Belle/Belle II 上轻强子相关实验进展

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2025年轻强子专题研讨会

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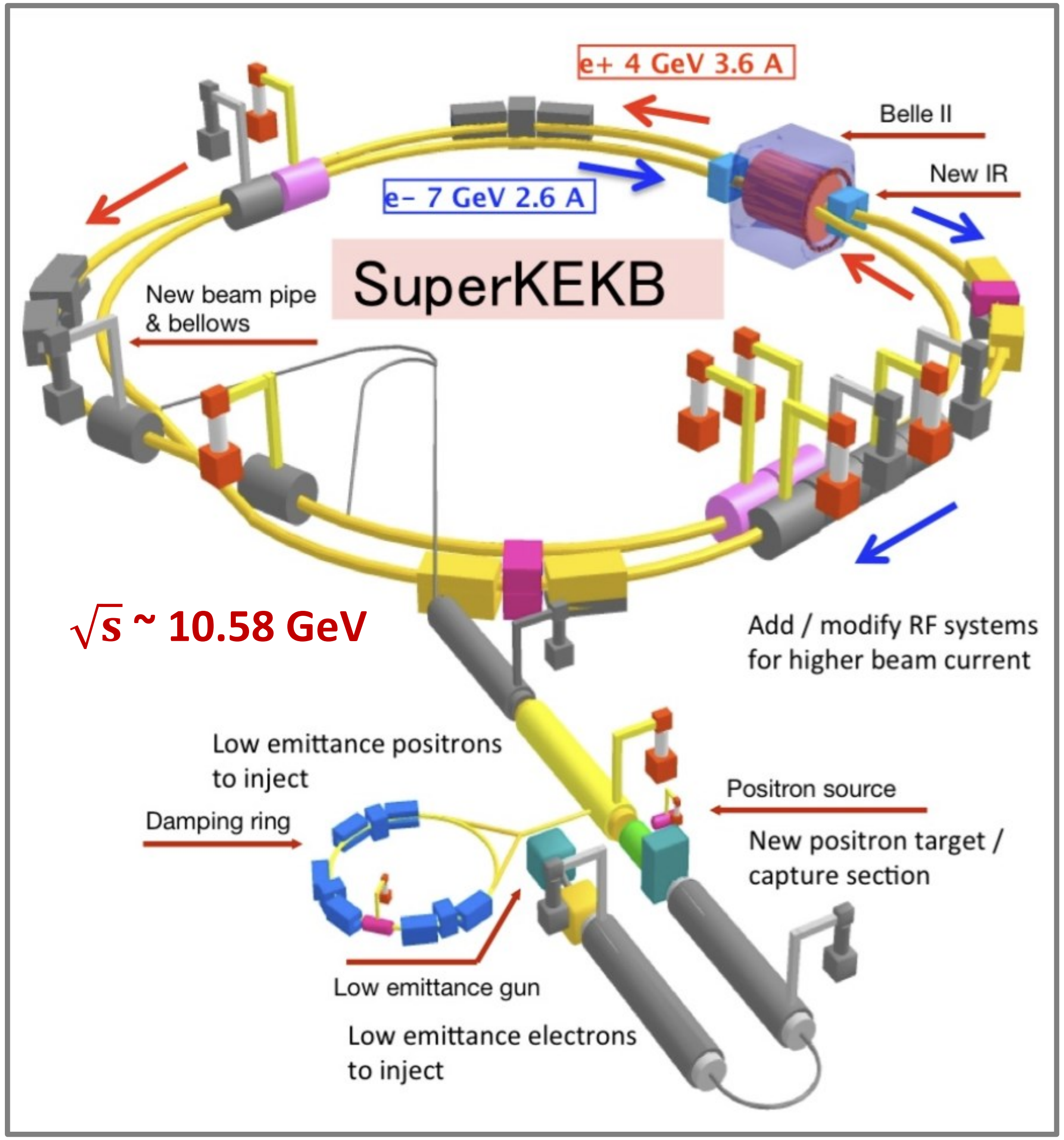
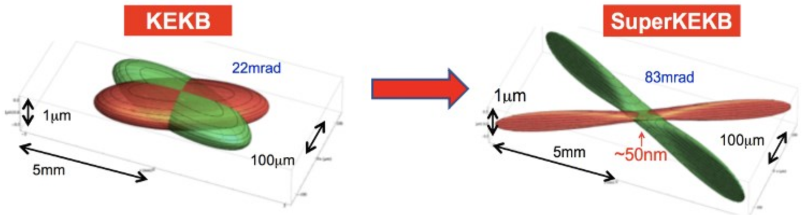
SuperKEKB and Belle II

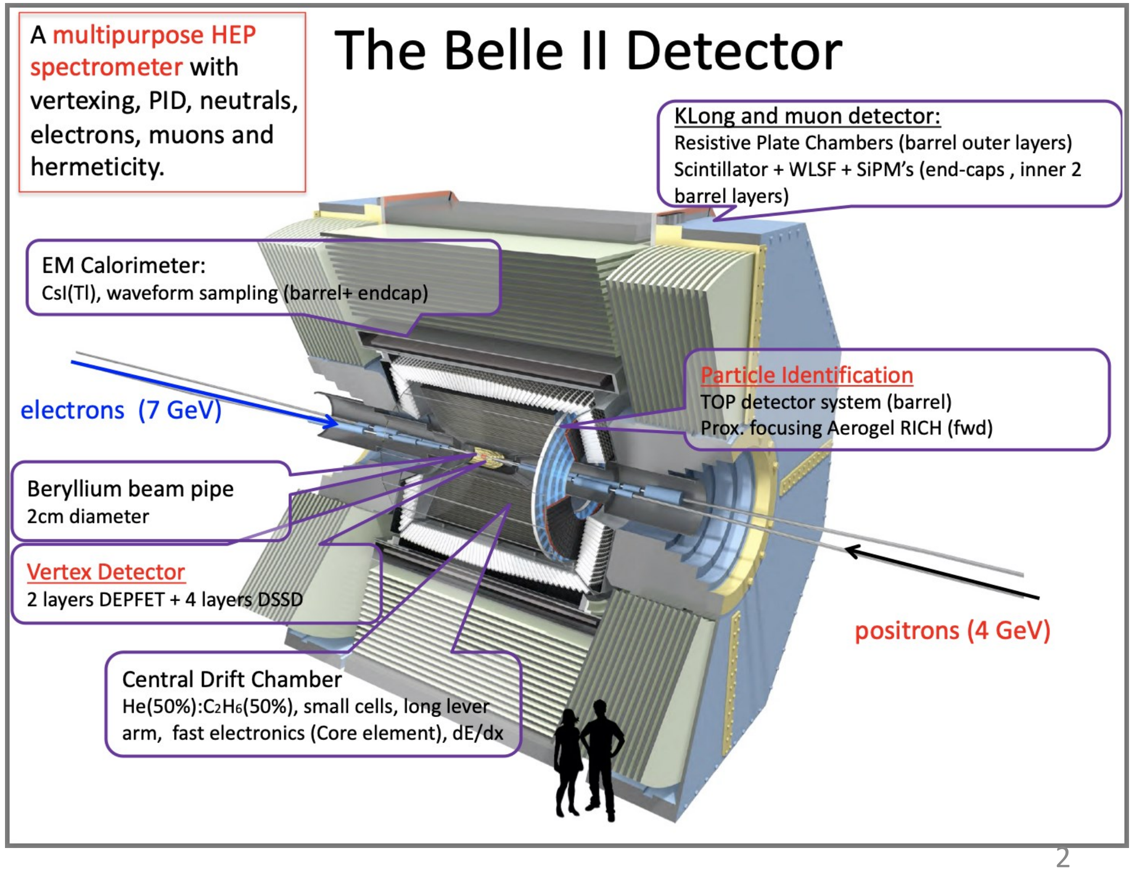




Nano-beam design:

Beam squeezing: ×20 smaller; Beam current: ×2 larger

Target peak luminosity: KEKB×30

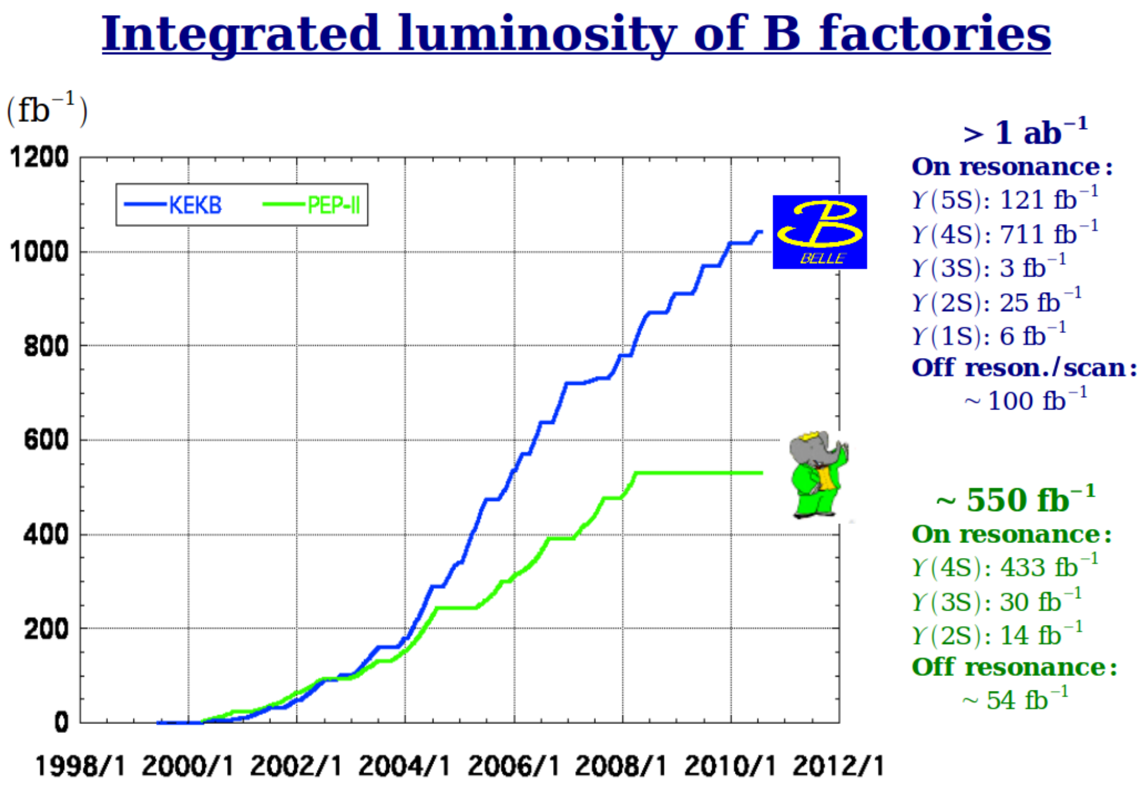


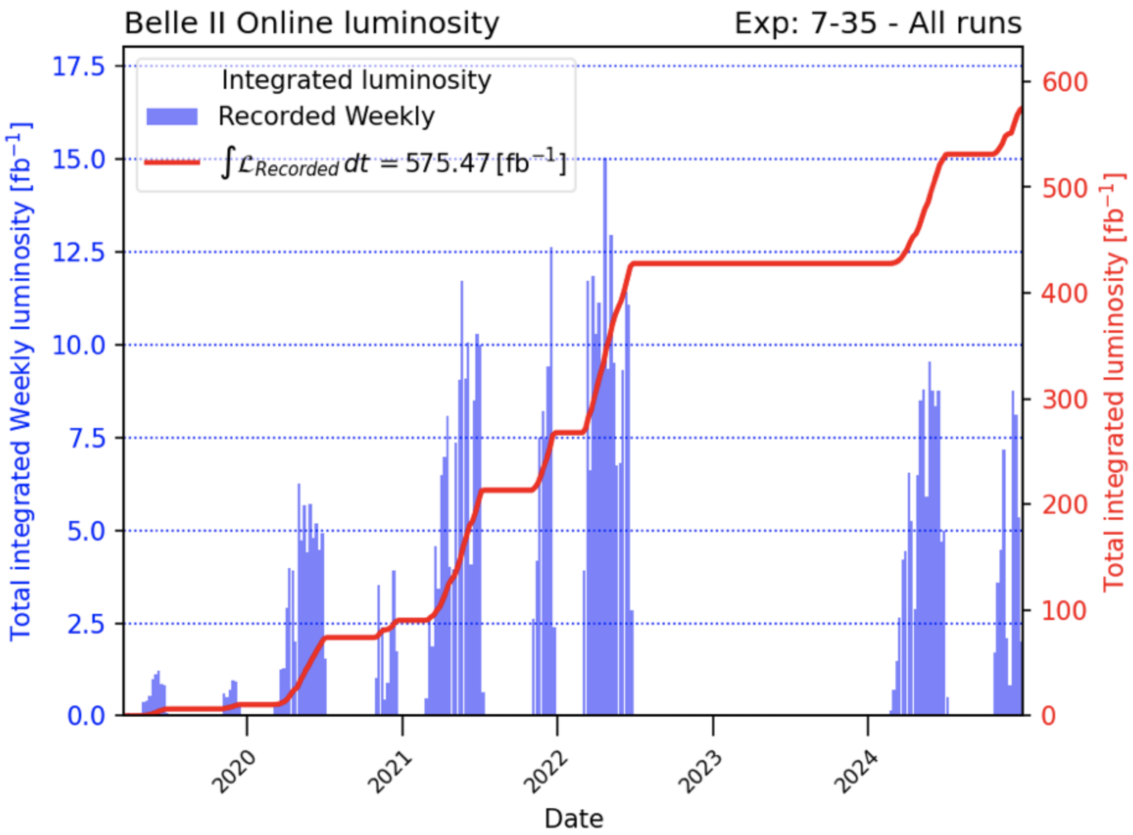
Belle and Belle II Datasets

- **Belle (1999 - 2012)**
- **Belle II RUN-I (2019 - 2023)**
- **Belle II RUN-II (2024 - 2025)**

In December 2024

WORLD RECORD: $5.1 \times 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$


Integrated luminosity of B factories



Most data at or near the $\Upsilon(4S)$ resonance, some below/above $\Upsilon(4S)$.

