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# Study of the $B^- \rightarrow K^- \eta \eta_c$ decay due to the $D\bar{D}$ bound state

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Motivation

Formalism

Results & Discussion

Summary

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Results & Discussion

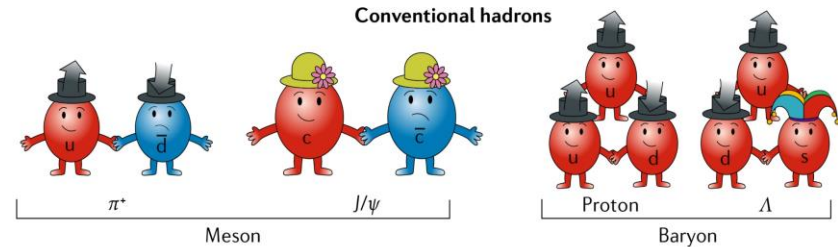
Summary

# Exotic state

➤ **Hadron**: strong interaction, composites of quarks and gluons

➤ **Traditional quark model**:

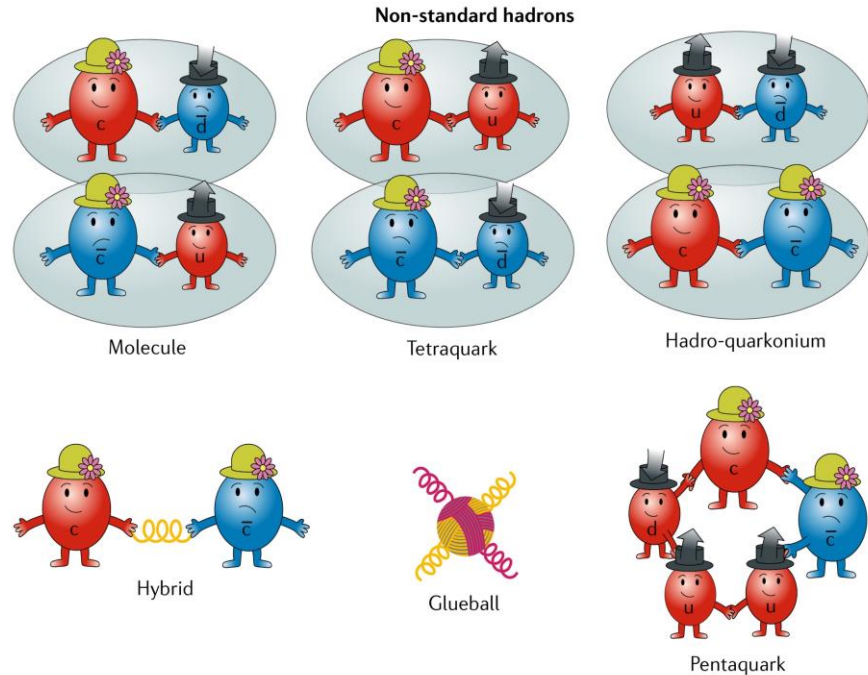
Hadron { Meson ( $q\bar{q}$ )  
Baryon ( $qqq, \bar{q}\bar{q}\bar{q}$ )



Phys. Lett. 8, 214 (1964)  
CERN-TH-401, CERN-TH-412

➤ **Quantum Chromodynamics**:

Exotic state { Molecular ( $M\bar{M}, B\bar{B}...$ )  
Tetraquark ( $qq\bar{q}\bar{q}...$ )  
Pentaquark ( $qqqq\bar{q}...$ )  
Hybrid ( $q\bar{q}g, qqqg...$ )  
Glueball ( $gg, ggg...$ )  
...



# Exotic state

- 2003, Belle Collaboration,  $X(3872)$  in the  $J/\psi\pi^+\pi^-$  invariant mass distribution  
[Belle] Phys. Rev. Lett. 91, 262001 (2003)  
LHCb, Belle, BABAR, BESIII... many exotic states have been observed experimentally
- Exotic states near the **thresholds** of a pair of hadrons:
  - $X(3872) \leftrightarrow D\bar{D}^*$  [Belle] Phys. Rev. Lett. 91, 262001 (2003)
  - $Z_c(3900) \leftrightarrow D\bar{D}^*$  [BESIII] Phys. Rev. Lett. 110, 252001 (2013)
  - $Z_{cs}(3985) \leftrightarrow D\bar{D}_s^*$  [BESIII] Phys. Rev. Lett. 126, 102001 (2021)
  - $P_{cs}(4459) \leftrightarrow \Xi_c\bar{D}^*$  [LHCb] Sci. Bull. 66, 1278 (2021)
  - $T_{cc}^+ \leftrightarrow DD^*$  [LHCb] Nature Phys. 18, 751 (2022) ...

