第二届武汉高能物理青年论坛

The review of QCD axion

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Outline

- ⭐ The $\boldsymbol{\theta}$-vacua and strong CP problem
- ⭐ The Peccei Quinn mechanism
- ⭐ PQWW, DFSZ and KSVZ model
- ⭐ Axion effective Lagrangian and mass
- ⭐ Axion and other open issues of the SM

The $\boldsymbol{\theta}$-vacua and strong CP problem

★ QCD vacuum structure

The vacuum configurations of gluon is gauge equivalent to G_μ=0. In temporal gauge, this means

$$G_j = \mathrm{i}g^{-1}U\partial_j U^{-1}$$

$U = U(\mathbf{x}) \in SU(3)_C$

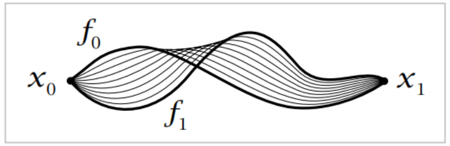
$U(\mathbf{x})$ is a map from $\mathbb{R}^3$ to $SU(3)_C$. Such maps, and hence the vacuum configurations, can be classified by elements of $\pi_3(SU(3)_C) \cong \mathbb{Z}$.

These integers are called winding number

By a **path** in a space X we mean a continuous map $f: I \to X$ where I is the unit interval $[0,1]$. The idea of continuously deforming a path, keeping its endpoints fixed, is made precise by the following definition. A **homotopy** of paths in X is a family $f_t: I \to X$, $0 \le t \le 1$, such that

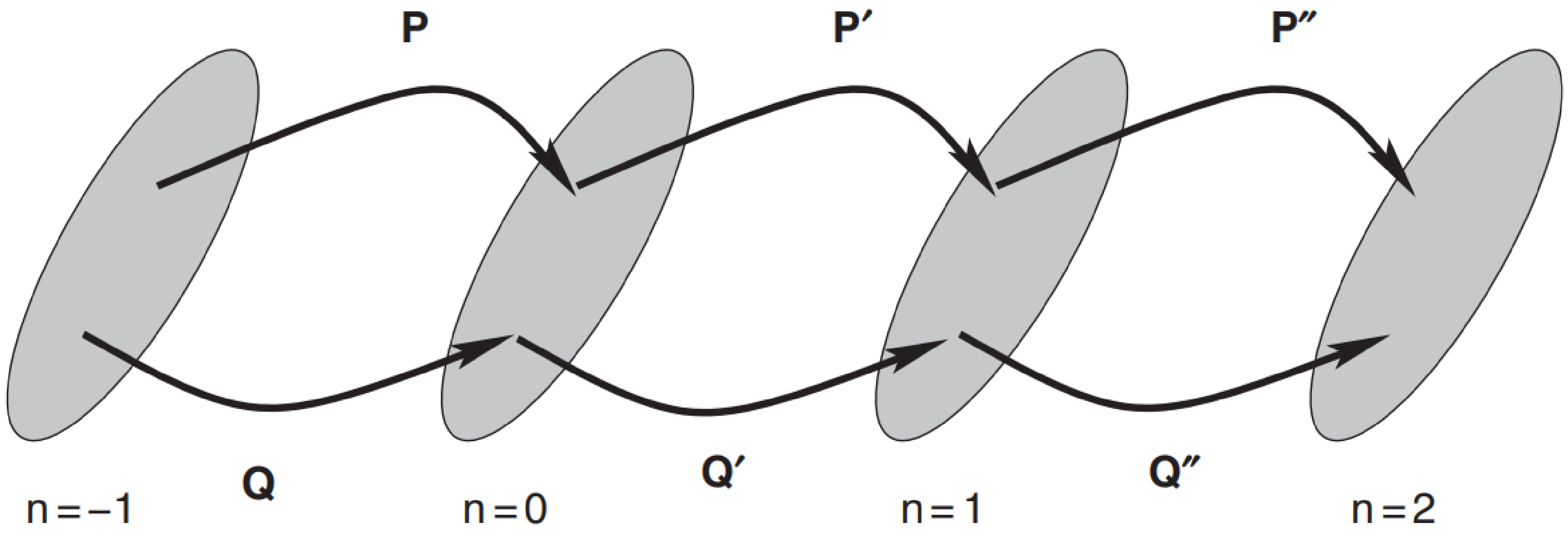
(1) The endpoints $f_t(0) = x_0$ and $f_t(1) = x_1$ are independent of t.

