# 2025 年轻强子专题研讨会

## Study of the $f_0(1710)$ and $a_0(1710)$ states


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**arXiv: 2306.06395; arXiv: 2402.02539**

**2025.5. 安阳**

# Outline

1. Introduction
2. Study of $D_s^+ \to K_S^0 K^+ \pi^0$
3. Investigation of $D_s^+ \to K_S^0 K_S^0 \pi^+$
4. Summary

# §1. Introduction

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
*California Institute of Technology, Pasadena, California*

Received 4 January 1964

anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc. It is assuming that the lowest baryon configuration $(q\,q\,q)$ gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\,\bar{q})$ similarly gives just **1** and **8**.

AN $SU_3$ MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

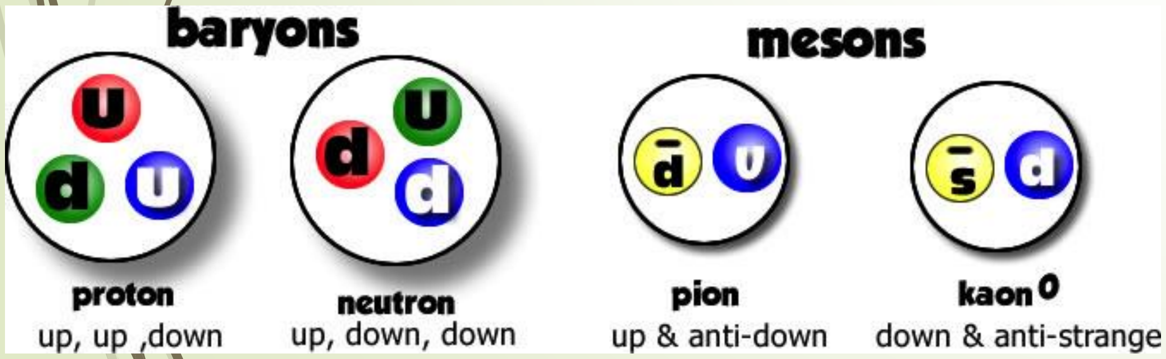
CERN LIBRARIES, GENEVA

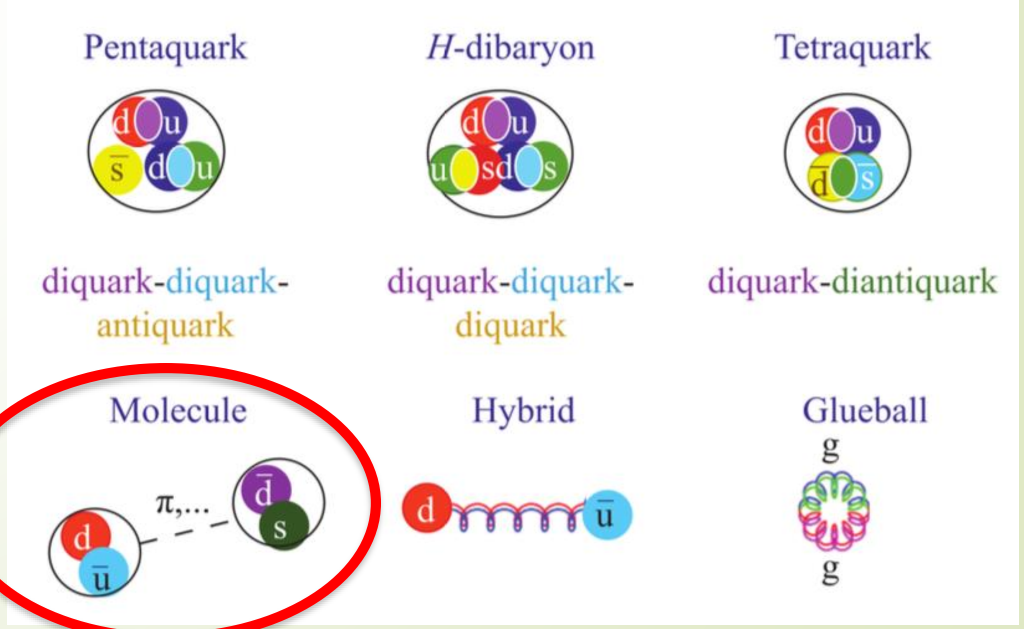
G. Zweig *)
CERN - Geneva

In general, we would expect that baryons are built not only from the product of three aces, AAA, but also from $\bar{A}$AAAA, $\bar{A}\bar{A}$AAAAA, etc., where $\bar{A}$ denotes an anti-ace. Similarly, mesons could be formed from $\bar{A}$A, $\bar{A}\bar{A}$AA etc. For the low mass mesons and baryons we will assume the simplest possibilities, $\bar{A}$A and AAA, that is, "deuces and treys".

Quark model

Exotic States



# Molecular nature

PHYSICAL REVIEW LETTERS **126**, 152001 (2021)

**Explaining the Many Threshold Structures in the Heavy-Quark Hadron Spectrum**

Xiang-Kun Dong[1,2] Feng-Kun Guo[1,2,*] and Bing-Song Zou[1,2,3]

Bing-Song Zou's talk

F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao and B.-S. Zou, Rev. Mod. Phys. 90, 015004 (2018)
H.-X. Chen, W. Chen, X. Liu and S.-L. Zhu, Phys. Rept. 639, 1 (2016)
N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo and C. Z. Yuan, Phys. Rept. 873, 1 (2020)

$f_0$(1710) was **discovered** about **40 years ago**:

*A. Etkin, et al., Phys. Rev. D 25, 1786 (1982)*

*C. Edwards, et al., Phys. Rev. Lett. 48, 458 (1982)*

$K^*\bar{K}^*$ **molecular state:** Coupled channel approach

L. S. Geng and E. Oset, Phys. Rev. D 79, 074009 (2009)

ELSEVIER Nuclear Physics A 620 (1997) 438–456

Chiral symmetry amplitudes in the $S$-wave isoscalar and isovector channels and the $\sigma$, $f_0(980)$, $a_0(980)$ scalar mesons

J.A. Oller, E. Oset

$K\bar{K}$ **molecular state**

But, its isovector partner $a_0$(1710) were **NOT** found for a long time……..

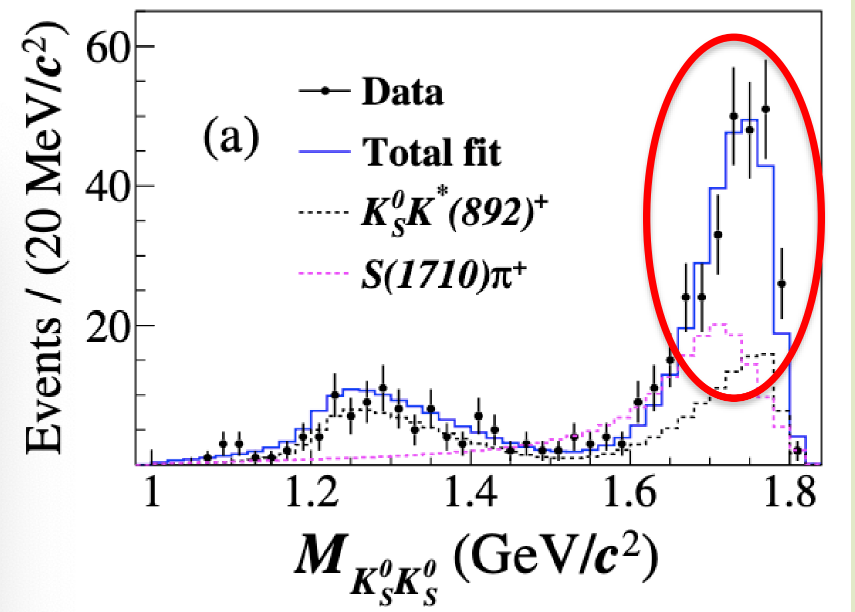
# Recent Findings from BESIII

PHYSICAL REVIEW D **105,** L051103 (2022)

**Letter**

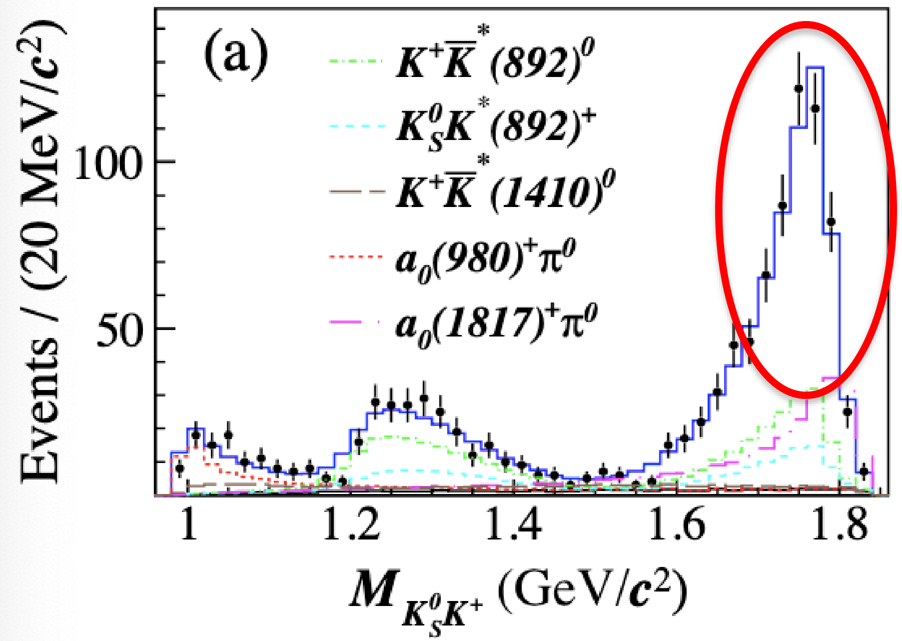
**Study of the decay $D_s^+ \to K_S^0 K_S^0 \pi^+$ and observation of an isovector partner to $f_0(1710)$**

