Form factors of light pseudoscalar mesons from the perturbative QCD approach

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河南省 · 安阳市
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Outline

1. Form Factors
2. The perturbative QCD approach
   - Three scale factorization
   - The soft-transversal dynamics
3. π,K,$\eta^{(\prime)}$ form factors
   - Electromagnetic form factor of π and K
   - Transition form factor of π and $\eta^{(\prime)}$
4. Conclusion

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"In elementary particle physics and mathematical physics, in particular in effective field theory, a **form factor** is a function that encapsulates the properties of a certain interaction without including all of the underlying physics, but instead, providing the momentum dependence of a suitable matrix elements. Its further measured experimentally in confirmation or specification of a theory."

Momenta Redistribution

$\Downarrow$ QCD is believed to exhibit confinement

hadron structures $\otimes$ hard scattering

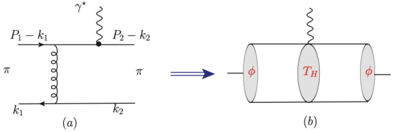
$\Downarrow$ decoupling of LD and SD interactions

factorisation theorem, EFT; CKM, g-2, B anomalies

Form Factors

PION is the lightest Glodstone boson and one of the most simplest hadrons, hence the ideal probe to extremely rich QCD dynamics.

- (spacelike) electromagnetic form factor
  $\langle \pi^-(p_2)|J_\mu^{em}|\pi^-(p_1)\rangle = e_q(p_1+p_2)_\mu F_\pi(Q^2)$
- the interaction distance of J_μ^{em} is decided by the external reason Q^2.
- Separate the hard partonic physics out of the hadronic physics (soft, nonperturbative objects) in exclusive processes **Factorization**



$$\frac{d\sigma}{d\Omega} = \int_u^1 \frac{d\zeta}{\zeta} \mathcal{H}^{(t)}(\zeta)\psi^{(t)}(\frac{u}{\zeta})$$

- The universal nonperturbative objects, studied by QCD-based analytical (QCDSRs, χPT, DSE, instanton) and numerical approaches (LQCD)
- also by data-driven method

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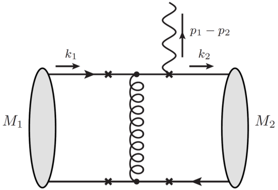
Three scale factorization

- end-point singularities appear in exclusive QCD processes
  $m_{1,2}^2 \ll Q^2$, light-cone coordinate $p_1 = (\frac{Q}{\sqrt{2}}, 0, 0_{\mathrm{T}})$, $p_2 = (0, \frac{Q}{\sqrt{2}}, 0_{\mathrm{T}})$,
  (anti-)valence quarks: $k_1 = x_1 p_1, \bar{k}_1 = \bar{x}_1 p_1$

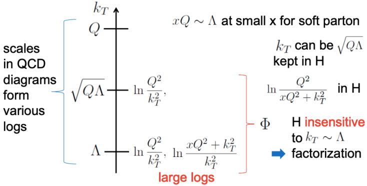


$\phi \propto u(1-u), \quad m_0^\pi \phi^{P,\sigma} \propto 1$

$\propto \sum_t \int du_1\, du_2 \kappa_t(u_i) \frac{\alpha_s(\mu)\phi_1^t(u_1)\phi_2^t(u_2)}{(u_1 u_2 Q^2)(u_2 Q^2)}$

- pick up k_T in the internal propagators
  $\mathcal{M} \propto \sum_{t=2,3,4} \int du_1\, du_2\, dk_{1\,T} dk_{2\,T} K_t(u_i) \frac{\alpha_s(\mu)\phi_1^t(u_1)\phi_2^t(u_2)}{[u_1 u_2 Q^2 - (\Delta k_T)^2][u_2 Q^2 - k_{2\,T}^2]}$
- end-point singularity at leading and subleading powers
  $\mathcal{H} \propto \frac{\alpha_s(\mu)}{u_1 u_2 Q^2 - k_T^2} \sim \frac{\alpha_s(\mu)}{u_1 u_2 Q^2} - \frac{\alpha_s(\mu) k_T^2}{(u_1 u_2 Q^2)^2} + \cdots$
- the power suppressed TMD terms becomes important at the end-points