(2) The associated map $F: I \times I \to X$ defined by $F(s,t) = f_t(s)$ is continuous.



The $\boldsymbol{\theta}$-vacua and strong CP problem

★ **QCD vacuum structure**



Weinberg EJ. *Classical Solutions in Quantum Field Theory: Solitons and Instantons in High Energy Physics*. Cambridge University Press; 2012.

The $\boldsymbol{\theta}$-vacua and strong CP problem

★ **QCD vacuum structure**

The winding number can be calculated by a homomorphism

$$N[U] = \frac{1}{24\pi^2}\epsilon^{ijk}\int d^3x\, \mathrm{tr}\, \underline{U^{-1}\partial_i U U^{-1}\partial_j U U^{-1}\partial_k U}$$

$\rightarrow G_i G_j G_k$

This homomorphism is related to chiral anomaly

$$\partial_\mu J^\mu_A = \frac{g^2}{16\pi^2}\mathrm{Tr}\; G_{\mu\nu}\tilde{G}^{\mu\nu}$$

$$\mathrm{Tr}\; G_{\mu\nu}\tilde{G}^{\mu\nu} = \partial_\mu \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left(G_\nu G_{\rho\sigma} + \frac{2ig}{3}G_\nu G_\rho G_\sigma\right)$$

The $\boldsymbol{\theta}$-vacua and strong CP problem

★ QCD vacuum structure

The charge associated with the axial current is

$G_{jk} = 0$

$$Q_A = \int d^3x J_A^0 = \frac{g^2}{16\pi^2} \int d^3x \epsilon^{ijk} \, \mathrm{tr} \left(G_i G_{jk} + \frac{2ig}{3} G_i G_j G_k \right)$$

Now let $G_\mu(t, \mathbf{x})$ be a path parameterized by t, interpolate between two vacuum configurations with winding numbers N_1 and N_2

ΔN is gauge invariant while N is not

$$\frac{g^2}{16\pi^2} \int d^4x \, \mathrm{Tr} \, G_{\mu\nu} \tilde{G}^{\mu\nu} = N_2 - N_1$$

The $\boldsymbol{\theta}$-vacua and strong CP problem

☆ **$\boldsymbol{\theta}$-vacua**

There exists one quantum vacuum state for each winding number, called n-vacua. Large gauge transformation takes one n-vacuum to the next

$$T|n\rangle = |n+1\rangle$$

The true vacuum is also a eigenvector of T

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{-in\theta}|n\rangle \quad \rightarrow \quad T|\theta\rangle = \mathrm{e}^{\mathrm{i}\theta}|\theta\rangle$$

The $\boldsymbol{\theta}$-vacua and strong CP problem

☆ $\boldsymbol{\theta}$-vacua

The θ-vacua to θ'-vacua amplitude is

$$\langle\theta'|\theta\rangle = 2\pi\delta(\theta-\theta')\sum_k \mathrm{e}^{\mathrm{i}k\theta}\langle k|0\rangle$$

$$\langle k|0\rangle = \int [\mathrm{d}G]\mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x\mathcal{L}}$$

The factor of $e^{ik\theta}$ is equivalent to adding to the Lagrangian density a term of

$$\mathrm{e}^{\mathrm{i}k\theta} = \exp\left(\frac{\mathrm{i}\theta g^2}{16\pi^2}\int \mathrm{d}^4x\,\mathrm{Tr}\,G_{\mu\nu}\tilde{G}^{\mu\nu}\right)$$

