2025年轻强子专题研讨会

2025.5.8 –12

$\bar{K}N$ interaction and $\Lambda(1405)$ in a renormalizable framework of Chiral EFT

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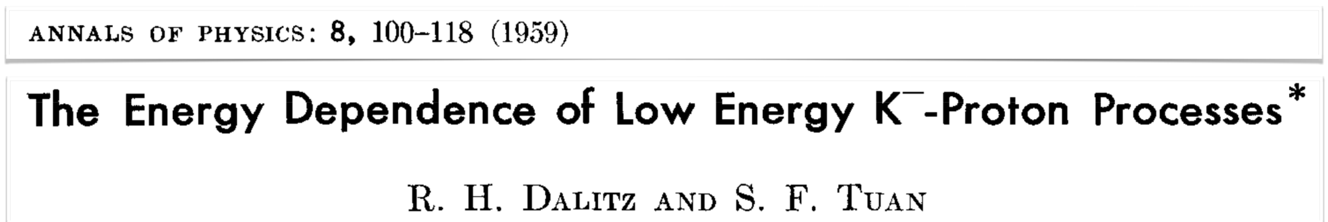
OUTLINE

- Introduction
- Theoretical framework
- Results and discussion
- Summary

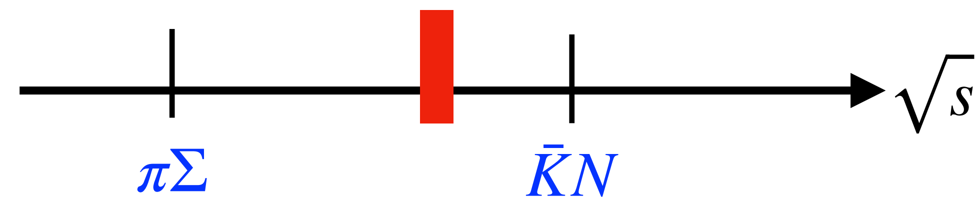
$\bar{K}N$ interaction

- $\bar{K}N$ interaction is strongly attractive (I=0)
  - **Exotic** $\Lambda(1405)$ resonance → $\bar{K}N$ amplitude in free space

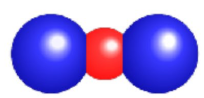

ANNALS OF PHYSICS: 8, 100–118 (1959)

The Energy Dependence of Low Energy K⁻-Proton Processes*

R. H. DALITZ AND S. F. TUAN

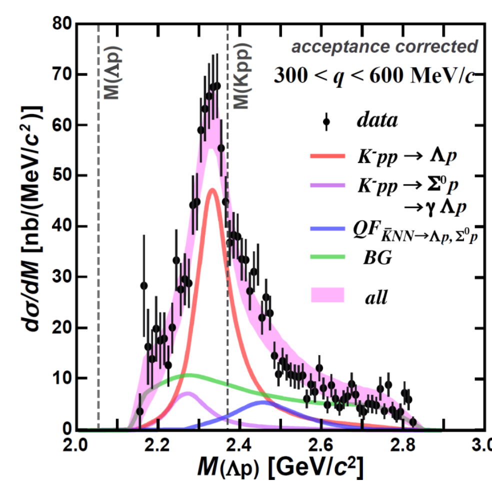


  - **New form** of nuclei/atoms: $\bar{K}NN$, $\bar{K}NNN$, multi-$\bar{K}$ N/A J-PARC, DAΦNE, GSI…