existence of an isospin one partner of the $f_0(1710)$



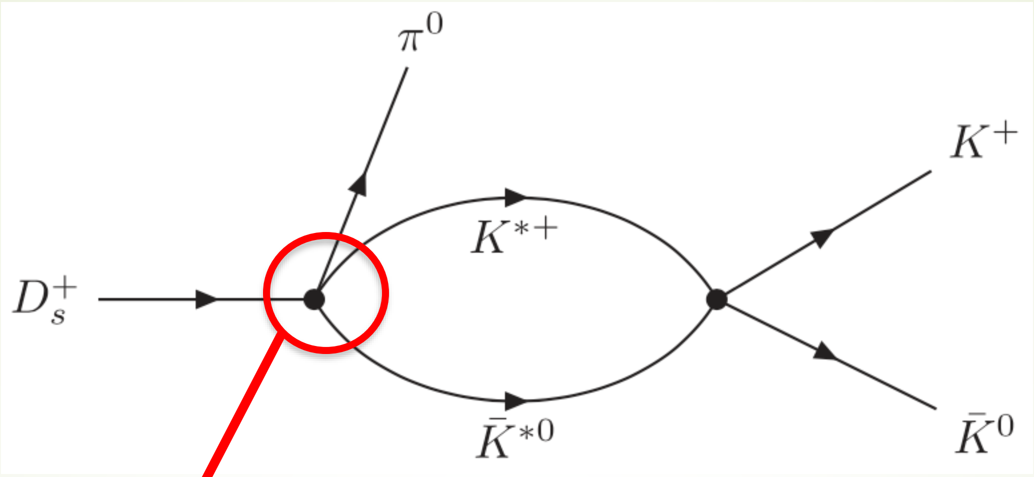

PHYSICAL REVIEW LETTERS **129,** 182001 (2022)

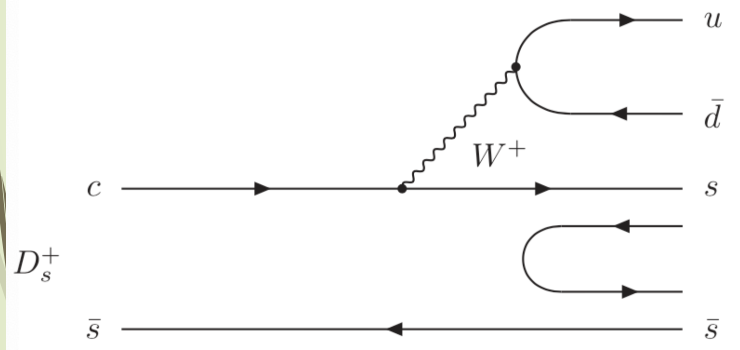
**Observation of an $a_0$-like State with Mass of 1.817 GeV in the Study of $D_s^+ \to K_S^0 K^+ \pi^0$ Decays**

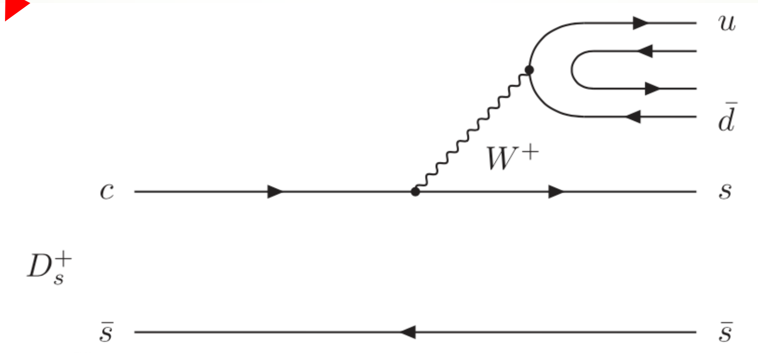


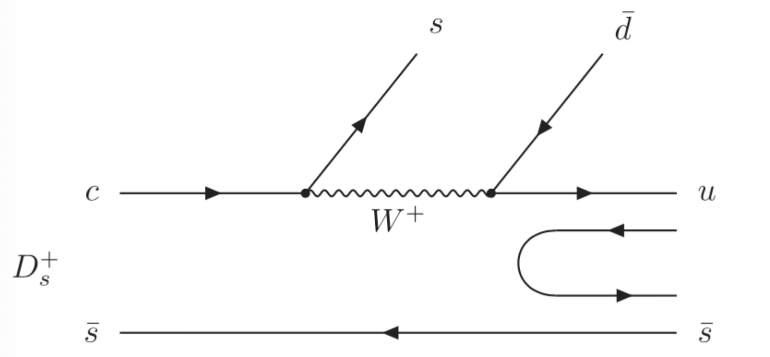
# §2. Study of $D_s^+ \to K_S^0 K^+ \pi^0$