θ term

The θ-vacua and strong CP problem

☆ Strong CP problem

The θ term is CP odd

$$CP: \ \mathrm{Tr}\, G_{\mu\nu}\tilde{G}_{\mu\nu} \rightarrow -\mathrm{Tr}\, G_{\mu\nu}\tilde{G}_{\mu\nu}$$

The θ term can be transformed to the phase of the mass term for the quarks by $U(1)_A$

$$U(1)_A: \bar{q}_L M q_R + h.c. \rightarrow \bar{q}_L M \mathrm{e}^{-2\mathrm{i}\alpha} q_R + h.c.$$

$$\frac{\theta g^2}{32\pi^2} G^{a\mu\nu}\tilde{G}^a_{\mu\nu} \rightarrow (\theta + 2N\alpha)\frac{g^2}{32\pi^2} G^{a\mu\nu}\tilde{G}^a_{\mu\nu}$$

The $\boldsymbol{\theta}$-vacua and strong CP problem

☆ Strong CP problem

Now the CP odd term is the imaginary part of mass term for quark

$$\mathcal{L}_{\not{CP}} = -\mathrm{i}\bar{\theta}\frac{m_u m_d}{m_u + m_d}(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d)$$

$\bar{\theta} = \theta + \arg\det M$

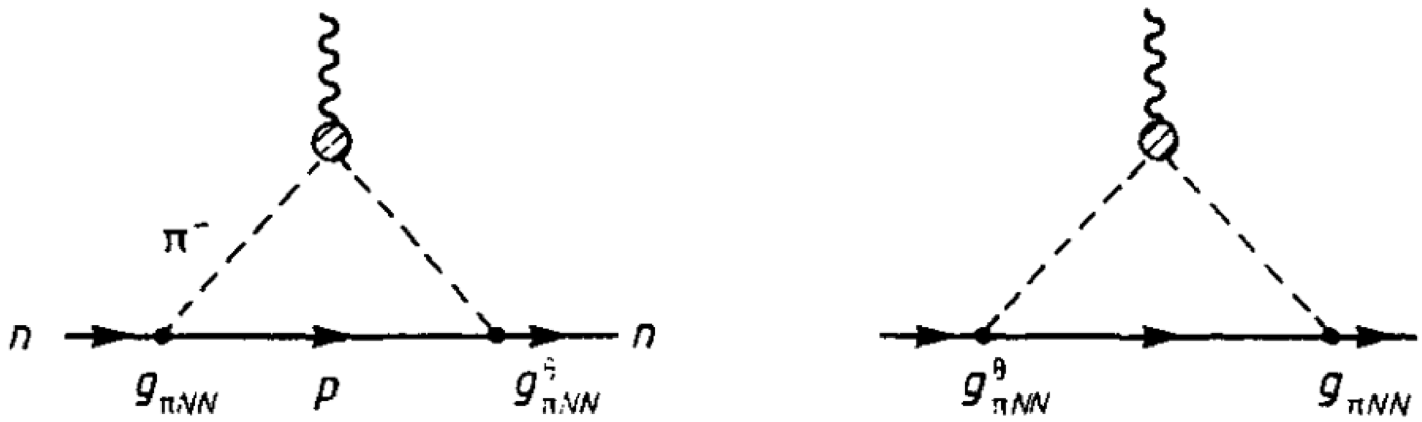
This term will generate a CP-violating pion-nucleon interaction

$$\mathcal{L}_{\pi NN} = g^{\theta}_{\pi NN}\bar{N}\tau^a N\pi^a$$

The $\boldsymbol{\theta}$-vacua and strong CP problem

★ **Strong CP problem**

It contributes to neutron's electric dipole moment



David Bailin and Alexander Love. *Introduction to Gauge Field Theory*. Taylor and Francis Groupr, 1993.

The $\boldsymbol{\theta}$-vacua and strong CP problem

⭐ Strong CP problem

The experimental data gives[1,2]

$$|\bar{\theta}| < 10^{-10}$$

The strong CP problem: why $\bar{\theta}$ is so small?