E15@J-PARC,

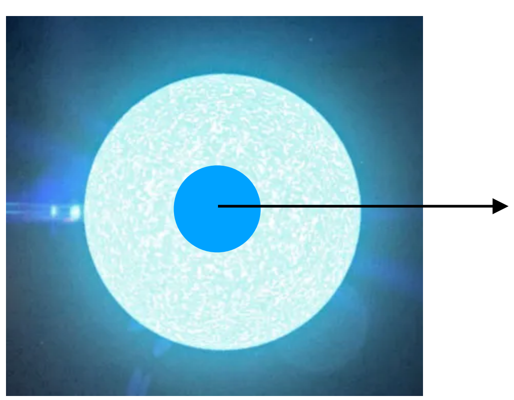
PRC102(2020)044002



$B_{K^-pp} \sim 40$ MeV

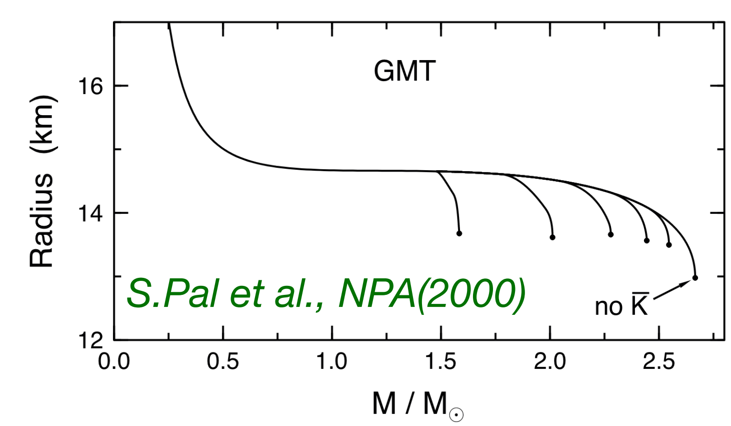
$\Gamma_{K^-pp} \sim 100$ MeV

  - **Kaon-condensate** could change EoS of neutron star



Inner core

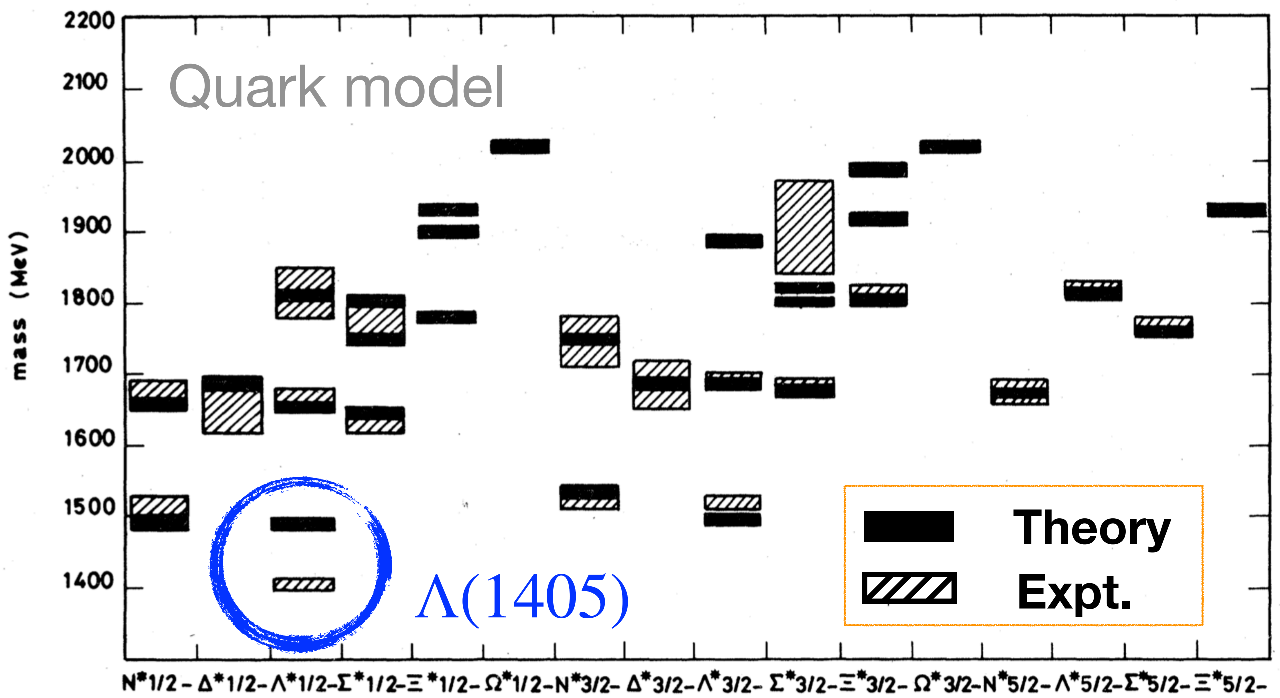
- hyperonic matter
- Kaon condensate
- Quark matter



