Physics with pion beam:

Structure of pion and nucleon

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第一届介子束流物理研讨会

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Collaborators: Enzo Barone, Stan Brodsky, Tao Huang, Jacques Soffer, Ivan Schmidt
and students: Zhun Lu, Jun She, Bo-Wen Xiao, Xiaonan Liu

Probing pion Boer–Mulders function and proton transversity in pion+p Drell-Yan Process

COMPASS pion+p Drell-Yan process

can also measure

the pretzelosity distributions of the nucleon.

Physics Letters B 696 (2011) 513–517

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Single spin asymmetry in πp Drell–Yan process

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Probing pion Boer-Mulders function and proton pretzelosity with pion+p Drell-Yan Process

COMPASS pion+p Drell-Yan process

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THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Boer–Mulders function of the pion and pretzelosity distribution of the proton in the polarized pion-proton Drell–Yan process at COMPASS

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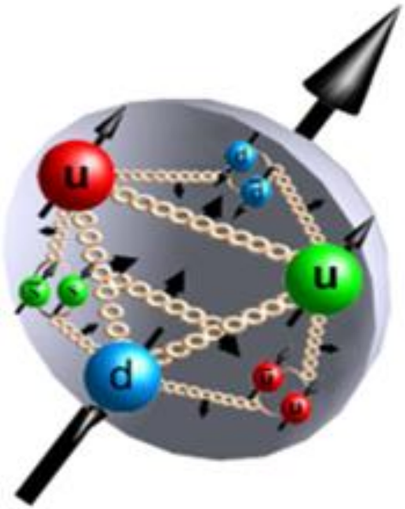
The structure of nucleon



- The most abundant piece of matter in our world.
- A very mysterious object with many puzzles: proton spin crisis, sea content of the nucleon, 3-dimentional structure of the nucleon.

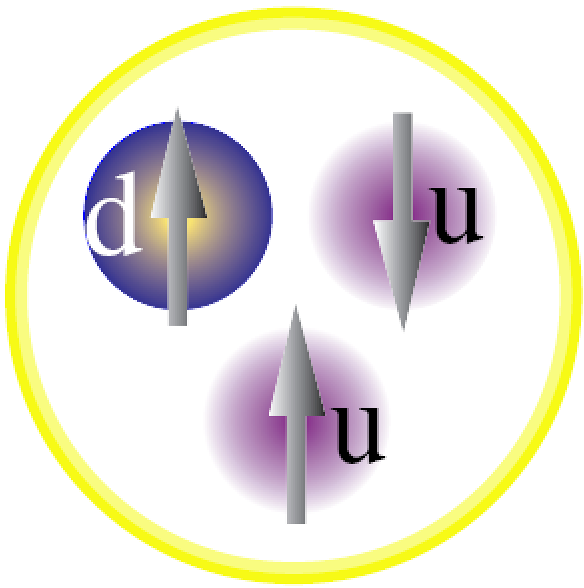
The Proton “Spin Crisis”

These results could be argued to imply that the sum of the spins carried by the quarks in a proton was consistent with zero, rather than with 1/2 as given in the quark model, suggesting a “Spin Crisis” in the parton model.

M.Anselmino, A.Efremov, E.Leader
Physics Reports 261 (1995) page 4

The structure of the nucleon

Constituent Quarks

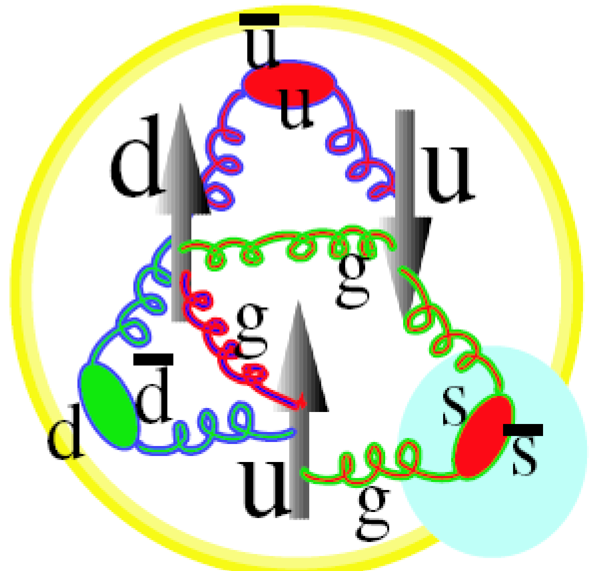


($Q^2 = 0\ GeV^2$)

baryon octet

masses,magn. momenta

Parton Distributions



($Q^2 > 1\ GeV^2$)

structure functions

momentum, spin

The most simple case:

The Pion Spin Structure

Based on collaborated works with T.Huang and Q.-X.Shen

[1] T. Huang, B.Q. Ma, and Q.X. Shen, Phys. Rev. **D 49**, 1490 (1994).