k_T Factorization Soft+colinear divergence appears double logarithmic term $\alpha_s ln^2(Q/k_T)$



consider contribution from the iTMD

$$\frac{f_\pi m_0^{\mathcal{P}}}{2\sqrt{6}}\phi^p(u,\mu) = \int \frac{d^2\vec{k}_T}{16\pi^3}\phi^p_{2p}(u,\vec{k}_T) + \int \frac{d^2\vec{k}_{T1}}{16\pi^3}\frac{d^2\vec{k}_{T2}}{4\pi^2}\phi^p_{3p}(u,\vec{k}_{T1},\vec{k}_{T2}).$$

$$\psi^p_{2p}(u,\vec{k}_T) = \frac{f_\pi m_0^P}{2\sqrt{6}}\phi^p_{2p}(u,\mu)\Sigma(u,\vec{k}_T),$$

$$\psi^p_{3p}(u,\vec{k}_{1\,T},\vec{k}_{2\,T}) = \frac{f_\pi m_0^P}{2\sqrt{6}}\eta_{3\pi}\phi^p_{3p}(u,\mu)\Sigma'(\alpha_i,\vec{k}_{1\,T},\vec{k}_{2\,T}).$$

The soft-transversal dynamics

Two-particlie Fock state

$$\Sigma\left(u,\mathbf{k}_T\right) = 16\pi^2\beta^2 g(u)\,\mathrm{Exp}\left[-\beta^2 k_T^2 g(u)\right],\, g(u) = 1/(u\bar{u})$$

$$\int \frac{d^2k_\perp}{16\pi^3}\Sigma\left(u,\mathbf{k}_T\right) = 1$$

$$\beta_\pi^2 = \frac{1}{8\pi^2 f_\pi^2\left(1+a_2^\pi+a_4^\pi+\cdots\right)}$$

$$\psi\left(u,\mathbf{b}_T\right) = \frac{f_\pi}{2\sqrt{6}}\varphi(u,\mu)\hat{\Sigma}\left(u,\mathbf{b}_T\right),\, \hat{\Sigma}\left(u,\mathbf{b}_T\right) = 4\pi\,\mathrm{Exp}\left[-\frac{b_T^2}{4\beta^2 g(u)}\right]$$

Three-particle Fock state

$$\psi_{3p}\left(u,\mathbf{k}_{1T},\mathbf{k}_{2T}\right) = \frac{f_\pi m_0^{\mathcal{P}}}{2\sqrt{6}}\varphi_{3p}(u,\mu)\int_0^u d\alpha_1\int_0^{\bar{u}} d\alpha_2\,\frac{\Sigma'\left(\alpha_i,\mathbf{k}_{1T},\mathbf{k}_{2T}\right)}{1-\alpha_1-\alpha_2}$$

three-particle iTMD Gaussian function is:

$$\Sigma'\left(\alpha_i,\mathbf{k}_{1T},\mathbf{k}_{2T}\right) = \frac{64\pi^3\beta'^4}{\alpha_1\alpha_2(1-\alpha_1-\alpha_2)}\,\mathrm{Exp}\left[-\beta'^2\left(\frac{k_{1T}^2}{\alpha_1}+\frac{k_{2T}^2}{\alpha_2}+\frac{(k_{1T}+k_{2T})^2}{1-\alpha_1-\alpha_2}\right)\right]$$

$$\hat{\Sigma}'\left(u,\mathbf{b}_1,\mathbf{b}_2\right) = 4\pi Exp[-\frac{2\alpha_3(b_1^2+b_2^2)+(\alpha_1+\alpha_2)(b_1-b_2)^2}{16\beta'^2}]$$

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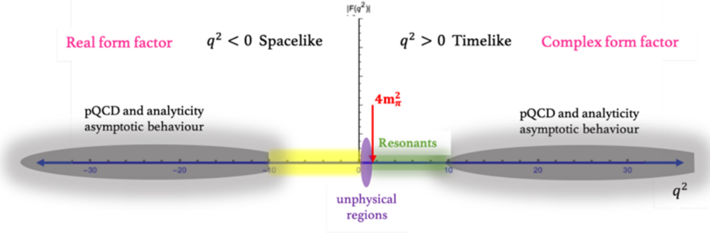
Measurements of F_π in different energy regions