Play an important role in the strangeness nuclear physics

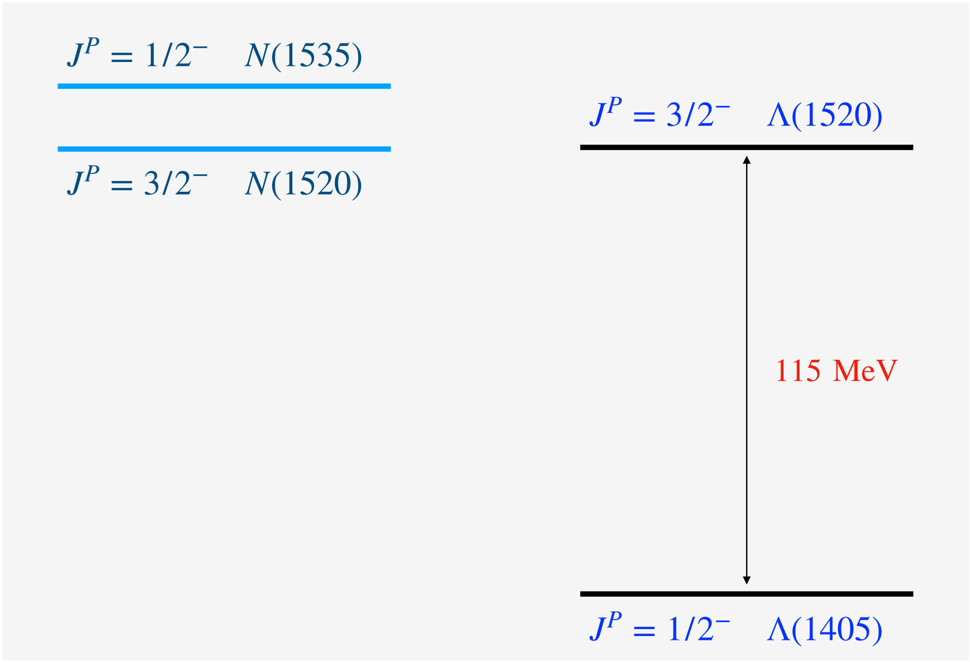
$\Lambda(1405)$ resonance

- $\Lambda(1405)$ state is an exotic candidate



N. Isgur and G. Karl, Phys.Rev.D 18 (1978) 4187

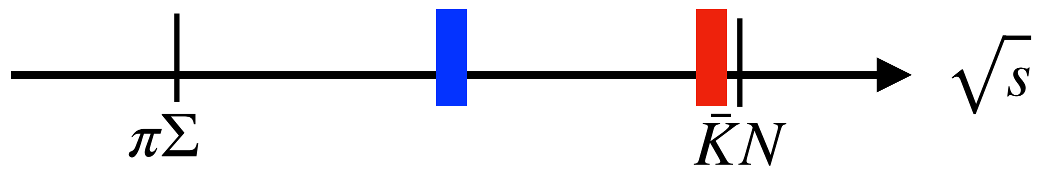
- **Lightest excited baryon with** $J^P = \frac{1}{2}^-$