1. C. A. Baker *et al.*, *Phys. Rev. Lett.* **97** (2006) 131801
2. J. Engel et al., *Prog. Part. Nucl. Phys.* **71** (2013) 21–74,

The Peccei Quinn mechanism

★ Peccei and Quinn's proof

Peccei and Quinn proved that $\bar{\theta}$ is vanished when the Lagrangian possesses a chiral U(1) symmetry.[1,2]

In single-flavor toy model

$$\psi \to \exp(-\mathrm{i}\alpha\gamma_5)\psi$$
$$\varphi \to \exp(2\mathrm{i}\alpha)\varphi$$

$$\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a} + i\bar{\psi}D_\mu\gamma^\mu\psi + G\bar{\psi}_L\psi_R\varphi + G^*\bar{\psi}_R\psi_L\varphi^*$$
$$- |\partial_\mu\varphi|^2 - \mu^2|\varphi|^2 - h|\varphi|^4; \quad \mu^2 < 0.$$

1. Roberto D Peccei and Helen R Quinn, *Phys. Rev. Lett,* 38(25):1440, 1977.
2. Roberto D Peccei and Helen R Quinn, *Phys. Rev. D*, 16(6):1791, 1977.

The Peccei Quinn mechanism

☆ Peccei and Quinn's proof

The CP violating phase is

$$\bar{\theta} = \theta + \arg G + \arg\langle\varphi\rangle$$

The LO of effective potential can be calculated

$$A = \arg\varphi - \arg\langle\varphi\rangle$$

$$V_{\text{eff}} = U(|\varphi|) - K|G\varphi|\cos\left(\bar{\theta} + A\right)$$

It implies that $\bar{\theta} = 0$

The phase of scalar $a = A/|\langle\varphi\rangle|$ is called axion

The Peccei Quinn mechanism

☆ **Vafa-Witten theorem**

Parity cannot be spontaneously broken in QCD[1]

$$E(0) \leq E(\lambda) = 1 + \lambda\langle G\tilde{G}\rangle + O(\lambda^2)$$

$$E(\bar{\theta} + A) \geq E(\bar{\theta}) \Rightarrow \bar{\theta} = 0$$

1. C. Vafa, E. Witten, Phys. Rev. Lett. 53 (1984) 535.

The Peccei Quinn mechanism

★ **PQ quality problem**

PQ symmetry may be violated by effective operator by gravitational corrections. For example

$$g_5 \frac{|\Phi|^4 \left(\Phi + \Phi^*\right)}{M_{\mathrm{Pl}}}$$

These violations result in a non-vanished $\bar{\theta}$

PQWW, DFSZ and KSVZ model

☆ PQWW model

The first realistic axion model is PQWW model[1]

$$-\mathcal{L}_Y = y_{ij}^{(u)} \bar{Q}_i H_u u_{jR} + y_{ij}^{(d)} \bar{Q}_i H_d d_{jR} + h.c.$$

H_u is independent with H_d for the independence of $U(1)_{PQ}$ and $U(1)_Y$.

The axion the combination of the phases of H's

$$a = \frac{v_d}{v} a_u + \frac{v_u}{v} a_d$$

$$v = \sqrt{v_u^2 + v_d^2}$$

$$f_a = \frac{v_u v_d}{3v}$$

1. Weinberg PhysRevLett.40.223, Wilczek PhysRevLett.40.279, 1978

PQWW, DFSZ and KSVZ model

★ DFSZ model

DFSZ model requires the additional SM-singlet scalar to PQWW model[1]

$$V(H_u, H_d, \phi) = \tilde{V}_{moduli}(|H_u|, |H_d|, |\phi|, |\phi_u^\dagger \tilde{\phi}_d|) + \lambda H_u^\dagger \tilde{H}_d \phi^2 + h.c$$