[2] B. Q. Ma, Z. Phys. **A 345**, 321 (1993).

[3] B.Q. Ma and T.Huang, J. Phys. **G 21**, (765) (1995).

Fu-Guang Cao, Tao Huang, and Bo-Qiang Ma, Phys.Rev.D 53 (1996) 6582-6585.

Fu-Guang Cao, Jun Cao, Tao Huang, and Bo-Qiang Ma, Phys.Rev.D 55 (1997) 7107-7113.

Jun Cao, Fu-Guang Cao, Tao Huang, Bo-Qiang Ma, Phys. Rev. D 58 (1998) 113006.

BIHEP-TH-90-37

PHYSICAL REVIEW D VOLUME 49, NUMBER 3 1 FEBRUARY 1994

Analysis of the pion wave function in the light-cone formalism

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(Received 22 January 1991; revised manuscript received 12 August 1993)

We analyze several general constraints on the pionic valence-state wave function. It is found that the present model wave functions used in the light-cone formalism of perturbative quantum chromodynamics have failed to reproduce the Chernyak-Zhitnitsky (CZ) distribution amplitude which is required to fit the pionic form factor data and the reasonable valence-state structure function which does not exceed the pionic structure function data for $x \to 1$ simultaneously. A possible model wave function which can satisfy all the general constraints has been suggested and analyzed.

PACS number(s): 12.38.−t, 12.39.−x, 13.60.−r

calculation. Also, we have shown that there are two higher helicity $(\lambda_1+\lambda_2=\pm 1)$ components in the light-cone wave function for the pion as a natural consequence from the Melosh rotation and it is speculated that these components should be incorporated into the perturbative quantum chromodynamics. Some progress has been

Pion Spin-Space Wave Function in Rest Frame

In the pion rest frame, the instant-form spin space wave-function of pion is

$$\chi_T = (\chi_1^{\uparrow}\chi_2^{\downarrow} - \chi_2^{\uparrow}\chi_1^{\downarrow})/\sqrt{2},$$

in which $\chi_i^{\uparrow,\downarrow}$ are the two-component Pauli spinors.

Melosh Rotation for Spin-1/2 Particle

The connection between spin states in the rest frame and infinite momentum frame

Or between spin states in the conventional equal time dynamics and the light-front dynamics

$$\chi^{\uparrow}(T) = w[(q^{+} + m)\chi^{\uparrow}(F) - q^{R}\chi^{\downarrow}(F)];$$

$$\chi^{\downarrow}(T) = w[(q^{+} + m)\chi^{\downarrow}(F) + q^{L}\chi^{\uparrow}(F)].$$

Huang, Ma & Shen, PRD49 (1994) 1490

Pion as the Lowest Valence State Wave Function in Light-Cone

$$|\psi^{\pi}_{q\bar{q}}> = \psi(x, \mathbf{k}\_, \uparrow, \downarrow)|\uparrow\downarrow> + \psi(x, \mathbf{k}\_, \downarrow, \uparrow)|\downarrow\uparrow>$$

$$+\psi(x, \mathbf{k}\_, \uparrow, \uparrow)|\uparrow\uparrow> + \psi(x, \mathbf{k}\_, \downarrow, \downarrow)|\downarrow\downarrow>,$$

where

$$\psi(x, \mathbf{k}\_, \lambda_1, \lambda_2) = C_0^F(x, \mathbf{k}\_, \lambda_1, \lambda_2)\varphi(x, \mathbf{k}\_).$$

Here $\varphi(x, \mathbf{k}\_)$ is the momentum space wave function in the light-cone formalism.