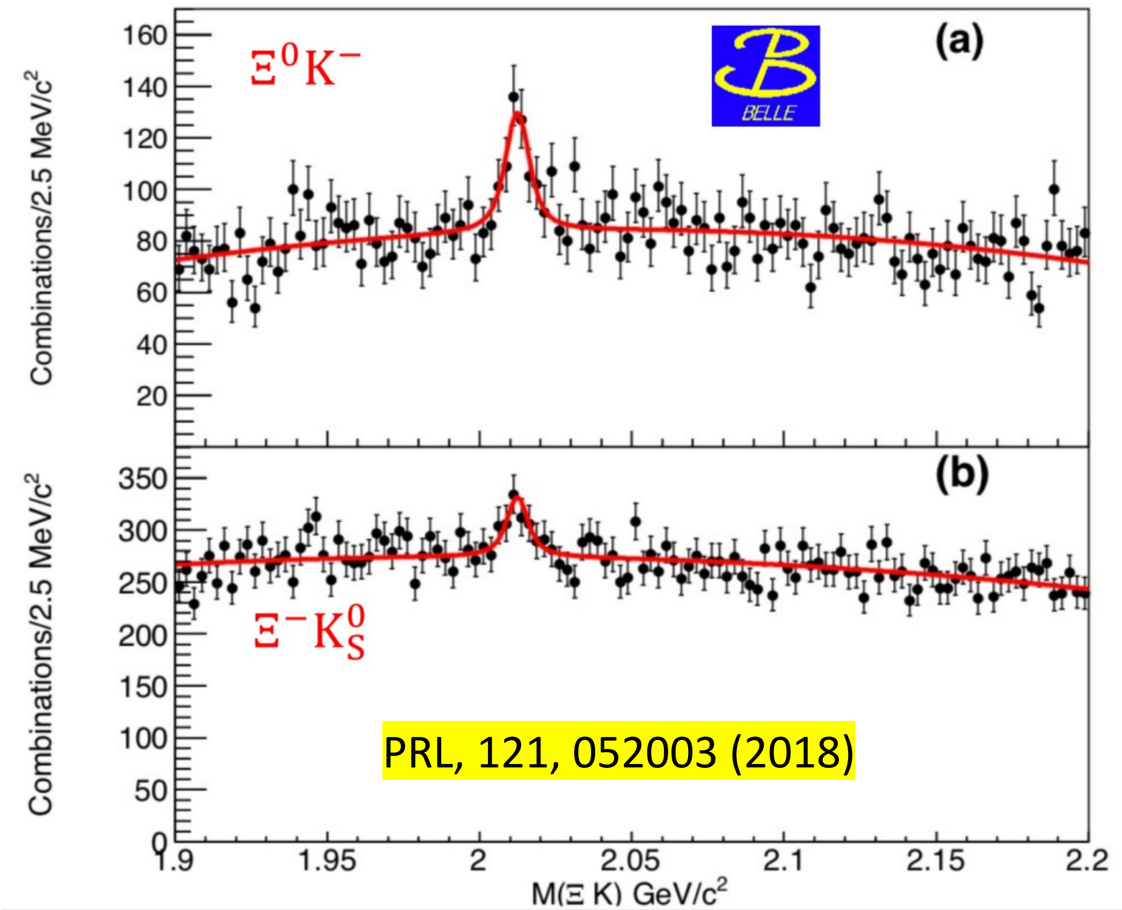
Outline:

- $\Omega(2012)^- \to \Xi(1530)\bar{K} \to \Xi\pi\bar{K}$ **[PLB 860, 139224 (2025)]**
- Peak at $\Lambda\eta$ threshold in $\Lambda_c^+ \to pK^-\pi^+$ **[PRD 108, L031104 (2023)]**
- Peak at $\bar{K}N$ threshold in $\Lambda_c^+ \to \Lambda\pi^+\pi^+\pi^-$ **[PRL 130, 151903 (2023)]**
- $\phi(2170)$ in $e^+e^- \to \eta\phi$ **[PRD 107, 012006 (2023)]**

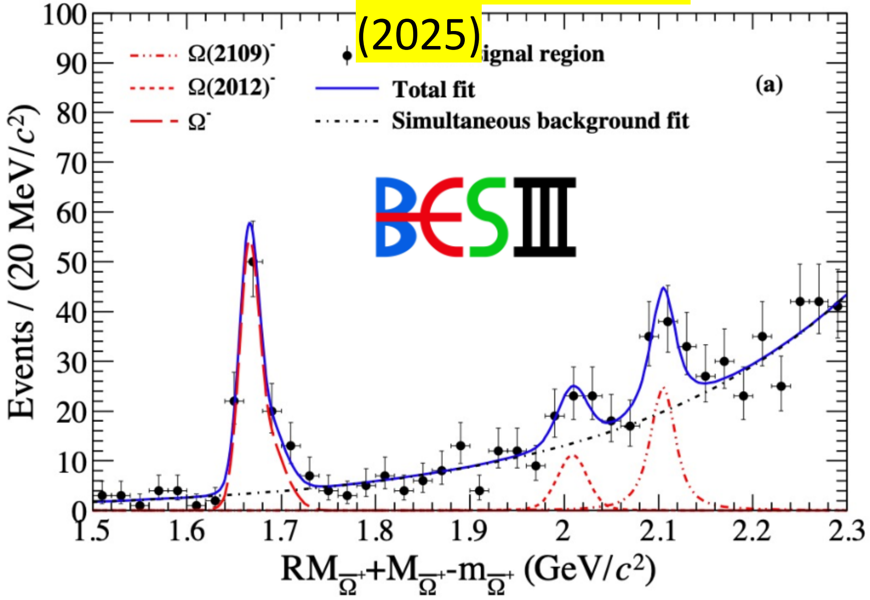
$$\boldsymbol{\Omega(2012)^{-} \to \Xi(1530)\bar{K} \to \Xi\pi\bar{K}}$$

Discovery of $\Omega(2012)^{-}$

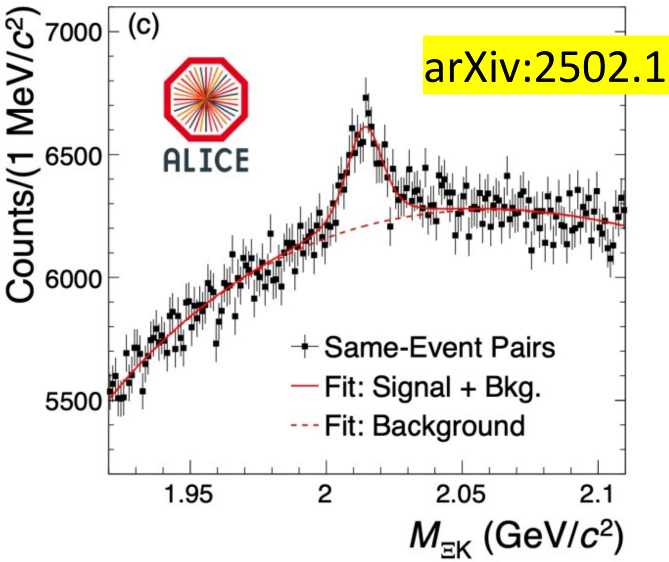
$\Omega(2012)$ was first observed by Belle in two-body (ΞK) decays, Confirmed by BESIII (low statistics) and ALICE (15σ).



PRL, 121, 052003 (2018)



PRL 134, 131903 (2025)



arXiv:2502.18063

The $\Omega(2012)$ was interpreted as **a standard baryon or a $\Xi(1530)\bar{K}$ molecule**.

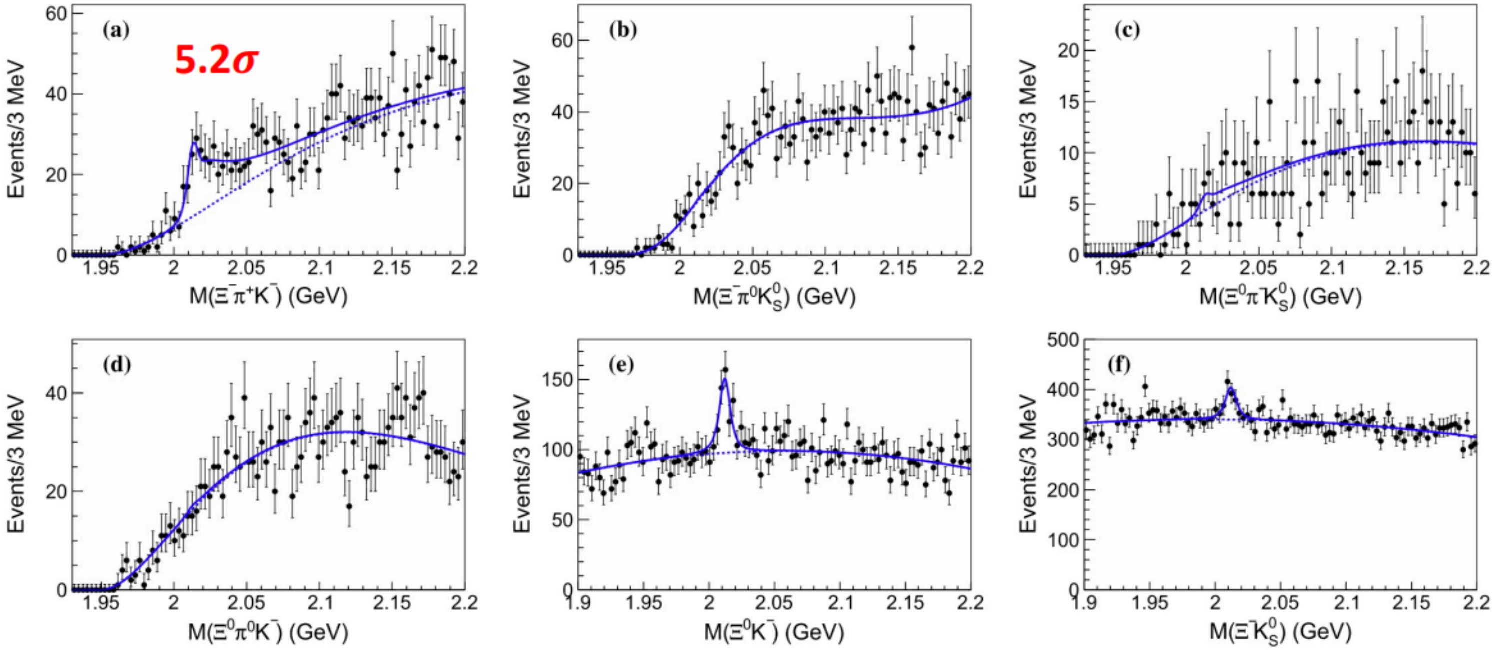
$\Omega(2012)^- \to \Xi(1530)\bar{K} \to \Xi\pi\bar{K}$

[PLB 860, 139224 (2025)]

The Flatté-like function [PRD 81, 094028 (2010)]

$$T_n(M) \equiv \frac{g_n k_n(M)}{|M - m_{\Omega(2012)^-} + \frac{1}{2}\sum_{j=2,3} g_j[\kappa_j(M) + ik_j(M)]|^2}$$

- g_n is the effective coupling of to the *n*–body final state.
- k_n and κ_n parameterize the real and imaginary parts of the $\Omega(2012)^-$self-energy.



The mass and ratio of effective couplings :

$\Omega(2012)^-$ mass	(2012.5±0.7±0.5) MeV
g_3/g_2	$22.9^{+17.9}_{-22.4}\pm2.2$

$$\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}} = \frac{\mathcal{B}(\Omega(2012)^- \to \Xi(1530)\bar{K} \to \Xi\pi\bar{K})}{\mathcal{B}(\Omega(2012)^- \to \Xi\bar{K})}$$

0.99±0.26±0.06

Mode	ε (%)	Y
$\Omega(2012)^- \to \Xi(1530)^0 K^- \to \Xi^-\pi^+K^-$	6.97 ± 0.07	267 ± 60
$\Omega(2012)^- \to \Xi(1530)^- \bar{K}^0 \to \Xi^-\pi^0\bar{K}^0$	1.06 ± 0.01	7 ± 2
$\Omega(2012)^- \to \Xi(1530)^- \bar{K}^0 \to \Xi^0\pi^-\bar{K}^0$	1.74 ± 0.02	23 ± 5
$\Omega(2012)^- \to \Xi(1530)^0 K^- \to \Xi^0\pi^0K^-$	0.63 ± 0.01	12 ± 3
$\Omega(2012)^- \to \Xi^0 K^-$	4.00 ± 0.04	242 ± 40
$\Omega(2012)^- \to \Xi^- \bar{K}^0$	15.5 ± 0.16	293 ± 65

Our result is **consistent with the molecular model of Ω(2012),** which predicts comparable rates for Ω(2012) decay to Ξ(1530)$\bar{K}$ and Ξ$\bar{K}$ [PRD 98 (2018) 054009, PRD 98 (2018) 056013, PRD 98 (2018) 076012]