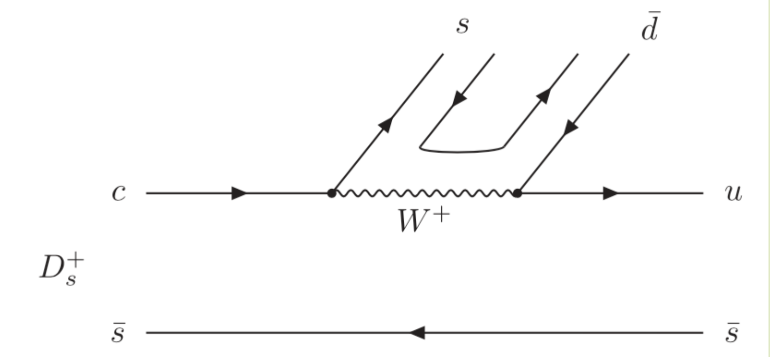
**Final state interaction formalism**



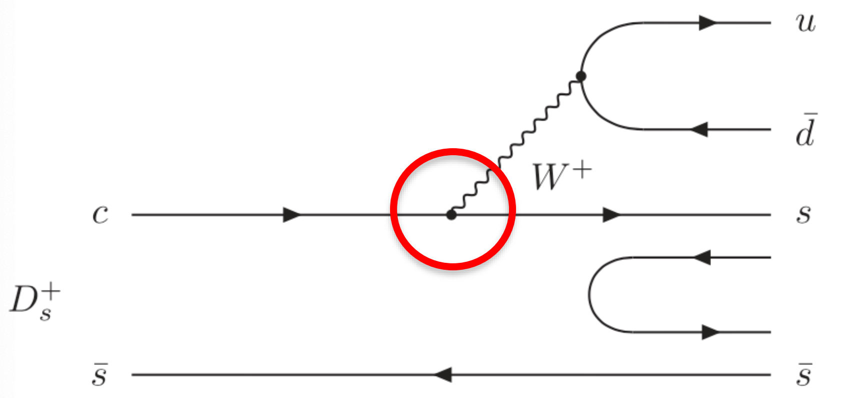



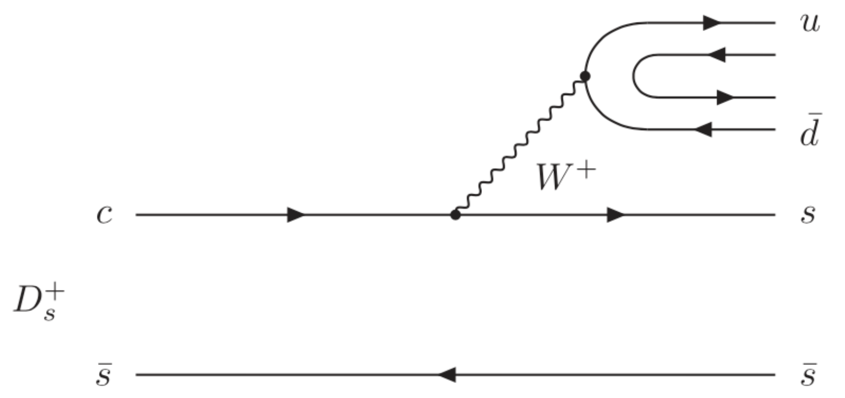



# (1) Quark level: external and internal W-emission mechanisms



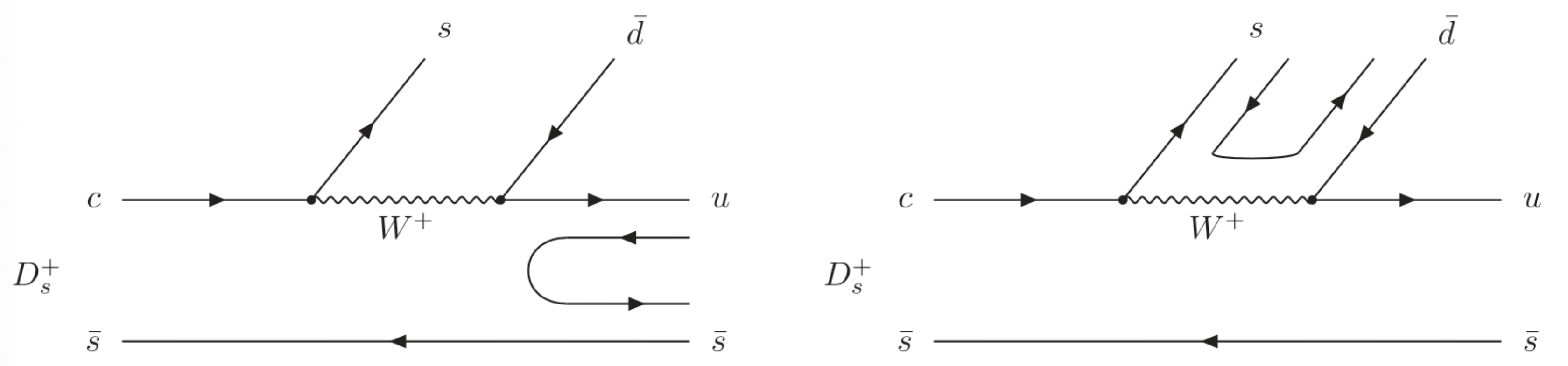




External W-emission mechanisms

$$
\begin{aligned}
|H^{(1a)}\rangle =& \underline{V_P^{(1a)} V_{cs} V_{ud}}(u\bar{d} \to \pi^+)|s(\bar{u}u + \bar{d}d + \bar{s}s)\bar{s}\rangle \\
&+ V_P^{*(1a)} V_{cs} V_{ud}(u\bar{d} \to \rho^+)|s(\bar{u}u + \bar{d}d + \bar{s}s)\bar{s}\rangle \\
=& V_P^{(1a)} V_{cs} V_{ud}\pi^+(M\cdot M)_{33} + V_P^{*(1a)} V_{cs} V_{ud}\rho^+(M\cdot M)_{33}
\end{aligned}
$$

$$
M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}
$$

$$
\begin{aligned}
|H^{(1b)}\rangle =& V_P^{(1b)} V_{cs} V_{ud}(s\bar{s} \to \frac{-2}{\sqrt{6}}\eta)|u(\bar{u}u + \bar{d}d + \bar{s}s)\bar{d}\rangle \\
&+ V_P^{*(1b)} V_{cs} V_{ud}(s\bar{s} \to \phi)|u(\bar{u}u + \bar{d}d + \bar{s}s)\bar{d}\rangle \\
=& V_P^{(1b)} V_{cs} V_{ud}\frac{-2}{\sqrt{6}}\eta(M\cdot M)_{12} + V_P^{*(1b)} V_{cs} V_{ud}\phi(M\cdot M)_{12}
\end{aligned}
$$




## Internal W-emission mechanisms

$$\begin{aligned}|H^{(2a)}\rangle =& V_P^{(2a)} V_{cs} V_{ud} (s\bar{d} \to \bar{K}^0) |u(\bar{u}u + \bar{d}d + \bar{s}s)\bar{s}\rangle \\ &+ V_P^{*(2a)} V_{cs} V_{ud} (s\bar{d} \to \bar{K}^{*0}) |u(\bar{u}u + \bar{d}d + \bar{s}s)\bar{s}\rangle \\ =& V_P^{(2a)} V_{cs} V_{ud} \bar{K}^0 (M \cdot M)_{13} + V_P^{*(2a)} V_{cs} V_{ud} \bar{K}^{*0} (M \cdot M)_{13}\end{aligned}$$

$$\begin{aligned}|H^{(2b)}\rangle =& V_P^{(2b)} V_{cs} V_{ud} (u\bar{s} \to K^+) |s(\bar{u}u + \bar{d}d + \bar{s}s)\bar{d}\rangle \\ &+ V_P^{*(2b)} V_{cs} V_{ud} (u\bar{s} \to K^{*+}) |s(\bar{u}u + \bar{d}d + \bar{s}s)\bar{d}\rangle \\ =& V_P^{(2b)} V_{cs} V_{ud} K^+ (M \cdot M)_{32} + V_P^{*(2b)} V_{cs} V_{ud} K^{*+} (M \cdot M)_{32}\end{aligned}$$

# Hadronization

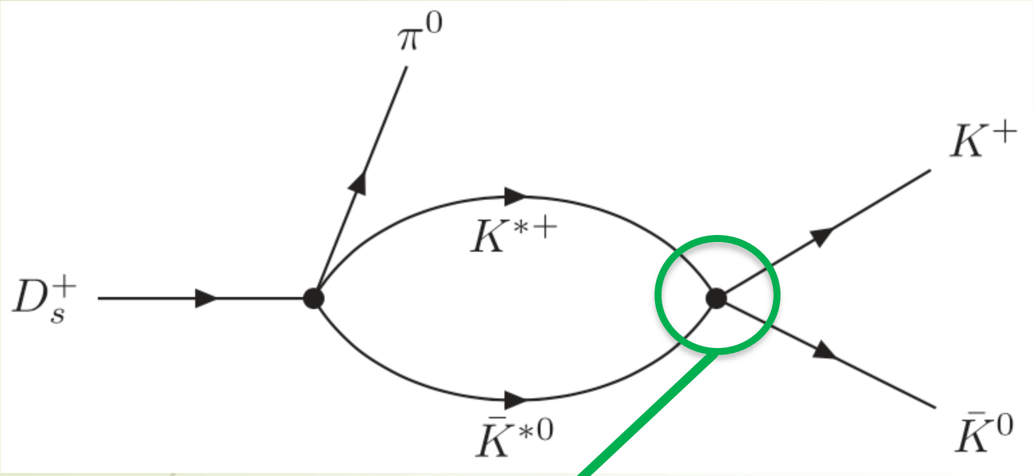
$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}$$

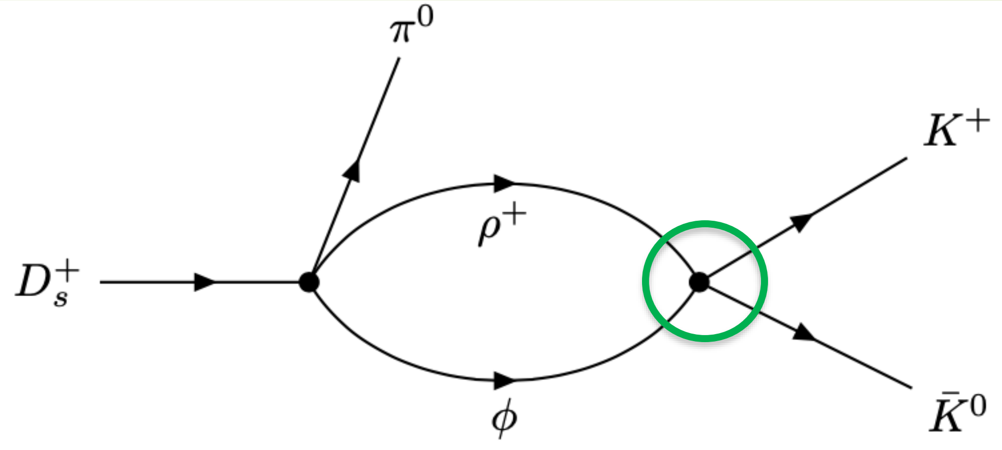
$$P = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

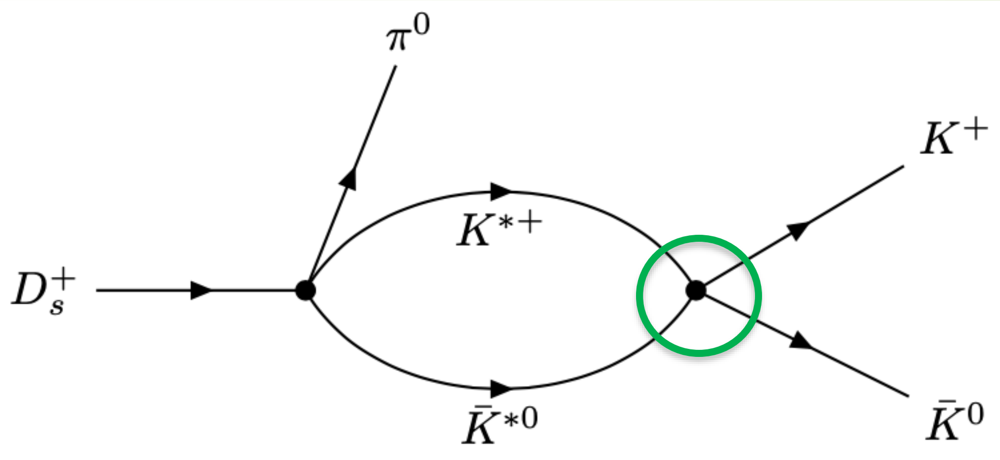
$$V = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}$$

$$\begin{aligned} |H\rangle =& |H^{(1b)}\rangle + |H^{(2a)}\rangle + |H^{(2b)}\rangle \\ =& \frac{1}{\sqrt{2}}(V_P^{*(1b)'} - V_P^{*(1b)})V_{cs}V_{ud}\rho^+\phi\pi^0 + \frac{1}{\sqrt{2}}(V_P^{(2a)} - V_P^{(2b)})V_{cs}V_{ud}K^+\bar{K}^0\pi^0 \\ &+ \frac{1}{\sqrt{2}}(V_P^{*(2a)} - V_P^{*(2b)})V_{cs}V_{ud}K^{*+}\bar{K}^{*0}\pi^0, \\ =& \frac{1}{\sqrt{2}}V_P^{*'}V_{cs}V_{ud}\rho^+\phi\pi^0 + \frac{1}{\sqrt{2}}V_PV_{cs}V_{ud}\boxed{K^+\bar{K}^0\pi^0} + \frac{1}{\sqrt{2}}V_P^{*}V_{cs}V_{ud}K^{*+}\bar{K}^{*0}\pi^0 \end{aligned}$$

