



# BESIII上轻介子衰变的研究进展



2025年轻强子专题研讨会 河南 安阳 5月8日-12日



# $\eta$ Physics

#### **Standard Model Tests:**

- Chiral symmetry and anomalies
- Extract  $\eta \eta'$  mixing angle and quark mass ratio
- Theory inputs to HLbL for  $(g-2)_{\mu}$
- QCD scalar dynamics

#### **Fundamental Symmetry Tests:**

- C, CP violations
- P, CP violations
- Lepton flavor violations

#### **BSM Physics in Dark Sector:**

- Vector bosons (B boson, dark photon and X boson)
- Dark scalars
- Pseudoscalars (ALPs)
- BSM weak decays

	•
Channel	Expt. branching rat
$\eta  ightarrow 2\gamma$	39.41(20)%
$\eta \rightarrow 3\pi^0$	32.68(23)%
$\eta \to \pi^0 \gamma \gamma$	$2.56(22) \times 10^{-4}$
$\eta \rightarrow \pi^0 \pi^0 \gamma \gamma$	$< 1.2 \times 10^{-3}$
$\eta  ightarrow 4\gamma$	${<}2.8 imes10^{-4}$
$\eta  ightarrow \pi^+\pi^-\pi^0$	22.92(28)%
$\eta  ightarrow \pi^+\pi^-\gamma$	4.22(8)%
	_
$\eta  ightarrow \pi^+\pi^-\gamma\gamma$	$<2.1 \times 10^{-3}$
$\eta  ightarrow e^+ e^- \gamma$	$6.9(4) \times 10^{-3}$
$\eta  ightarrow \mu^+ \mu^- \gamma$	$3.1(4) \times 10^{-4}$
$\eta  ightarrow e^+e^-$	$<7 \times 10^{-7}$
$\eta  ightarrow \mu^+ \mu^-$	$5.8(8)  imes 10^{-6}$
0 0 1 1	
$\eta  ightarrow \pi^{0}\pi^{0}\ell^{+}\ell^{-}$	
$\eta  ightarrow \pi^+ \pi^- e^+ e^+$	$2.68(11) \times 10^{-4}$
+ - + -	$2.6 10^{-4}$
$\eta \to \pi^+\pi^-\mu^+\mu^-$	$<3.6 \times 10^{-4}$
·····	$2 40(22) \times 10^{-5}$
$\eta \rightarrow e^+e^-e^+e^-$	$2.40(22) \times 10^{-4}$
$\eta \to e^+e^-\mu^+\mu^-$	$< 1.0 \times 10$
$\frac{\eta \to \mu^+ \mu^- \mu^+ \mu^-}{\eta \to \pi^+ \pi^- \pi^0 \nu}$	$< 5.0 \times 10^{-4}$
$\eta \rightarrow \pi^{\pm} a^{\mp} v$	$< 1.7 \times 10^{-4}$
$\eta \rightarrow \pi^+ \pi^-$	$< 1.7 \times 10$ $< 4.4 \times 10^{-6}$ [56]
$\eta \rightarrow \pi \eta$	$< 3.4 \times 10^{-4}$
$\eta \rightarrow 2\pi$ $n \rightarrow 4\pi^0$	$< 5.3 \times 10^{-7}$
$\eta \rightarrow - \eta \iota$	$\sim 0.3 \times 10$

Discussion 0 Chiral anomaly,  $\eta - \eta'$  mixing  $m_u - m_d$  $\chi$  PT at  $\mathcal{O}(p^6)$ , leptophobic B boson, light Higgs scalars  $\chi$  PT, axion-like particles (ALPs) <10<sup>-11</sup>[55]  $m_u - m_d$ , C/CP violation, light Higgs scalars Chiral anomaly, theory input for singly-virtual TFF and  $(g-2)_{\mu}$ , P/CP violation  $\chi$  PT, ALPs Theory input for  $(g - 2)_{\mu}$ , dark photon, protophobic X boson Theory input for  $(g - 2)_{\mu}$ , dark photon Theory input for  $(g - 2)_{\mu}$ , BSM weak decays Theory input for  $(g - 2)_{\mu}$ , BSM weak decays, **P**/**CP** violation C/CP violation, ALPs Theory input for doubly-virtual TFF and  $(g - 2)_{\mu}$ , P/CP violation, ALPs Theory input for doubly-virtual TFF and  $(g - 2)_{\mu}$ , P/CP violation, ALPs Theory input for  $(g-2)_{\mu}$ Theory input for  $(g-2)_{\mu}$ Theory input for  $(g-2)_{\mu}$ Direct emission only Second-class current P/CP violation *P*/*CP* violation P/CP violation

