\ (3H + \Upsilon)\dot{\phi}_w + V'(\phi_w) &= 0.\end{aligned}$$

with

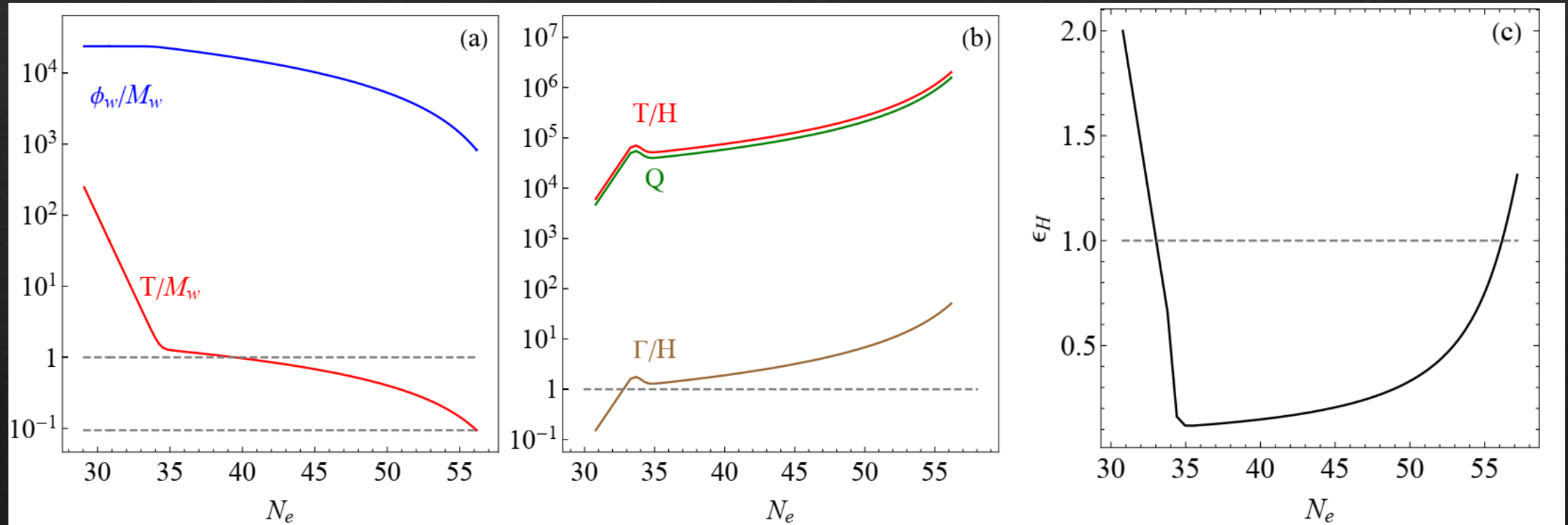
$$V_w(\phi_w) = \lambda\phi_w^4, \quad \Upsilon \simeq C_T T, \quad C_T = \frac{g^2}{h^2} \frac{3}{1 - 0.34 \ln h}$$

- After reheating, radiation is initially diluted as $\rho_R \sim a^{-4}$ until the dissipative source term becomes comparable. When $Q \gg 1$, we can see from $\epsilon_w < 1 + Q$ that the radiation smoothly takes over as the dominant component:

$$\frac{\rho_R}{\rho_w} \simeq \frac{\epsilon_w}{2} \frac{Q}{(1+Q)^2} \simeq \frac{\epsilon_H}{2} \frac{Q}{1+Q}$$

The Model - Warm inflation

- Dynamics of the second stage of warm inflation:



Parameters setting: $M_w = 5.36 \times 10^{12} \text{ GeV}$, $g \simeq 0.13$, $h \simeq 0.21$ ($C_T \simeq 0.77$), $g_* \simeq 12.5$