Huang, Ma & Shen, PRD49 (1994) 1490

The Spin Component Coefficients

The spin component coefficients C_0^F have the forms,

$$C_0^F(x, q, \uparrow, \downarrow) = w_1 w_2[(q_1^+ + m)(q_2^+ + m) - \mathbf{q}_\perp^2]/\sqrt{2};$$

$$C_0^F(x, q, \downarrow, \uparrow) = -w_1 w_2[(q_1^+ + m)(q_2^+ + m) - \mathbf{q}_\perp^2]/\sqrt{2};$$

$$C_0^F(x, q, \uparrow, \uparrow) = w_1 w_2[(q_1^+ + m)q_2^L - (q_2^+ + m)q_1^L]/\sqrt{2};$$

$$C_0^F(x, q, \downarrow, \downarrow) = w_1 w_2[(q_1^+ + m)q_2^R - (q_2^+ + m)q_1^R]/\sqrt{2}.$$

C_0^F satisfy the relation

$$\Sigma_{\lambda_1, \lambda_2} C_0^F(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) C_0^F(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) = 1.$$

From field theory vertex calculation

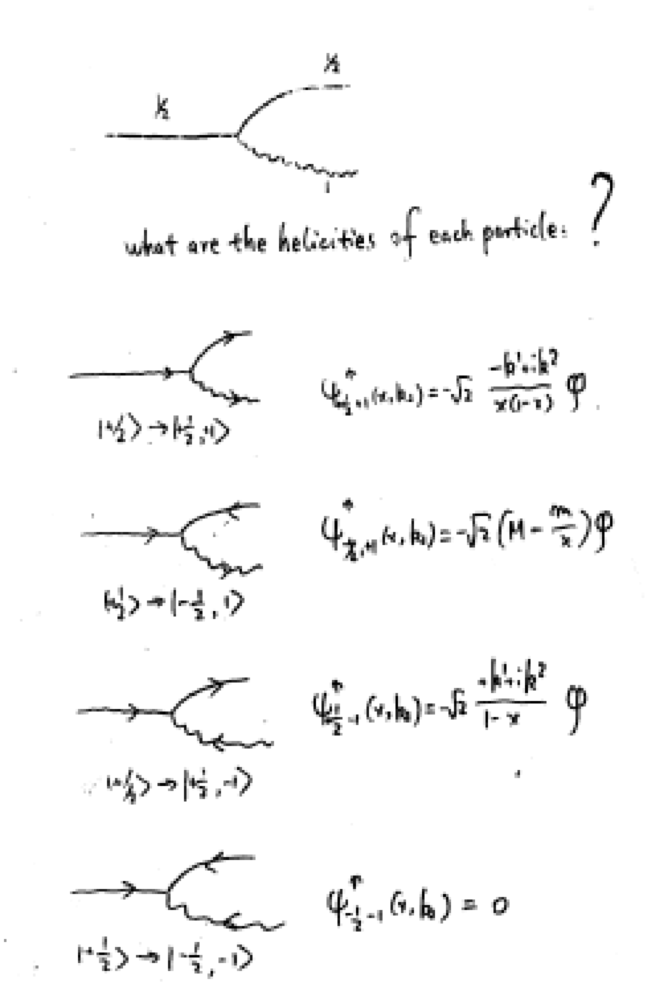
$$\frac{\overline{v}(p_2^+, p_2^-, -\mathbf{k}_\perp)}{\sqrt{p_2^+}} \gamma_5 \frac{u(p_1^+, p_1^-, \mathbf{k}_\perp)}{\sqrt{p_1^+}},$$

$$\begin{cases} \frac{\overline{v}_\downarrow}{\sqrt{p_2^+}} \gamma_5 \frac{u_\uparrow}{\sqrt{p_1^+}} &= -\frac{2mP^+}{4mx(1-x)P^{+2}}, \\ \frac{\overline{v}_\downarrow}{\sqrt{p_2^+}} \gamma_5 \frac{u_\uparrow}{\sqrt{p_1^+}} &= +\frac{2mP^+}{4mx(1-x)P^{+2}}, \\ \frac{\overline{v}_\uparrow}{\sqrt{p_2^+}} \gamma_5 \frac{u_\uparrow}{\sqrt{p_1^+}} &= +\frac{2(k_1+ik_2)P^+}{4mx(1-x)P^{+2}}, \\ \frac{\overline{v}_\downarrow}{\sqrt{p_2^+}} \gamma_5 \frac{u_\downarrow}{\sqrt{p_1^+}} &= +\frac{2(k_1-ik_2)P^+}{4mx(1-x)P^{+2}}, \end{cases}$$

Xiao & Ma, PRD71(2005)014034

A QED Example of Relativistic Spin Effect

S.J. Brodsky, D.S. Hwang, B.-Q. Ma, I. Schmidt, Nucl. Phys. B 593 (2001) 311



The lowest spin states of a composite system must contain the orbital angular momentum contribution.