The axion is the combination of three phases

$$v_a^2 \equiv \mathcal{X}_u^2 v_u^2 + \mathcal{X}_d^2 v_d^2 + \mathcal{X}_\phi^2 v_\phi^2, \quad a = \frac{1}{v_a}(\mathcal{X}_u v_u a_u + \mathcal{X}_d v_d a_d + \mathcal{X}_\phi v_\phi a_\phi)$$

In the limit $v_\phi \gg v_u, v_d$,

$$f_a \simeq v_\phi / 6, \quad a \simeq a_\phi$$

1. M. Dine et al., Physics Letters B 104, 199 (1981)

PQWW, DFSZ and KSVZ model

★ KSVZ model

The KSVZ model extends the SM field content with a heavy quark and a SM-singlet[1]

$$-\mathcal{L}_Y = y_Q \bar{Q}_L Q_R \phi + h.c.$$

The axion is just the phase of ϕ with $f_a = 2v$

1. J. E. Kim, Phys. Rev. Lett. **43**, 103 (1979)

Axion effective Lagrangian and mass

☆ Axion effective Lagrangian

All of the axion model contain the quark-scalar Yukawa term

$$-\mathcal{L}_Y \supset y_q \bar{q}_L q_R \phi + h.c.$$

With a local quark $U(1)_A$ rotation

$$q \to \mathrm{e}^{-\mathrm{i}\gamma_5 a/f_a} q$$

Now the axion is disentangled from the Yukawa term with some additional term

Axion effective Lagrangian and mass

★ Axion effective Lagrangian

$$\Delta\mathcal{L} = \frac{g_s^2}{32\pi}\frac{a}{f_a}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} + \frac{e^2}{32\pi}\frac{E}{N}\frac{a}{f_a}F^a_{\mu\nu}\tilde{F}^{a\mu\nu} + c_q^0\frac{\partial_\mu a}{2f_a}\bar{q}\gamma^\mu\gamma_5 q$$

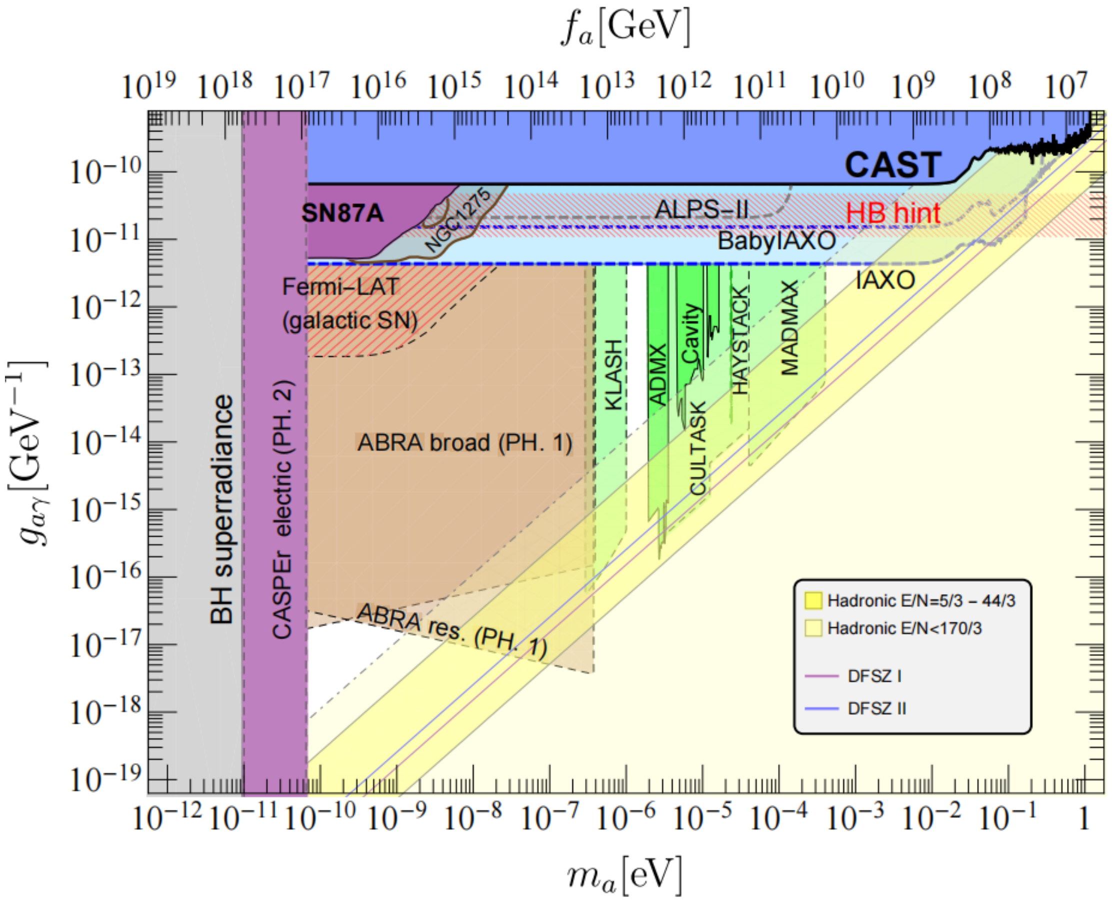
The axion effective Lagrangian (in quark level) reads

