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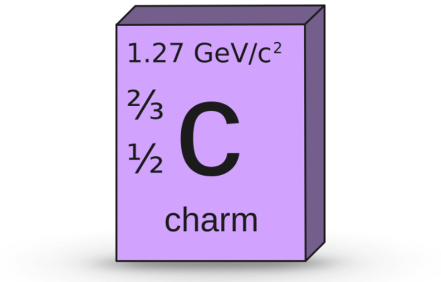

第二届武汉高能物理青年论坛

Wu han, China

Content

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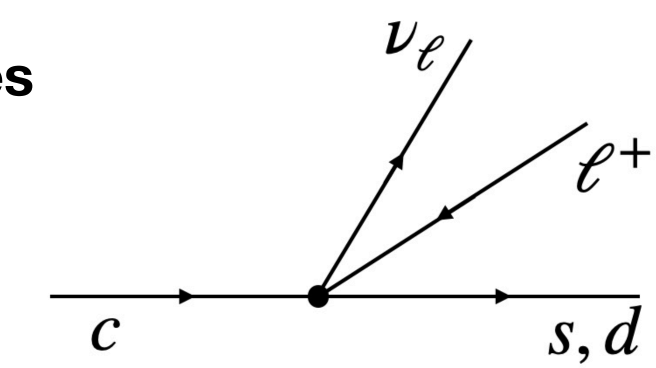
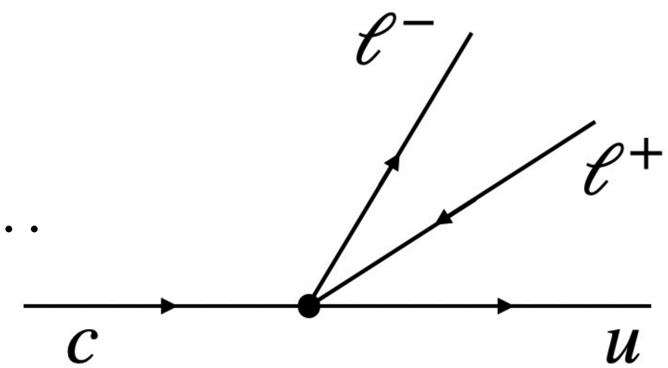
1. What and Why Electronic Semi-inclusive Charm decay?

What and Why Electronic Semi-inclusive Charm decay

- **Experimental detection of partial final state particles**
  - ➡ $D \to e^+ X$ ($D \to e^+ \nu_e X$, only e^+ is detected)



- **Sum of a group of exclusive channels**
  - ➡ $D^0 \to e^+ X_s = D \to e^+ \nu_e K^-, e^+ \nu_e K^- \pi^0, e^+ \nu_e \bar{K}^0 \pi^-, \ldots$
  - ➡ $D^0 \to e^+ X_d = D \to e^+ \nu_e \pi^-, e^+ \nu_e \pi^- \pi^0, e^+ \nu_e \pi^- \pi^+ \pi^-, \ldots$



- **Compared to exclusive decays: Better theoretical control**
- **Compared to beauty decays: More sensitive to power corrections**

What and Why Electronic Semi-inclusive Charm decay

Charmed hadron lifetimes:
theory vs experiment

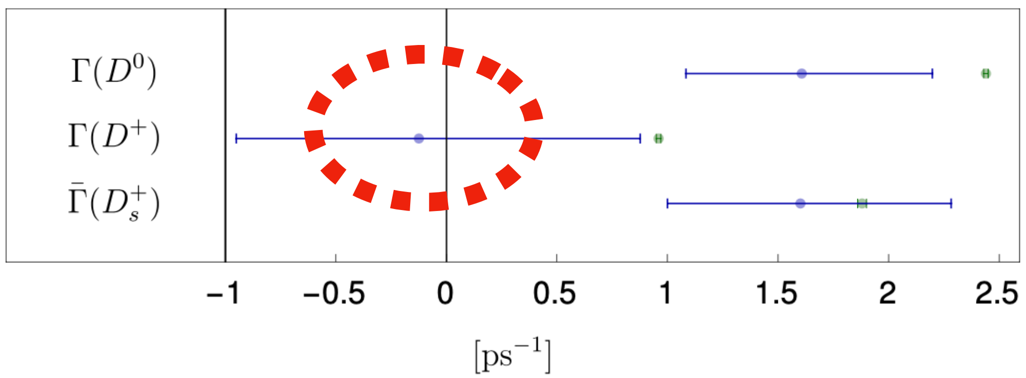


Fig 1 [Lenz et al, ’22]

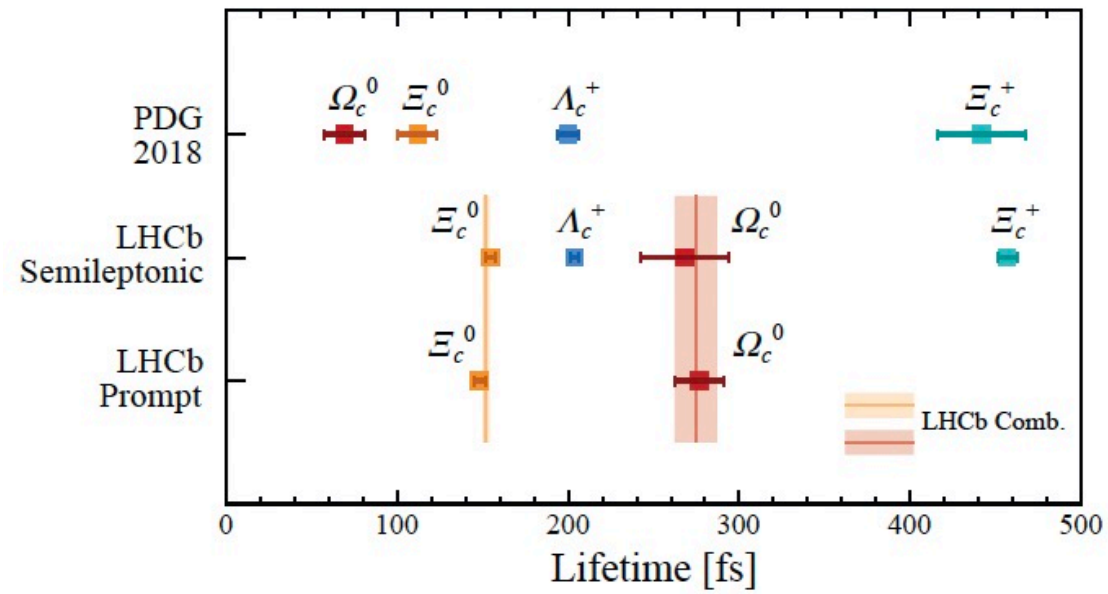


Fig 2

$$\mathcal{O}(1/m_c^3) \Rightarrow \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0) > \tau(\boldsymbol{\Omega_c^0}),$$
$$\mathcal{O}(1/m_c^4) \Rightarrow \tau(\boldsymbol{\Omega_c^0}) > \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0),$$
$$\mathcal{O}(1/m_c^4) \text{ with } \alpha \Rightarrow \tau(\Xi_c^+) > \tau(\boldsymbol{\Omega_c^0}) > \tau(\Lambda_c^+) > \tau(\Xi_c^0).$$

[Cheng, ’21]

What and Why Electronic Semi-inclusive Charm decay

Charmed hadron lifetimes: theory vs experiment

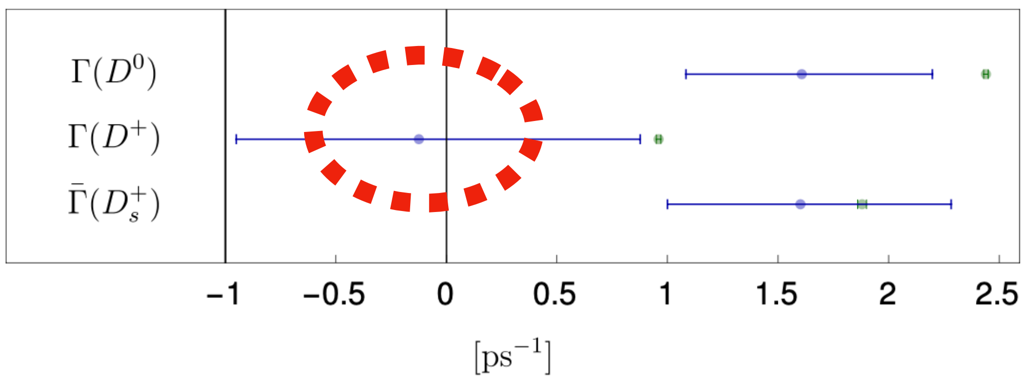


Fig 1 [Lenz et al, '22]

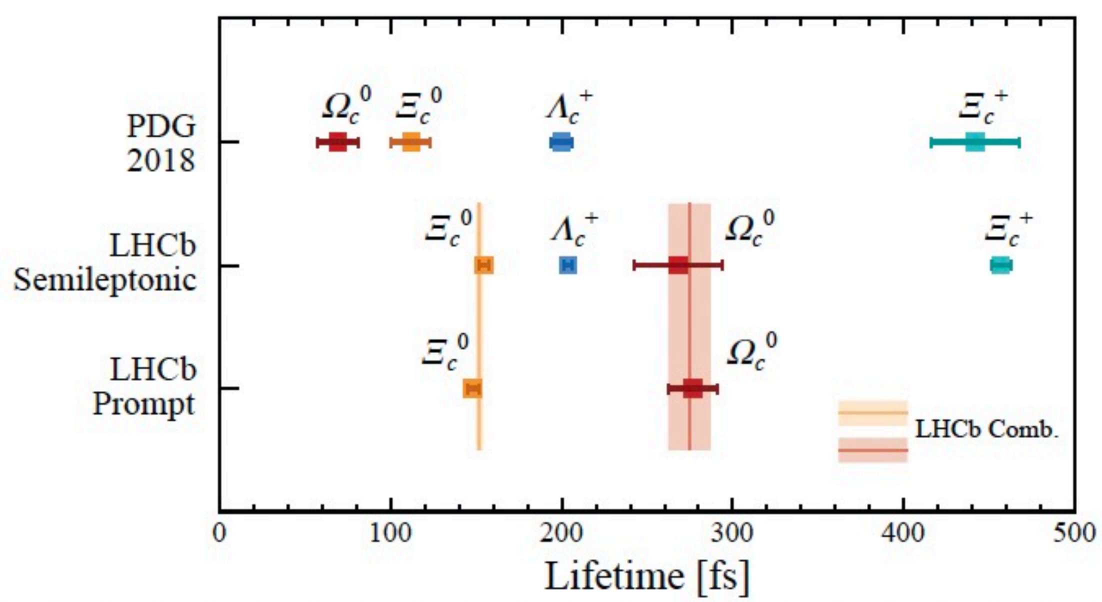


Fig 2

$$\mathcal{O}(1/m_c^3) \Rightarrow \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0) > \tau(\boldsymbol{\Omega_c^0}),$$
$$\mathcal{O}(1/m_c^4) \Rightarrow \tau(\boldsymbol{\Omega_c^0}) > \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0),$$
$$\mathcal{O}(1/m_c^4) \text{ with } \alpha \Rightarrow \tau(\Xi_c^+) > \tau(\boldsymbol{\Omega_c^0}) > \tau(\Lambda_c^+) > \tau(\Xi_c^0).$$

[Cheng, '21]

Solutions/hints

- Dependence on identical hadronic parameters in HQET, $\langle H_c | O_i | H_c \rangle$
- Extraction in the inclusive decay spectrum and application to lifetime

" Again a more precise experimental determination of μ_π^2 from fits to semi-leptonic D^+, D^0 and D_s^+meson decays — as it was done for the B^+ and B^0 decays — would be very desirable. " [Lenz et al, '22]

2. Theoretical Framework

Theoretical Framework

- **Optical theorem**

$$\sum \langle D|H|X\rangle\langle X|H|D\rangle \propto \mathrm{Im}\int \mathrm{d^4x}\langle \mathrm{D}|\mathrm{T}\{\mathrm{H(x)H(0)}\}|\mathrm{D}\rangle$$

- **Operator product expansion (OPE)**

  - ☆ Short distance : $x \sim 1/m_c$
  - ☆ Fluctuation in D meson $\sim \Lambda_{\mathrm{QCD}}$

$$T\{H(x)H(0)\} = \sum_n C_n(x)O_n(0) \to 1 + \frac{\Lambda_{\mathrm{QCD}}}{m_c} + \frac{\Lambda^2_{\mathrm{QCD}}}{m_c^2} + \ldots$$

Systematic OPE in HQET.

- **Heavy Quark Effective Theory**

$$h_v(x) \equiv e^{-im_c v\cdot x}\frac{1+\gamma\cdot v}{2}c(x) \qquad v = (1,0,0,0)$$

$$L \ni \bar{h}_v iv\cdot Dh_v - \bar{h}_v\frac{D_\perp^2}{2m_c}h_v - a(\mu)g\bar{h}_v\frac{\sigma\cdot G}{4m_c}h_v + \ldots$$

Subtract the big intrinsic momentum,

Leave only ~Λ_{QCD} degrees of freedom.

Theoretical Framework

- **OPE**

$$T\{H(x)H(0)\} = \sum_n C_n(x) O_n(0)$$

$C_n(x)$

- ☆ LO: $\alpha_s^0(m_c)$
- ☆ NLO: $\alpha_s(m_c)$
- ☆ **…**

$O_n(0)$

- ☆ Dim-3: $\bar{h}_v h_v$ $(\bar{c}\gamma^\mu c) \rightarrow$ **partonic decay rate.**
- ☆ Dim-5: $\bar{h}_v D_\perp^2 h_v,\ g\bar{h}_v \sigma \cdot G h_v$.
- ☆ Dim-6: $\bar{h}_v D_\mu (v \cdot D) D^\mu h_v,\ (\bar{h}_v \Gamma_1 q)(\bar{q}\Gamma_2 h_v), \ldots$
- ☆ **…**

- **Contribute to inclusive decay rate and lifetime**

1. Matrix elements of the **same operators** (SL& NL)
2. Only different short-distance coefficients

$$\lambda_1 \equiv \frac{1}{4m_D}\langle D|\bar{h}_v (iD)^2 h_v|D\rangle = -\mu_\pi^2$$

$$\lambda_2 \equiv \frac{1}{16(s_c \cdot s_q)}\frac{1}{2m_D}\langle D|\bar{h}_v g\sigma \cdot G h_v|D\rangle = \frac{\mu_G^2}{3}$$

Theoretical Framework

- **Structure functions**

$$\frac{d\Gamma}{d\hat{E}_\ell\, d\hat{q}^2\, d\hat{u}} = \frac{G_F^2 m_b^5 |V_{cb}|^2}{16\pi^3}\theta(\hat{u}_+ - \hat{u})\theta(\hat{E}_\ell)\theta(\hat{q}^2) \times$$
$$\times \left\{ \hat{q}^2\, W_1 - \left[2\hat{E}_\ell^2 - 2\hat{E}_\ell \hat{q}_0 + \frac{\hat{q}^2}{2}\right] W_2 + \hat{q}^2(2\hat{E}_\ell - \hat{q}_0)\, W_3 \right\},$$

$$W_i = W_i^{(0)} + \frac{\mu_\pi^2}{2m_c^2} W_i^{(\pi,0)} + \frac{\mu_G^2}{2m_c^2} W_i^{(G,0)} + \frac{\alpha_s}{\pi}\left[C_F W_i^{(1)} + C_F \frac{\mu_\pi^2}{2m_c^2} W_i^{(\pi,1)} + \frac{\mu_G^2}{2m_c^2} W_i^{(G,1)} \right] + \ldots$$

$$W_i^{(1)} = w_i^{(0)} \left\{ \mathcal{S}_i\, \delta(\hat{u}) - \left[\frac{\ln \hat{u}}{\hat{u}}\right]_+ - \left(\frac{7}{4} - 2\ln w\right)\left[\frac{1}{\hat{u}}\right]_+ + w\, B(\hat{q}^2, \hat{u})\, \theta(\hat{u}) \right\} + \mathcal{R}_i^{(1)}\, \theta(\hat{u}),$$

$$\mathcal{S}_i = -\frac{5}{4} - \frac{\pi^2}{3} - \mathrm{Li}_2(1-w) - 2\ln^2 w - \frac{5w-4}{2(1-w)}\ln w + \frac{\ln w}{2(1-w)}\delta_{i2}$$

$$\mathcal{R}_1^{(1)} = \frac{3}{4} + \frac{\hat{u}(12 - w - \hat{u})}{2\tilde{\lambda}} + \left(w + \frac{\hat{u}}{2} - \frac{\hat{u}(2\hat{u}+3w)}{\tilde{\lambda}}\right)\mathcal{I}_1$$

$$\mathcal{R}_2^{(1)} = \frac{6\hat{u}(\hat{u}^2 - (3-w)\hat{u} - 12 + 13w)}{\tilde{\lambda}^2} + \frac{\hat{u} - 38 + 21w}{\tilde{\lambda}}$$
$$-4\frac{\frac{w}{2}\hat{u}^3 + (2w^2 - 6)\hat{u}^2 + (7 - 3w + \frac{5}{2}w^2)w\hat{u} + w^3(w-4)}{\tilde{\lambda}^2}\mathcal{I}_1$$

$$\mathcal{R}_3^{(1)} = \frac{3\hat{u} - 8 + 5w}{\tilde{\lambda}} + \frac{\hat{u}^2 - (6-w)\hat{u} + 4w}{\tilde{\lambda}}\mathcal{I}_1$$

$$\mathcal{I}_1 = \frac{1}{\sqrt{\tilde{\lambda}}}\ln\frac{\hat{u} + w + \sqrt{\tilde{\lambda}}}{\hat{u} + w - \sqrt{\tilde{\lambda}}}$$