#### Phys. Rept. 945 (2022) 1-105

# $\eta'$ Physics

#### **Standard Model Tests:**

- Chiral symmetry and anomalies ٠
- Extract  $\eta \eta'$  mixing angle and quark mass ratio ٠
- Theory inputs to HLbL for  $(g 2)_{\mu}$
- QCD scalar dynamics ٠

#### **Fundamental Symmetry Tests:**

- C, CP violations ٠
- P, CP violations ٠
- Lepton flavor violations

#### **BSM Physics in Dark Sector:**

Vector bosons (B boson, dark photon and X • boson)

- Dark scalars •
- Pseudoscalars (ALPs) ٠
- BSM weak decays ٠

Channel	Expt. branching ratio	Discussion	
$\eta'  ightarrow \eta \pi^+ \pi^-$	42.6(7)%	Large- $N_c \chi$ PT, light Higgs scalars	
$\eta'  o \pi^+ \pi^- \gamma$	28.9(5)%	Chiral anomaly, theory input for singly-virtual TFF and $(g - 2)_{\mu}$ , <i>P/CP</i> violation	
$\eta'  ightarrow \eta \pi^0 \pi^0$	22.8(8)%	Large- $N_c \chi PT$	
$\eta'  ightarrow \omega \gamma$	2.489(76)% [58]	Theory input for singly-virtual TFF and $(g-2)_{\mu}$	
$\eta'  ightarrow \omega e^+ e^-$	$2.0(4)  imes 10^{-4}$	Theory input for doubly-virtual TFF and $(g-2)_{\mu}$	
$\eta'  ightarrow 2\gamma$	2.331(37)% [58]	Chiral anomaly, $\eta - \eta'$ mixing	
$\eta'  ightarrow 3\pi^0$	2.54(18)% (*)	$m_u - m_d$	
$\eta'  ightarrow \mu^+ \mu^- \gamma$	$1.09(27)  imes 10^{-4}$	Theory input for $(g-2)_{\mu}$ , dark photon	
$\eta'  ightarrow e^+ e^- \gamma$	$4.73(30) \times 10^{-4}$	Theory input for $(g-2)_{\mu}$ , dark photon	
$\eta'  ightarrow \pi^+ \pi^- \mu^+ \mu^+$	$<2.9 \times 10^{-5}$	Theory input for doubly-virtual TFF and $(g-2)_{\mu}$ ,	
	(11)	P/CP violation, dark photon, ALPs	
$\eta'  ightarrow \pi^+\pi^- e^+ e^-$	$2.4\binom{+1.3}{-1.0}  imes 10^{-3}$	Theory input for doubly-virtual TFF and $(g-2)_{\mu}$ ,	
		P/CP violation, dark photon, ALPs	
$\eta' \to \pi^0 \pi^0 \ell^+ \ell^-$		C/CP violation, ALPs	
$\eta'  ightarrow \pi^+ \pi^- \pi^0$	$3.61(17) \times 10^{-3}$	$m_u - m_d$ , C/CP violation,	
	_	light Higgs scalars	
$\eta'  ightarrow 2(\pi^+\pi^-)$	$8.4(9)  imes 10^{-5}$	Theory input for doubly-virtual TFF and $(g-2)_{\mu}$	
$\eta'  ightarrow \pi^+ \pi^- 2 \pi^0$	$1.8(4) \times 10^{-4}$		
$\eta' \rightarrow 2(\pi^+\pi^-)\pi^0$	$< 1.8 \times 10^{-3}$	ALPs	
$\eta' \to K^{\pm} \pi^{\mp}$	$< 4 \times 10^{-5}$	Weak interactions	
$\eta' \to \pi^{\pm} e^{\mp} \nu_e$	$<2.1 \times 10^{-4}$	Second-class current	
$\eta'  o \pi^0 \gamma \gamma$	$3.20(24) \times 10^{-3}$	Vector and scalar dynamics, B boson,	
		light Higgs scalars	
$\eta' \to \eta \gamma \gamma$	$8.3(3.5) \times 10^{-5}$ [59]	Vector and scalar dynamics, B boson,	
		light Higgs scalars	
$\eta'  ightarrow 4\pi^0$	$<4.94 \times 10^{-5}$ [60]	(S-wave) P/CP violation	
$\eta' \rightarrow e^+e^-$	$< 5.6 \times 10^{-9}$	Theory input for $(g - 2)_{\mu}$ , BSM weak decays	
$\eta' \rightarrow \mu^+ \mu^-$		Theory input for $(g-2)_{\mu}$ , BSM weak decays	
$\eta' \to \ell^+ \ell^- \ell^+ \ell^-$		Theory input for $(g-2)_{\mu}$	
$\eta' \rightarrow \pi^+ \pi^- \pi^0 \gamma$		<i>B</i> boson	
$\eta' \rightarrow \pi^+ \pi^-$	$- < 1.8 \times 10^{-5}$	<i>P/CP</i> violation	
$\eta'  ightarrow 2\pi^0$	$< 4 \times 10^{-4}$	P/CP violation	