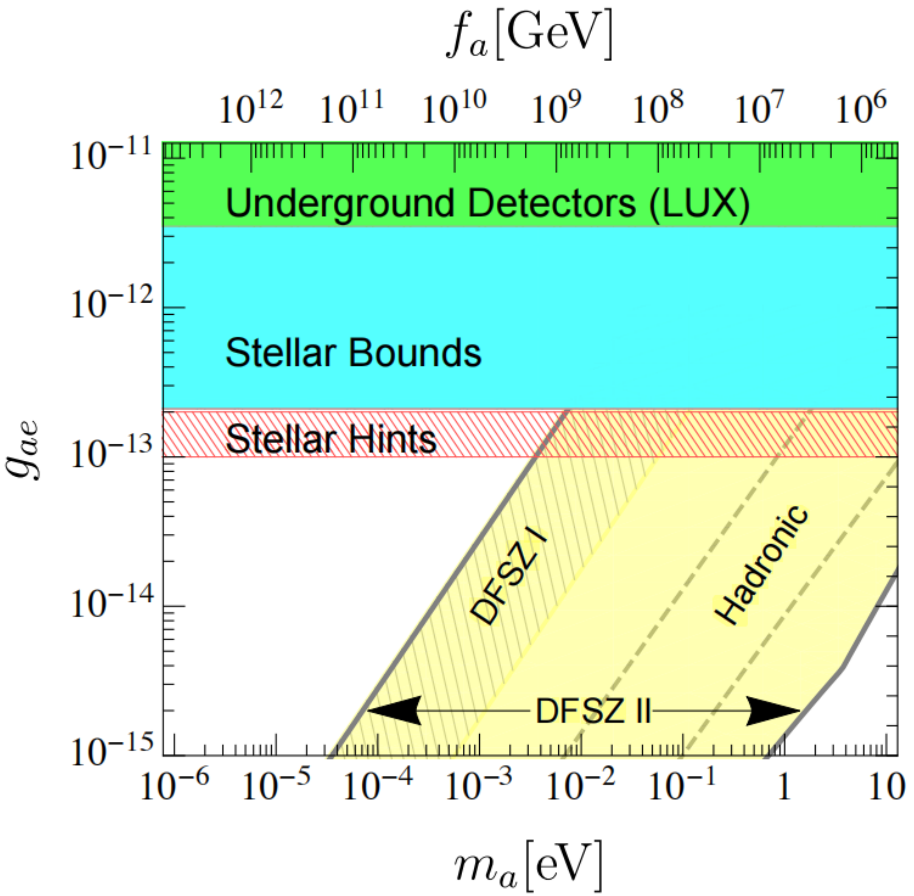
$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{g_s^2}{32\pi^2}G\tilde{G} + \frac{1}{4}g^0_{a\gamma}aF\tilde{F} + \frac{\partial_\mu a}{2f_a}\bar{q}c_q^0\gamma^\mu\gamma_5 q - \bar{q}_L M_q q_R + \text{ h.c.}$$