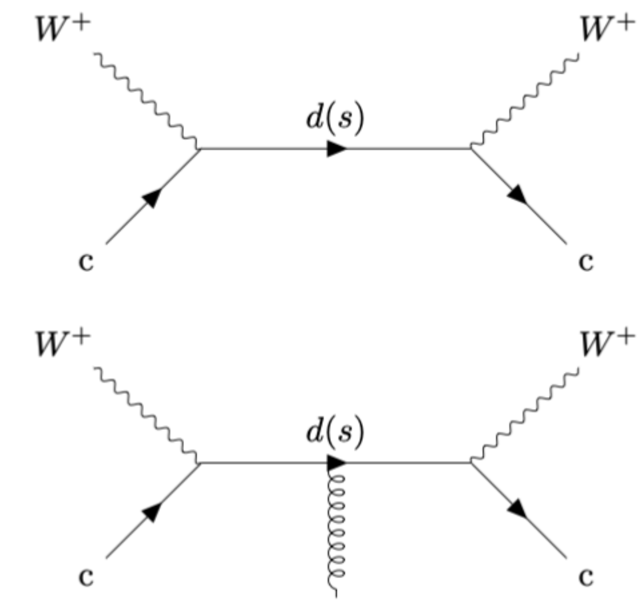


Fig 5. Leading Order

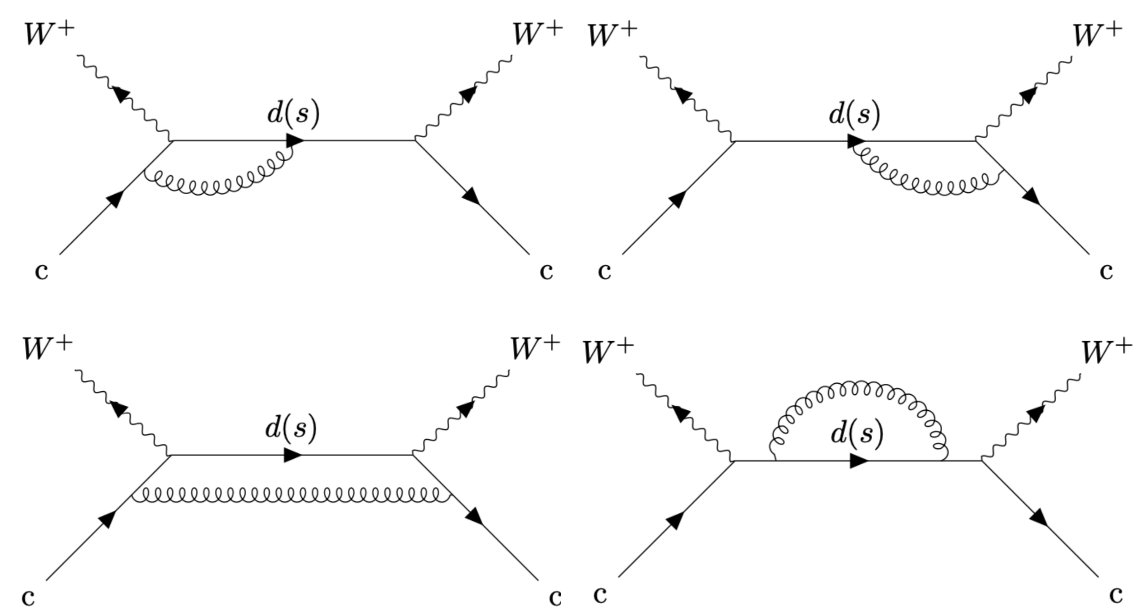


Fig 6. Next to Leading Order

NLO dim-3

[Capdevila et al. '22]

Theoretical Framework

- **Structure functions**

$$
\begin{aligned}
W_1^{(\pi,1)}=&w\Big[B_1-\frac{C}{2}+\frac{5w-2}{12}\Big[\frac{1}{\hat{u}^2}\Big]_++\Big(\frac{16+3w-10w^2}{12}-\frac{8w^3-w^2-14w+8}{6(1-w)}\ln w\Big)\delta'(\hat{u})\Big]\\
&-\frac{4}{3}(2-w)\Big(\Big[\frac{\ln\hat{u}}{\hat{u}}\Big]_++L_w\delta(\hat{u})\Big)+\Big(\frac{8}{3}(2-w)\ln w-\frac{4+18w-13w^2}{6w}\Big)\Big[\frac{1}{\hat{u}}\Big]_++\mathcal{R}_1^{(\pi)}\theta(\hat{u})\\
&+\Big(\frac{13w}{12}-\frac{1}{6}-\frac{1}{3w}-\frac{w^2}{12}+\frac{w^3}{4}+\frac{4+6w-13w^2+3w^3+2w^5}{3w(1-w)}\ln w\Big)\delta(\hat{u}) \qquad \text{(A.1)}
\end{aligned}
$$

$$
\begin{aligned}
W_2^{(\pi,1)}=&\,4B_2+6C+\frac{9w-10}{3}\Big[\frac{1}{\hat{u}^2}\Big]_++\Big(\frac{4+6w+16w^2}{3}\ln w-\frac{22-21w+10w^2}{3}\Big)\delta'(\hat{u})\\
&+\Big(w^2+\frac{116}{3w^2}-7w-\frac{50}{w}+\frac{88}{3}-4\,\frac{42-34w+17w^2-6w^3+2w^4}{3w^2}\ln w\Big)\delta(\hat{u})\\
&+\Big(\frac{10}{3}-\frac{68}{3w}+\frac{28}{w^2}\Big)\Big[\frac{1}{\hat{u}}\Big]_++\mathcal{R}_2^{(\pi)}\theta(\hat{u}) \qquad \text{(A.2)}
\end{aligned}
$$

$$
\begin{aligned}
W_3^{(\pi,1)}=&\,2B_3+C+\Big(\frac{7w}{6}-1\Big)\Big[\frac{1}{\hat{u}^2}\Big]_++\Big(\frac{5}{3}(1-w)w+\frac{w(6+3w-8w^2)}{3(1-w)}\ln w\Big)\delta'(\hat{u})\\
&+2\Big[\frac{\ln\hat{u}}{\hat{u}}\Big]_++\Big(\frac{19}{6}-\frac{2}{w}+\frac{4}{w^2}-4\ln w\Big)\Big[\frac{1}{\hat{u}}\Big]_++\Big[2L_w+\frac{w^2}{2}+\frac{14}{3w^2}-\frac{11w}{6}-\frac{20}{3w}\\
&+\frac{41}{6}+\Big(\frac{7w-6}{1-w}+\frac{4}{3}w-\frac{4}{3}w^2-\frac{8}{w^2}+\frac{4}{w}\Big)\ln w\Big]\delta(\hat{u})+\mathcal{R}_3^{(\pi)}\theta(\hat{u}) \qquad \text{(A.3)}
\end{aligned}
$$

$$
\begin{aligned}
B_i&=\tfrac{w^2}{6}\Big(\Big[\tfrac{7}{4}-2L_w-\tfrac{2-w}{1-w}\ln w+\delta_{i2}\tfrac{\ln w}{1-w}\Big]\delta''(\hat{u})-4\Big[\tfrac{\ln\hat{u}}{\hat{u}^3}\Big]_++(8\ln w-1)\Big[\tfrac{1}{\hat{u}^3}\Big]_+\Big),\\
C&=\tfrac{2(2-w)}{3}\Big(-\Big[\tfrac{\ln\hat{u}}{\hat{u}^2}\Big]_++2\ln w\Big[\tfrac{1}{\hat{u}^2}\Big]_++L_w\,\delta'(\hat{u})\Big) \qquad \text{(A.4)}\\
L_w&=\mathrm{Li}_2(1-w)+2\ln^2 w+\tfrac{\pi^2}{3}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{R}_1^{(\pi)}=&\frac{(4\hat{u}-w)(2-w)\hat{u}+2w^3}{3\hat{u}^3}\ln\frac{\hat{u}}{w^2}+\Big[\frac{2w^6}{3\hat{u}^3}+\frac{7w^5}{3\hat{u}^2}-\frac{14-5\hat{u}}{3\hat{u}^2}w^4-\frac{13\hat{u}+32}{6\hat{u}}w^3\\
&-\frac{23\hat{u}^2-36\hat{u}-48}{6\hat{u}}w^2-(13\hat{u}^2-58\hat{u}+36)\frac{w}{6}-\frac{\hat{u}}{6}(3\hat{u}^2-26\hat{u}+8)\Big]\frac{\mathcal{I}_1}{\tilde{\lambda}} \qquad \text{(A.5)}\\
&-\frac{4w^2}{3\hat{u}^2}+\frac{2\hat{u}^2+2\hat{u}w-13\hat{u}+17w-28}{3\tilde{\lambda}}+\frac{4w}{3\hat{u}^2}+\frac{2}{3\hat{u}w}-\frac{7\hat{u}+8}{12\hat{u}}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{R}_2^{(\pi)}=&\frac{12(2-w)\hat{u}+8w^2}{3\hat{u}^3}\ln\frac{\hat{u}}{w^2}+\Big[w\Big(\frac{8w^4}{3\hat{u}^3}-\frac{40}{3}-\frac{14w}{3}-2\hat{u}+\frac{(4w-8)w^2}{\hat{u}^2}-4\frac{8-8w+w^2}{\hat{u}}\Big)\\
&+68+60\hat{u}-\frac{4}{\tilde{\lambda}}(15\hat{u}^3-35\hat{u}^2-76\hat{u}+14w+63w\hat{u}+19w\hat{u}^2)\Big]\frac{\mathcal{I}_1}{\tilde{\lambda}}+\frac{16(1+2\hat{u})}{3\hat{u}^2}\\
&-\frac{16w}{3\hat{u}^2}-\frac{28}{\hat{u}w^2}+\frac{68}{3\hat{u}w}-\frac{2\left(9\hat{u}^2+50\hat{u}w-201\hat{u}+86w-78\right)}{3\hat{u}\tilde{\lambda}} \qquad \text{(A.6)}\\
&-\frac{4\left(2\hat{u}^2w+2\hat{u}^3+11\hat{u}^2+49\hat{u}w-81\hat{u}+45w-28\right)}{\tilde{\lambda}^2}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{R}_3^{(\pi)}=&-\frac{2(3\hat{u}^2-(2-w)\hat{u}-2w^2)}{3\hat{u}^3}\ln\frac{\hat{u}}{w^2}+\Big(8w-\frac{13}{3}w^2-4-\frac{10}{3}\hat{u}(w-2)-\hat{u}^2\\
&+\frac{10\hat{u}(w-2)w^3+4w^5}{3\hat{u}^3}\Big)\frac{\mathcal{I}_1}{\tilde{\lambda}}-\frac{2\left(7\hat{u}^2+11\hat{u}w-19\hat{u}+17w-16\right)}{3\hat{u}\tilde{\lambda}} \qquad \text{(A.7)}\\
&-\frac{8w}{3\hat{u}^2}+\frac{8(\hat{u}+1)}{3\hat{u}^2}-\frac{4}{\hat{u}w^2}+\frac{2}{\hat{u}w}
\end{aligned}
$$

NLO dim-5: μ_π^2

$$
\begin{aligned}
W_1^{(G,1)}=&-\frac{2}{3}w\Big[G_1+\Big(\tfrac{C_F}{4}\Big(1+8w-5\frac{w^2\ln w}{1-w}\Big)-\tfrac{C_A}{4}(1+2w)\Big)\delta'(\hat{u}) \qquad \text{(A.9)}\\
&+C_F(5+\tfrac{2}{w^2}-\tfrac{2}{w})\Big(\Big[\frac{\ln\hat{u}}{\hat{u}}\Big]_+-2\ln w\Big[\frac{1}{\hat{u}}\Big]_++L_w\delta(\hat{u})\Big)\Big]\\
&-\frac{2}{3}\Big(\tfrac{C_A}{4}(8-5w)+C_F\Big(\frac{4}{w}-3+\frac{5w}{4}\Big)\Big)\Big[\frac{1}{\hat{u}}\Big]_+\\
&-\frac{1}{3}\Big(C_A\tfrac{5w^3-34w^2+51w-20}{2(w-1)^2}+C_F\tfrac{10w^5-21w^4+7w^3-10w^2+28w-16}{(w-1)^2w}\Big)\ln w\,\delta(\hat{u})\\
&-\frac{1}{3}\Big(C_A\tfrac{2w^4+2w^3-3w^2+5w-4}{2(1-w)w}+C_F\tfrac{35w^3-25w^2-10w-8}{4(1-w)}\Big)\delta(\hat{u})+\mathcal{R}_1^{(G)}\theta(\hat{u})
\end{aligned}
$$

$$
\begin{aligned}
W_2^{(G,1)}=&-\frac{8}{3}\Big[G_2+\Big(C_F\Big(\frac{1}{w}-\frac{11}{4}+2w-(1-\frac{5w}{4})\ln w\Big)+\tfrac{C_A}{4}(3-2w)\Big)\delta'(\hat{u})\\
&+2C_F\tfrac{w^2-w-1}{w^3}\Big(\Big[\frac{\ln\hat{u}}{\hat{u}}\Big]_+-2\ln w\Big[\frac{1}{\hat{u}}\Big]_++L_w\delta(\hat{u})\Big)\\
&+\Big(C_A(\tfrac{8}{w}-\tfrac{9}{2w^2}-\tfrac{2}{w^3}-\tfrac{9}{4})+C_F(\tfrac{7}{w^2}-\tfrac{6}{w^3}-\tfrac{5}{2})\Big)\Big[\frac{1}{\hat{u}}\Big]_+ \qquad \text{(A.10)}\\
&+\Big(C_A\tfrac{9w^3-56w^2+40w+16}{4w^3}+C_F\tfrac{5w^4-6w^3+3w^2-12w+12}{w^3}\Big)\ln w\,\delta(\hat{u})\\
&-\Big(C_A\tfrac{2w^3+4w^2-23w+16}{4w^2}+C_F\tfrac{35w^4-98w^3+134w^2-120w+32}{8w^3}\Big)\delta(\hat{u})\Big]+\mathcal{R}_2^{(G)}\theta(\hat{u})
\end{aligned}
$$

$$
\begin{aligned}
W_3^{(G,1)}=&-\frac{4}{3}\Big[G_3+\Big(C_F\Big(\frac{1}{4}+\frac{5w}{2}-\frac{5w^2\ln w}{4(1-w)}\Big)-\tfrac{C_A}{4}(1+w)\Big)\delta'(\hat{u})\Big]\\
&-\frac{2}{3}C_F\tfrac{5w^2+4w+4}{w^2}\Big(\Big[\frac{\ln\hat{u}}{\hat{u}}\Big]_+-2\ln w\Big[\frac{1}{\hat{u}}\Big]_++L_w\delta(\hat{u})\Big)\\
&-\frac{2}{3}\Big(C_A(\tfrac{2}{w^2}+\tfrac{5}{w}-\tfrac{7}{2})+C_F(\tfrac{4}{w^2}-\tfrac{5}{4})\Big)\Big[\frac{1}{\hat{u}}\Big]_+ \qquad \text{(A.11)}\\
&-\frac{1}{3}\Big(C_A\tfrac{7w^4-40w^3+49w^2-6w-8}{(w-1)^2w^2}+C_F\tfrac{(20w^5-37w^4-w^3+6w^2+24w-16)}{(w-1)^2w^2}\Big)\ln w\,\delta(\hat{u})\\
&-\frac{2}{3}\Big(C_A\tfrac{w^2-w+1}{1-w}+C_F\tfrac{35w^4-85w^3+66w^2-8w-16}{4(1-w)w^2}\Big)\delta(\hat{u})+\mathcal{R}_3^{(G)}\theta(\hat{u})
\end{aligned}
$$

$$
\begin{aligned}
G_i=&\Big(1+\frac{5}{2}w-4\delta_{i2}\Big)\Big[C_F\Big(\frac{3-8\ln w}{4}\Big[\frac{1}{\hat{u}^2}\Big]_++\Big[\frac{\ln\hat{u}}{\hat{u}^2}\Big]_+-L_w\delta'(\hat{u})\Big)+\tfrac{C_A}{2}\ln\frac{\mu}{m_b}\delta'(\hat{u})\Big]\\
&+C_A\Big[\frac{1+w}{2}\Big[\Big[\frac{1}{\hat{u}^2}\Big]_++\ln w\,\delta'(\hat{u})\Big]-\delta_{i2}\Big(\frac{1+2w}{2w}\Big[\frac{1}{\hat{u}^2}\Big]_++\frac{\ln w}{w}\delta'(\hat{u})\Big)\Big]-\tfrac{3C_A}{4}\frac{w_i^{(G,0)}}{w_i^{(0)}}\ln\frac{\mu}{m_b}\delta(\hat{u})\\
&+C_A\Big(\frac{1+4w}{2w^2}-\frac{1+2w}{w^3}\delta_{i2}\Big)\Big[\Big[\frac{\ln\hat{u}}{\hat{u}}\Big]_+-2\ln w\Big[\frac{1}{\hat{u}}\Big]_++L_w\delta(\hat{u})\Big] \qquad \text{(A.12)}
\end{aligned}
$$

where

$$
w_1^{(G,0)}=-\frac{2}{3}(4-5w),\qquad w_2^{(G,0)}=0,\qquad w_3^{(G,0)}=\frac{10}{3}, \qquad \text{(A.13)}
$$