- Variety of theoretical studies
  - ✓ Chiral SU(3) quark model *F. Huang, PRC2007…*
  - ✓ QCD sum rules *L.S. Kisslinger, EPJA2011…*
  - ✓ Phenomenological potential model *A. Cieplý, NPA2015…*
  - ✓ Skyrme model *T. Ezoe, PRD2020…*
  - ✓ Hamiltonian effective field theory *Z.-W. Liu, PRD2017…*
  - ✓ Chiral unitary approach *N.Kaiser,NPA1995; E.Oset,NPA1998; J.A.Oller&U.-G.Meißner,PLB2001…*

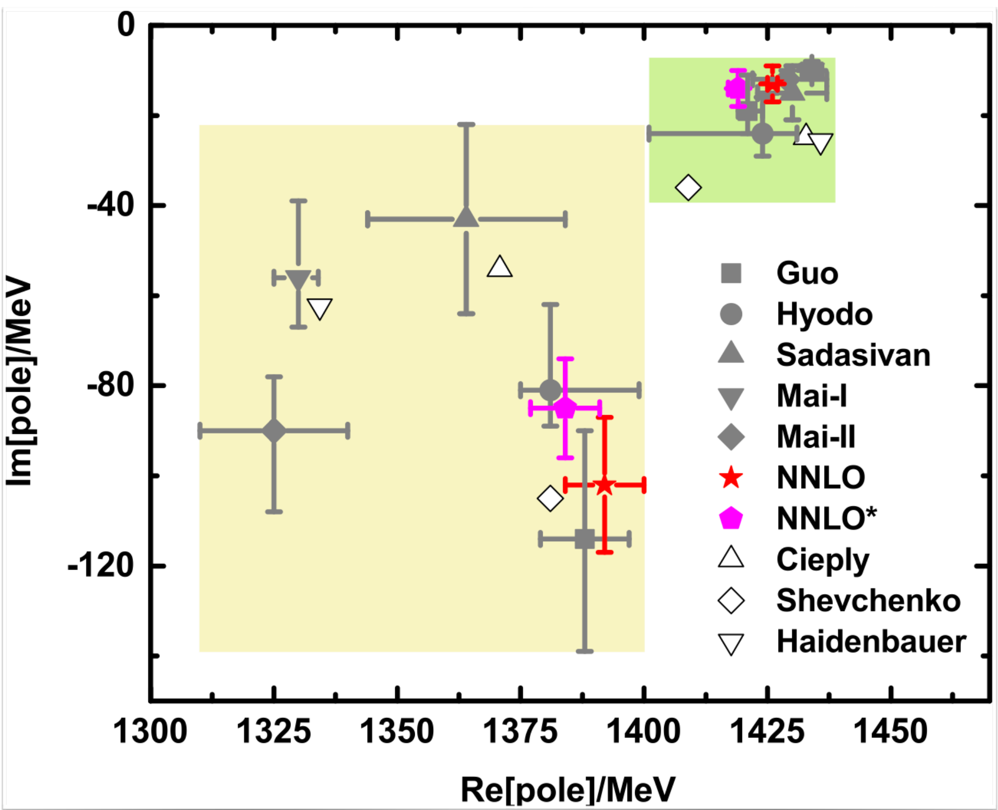
Structure of $\Lambda(1405)$ resonance

☐ Double-pole predicted by **chiral unitary approach**



$\Lambda(1405)\ 1/2^-$ — $I(J^P) = 0(\frac{1}{2}^-)$ Status: ****

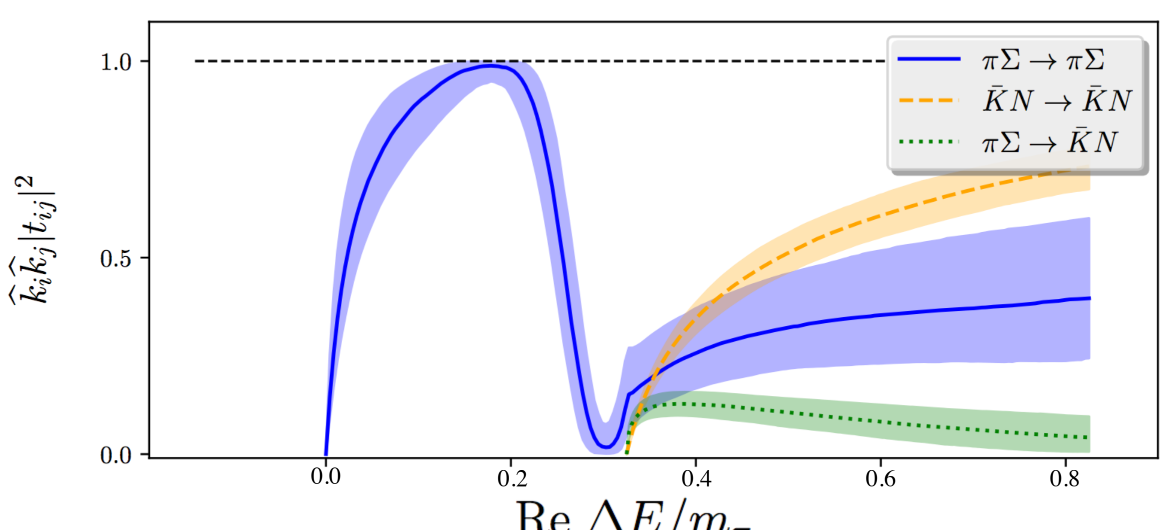
$\Lambda(1380)\ 1/2^-$ — $J^P = \frac{1}{2}^-$ Status: **