## (2) Final state interaction



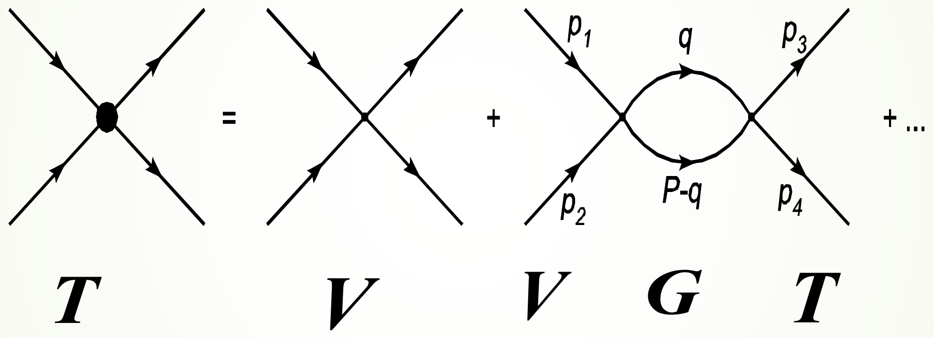

S-wave interactions

$$T = [1 - VG]^{-1} V$$

$$\begin{aligned} & t_{S\text{-wave}}(M_{12})|_{\bar{K}^0 K^+ \pi^0} \\ & = \frac{1}{\sqrt{2}} \mathcal{C}_1 G_{\rho^+\phi}(M_{12}) T_{\rho^+\phi \to K^+\bar{K}^0}(M_{12}) \\ & \boxed{+ \frac{1}{\sqrt{2}} \mathcal{C}_2 + \frac{1}{\sqrt{2}} \mathcal{C}_2 G_{K^+\bar{K}^0}(M_{12}) T_{K^+\bar{K}^0 \to K^+\bar{K}^0}(M_{12})} \\ & + \frac{1}{\sqrt{2}} \mathcal{C}_3 G_{K^{*+}\bar{K}^{*0}}(M_{12}) T_{K^{*+}\bar{K}^{*0} \to K^+\bar{K}^0}(M_{12}), \end{aligned}$$

➨ **Coupled Channel Unitary Approach**： solving Bethe-Salpeter equations, which take on-shell approximation for the loops.

$$T = V + V\,G\,T,\; T = [1 - V\,G]^{-1}\,V$$




D. L. Yao, L. Y. Dai, H. Q. Zheng and Z. Y. Zhou, Rept. Prog. Phys. 84, 076201 (2021)

where V matrix (potentials) can be evaluated from the interaction Lagrangians.

*J. A. Oller and E. Oset, Nucl. Phys. A 620 (1997) 438*
*E. Oset and A. Ramos, Nucl. Phys. A 635 (1998) 99*
*J. A. Oller and U. G. Meißner, Phys. Lett. B 500 (2001) 263*

$G$ is a diagonal matrix with the loop functions of each channels:

$$G_{ll}(s) = i\int \frac{d^4q}{(2\pi)^4} \frac{2M_l}{(P-q)^2 - m_{l1}^2 + i\varepsilon} \frac{1}{q^2 - m_{l2}^2 + i\varepsilon}$$

The coupled channel scattering amplitudes **T matrix satisfy the unitary** :

$$\mathrm{Im}\, T_{ij} = T_{in}\, \sigma_{nn}\, T_{nj}^*$$

$$\sigma_{nn} \equiv \mathrm{Im}\, G_{nn} = -\frac{q_{cm}}{8\pi\sqrt{s}}\theta(s - (m_1 + m_2)^2))$$

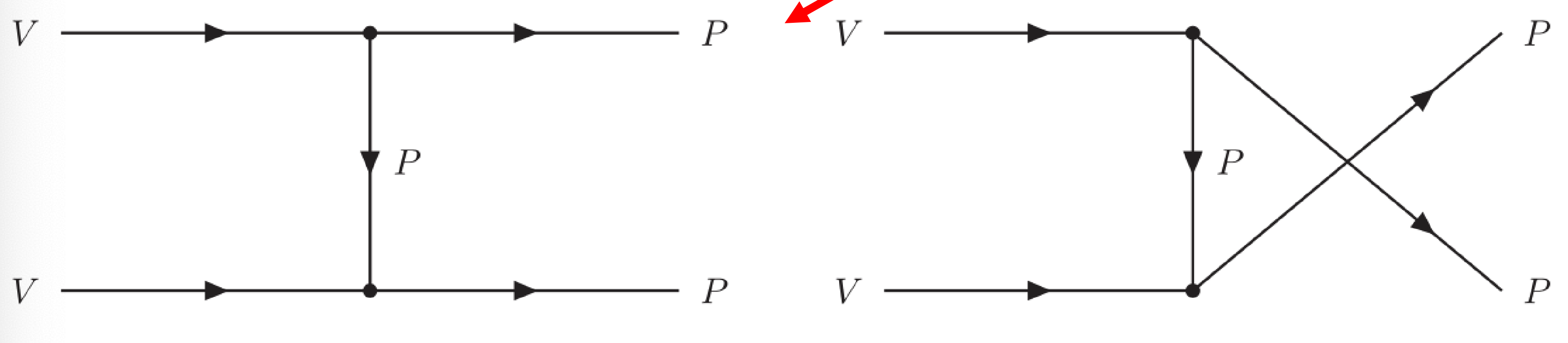
To search the poles of the resonances, we should extrapolate the scattering amplitudes to the second Riemann sheets:

$$G_{ll}^{II}(s) = G_{ll}^{I}(s) + i\,\frac{q_{cm}}{4\pi\sqrt{s}}$$

*Z. Y. Wang, Y. W. Peng, J. Y. Yi, W. C. Luo and* ***CWX****, Phys. Rev. D 107 (2023) 116018.*

$v_{VV\to VV}$ $v_{PP\to PP}$

$v_{VV\to PP}$




(a) t-channel. (b) u-channel.

$$v_{K^{*+}\bar{K}^{*0}\to K^+\bar{K}^0} = \left(\frac{2}{t-m_\pi^2} - \frac{6}{t-m_\eta^2}\right) g^2 \epsilon_{1\mu} k_3^\mu \epsilon_{2\nu} k_4^\nu,$$

$$v_{K^{*+}\bar{K}^{*0}\to \pi^+\eta} = -2\sqrt{6}\left(\frac{g^2}{t-m_K^2}\epsilon_{1\mu}k_3^\mu\epsilon_{2\nu}k_4^\nu + \frac{g^2}{u-m_K^2}\epsilon_{1\mu}k_4^\mu\epsilon_{2\nu}k_3^\nu\right),$$

$$v_{\rho^+\omega\to K^+\bar{K}^0} = -2\sqrt{2}\left(\frac{g^2}{t-m_K^2}\epsilon_{1\mu}k_3^\mu\epsilon_{2\nu}k_4^\nu + \frac{g^2}{u-m_K^2}\epsilon_{1\mu}k_4^\mu\epsilon_{2\nu}k_3^\nu\right),$$

$$v_{\rho^+\omega\to\pi^+\eta} = 0,$$

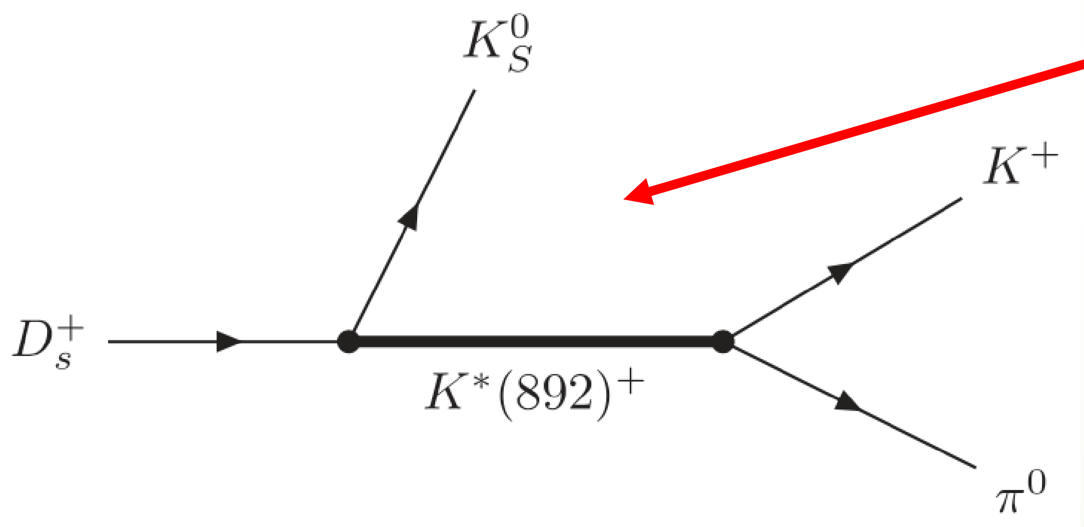
$$v_{\rho^+\phi\to K^+\bar{K}^0} = 4\left(\frac{g^2}{t-m_K^2}\epsilon_{1\mu}k_3^\mu\epsilon_{2\nu}k_4^\nu + \frac{g^2}{u-m_K^2}\epsilon_{1\mu}k_4^\mu\epsilon_{2\nu}k_3^\nu\right),$$