$$
\begin{aligned}
\mathcal{R}_1^{(G)}=&\frac{C_A}{3}\Big[\tfrac{1}{2}+\tfrac{\hat{u}+13w-16}{\tilde{\lambda}}+\tfrac{4w+1}{\hat{u}w}\ln\tfrac{\hat{u}}{w^2}+\Big(\tfrac{4w+1-6\hat{u}}{\hat{u}}+2\tfrac{3\hat{u}(\hat{u}-3+w)+4w}{\tilde{\lambda}}\Big)\mathcal{I}_1\Big] \qquad \text{(A.14)}\\
&+\frac{C_F}{3}\Big[\tfrac{15\hat{u}-5\hat{u}w-5\hat{u}^2-11w+20}{\tilde{\lambda}}-\tfrac{4w}{\hat{u}\tilde{\lambda}}-\tfrac{10w}{\hat{u}}+\tfrac{8}{\hat{u}w}+\tfrac{11\hat{u}+24}{4\hat{u}}+\Big(\tfrac{5w^2}{\hat{u}^2}+\tfrac{2(5\hat{u}+1)w}{\hat{u}^2}+\tfrac{4-4w}{\hat{u}w}\Big)\ln\frac{\hat{u}}{w^2}\\
&+\Big(\tfrac{8-3\hat{u}^2-13\hat{u}w+10\hat{u}-12w}{\tilde{\lambda}}+\tfrac{5w^3}{\hat{u}^2}+\tfrac{(15\hat{u}+2)w^2}{\hat{u}^2}+\tfrac{3(5\hat{u}-8)w}{2\hat{u}}+\tfrac{5\hat{u}}{2}-2\Big)\mathcal{I}_1\Big]
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{R}_2^{(G)}=&4C_A\Big[\tfrac{16-13\hat{u}^2-25\hat{u}w+51\hat{u}-29w}{\tilde{\lambda}^2}+\tfrac{22-15\hat{u}w-9\hat{u}^2+112\hat{u}-32w}{6\tilde{\lambda}\hat{u}}+\tfrac{w}{3\tilde{\lambda}\hat{u}^2}+\tfrac{16\hat{u}-1}{3\hat{u}^2w}-\tfrac{3}{\hat{u}w^2}-\tfrac{4}{3\hat{u}w^3}\\
&+\tfrac{4w^2-3w-2}{3w^3\hat{u}}\ln\frac{\hat{u}}{w^2}+\Big(\tfrac{14\hat{u}^2-26\hat{u}w+58\hat{u}-3w-2}{3\tilde{\lambda}\hat{u}}-\tfrac{2\left(3\hat{u}^2w+3\hat{u}^3-5\hat{u}^2+20\hat{u}w-25\hat{u}\right)}{\tilde{\lambda}^2}-\tfrac{8w}{\tilde{\lambda}^2}+\tfrac{4}{3\hat{u}}\Big)\mathcal{I}_1\Big]\\
&+4C_F\Big[\tfrac{5\hat{u}^2w+42\hat{u}w+5\hat{u}^3-4\hat{u}^2-55\hat{u}+39w-36}{\tilde{\lambda}^2}+\tfrac{4w}{\tilde{\lambda}^2\hat{u}}+\tfrac{53\hat{u}w-20\hat{u}^2-155\hat{u}+44w-52}{6\tilde{\lambda}\hat{u}}+\tfrac{14}{3\hat{u}w^2}-\tfrac{4}{\hat{u}w^3}-\tfrac{10}{3\hat{u}}\\
&+\tfrac{4\hat{u}\left(w^2-w-1\right)+(5w-6)w^3}{3\hat{u}^2w^3}\ln\frac{\hat{u}}{w^2}+\Big(\tfrac{23\hat{u}^2w+13\hat{u}^3-37\hat{u}^2+47\hat{u}w-58\hat{u}+20w-8}{\tilde{\lambda}^2} \qquad \text{(A.15)}\\
&+\tfrac{25\hat{u}^2w+15\hat{u}^3-114\hat{u}^2+76\hat{u}w-150\hat{u}+16w-8}{6\tilde{\lambda}\hat{u}}+\tfrac{5w^2}{3\hat{u}^2}+\tfrac{(5\hat{u}-6)w}{3\hat{u}^2}-\tfrac{5\hat{u}+8}{2\hat{u}}\Big)\mathcal{I}_1\Big]
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{R}_3^{(G)}=&\frac{4C_A}{3}\Big[\tfrac{15\hat{u}-3\hat{u}^2-3\hat{u}w-5w-2}{2\tilde{\lambda}\hat{u}}+\tfrac{1}{\hat{u}w^2}+\tfrac{5}{2\hat{u}w}+\tfrac{1+4w}{2w^2\hat{u}}\ln\tfrac{\hat{u}}{w^2}+\tfrac{w-5\hat{u}-2w\hat{u}+4w^2}{2\tilde{\lambda}\hat{u}}\mathcal{I}_1\Big] \qquad \text{(A.16)}\\
&+\frac{4C_F}{3}\Big[\tfrac{2\hat{u}^2+7\hat{u}w-9\hat{u}+3w}{\tilde{\lambda}\hat{u}}+\tfrac{2}{\hat{u}w^2}-\tfrac{5}{\hat{u}}+\Big(\tfrac{5w}{2\hat{u}^2}+\tfrac{5\hat{u}+2}{2\hat{u}^2}+\tfrac{2}{\hat{u}w^2}+\tfrac{2}{\hat{u}w}\Big)\ln\tfrac{\hat{u}}{w^2}\\
&+\Big(\tfrac{5\hat{u}^2+5\hat{u}w-16\hat{u}+12w-12}{2\tilde{\lambda}}+\tfrac{5w^2}{2\hat{u}^2}+\tfrac{(5\hat{u}+1)w}{\hat{u}^2}-\tfrac{5\hat{u}+8}{4\hat{u}}\Big)\mathcal{I}_1\Big]
\end{aligned}
$$

NLO dim-5: μ_G^2

[Capdevila et al. '22]

Theoretical Framework

Analytical Result

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dy} = \{2(3-2y)y^2\theta(1-y) - \frac{2\lambda_1}{m_b^2}\left[-\frac{5}{3}y^3\theta(1-y) + \frac{1}{6}\delta(1-y)\theta(1^+-y) + \frac{1}{6}\delta'(1-y)\theta(1^+-y)\right] - \frac{2\lambda_2}{m_b^2}\left[-y^2(6+5y)\theta(1-y) + \frac{11}{2}\delta(1-y)\theta(1^+-y)\right]\}$$

LO: dim-3 and dim-5 $y = \frac{2E_e}{m_c}$

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dy} \supset \frac{1}{360}\Big(5y\left(-82+y\left(153-86y+8\pi^2(-3+2y)\right)\right) - 24y^3\left(-50+y\left(5+y(-28+9y)\right)\right)\mathrm{ArcCoth}\left[1+2\sqrt{y}\right] + 12y^3\left(125-2(-5+y)y(-1+9y)\right)\mathrm{ArcTanh}\left[1-2\sqrt{y}\right] - 816\,\mathrm{Log}\left[1-\sqrt{y}\right] - 600\,\mathrm{Log}\left[1+\sqrt{y}\right] - 108\,\mathrm{Log}\left[\left(1+\sqrt{y}\right)^2\right] + 406\,\mathrm{Log}[1-y] + 4\Big(6y^4(1+y)(5+9y)\mathrm{ArcTanh}[1-2y] + 6y^3\left(25+9y^2\right)\mathrm{Log}\left[1+\sqrt{y}\right] + 90y\,\mathrm{Log}[1-y] - 105y^2\mathrm{Log}[1-y] - 260y^3\mathrm{Log}[1-y] - 180y^5\mathrm{Log}[1-y] + 150\,\mathrm{Log}[1-y]^2 - 360y\,\mathrm{Log}[1-y]^2 + 135y^2\mathrm{Log}[1-y]^2 + 30y^3\mathrm{Log}[1-y]^2 - 75y^3\mathrm{Log}[y] + 153y^5\mathrm{Log}[y] - 30y^3\mathrm{Log}\left[y^5\right]\Big) + \ldots$$

... **NLO: dim-3**

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dy} \supset \frac{1}{810}\Big(480\pi^2 + 695y + \frac{7615y^2}{2} - 165\pi^2y^2 - \frac{545y^3}{2} + 540\pi^2y^3 - 6030y^4 + 795\pi^2y^4 + 1800y^5 - 1230\pi^2y^5 + 300\pi^2y^6 + 1440\,i\,\pi\,\mathrm{Log}[2] + 2880y\,\mathrm{Log}[2] - 7740y^2\mathrm{Log}[2] + \ldots$$

... **NLO: dim-5**

Some tips

- Up to finite power, the obtained differential decay rate is **NOT** the experimental spectrum
- Observables require integration over final states

$$\Gamma = \int \frac{d\Gamma}{dy}dy$$

$$\langle E_\ell \rangle = \frac{1}{\Gamma}\int \frac{d\Gamma}{dy}E_\ell\, dy$$

$$\langle E_\ell^2 \rangle = \frac{1}{\Gamma}\int \frac{d\Gamma}{dy}E_\ell^2\, dy$$

...

3. Experimental status

Experimental status

CLEO measurements

$D^0 \to e^+ X$ $\quad$ $D^+ \to e^+ X$ $\quad$ $D_s^+ \to e^+ X$

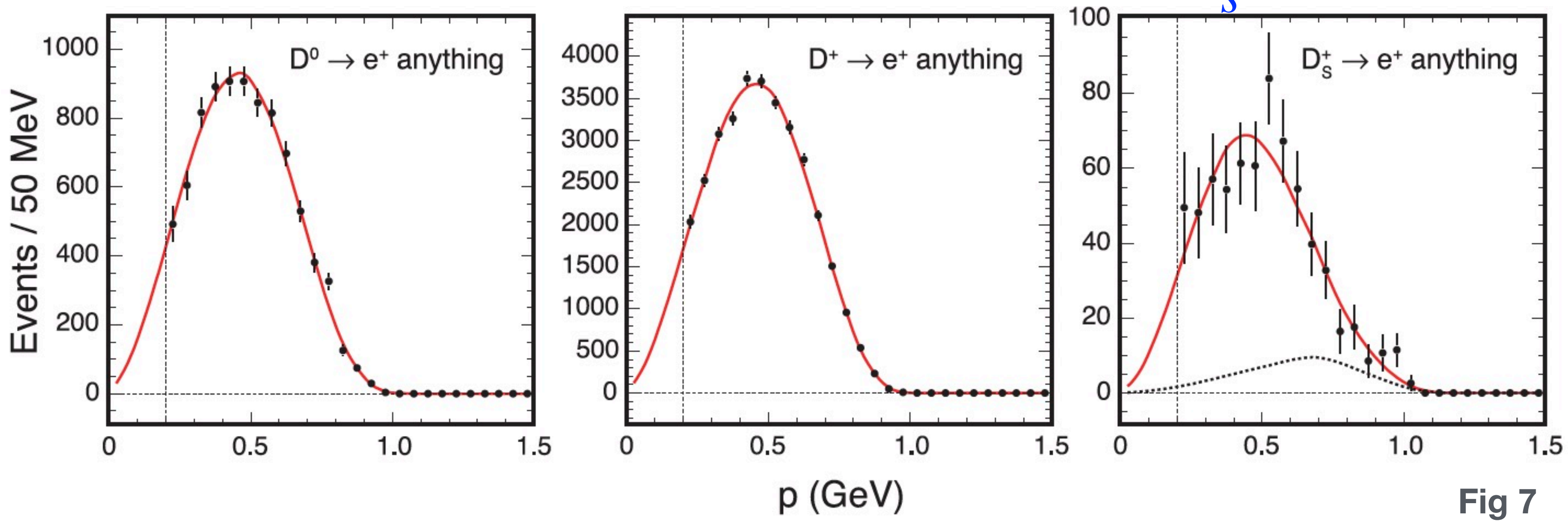


Fig 7

$$B\left(D^0 \to X e^+ \nu_e\right) = (6.46 \pm 0.09 \pm 0.11)\,\%$$

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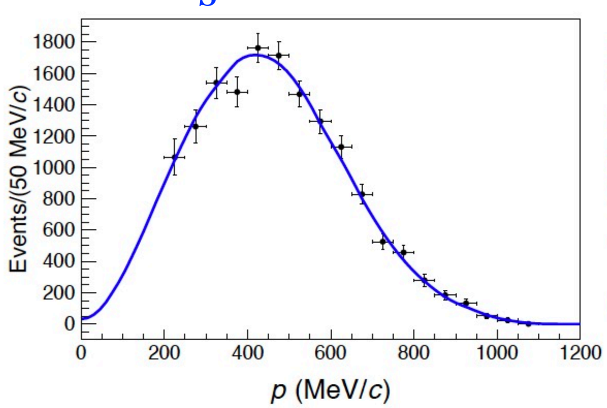
$$B\left(D_s^+ \to X e^+ \nu_e\right) = (6.52 \pm 0.39 \pm 0.15)\,\%$$

[CLEO, ’09]

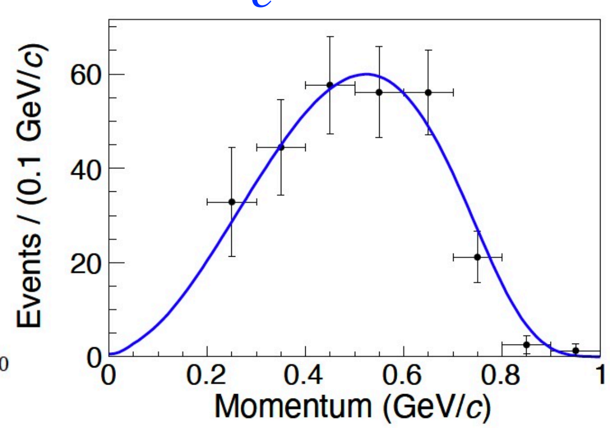
BESIII measurements

$D_s^+ \to e^+ X$



[BESIII, ’21]

$\Lambda_c^+ \to e^+ X$



$\Lambda_c^+ \to e^+ X$

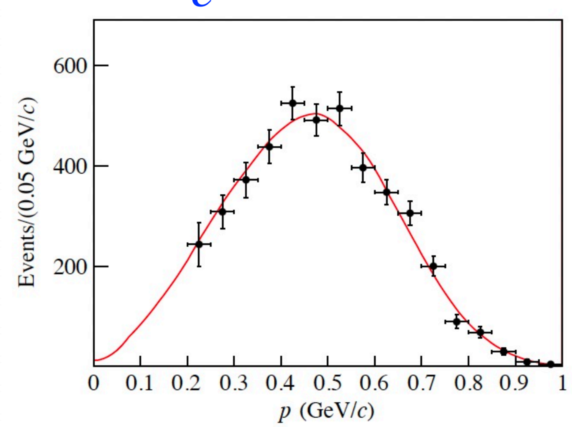


Fig 8

$$B\left(D_s^+ \to X e^+ \nu_e\right) = (6.30 \pm 0.13 \pm 0.10)\,\%$$

[BESIII, ’21]

$$B\left(\Lambda_c^+ \to X e^+ \nu_e\right) = (3.95 \pm 0.34 \pm 0.09)\,\%$$

[BESIII (567 pb^{-1}), ’18]

$$B\left(\Lambda_c^+ \to X e^+ \nu_e\right) = \left(4.06 \pm 0.10_{\text{stat.}} \pm 0.09_{\text{syst}}\right)\%$$

[BESIII (4.5 fb^{-1}), ’23]

Experimental status

Electronic energy momentum

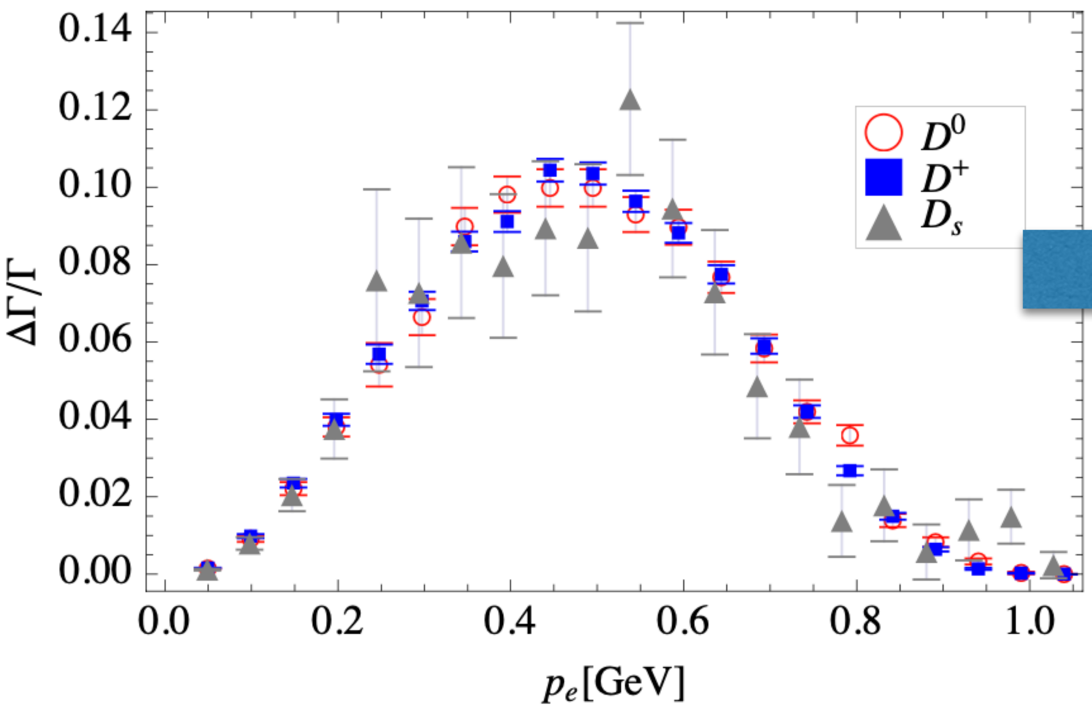


Fig 9

The laboratory frame of the D meson

$$\langle E_e \rangle_{lab}^{D^0} = 0.465(3)\text{GeV}\,,$$
$$\langle E_e \rangle_{lab}^{D^+} = 0.459(1)\text{GeV}\,,$$
$$\langle E_e \rangle_{lab}^{D_s} = 0.466(12)\text{GeV}\,,$$

$$\langle E_e^2 \rangle_{lab}^{D^0} = 0.248(2)\text{GeV}^2\,,$$
$$\langle E_e^2 \rangle_{lab}^{D^+} = 0.242(1)\text{GeV}^2\,,$$
$$\langle E_e^2 \rangle_{lab}^{D_s} = 0.254(13)\text{GeV}^2\,.$$

D mesons rest frame

$$\langle E_\ell \rangle_{exp}^{D^0} = 0.459(3)\text{GeV}\,, \qquad \langle E_\ell^2 \rangle_{exp}^{D^0} = 0.240(2)\text{GeV}^2\,,$$
$$\langle E_\ell \rangle_{exp}^{D^+} = 0.455(1)\text{GeV}\,, \qquad \langle E_\ell^2 \rangle_{exp}^{D^+} = 0.236(1)\text{GeV}^2\,,$$
$$\langle E_\ell \rangle_{exp}^{D_s} = 0.456(11)\text{GeV}\,, \qquad \langle E_\ell^2 \rangle_{exp}^{D_s} = 0.239(12)\text{GeV}^2\,,$$

[Gambino,Kamenik, ’10]

4. Phenomenological Analysis (preliminary)

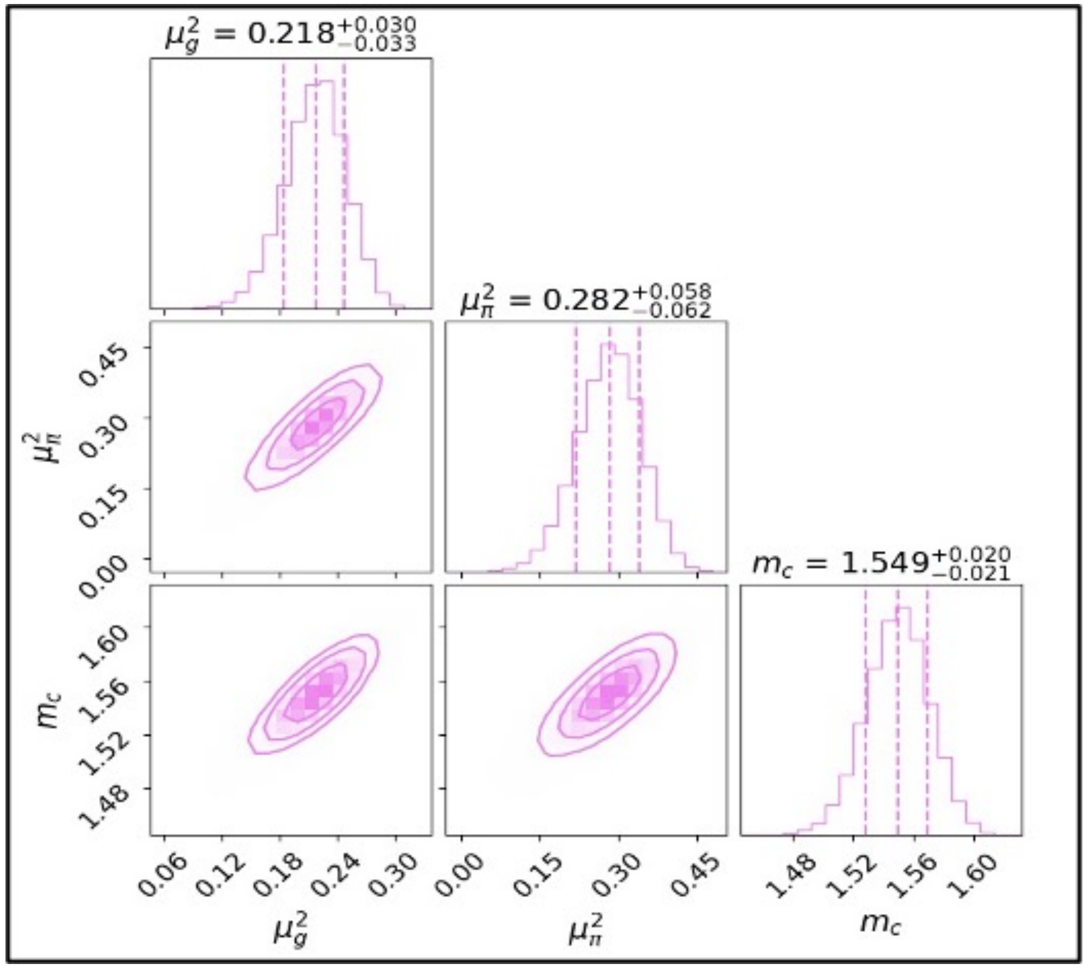
https://github.com/ChunHuangPhy/CompactOject