$$N_w = 22, N_1 = 3$$

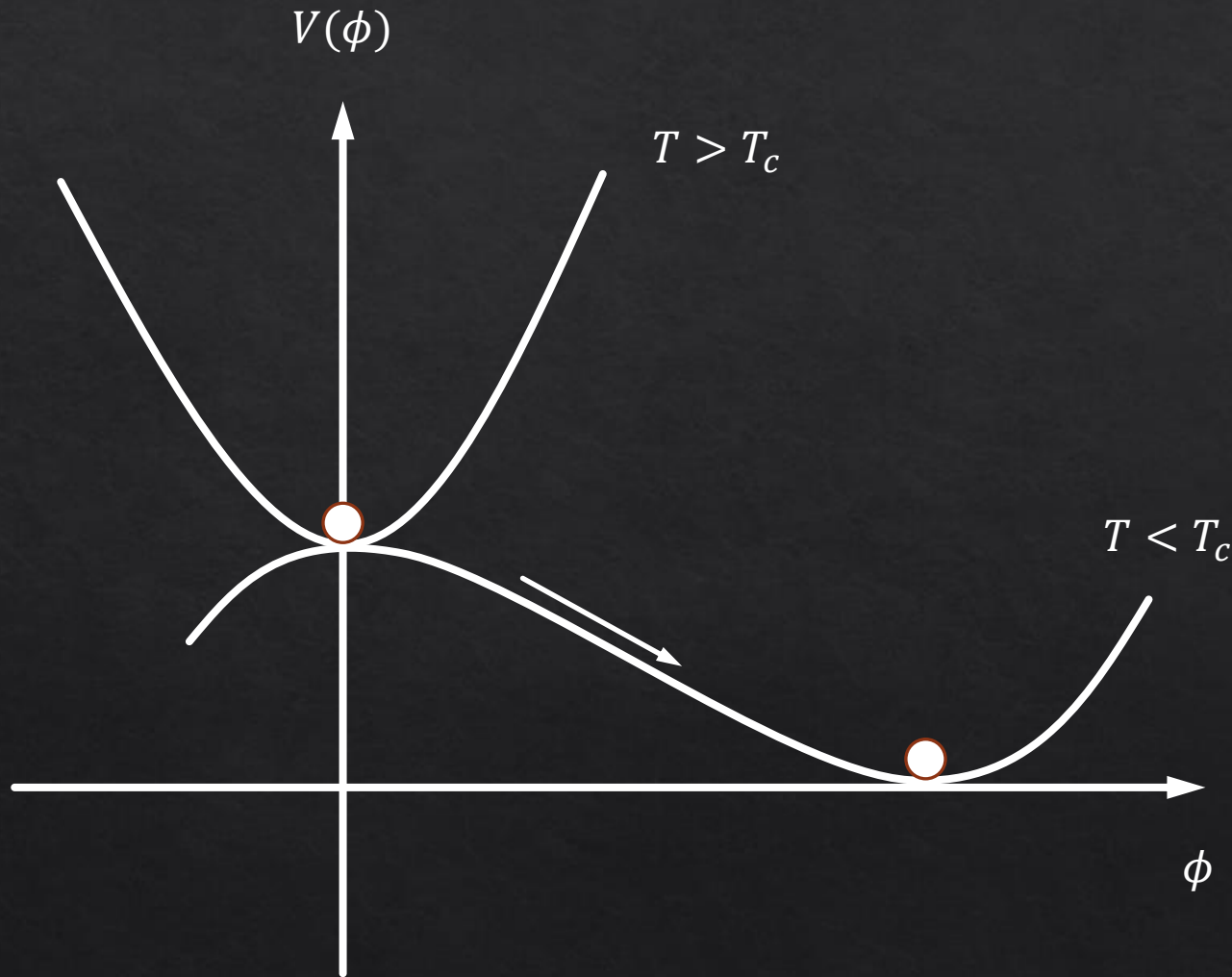
The Model - Warm inflation

- The dimensionless curvature power spectrum is:

$$\Delta_{\mathcal{R}}^2 = \frac{V_w (1+Q)^2}{24\pi^2 M_{\text{pl}}^4 \epsilon_w} \left(1 + 2n + \frac{2\sqrt{3}Q}{\sqrt{3+4\pi Q}} \frac{T}{H} \right) G(Q), \quad G(Q) \simeq 1 + 0.0185Q^{2.315} + 0.335Q^{1.364}.$$

- For $Q \sim T/H \gg 1$, the power spectrum is enhanced by nearly a factor Q^4 relative its quantum counterpart.
- In our example we find $\Delta_R^2 \sim 10^{-2}$ on scales crossing the horizon during warm inflation. This offers a natural setting for producing a significant abundance of PBHs that may potentially explain all of the dark matter.

The Model - Thermal inflation



$$V(\phi_t, T) \simeq V_0 + \frac{1}{2} (\alpha^2 T^2 - m^2) \phi_t^2 + \dots$$

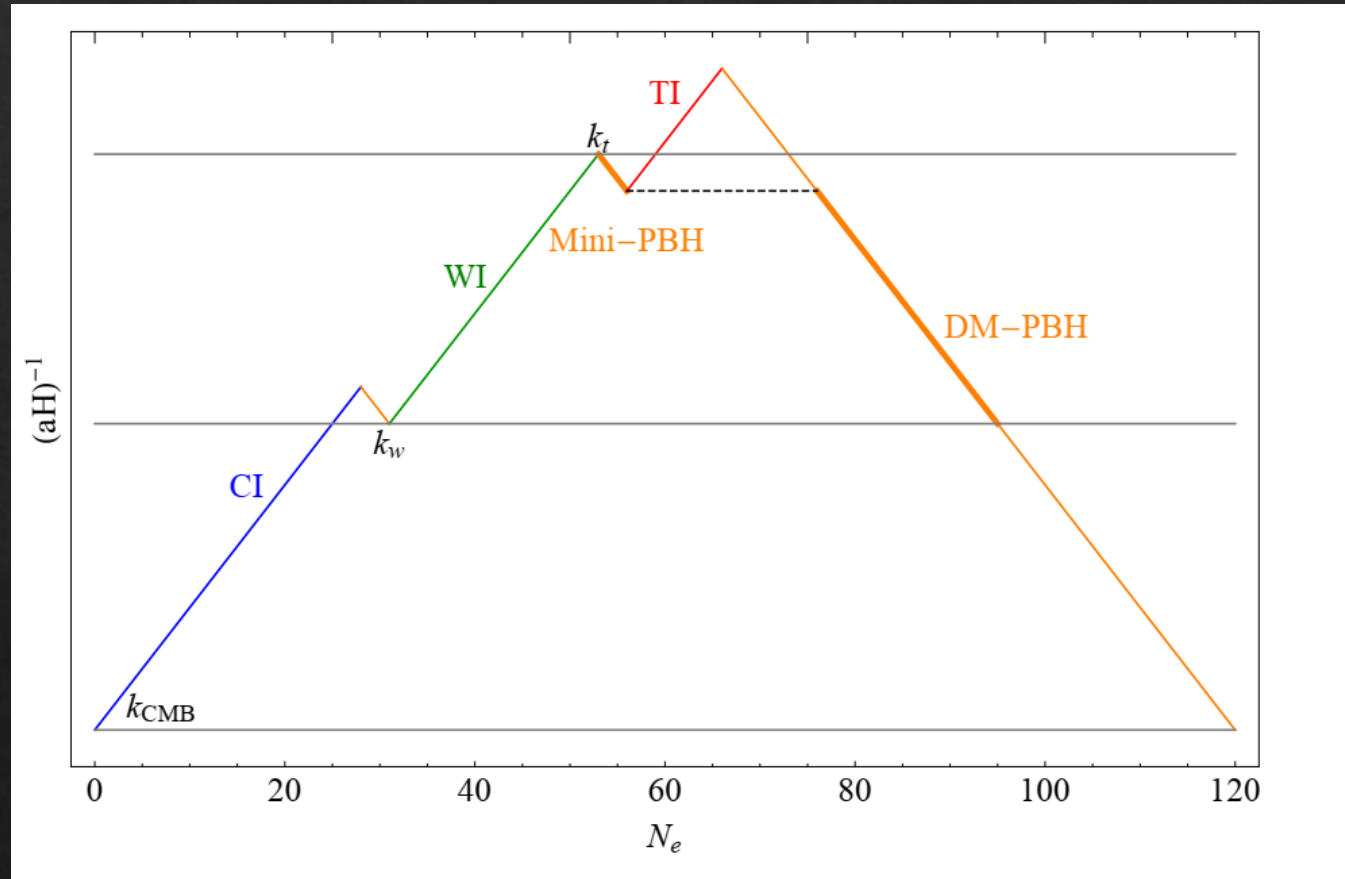
$$T_t = \left(\frac{30}{\pi^2 g_*} \right)^{1/4} V_0^{1/4}, \quad T_{\text{crit}} \equiv m/\alpha$$