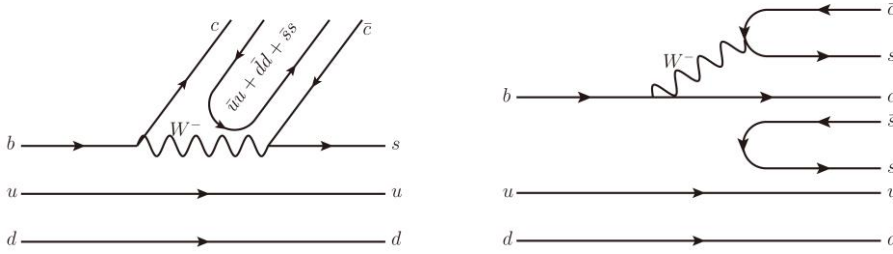
**Explanations:** compact tetraquark, pentaquark, or hexaquark, hadronic molecule, kinematic effect (threshold cusp, triangle singularity) ...
- **Hadronic molecule:**
  - Two or more hadrons bounded through the strong interactions
  - Mass around the threshold of a pair of hadrons
  - Quantum number  $J^{PC} \leftrightarrow S$ -wave of the composites [Rev. Mod. Phys. 90, 015003 (2018)]

# $D\bar{D}$ bound state

- 2007,  $\pi^+\pi^-$ ,  $\pi^0\pi^0$ ,  $K^+K^-$ ,  $K^0\bar{K}^0$ ,  $D^+D^-$ ,  $D^0\bar{D}^0$ ,  $D_s^+D_s^-$ ,  $\eta\eta$ ,  $\eta\eta_c$  coupled channels, unitary coupled-channel approach, [Phys. Rev. D 76, 074016 \(2007\)](#)  
 $D\bar{D}$  bound state with  $I(J^{PC}) = 0(0^{++})$ ,  $M \approx 3720$  MeV, denote as  $X(3700)$
- Unitary coupled-channel approach:  $\sqrt{s} = (3722 - i18)$  MeV [Eur. Phys. J. A 41, 85 \(2009\)](#)  
 Lattice QCD: binding energy  $B = (4.0_{-3.7}^{+5.0})$  MeV [J. High Energy Phys. 06, 035 \(2021\)](#)  
 QCD sum rule:  $M_{D\bar{D}} = (3.73 \pm 0.08)$  GeV [Phys. Rev. D 105, 094003 \(2022\)](#)  
 Nonrelativistic EFT:  $M = (3739.3 \pm 0.1)$  MeV [Phys. Rev. D 105, 034024 \(2022\)](#)
- Some theoretical studies of the experimental measured process support the existence of such a  $D\bar{D}$  bound state, e.g.,  $e^+e^- \rightarrow J/\psi D\bar{D}^{(*)}$ ,  $B \rightarrow D\bar{D}K$ ,  $\gamma\gamma \rightarrow D\bar{D}$ .  
[Eur. Phys. J. A 36, 189 \(2008\)](#), [Eur. Phys. J. C 76, 121 \(2016\)](#), [Phys. Lett. B 827, 136982 \(2022\)](#),  
[Eur. Phys. J. A 57, 38 \(2021\)](#), [Phys. Rev. D 103, 054008 \(2021\)](#)
- Some process suggested to search for the  $D\bar{D}$  bound state:  
 $\psi(3770)/\psi(4040) \rightarrow \gamma X(3700) \rightarrow \gamma\eta\eta'$ ,  $e^+e^- \rightarrow J/\psi X(3700) \rightarrow J/\psi\eta\eta'$ ,  
 $\psi(3770) \rightarrow \gamma D\bar{D}$ ,  $\Lambda_b \rightarrow \Lambda D\bar{D}$ ,  $B^+ \rightarrow K^+\eta\eta$   
[Eur. Phys. J. A 41, 85 \(2009\)](#), [Eur. Phys. J. A 49, 52 \(2013\)](#), [Eur. Phys. J. C 80, 510 \(2020\)](#),  
[Phys. Rev. D 103, 114013 \(2021\)](#), [Phys. Rev. D 108, 054004 \(2023\)](#)

# $\Lambda_b \rightarrow \Lambda D \bar{D}$

The internal (left) and external (right)  $W^-$  emission:



Phys. Rev. D 103, 114013 (2021)

$$t_{\Lambda_b \rightarrow \Lambda D^0 \bar{D}^0}^{S\text{-wave}} = V_p (1 + G_{D^+ D^-} t_{D^+ D^- \rightarrow D^0 \bar{D}^0} + G_{D^0 \bar{D}^0} t_{D^+ D^- \rightarrow D^0 \bar{D}^0} + (1 + C) G_{D_s^+ D_s^-} t_{D_s^+ D_s^- \rightarrow D^0 \bar{D}^0})$$

$$t_{\Lambda_b \rightarrow \Lambda D^+ D^-}^{S\text{-wave}} = V_p (1 + G_{D^+ D^-} t_{D^+ D^- \rightarrow D^+ D^-} + G_{D^0 \bar{D}^0} t_{D^0 \bar{D}^0 \rightarrow D^+ D^-} + (1 + C) G_{D_s^+ D_s^-} t_{D_s^+ D_s^- \rightarrow D^+ D^-})$$