Peak at $\Lambda\eta$ *threshold in* $\Lambda_c^+ \to pK^-\pi^+$

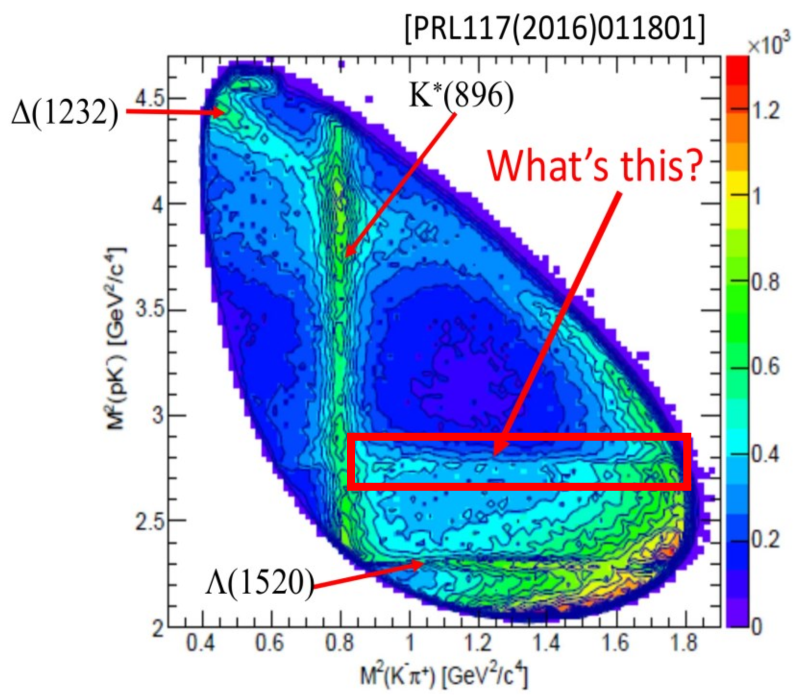
A peak at Λη threshold

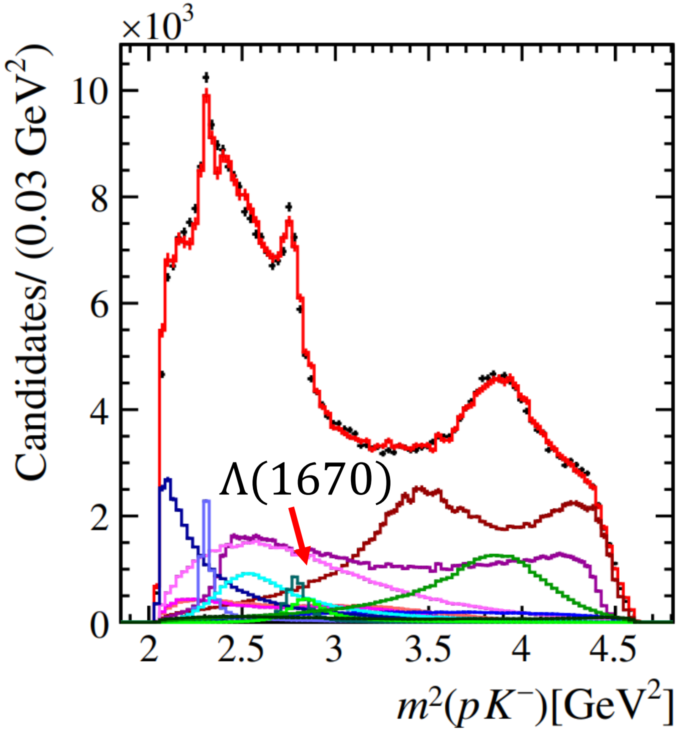
- ➢ A trace of a peak structure is observed in the pK^- mass spectrum in the previous analysis of $\Lambda_c^+ \to pK^-\pi^+$ decay by the Belle.

PRL, 117, 011801 (2016)

- ➢ LHCb performed an amplitude analysis of $\Lambda_c^+ \to pK^-\pi^+$. A similar structure is also seen. LHCb explained the structure using a BW form with fixed mass and width.

PRD 108, 012023 (2023)





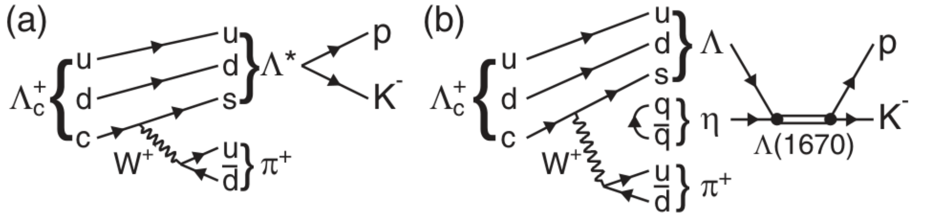


FIG. 1. Feynman diagrams for (a) a new Λ^* resonance and (b) a visible $\Lambda\eta$ threshold cusp enhanced by the $\Lambda(1670)$ pole in $\Lambda_c^+ \to pK^-\pi^+$ decay.

Two approach to describe this peak:

① **BW function**

② **Flatt´e function**

From the perspective of a new resonance

[PRD 108, L031104 (2023)]

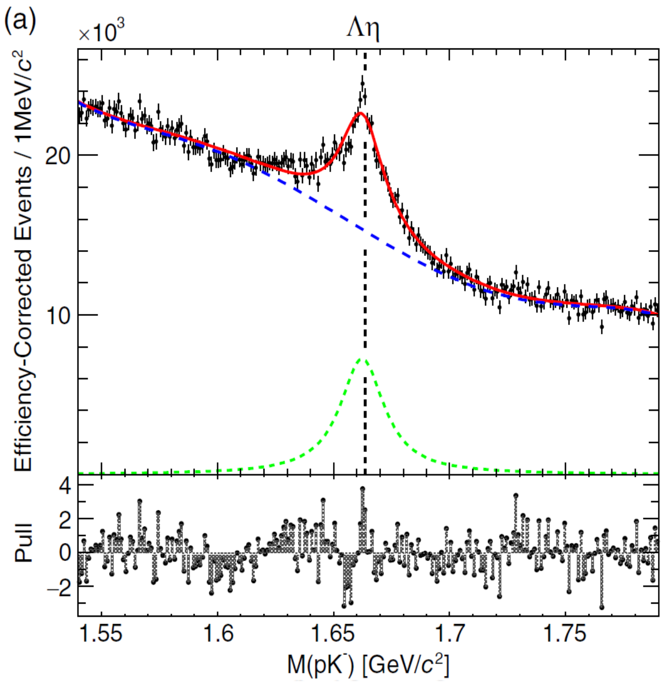
- We perform a binned least-χ^2 fit to the efficiency-corrected $M(pK^-)$distribution

- Fit to $M(pK^-)$ distribution using non-relativistic BW function.

$$\frac{dN}{dm} \propto |BW(m)|^2 = \left|\frac{1}{(m-m_0)+i\frac{\Gamma_0}{2}}\right|^2$$

$m_0 = (1662.4 \pm 0.3)$ MeV
$\Gamma_0 = (22.6 \pm 1.5)$ MeV

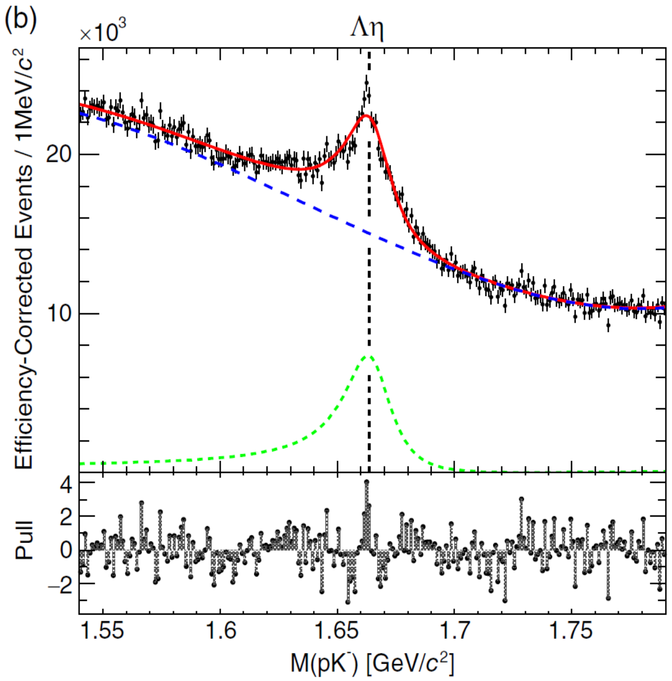
Reduced $\chi^2/\text{ndf} = 328/24$
$= 1.35$



- Not very good especially near the peak.

- Fit to $M(pK^-)$ using BW with complex constant added coherently, leading to constructive interference below the $\Lambda\eta$ threshold and destructive above that.

$$\frac{dN}{dm} \propto \left|BW(m) + re^{i\theta}\right|^2 = \left|\frac{1}{(m-m_0)+i\frac{\Gamma_0}{2}} + re^{i\theta}\right|^2$$



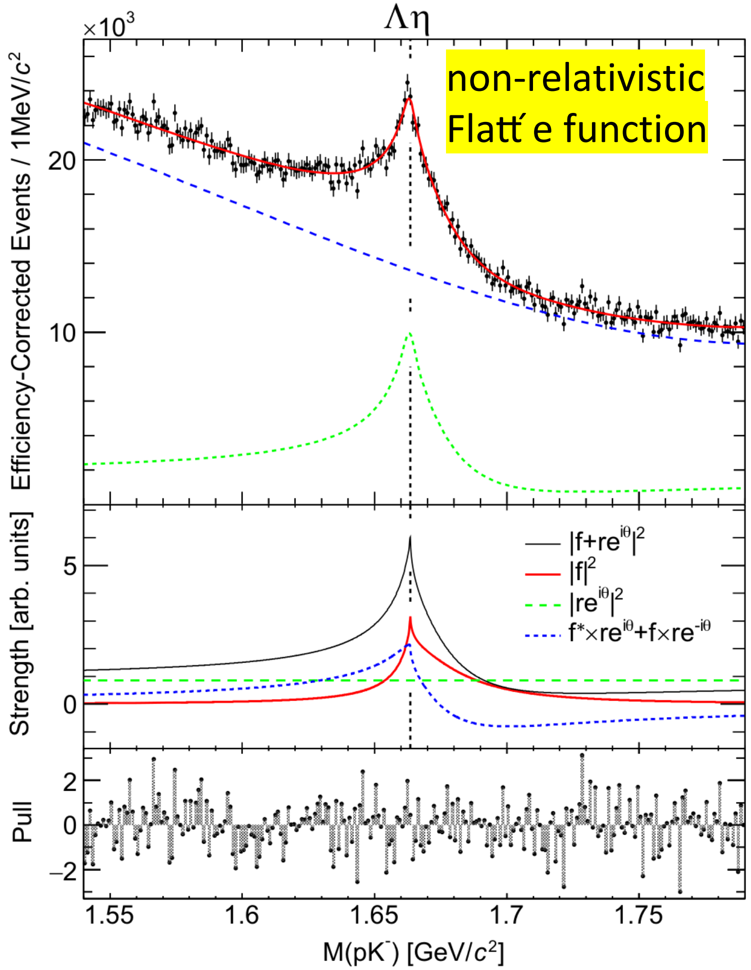
$m_0 = (1665.4 \pm 0.5)$ MeV
$\Gamma_0 = (23.8 \pm 1.2)$ MeV

Reduced $\chi^2/\text{ndf} = 308/243$
$= 1.27$

From the perspective of a cusp at $\Lambda\eta$ threshold

- Another possibility is that the peak structure is a cusp at the $\Lambda\eta$ threshold enhanced by the $\Lambda(1670)$ pole nearby.

$$\frac{dN}{dm} \propto |f(m)|^2 = \left|\frac{1}{m - m_f + \frac{i}{2}(\Gamma' + \bar{g}_{\Lambda\eta} k)}\right|^2$$



- The best fit with χ^2/ndf=1.06 (257/243) is obtained at m_f=1674.4 MeV/c^2.
- The measured: $\Gamma' = (27.2 \pm 1.9^{+5.0}_{-3.9})$ MeV, $\bar{g}_{\Lambda\eta} = \left(258 \pm 23^{+61}_{-75}\right) \times 10^{-3}$

	Our measurement	$\Lambda(1670)$ [PRD 103, 052005 (2021)]
mass	Fix m_f =1674.4 MeV/c^2	$(1674.3 \pm 0.8 \pm 4.9)$MeV/c^2
Total width	$(50.3 \pm 2.9^{+4.2}_{-4.0})$ MeV	$(36.1 \pm 2.4 \pm 4.8)$MeV

- The fit result with the Flatté function to which the constant is coherently added shows the best reduced χ^2/ndf of 1.06 (257/243, p = 0.25), while 1.27 (308/243, p = 3.1×10^{-3}) from the best BW fit.

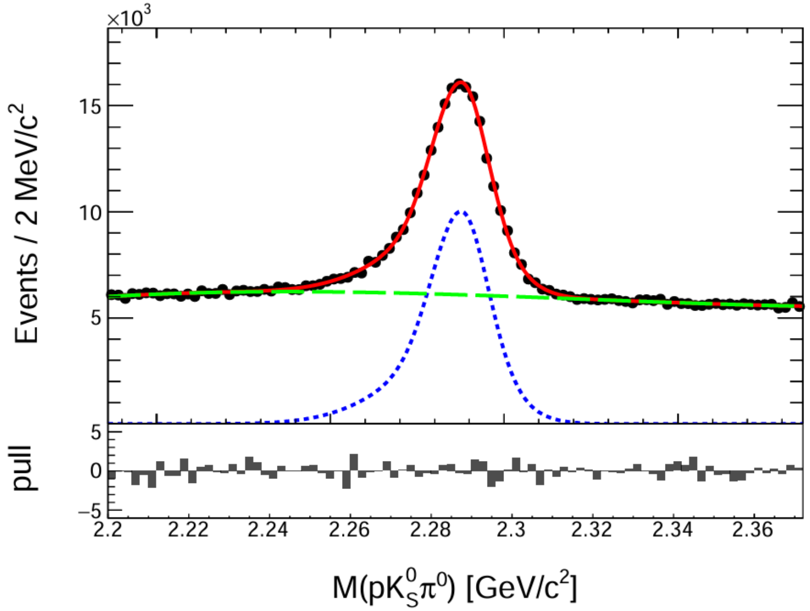
- The best fit explains the structure as a cusp at the $\Lambda\eta$ threshold.
- The obtained parameters are consistent with the known properties of $\Lambda(1670)$.