J.-X. Lu, et al., PRL130(2023)071902

- ✓ Pole 1: Λ(1405) is around 1420 MeV
- ✓ Pole 2: Λ(1380) needs further studies to fix its position

☐ Double-pole structure verified by LQCD



$m_\pi \approx 200\ \text{MeV}, m_K \approx 487\ \text{MeV}$

$$\text{Lower Pole}: \ E_1 = 1392(9)_{\text{stat}}(2)_{\text{model}}(16)_{\text{a}}\ \text{MeV}$$

$$\text{Higher Pole}: \ E_2 = 1455(13)_{\text{stat}}(2)_{\text{model}}(17)_{\text{a}} - i11.5(4.4)_{\text{stat}}(4.0)_{\text{model}}(0.1)_{\text{a}}\ \text{MeV}$$

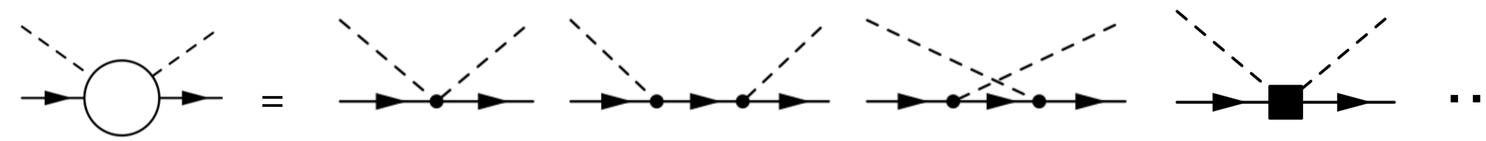
Baryon Scattering Coll., PRL132(2024)051901

Chiral Unitary approach

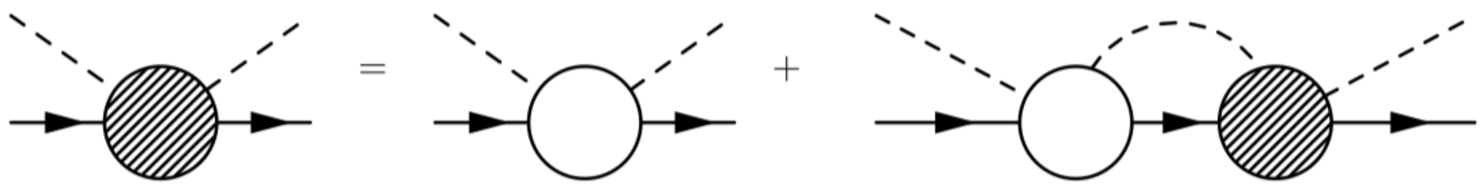
- **Chiral symmetry** of low-energy QCD + **Unitary Relation**