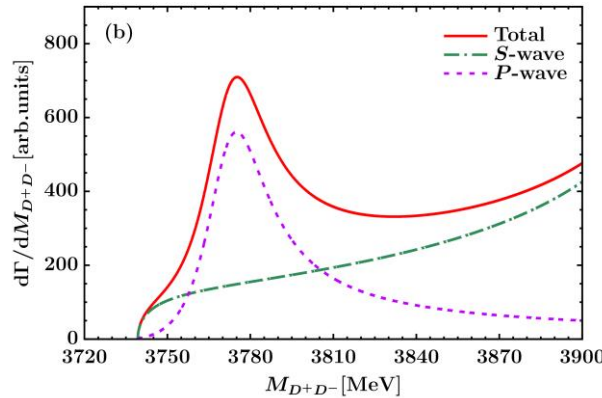
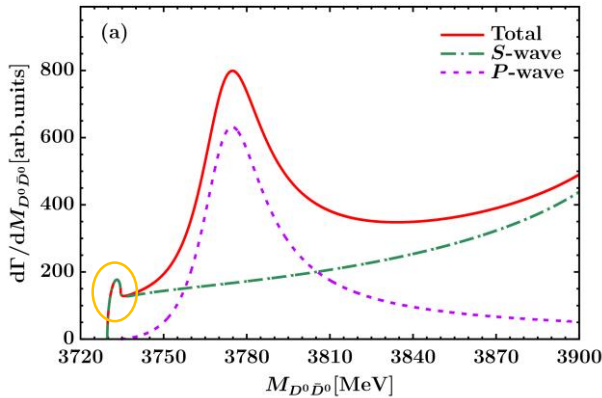
The intermediate state  $\psi(3770)$ :

$$t^{P\text{-wave}} = \frac{\beta \times V_p \times M_{\psi(3770)} \tilde{p}_D}{M_{D\bar{D}}^2 - M_{\psi(3770)}^2 + i M_{\psi(3770)} \tilde{\Gamma}_{\psi(3770)}}$$

BS equation:  $t_{i \rightarrow j} = [1 - VG]^{-1} V$

The  $D^0 \bar{D}^0$  (left) and  $D^+ D^-$  (right) invariant mass distributions:

J. High Energy Phys. 07,140 (2024)



- An enhancement near  $D\bar{D}$  threshold
- Partial wave analysis

- Sizeable coupling between  $X(3700)$  and  $\eta\eta_c$ ,  $M_{X(3700)} - (M_\eta + M_{\eta_c}) \approx 200$  MeV
- A clear peak in the  $\eta\eta_c$  invariant mass distribution →  $B^- \rightarrow K^- \eta\eta_c$  decay

Motivation

Formalism

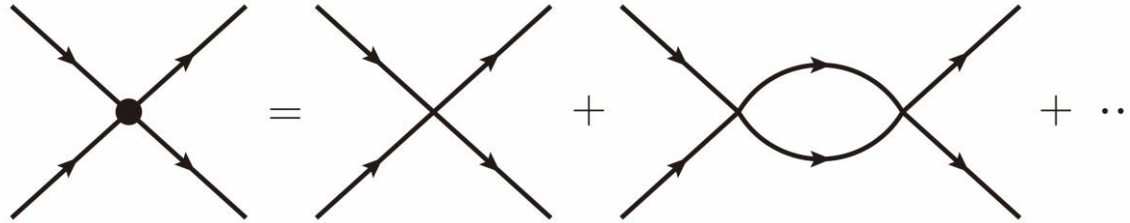
Results & Discussion

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# Unitary coupled-channel approach

- Bethe-Salpeter equation:



Scattering amplitude:  $T = V + VGT \rightarrow T = [1 - VG]^{-1}V$

$V$ : transition potential between the coupled channels

$G$ : loop function for the two-meson propagation in the  $l$ th coupled channel

$$G_l(s) = \frac{1}{16\pi^2} \left[ \alpha_l + \ln \frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s} \ln \frac{m_2^2}{m_1^2} \right. \\ \left. + \frac{\mathbf{p}}{\sqrt{s}} \times \left( \ln \frac{s - m_2^2 + m_1^2 + 2\mathbf{p}\sqrt{s}}{-s + m_2^2 - m_1^2 + 2\mathbf{p}\sqrt{s}} + \ln \frac{s + m_2^2 - m_1^2 + 2\mathbf{p}\sqrt{s}}{-s - m_2^2 + m_1^2 + 2\mathbf{p}\sqrt{s}} \right) \right]$$

Prog. Part. Nucl. Phys. 45, 157 (2000)

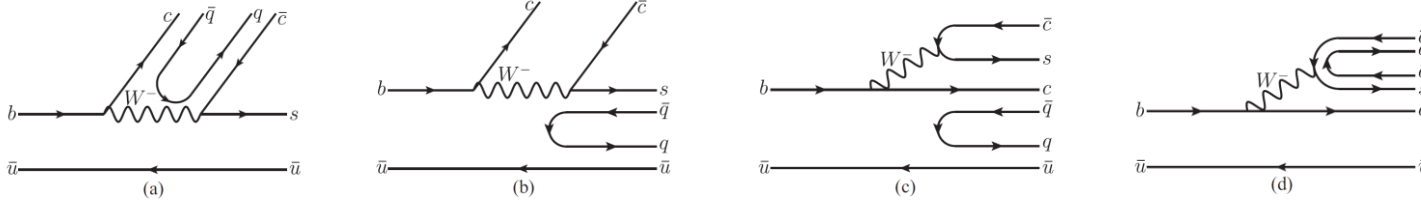
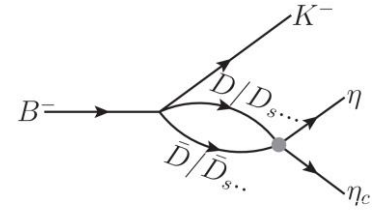
- **Pole** in the complex energy plane ( $T$ )  $\leftrightarrow$  bound state of a pair of hadrons

$$\sqrt{s}_p = (M_R - i\Gamma_R/2) \text{ MeV} \quad M_R \leftrightarrow \text{mass}; \quad \Gamma_R \leftrightarrow \text{width}$$