Pnys. керт. 945 (2022) 1-105

# Source of $\eta/\eta'$ events

**New Proposals** 



### $\eta/\eta'$ sample from J/ $\psi$ decays at BESIII



- High production rate of  $\eta/\eta'$  in J/  $\Psi$  decays
  - radiative decays:  $5.2 \times 10^7 \, \eta'$ ,  $1.1 \times 10^7 \, \eta$
  - hadronic decays:  $6.5 \times 10^6 \,\eta'$ ,  $2.5 \times 10^7 \,\eta$
- Unique opportunity to investigate the decays of  $\eta/\eta'$

 $\rightarrow \eta' \rightarrow \pi^+\pi^-\eta \rightarrow 2.2 \times 10^7 \eta$ 

# BESIII: an important role in $\eta/\eta'$ decays

Decay channel	Physics	Publication
η→π⁺π <sup>−</sup> π <sup>0</sup> , η/η′→π <sup>0</sup> π <sup>0</sup> π <sup>0</sup>	Matrix elements, m <sub>u</sub> -m <sub>d</sub> , C-inv	PRD92, 012014(2015)
η'→ωe⁺e <sup>-</sup>	First observation, BR	PRD92, 051101(2015)
η′→Кπ	Weak decay, UL	PRD93, 072008 (2016)
η'→ρπ	First observation, BR	PRL118, 012001(2017)
η΄→γγπ <sup>0</sup>	BR, B boson	PRD96, 012005(2017)
η' <b>→</b> γπ⁺π <sup>−</sup>	BR, decay dynamic (box anomaly)	PRL120, 242003(2018)
ŋ′→π⁺π⁻ŋ, ŋ′→π <sup>0</sup> π <sup>0</sup> ŋ	Matrix elements, cusp effect	PRD97, 012003(2018)
$\omega \rightarrow \pi^+ \pi^- \pi^0$	Dalitz plot analysis	PRD98, 112007(2018)
Р→үү	BRs, chiral anomaly	PRD97, 072014(2018)
η'→γγη	UL	PRD100, 052015(2019)
Absolute BF of η' decays	BRs	PRL122, 142002(2019)
η΄→π <sup>0</sup> π <sup>0</sup> π <sup>0</sup> π <sup>0</sup>	CP-Vio, UL	PRD101, 032001(2020)
η'→π⁺π <sup>−</sup> e⁺e <sup>−</sup>	BR, CP-viol assymm	PRD103, 092005(2021)
η'→π <sup>+</sup> π <sup>-</sup> u <sup>+</sup> u <sup>-</sup>	BR, decay dynamic	PRD103, 072006(2021)
Absolute BF of η decays	BRs	PRD104, 092004(2021)
η'→e⁺e⁻e⁺e⁻	BR, TFF	PRD105, 112010(2022)
η′→ηπ <sup>0</sup> π <sup>0</sup>	Cusp effect	PRL130, 081901(2023)
η→π⁺π <sup>−</sup> π <sup>0</sup> , π <sup>0</sup> π <sup>0</sup> π <sup>0</sup>	Matrix elements, cusp effect	PRD107,092007(2023)
η'→2(π <sup>+</sup> π <sup>-</sup> ),π <sup>+</sup> π <sup>-</sup> π <sup>0</sup> π <sup>0</sup> , 2(π <sup>0</sup> π <sup>0</sup> )	VMD, CP-Vio	PRD 109, 032006 (2024)
η'→π <sup>+</sup> π <sup>-</sup> e <sup>+</sup> e <sup>-</sup> , π <sup>+</sup> π <sup>-</sup> u <sup>+</sup> u <sup>-</sup>	BR, decay dynamic, CP-Vio	JHEP07, 135 (2024), arXiv:2501.10130v1
η/η′→γe⁺e <sup>_</sup>	TFF	PRD109, 072001 (2024)
η/η′→/+/ <sup>_</sup> /+/ <sup>_</sup>	BR, TFF	PRD111,052002 (2025)