$$v_{\rho^+\phi\to\pi^+\eta} = 0,$$

CWX, J. J. Wu,
arXiv: 2406.08313

## (3) P-wave state contribution

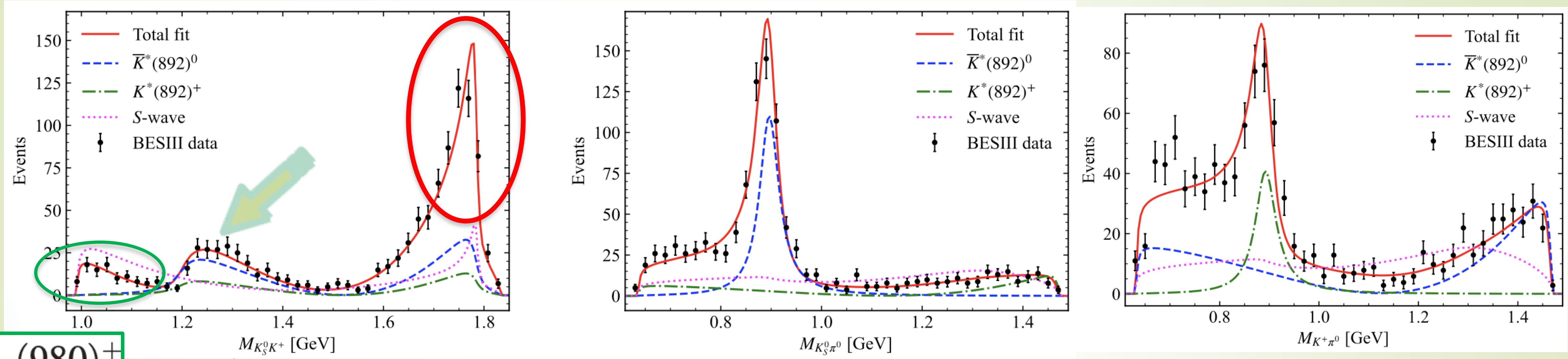
$$\frac{d^2\Gamma}{dM_{12}dM_{13}} = \frac{1}{(2\pi)^3}\frac{M_{12}M_{13}}{8m^3_{D_s^+}} \times \left(\left|t_{S\text{-wave}} + t_{\bar{K}^*(892)^0} + t_{K^*(892)^+}\right|^2\right)$$




$$t_{K^*(892)^+}(M_{12}, M_{13}) = \frac{\mathcal{D}_2 e^{i\phi_{K^*(892)^+}}}{M^2_{23} - m^2_{K^*(892)^+} + im_{K^*(892)^+}\Gamma_{K^*(892)^+}} \times \left[(m^2_{K^+} - m^2_{\pi^0})\frac{m^2_{D_s^+} - m^2_{K_S^0}}{m^2_{K^*(892)^+}} - M^2_{12} + M^2_{13}\right]$$

# (4) results

$a_0(1710)^+$




$a_0(980)^+$

| Parameter | $\mu$ | $\mathcal{C}_1$ | $\mathcal{C}_2$ | $\mathcal{C}_3$ |
|---|---|---|---|---|
| Fit | $0.716 \pm 0.013$ GeV | $47518.79 \pm 7523.18$ | $1595.34 \pm 138.51$ | $46454.25 \pm 3868.04$ |
| | $\mathcal{D}_1$ | $\mathcal{D}_2$ | $\phi_{\bar{K}^*(892)^0}$ | $\phi_{K^*(892)^+}$ |
| | $61.65 \pm 2.33$ | $40.43 \pm 2.95$ | $1.46 \pm 0.12$ | $1.67 \pm 0.15$ |

$$a_{K^{*+}\bar{K}^{*0}} = -1.91, \quad a_{\rho^+\omega} = -1.82, \quad a_{\rho^+\phi} = -2.02, \quad a_{K^+\bar{K}^0} = -1.59, \quad a_{\pi^+\eta} = -1.63.$$

| | This work | Reference [63] | Reference [26] | Reference [29] | Reference [27] |
|---|---|---|---|---|---|
| Parameter | $\mu = 0.716$ | $q_{\max} = 0.931$, $q_{\max} = 1.08$ | $\mu = 1.00$ | $q_{\max} = 1.00$ | $q_{\max} = 1.00$, $g_1 = 4.596$ |
| $a_0(980)$ | $1.0419 + 0.0345i$ | $1.0029 + 0.0567i$, $0.9745 + 0.0573i$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $a_0(1710)$ | $1.7936 + 0.0094i$ | $\cdots$ | $1.780 - 0.066i$ | $1.72 - 0.10i$ | $1.76 \pm 0.03i$ |

# Partial decay widths

| $\Gamma_{a_0(980)^+\to K^+\bar{K}^0}$ | $\Gamma_{a_0(980)^+\to\pi^+\eta}$ |
|---|---|
| 28.38 MeV | 43.60 MeV |

| $\Gamma_{a_0(1710)^+\to\rho^+\omega}$ | $\Gamma_{a_0(1710)^+\to K^+\bar{K}^0}$ | $\Gamma_{a_0(1710)^+\to\pi^+\eta}$ |
|---|---|---|
| 19.65 MeV | 0.54 MeV | 0.05 MeV |

$$\Gamma_{R\to i} = -\frac{1}{16\pi^2}\int_{E_{\min}}^{E_{\max}} dE\,\frac{q_{cmi}}{E^2}4M_R\mathrm{Im}T_{ii}$$

$$\Gamma_{R\to j} = -\frac{1}{16\pi^2}\int_{E_{\min}}^{E_{\max}} dE\,\frac{q_{cmj}}{E^2}4M_R\frac{(\mathrm{Im}T_{ji})^2}{\mathrm{Im}T_{ii}}$$

Eur. Phys. J. C (2022) 82:509
https://doi.org/10.1140/epjc/s10052-022-10460-4

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

**Two dynamical generated $a_0$ resonances by interactions between vector mesons**

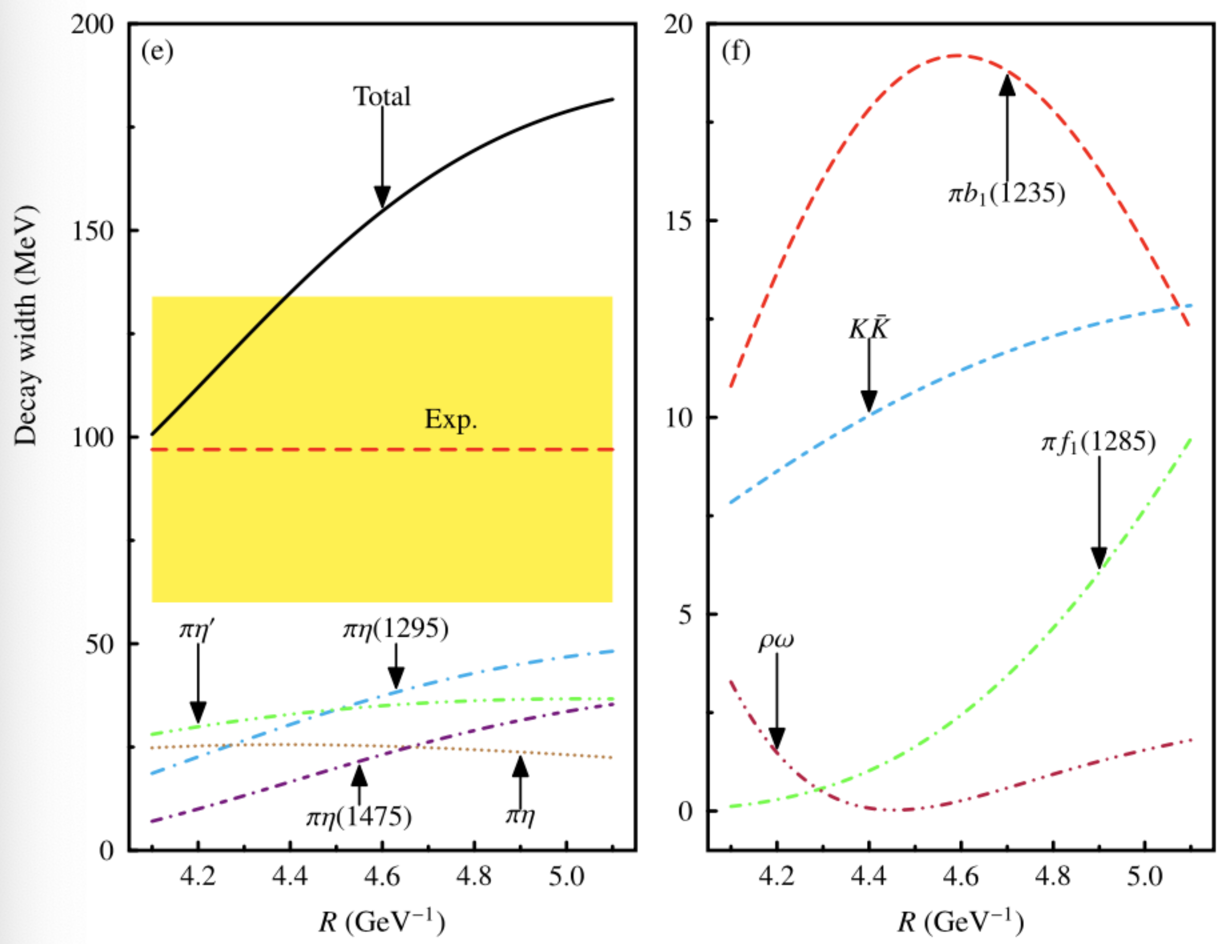
Zheng-Li Wang[1,2,a], Bing-Song Zou[1,2,3,b]