# $B^- \rightarrow K^- \eta \eta_c$

Three steps: weak decay  $\rightarrow$  hadronization  $\rightarrow$  final-state interactions

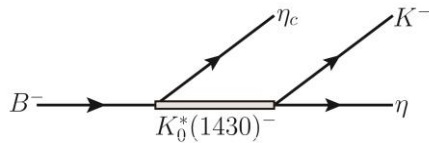
The internal (a-b) and external (c-d)  $W^-$  emission:



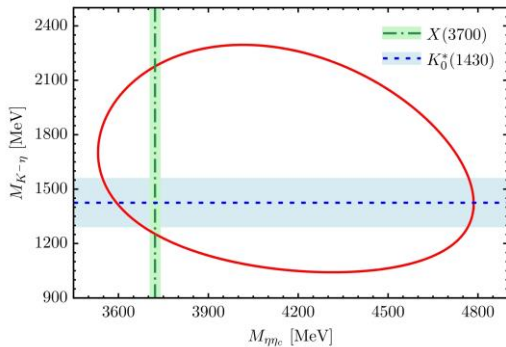
Rescattering amplitude:  $T_{X(3700)} = V_p V_{cb} V_{cs} \left( (1 + C) G_{D^0 \bar{D}^0} t_{D^0 \bar{D}^0 \rightarrow \eta \eta_c} + G_{D^+ D^-} t_{D^+ D^- \rightarrow \eta \eta_c} \right. \\ \left. + (1 + C) G_{D_s^+ D_s^-} t_{D_s^+ D_s^- \rightarrow \eta \eta_c} + 3/\sqrt{6} G_{\eta' \eta_c} t_{\eta' \eta_c \rightarrow \eta \eta_c} \right)$

Eur. Phys. J. A 41, 85 (2009)

The intermediate state  $K^*(1430)$ :



$$T_{K^*} = \frac{V_p \times \beta \times M_{K_0^*(1430)}^2}{M_{K^- \eta}^2 - M_{K_0^*(1430)}^2 + i M_{K_0^*(1430)} \Gamma_{K_0^*(1430)}}$$



Doubly differential decay width:

$$\frac{d^2 \Gamma}{dM_{\eta \eta_c} dM_{K^- \eta}} = \frac{1}{(2\pi)^3} \frac{M_{K^- \eta} M_{\eta \eta_c}}{8M_{B^-}^3} |T_{X(3700)} + e^{i\varphi} T_{K^*}|^2$$

Differential decay width:  $\frac{d\Gamma}{dM_{12}} = \int \frac{d^2 \Gamma}{dM_{12} dM_{23}} dM_{23}$