Axion effective Lagrangian and mass

★ **Axion effective Lagrangian**

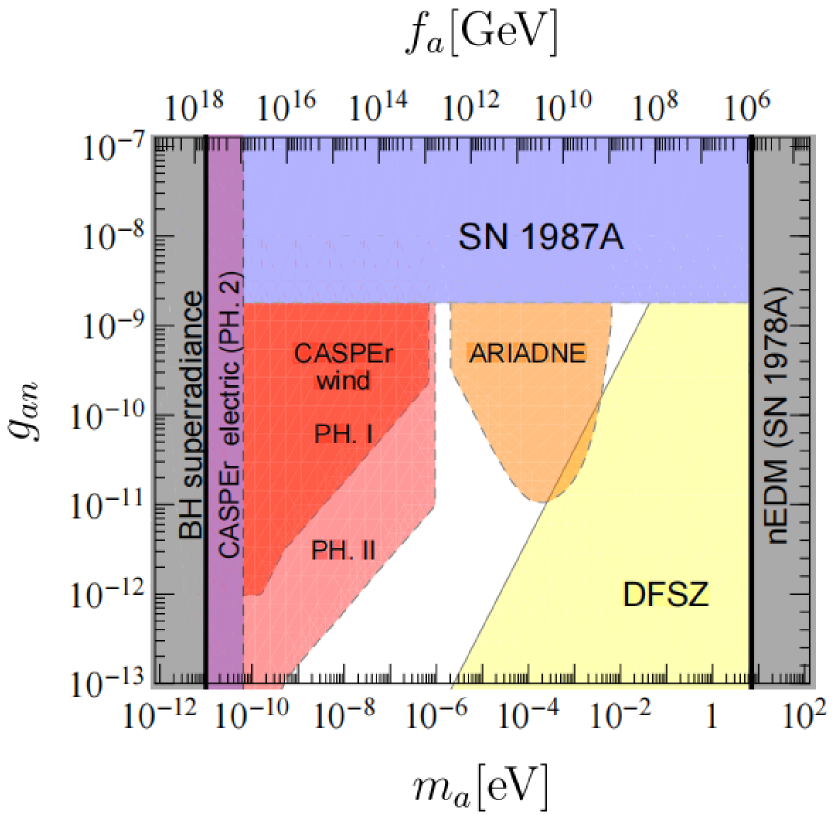
The axion can be probed by $g_{a\gamma}$ and g_{af}