$$\Delta s_{\text{non-rel}} + L_{\text{non-rel}} = \Delta s_{\text{rel}} + L_{\text{rel}}$$

What is Δq measured in DIS

- Δq is defined by $\Delta q \; s_{\mu} = \langle p, s | \bar{q} \gamma_{\mu} \gamma_5 q | p, s \rangle$

$$\Delta q = \langle p, s | \bar{q} \gamma^{+} \gamma_5 q | p, s \rangle$$

- Using light-cone Dirac spinors

$$\Delta q = \int_0^1 \mathrm{d}x \left[q^{\uparrow}(x) - q^{\downarrow}(x) \right]$$

- Using conventional Dirac spinors

$$\Delta q = \int \mathrm{d}^3 \vec{p} M_q \left[q^{\uparrow}(\vec{p}) - q^{\downarrow}(\vec{p}) \right]$$

$$M_q = \frac{(p_0 + p_3 + m)^2 - \vec{p}_{\perp}^{\,2}}{2(p_0 + p_3)(p_0 + m)}$$

Thus Δq is the light-cone quark spin, or quark spin in IMF, not that in the rest frame of the proton

The proton spin crisis

& the Melosh-Wigner rotation

- **It is shown that the proton "spin crisis" or "spin puzzle" can be understood by the relativistic effect of quark transversal motions due to the Melosh-Wigner rotation.**
- **The quark helicity Δq measured in polarized deep inelastic scattering is actually the quark spin in the infinite momentum frame or in the light-cone formalism, and it is different from the quark spin in the nucleon rest frame or in the quark model.**

B.-Q. Ma, J.Phys. G 17 (1991) L53

B.-Q. Ma, Q.-R. Zhang, Z.Phys.C 58 (1993) 479-482

Quark spin sum is not a Lorentz invariant quantity

Thus the quark spin sum equals to the proton in the rest frame does not mean that it equals to the proton spin in the infinite momentum frame

$$\sum_q \vec{s}_q = \vec{S}_p \quad \text{in the rest frame}$$

does not mean that

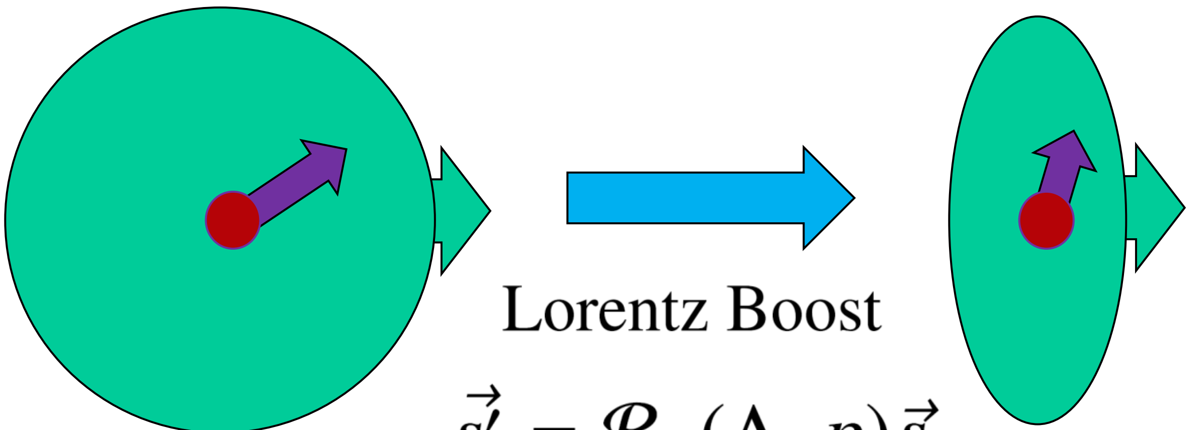
$$\sum_q \vec{s}_q = \vec{S}_p \quad \text{in the infinite momentum frame}$$

Therefore it is not a surprise that the quark spin sum measured in DIS does not equal to the proton spin

B.-Q. Ma, J.Phys.G 17 (1991) L53-L58

B.-Q. Ma, Q.-R. Zhang, Z.Phys.C 58 (1993) 479-482

An intuitive picture to understand the spin puzzle



$$\vec{s}' = \mathcal{R}_w(\Lambda, p)\vec{s}$$

Rest Frame

$$\Sigma \vec{s} = \vec{S}_p$$

Infinite Momentum Frame

$$\Sigma \vec{s}' \neq \vec{S}_p$$

A general consensus

The quark helicity Δq defined in the infinite momentum frame is generally not the same as the constituent quark spin component in the proton rest frame, just like that it is not sensible to compare apple with orange.