Motivation

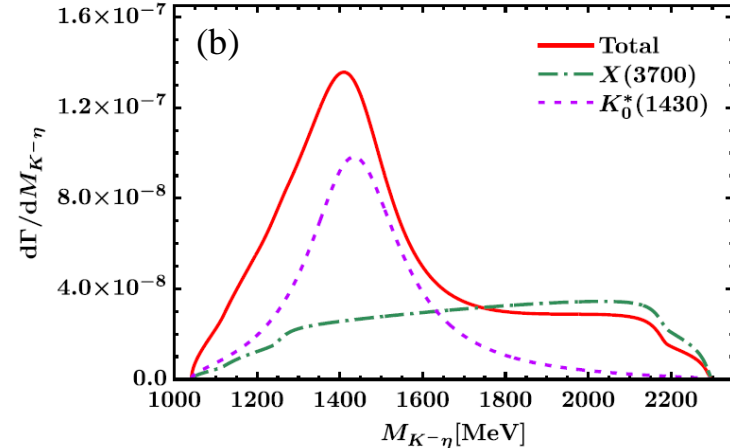
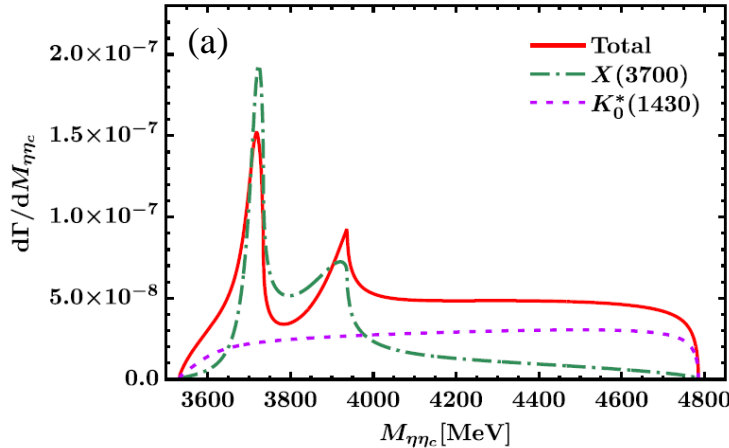
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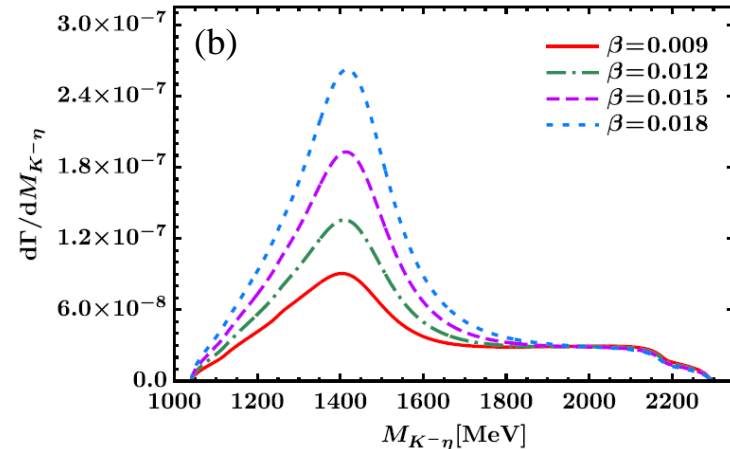
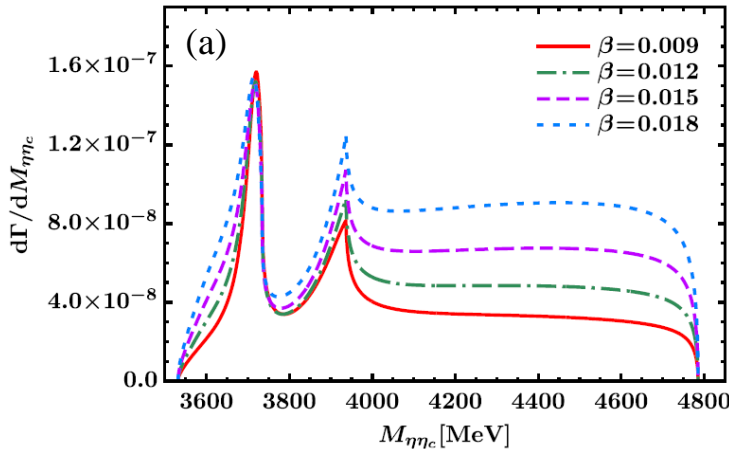
# Results and discussion

The  $\eta\eta_c$  (a) and  $K^-\eta$  (b) **invariant mass distributions** (fixed parameter):



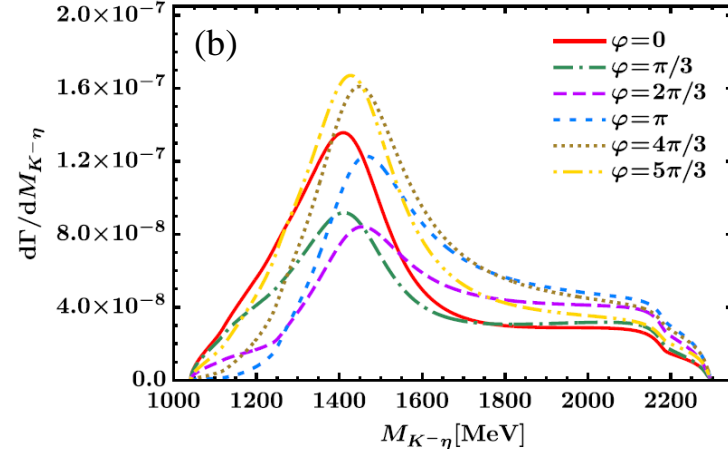
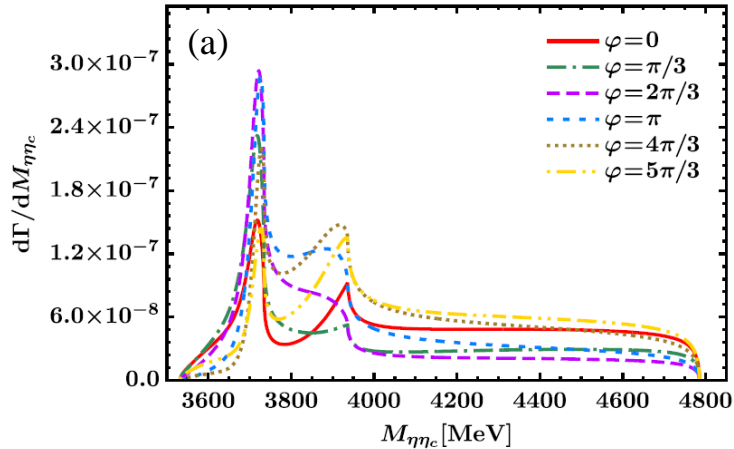
➤ a clear peak around 3720 MeV in the  $\eta\eta_c$  IMD associated with the  $D\bar{D}$  bound state

The  $\eta\eta_c$  (a) and  $K^-\eta$  (b) invariant mass distributions with  $\beta = 0.009, 0.012, 0.015, 0.018$ :