Phenomenological Analysis (preliminary)

CompactObject

[Huang et al, '2024]

⭐ **For the first time,** we systematically extract the **Model-independent** fundamental parameters of HQE from experimental data on semi-leptonic inclusive decays of D mesons.



$$\mu_\pi^2(D) = (0.48 \pm 0.20)\mathrm{GeV}^2$$

$$\mu_G^2(D) = (0.34 \pm 0.10)\mathrm{GeV}^2$$

[Lenz et al, '22]

$$\mu_\pi^2(D) = (0.282^{+0.058}_{-0.062})\mathrm{GeV}^2$$

$$\mu_G^2(D) = (0.218^{+0.030}_{-0.033})\mathrm{GeV}^2$$

$$m_c = (1.549^{+0.020}_{-0.021})\mathrm{GeV}$$

[Our results]

☑ **Higher** precision

☑ **Stronger** constraints

Phenomenological Analysis (preliminary)

CompactObject

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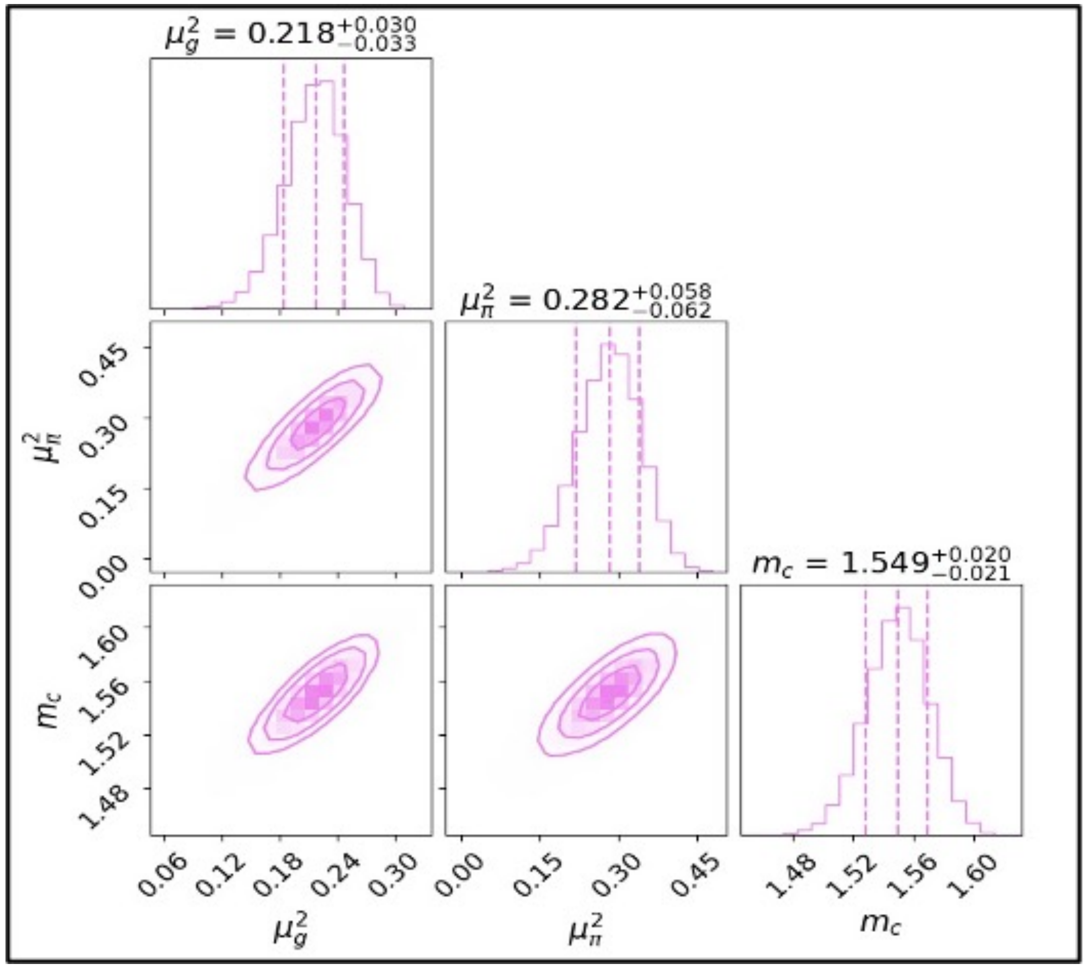

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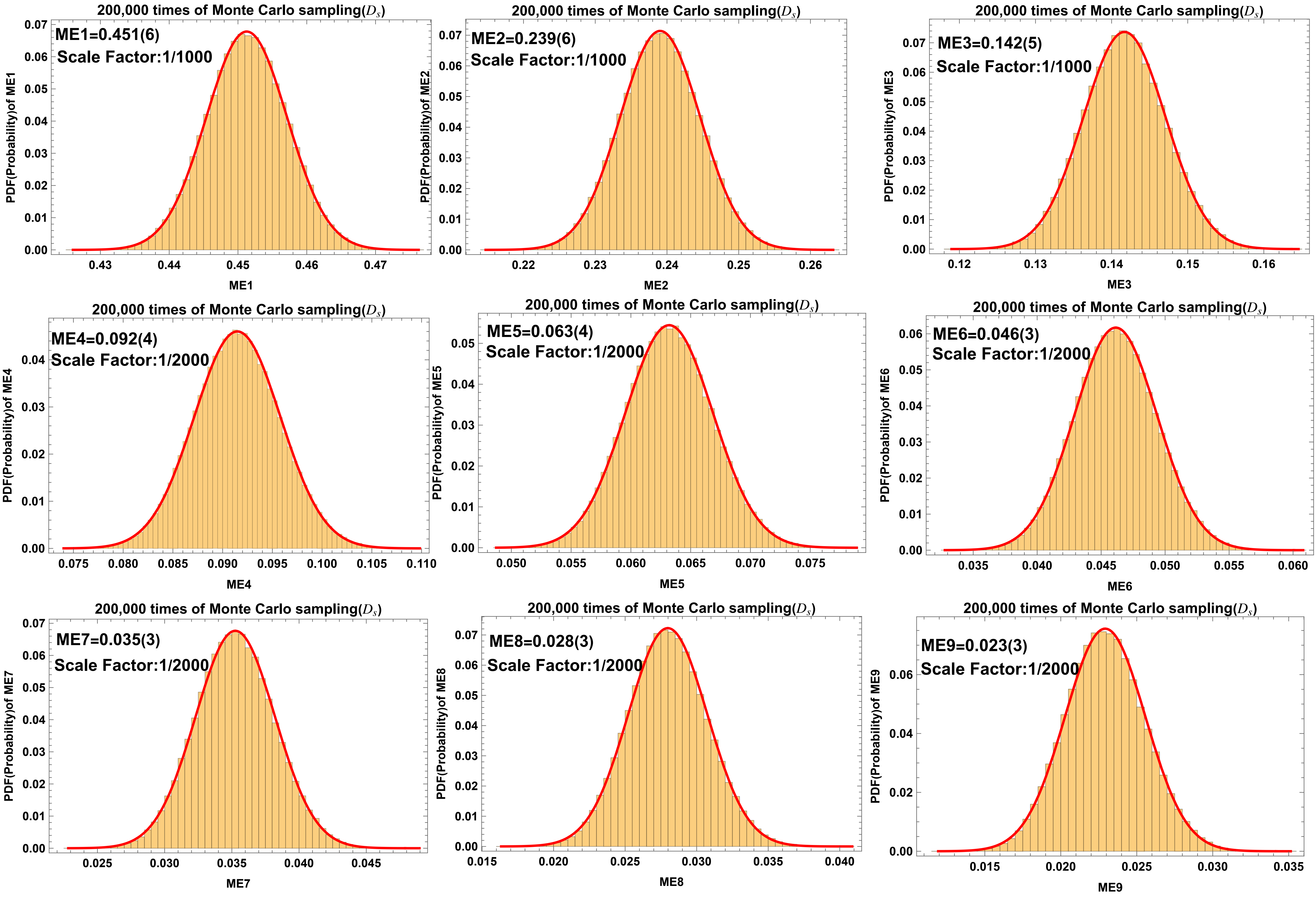
[Our results]

☑ **Higher** precision

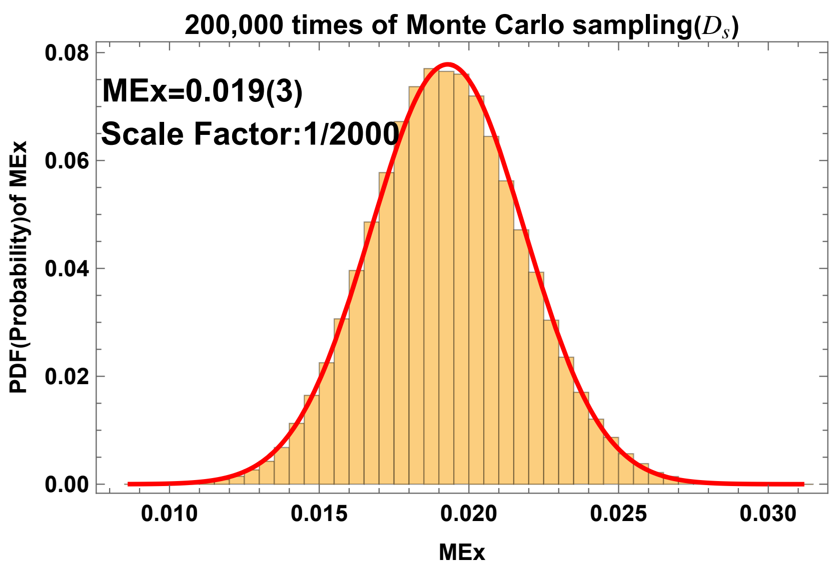
☑ **Stronger** constraints

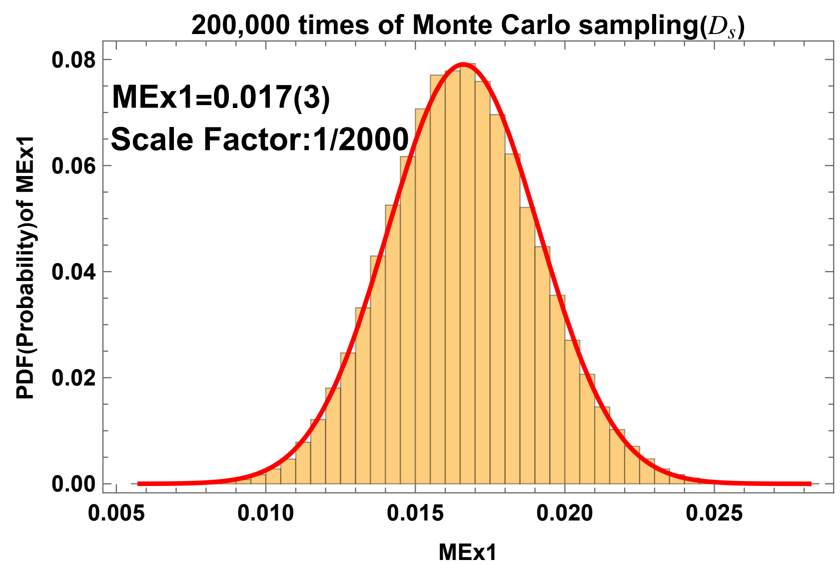
Lab Frame

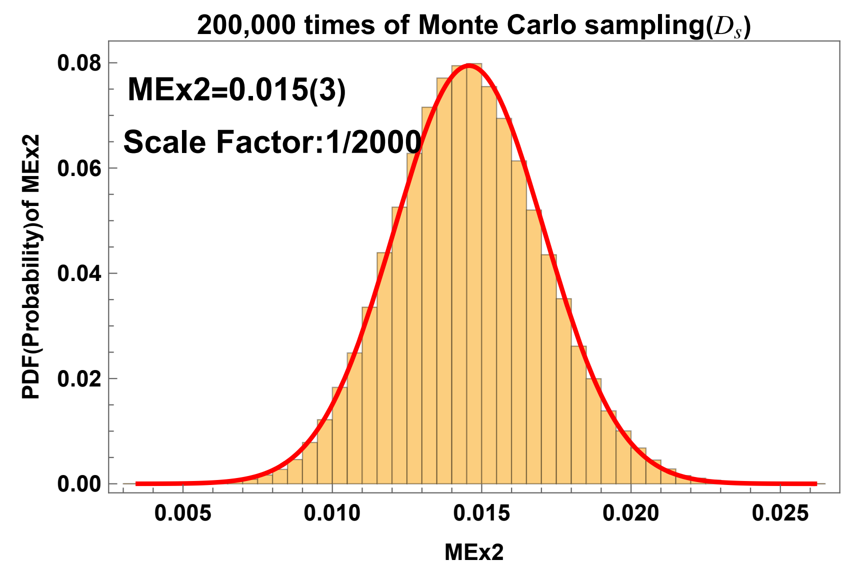


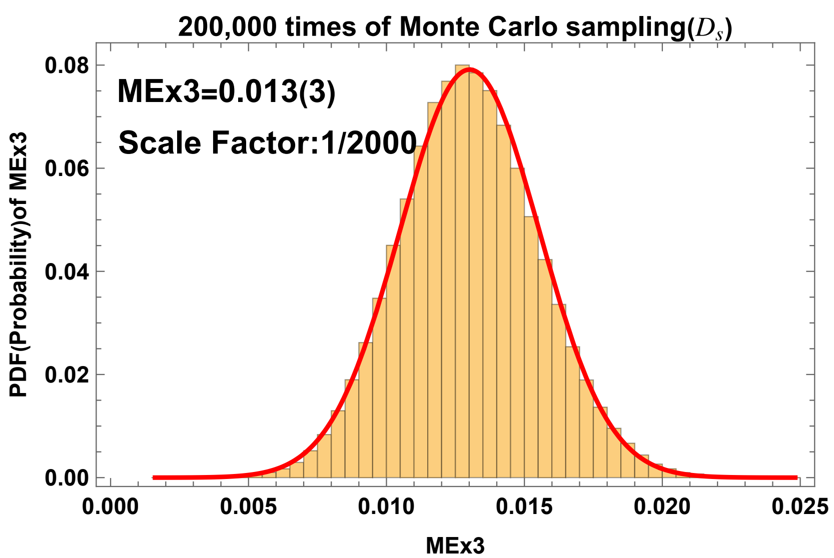
200,000 times of Monte Carlo sampling(D_s)
ME1=0.451(6)
Scale Factor:1/1000
PDF(Probability)of ME1
ME1
200,000 times of Monte Carlo sampling(D_s)
ME2=0.239(6)
Scale Factor:1/1000
PDF(Probability)of ME2
ME2
200,000 times of Monte Carlo sampling(D_s)
ME3=0.142(5)
Scale Factor:1/1000
PDF(Probability)of ME3
ME3
200,000 times of Monte Carlo sampling(D_s)
ME4=0.092(4)
Scale Factor:1/2000
PDF(Probability)of ME4
ME4
200,000 times of Monte Carlo sampling(D_s)
ME5=0.063(4)
Scale Factor:1/2000
PDF(Probability)of ME5
ME5
200,000 times of Monte Carlo sampling(D_s)
ME6=0.046(3)
Scale Factor:1/2000
PDF(Probability)of ME6
ME6
200,000 times of Monte Carlo sampling(D_s)
ME7=0.035(3)
Scale Factor:1/2000
PDF(Probability)of ME7
ME7
200,000 times of Monte Carlo sampling(D_s)
ME8=0.028(3)
Scale Factor:1/2000
PDF(Probability)of ME8
ME8
200,000 times of Monte Carlo sampling(D_s)
ME9=0.023(3)
Scale Factor:1/2000
PDF(Probability)of ME9
ME9

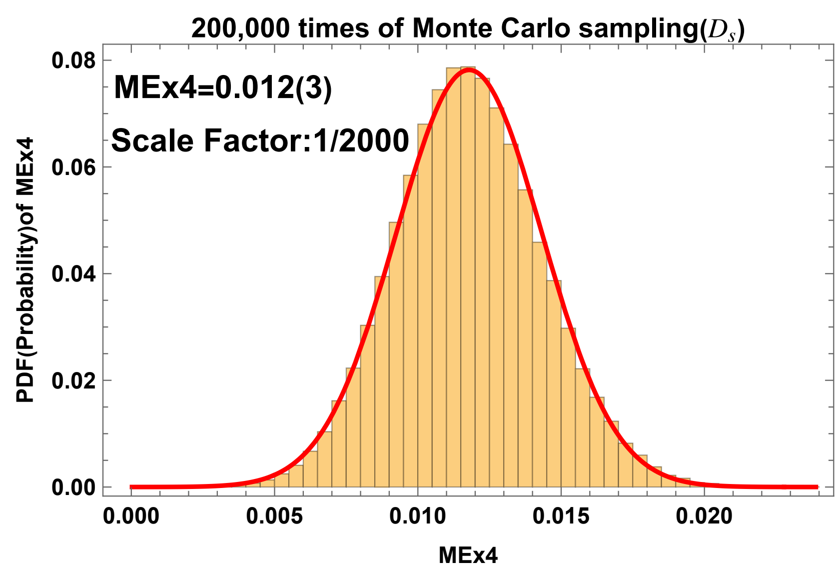
Lab Frame

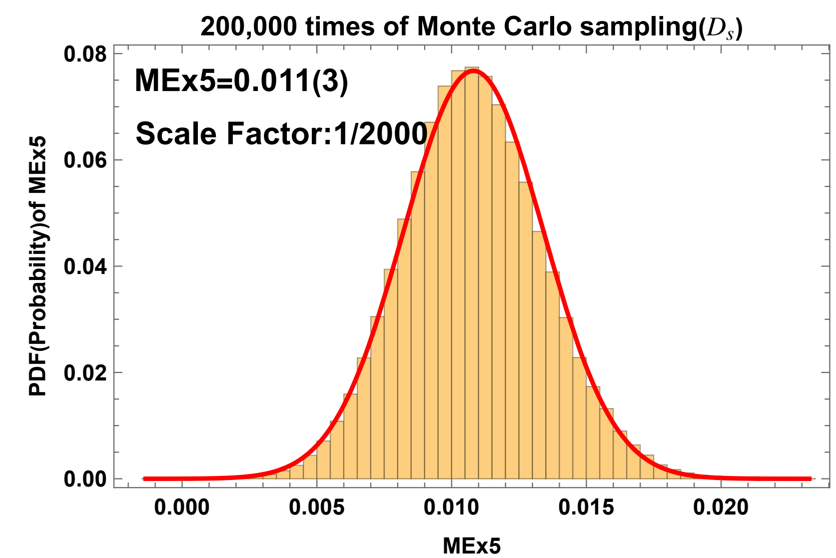

200,000 times of Monte Carlo sampling(D_s)
MEx=0.019(3)
Scale Factor:1/2000
PDF(Probability)of MEx
MEx