H.-Y.Cheng, hep-ph/0002157，
Chin.J.Phys.38:753,2000

Chances：New Research Directions

- New quantities：Transversity, Generalized Parton Distributions, Collins Functions, Silver Functions, Boer-Mulders Functions, Pretzelosity

- Hyperon Physics：The spin structure of Lambda and Sigma hyperons

B.-Q. Ma, I. Schmidt, J.-J. Yang
Phys. Lett. B 477 (2000) 107
Phys. Rev. D 61 (2000) 034017

Leading-Twist TMD PDFs

Nucleon Spin
Quark Spin

Nucleon Polarization		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
	U	f_1		$h_1^{\perp}$ Boer-Mulders
	L		g_1 Helicity	$h_{1L}^{\perp}$ Long-Transversity
	T	$f_{1T}^{\perp}$ Sivers	g_{1T} Trans-Helicity	h_1 Transversity $h_{1T}^{\perp}$ Pretzelosity

What is “Pretzelosity” 麻花度？

J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008

Connection with Quark Orbital Angular Momentum

- The rotation factor for $\vec{x} \times -i\nabla$ is $\frac{p_\perp^2}{(x\mathcal{M}_D+m_q)^2+p_\perp^2}$
  B.-Q. Ma, I. Schmidt, Phys. Rev. **D 58**, 096008 (1998).
- a simple relation between the pretzelosity and the quark orbital angular momentum

$$L^{qv}(x,\mathbf{p}_\perp) = -h_{1T}^{\perp(1)qv}(x,\mathbf{p}_\perp) = h_1^{qv}(x,\mathbf{p}_\perp) - g_1^{qv}(x,\mathbf{p}_\perp), \quad (21)$$

or at the integration level

$$L^{qv}(x) = \int d^2\mathbf{p}_\perp L^{qv}(x,\mathbf{p}_\perp) = -h_{1T}^{\perp(1)qv}(x) = h_1^{qv}(x) - g_1^{qv}(x).$$

- A measurement of pretzelosity may reveal the information on the quark orbital angular momentum.

The Melosh-Wigner Rotation in Transversity

$$2\,\delta q = \langle\, p, \uparrow | \bar{q}_{\lambda} \gamma^{\perp} \gamma^{+} q_{-\lambda} | p, \downarrow \rangle$$

$$\delta q(x) = \int [\mathrm{d}^2 \boldsymbol{k}_{\perp}] \tilde{M}_q(x, \boldsymbol{k}_{\perp}) \Delta q_{\mathrm{RF}}(x, \boldsymbol{k}_{\perp})$$

$$\tilde{M}_q(x, \boldsymbol{k}_{\perp}) = \frac{(k^{+} + m)^2}{(k^{+} + m)^2 + \boldsymbol{k}_{\perp}^2}$$

I.Schmidt&J.Soffer, Phys.Lett.B 407 (1997) 331

B.-Q. Ma, I. Schmidt, J. Soffer, Phys.Lett. B 441 (1998) 461-467.

The Melosh-Wigner Rotation in Quark Orbital Angular Moment

$$\hat{L}_q = -i\left(k_1 \frac{\partial}{\partial k_2} - k_2 \frac{\partial}{\partial k_1}\right).$$

$$L_q(x) = \int [d^2 k_\perp] M_L(x, k_\perp) \Delta q_{QM}(x, k_\perp)$$

$$M_L(x, k_\perp) = \frac{k_\perp^2}{(k^+ + m)^2 + k_\perp^2}$$

Ma&Schmidt, Phys.Rev.D 58 (1998) 096008

The Melosh-Wigner Rotation in five 3dPDFs

分布函数	Melosh转动因子（$W_D(D=V,\ S)$）
g_{1L}	$[(x\mathscr{M}_D+m_q)^2-p_\perp^2]/[(x\mathscr{M}_D+m_q)^2+p_\perp^2]$
g_{1T}	$2M_N(x\mathscr{M}_D+m_q)/[(x\mathscr{M}_D+m_q)^2+p_\perp^2]$
h_1	$(x\mathscr{M}_D+m_q)^2/[(x\mathscr{M}_D+m_q)^2+p_\perp^2]$
$h_{1L}^\perp$	$-2M_N(x\mathscr{M}_D+m_q)/[(x\mathscr{M}_D+m_q)^2+p_\perp^2]$
$h_{1T}^\perp$	$-2M_N^2/[(x\mathscr{M}_D+m_q)^2+p_\perp^2]$