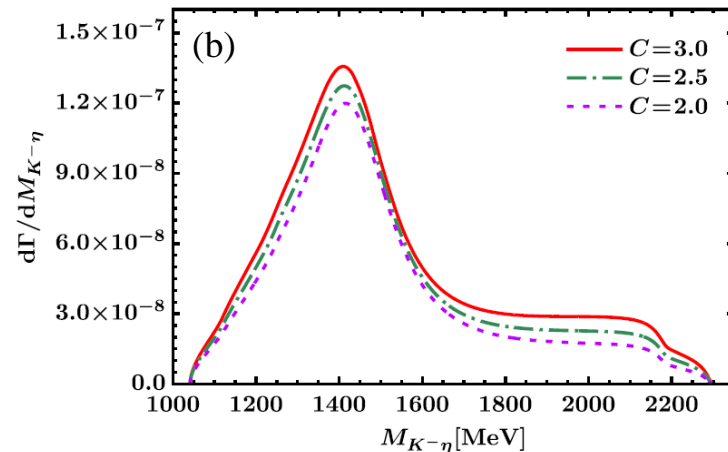
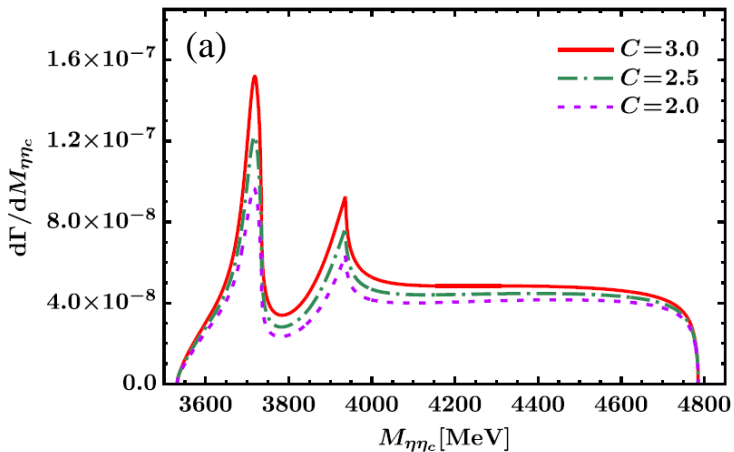


# Results and discussion

The  $\eta\eta_c$  (a) and  $K^-\eta$  (b) invariant mass distributions with  $\varphi = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$ :



The  $\eta\eta_c$  (a) and  $K^-\eta$  (b) invariant mass distributions with  $C = 3.0, 2.5, 2.0$ :



➤ Within the variation ranges of parameters, always a clear peak in the  $\eta\eta_c$  IMD

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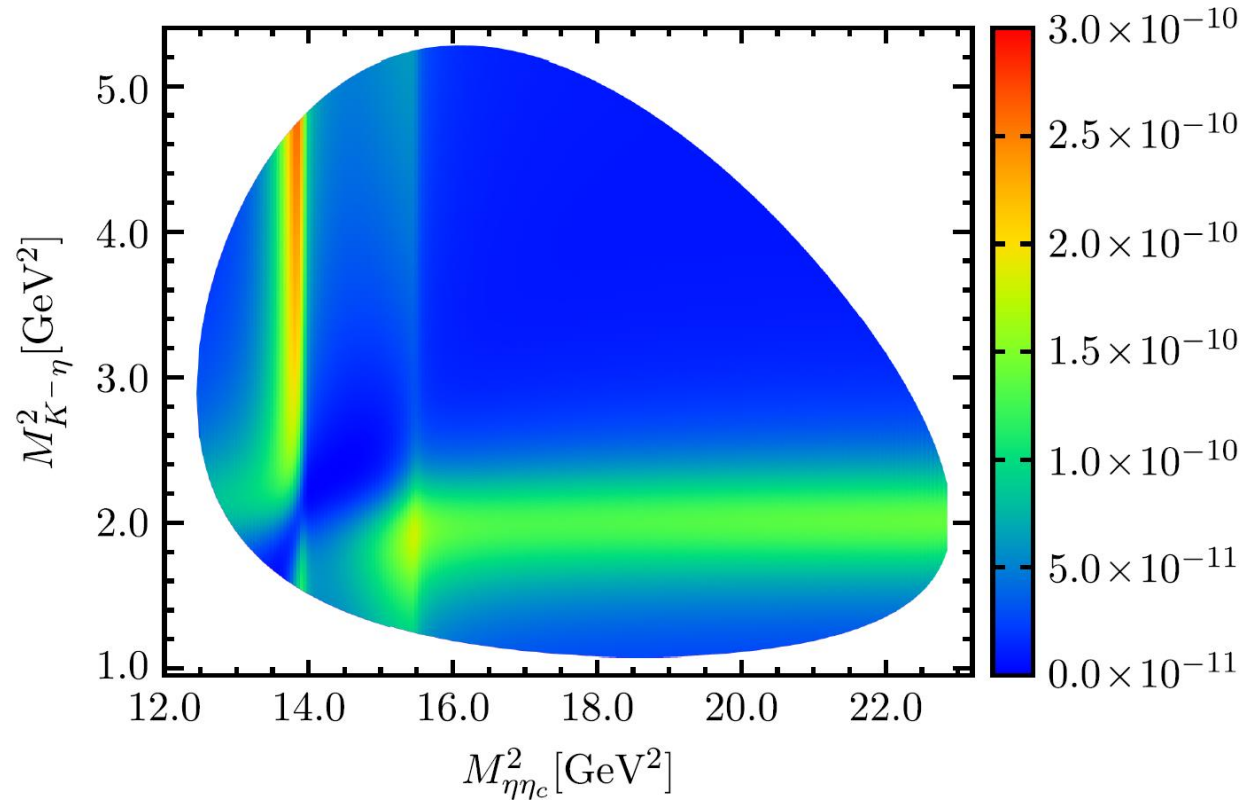
Summary