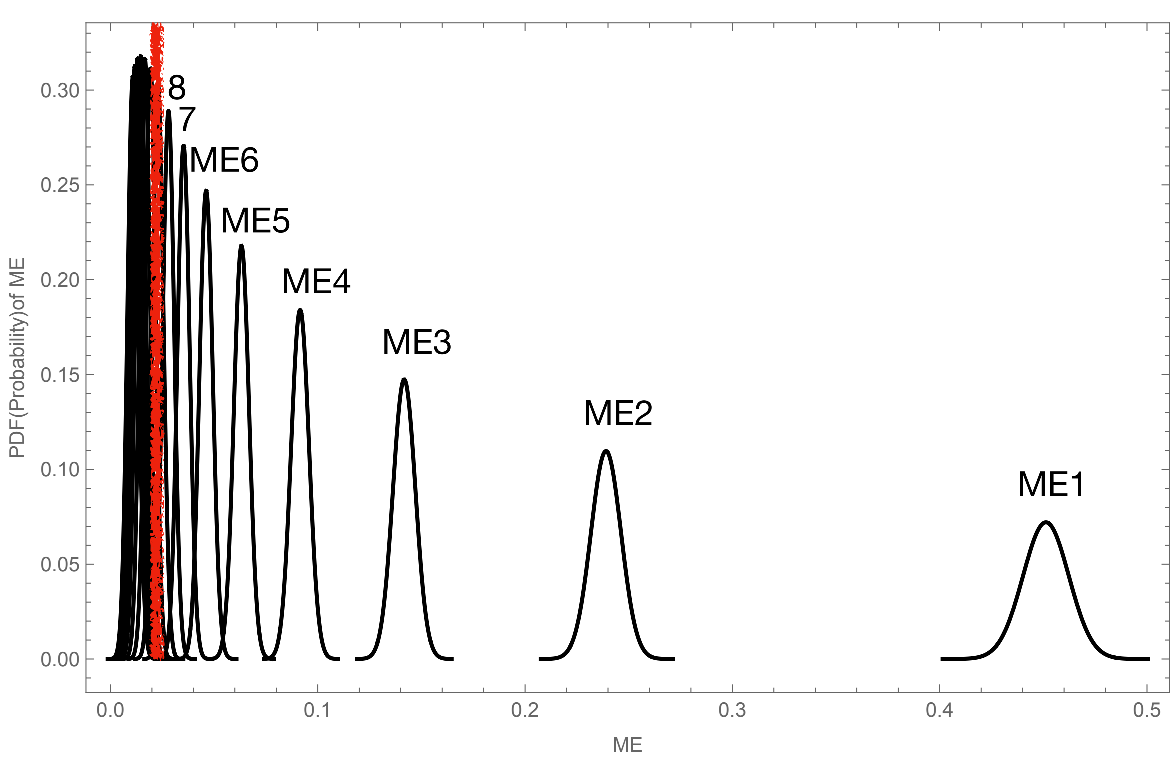

200,000 times of Monte Carlo sampling(D_s)
MEx1=0.017(3)
Scale Factor:1/2000
PDF(Probability)of MEx1
MEx1


200,000 times of Monte Carlo sampling(D_s)
MEx2=0.015(3)
Scale Factor:1/2000
PDF(Probability)of MEx2
MEx2

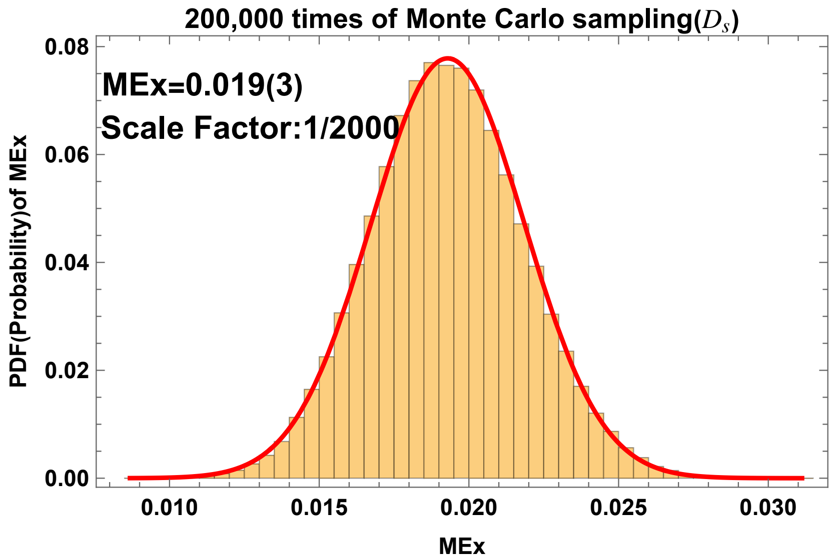

200,000 times of Monte Carlo sampling(D_s)
MEx3=0.013(3)
Scale Factor:1/2000
PDF(Probability)of MEx3
MEx3

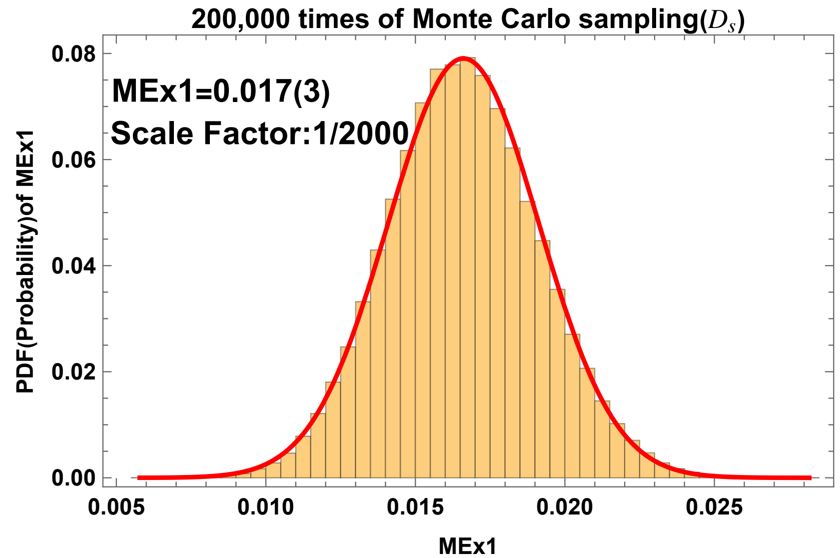

200,000 times of Monte Carlo sampling(D_s)
MEx4=0.012(3)
Scale Factor:1/2000
PDF(Probability)of MEx4
MEx4


200,000 times of Monte Carlo sampling(D_s)
MEx5=0.011(3)
Scale Factor:1/2000
PDF(Probability)of MEx5
MEx5


8
7
ME6
ME5
ME4
ME3
ME2
ME1
PDF(Probability)of ME
ME

Lab Frame


200,000 times of Monte Carlo sampling(D_s)
MEx=0.019(3)
Scale Factor:1/2000
PDF(Probability)of MEx
MEx
0.00
0.02
0.04
0.06
0.08
0.010
0.015
0.020
0.025
0.030


200,000 times of Monte Carlo sampling(D_s)
MEx1=0.017(3)
Scale Factor:1/2000
PDF(Probability)of MEx1
MEx1
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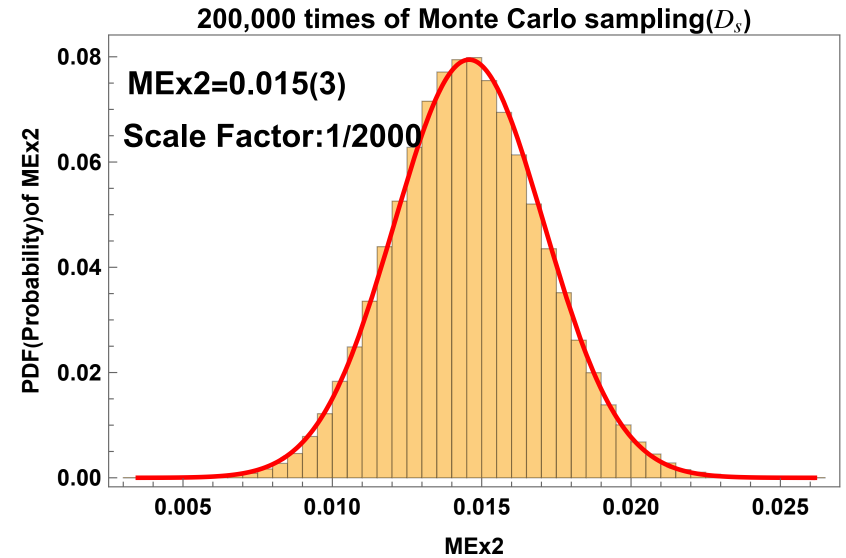
0.02
0.04
0.06
0.08
0.005
0.010
0.015
0.020
0.025


200,000 times of Monte Carlo sampling(D_s)
MEx2=0.015(3)
Scale Factor:1/2000
PDF(Probability)of MEx2
MEx2
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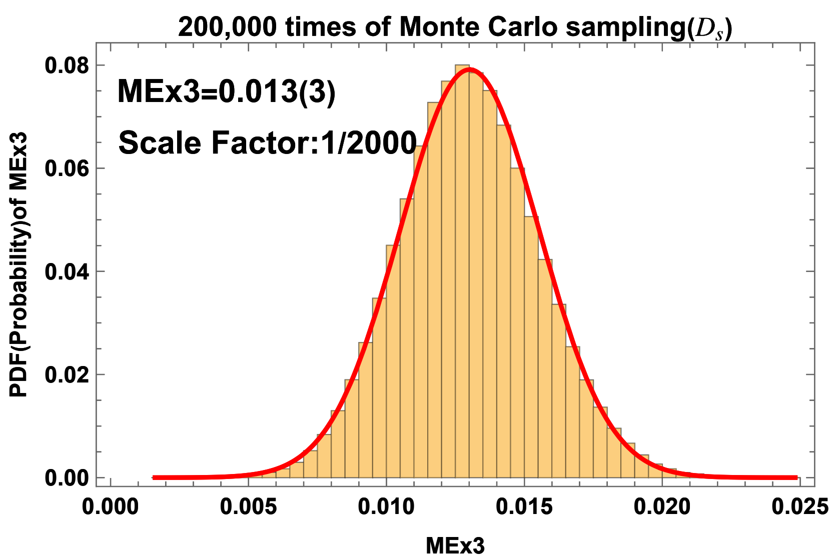
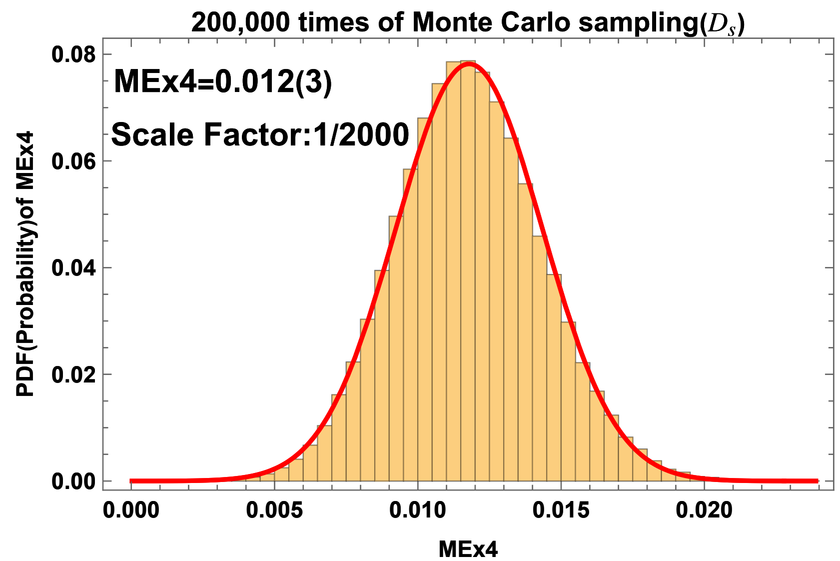
0.02
0.04
0.06
0.08
0.005
0.010
0.015
0.020
0.025


200,000 times of Monte Carlo sampling(D_s)
MEx3=0.013(3)
Scale Factor:1/2000
PDF(Probability)of MEx3
MEx3
0.00
0.02
0.04
0.06
0.08
0.000
0.005
0.010
0.015
0.020
0.025


200,000 times of Monte Carlo sampling(D_s)
MEx4=0.012(3)
Scale Factor:1/2000
PDF(Probability)of MEx4
MEx4
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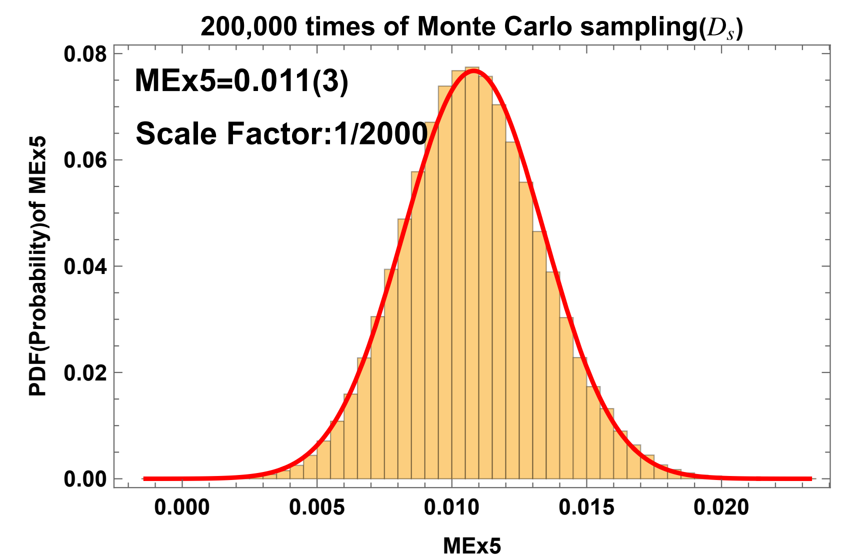
0.02
0.04
0.06
0.08
0.000
0.005
0.010
0.015
0.020