$$N_t \simeq \log \frac{T_t}{T_{\text{crit}}} = \log \frac{V_0^{1/4}}{m} + \frac{1}{4} \log \left(\frac{30}{\pi^2 g_*} \right) + \log \alpha$$

$$N_t \approx 10$$

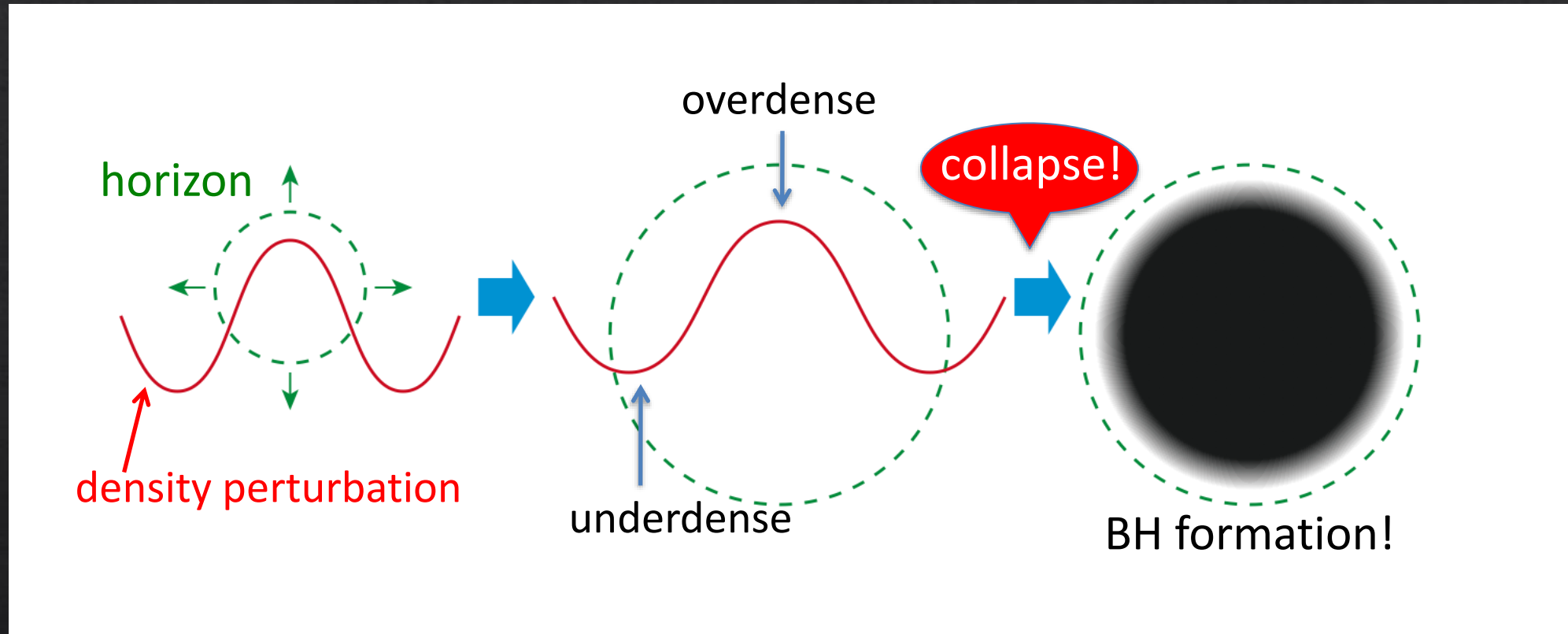
Primordial Black Holes

- Evolution of the Hubble horizon in this inflationary trilogy scenario:



Primordial Black Holes

- PBHs are formed by collapse of overdense regions in radiation dominated universe.



[copied from Kitajima's slides]

PBH mass $\approx$ horizon mass at the formation:
$$M_{\text{PBH}} \sim \frac{4\pi}{3} \rho(t_{\text{PBH}}) H^{-3}(t_{\text{PBH}})$$

Primordial Black Holes

$$\gamma \simeq 0.2$$

- The mass of a PBH:

$$\Delta_{\delta}^2 = \frac{4(1+w)^2}{(5+3w)^2} \left(\frac{k}{aH} \right)^4 \Delta_{\mathcal{R}}^2$$

$$M_{\text{PBH}} = 1.55 \times 10^{24} M_{\odot} \left(\frac{\gamma}{0.2} \right) \left(\frac{g_{\star}}{106.75} \right)^{1/6} (1+z)^{-2} \xi^2, \quad \xi = \begin{cases} 1, & \text{DM - PBHs} \\ e^{N_t}, & \text{mini - PBHs} \end{cases}$$

- The present fraction of dark matter in the form of PBHs:

$$f_{\text{PBH}} = 1.68 \times 10^8 \left(\frac{\gamma}{0.2} \right)^{1/2} \left(\frac{g_{\star}}{106.75} \right)^{-1/4} \left(\frac{M_{\text{PBH}}}{M_{\odot}} \right)^{-1/2} \beta_{\text{PBH}} \xi^{-3}.$$