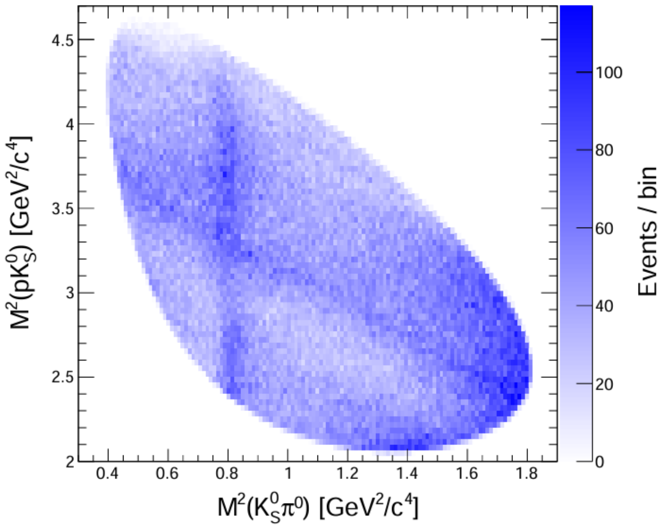
(See Duan, Bayar and Oset for a theoretical interpretation of this result. Phys. Lett. B 857 (2024), 139003)

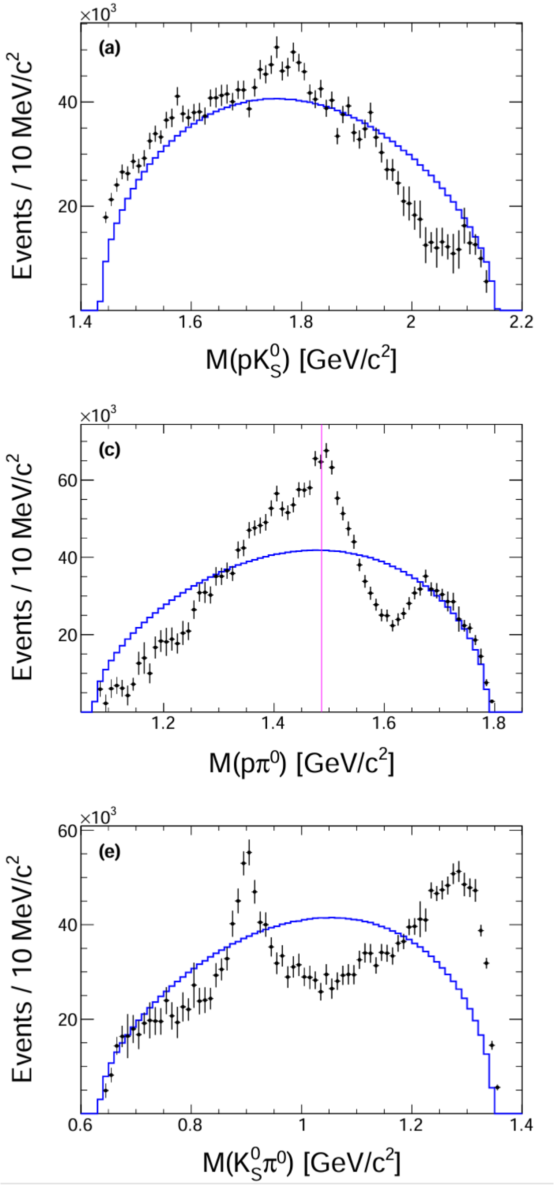
First identification of a threshold cusp in hadrons from the spectrum shape

Peak at $p\eta$ threshold in $\Lambda_c^+ \to pK_S^0\pi^0$

arXiv:2503.04371





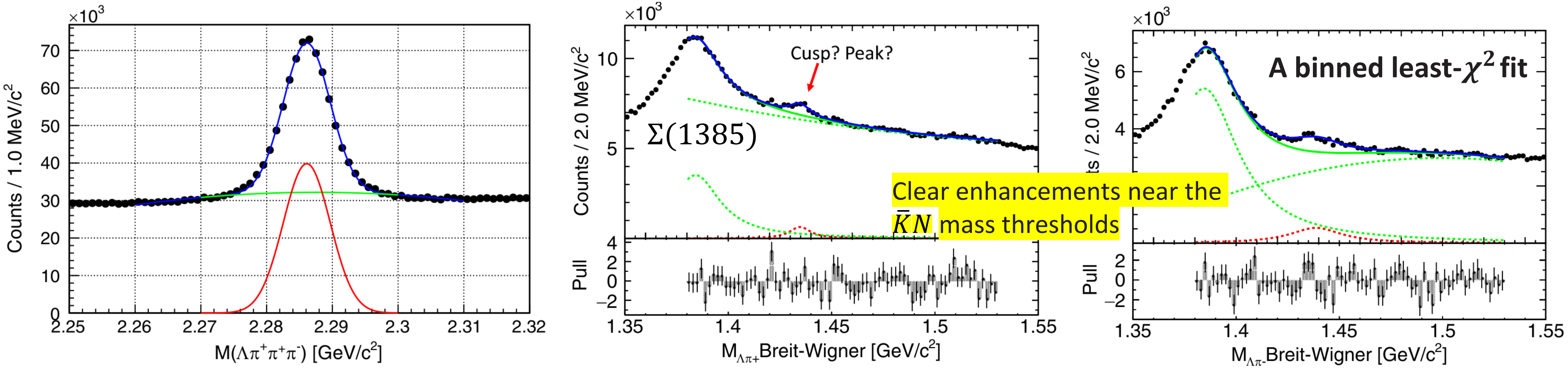


- A clear **peaking structure near the $p\eta$ mass threshold** is evident in the $M(p\pi^0)$ distribution.
- The same effect was observed in the $\Lambda_c^+ \to pK_S^0\eta$ study
- The similarity of this effect and the $\Lambda\eta$ threshold cusp, which was found to be amplified by the $\Lambda(1670)$ in the pK^- system
- Suggesting that the peak near the $p\eta$ threshold may also **be attributed to a threshold cusp enhanced by the N(1535)+.**
- **A further analysis is planned for the near future**

Peak at $\bar{K}N$ threshold in $\Lambda_c^+ \to \Lambda\pi^+\pi^+\pi^-$

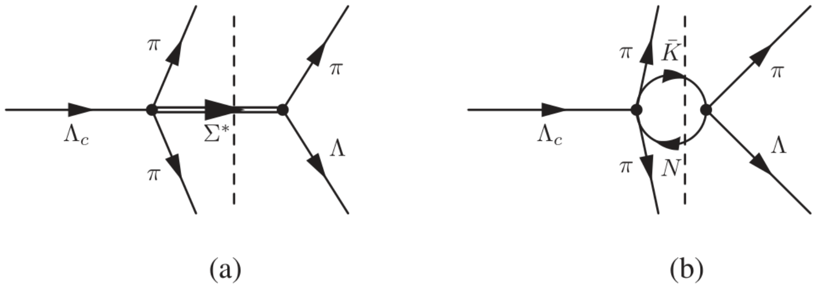
Investigation of the $\Lambda\pi^{\pm}$ substructure [PRL 130, 151903 (2023)]

Cusp candidates are observed in $\Lambda\pi^{\pm}$ invariant mass spectra in $\Lambda_c^+ \to \Lambda\pi^+\pi^+\pi^-$ decay



A binned least-χ^2 fit

Clear enhancements near the $\bar{K}N$ mass thresholds



(a) search for Σ^* resonances

(b) study $\bar{K}N$ rescattering with a cusp

- **To interpret the signals as Σ^* resonances, we use a nonrelativistic Breit-Wigner**

$$f_{\rm BW} = \frac{\Gamma/2}{(E-E_{\rm BW})^2+\Gamma^2/4},$$

Mode	$E_{\rm BW}$ (MeV/c^2)	Γ (MeV/c^2)	χ^2/NDF
$\Lambda\pi^+$	1434.3 ± 0.6	11.5 ± 2.8	74.4/68
$\Lambda\pi^-$	1438.5 ± 0.9	33.0 ± 7.5	92.3/68

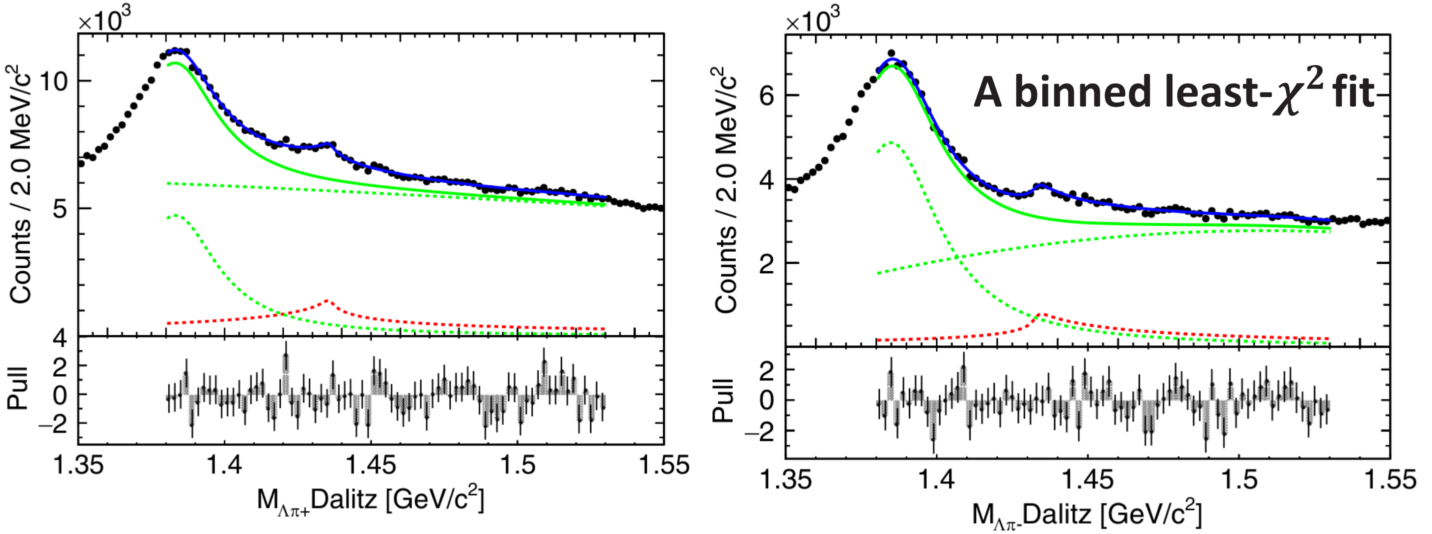
- Significance 7.5(6.2)σ
- This interpretation implies the existence of an exotic state, Σ(1435).

Investigation of the $\Lambda\pi^{\pm}$ substructure

- **Dalitz model (describe a $\bar{K}N$ cusp) [Czech. J. Phys. B32, 1021 (1982)]**
- **$\bar{K}N$ cusp is related to the $\bar{K}N$ scattering length $A = a + ib$ and decay momentum k/|k|.**

$$f_D = \frac{4\pi b}{(1+kb)^2 + (ka)^2}, \qquad E > m_{\bar{K}N}$$
$$= \frac{4\pi b}{(1+\kappa a)^2 + (\kappa b)^2}, \qquad E < m_{\bar{K}N},$$

Mode	a (fm)	b (fm)	χ^2/NDF
$\Lambda\pi^+$	0.48 ± 0.32	1.22 ± 0.83	$68.9/68$
$\Lambda\pi^-$	1.24 ± 0.57	0.18 ± 0.13	$78.1/68$



A binned least-χ^2 fit

Dalitz model gives slightly better χ^2, but the difference is not significant.

- Many theories predict a cusp here.
  - Due to the attraction between $\bar{K}$ and N in the I=1 channel
- Obtained scattering lengths are larger than most theories, but with large uncertainties
  (Also, form factor is ignored.)

- Peak/cusp at $\bar{K}N$ threshold in $\Lambda_c \rightarrow \Lambda\pi^+\pi^+\pi^-$
  - Peak? Cusp?
  - Cannot be identified from the spectrum only due to poor S/N.
- More studies should be done with Belle II and other experiments.

$\boldsymbol{\phi(2170)}$ ***in*** $\boldsymbol{e^+e^- \to \eta\phi}$

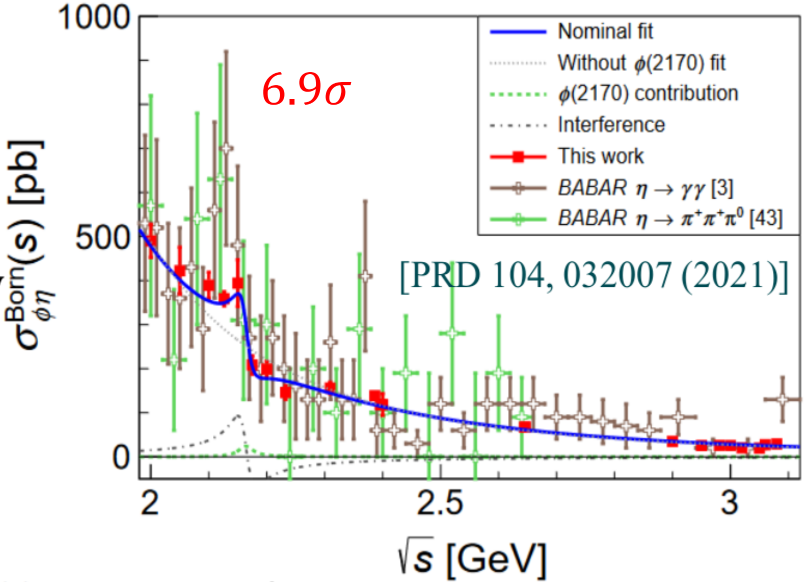
History

- ➢ Babar studied the $e^+e^- \to \eta\phi$ process vis ISR using a 232fb^{-1} data sample and an excess was observed around 2.1 GeV/c^2, called the ϕ''.