- Spacelike data is available in the narrow region $q^2 \in [-2.5, -0.25]\mathrm{GeV}^2$
  Jefferson Lab 2006,2008, $\cdots$, NA7 1996, CLEO 2005
- Timelike data is dominated by the resonant states, have not extend to large momentum transfers (perturbative QCD available)



Whole region of momentum transfers for electromagnetic form factor

- Mismatch between the QCD based calculation and the available data
- could be restored by employing the dispersion relation
- pQCD prediction at large $|q^2|$ is indispensable

Dispersion Relation

- spacelike data is available in the narrow region $q^2 \in [-2.5, -0.25]\mathrm{GeV}^2$
- the mismatch destroys the direct extracting programme from $F_\pi(q^2 < 0)$
- timelike data $F_\pi(q^2 > 0)$ provides another opportunity

Standard dispersion relation:

$$\mathcal{F}_\pi^{pQCD}(q^2) = \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\mathcal{F}_P(s)}{s - q^2 - i\epsilon}, \qquad q^2 > s_0$$

modulus squared dispersion integral:

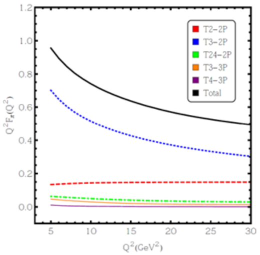
[S. Cheng, A. Khodjamirian and A. V. Rusov, PRD 102 (2020) 074022
J. Chai, S. Cheng and J. Hua, EPJC 83 (2023) no.7, 556.]

$$\mathcal{F}_\pi^{\mathrm{pQCD}}(\mathbf{q^2}) = \exp\left[\frac{\mathbf{q^2}\sqrt{\mathbf{s_0} - \mathbf{q^2}}}{\mathbf{2\pi}} \int\limits_{\mathbf{4m_\pi^2}}^{\infty} \mathbf{ds} \frac{\ln|\mathcal{F}_\pi(\mathbf{s})|^2}{\mathbf{s}\sqrt{\mathbf{s} - \mathbf{s_0}}\,(\mathbf{s} - \mathbf{q^2})}\right].$$

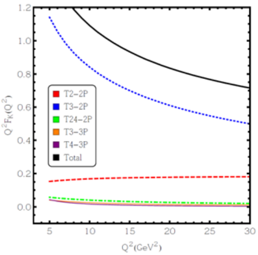
$$|\mathcal{F}_\pi(s)|^2 = \Theta(s_{\max} - s)\,|\mathcal{F}_{\pi,\mathrm{Inter.}}^{\mathrm{data}}(s)|^2 + \Theta(s - s_{\max})\,|\mathcal{F}_\pi^{\mathrm{pQCD}}(s)|^2$$

Pion LCDAs

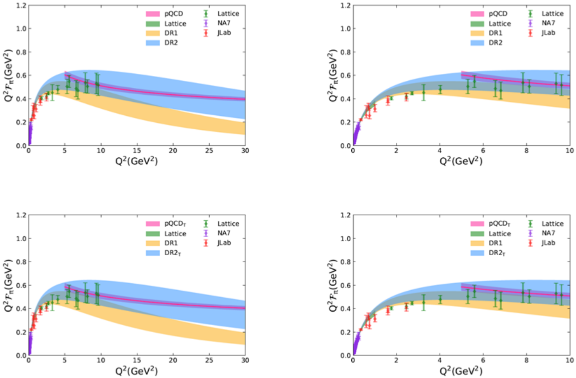
- Three sources of high twist LCDAs
  - † "bad" components in WFs in particular of those with " wrong" spin projection
  - † transversal motion of $q(\bar{q})$ in the leading twist components
    given by the integrals with additional factors of $k_\perp^2$
  - † higher Fock states with additional g and $q\bar{q}$ pairs
- higher twist contributions to exclusive QCD processes are commonly power suppressed $\mathcal{O}(1/Q)$
- but twist 3 contribution are dominate in the π, K evolved processes due to chiral enhancement $\mathcal{O}(m_0/(x_i Q))$