# Recent highlights of eta/eta' at BESIII

- Decay dynamics
- Transition Form Factors
- Dark force

### **Evidence of cusp effect in** $\eta' \rightarrow \pi^0 \pi^0 \eta$

BESIII: PRL130, 081901(2023)

- Investigation on  $\pi\pi$  and  $\pi\eta$  final interactions
- The cusp effect  $(\pi^+\pi^- \to \pi^0\pi^0 \to \pi^+\pi^-)$  is sizable in this decay



 B. Kubis, S. P. Schneider, EPJC 62, 511 (2009)
 One and two-loop level contributions based on NREFT are introduced

 $\mathcal{M}_{\eta' o \eta \pi^0 \pi^0} = \mathcal{M}_N^{tree} + \mathcal{M}_N^{one-loop} + \mathcal{M}_N^{two-loop} + \cdots$ 

η

- Fits at different cases
- > Cusp effect with ~3.5  $\sigma$ !

 $\pi^0$ 

### Cusp structure in $\eta \to \pi^0 \pi^0 \pi^0$



### Matrix elements for $\eta \to \pi^0 \pi^0 \pi^0$

#### BESIII: PRD 107, 092007 (2023)

https://www.hepdata.net/record/141642

 $\alpha$  is consistent with A2  $(-0.0302 \pm 0.0008_{stat.})$  in 2.8 $\sigma$   $\beta(A_2) = -0.0070 \pm 0.0010_{stat.}$  $\gamma(A_2) = -0.0023 \pm 0.0040_{stat.}$ 



### Matrix elements for $\eta \to \pi^+ \pi^- \pi^0$

#### BESIII: PRD 107, 092007 (2023)

SM: Isospin violating process, C conserved, EM effects suppressed  $\Rightarrow$  ideal process to extract  $m_u - m_d$ 

$$|A(X,Y)|^2 \propto 1 + aY + bY^2 + cX + dX^2 + eXY + fY^3 + gX^2Y + \cdots$$

$$X = \frac{\sqrt{3}}{Q}(T_{\pi^+} - T_{\pi^-}), Y = \frac{3T_{\pi^0}}{Q} - 1,$$

 $a = -1.086 \pm 0.006 \pm 0.001,$   $b = 0.162 \pm 0.006 \pm 0.003,$   $d = 0.083 \pm 0.007 \pm 0.001,$   $f = 0.118 \pm 0.011 \pm 0.003,$  $g = -0.053 \pm 0.017 \pm 0.003.$ 

 $c = (-0.086 \pm 2.986) \times 10^{-3}, e = -0.001 \pm 0.007$ no C symmetry breaking





### Dalitz plot Asymmetries in $\eta \rightarrow \pi^+ \pi^- \pi^0$

BESIII: PRD 107, 092007 (2023)

**BSM:** C broken, isospin either conserved or broken

$$\mathcal{M}(s,t,u) = \mathcal{M}_1^C(s,t,u) + \mathcal{M}_0^{\mathcal{C}}(s,t,u) + \mathcal{M}_2^{\mathcal{C}}(s,t,u)$$

S. Gardner, J. Shi, PRD 101 (2020) 115038 H. Akdag, T. Isken, B. Kubis, JHEP 02 (2022)137 J. Shi, J. Liang, S. Gardner PR 110 (2024) 055039

> The interferences give rise to mirror symmetry breaking (permille level) in the Dalitz plot



C	overall C/CP-violation	on ∆l = 2	$\Delta I = 0$
Experiment	$A_{LR}(\%)$	$A_Q(\%)$	$A_S(\%)$
This work	$0.114 \pm 0.131 \pm 0.001$	$-0.035\pm0.131\pm0.011$	$-0.070\pm0.131\pm0.009$
KLOE-2 [11]	$-0.050\pm0.045^{+0.050}_{-0.110}$	$0.018 \pm 0.045 ^{+0.048}_{-0.023}$	$0.004 \pm 0.045 ^{+0.031}_{-0.035}$
Jane [40]	$0.28\pm0.26$	$-0.30\pm0.25$	$0.20\pm0.25$
Layter $[24]$	$-0.05\pm0.22$	$-0.07\pm0.22$	$0.10\pm0.22$
Gormley [41]	$1.5\pm0.5$	-	$0.5\pm0.5$