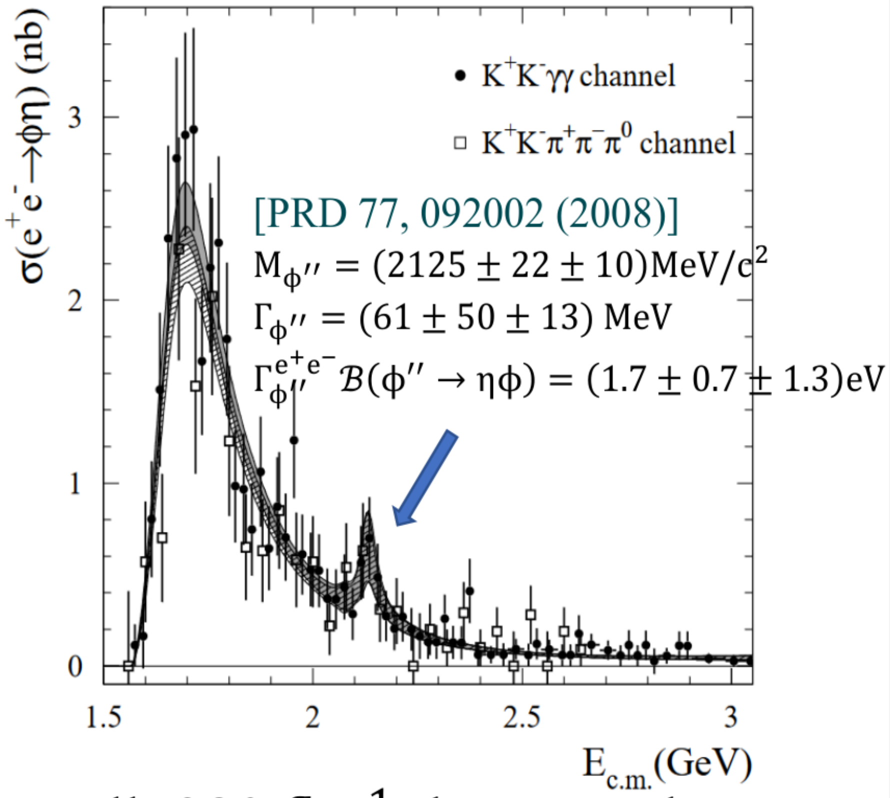
[PRD 76, 092005 (2007), PRD 77, 092002 (2008)]

- ➢ BESIII also measured the Born cross section of $\mathrm{e^+e^-} \to \eta\phi$ and determined the resonant parameters of $\phi(2170)$.

$$M_{\phi(2170)} = (2163.5 \pm 6.2 \pm 3.0)\ \mathrm{MeV}/c^2$$
$$\Gamma_{\phi(2170)} = (31.1^{+21.1}_{11.6} \pm 1.1)\ \mathrm{MeV}$$
$$\Gamma^{e^+e^-}_{\phi(2170)}\mathcal{B}(\phi(2170) \to \phi\eta) = (0.24^{+0.12}_{-0.07})\ \mathrm{eV}$$
$$\text{or } (10.11^{+3.87}_{-3.13})\ \mathrm{eV}$$



[PRD 104, 032007 (2021)]



[PRD 77, 092002 (2008)]

$M_{\phi''} = (2125 \pm 22 \pm 10)\mathrm{MeV}/c^2$

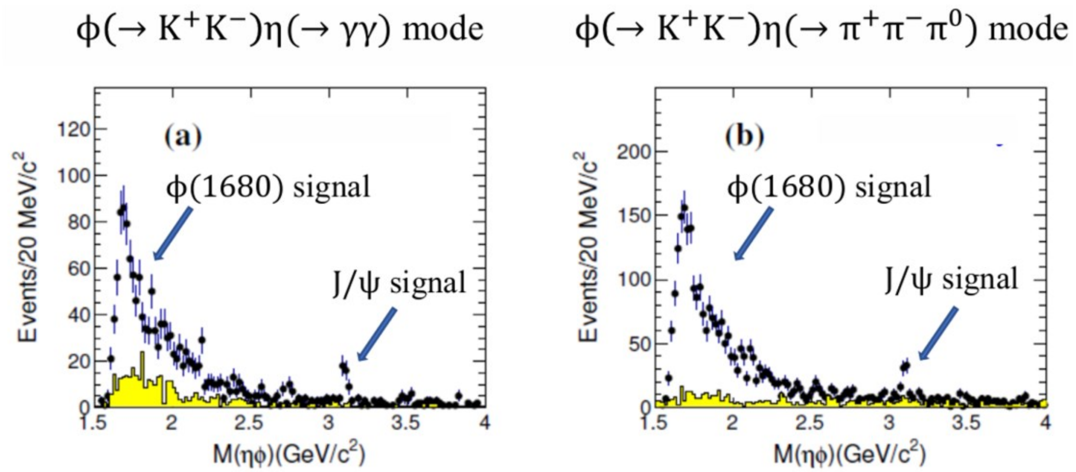
$\Gamma_{\phi''} = (61 \pm 50 \pm 13)$ MeV

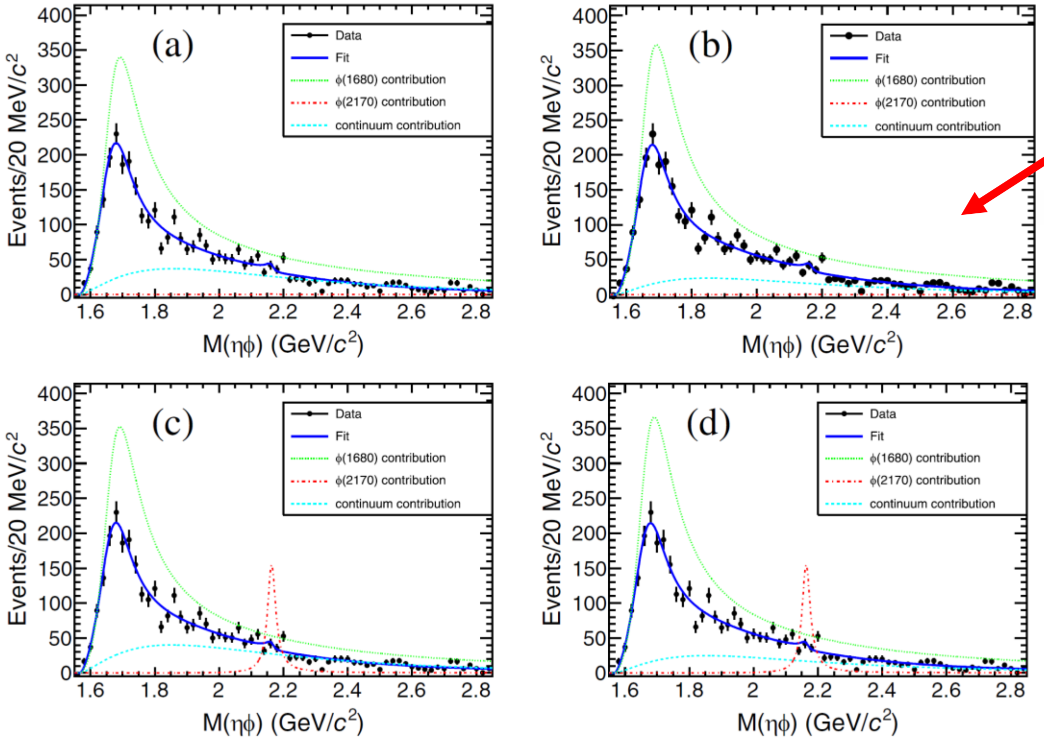
$\Gamma^{e^+e^-}_{\phi''}\mathcal{B}(\phi'' \to \eta\phi) = (1.7 \pm 0.7 \pm 1.3)\mathrm{eV}$

- ➢ In this analysis, we study the process $\mathrm{e^+e^-} \to \eta\phi$ via ISR using all 980 fb^{-1} data sample.

M($\eta\phi$) distributions from ISR production

[PRD 107, 012006 (2023)]





- **No significant $\phi(2170)$ signal is seen**
- **Clear J/ψ and $\phi(1680)$ signals.**

Perform unbinned maximum likelihood fit to the $M(\eta\phi)$

- The parametrization for the cross section of $e^+e^- \to \eta\phi$

phase space of final state   ϕ(1680) amplitude

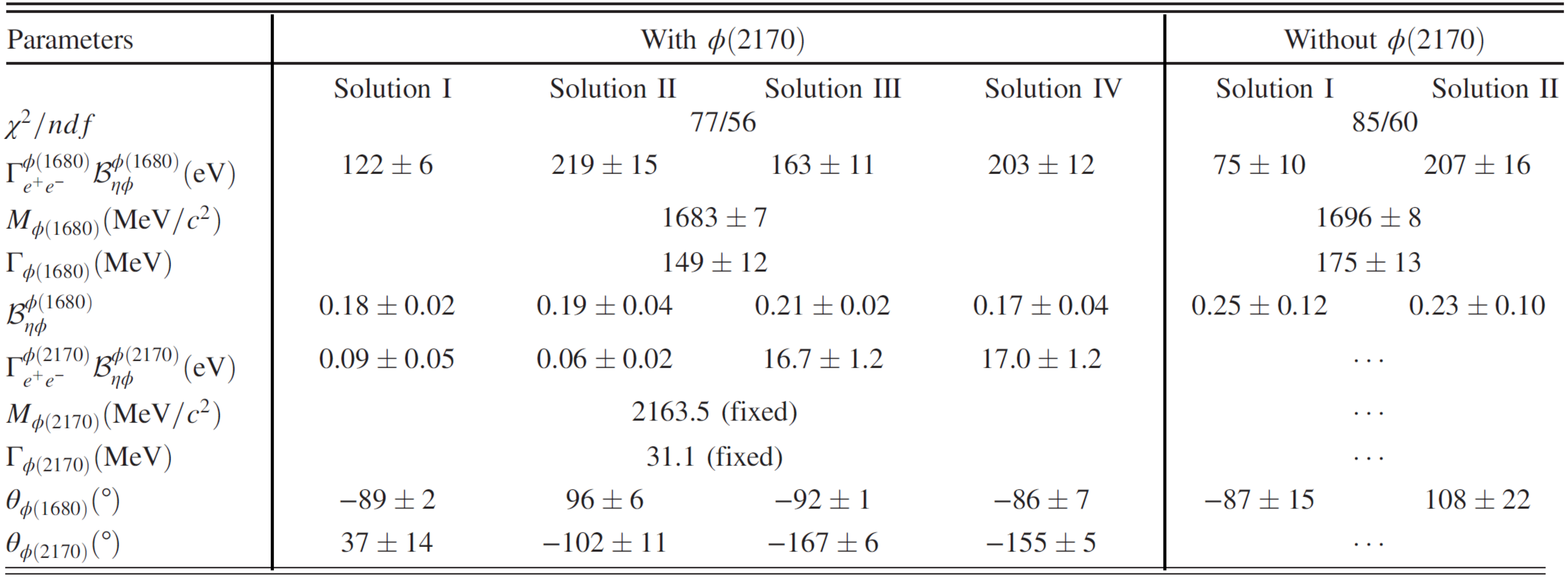
$$\sigma_{\eta\phi}(\sqrt{s}) = 12\pi P_{\eta\phi}(\sqrt{s})|A_{\eta\phi}^{n.r.}(\sqrt{s}) + A_{\eta\phi}^{\phi(1680)}(\sqrt{s}) + A_{\eta\phi}^{\phi(2170)}(\sqrt{s})|^2$$

Non-resonant contribution   ϕ(2170) amplitude

- We assume that the $\phi(2170)$ exists, and fix its mass and width based on the values measured by BESIII. There are four solutions of equivalent quality, having the same $M_{\phi(1680)}$ and $\Gamma_{\phi(1680)}$.
- The statistical significance of ϕ(2170) is only 1.7σ.

Extraction of Resonant Parameters

- Fit results with both ϕ(1680) and ϕ(2170) included, or without ϕ(2170). The mass and width of ϕ(2170) are fixed from the prior BESIII measurement.



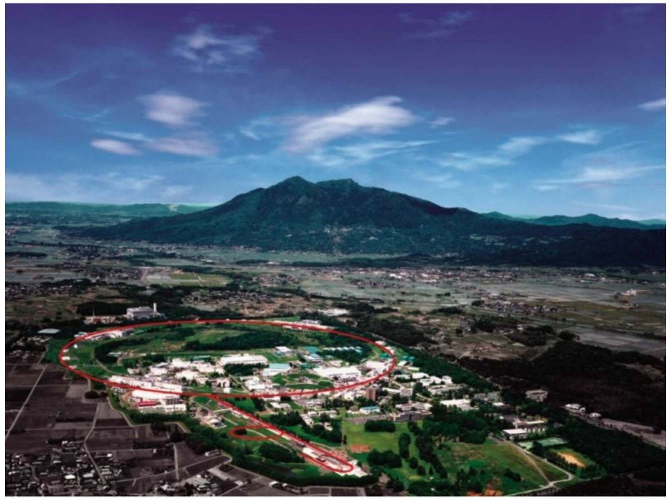
Parameters	With $\phi(2170)$				Without $\phi(2170)$	
	Solution I	Solution II	Solution III	Solution IV	Solution I	Solution II
χ^2/ndf	77/56				85/60	
$\Gamma^{\phi(1680)}_{e^+e^-}\mathcal{B}^{\phi(1680)}_{\eta\phi}$ (eV)	122 ± 6	219 ± 15	163 ± 11	203 ± 12	75 ± 10	207 ± 16
$M_{\phi(1680)}$ (MeV/c^2)	1683 ± 7				1696 ± 8	
$\Gamma_{\phi(1680)}$ (MeV)	149 ± 12				175 ± 13	
$\mathcal{B}^{\phi(1680)}_{\eta\phi}$	0.18 ± 0.02	0.19 ± 0.04	0.21 ± 0.02	0.17 ± 0.04	0.25 ± 0.12	0.23 ± 0.10
$\Gamma^{\phi(2170)}_{e^+e^-}\mathcal{B}^{\phi(2170)}_{\eta\phi}$ (eV)	0.09 ± 0.05	0.06 ± 0.02	16.7 ± 1.2	17.0 ± 1.2	...	
$M_{\phi(2170)}$ (MeV/c^2)	2163.5 (fixed)				...	
$\Gamma_{\phi(2170)}$ (MeV)	31.1 (fixed)				...	
$\theta_{\phi(1680)}$ (°)	-89 ± 2	96 ± 6	-92 ± 1	-86 ± 7	-87 ± 15	108 ± 22
$\theta_{\phi(2170)}$ (°)	37 ± 14	-102 ± 11	-167 ± 6	-155 ± 5	...	