# Summary

- Sizeable coupling between  $X(3700)$  and  $\eta\eta_c$ ,  $M_{X(3700)} - (M_\eta + M_{\eta_c}) \approx 200$  MeV  
Many exotic states discovered in the  $B$ -meson three-body decay  
→ Search for the  $D\bar{D}$  bound state in the  $B^- \rightarrow K^- \eta\eta_c$  process
- Unitary coupled-channel approach, calculate the  $\eta\eta_c$  invariant mass distributions  
 $X(3700)$ :  $S$ -wave pseudoscalar meson-pseudoscalar meson interactions  
Intermediate resonance  $K_0^*(1430)$
- $\eta\eta_c$  invariant mass distribution:  
A **clear peak** appears around 3720 MeV associated with the  $D\bar{D}$  bound state  $X(3700)$   
Intermediate resonance  $K_0^*(1430)$  gives a smooth contribution
- A more precise measurement of the  $B^- \rightarrow K^- \eta\eta_c$  decay at the Belle II and LHCb experiments in the future, to confirm the existence of such a predicted  $D\bar{D}$  bound state and measure its mass and width

*Thank you for your attention!*

Doubly differential decay width of the  $B^- \rightarrow K^- \eta \eta_c$  decay in the  $(M_{\eta\eta_c}^2, M_{K^- \eta}^2)$  plane



- The  $X(3700)$  and  $K_0^*(1430)$  resonances can be clearly seen



# Backup

$$B^\pm \rightarrow K^\pm \eta_c \eta$$

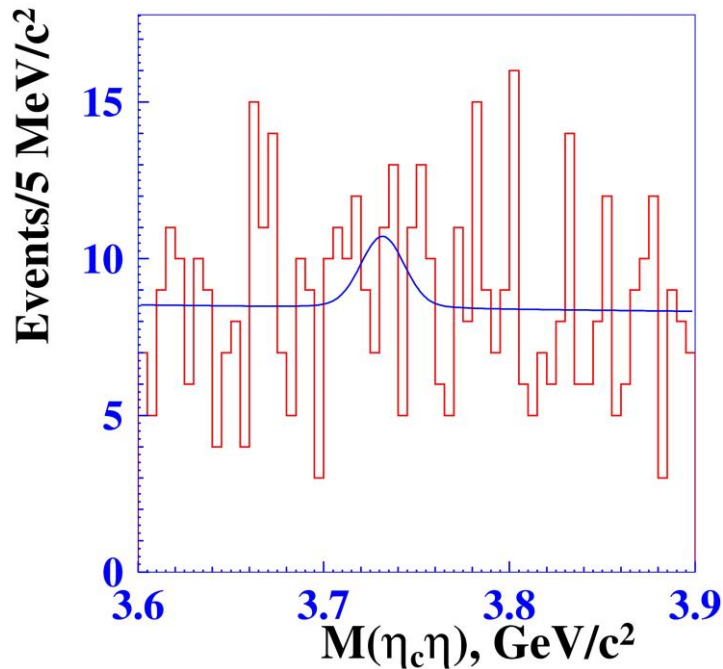
Belle Collaboration:  $772 \times 10^6$   $B\bar{B}$  pairs collected at  $\Upsilon(4S)$

No obvious  $X(3730)$  signal, the upper limits of the branching ratios:

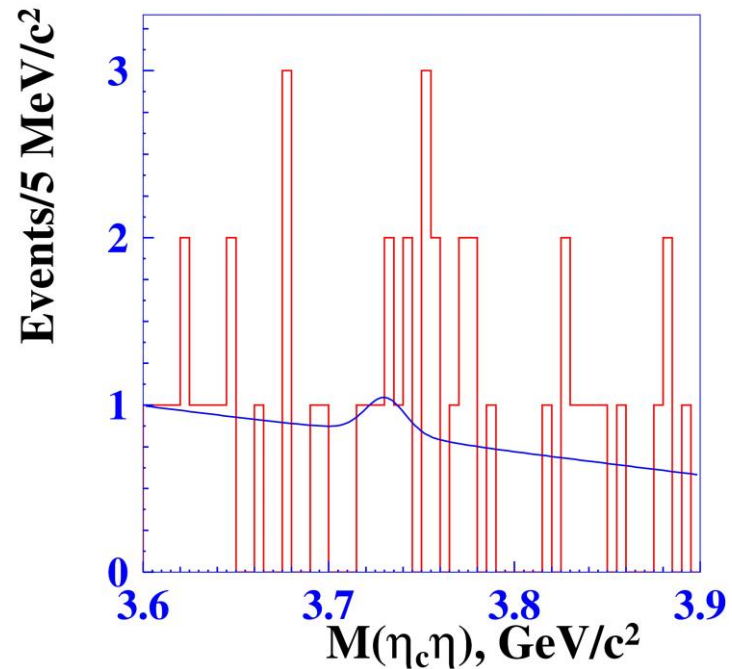
$$\mathcal{B}(B^\pm \rightarrow K^\pm \eta_c \eta) < 2.2 \times 10^{-4}$$