### Amplitude analysis for $\eta' \rightarrow 4\pi$

#### BESIII: PRD 109, 032006 (2024)

Loop and counter term at  $O(p^6)$ 

F. K. Guo, B. Kubis, A. Wirzba, PRD 85,014014 (2012)

 $Br(\eta' \to 2(\pi^+\pi^-)) = (1.0 \pm 0.3) \times 10^{-4}$  $Br(\eta' \to \pi^+\pi^-2\pi^0) = (2.4 \pm 0.7) \times 10^{-4}$ 







 $Br(\eta' \to \pi^+ \pi^- \pi^+ \pi^-) = (8.56 \pm 0.25 \pm 0.23) \times 10^{-5}$  $Br(\eta' \to \pi^+ \pi^- \pi^0 \pi^0) = (2.12 \pm 0.12 \pm 0.10) \times 10^{-4}$ 

### Amplitude analysis results for $\eta' \rightarrow 2(\pi^+\pi^-)$ BESIII: PRD 109, 032006 (2024)



First measurement of the doubly virtual isovector form factor

$$\alpha = \frac{c_3}{c_1 - c_2} = 1.22 \pm 0.33 \pm 0.04$$

If  $\alpha\simeq$  1, triangle anomaly would be dominated



### Search for rare decay $\eta' \rightarrow \pi^0 \pi^0 \pi^0 \pi^0$

BESIII: PRD 109, 032006 (2024)

- CP-violation S-wave, induced by the QCD Lagrangian  $\theta$ -term  $\Rightarrow$  Br $\sim 10^{-23}$
- CP-conserving higher order  $\Rightarrow$  Br $\sim 10^{-8}$  F. K. Guo, B. Kubis, A. Wirzba, PRD 85,014014 (2012)



0.7

0.8

 $M(\pi^0\pi^0\pi^0\pi^0)$  (GeV/c<sup>2</sup>)

• With 10 billion J/ $\psi$ , the UL at 90% CL is set as  $1.24 \times 10^{-5}$ 

1.1

1.0

### **Transition form factor of** $\eta/\eta'$

• Important input for HLbL contributions

#### Pseudoscalar TFFs are experimentally accessible in three different processes

 $e^+$ 

 $e^{-}$ 

Dalitz decays 0<q<sup>2</sup><M<sup>2</sup>

Annihilation process q<sup>2</sup> > M<sup>2</sup>

Two photon process





### Transition form factor of $\eta/\eta' \rightarrow \gamma e^+ e^-$ BESIII: PRD 109, 072001 (2024)

$$\frac{d\Gamma(P \to \gamma l^+ l^-)}{dq^2 \Gamma_{\gamma\gamma}} = \frac{2\alpha}{3\pi} \frac{1}{q^2} \sqrt{1 - \frac{4m_l^2}{q^2}} (1 + \frac{2m_l^2}{q^2}) (1 - \frac{q^2}{M_P^2})^3 |F_P(q^2, 0)|^2 \qquad \underbrace{P = \frac{|F(q^2)|^2}{\gamma^*} e^{-\frac{q^2}{\gamma^*}} e^{-\frac{q^2}$$

### Amplitude analysis result of $\eta' \rightarrow \pi^+ \pi^- l^+ l^-$ Besili: JHEP 07, 135 (2024)







 $\pi^+\pi^-e^+e^-$ 

Study of  $\eta \rightarrow \pi^+ \pi^- l^+ l^-$ 

#### arXiv:2501.10130v1 [hep-ex] Accepted by PRD

	$\mathcal{B}(\eta \to \pi^+ \pi^- e^+ e^-)$	$\mathcal{B}(\eta \to \pi^+ \pi^- \mu^+ \mu^-)$
	$(10^{-4})$	$(10^{-9})$
Unitary $\chi PT$ [1]	$2.99\substack{+0.06 \\ -0.09}$	$7.50^{+1.80}_{-0.70}$
Hidden gauge [2]	$3.14\pm0.17$	$8.65\pm0.39$
VMD [2]	$3.02\pm0.12$	$8.64\pm0.25$
CMD-2 [4]	$3.7^{+2.5}_{-1.8} \pm 3.0$	
WASA [5]	$4.3^{+0.2}_{-1.6} \pm 0.4$	$< 3.6 \times 10^5$
KLOE [6]	$2.68 \pm 0.09 \pm 0.07$	