  *J.A.Oller et al.,PPNP45(2000)157-242; T.Hyodo et al.,PPNP*120 (2021)103868 …

- Interaction kernel V: calculate in ChPT order by order
  - Leading, next-to-leading order, …



- Scattering T-matrix: solve scattering equations



  - Lippmann-Schwinge equation or Bethe-Salpeter equation

$$T(p',p) = V(p',p) + i\int \frac{d^4k}{(2\pi)^4} V(p',k)\, G(k)\, T(k,p)$$

- On-shell factorization $\rightarrow V(p',p) + V(p',p)\left(i\int \frac{d^4k}{(2\pi)^4} G(k) \right) T(p',p)$

  Neglecting off-shell effect

  → cause troubles in the study of three-body interaction?

- Finite cutoff or subtraction constant to renormalize the loop integral

$$G^R(E,\Lambda) \text{ or } G^R(E,\alpha_i)$$

Cutoff / Model dependence

In this work

- Facing the rapid progress of precision experiments, **a model-independent formalism would be needed** ALICE, AMADEUS, J-PARC, STAR…
- We tentatively propose **a renormalizable framework** of Chiral EFT for meson-baryon scattering by using **time-ordered perturbation theory** with the covariant chiral Lagrangians
  - Apply to the SU(2) sector: pion-nucleon scattering

    XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C80 (2020) 406
  - Extend to the SU(3) sector: $\bar{K}N$ scattering and $\Lambda(1405)$ state

    XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C81 (2021) 582
  - Investigate the light-quark mass dependence of $\Lambda(1405)$

    XLR, Phys. Lett. B 855 (2024) 138802

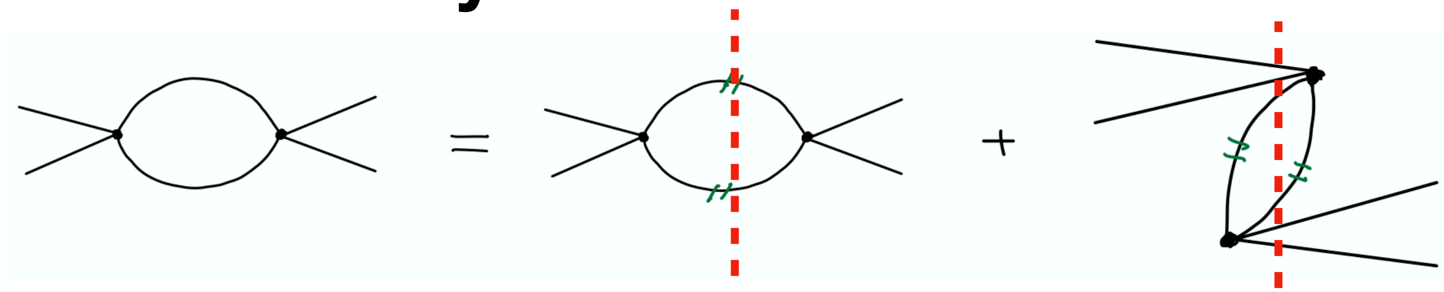
Theoretical framework

Time-ordered perturbation theory

Definition

S. Weinberg, Phys.Rev.150(1966)1313
G.F. Sterman, "An introduction to quantum field theory", Cambridge (1993)

- Re-express the Feynman integral in a form that makes the connection with on-mass-shell (off-energy shell) state explicit.
  - ✓ Instead the propagators for internal lines as the energy denominators for intermediate states
- **TOPT or old-fashioned perturbation theory**