[**SC**, 1905.05059]



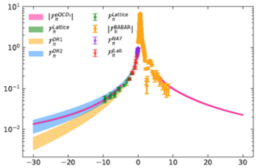
Electromagnetic form factor of π

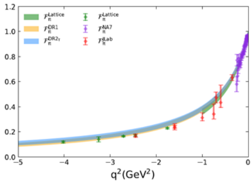


Set	IA	IB	IIA	IIB
$m_0^\pi(\mathrm{GeV})$	1.37 ± 0.10	1.30 ± 0.10	1.54 ± 0.06	1.84 ± 0.07

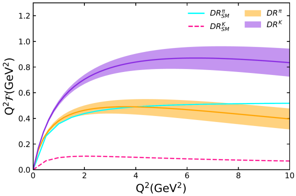
Electromagnetic form factor of π

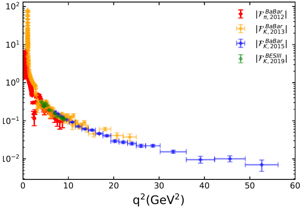
- the precise pQCD calculation
- modular dispersion relation with e^+e^- annihilation data
- a comprehensive description of $F_\pi(q^2)$ in the whole kinematics





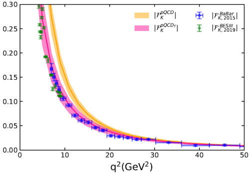
- the slight derivation is still there despite its sensitive to iTMD in the small q^2

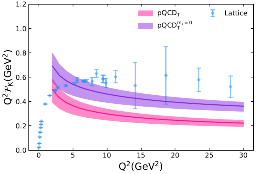




- **Pion Consistency:** Pion EMFF from standard and modular methods match, confirming GS/KS models explain the imaginary part and supporting the "no zeros in complex plane" assumption for pion form factor.
- **Kaon Discrepancy:** Modular kaon result is larger than standard. With SU(3) breaking, it nears pion's value. This shows: BaBar's model may miss kaon's imaginary part, and modular relation might not fit kaon (due to S- wave near f_0 resonance).
- **Kaon Analysis Approach:** Avoid dispersion relations for kaon EMFF. Adapt pQCD calculations to the data in $10 \leqslant q^2 \leqslant 60 GeV^2$

Electromagnetic form factor of K

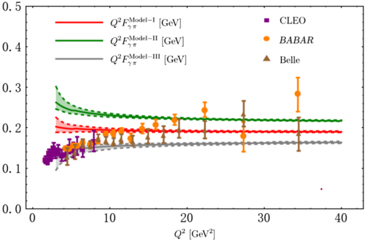




- † $m_0^K(1\,GeV) = 1.90 \pm 0.09$GeV is well-known from the CHPT relation
- † fit the transversal-size parameter $\beta_K^2 = 0.30 \pm 0.05\,GeV^{-2}$ from timelike data settle for the second best and take $\beta_K^2 = \beta_K'^2$
- † the iTMDs is indispensable to explain the data in the intermediate q^2
- † iTMDs-improved pQCD result of spacelike FF is small than the lattice data
  - ‡ agrees with results obtained from the DSE approach and the collinear QCD factorization
  - ‡ large SU(3) flavor breaking emerges an additional term proportional to m_s in the twist-three LCDAs

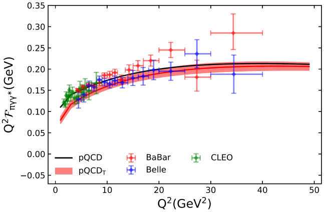
Transition form factor of π

$F_{\pi\gamma\gamma^\star}$ is the theoretically most clean observable $\propto a_n^\pi$



- Model-I [Brodsky, Teramond 0707.3859, RQCD 1903.08038]
- Model-II [SC, Khodjamirian, Rosov 2007.05550]
- Model-III [Mikhailov, Pimikov, Stefanis 1604.06391]

† NLO pQCD calculation with the iTMD contribution, modification in the small and intermediate regions is significant