200,000 times of Monte Carlo sampling(D_s)
MEx5=0.011(3)
Scale Factor:1/2000
PDF(Probability)of MEx5
MEx5
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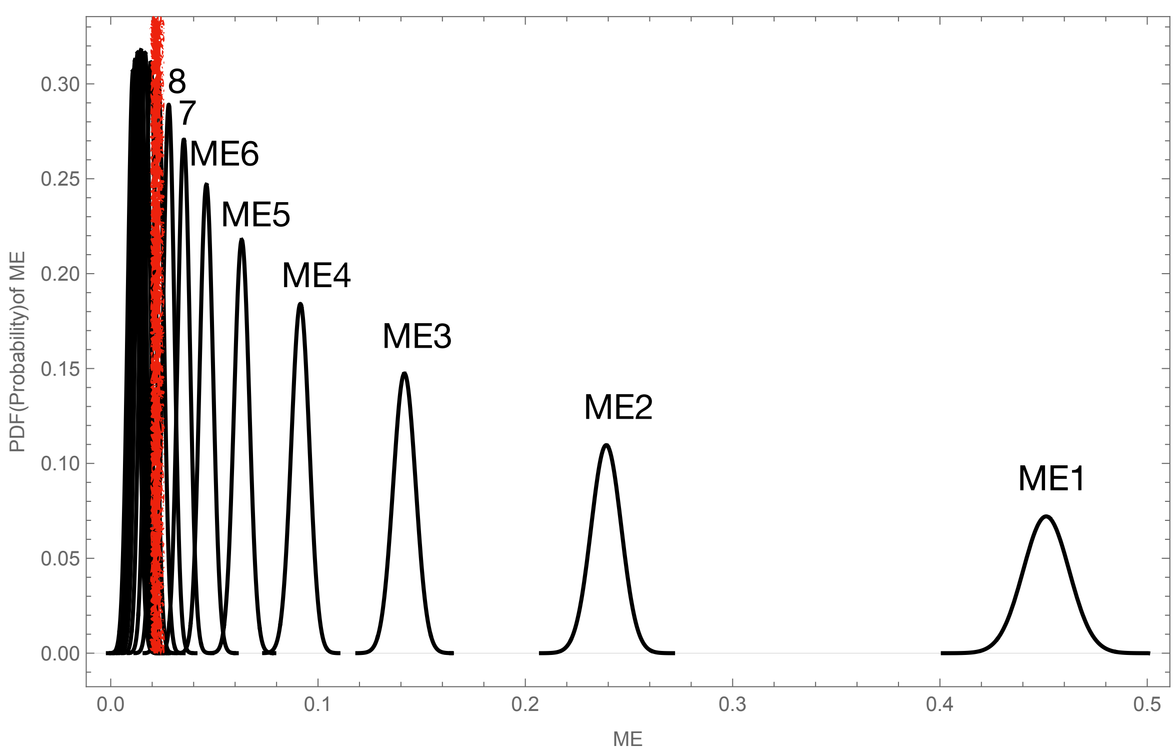
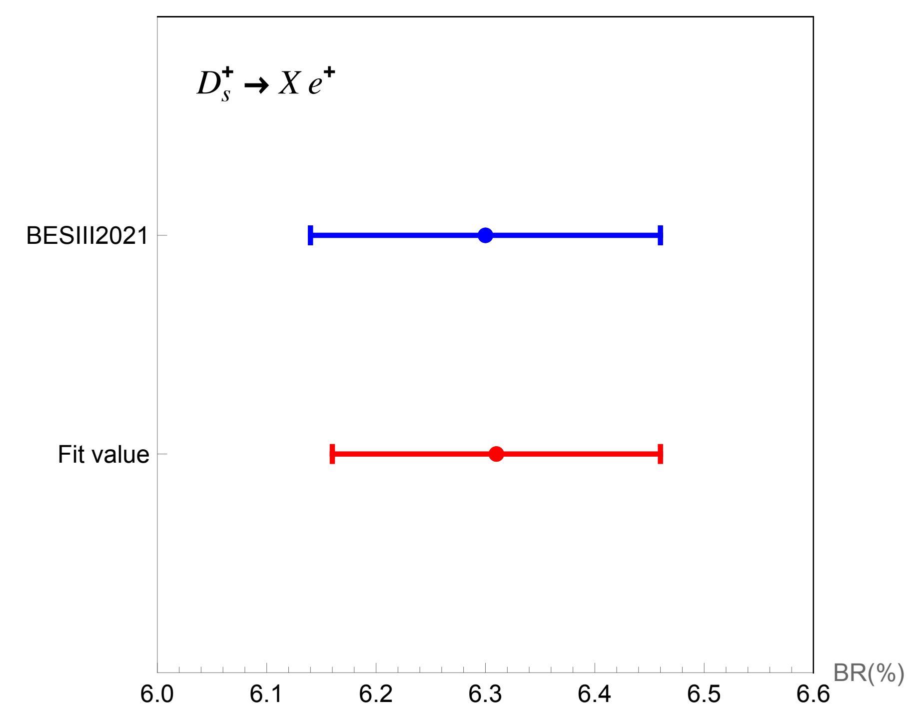
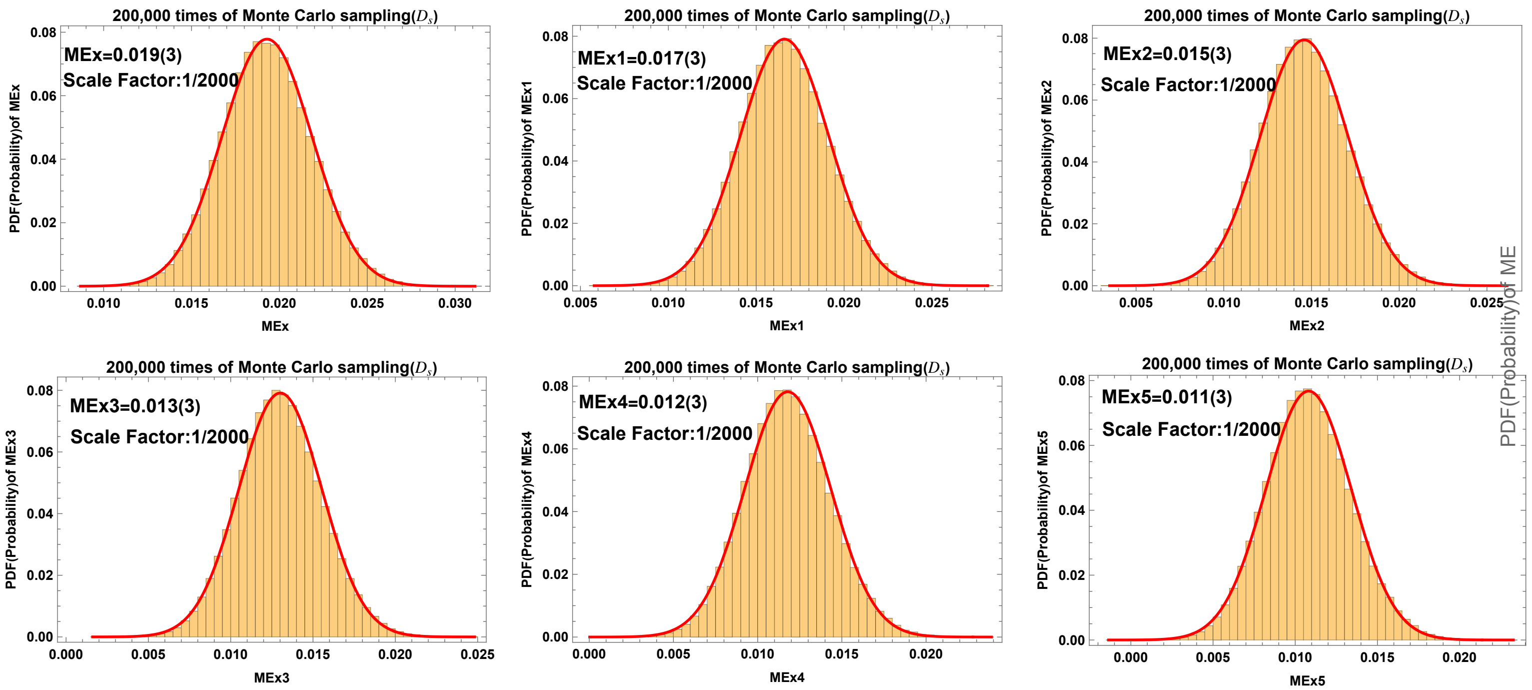
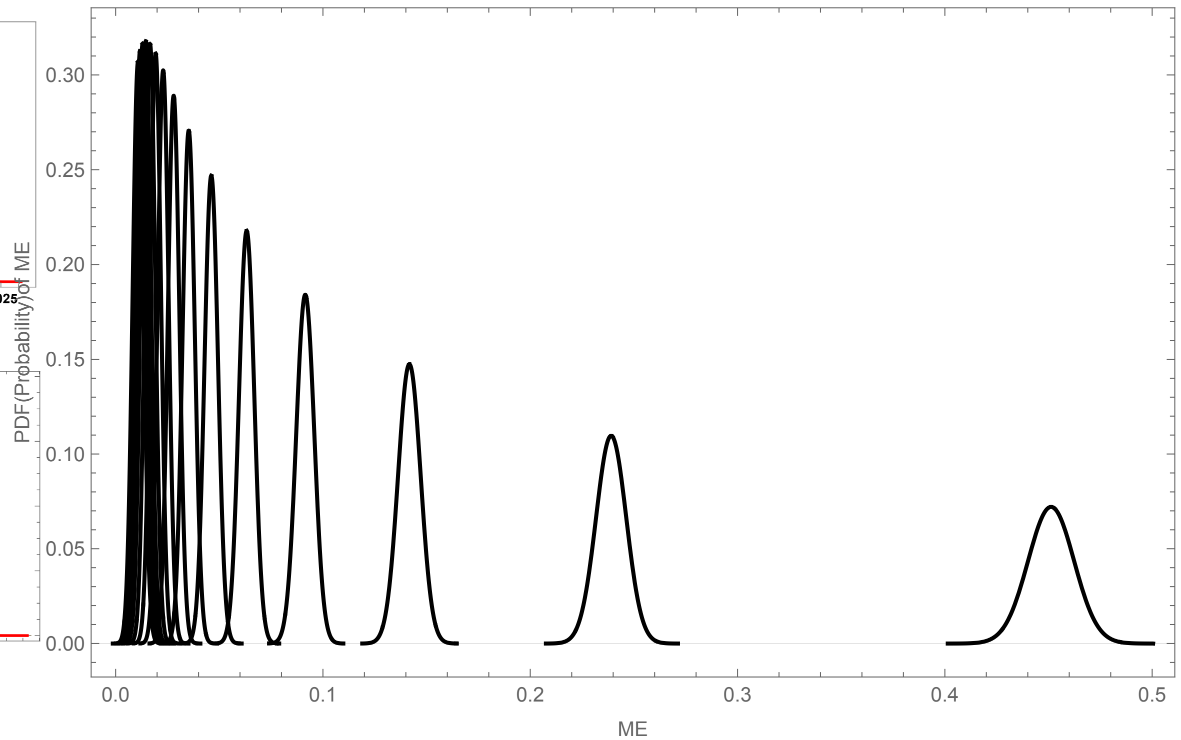
0.04
0.06
0.08
0.000
0.005
0.010
0.015
0.020


8
7
ME6
ME5
ME4
ME3
ME2
ME1
PDF(Probability)of ME
ME
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.0
0.1
0.2
0.3
0.4
0.5


$D_s^+ \to X e^+$
BESIII2021
Fit value
BR(%)
6.0
6.1
6.2
6.3
6.4
6.5
6.6

Lab Frame



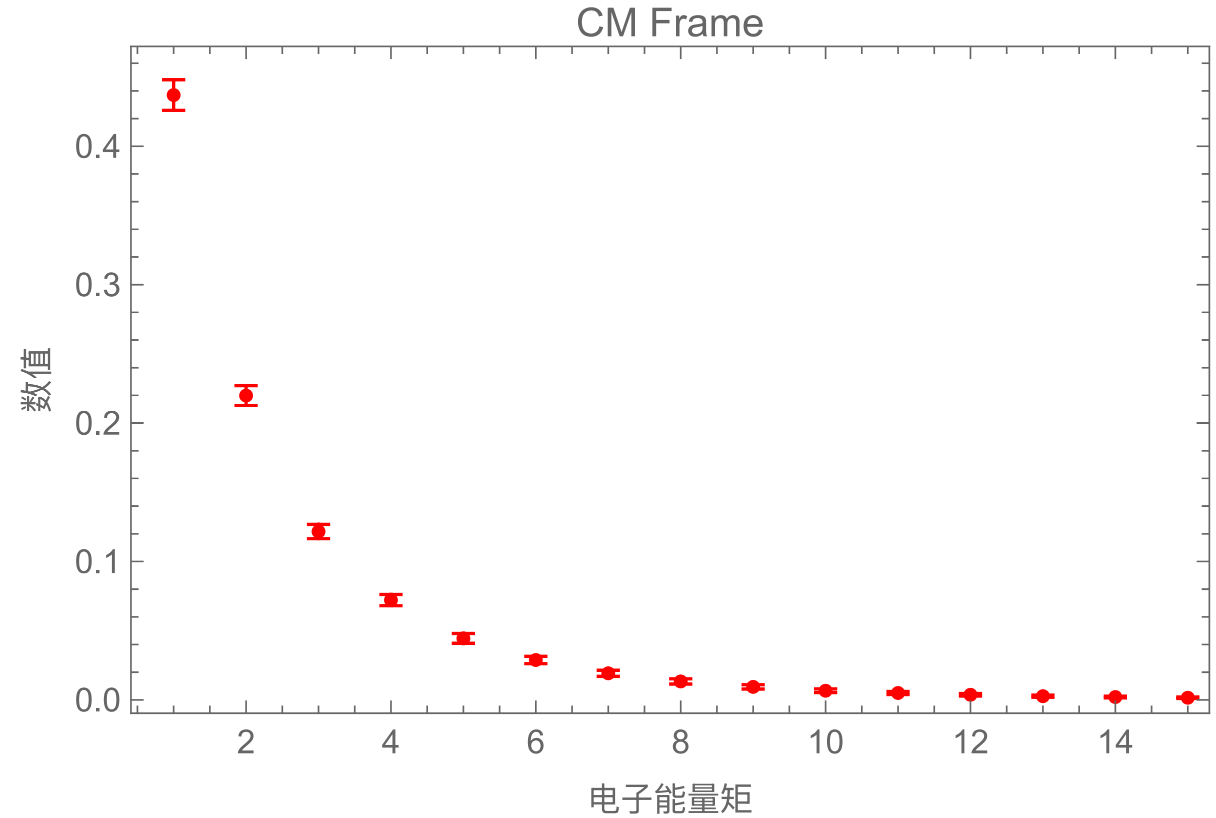


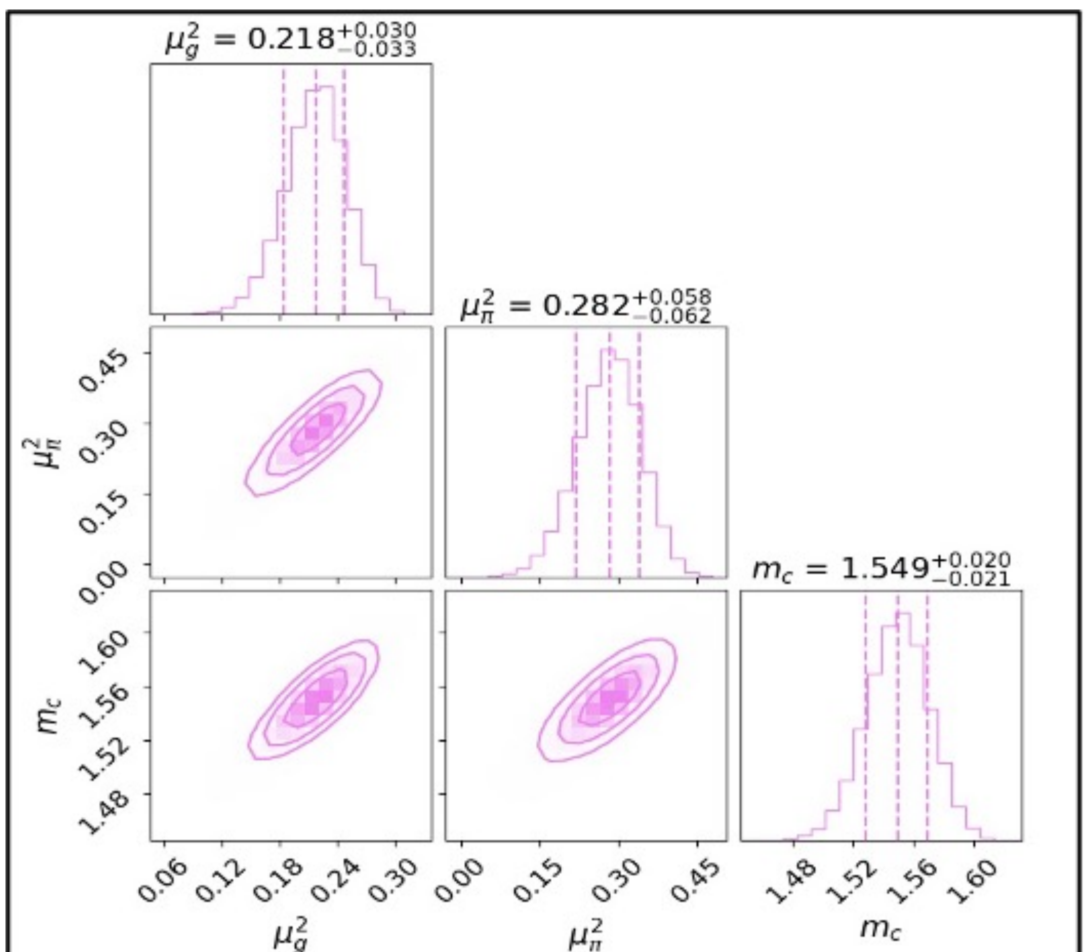
|//MatrixForm=

	ME1	ME2	ME3	ME4	ME5	ME6	ME7	ME8	ME9	MEx	MEx1	MEx2	MEx3	MEx4	MEx5
ME1	1.	0.936881	0.830061	0.725193	0.631639	0.551242	0.483639	0.427467	0.380973	0.342399	0.310188	0.283049	0.259952	0.240092	0.222844
ME2	0.936881	1.	0.967706	0.90011	0.821025	0.742009	0.668842	0.603852	0.547373	0.49875	0.456954	0.420911	0.389647	0.362335	0.338296
ME3	0.830061	0.967706	1.	0.978961	0.930231	0.86908	0.804952	0.743163	0.686227	0.634967	0.58931	0.548778	0.512763	0.480654	0.4519
ME4	0.725193	0.90011	0.978961	1.	0.984712	0.947972	0.900444	0.849159	0.798258	0.749891	0.704974	0.66374	0.626072	0.5917	0.560306
ME5	0.631639	0.821025	0.930231	0.984712	1.	0.988516	0.96047	0.923459	0.882565	0.840891	0.800177	0.761306	0.72466	0.690341	0.658301
ME6	0.551242	0.742009	0.86908	0.947972	0.988516	1.	0.991273	0.969692	0.940677	0.907884	0.873633	0.839321	0.805755	0.773375	0.742402
ME7	0.483639	0.668842	0.804952	0.900444	0.96047	0.991273	1.	0.993296	0.976489	0.953456	0.926851	0.898428	0.869318	0.840238	0.811635
ME8	0.427467	0.603852	0.743163	0.849159	0.923459	0.969692	0.993296	1.	0.994772	0.981464	0.962876	0.940968	0.917089	0.892162	0.866815
ME9	0.380973	0.547373	0.686227	0.798258	0.882565	0.940677	0.976489	0.994772	1.	0.995845	0.985106	0.969834	0.951507	0.931183	0.909623
MEx	0.342399	0.49875	0.634967	0.749891	0.840891	0.907884	0.953456	0.981464	0.995845	1.	0.996631	0.987795	0.975026	0.959462	0.941949
MEx1	0.310188	0.456954	0.58931	0.704974	0.800177	0.873633	0.926851	0.962876	0.985106	0.996631	1.	0.997214	0.989811	0.978963	0.965565
MEx2	0.283049	0.420911	0.548778	0.66374	0.761306	0.839321	0.898428	0.940968	0.969834	0.987795	0.997214	1.	0.997655	0.991352	0.982006
MEx3	0.259952	0.389647	0.512763	0.626072	0.72466	0.805755	0.869318	0.917089	0.951507	0.975026	0.989811	0.997655	1.	0.997995	0.992554
MEx4	0.240092	0.362335	0.480654	0.5917	0.690341	0.773375	0.840238	0.892162	0.931183	0.959462	0.978963	0.991352	0.997995	1.	0.998263
MEx5	0.222844	0.338296	0.4519	0.560306	0.658301	0.742402	0.811635	0.866815	0.909623	0.941949	0.965565	0.982006	0.992554	0.998263	1.

CM Frame

```
= Solve[ME1 == Around[0.451, 0.011], ME1CM] (*CM系下的电子能量一阶矩*)
  Solve[ME2 == Around[0.239, 0.007], ME2CM] (*CM系下的电子能量二阶矩*)
  Solve[ME3 == Around[0.142, 0.005], ME3CM] (*CM系下的电子能量三阶矩*)
  Solve[ME4 == Around[0.092, 0.004], ME4CM] (*CM系下的电子能量四阶矩*)
  Solve[ME5 == Around[0.063, 0.004], ME5CM] (*CM系下的电子能量五阶矩*)
  Solve[ME6 == Around[0.046, 0.003], ME6CM] (*CM系下的电子能量六阶矩*)
  Solve[ME7 == Around[0.035, 0.003], ME7CM] (*CM系下的电子能量七阶矩*)
  Solve[ME8 == Around[0.028, 0.003], ME8CM] (*CM系下的电子能量八阶矩*)
  Solve[ME9 == Around[0.023, 0.003], ME9CM] (*CM系下的电子能量九阶矩*)
  Solve[MEx == Around[0.019, 0.003], MExCM] (*CM系下的电子能量十阶矩*)
  Solve[MEx1 == Around[0.017, 0.003], MEx1CM] (*CM系下的电子能量十一阶矩*)
  Solve[MEx2 == Around[0.015, 0.003], MEx2CM] (*CM系下的电子能量十二阶矩*)
  Solve[MEx3 == Around[0.013, 0.003], MEx3CM] (*CM系下的电子能量十三阶矩*)
  Solve[MEx4 == Around[0.012, 0.003], MEx4CM] (*CM系下的电子能量十四阶矩*)
  Solve[MEx5 == Around[0.011, 0.003], MEx5CM] (*CM系下的电子能量s五阶矩*)
```





$$\mu_\pi^2(D) = (0.48 \pm 0.20)\mathrm{GeV}^2$$

$$\mu_G^2(D) = (0.34 \pm 0.10)\mathrm{GeV}^2$$

[Lenz et al, '22]

$$\mu_\pi^2(D) =$$

$$\mu_G^2(D) =$$

$$m_c =$$

...