Advantages

- Explicitly show the unitarity
- Easily to tell the contributions of a particular diagram

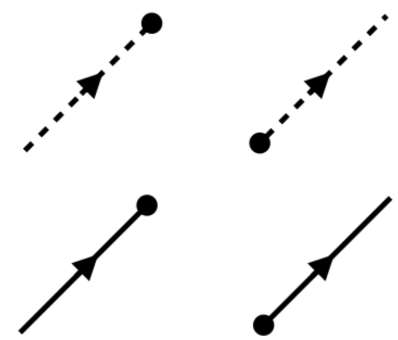
Obtain the rules for time-ordered diagrams

- Perform Feynman integrations over the zeroth components of the loop momenta
- Decompose Feynman diagram into sums of time-ordered diagrams
- Match to the rules of time-ordered diagrams

Diagrammatic rules in TOPT

XLR, PoS(CD2021)007

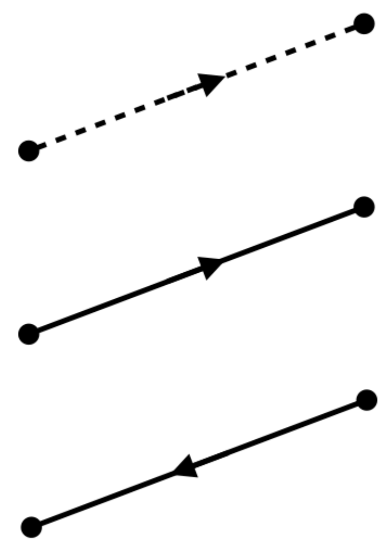
- External lines



Spin 0 boson (in, out) — **1**

Spin 1/2 fermion (in, out) — $u(\mathbf{p}), \quad \bar{u}(\mathbf{p}')$

- Internal lines



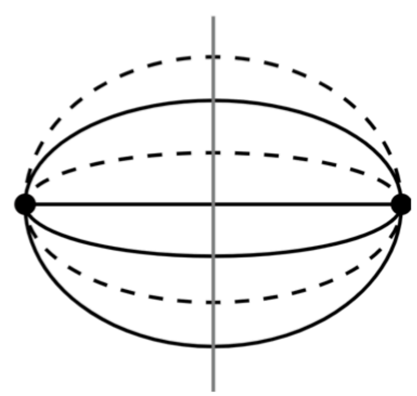
Spin 0 (anti-)boson — $\frac{1}{2\,\epsilon_q}$ $\quad \epsilon_q \equiv \sqrt{\mathbf{q}^2 + M^2}$

Spin 1/2 fermion — $\frac{m}{\omega_p} \sum u(\mathbf{p})\bar{u}(\mathbf{p})$ $\quad \omega_p \equiv \sqrt{\mathbf{p}^2 + m^2}$

anti-fermion — $\frac{m}{\omega_p} \sum u(\mathbf{p})\bar{u}(\mathbf{p}) - \gamma_0$

- Intermediate state

A set of lines between two vertices



$$\frac{1}{E - \sum_i \omega_{p_i} - \sum_j \epsilon_{q_j} + i\epsilon}$$

- Interaction vertices: **the standard Feynman rules**
  - Take care of zeroth components of integration momenta

✓ particle $p^0 \to \omega(p, m)$

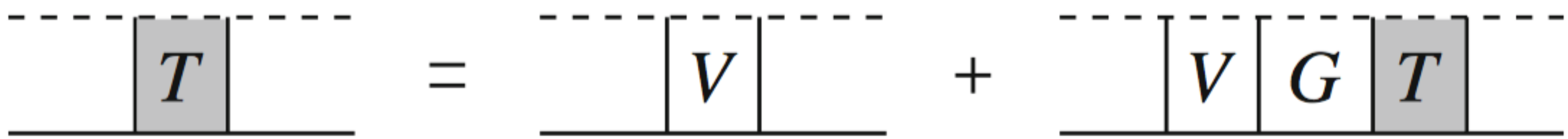
✓ antiparticle $p^0 \to -\omega(p, m)$

Meson–baryon scattering in TOPT

Interaction kernel / potential V

- **Define:** sum up the one-meson and one-baryon **irreducible diagrams**
- **Power counting:** Q/Λ_χ systematic ordering of all graphs