 $Br(\eta \to \pi^+\pi^-e^+e^-) = (3.07 \pm 0.12 \pm 0.19) \times 10^{-4}$ 

- Allow to access the decay dynamic but with limited statistics
- No event left in for  $\pi^+\pi^-\mu^+\mu^-$  hypothesis in  $\eta$  signal region and the UL is set as  $4.0 \times 10^{-7}$  at the 90% CL.

### **CP Asymmetry in** $\eta/\eta' \to \pi^+ \pi^- l^+ l^-$ JHEP 07, 135 (2024), arXiv:2501.10130v1

• A new sources of CP violation beyond the CKM phase and outside flavor-changing processes





### **Double Dalitz decays** $\eta/\eta' \rightarrow l^+ l^- l^+ l^-$





T. Petri, arXiv:1010.2378

 $8.626(33) \times 10^{-7}$ 

 $\rightarrow e^+ e^- \mu^+ \mu^-$ 

 $7.968(31) \times 10^{-7}$ 

CPC 42 (2018) 023109

 $6.39(91) \times 10^{-7}$ 

 $< 1.75 \times 10^{-6}$ 

#### $d|F(q^2)|$ Slope parameter: $b_{\eta/\eta'} =$ $dq^2$ $|q^2=0|$ $\mathsf{BESIII}(\eta'{\rightarrow}\gamma e^{\scriptscriptstyle +}e^{\scriptscriptstyle -})$ This Work BESIII( $\eta' \rightarrow \pi^+ \pi^- l^+ l^-$ ) A2 BESIII(2015) NA60 Lepton-G TPC/Two-Gamma TPC/Two-Gamma CELLO CELLO CLEO CLEO VMD VMD Quark Loop Quark Loop Brodsky-Lepage Brodsky-Lepage 1-loop ChPT 1-loop ChPT Dispersion Dispersion P.A.fit to data P.A.fit to data

2.2

2

2.4

2.6

0.8 1 1.2 1.4 1.6 1.8 2 2.2 2.4 2.6  $b_{n'} (\text{GeV}/c^2)^{-2}$ 

 $b_{\eta} = 1.781 \pm 0.123 \pm 0.033 \, (\text{GeV/c}^2)^{-2}$ 

 $b_{\eta} (GeV/c^2)^{-2}$ 

1.6 1.8

1.2

1.4



### **BSM Physics in Dark Sector**

JHEP 07 (2024) 135



• Axion-like particle in  $\eta' \rightarrow \pi^+\pi^- a$ ,  $a \rightarrow e^+e^-$ 



#### **BSM Physics in Dark Sector**

#### PRD 109 (2024) 072001

• Dark photon in  $\eta/\eta' \rightarrow \gamma A', A' \rightarrow e^+e^-$ 



# Summary

- $\eta/\eta'$ : an important tool for particle physics
- BESIII: an unique place for light mesons
  - Significant progresses achieved on decay mechanisms, TFFs ... with unprecedented statistics
- Together with other experiments, the light meson physics will be into a precision era

# Thanks for your attention!!!

### **Decay Amplitude of** $\eta' \rightarrow \pi^+ \pi^- l^+ l^-$

$$\overline{\left|\mathcal{A}_{\eta'\to\pi^+\pi^-l^+l^-}\right|^2}(s_{\pi\pi},s_{ll},\theta_{\pi},\theta_1,\phi) = \frac{e^2}{8k^2}|M(s_{\pi\pi},s_{ll})|^2 \times \lambda\left(m_{\eta'}^2,s_{\pi\pi},s_{ll}\right) \times \left[1-\beta_1^2\sin^2\theta_1\sin^2\phi_1s_{\pi\pi}\beta_{\pi}^2\sin^2\theta_{\pi}\right]$$

 $M(s_{\pi\pi}, s_{ll}) = M_{mix} \times VMD(s_{\pi\pi}, s_{ll})$ 

A. Faessler, C. Fuchs, M. I. Krivoruchenko, PRC 61, 035206 (2000) B. Borasoy, R. Nissler, EPJA 33, 95 (2007) T. Petri, arXiv:1010.2378

contains the information of the decaying particle and the form factor