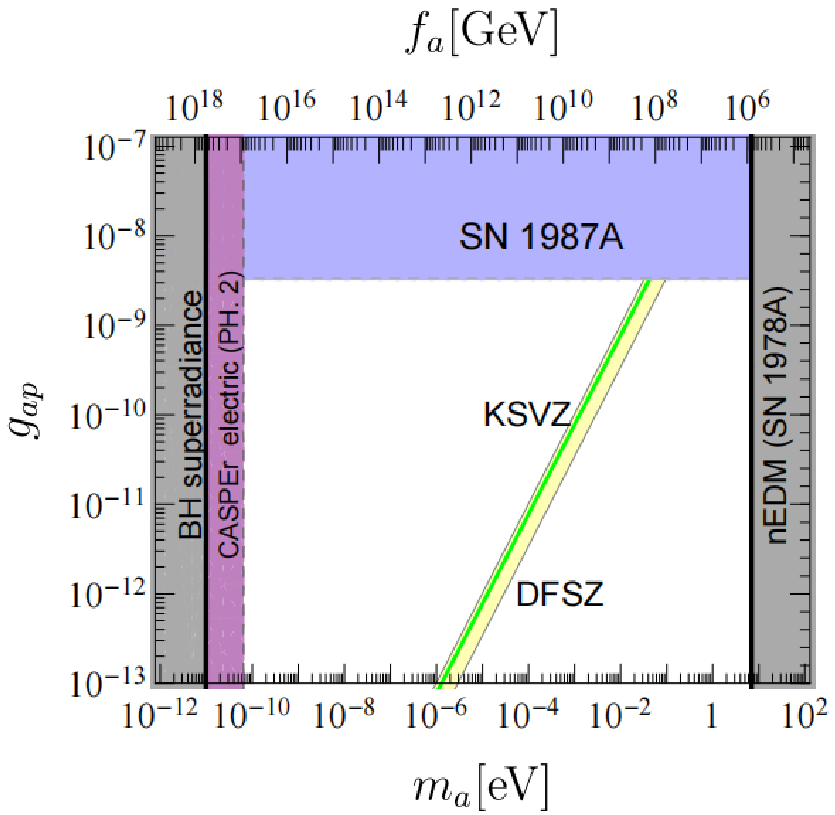




Axion effective Lagrangian and mass

★ **Axion effective Lagrangian**





arXiv:2003.01100v4

Axion effective Lagrangian and mass

☆ Axion effective Lagrangian

$K^+ \to \pi^+ + a, \quad J/\psi \to a + \gamma$

The bound from particle physics experiment is $f_a > 10^4\text{GeV}$

However, for an axion being so light, the stringent bounds emerge from astrophysics and cosmology

$$10^9\text{Gev} \leq f_a \leq 10^{12}\text{Gev}$$

Axion effective Lagrangian and mass

★ Axion mass

The mass of axion comes from the anomaly term, it can be calculated by CHPT[1]

$$\mathcal{L}_{\text{eff}} = v^3 \text{Tr}(M\tilde{\Sigma} + M\tilde{\Sigma}^\dagger) + K\cos(\sqrt{6}\pi_9/f_9 + a/f_a)$$

$$\tilde{\Sigma} = \exp[\mathrm{i}(2\pi_9/f_9\sqrt{6})I]\Sigma$$

$$v \sim \Lambda_{QCD}, K \sim \Lambda_{QCD}^4$$

The physical axion is the mixing among a, π^3, π^8, π^9, with mass

$$m_A^2 = \frac{K}{f_a^2}\frac{1}{1 + K\text{Tr}M^{-1}/2v^3}$$

1. Kiwoon Choi et al., Physics Letters B, 181(1):145-149, 1986.

Axion effective Lagrangian and mass

★ **Axion mass**

In QCD, $\Lambda_{QCD} \sim 100\text{Mev} \gg m_u, m_d \sim 1\text{Mev}$

$$m_A^2 = \frac{2v^3}{f_a^2 \mathrm{Tr} M^{-1}} = \frac{m_u m_d}{(m_u + m_d)^2} \frac{f_\pi^2 m_\pi^2}{f_a^2}$$

$2(m_u + m_d)v^3 = f_\pi^2 m_\pi^2$

This is the well-known formula for the axion mass.

Axion and other open issues

★ Axions and neutrino masses

The RH neutrino and the PQ symmetry breaking scales naturally fall in the same intermediate range $M_R \sim f_a \sim 10^9\text{Gev} - 10^{12}\text{Gev}$

A common origin for mass of RH neutrino and axion decay constant: $N_R N_R \Phi$

Classical article: M. Shin, Phys. Rev. Lett. 59, 2515 (1987)

Composite axion and sterile neutrino: arXiv:2310.08557v1

Flavor violated axion neutrino model: arXiv:2408.05903v1 $0.0011\text{eV} \lesssim m_3 \lesssim 0.0029\text{eV}$

Axion and other open issues

- **Axions and the baryon asymmetry**

  Axion carries PQ charge and generates net PQ charge, and these charge can be transformed to net baryon number

  Axiogenesis: 10.1103/PhysRevLett.124.111602

  Affleck-Dine baryogenesis: arXiv:1906.05286

Axion and other open issues

- **Axions and inflation**

The potential of axion is slow, it can play the role as inflaton

Natural Inflation: 10.1103/PhysRevLett.114.151303

SMASH model : arXiv:1610.01639