What did we do?

Theory:

Experiment:

Phenomenological Analysis:

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What did we do?

Theory:

$D \to X_{d,s} + e^+$ — Optical Theorem / OPE → { Wilson Coef, Loacl Operators } — **Analytical Integral** → **Observables**

HQET

Experiment:

Phenomenological Analysis:

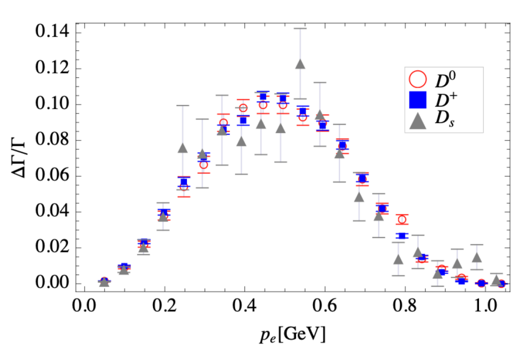
What did we do?

Theory:

$D \to X_{d,s} + e^+$ — Optical Theorem / OPE → { Wilson Coef, Loacl Operators } — **Analytical Integral** → **Observables**

HQET

Experiment:



→ **Observables**

Phenomenological Analysis:

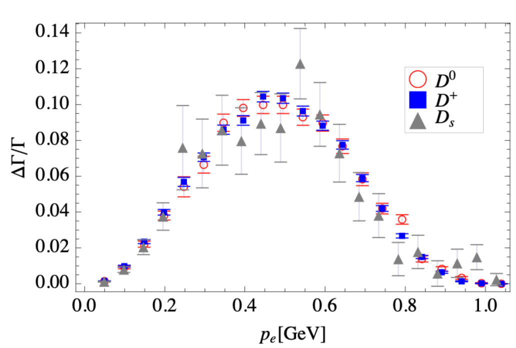
What did we do?

Theory:

$D \to X_{d,s} + e^+$ —Optical Theorem / OPE→ { Wilson Coef, Loacl Operators } —Analytical Integral→ Observables

HQET

Experiment:



→ Observables

Phenomenological Analysis:

Observables

Observables

—Bayesian analysis→

$\mu_\pi^2(D)$

$\mu_G^2(D)$

m_c

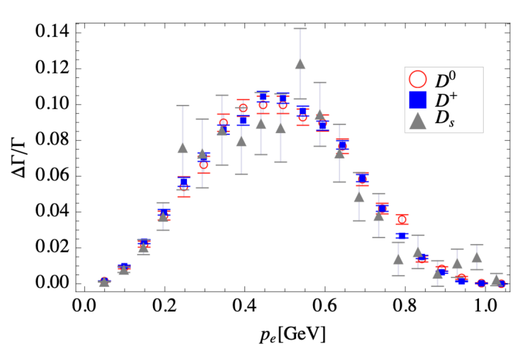
What did we do?

Theory:

$D \to X_{d,s} + e^{+}$ —— Optical Theorem / OPE ——> { Wilson Coef / Loacl Operators } —— **Analytical Integral** ——> **Observables**

HQET

Experiment:



——> **Observables**

Phenomenological Analysis:

Observables

Observables

—— **Bayesian analysis** ——>

$\mu_\pi^2(D)$

$\mu_G^2(D)$

m_c

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Wishlist

- Precision measurements of leptonic energy spectrum **in the rest frame** of charmed hadrons

- q^2 spectrum, good for **higher-dimensional operators**

- Separate X_d, X_s, to give **first measurements** of V_{cd}, V_{cs}

- Rare decays: $D \to X_u \ell \ell$

Thank you!

Wen-jie Song (宋雯捷), Dong-Xiao (肖栋) , Ying-ao Tang (唐迎澳), Ji-xin Yu(余纪新), Yong-Zheng(郑勇)，Bo-nan zhang(张博楠)，Yin-fa Shen (沈胤发) et, al

Appendix

Summary: observable

$\Gamma = \frac{\frac{G_F^2 m_c^5 |V_{CKM}|}{192\pi^3}}{\text{Cons}}$

$\langle E_e \rangle = \frac{G_F^2 m_c^5 |V_{CKM}|}{192\pi^3 \Gamma}$

$\langle E_e^2 \rangle = \frac{G_F^2 m_c^5 |V_{CKM}}{192\pi^3 \Gamma}$

M1=M1
M2=M1+M2
M3=M1+M2+M3
M4=M1+M2+M3+M4

Cons= $2.28252 \times 10^{-14}\ mc^5$

Prior Distribution for Free Parameters

$m_c \in$ [1, 2] GeV

$\mu_\pi^2 \in$ [-0.28, 1.08] GeV^2

$\mu_G^2 \in$ [0.04, 0.64] GeV^2

A: Daniel King, Alexander Lenz at al. "Revisiting inclusive decay widths of charmed mesons", JHEP08(2022)241.

Fixed $\alpha_s(\bar{m}_c = 1.273 GeV)$ = 0.378387

Prof Qin's code：running from $\alpha_s(m_z = 91.1880 GeV) = 0.1179$ at four loop level.

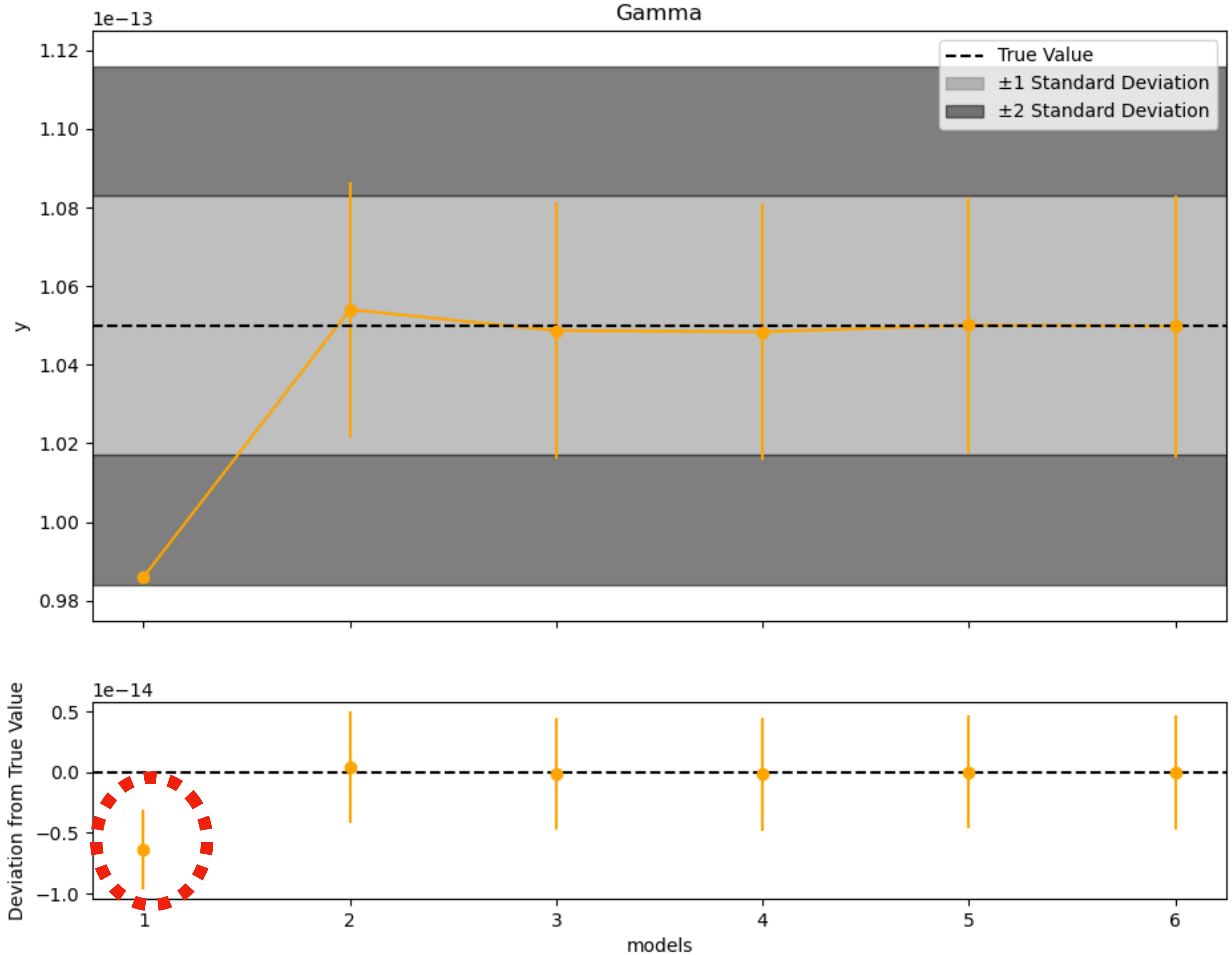
Note!

M1=M1	Tree Level dim-3 operator
M2=M1+M2	Tree Level : dim-3 operator+ μ_π^2
M3=M1+M2+M3	Tree Level : dim-3 operator+ μ_π^2+ μ_G^2
M4=M1+M2+M3+M4	Tree Level : dim-3 operator+ μ_π^2 + μ_G^2 +dim-3（NLO)

Phenomenological Analysis (preliminary)

CompactObject

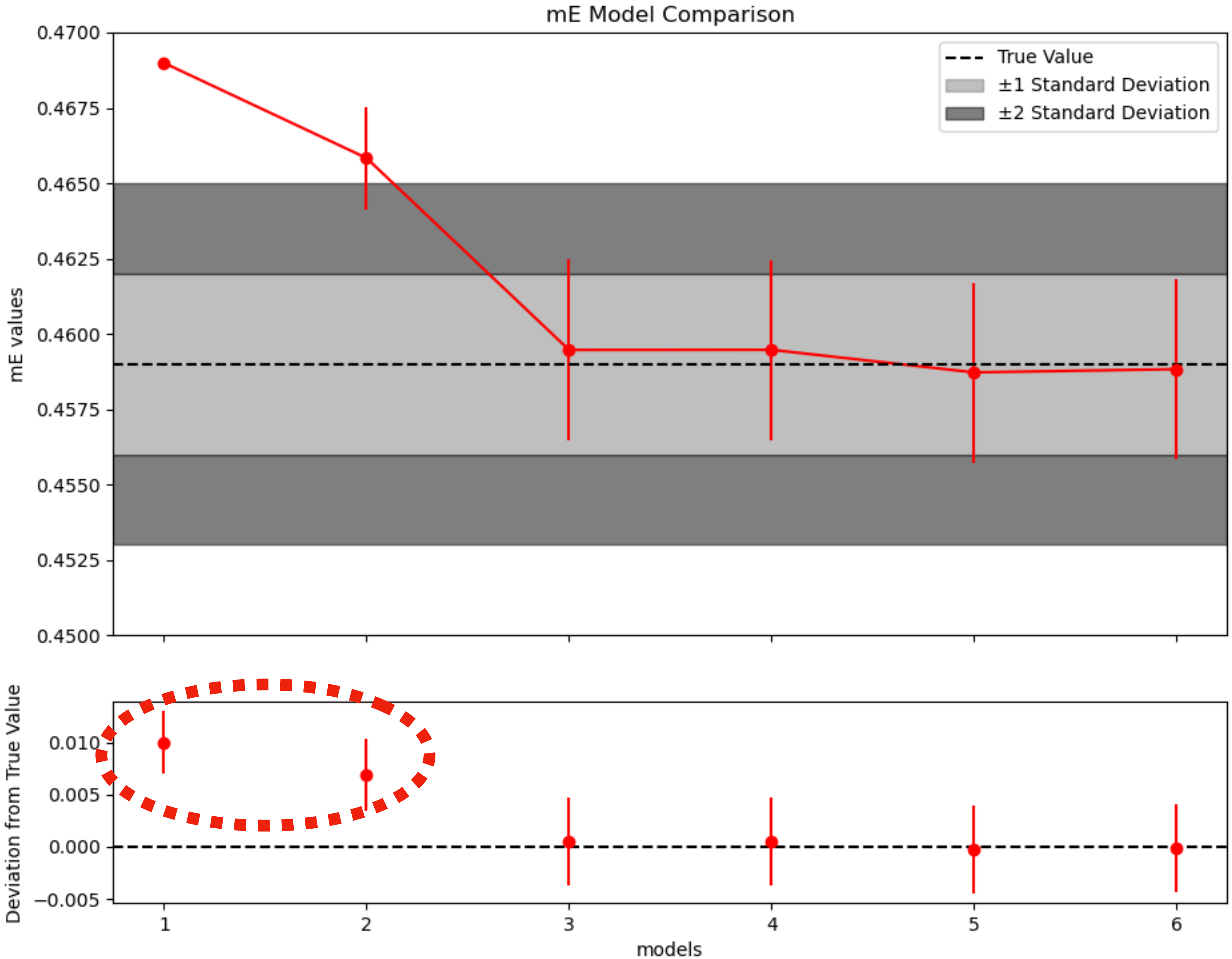
[Huang et al, ’2024]



Phenomenological Analysis (preliminary)

CompactObject

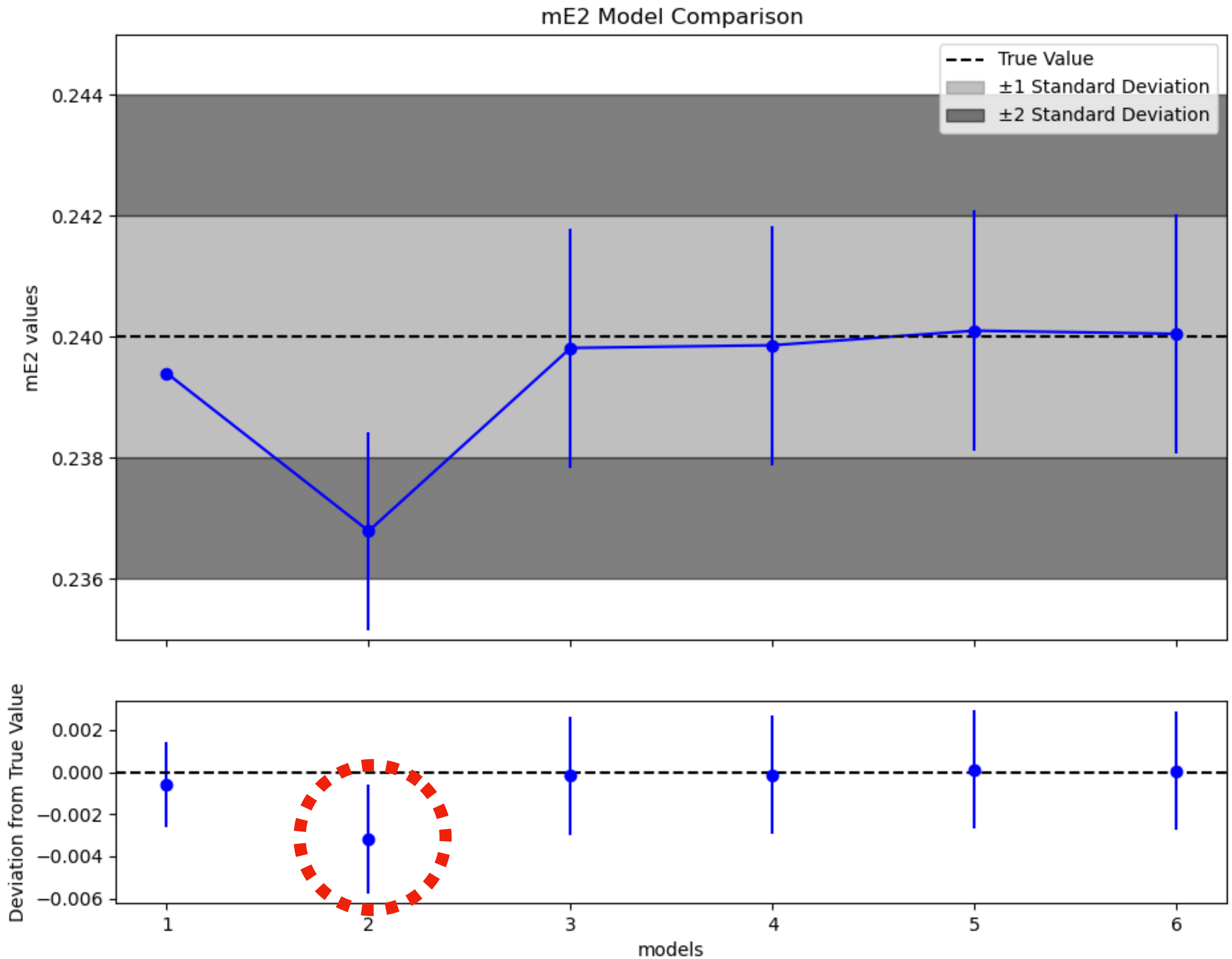
[Huang et al, '2024]