[Belle] *J. High Energy Phys.* 06, 132 (2015)

$$\mathcal{B}(B^\pm \rightarrow K^\pm X(3730)) \times \mathcal{B}(X(3730) \rightarrow \eta_c \eta) < 4.6 \times 10^{-5}$$



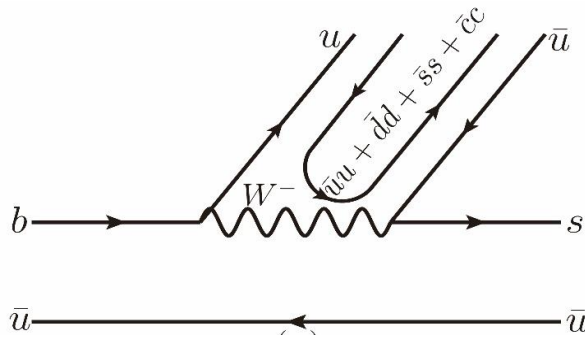
$\eta \rightarrow \gamma\gamma$



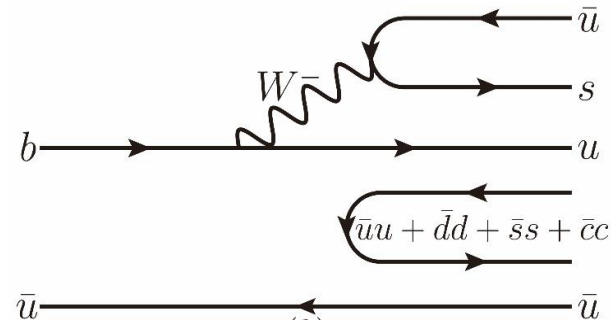
$\eta \rightarrow \pi^+ \pi^- \pi^0$

# Backup

Other quark level diagrams:



(a) Internal  $W^-$  emission



(b) External  $W^-$  emission

Components of the meson systems after hadronization:

$$|H\rangle^{3a} = V_p V_{ub} V_{us} \left( \frac{1}{3} \eta \eta + \frac{1}{2} \pi^0 \pi^0 + \frac{1}{6} \eta' \eta' + \frac{1}{\sqrt{18}} \eta \eta' + \pi^+ \pi^- + K^+ K^- + D^0 \bar{D}^0 \right) K^-$$

$$|H\rangle^{3b} = C \times V_p V_{ub} V_{us} \left( \frac{1}{3} \eta \eta + \frac{1}{2} \pi^0 \pi^0 + \frac{1}{6} \eta' \eta' + \frac{1}{\sqrt{18}} \eta \eta' + \pi^+ \pi^- + K^+ K^- + D^0 \bar{D}^0 \right) K^-$$

$$|V_{ub}| = 0.00369 \pm 0.00011$$

$$|V_{us}| = 0.22500 \pm 0.00067$$

$$|V_{cb}| = 0.04182^{+0.00085}_{-0.00074}$$

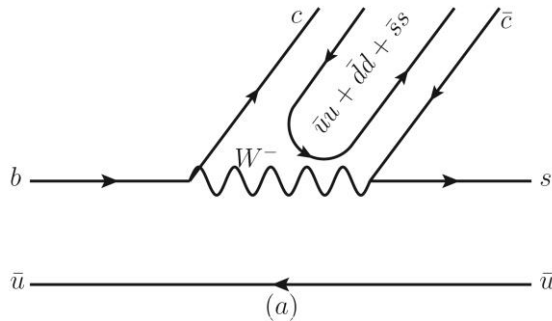
$$|V_{cs}| = 0.97349 \pm 0.00016$$

# Backup

## Hadronization:

L. R Dai, et al. Eur. Phys. J. C 76, 121 (2016)

e.g.



$$\sum_k q_i (\bar{q}_k q_k) \bar{q}_j = \sum_k M_{ik} M_{kj} = (M^2)_{ij}$$

$$|H\rangle^a = V_p V_{cb} V_{cs}^* (D^0 \bar{D}^0 + D^+ D^- + D_s^+ D_s^-) K^-$$

$q\bar{q}$  matrix expressed in terms of the physical pseudoscalar mesons:

$$M \Rightarrow \begin{pmatrix} \frac{\eta}{\sqrt{3}} + \frac{\pi^0}{\sqrt{2}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & \frac{\eta}{\sqrt{3}} - \frac{\pi^0}{\sqrt{2}} + \frac{\eta'}{\sqrt{6}} & K^0 & D^- \\ K^- & \bar{K}^0 & \frac{\sqrt{2}}{\sqrt{3}} \eta' - \frac{\eta}{\sqrt{3}} & D_s^- \\ D^0 & D^+ & D_s^+ & \eta_c \end{pmatrix}$$