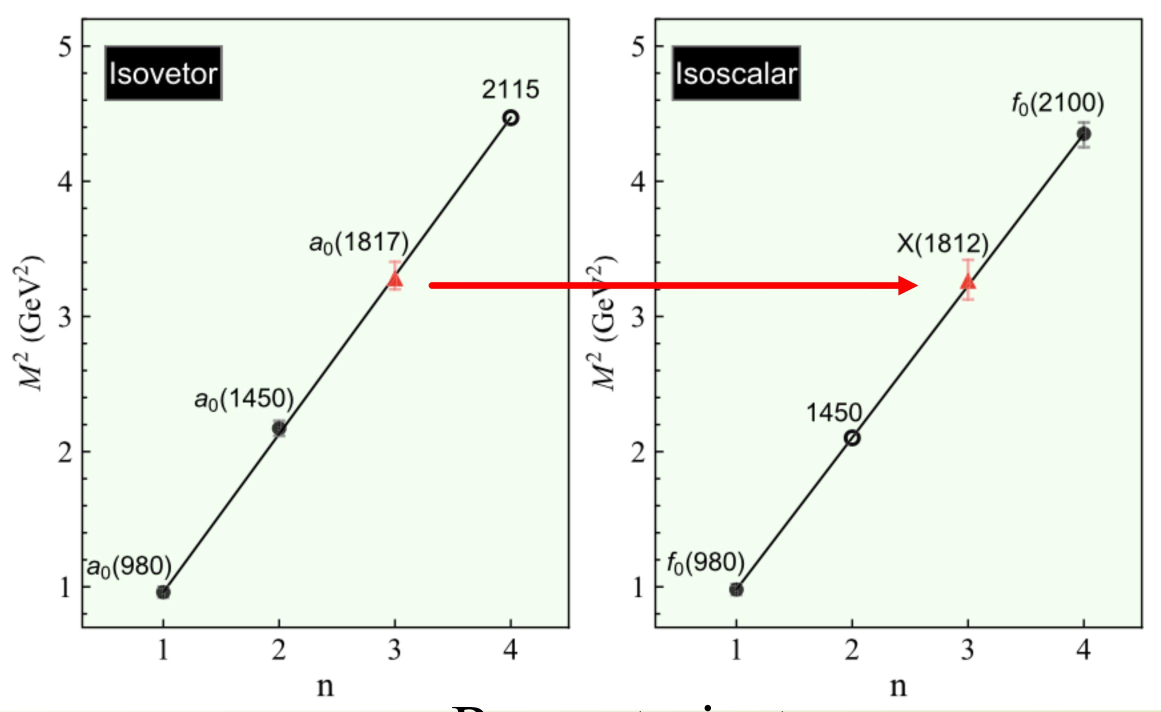
| $\Gamma(K^*\bar{K}^*\to\rho\omega)$ | $\Gamma(K^*\bar{K}^*\to K\bar{K})$ | $\Gamma(K^*\bar{K}^*\to\pi\eta)$ |
|---|---|---|
| 61.0 MeV | 74.4 MeV | 66.9 MeV |
| $\Gamma(\rho\phi\to\rho\omega)$ | $\Gamma(\rho\phi\to K\bar{K})$ | $\Gamma(\rho\phi\to\pi\eta)$ |
| 60.8 MeV | 74.2 MeV | 66.6 MeV |

PHYSICAL REVIEW D **105,** 114014 (2022)

# Newly observed $a_0(1817)$ as the scaling point of constructing the scalar meson spectroscopy

Dan Guo,[1,2,*] Wei Chen,[4,†] Hua-Xing Chen,[5,‡] Xiang Liu,[1,2,3,7,§] and Shi-Lin Zhu[6,∥]




Regge trajectory

*E. Oset, L. R. Dai and L. S. Geng, Sci. Bull. 68 (2023) 243-246.*

## Branching ratios

$$\frac{\mathcal{B}(D_s^+ \to K^*(892)^+ K_S^0, K^*(892)^+ \to K^+\pi^0)}{\mathcal{B}(D_s^+ \to \bar{K}^*(892)^0 K^+, \bar{K}^*(892)^0 \to K_S^0\pi^0)} = 0.40^{+0.002}_{-0.003}$$

$$\frac{\mathcal{B}(D_s^+ \to a_0(980)^+ \pi^0, a_0(980)^+ \to K_S^0 K^+)}{\mathcal{B}(D_s^+ \to \bar{K}^*(892)^0 K^+, \bar{K}^*(892)^0 \to K_S^0\pi^0)} = 0.53^{+0.06}_{-0.08},$$

$$\frac{\mathcal{B}(D_s^+ \to a_0(1710)^+ \pi^0, a_0(1710)^+ \to K_S^0 K^+)}{\mathcal{B}(D_s^+ \to \bar{K}^*(892)^0 K^+, \bar{K}^*(892)^0 \to K_S^0\pi^0)} = 0.41^{+0.04}_{-0.05}.$$

$$\mathcal{B}(D_s^+ \to \bar{K}^*(892)^0 K^+, \bar{K}^*(892)^0 \to K_S^0\pi^0) = (4.77 \pm 0.38 \pm 0.32) \times 10^{-3}$$

Our predictions

$$\mathcal{B}(D_s^+ \to K^*(892)^+ K_S^0, K^*(892)^+ \to K^+\pi^0) = \underline{(1.91 \pm 0.20^{+0.01}_{-0.01}) \times 10^{-3}}$$

$$\mathcal{B}(D_s^+ \to a_0(980)^+ \pi^0, a_0(980)^+ \to K_S^0 K^+) = \boxed{(2.53 \pm 0.26^{+0.27}_{-0.38}) \times 10^{-3}}$$

$$\mathcal{B}(D_s^+ \to a_0(1710)^+ \pi^0, a_0(1710)^+ \to K_S^0 K^+) = \boxed{(1.94 \pm 0.20^{+0.18}_{-0.24}) \times 10^{-3}}$$

BESIII measurements

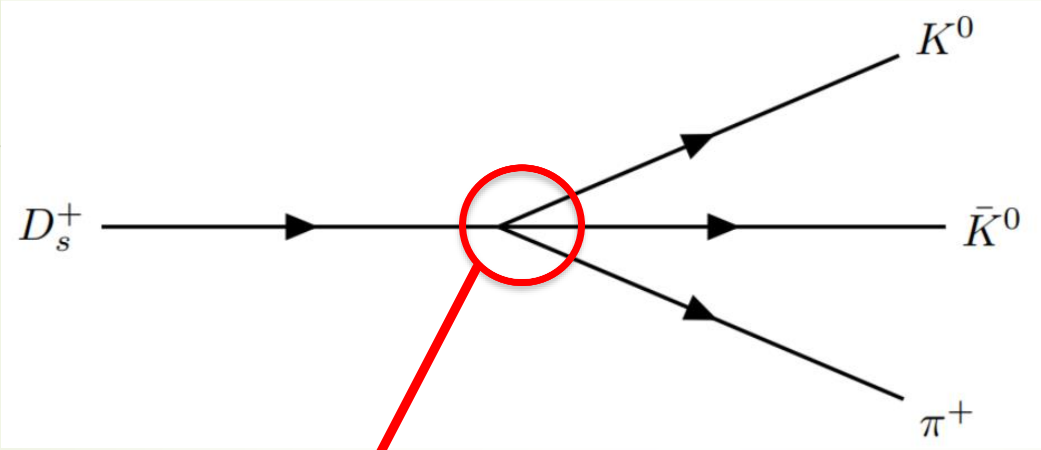
$$\mathcal{B}(D_s^+ \to K^*(892)^+ K_S^0, K^*(892)^+ \to K^+\pi^0) = \underline{(2.03 \pm 0.26 \pm 0.20) \times 10^{-3}}$$

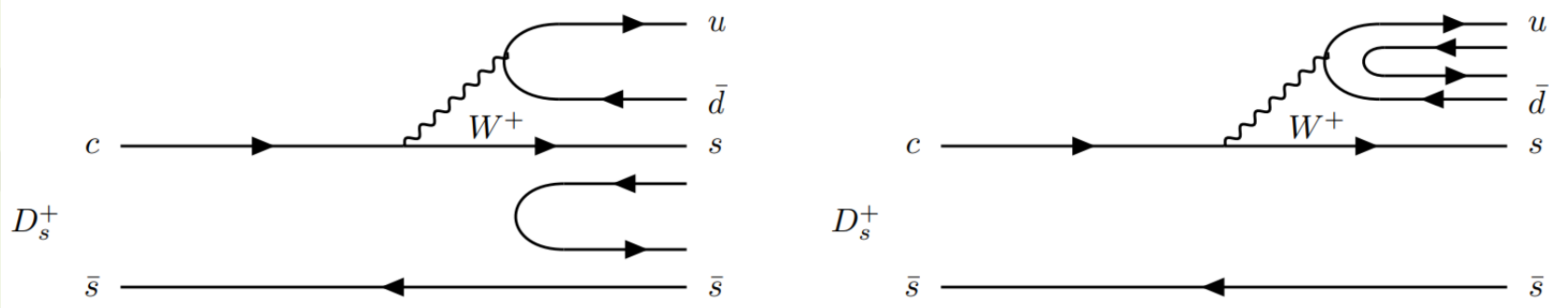
$$\mathcal{B}(D_s^+ \to a_0(980)^+ \pi^0, a_0(980)^+ \to K_S^0 K^+) = \boxed{(1.12 \pm 0.25 \pm 0.27) \times 10^{-3}}$$

$$\mathcal{B}(D_s^+ \to a_0(1710)^+ \pi^0, a_0(1710)^+ \to K_S^0 K^+) = \boxed{(3.44 \pm 0.52 \pm 0.32) \times 10^{-3}}$$