$\mathscr{M}_D^2=\frac{m_q^2+p_\perp^2}{x}+\frac{m_D^2+p_\perp^2}{1-x}$是旁观双夸克的不变质量。

Huang, Ma & Shen, PRD49 (1994) 1490

Pion as the Lowest Valence State Wave Function in Light-Cone

$$|\psi^{\pi}_{q\bar{q}}> = \psi(x, \mathbf{k}\_, \uparrow, \downarrow)|\uparrow\downarrow> + \psi(x, \mathbf{k}\_, \downarrow, \uparrow)|\downarrow\uparrow>$$

$$+\psi(x, \mathbf{k}\_, \uparrow, \uparrow)|\uparrow\uparrow> + \psi(x, \mathbf{k}\_, \downarrow, \downarrow)|\downarrow\downarrow>,$$

where

$$\psi(x, \mathbf{k}\_, \lambda_1, \lambda_2) = C_0^F(x, \mathbf{k}\_, \lambda_1, \lambda_2)\varphi(x, \mathbf{k}\_).$$

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Huang, Ma & Shen, PRD49 (1994) 1490

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$$C_0^F(x,q,\downarrow,\uparrow) = -w_1 w_2[(q_1^+ + m)(q_2^+ + m) - \mathbf{q}_\perp^2]/\sqrt{2};$$

$$C_0^F(x,q,\uparrow,\uparrow) = w_1 w_2[(q_1^+ + m)q_2^L - (q_2^+ + m)q_1^L]/\sqrt{2};$$

$$C_0^F(x,q,\downarrow,\downarrow) = w_1 w_2[(q_1^+ + m)q_2^R - (q_2^+ + m)q_1^R]/\sqrt{2}.$$

C_0^F satisfy the relation

$$\Sigma_{\lambda_1,\lambda_2} C_0^F(x,\mathbf{k}_\perp,\lambda_1,\lambda_2) C_0^F(x,\mathbf{k}_\perp,\lambda_1,\lambda_2) = 1.$$

From field theory vertex calculation

$$\frac{\overline{v}(p_2^+, p_2^-, -\mathbf{k}_\perp)}{\sqrt{p_2^+}}\gamma_5\frac{u(p_1^+, p_1^-, \mathbf{k}_\perp)}{\sqrt{p_1^+}},$$

$$\begin{cases}
\frac{\overline{v}_\downarrow}{\sqrt{p_2^+}}\gamma_5\frac{u_\uparrow}{\sqrt{p_1^+}} = -\frac{2mP^+}{4mx(1-x)P^{+2}}, \\
\frac{\overline{v}_\downarrow}{\sqrt{p_2^+}}\gamma_5\frac{u_\uparrow}{\sqrt{p_1^+}} = +\frac{2mP^+}{4mx(1-x)P^{+2}}, \\
\frac{\overline{v}_\uparrow}{\sqrt{p_2^+}}\gamma_5\frac{u_\uparrow}{\sqrt{p_1^+}} = +\frac{2(k_1+ik_2)P^+}{4mx(1-x)P^{+2}}, \\
\frac{\overline{v}_\downarrow}{\sqrt{p_2^+}}\gamma_5\frac{u_\downarrow}{\sqrt{p_1^+}} = +\frac{2(k_1-ik_2)P^+}{4mx(1-x)P^{+2}},
\end{cases}$$

Xiao & Ma, PRD71(2005)014034

Pion Boer–Mulders Function

PHYSICAL REVIEW D, VOLUME 70, 094044

Nonzero transversity distribution of the pion in a quark-spectator-antiquark model

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PHYSICS LETTERS B

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Flavor separation of the Boer–Mulders function from unpolarized $\pi^- p$ and $\pi^- D$ Drell–Yan processes

Zhun Lu [a], Bo-Qiang Ma [b,*], Ivan Schmidt [a,*]

Pion Boer–Mulders Function

PHYSICAL REVIEW D **84**, 034010 (2011)

sin2ϕ azimuthal asymmetry in single longitudinally polarized πN Drell-Yan process