- The upper limits at 90% C.L. on the $\Gamma^{e^+e^-}_{\phi(2170)}\mathcal{B}^{\eta\phi}_{\phi(2170)}$ are determined to be < 0.17eV or < 18.6eV, both are consistent the BESIII measurement.

[PRD 104, 032007 (2021)]

Summary

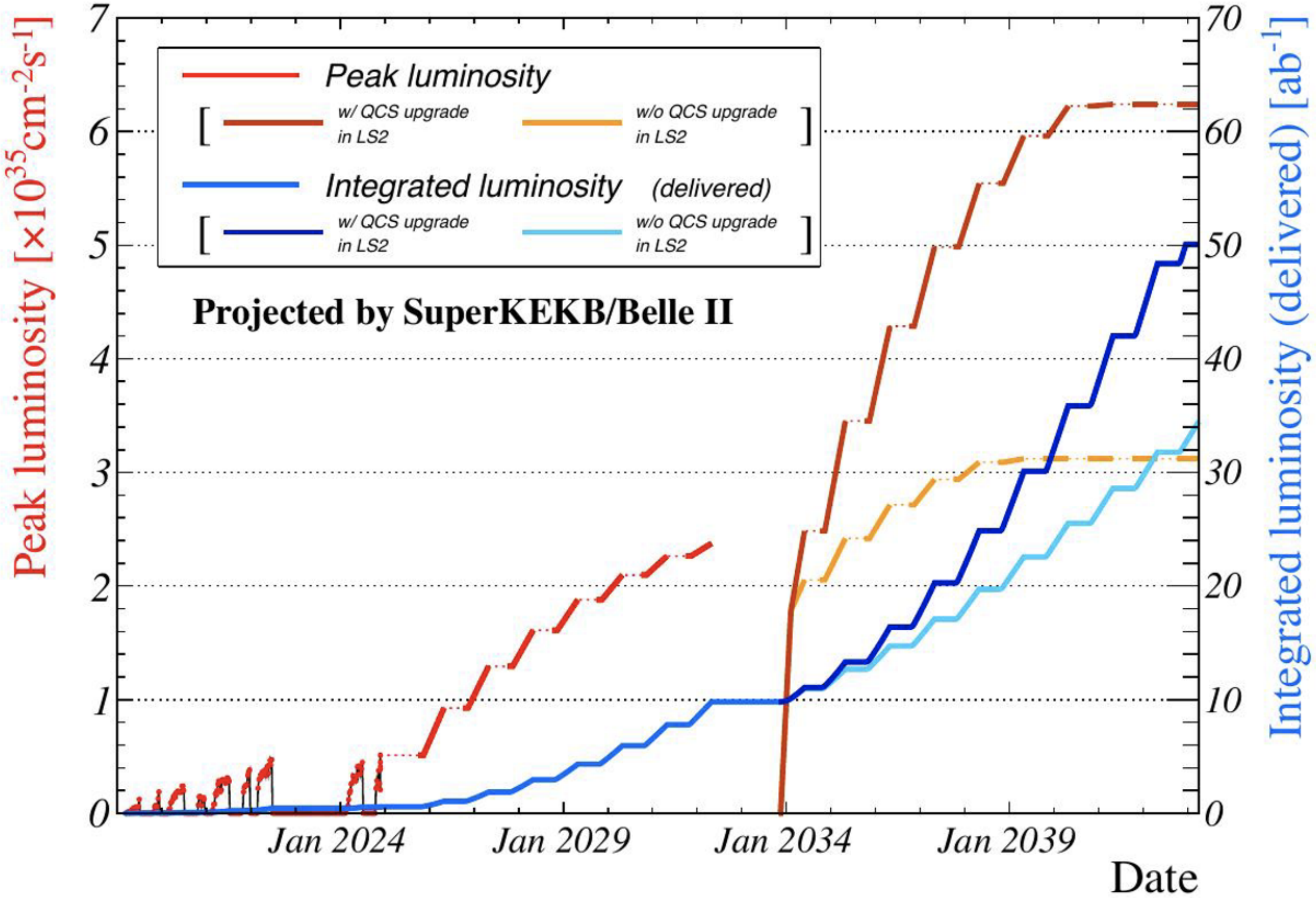
- Belle II and Belle hold a unique data sample. Some interesting measurement has been already performed in light hadron, such as
  - ➢ Discovered of a new $\Omega(2012)$ three-body decay, first determined a threshold cusp in experiments, measured resonant parameters of $\phi(1680)$.
- Only 1% of target luminosity collected so far. Stay tuned for more exciting results from Belle & Belle II.



Thanks for your attention!

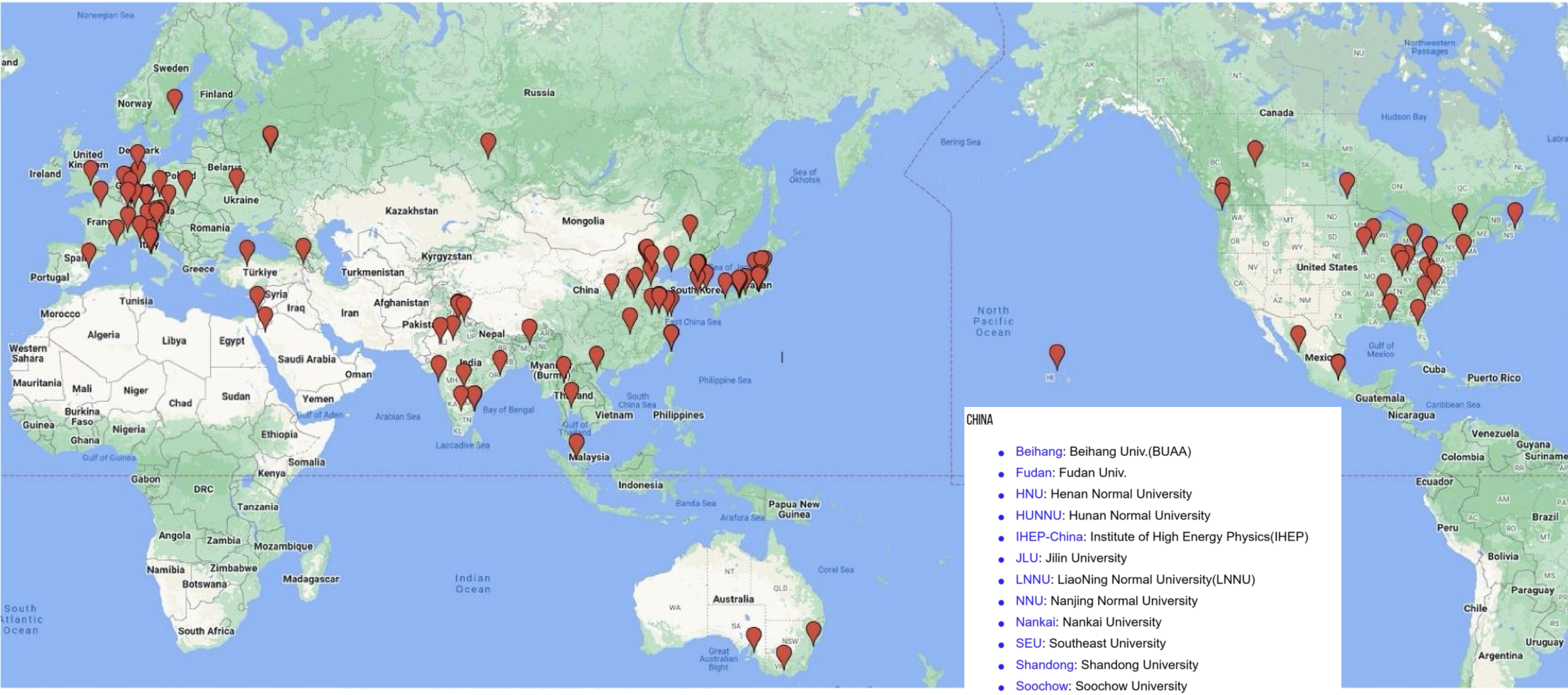
Backup slides

Data-taking plan at Belle II



- *Until 2026, about 1 ab^{-1} data, comparable to Belle*
- *Until 2029, about 4 ab^{-1} data.*

International Belle II collaboration

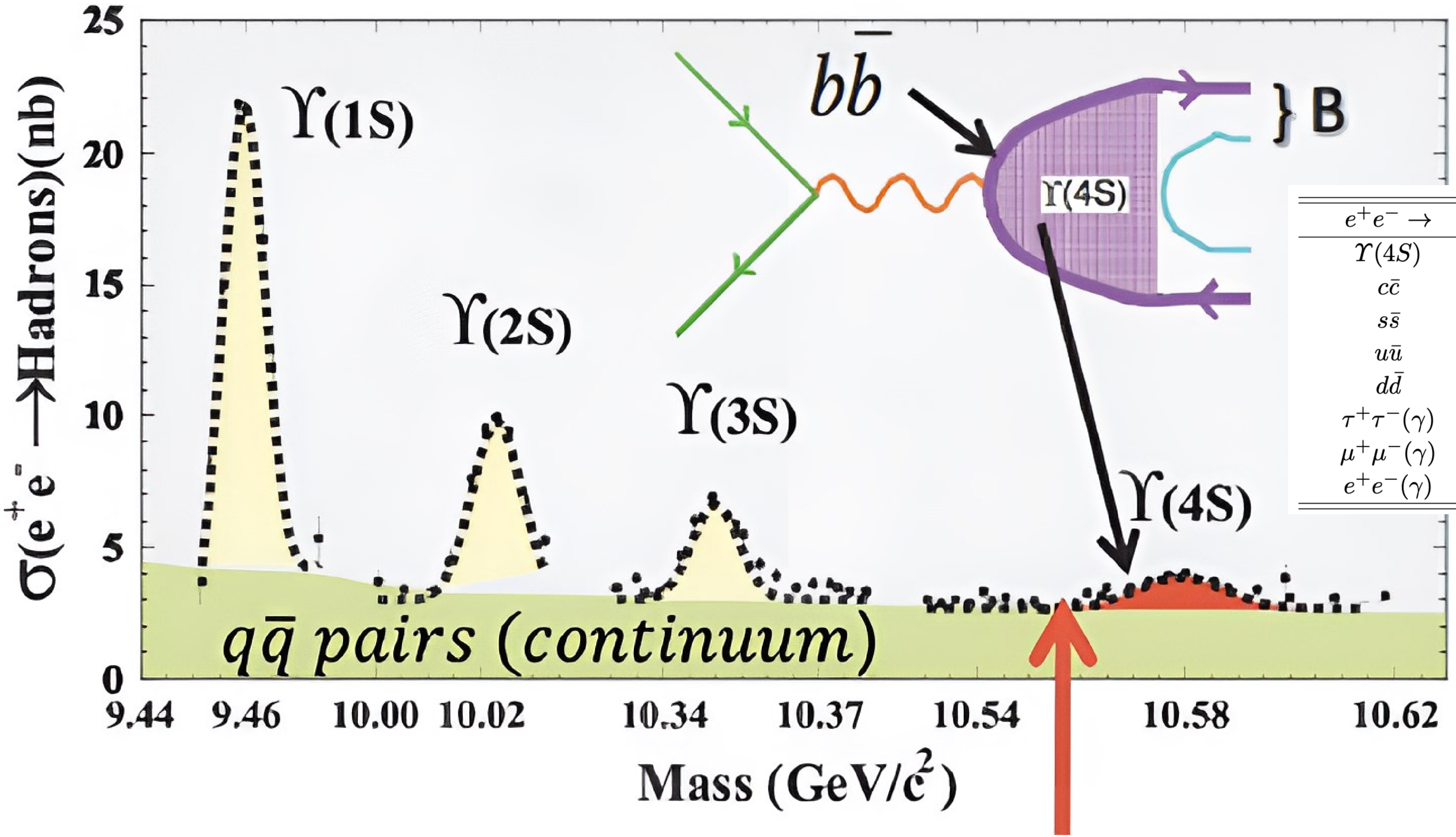


CHINA

- Beihang: Beihang Univ.(BUAA)
- Fudan: Fudan Univ.
- HNU: Henan Normal University
- HUNNU: Hunan Normal University
- IHEP-China: Institute of High Energy Physics(IHEP)
- JLU: Jilin University
- LNNU: LiaoNing Normal University(LNNU)
- NNU: Nanjing Normal University
- Nankai: Nankai University
- SEU: Southeast University
- Shandong: Shandong University
- Soochow: Soochow University
- USTC: Univ. of Science and Technology of China(USTC)
- XJTU: Xi'an Jiaotong University
- ZZU: Zhengzhou University

Belle II now has grown to 1229 researchers from 28 countries/regions.