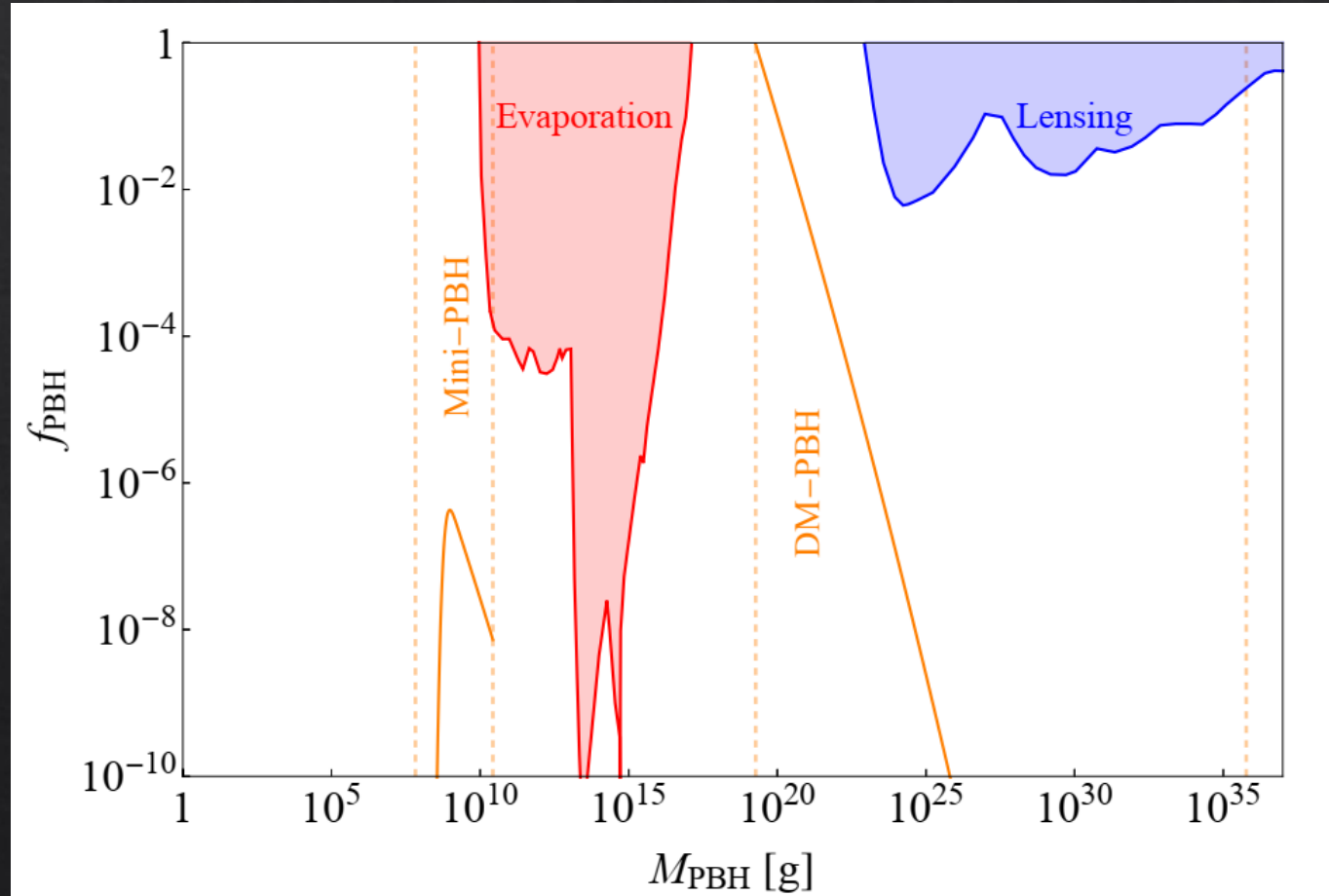
with

$$\beta_{\text{PBH}} = \gamma \int_{\delta_c}^1 P(\delta) d\delta \simeq \gamma \frac{\sigma}{\sqrt{2\pi} \delta_c} e^{-\frac{\delta_c^2}{2\sigma^2}},$$

$$\sigma^2 = \frac{8}{81} A_s \left(\frac{k_{\text{PBH}}}{k_w} \right)^{n_s-1} \left(\Gamma \left[\frac{n_s+3}{2}, \left(\frac{k_w}{k_{\text{PBH}}} \right)^2 \right] - \Gamma \left[\frac{n_s+3}{2}, \left(\frac{k_t}{k_{\text{PBH}}} \right)^2 \right] \right).$$

Primordial Black Holes

- PBH redictions of the inflation trilogy scenario:



Thanks for your attention!

Backup

$$M_{\text{PBH}} = \gamma \frac{4\pi}{3} H^{-3}(t_{\text{PBH}}) \rho(t_{\text{PBH}}) = 4\pi M_{\text{pl}}^2 H^{-1}(t_{\text{PBH}})$$

$$f_{\text{PBH}} = \Omega_{\text{DM},0}^{-1} \left(\frac{H}{H_0} \right)^2 (1+z)^{-3} \beta_{\text{PBH}}.$$

$$M_{\text{PBH}} = \gamma \frac{4\pi M_{\text{pl}}^2}{H}, \quad H \propto a^{-2} = (1+z)^2$$

$$f_{\text{PBH}} = \frac{\rho_{\text{PBH},0}}{\rho_{\text{DM},0}} \quad \beta_{\text{PBH}} = \frac{\rho_{\text{PBH}}}{\rho_{\text{tot}}} = \frac{\rho_{\text{PBH}}}{3M_{\text{pl}}^2 H^2}$$

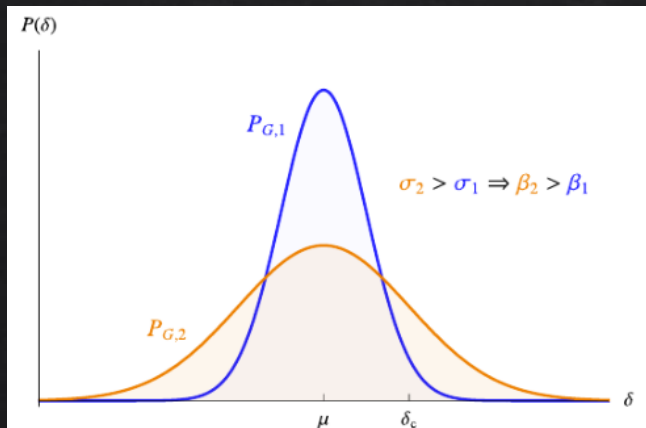
$$f_{\text{PBH}} = \frac{\rho_{\text{PBH},0}}{\rho_{\text{DM},0}}$$

$$f_{\text{PBH}} = \frac{\rho_{\text{PBH},0}}{\rho_{\text{DM},0}} = \frac{\rho_{\text{PBH},0}}{\Omega_{\text{DM},0} \rho_{\text{c},0}} = \frac{\rho_{\text{PBH},0}}{\Omega_{\text{DM},0} H_0^2 3M_{\text{pl}}^2} = \frac{\rho_{\text{PBH}}}{\Omega_{\text{DM},0} H_0^2 3M_{\text{pl}}^2} \left(\frac{a}{a_0} \right)^3$$