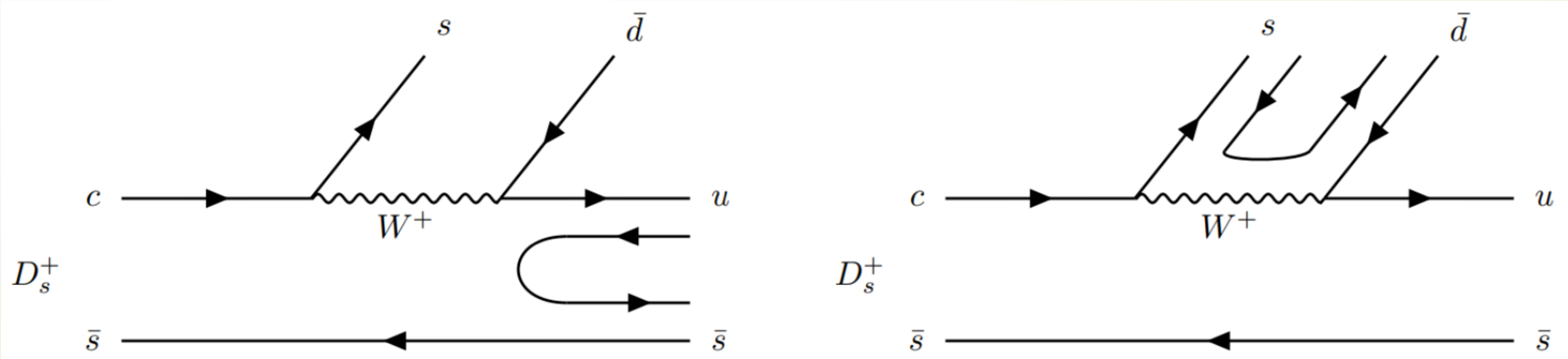
# §3. Investigation of $D_s^+ \to K_S^0 K_S^0 \pi^+$

**Final state interaction formalism**

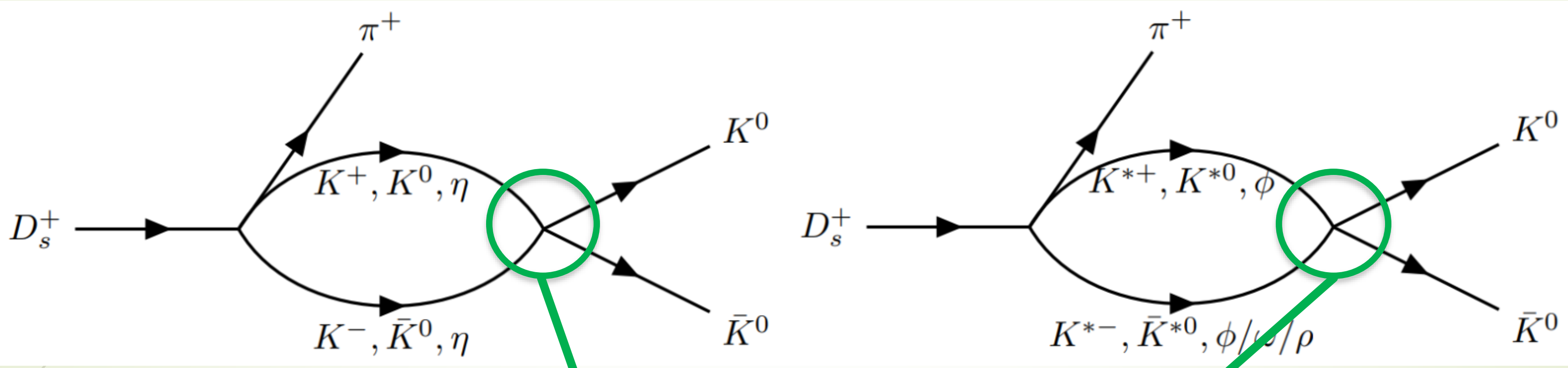



External W-emission mechanisms




Internal W-emission mechanisms



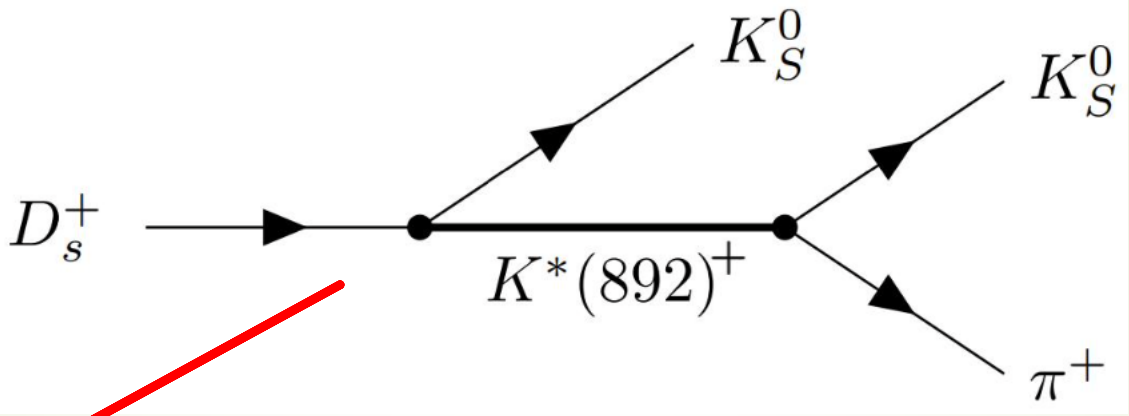

S-wave final state interactions

$$T = [1 - VG]^{-1} V$$

$$\begin{aligned} t(M_{12})|_{K^0\bar{K}^0\pi^+} = & C_1 G_{K^+K^-}(M_{12}) T_{K^+K^-\to K^0\bar{K}^0}(M_{12}) + C_2 + C_2 G_{K^0\bar{K}^0}(M_{12}) T_{K^0\bar{K}^0\to K^0\bar{K}^0}(M_{12}) \\ & + \frac{2}{3} C_3 G_{\eta\eta}(M_{12}) T_{\eta\eta\to K^0\bar{K}^0}(M_{12}) + C_4 G_{K^{*+}K^{*-}}(M_{12}) T_{K^{*+}K^{*-}\to K^0\bar{K}^0}(M_{12}) \\ & + C_5 G_{K^{*0}\bar{K}^{*0}}(M_{12}) T_{K^{*0}\bar{K}^{*0}\to K^0\bar{K}^0}(M_{12}) + C_6 G_{\phi\phi}(M_{12}) T_{\phi\phi\to K^0\bar{K}^0}(M_{12}) \\ & + \frac{1}{\sqrt{2}} C_7 G_{\omega\phi}(M_{12}) T_{\omega\phi\to K^0\bar{K}^0}(M_{12}) + \frac{1}{\sqrt{2}} C_8 G_{\rho^0\phi}(M_{12}) T_{\rho^0\phi\to K^0\bar{K}^0}(M_{12}), \end{aligned}$$