Belle II physics



$e^+e^- \rightarrow$	Cross section [nb]
$\Upsilon(4S)$	1.05 ± 0.10
$c\bar{c}$	1.30
$s\bar{s}$	0.38
$u\bar{u}$	1.61
$d\bar{d}$	0.40
$\tau^+\tau^-(\gamma)$	0.919
$\mu^+\mu^-(\gamma)$	1.148
$e^+e^-(\gamma)$	300 ± 3

Belle II physics

The Belle II Physics Book: [PTEP 2019 (2019) 12, 123C01]

$\Upsilon(10753)$ study

Dark Higgs

τ physics

A_{CP} in Charm

Charmed baryon

CKM matrix element

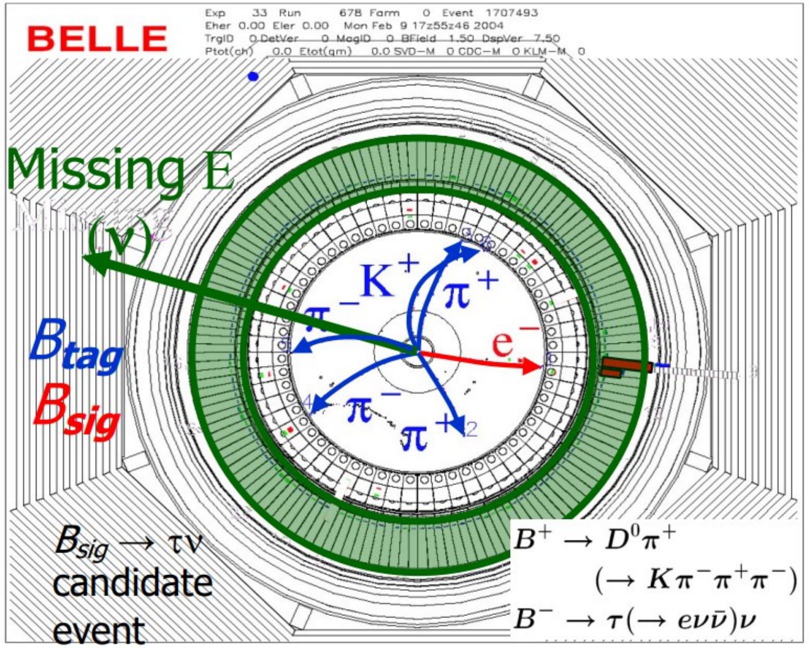
Lepton-Flavor universality

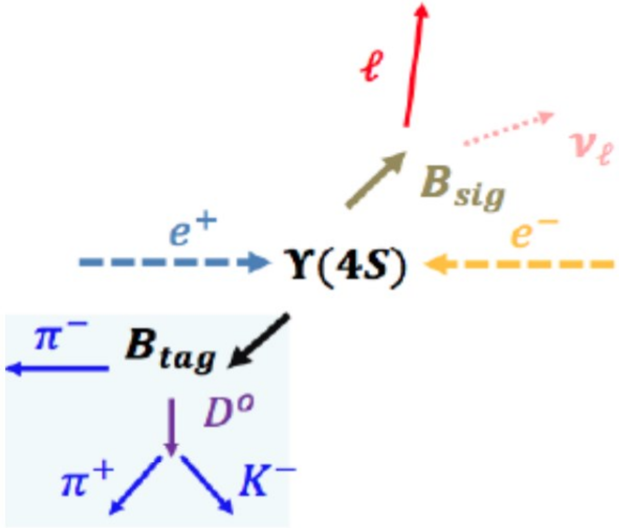
B rare decays

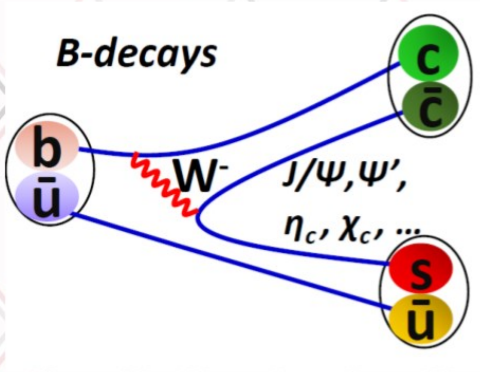
Unique capabilities of Belle/Belle II

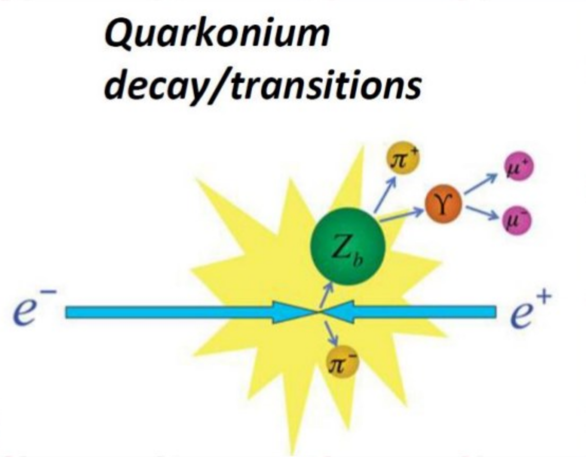
- **Beam energy constraint**
- **Clean experimental environment:** high B, D, K, τ lepton reconstruction efficiency
- Long lived particles (e.g. **K_S**), **π^0s** and **photons** well reconstructed
- Capability of **inclusive measurements**
- **BB produced in quantum correlated state:** high flavour tagging effective efficiency (30% vs 5%@LHCb)
- The **full reconstruction of one B (B_{tag})** constraints the 4-momentum of the other B (B_{sig})
- Reconstruction of **channels with missing energy**

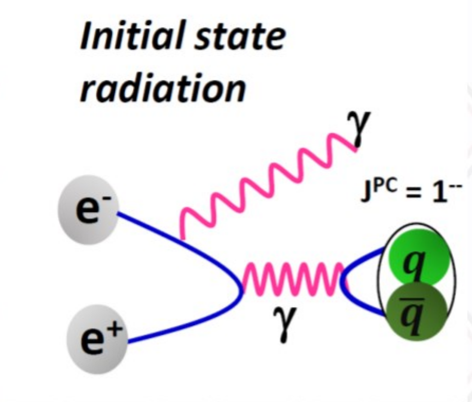
$$\boxed{p_\nu = p_{e^+e^-} - p_{B_{tag}} - p_{B_{sig}}}$$



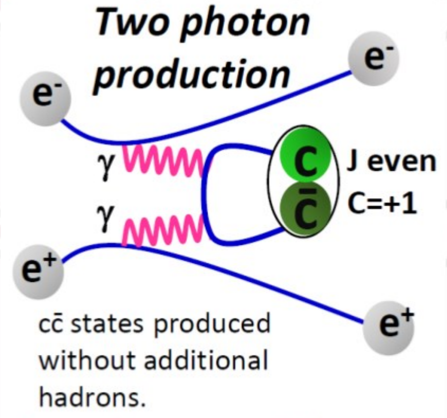



B-decays

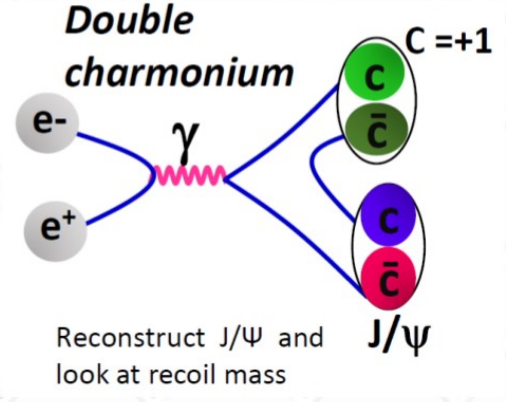

Quarkonium decay/transitions


Initial state radiation

- ISR allows to scan the mass of the final state in a very clean environment
- Photon quantum numbers must be conserved in final states to be created


Two photon production

cc̄ states produced without additional hadrons.


Double charmonium

Reconstruct J/Ψ and look at recoil mass

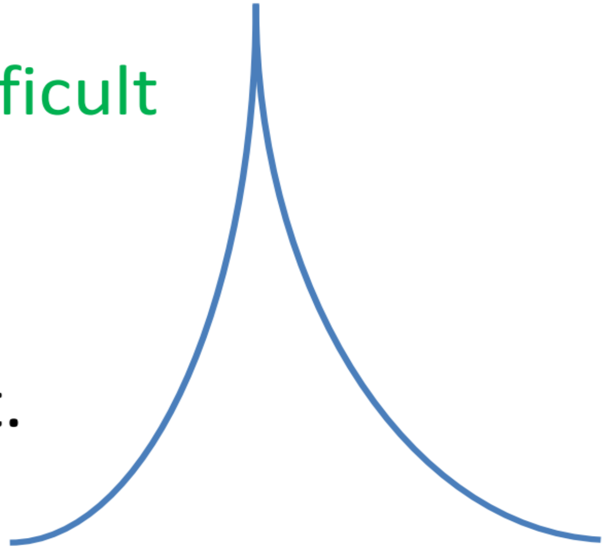
Also, particle production in fragmentation q→hX

- Different sub processes access different J^{PC} states and mass ranges

- Very rare process of creating double charmonium pairs, but dominant prompt J/ψ production process

Virtual state & threshold cusp

- Molecular type state -- when interaction is not strong enough to make a bound state, there would be a virtual state.
  - E < 0 (bound??), but in different Riemann sheet
  - Appears as threshold cusp instead of usual Breit-Wigner peak (in the narrow sense).
  - However, identification is rather difficult due to experimental resolution
- Are there really such states?
  - Pointing shape is not confirmed yet.



"threshold cusp" 指在物理过程的**能量阈值**（threshold）附近，某些观测量（如散射截面、衰变率、能谱分布等）出现**非解析的尖点状突变**（cusp）。这种现象通常源于新反应通道的打开或多粒子动力学的临界行为。

Fitting the M($\eta\phi$)

- The parametrization for the cross section of $e^+e^- \to \eta\phi$

$$\sigma_{\eta\phi}(\sqrt{s}) = 12\pi\mathcal{P}_{\eta\phi}(\sqrt{s})\left|A_{\eta\phi}{}^{n.r.}(\sqrt{s}) + A_{\eta\phi}{}^{\phi(1680)}(\sqrt{s}) + A_{\eta\phi}{}^{\phi(2170)}(\sqrt{s})\right|^2$$

$A_{\eta\phi}{}^{n.r.}(\sqrt{s}) = a_0/s^{a_1}$ is used to describe the non-resonant contribution

$$A_{\eta\phi}^{\phi(1680)}(\sqrt{s}) = \sqrt{\mathcal{B}_{\phi(1680)}^{\eta\phi}\Gamma_{\phi(1680)}^{e^+e^-}}\frac{\sqrt{\Gamma_{\phi(1680)}/\mathcal{P}_{\eta\phi}(M_{\phi(1680)}^2)}e^{i\theta_{\phi(1680)}}}{M_{\phi(1680)}^2 - s - i\sqrt{s}\Gamma_{\phi(1680)}(\sqrt{s})}$$

$$\Gamma_{\phi(1680)}(\sqrt{s}) = \Gamma_{\phi(1680)}[\frac{\mathcal{P}_{KK^*(892)}(\sqrt{s})}{\mathcal{P}_{KK^*(892)}(M_{\phi(1680)})}\mathcal{B}_{\phi(1680)}^{KK^*(892)} + \frac{\mathcal{P}_{\eta\phi}(\sqrt{s})}{\mathcal{P}_{\eta\phi}(M_{\phi(1680)})}\mathcal{B}_{\phi(1680)}^{\eta\phi}$$
$$+(1 - \mathcal{B}_{\phi(1680)}^{\eta\phi} - \mathcal{B}_{\phi(1680)}^{KK^*(892)})].$$

$\mathcal{B}_{\phi(1680)}{}^{KK^*(892)} \approx 2 \times \mathcal{B}_{\phi(1680)}{}^{\eta\phi}$ from Ref. [1] directly

$$A_{\eta\phi}^{\phi(2170)}(s) = \sqrt{\mathcal{B}_{\phi(2170)}^{\eta\phi}\Gamma_{\phi(2170)}^{e^+e^-}}\frac{\sqrt{\Gamma_{\phi(2170)}/\mathcal{P}_{\eta\phi}(M_{\phi(2170)}^2)}e^{i\theta_{\phi(2170)}}}{M_{\phi(2170)}^2 - s - i\sqrt{s}\Gamma_{\phi(2170)}} \cdot \frac{B(p)}{B(p')}$$

k_3 and κ_3 are

PRD 81, 094028 (2010)

$$k_3(M) = \frac{g_l}{2\pi\mu_p}\int_0^{\sqrt{2\mu_p q(M)}} p^2 dp \times \frac{(q(M) - \frac{p^2}{2\mu_p})^{(2l+1)/2}}{(M_R - q(M) + \frac{p^2}{2\mu_p})^2 + \frac{g_l^2}{4}(q(M) - \frac{p^2}{2\mu_p})^{2l+1}},$$

$$\kappa_3(M) = \kappa(q(M)) + \kappa'(q(M)) - \kappa(q(m)) - \kappa'(q(m)),$$

$$\kappa(M) = \frac{1}{\pi\mu_p}\int_0^{\infty} p^2 dp \times \frac{M_R - q(M) + \frac{p^2}{2\mu_p}}{(M_R - q(M) + \frac{p^2}{2\mu_p})^2 + \frac{g_l^2}{4}(q(M) - \frac{p^2}{2\mu_p})^{2l+1}},$$

$$\kappa'(M) = -\frac{g_l}{2\pi\mu_p}\int_{\sqrt{2\mu_p q(M)}}^{\infty} p^2 dp \times \frac{(\frac{p^2}{2\mu_p} - q(M))^{(2l+1)/2}}{(M_R - q(M) + \frac{p^2}{2\mu_p})^2 + \frac{g_l^2}{4}(q(M) - \frac{p^2}{2\mu_p})^{2l+1}}.$$

Here, $q(M) = M(\Xi\pi\overline{K}) - m_\Xi - m_\pi - m_K$, $q(m) = m_{\Omega(2012)^-} - m_\Xi - m_\pi - m_K$, $\mu_p = \frac{m_K(m_\pi+m_\Xi)}{m_\Xi+m_\pi+m_K}$ is the reduced mass of the $\Xi\overline{K}$ system, $M_R = m_{\Xi(1530)} - m_\Xi - m_\pi$ is the mass of the unstable constituent, the coupling g_l is $\Gamma_R/E_R^{l+1/2}$ (Γ_R is the width of $\Xi(1530)$), the orbital angular momentum of $\overline{K}$ in the $\Xi(1530)\overline{K}$ system is $l = 1$, and p is the $\overline{K}$ momentum in the $\Xi(1530)\overline{K}$ center-of-mass system.

The functions k_2 and κ_2 are identical to k_3 and κ_3 with $\Xi(1530)$ replaced with Ξ, followed by $\Xi \longrightarrow \Lambda\pi$.

From the perspective of a cusp at $\Lambda\eta$ threshold

- ➢ Another possibility is that the peak structure is a cusp at the $\Lambda\eta$ threshold enhanced by the $\Lambda(1670)$ pole nearby.