$$\beta_{\text{PBH}} = \frac{\rho_{\text{PBH}}}{\rho_{\text{tot}}}$$

$$\Delta_{\mathcal{R}}^2(k) = A_s (k/k_w)^{n_s-1}$$

$$\sigma^2 = \int d \log k \Delta_{\delta}^2(k) W^2(k, k_{\text{PBH}}),$$

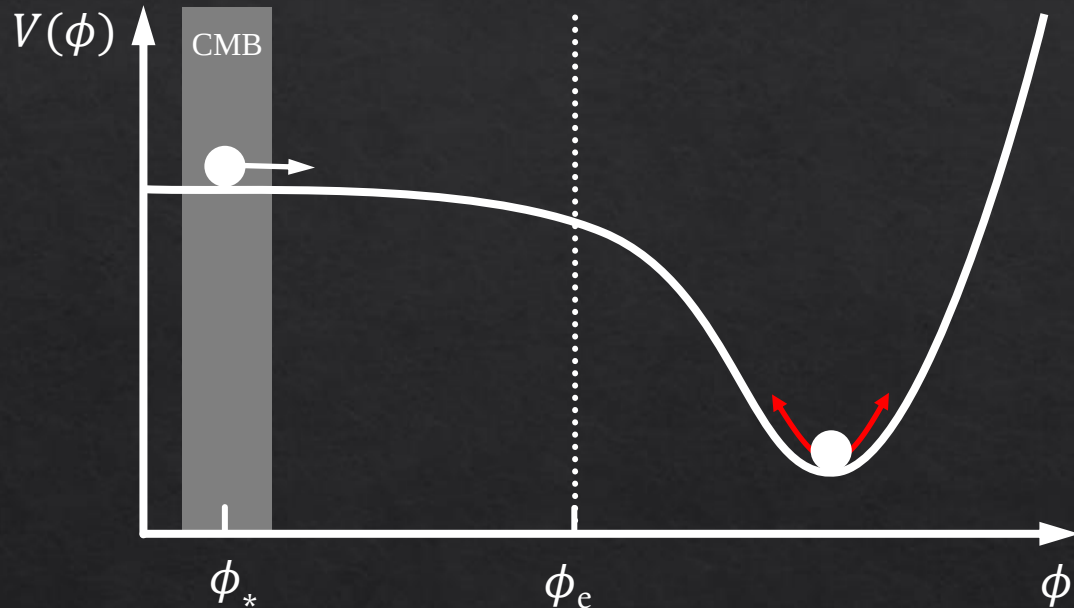


Gaussian window function: $W = \exp(-k^2/2k_{\text{PBH}}^2)$

Cold Inflation

$$H^2 \approx \frac{V}{3M_{\text{pl}}^2}$$

Slow-roll Inflation



EOM:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

Slow-roll conditions:

$$\epsilon_V = \frac{M_{\text{pl}}^2}{2} \left(\frac{V'}{V} \right)^2 \ll 1, \quad \eta_V = M_{\text{pl}}^2 \frac{V''}{V} \ll 1$$

- In cold inflation, interactions of the inflaton with other fields are considered negligible during inflation:

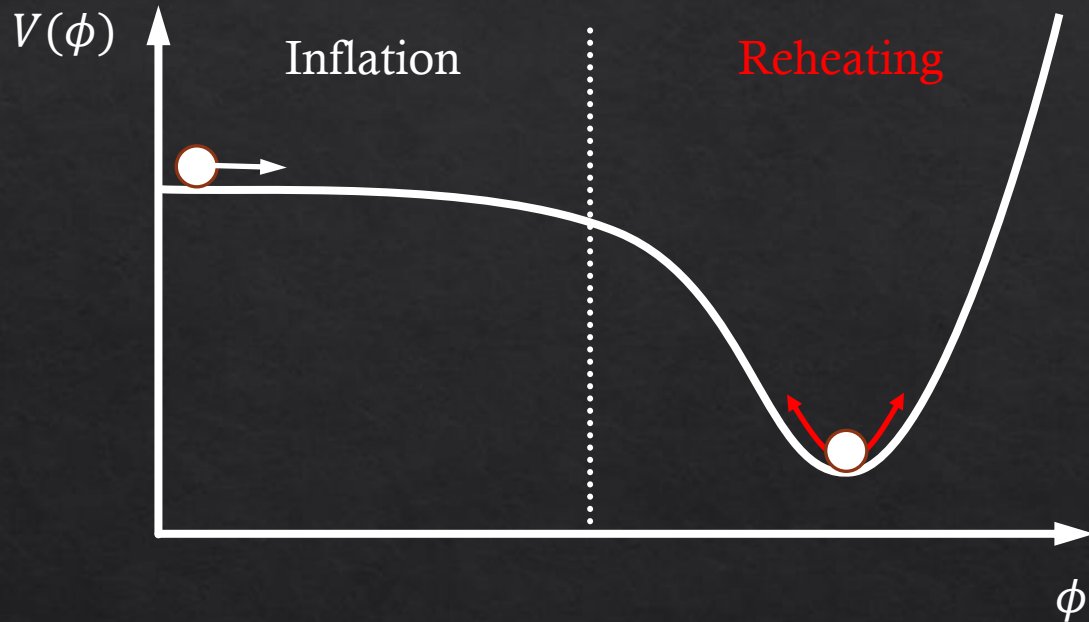
$$\dot{\rho}_r + 4H\rho_r = 0 \Rightarrow \rho_r \sim 1/a^4 \Rightarrow \text{Universe supercools}$$

“Radiation density redshifts away”

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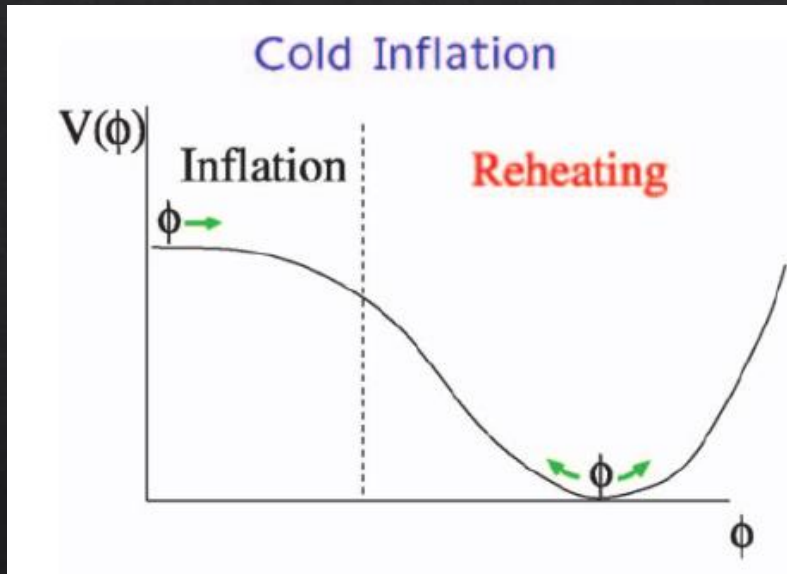
$$\dot{\rho}_r + 4H\rho_r = 0 \Rightarrow \rho_r \sim 1/a^4 \Rightarrow \text{Universe supercools during CI}$$