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(Received 28 April 2011; published 8 August 2011)

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Boer-Mulders function of the pion in the MIT bag model

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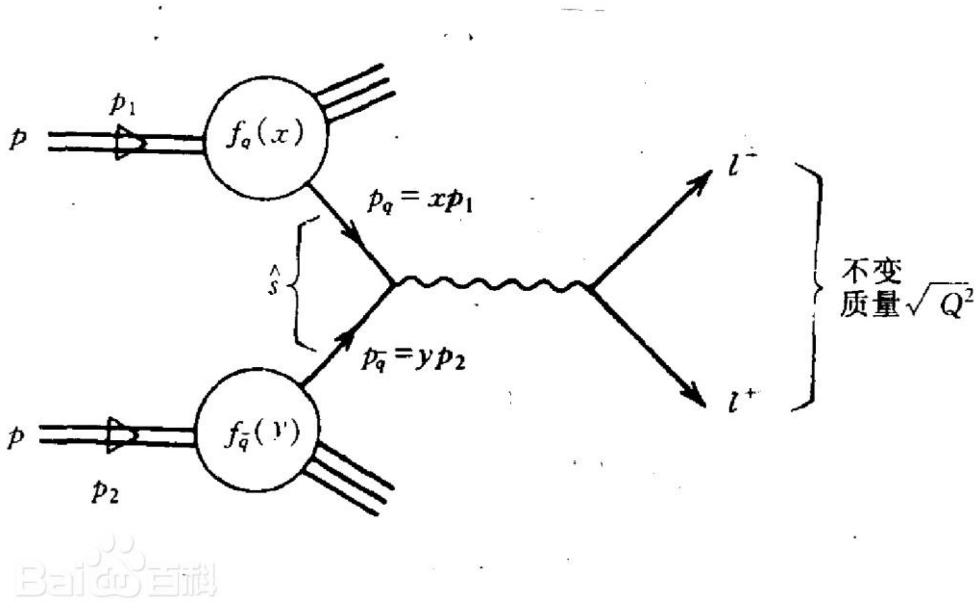
(Received 7 September 2012; published 16 November 2012)

Drell-Yan Process



- Drell-Yan process serves as a vital tool to probe the structure of colliding beams such as protons on targets.
- Pion as beam provides new opportunities to explore the spin and flavor structure of pions and nucleons.

Pion Boer–Mulders Function+ proton distributions in pion+p Drell-Yan process with COMPASS data

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https://doi.org/10.1140/epjc/s10052-021-09457-2

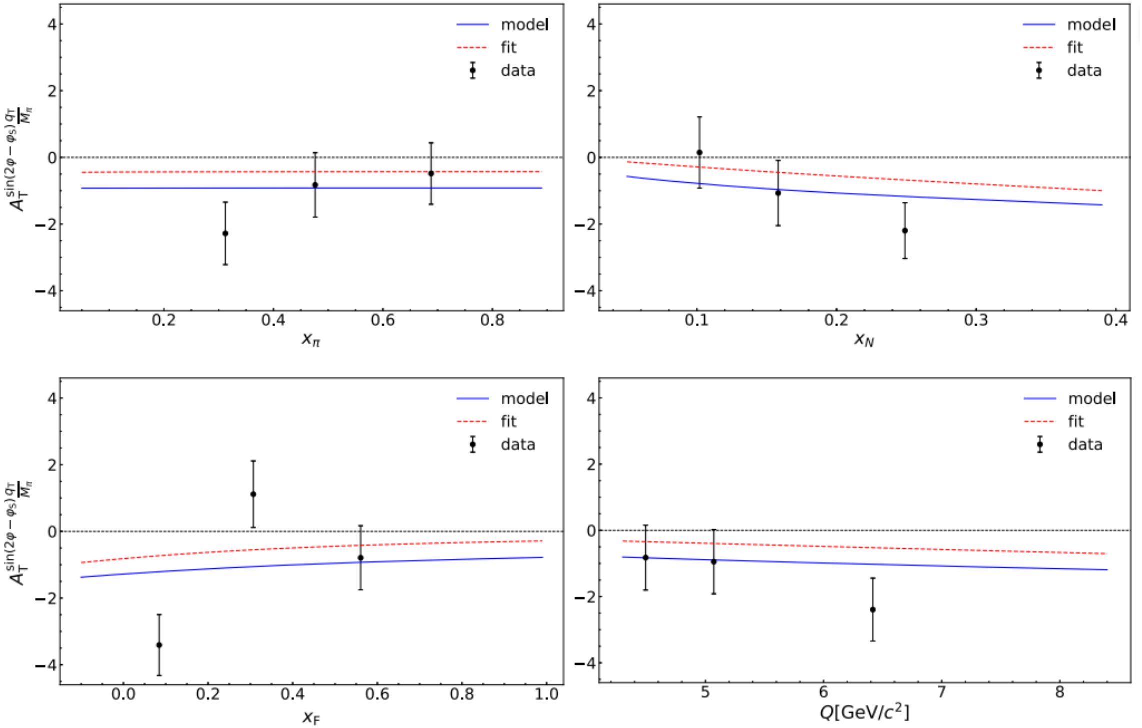
THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Boer–Mulders function of the pion and pretzelosity distribution of the proton in the polarized pion-proton Drell–Yan process at COMPASS