Eight channels contributed

# Also P-wave resonance contribution



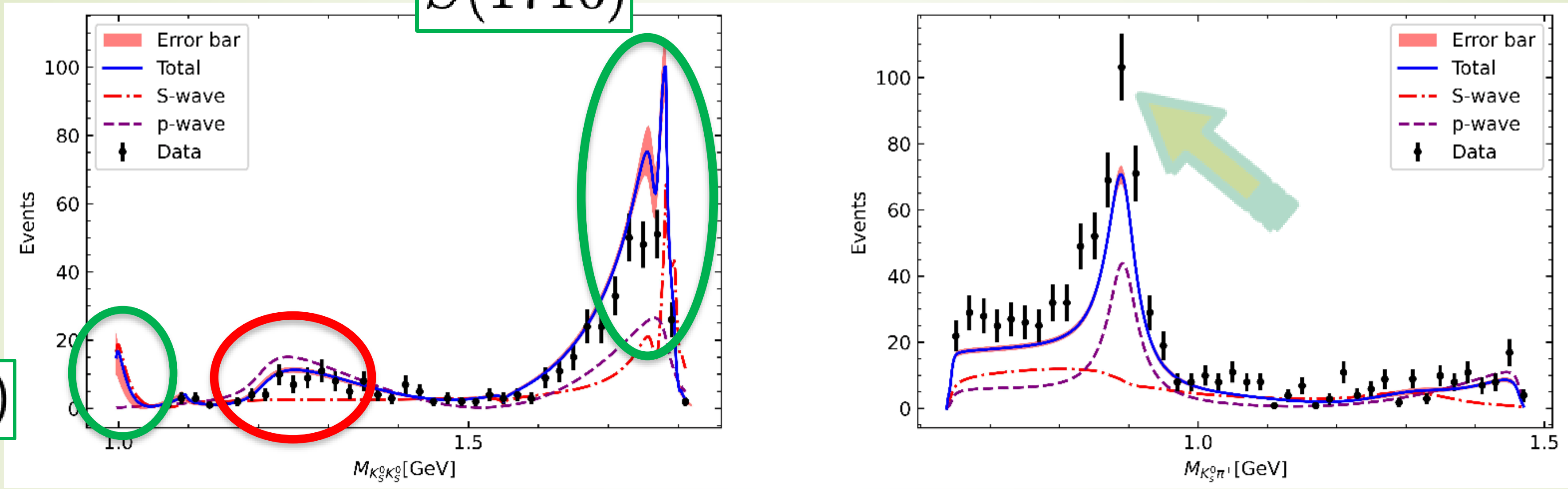

$$t_{K^*(892)^+}(M_{12},M_{23}) = \frac{\mathcal{D}e^{i\alpha_{K^*(892)^+}}}{M_{23}^2 - M_{K^*(892)^+}^2 + iM_{K^*(892)^+}\Gamma_{K^*(892)^+}} \times \left[\frac{(m_{D_s^+}^2 - m_{K_s^0}^2)(m_{K_s^0}^2 - m_{\pi^+}^2)}{M_{K^*(892)^+}^2} - M_{12}^2 + M_{13}^2\right]$$

$$\frac{d^2\Gamma}{dM_{12}dM_{23}} = \frac{1}{(2\pi)^3}\frac{M_{12}M_{23}}{8m_{D_s^+}^3}\frac{1}{2}\mid\mathcal{M}\mid^2$$

$$\mathcal{M} = t(M_{12})|_{K_S^0K_S^0\pi^+} + t_{K^*(892)^+}(M_{12},M_{23}) + \underline{(1\leftrightarrow 2)}$$

# Combined fit

$S(1710)$



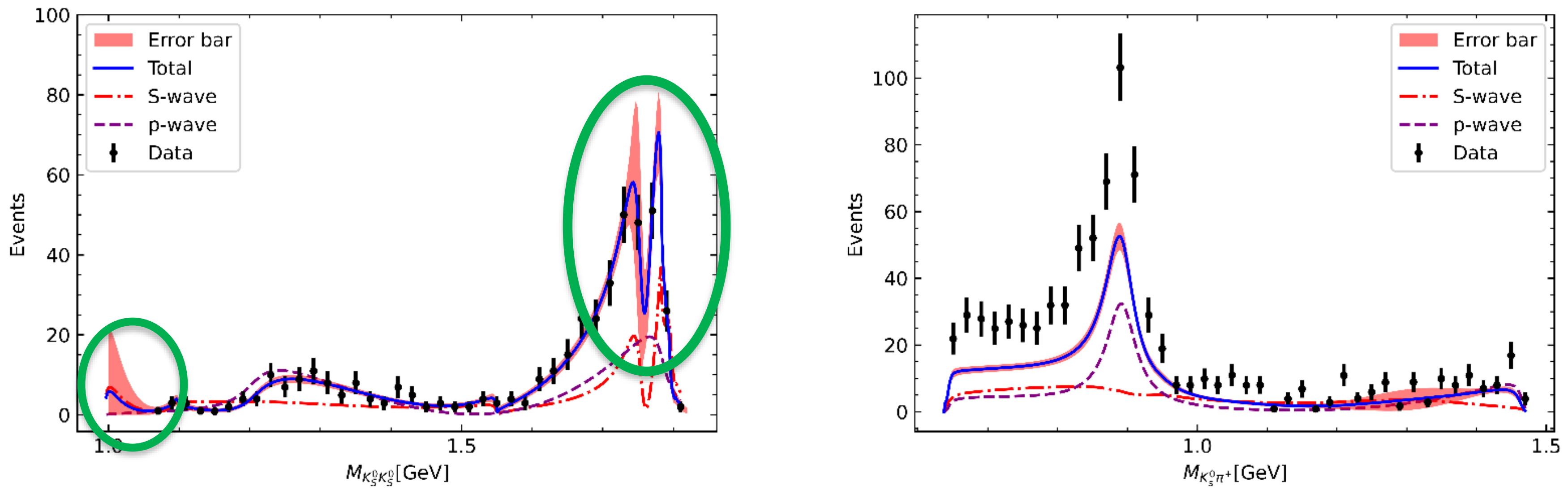

$S(980)$

Eleven coupled channels

$$(1)K^*\bar{K}^*\ (2)\rho\rho\ (3)\omega\omega\ (4)\omega\phi\ (5)\phi\phi\ (6)\pi\pi\ (7)K\bar{K}\ (8)\eta\eta$$

$I = 0$

# Only fit $K_S^0K_S^0$



| Parameters | $\mu$ | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|---|
| Fit | $0.648 \pm 0.01$ GeV | $8640.90 \pm 1115.80$ | $2980.71 \pm 638.37$ | $-1902.86 \pm 293.27$ |
| Parameters | $C_4$ | $C_5$ | $C_6$ | $C_7$ |
| Fit | $56906.35 \pm 10869.67$ | $-13433.15 \pm 5017.76$ | $-58284.22 \pm 7319.04$ | $102835.76 \pm 23333.56$ |
| Parameters | $C_8$ | $D$ | $\alpha_{K^*(892)^+}$ | $\chi^2/dof.$ |
| Fit | $202807.71 \pm 30750.45$ | $54.8 \pm 2.0$ | $0.0024 \pm 4.30$ | 2.55 |

| | This work | Ref. [64] | Ref. [96] | Ref. [43] | Ref. [62] | Ref. [44] |
|---|---|---|---|---|---|---|
| Parameters | $\mu = 0.648$ | $\mu = 0.716$ | $q_{max} = 0.931$ | $\mu = 1.0$ | $q_{max} = 1.0$ | $q_{max} = 1.0$ |
| $a_0(980)$ | $1.0598 + 0.024i$ | $1.0419 + 0.0345i$ | $1.0029 + 0.0567i$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $f_0(980)$ | $0.9912 + 0.003i$ | $\cdots$ | $0.9912 + 0.0135i$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $a_0(1710)$ | $1.7981 + 0.0018i$ | $1.7936 + 0.0094i$ | $\cdots$ | $1.780 - 0.066i$ | $1.72 - 0.010i$ | $1.76 \pm 0.03i$ |
| $f_0(1710)$ | $1.7676 + 0.0093i$ | $\cdots$ | $\cdots$ | $1.726 - 0.014i$ | $\cdots$ | $\cdots$ |

Consistent with our previous results in $D_s^+ \to K_S^0 K^+ \pi^0$

## Our results

$$\mathcal{B}(D_s^+ \to S(980)\pi^+, S(980) \to K_S^0 K_S^0) = (0.36 \pm 0.04^{+0.10}_{-0.06}) \times 10^{-3}$$
$$\mathcal{B}(D_s^+ \to S(1710)\pi^+, S(1710) \to K_S^0 K_S^0) = \underline{(1.66 \pm 0.17^{+1.38}_{-0.89}) \times 10^{-3}}$$

## BESIII measurements

$$B(D_s^+ \to K^*(892)K_S^0 \to K_S^0 K_S^0 \pi^+) = (3.0 \pm 0.3 \pm 0.1) \times 10^{-3};$$
$$B(D_s^+ \to S(1710)\pi^+ \to K_S^0 K_S^0 \pi^+) = \underline{(3.1 \pm 0.3 \pm 0.1) \times 10^{-3}}.$$

*Y. W. Peng, W. Liang, X. Xiong and* ***CWX****, Phys. Rev. D 110 (2024) 036013.*

L. R. Dai, E. Oset and L. S. Geng, Eur. Phys. J. C 82, 225 (2022)
X. Zhu, D. M. Li, E. Wang, L. S. Geng and J. J. Xie, Phys. Rev. D 105, 116010 (2022)

$$T_{K^+K^-\to K^0\bar{K}^0} = \frac{1}{2}\left(T^{I=0}_{K\bar{K}\to K\bar{K}} - T^{I=1}_{K\bar{K}\to K\bar{K}}\right)$$
$$T_{K^0\bar{K}^0\to K^0\bar{K}^0} = \frac{1}{2}\left(T^{I=0}_{K\bar{K}\to K\bar{K}} + T^{I=1}_{K\bar{K}\to K\bar{K}}\right)$$

$C_1 \neq C_2$ and $C_4 \neq C_5$

# §4. Summary

- We use the final state interaction formalism to investigate the Ds three-body weak decays
- In the final state interaction, $f_0/a_0(1710)$ and/or $f_0/a_0(980)$ generated (molecular nature)
- Related branching ratios are evaluated, some of which are consistent with the experiments.

**Hope future experiments and theories bring more clarifications on these issues…….**

第八届强子谱和强子结构研讨会

Jul 11 – 15, 2025

Asia/Shanghai timezone

会议计划于2025年7月11日至15日在桂林市桂林宾馆召开，其中11日报到注册。会议统一安排食宿，费用自理。会议收取注册费，教师及博士后1500元/人，学生1000元/人。

会议网址：https://indico.ihep.ac.cn/event/24044/ ，注册截止时间为6月20日。

*Thanks for your attention!*

感谢大家的聆听！