“Radiation density redshifts away”

Cold Inflation

- In cold inflation, interactions of the inflaton with other fields are considered

negligible during inflation:



- Shortcoming: eta-problem

$$\dot{\rho}_r + 4H\rho_r = \Upsilon \dot{\phi}^2$$

$\Rightarrow$ The radiation density redshifts away: $\rho_r \sim 1/a^4$

$\Rightarrow$ Universe supercools during inflation

• Curvature perturbations: $\Phi = H\delta\phi/\dot{\phi}$, $\delta\phi(\mathbf{x}, t) = \phi(\mathbf{x}, t) - \phi(t)$

$$P_R^2 = \frac{k^3}{2\pi^2} \int_{k'} \langle \Phi(k)\Phi(k') \rangle = P_R^2(k_0) \left(\frac{k}{k_0} \right)^{n_s-1}, \quad \text{amplitude} = P_R^2(k_0)$$

$\rightarrow$ nearly scale-invariant, but slightly red-tilted, $\sigma_8 \sim 10^{-9}$, no chance to get PBHs

Warm Inflation

- Strong dissipative effects lead to a blue-tilted primordial spectrum

$$\frac{\Upsilon}{3H} \equiv Q > 1$$

$$H^2 \simeq \frac{V(\phi)}{3M_{\text{pl}}^2}$$

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V'(\phi) = 0$$

$$\dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2 \quad H^2 = \frac{\rho_r + \dot{\phi}^2/2 + V(\phi)}{3M_{\text{pl}}^2}$$

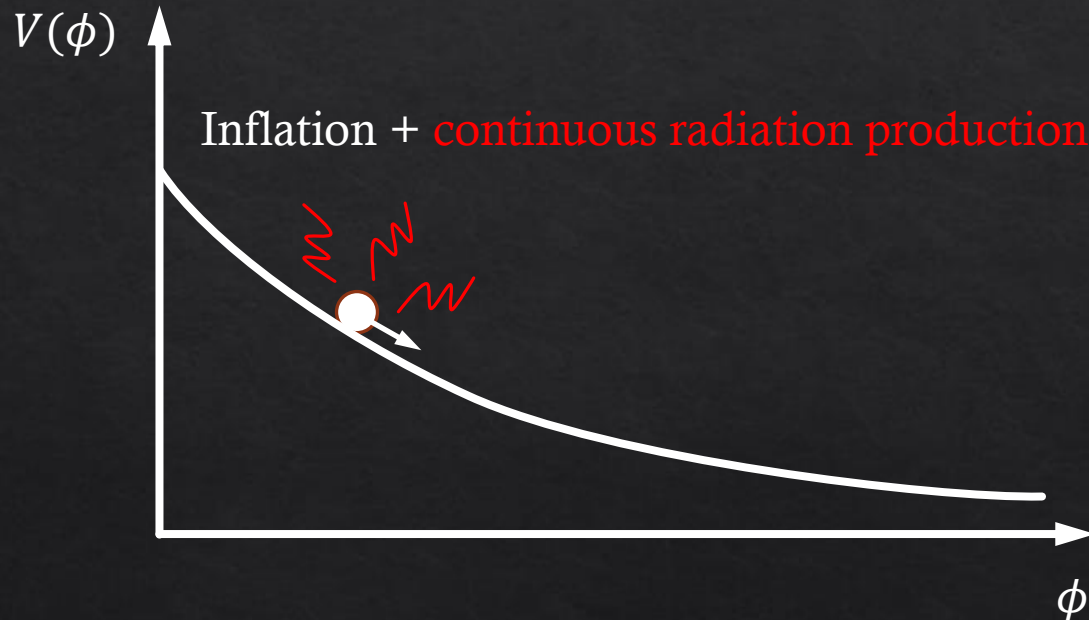
$$3H(1 + Q)\dot{\phi} \simeq -V'$$

$$4H\rho_r \simeq \Upsilon\dot{\phi}^2 \Rightarrow \rho_r = \frac{3}{4}Q\dot{\phi}^2$$

$$\rho_r = \frac{\pi^2}{30}g_*T^4$$

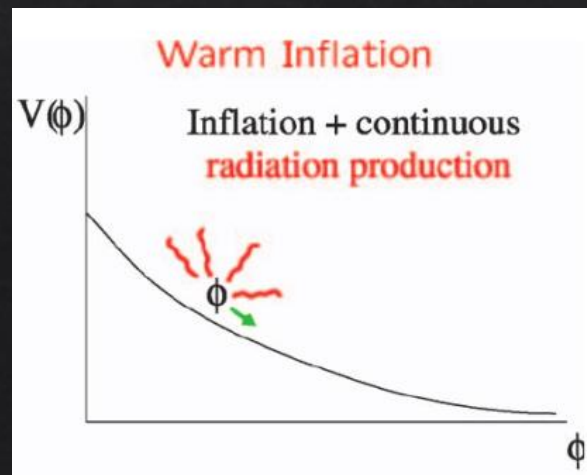
$$\epsilon_H = -\frac{\dot{H}}{H^2} \simeq \frac{1+Q}{2M_{\text{pl}}^2} \frac{\dot{\phi}^2}{H^2} \simeq \frac{\epsilon_V}{1+Q} < 1 \Rightarrow \epsilon_V < 1+Q$$

$$\eta_H = \frac{\dot{\epsilon}_H}{H\epsilon_H} < 1 \Rightarrow \eta_V < 1+Q$$



Warm Inflation

- Eta-problem can be alleviated in warm inflation due to the additional dissipative friction
- Strong dissipative effects lead to a blue-tilted primordial spectrum
- Requiring a single field to drive the full 50–60 e-folds of inflationary expansion may be too much