Xiaonan Liu[1], Bo-Qiang Ma [1,2,3,a]

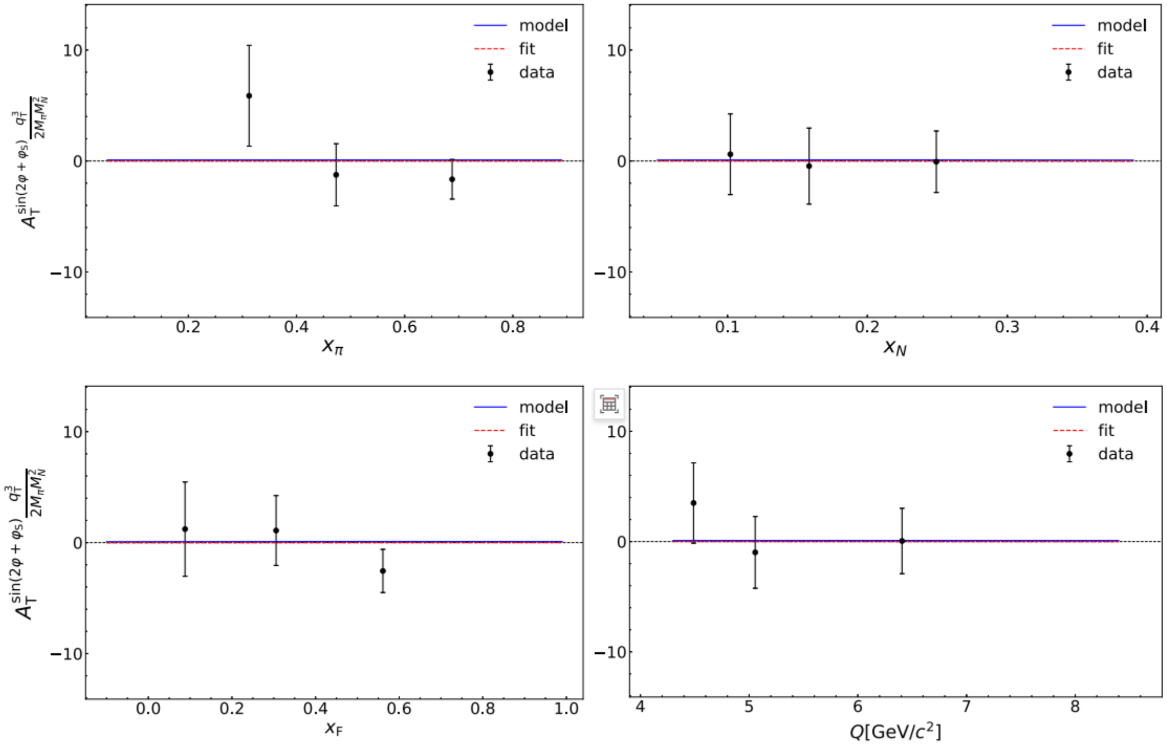
Comparison of model with data



Xiaonan Liu, B.-Q. Ma, arXiv: 2111.02766, EPJC 81 (2021) 635

Comparison of model with data

Eur. Phys. J. C (2021) 81:635 Page 9 of 11 **635**



Xiaonan Liu, B.-Q. Ma, arXiv: 2111.02766, EPJC 81 (2021) 635

Conclusions

- **The pion+proton (deteuron) Drell-Yan process provide a valuable chance to probe the pion spin structure and the nucleon structure.**

- **Pion as beam can provide new opportunities to investigate the quark spin and flavor structure of pions and nucleons.**


谢谢！