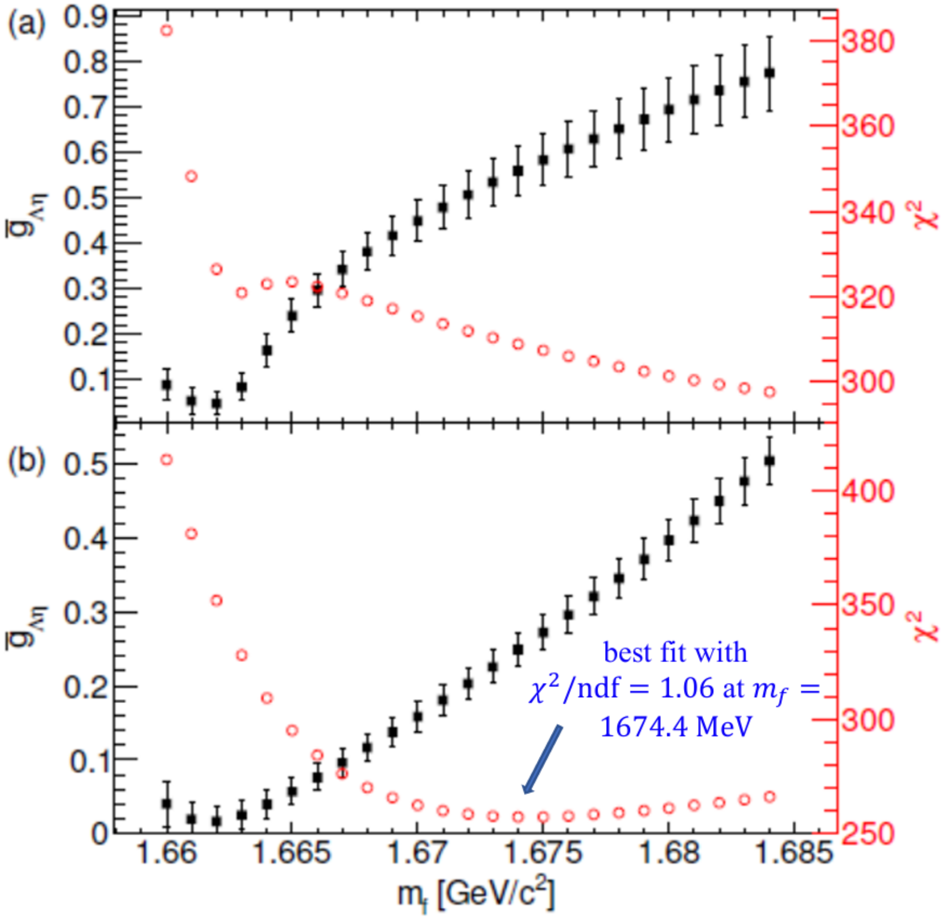
- ➢ We fit the efficiency-corrected $\mathrm{M}(\mathrm{pK}^-)$distribution using a non-relativistic Flatté function [PLB, 63, 224 (1976), EPJA, 23, 523 (2005)]:

$$\boxed{\frac{\mathrm{dN}}{\mathrm{dm}} \propto |\mathrm{f}(\mathrm{m})|^2 = \left|\frac{1}{\mathrm{m}-\mathrm{m_f}+\frac{i}{2}(\Gamma'+\bar{g}_{\Lambda\eta}k)}\right|^2}$$

- $\mathrm{m_f}$ is a parameter corresponding to the nominal mass of $\Lambda(1670)$.
- Γ' is a parameter for the sum of the partial widths of the decay modes other than Λη, and is approximated as a constant.
- k is the decay momentum in the Λη channel, and $\bar{g}_{\Lambda\eta}k$ represents the partial decay width of the Λη channel.

- ➢ We fix $\mathrm{m_f}$ when we perform a fit and repeat the fit with various $\mathrm{m_f}$ values.

- ➢ We take into account an interference with another S-wave amplitude such as a tail of $\Lambda(1405)$. We perform a binned least-χ^2 fit with the combined function, $\frac{\mathrm{dN}}{\mathrm{dm}} \propto \left|\mathrm{f}(\mathrm{m})+\mathrm{re}^{\mathrm{i}\theta}\right|^2$.

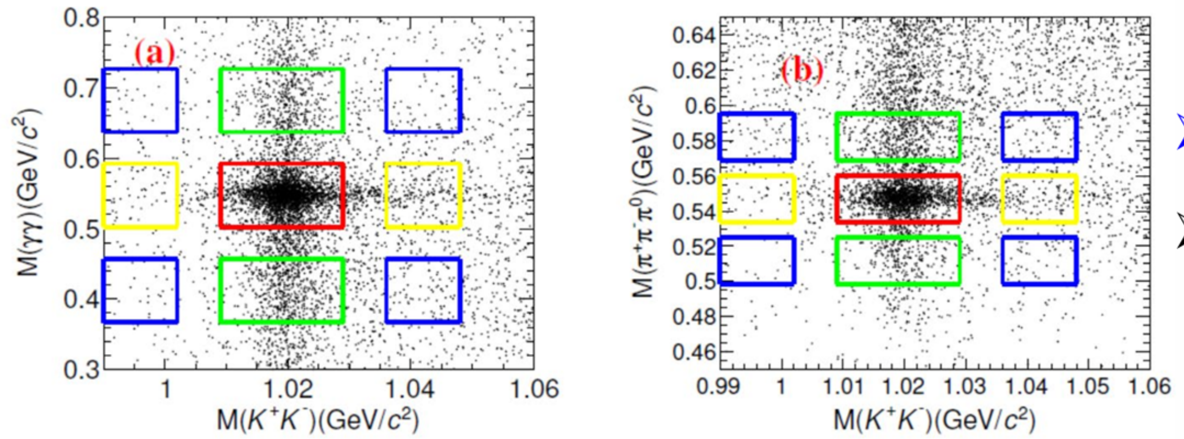


best fit with $\chi^2/\mathrm{ndf} = 1.06$ at $m_f =$ 1674.4 MeV

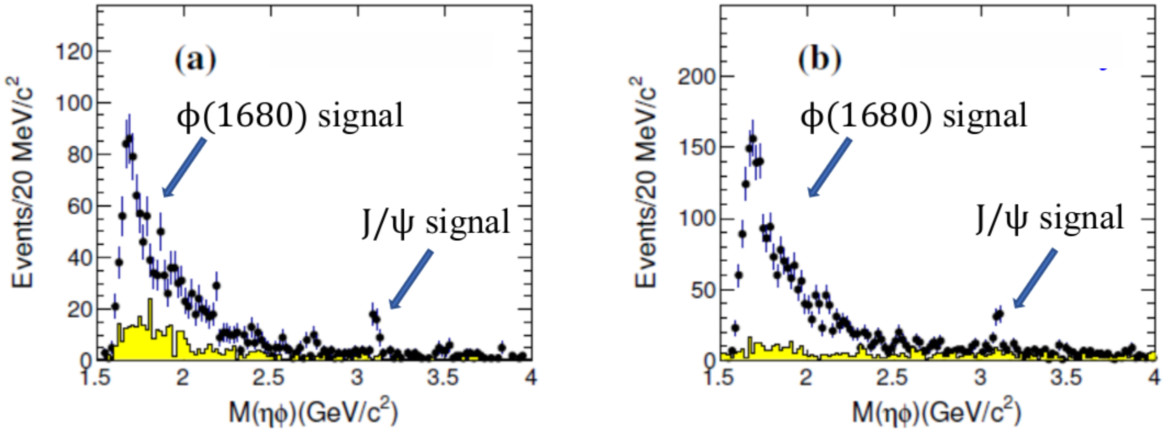
Figure :$\bar{g}_{\Lambda\eta}k$ and χ^2 from Flatté model (a) without and (b) with the interference as a function of fixed $\mathrm{m_f}$.

M($\eta\phi$) distributions from ISR production

[PRD 107, 012006 (2023)]



$\phi(\to K^+K^-)\eta(\to\gamma\gamma)$ mode $\quad$ $\phi(\to K^+K^-)\eta(\to\pi^+\pi^-\pi^0)$ mode

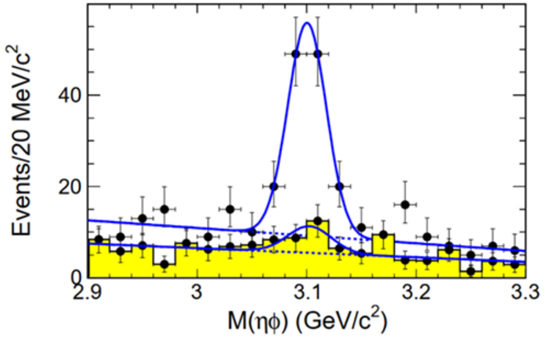


- The number of the ηϕ signal events we obtained is about seven times of that in the previous work. [PRD 77, 092002 (2008)]
- No significant ϕ(2170) signal is seen.
- There are clear J/ψ signals in both the $\pi^+\pi^-\pi^0$ mode and the $\gamma\gamma$ mode, and the branching fraction are determined to be

$$\mathcal{B}(J/\psi\to\eta\phi)=\frac{N_{\mathrm{sig}}^{\mathrm{fit}}}{\sigma_{\mathrm{ISR}}^{\mathrm{prod}}\times L\times\epsilon\times\mathcal{B}(\phi\to K^+K^-)\times\mathcal{B}(\eta\to\gamma\gamma/\pi^+\pi^-\pi^0)}$$

$$=(0.71\pm0.10\pm0.05\,)\times10^{-3},$$

which agrees well with the world average value $(0.74\pm0.08)\times10^{-3}$.



Fit to J/ψ signal

Cross section of $e^+e^- \rightarrow \eta\phi$

- The cross section of $e^+e^- \rightarrow \eta\phi$ for each $M_{\eta\phi}$ bin is calculated according to

$$\boxed{\sigma_i = \frac{n_i^{obs} - n_i^{bkg}}{L_i \times \Sigma_j \epsilon_{ij} \mathcal{B}_j}}$$

- The cross sections for $e^+e^- \rightarrow \eta\phi$ are around 2.6 nb and 0.4 nb at the $\phi(1680)$ and $\phi(2170)$ peaks, respectively.

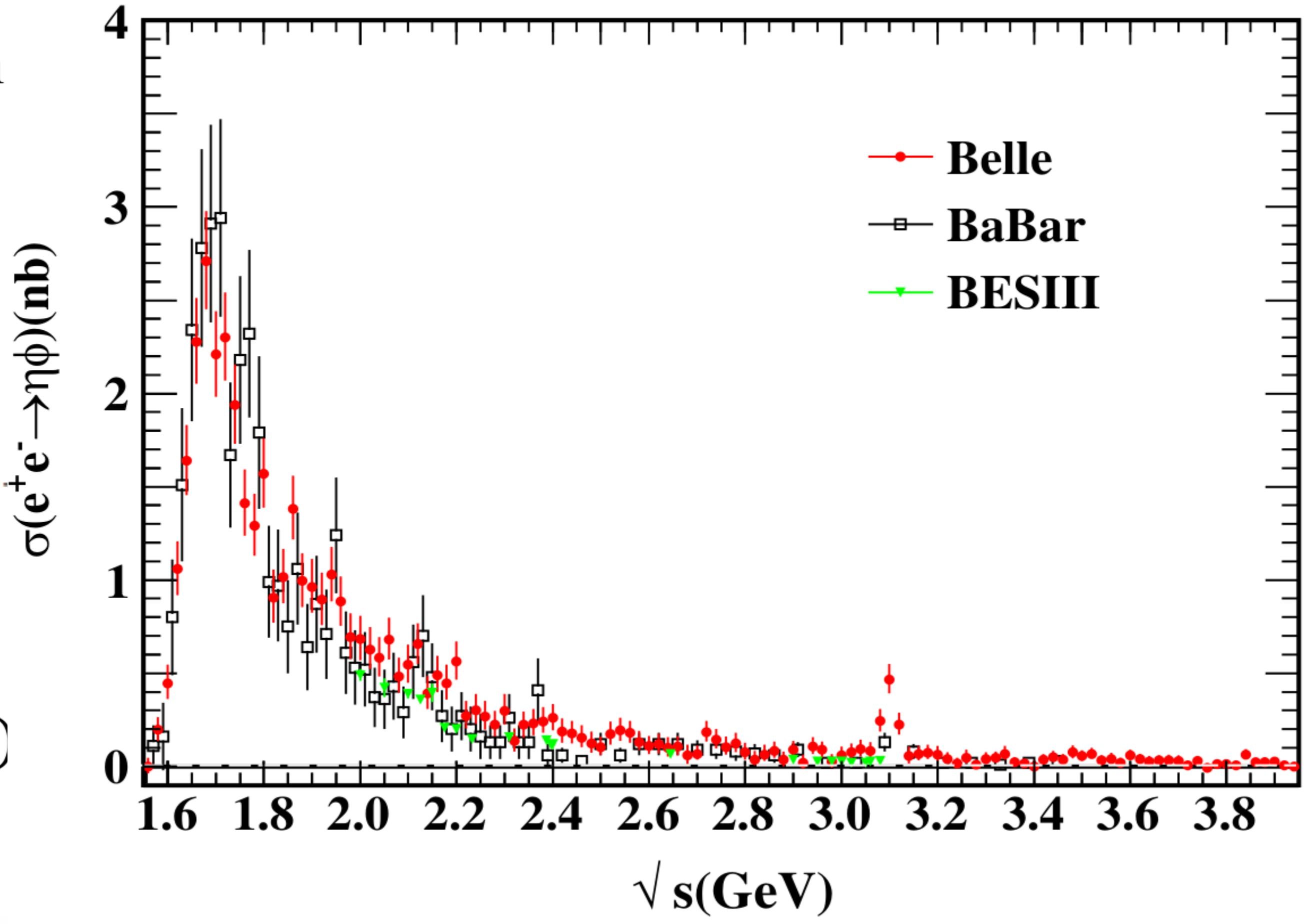


Figure: Cross section of $e^+e^- \rightarrow \eta\phi$ from threshold to 3.95 GeV.