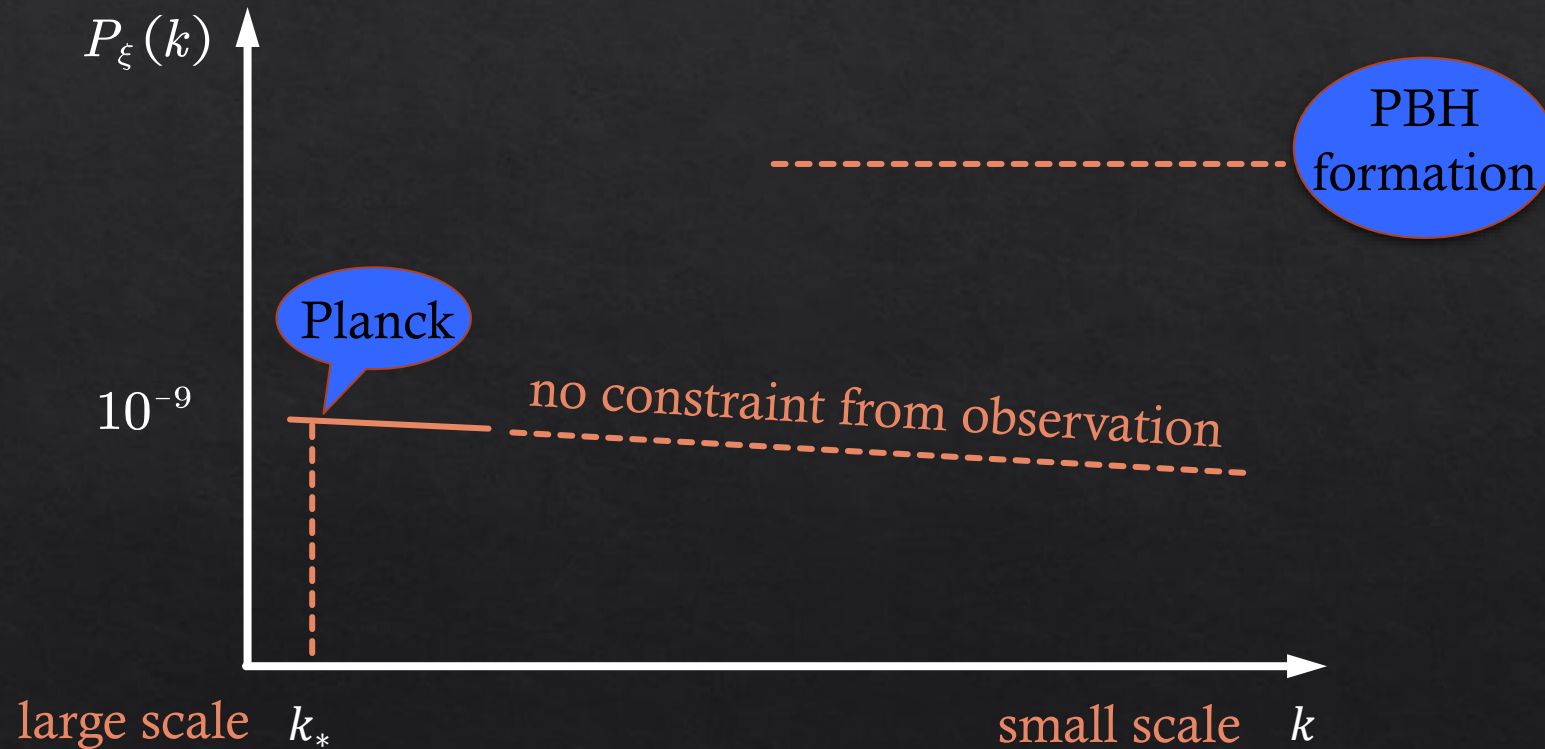


→ nearly scale-invariant, but slightly blue-tilted,
PR gets enhanced by large Q , get chance to get
PBHs

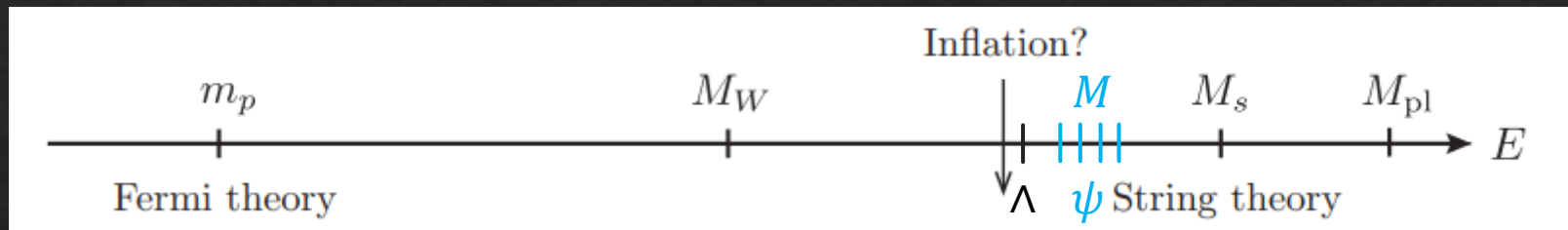
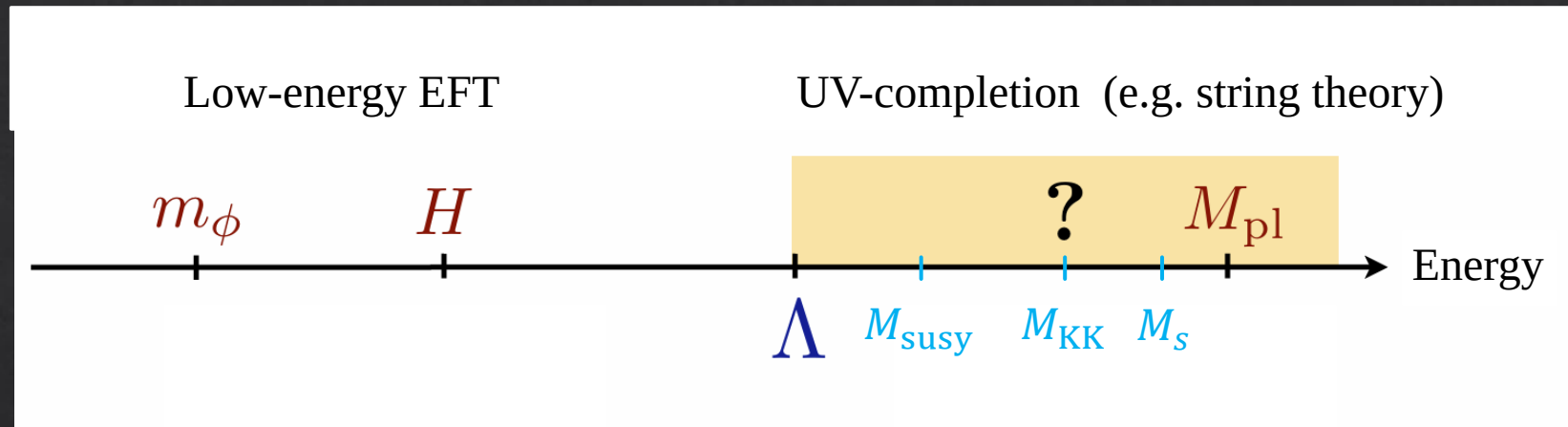
Primordial Black Holes

The power spectrum of curvature perturbation :

$P_\xi \sim 2 \times 10^{-9}$ with nearly scale-invariant [Planck obs.]