Transition form factors of $\eta^{(\prime)}$

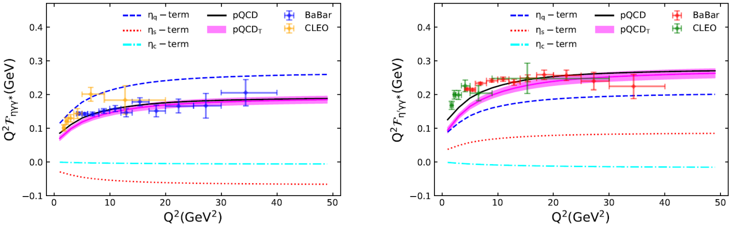
$F_{\eta^{(\prime)}\gamma\gamma^\star}$ serves as an sensitive probe for investigating flavor structure

$$\mathcal{F}_{\eta\gamma\gamma^*} = \cos\phi\frac{e_u^2+e_d^2}{\sqrt{2}}\mathcal{F}_{\eta_q\gamma\gamma^*} - \sin\phi e_s^2\mathcal{F}_{\eta_s\gamma\gamma^*} - 0.006 e_c^2\mathcal{F}_{\eta_c\gamma\gamma^*},$$

$$\mathcal{F}_{\eta'\gamma\gamma^*} = \sin\phi\frac{e_u^2+e_d^2}{\sqrt{2}}\mathcal{F}_{\eta_q\gamma\gamma^*} + \cos\phi e_s^2\mathcal{F}_{\eta_s\gamma\gamma^*} - 0.016 e_c^2\mathcal{F}_{\eta_c\gamma\gamma^*}.$$

inputs from [Feldmann:1998vh Escribano:2005qq,Escribano:2013kba Cao:2012nj Bali:2021qem,Hatton:2020qhk]

Decay constants、mixing angles、mass and gegenbauer. The default scale is 1GeV for η_q and η_s, while 3GeV for η_c



[J. Chai, SC, 2501.08783]

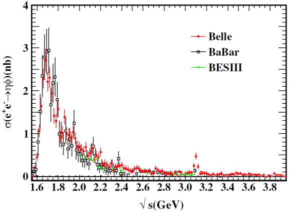
Transition form factor of $\eta^{(\prime)}$

- $\eta^{(\prime)}$ are dominated by η_q component, while a sizable η_s component in η'
- The η_c component contribution is negligible in magnitude and therefore plays no significant role in explaining the experimental data.
- iTMD-improved pQCD predictions favor the small ϕ, the large f_{η_s}, f_{η_q} and the small m_{η_q}, m_{η_s}
- In the perturbative QCD limit, $\mathcal{F}_{\eta_q\gamma\gamma^*} = \mathcal{F}_{\eta_s\gamma\gamma^*} = \mathcal{F}_{\eta_c\gamma\gamma^*} = \mathcal{F}_{\pi\gamma\gamma^*}$

$$\delta\mathcal{F} \equiv \mathcal{F}_{\eta\gamma\gamma^*} - \mathcal{F}_{\eta'\gamma\gamma^*} \xrightarrow{Q^2\to\infty} (0.071 \pm 0.032)\sqrt{2}f_\pi = 0.013 \pm 0.006$$ mainly the mixing angle

$$\delta\mathcal{F}(Q^2 = 112\,\mathrm{GeV}^2) = 0.25^{+0.02}_{-0.02} - 0.23^{+0.03}_{-0.03} = 0.02 \pm 0.02$$ [BaBar, PRD 84. 054001]

Besides,we can study the mixing of $\eta^{(\prime)}$ in $e^+e^- \to \eta\phi$ decays
[Belle:2022fhh]



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- **pQCD is a powerful approach to study an exclusive QCD process**
- **the LCDAs description of hadron oversights the soft transversal dynamics**
- **the universal soft function is actually a product of LCDAs and iTMDs**

- **we study the electromagnetic and transition form factors of light pseudoscalar mesons in the iTMDs-improved pQCD approach**
- **find the better agreements with the data and improve the prediction power down to a few** GeV^2
- **highly precise measurements are highly anticipated**

Thank you for your patience...