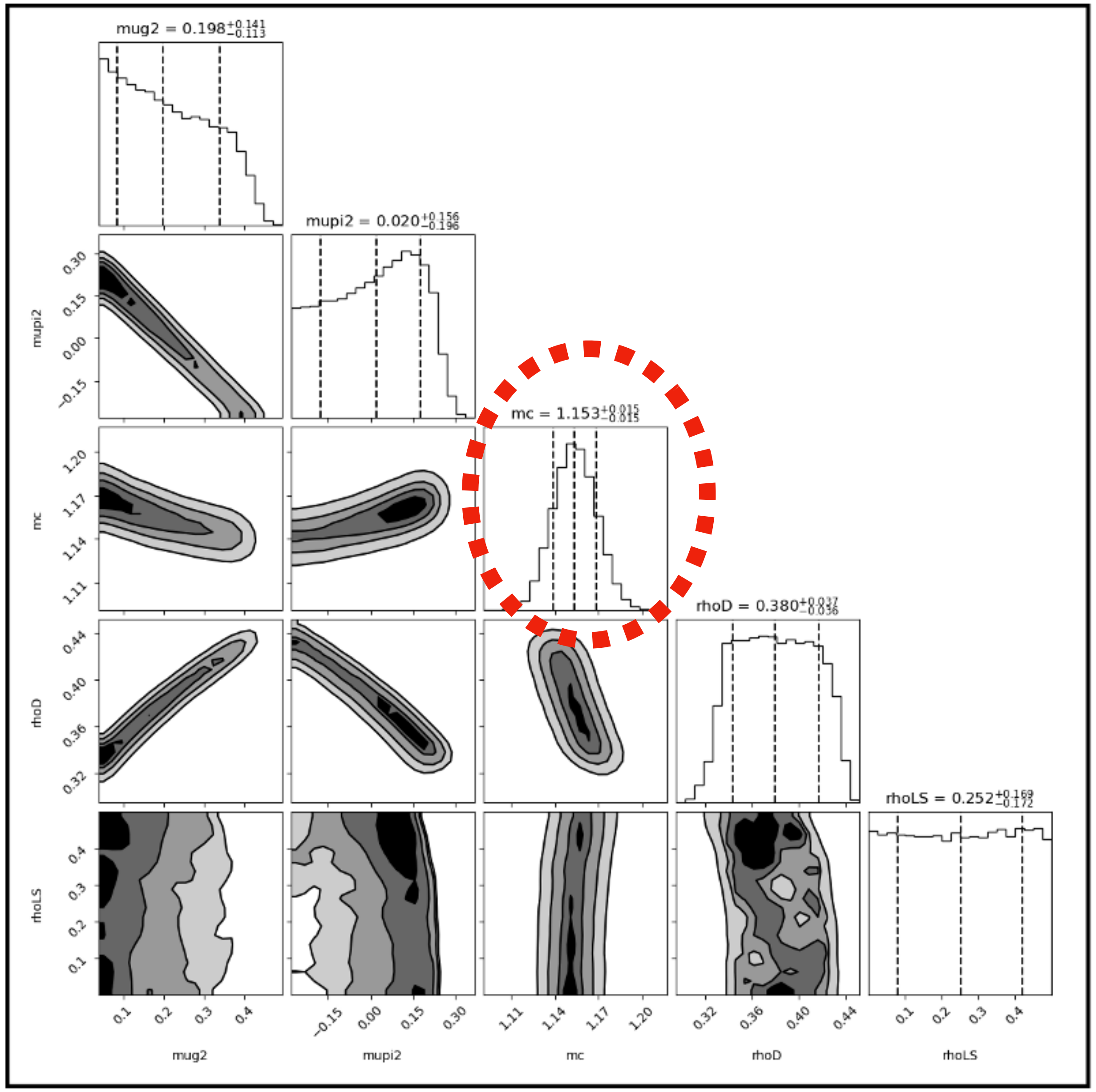
Phenomenological Analysis (preliminary)

CompactObject

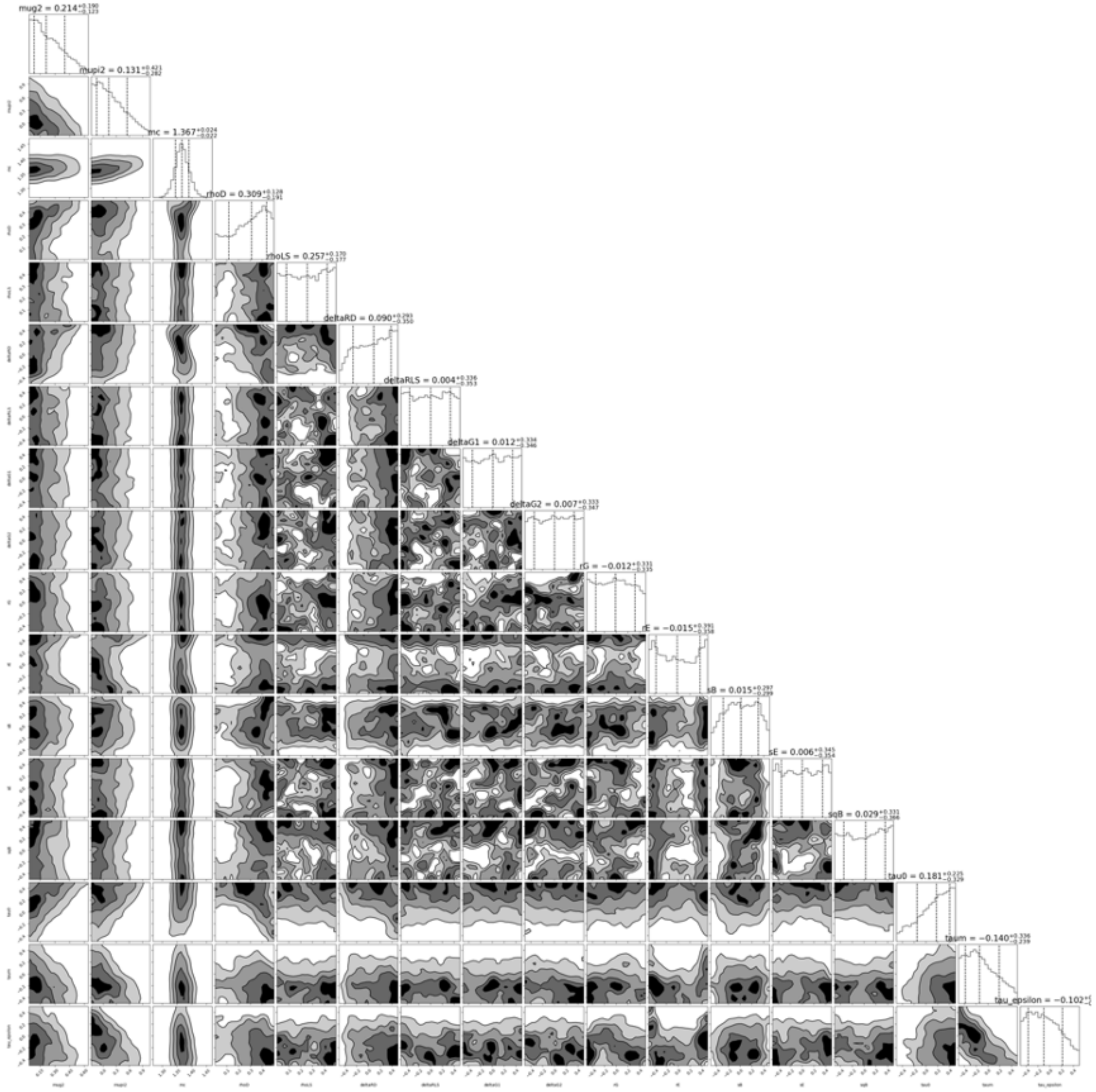
[Huang et al, ’2024]



Appendix



[M5]



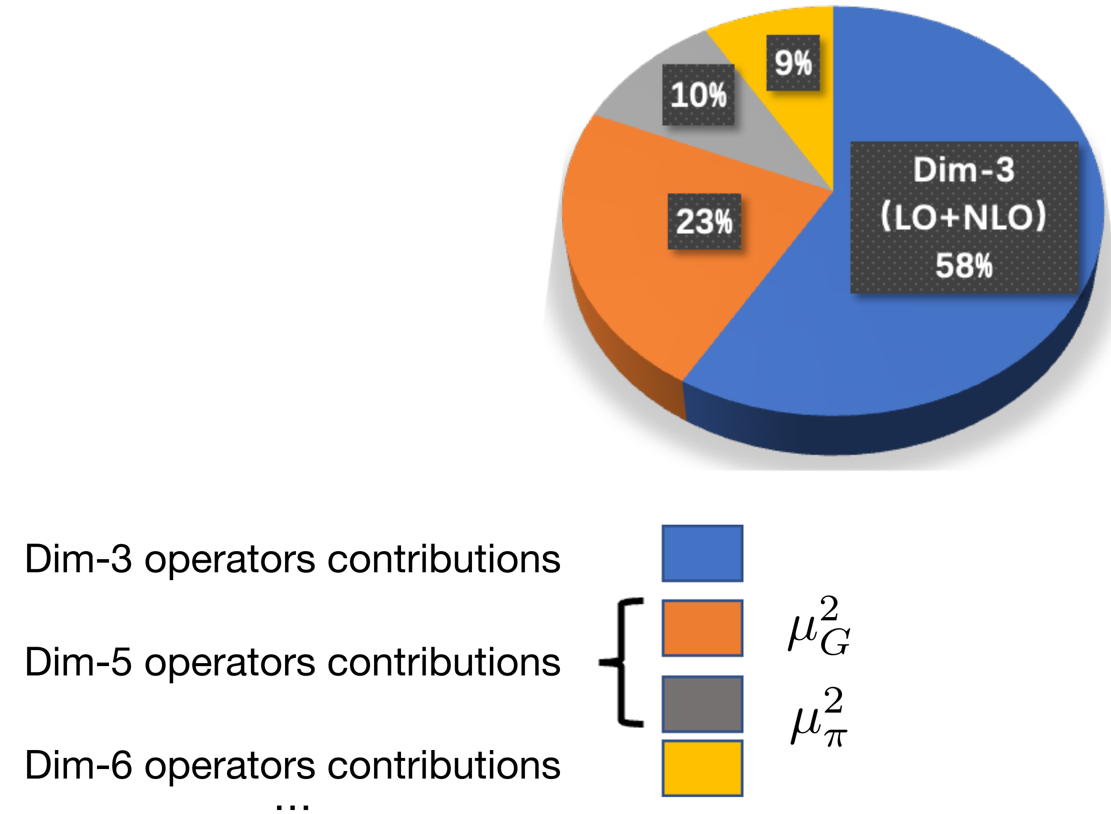
[M6]

Appendix

In the framework of the HQET,

SL:

$$\Gamma_{sl}^{D^+} = \Gamma_0 \Big[\underbrace{1.02}_{c_3^{\rm LO}} + \underbrace{0.16}_{\Delta c_3^{\rm NLO}} - 0.27\,\frac{\mu_\pi^2(D)}{\rm GeV^2} - 0.84\,\frac{\mu_G^2(D)}{\rm GeV^2} + 2.48\,\frac{\rho_D^3(D)}{\rm GeV^3} + \underbrace{0.00}_{\rm dim-7,VIA}$$
$$- 0.28\,\tilde{B}_1^q + 0.28\,\tilde{B}_2^q - 0.09\,\tilde{\epsilon}_1^q + 0.09\,\tilde{\epsilon}_2^q - 5.24\,\tilde{\delta}_1^{sq} + 5.24\,\tilde{\delta}_2^{sq} \Big]$$
$$= 1.02\,\Gamma_0 \Big[1 + 0.16 - 0.13\,\frac{\mu_\pi^2(D)}{0.48\,\rm GeV^2} - 0.28\,\frac{\mu_G^2(D)}{0.34\,\rm GeV^2} + 0.20\,\frac{\rho_D^3(D)}{0.082\,\rm GeV^3}$$
$$- \underbrace{0.00}_{\rm dim-6,7,VIA} - 0.005\,\frac{\delta\tilde{B}_1^q}{0.02} + 0.005\,\frac{\delta\tilde{B}_2^q}{0.02} + 0.004\,\frac{\tilde{\epsilon}_1^q}{-0.04} - 0.004\,\frac{\tilde{\epsilon}_2^q}{-0.04}$$
$$- 0.0118\,r_1^{sq} - 0.0088\,r_2^{sq} \Big], \qquad (4.13)$$



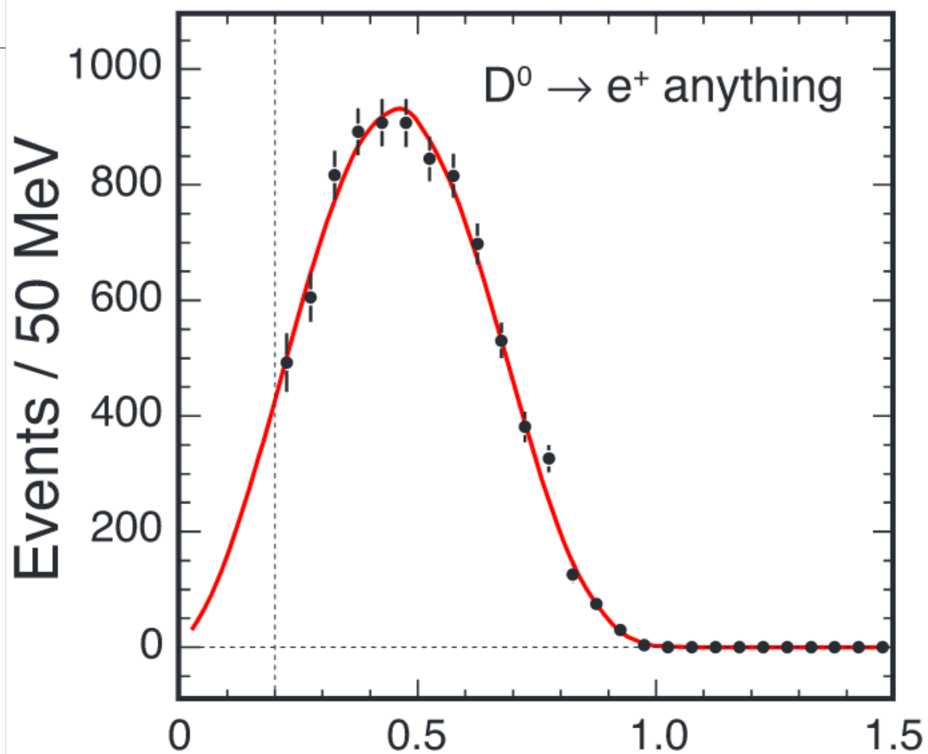
SL+NL:

$$\Gamma(D^+) = \Gamma_0 \Big[\underbrace{6.15}_{c_3^{\rm LO}} + \underbrace{2.95}_{\Delta c_3^{\rm NLO}} - 1.66\,\frac{\mu_\pi^2(D)}{\rm GeV^2} + 0.13\,\frac{\mu_G^2(D)}{\rm GeV^2} + 23.6\,\frac{\rho_D^3(D)}{\rm GeV^3}$$
$$-16.9\,\tilde{B}_1^q + 0.56\,\tilde{B}_2^q + 84.0\,\tilde{\epsilon}_1^q - 1.34\,\tilde{\epsilon}_2^q + \underbrace{6.76}_{\rm dim-7}$$
$$-0.06\,\tilde{\delta}_1^{qq} + 0.06\,\tilde{\delta}_2^{qq} - 16.8\,\tilde{\delta}_3^{qq} + 16.9\,\tilde{\delta}_4^{qq} - 29.3\,\tilde{\delta}_1^{sq} + 28.8\,\tilde{\delta}_2^{sq} + 0.56\,\tilde{\delta}_3^{sq} + 2.36\,\tilde{\delta}_4^{sq} \Big]$$
$$= 6.15\,\Gamma_0 \Big[1 + 0.48 - 0.13\,\frac{\mu_\pi^2(D)}{0.48\,\rm GeV^2} + 0.01\,\frac{\mu_G^2(D)}{0.34\,\rm GeV^2} + 0.31\,\frac{\rho_D^3(D)}{0.082\,\rm GeV^3}$$
$$- \underbrace{2.66}_{\rm dim-6,VIA} - 0.055\,\frac{\delta\tilde{B}_1^q}{0.02} + 0.002\,\frac{\delta\tilde{B}_2^q}{0.02} - 0.546\,\frac{\tilde{\epsilon}_1^q}{-0.04} + 0.009\,\frac{\tilde{\epsilon}_2^q}{-0.04} + \underbrace{1.10}_{\rm dim-7,VIA}$$
$$- 0.0000\,r_1^{qq} - 0.0000\,r_2^{qq} + 0.0011\,r_3^{qq} + 0.0008\,r_4^{qq}$$
$$- 0.0109\,r_1^{sq} - 0.0080\,r_2^{sq} - 0.0000\,r_3^{sq} + 0.0001\,r_4^{sq} \Big], \qquad (4.6)$$

JHEP08(2022)241

Discussion about Energy Spectrum

p(GeV)	$\Delta B(D^0 \to Xe^+\nu_e)(\%)$
0.200 – 0.250	0.347 ± 0.036
0.250 – 0.300	0.426 ± 0.030
0.300 – 0.350	0.576 ± 0.031
0.350 – 0.400	0.629 ± 0.030
0.400 – 0.450	0.640 ± 0.031
0.450 – 0.500	0.640 ± 0.031
0.500 – 0.550	0.596 ± 0.029
0.550 – 0.600	0.575 ± 0.029
0.600 – 0.650	0.492 ± 0.026
0.650 – 0.700	0.374 ± 0.023
0.700 – 0.750	0.269 ± 0.019
0.750 – 0.800	0.230 ± 0.017
0.800 – 0.850	0.089 ± 0.011
0.850 – 0.900	0.053 ± 0.008
0.900 – 0.950	0.021 ± 0.005
0.950 – 1.000	0.002 ± 0.002
1.000 – 1.050	...



$$\mathscr{B}\left(D^0 \to Xe^+\nu_e\right) = (6.46 \pm 0.09 \pm 0.11)\,\%$$

Channel	$\mathcal{B}(\%)$	Form factor	Comment
$D^0 \to K^{*-}e^+\nu_e$	$2.16(17)$[1]	SPOLE	$r_V = 1.62(8)$ and $r_2 = 0.83(5)$[17]
$D^0 \to K^-e^+\nu_e$	$3.50(5)$[5]	BK	$\alpha_{\rm BK} = 0.30(3)$[5]
$D^0 \to K_1^-e^+\nu_e$	$0.11(11)$	ISGW2	$\mathcal{B}$ from Ref. [10] scaled by Ref. [5]
$D^0 \to K_2^{*-}e^+\nu_e$	$0.11(11)$	ISGW2	$\mathcal{B}$ set to same as $D^0 \to K_1^-e^+\nu_e$
$D^0 \to \bar{K}\pi e^+\nu_e$	$0.12(3)$[17, 29]	PHSP	Nonresonant
$D^0 \to \pi^-e^+\nu_e$	$0.288(9)$[5]	BK	$\alpha_{\rm BK} = 0.21(7)$[5]
$D^0 \to \rho^-e^+\nu_e$	$0.16(2)$[2]	SPOLE	$r_V = 1.4(3)$ and $r_2 = 0.6(2)$[2]

arXiv:1004.0114v2 [hep-ph]

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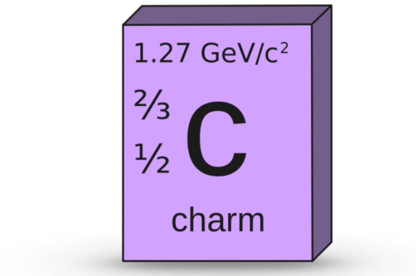
Supervisor：Fu-sheng Yu, Qin Qin



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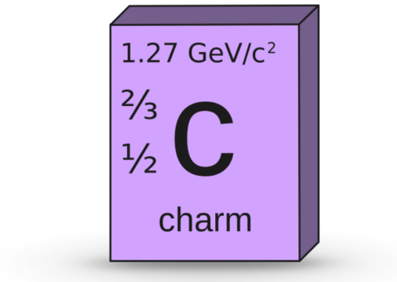

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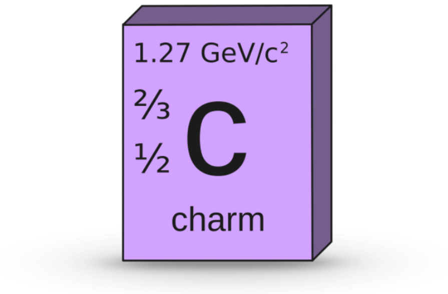

HFCPV2024

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