Cold Inflation



$$\mathcal{L}_{\text{eff}} = -\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) - \sum_n c_n V(\phi) \frac{\phi^{2n}}{\Lambda^{2n}} - \sum_n d_n \frac{(\partial \phi)^{2n}}{\Lambda^{4n}} + \dots$$

Cold Inflation

- Quantum fluctuations:

$$\delta\phi(\mathbf{x}) \sim H$$

- Primordial curvature power spectrum:

$$\Delta_{\mathcal{R}}^2(k) = \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k) = A_s(k_*) \left(\frac{k}{k_*} \right)^{n_s-1}, \quad A_s(k_*) \simeq 10^{-9}, \quad n_s \simeq 0.968$$

- nearly scale-invariant
- slightly red-tilted spectrum

$$\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') P_{\mathcal{R}}(k), \quad \Delta_{\mathcal{R}}^2(k) = \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k).$$

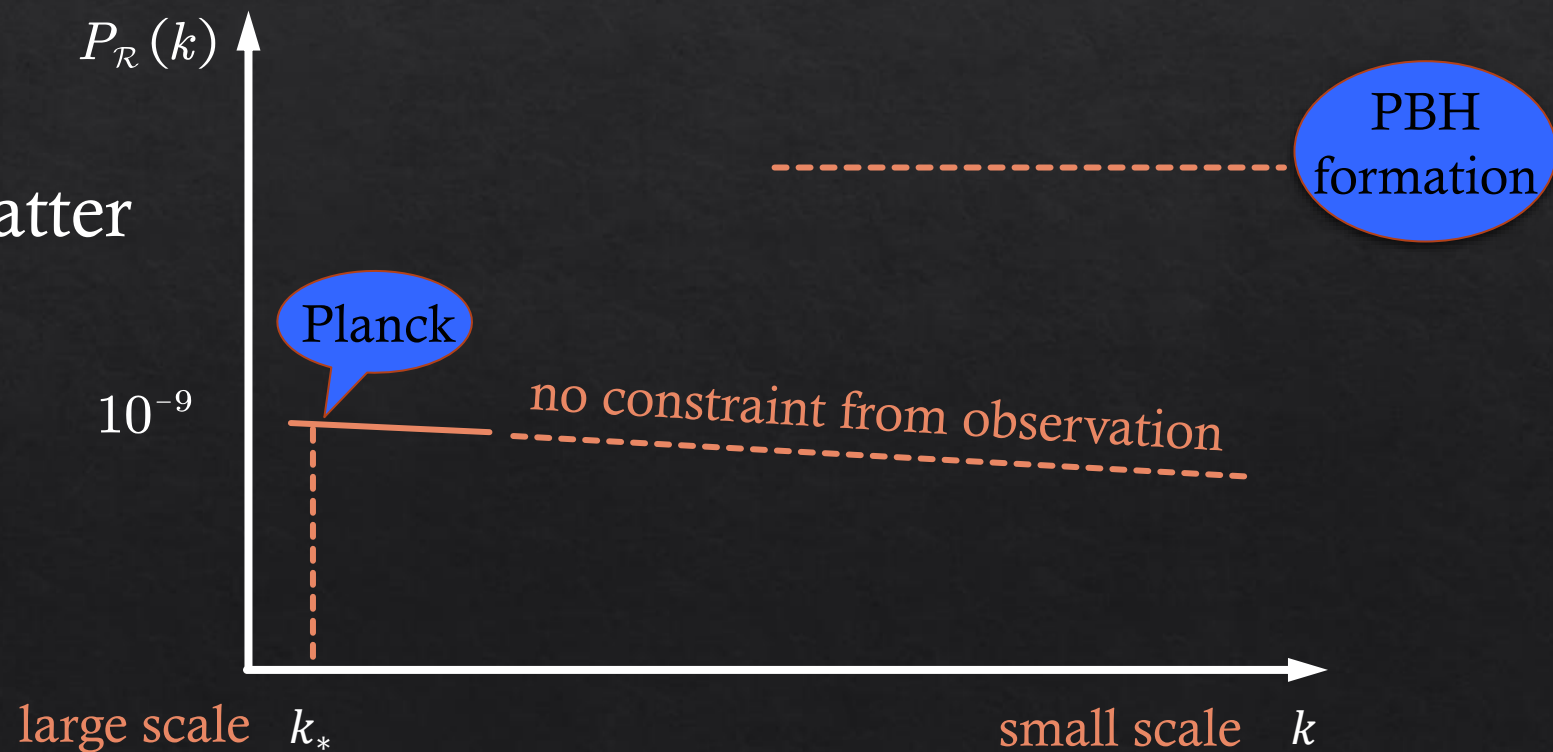
$$\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = \left(\frac{H}{\dot{\phi}} \right)^2 \langle \delta\phi_{\mathbf{k}} \delta\phi_{\mathbf{k}'} \rangle$$

$$\langle \delta\phi_{\mathbf{k}} \delta\phi_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \frac{2\pi^2}{k^3} \left(\frac{H}{2\pi} \right)^2, \quad \Delta_{\delta\phi}^2 = \left(\frac{H}{2\pi} \right)^2.$$

$$\Delta_{\mathcal{R}}^2(k) = \frac{H_*^2}{(2\pi)^2} \frac{H_*^2}{\dot{\phi}_*^2}.$$

Motivation

- Requiring a single field to drive the full 50-60 e-folds of inflationary expansion may be too much to ask for in a theory
- Avoid eta-problem
- Explain all of the dark matter



An aerial night photograph of a city. In the foreground, a multi-lane highway with glowing light trails from traffic curves through a dark, wooded hillside. Below the highway, several large, modern, multi-story buildings are illuminated. In the middle ground, a large, illuminated, dome-shaped structure, possibly a stadium or arena, stands out against the city lights. The background shows a vast urban landscape with numerous buildings and lights stretching to the horizon under a dark sky.

Thanks for your attention!