Scattering equation



- Coupled-channel integral equation for T-matrix

$$T_{M_jB_j,M_iB_i}(\boldsymbol{p}',\boldsymbol{p};E) = V_{M_jB_j,M_iB_i}(\boldsymbol{p}',\boldsymbol{p};E) + \sum_{MB}\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}V_{M_jB_j,MB}(\boldsymbol{p}',\boldsymbol{k};E)\,G_{MB}(E)\,T_{MB,M_iB_i}(\boldsymbol{k},\boldsymbol{p};E)$$

- **Meson-baryon Green function in TOPT**

$$G_{MB}(E) = \frac{m}{2\omega(k,M)\,\omega(k,m)}\frac{1}{E-\omega(k,M)-\omega(k,m)+i\epsilon}$$

Potential and scattering equation are obtained on an equal footing!

Leading order studies

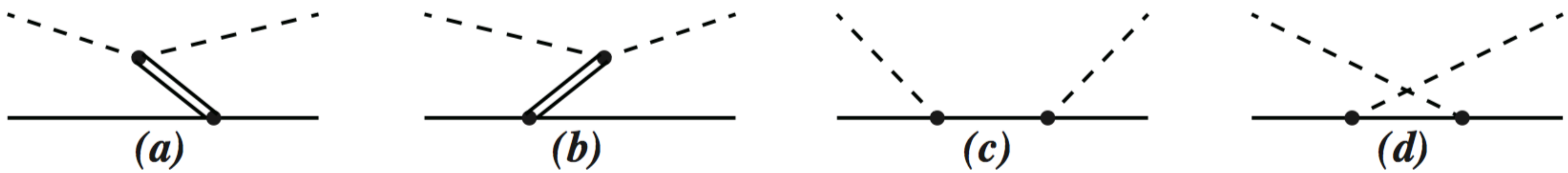
XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner,
Eur. Phys. J. C80 (2020) 406; Eur. Phys. J. C81 (2021) 582

Leading order potential

Chiral effective Lagrangian

$$\mathcal{L}_{\rm LO} = \frac{F_0^2}{4}\langle u_\mu u^\mu + \chi_+\rangle + \langle \bar{B}\left(i\gamma_\mu\partial^\mu - m\right)B\rangle + \frac{D/F}{2}\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]_\pm\rangle$$
$$-\frac{1}{4}\left\langle V_{\mu\nu}V^{\mu\nu} - 2\mathring{M}_V^2\left(V_\mu - \frac{i}{g}\Gamma_\mu\right)\left(V^\mu - \frac{i}{g}\Gamma^\mu\right)\right\rangle + g\,\langle \bar{B}\gamma_\mu[V^\mu, B]\rangle$$

Time ordered diagrams



- **Vector mesons included as explicit degrees of freedom**
  - ✓ One-vector meson exchange potential instead of the Weinberg-Tomozawa term
  - ✓ Improve the ultraviolet behaviour without changing the low-energy physics

LO potential in TOPT

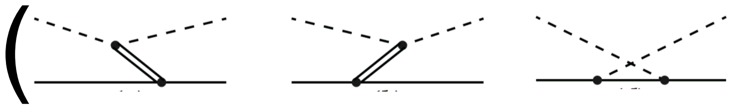
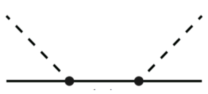
- Dirac spinor is decomposed as $u_B(p,s) = u_0 + [u(p) - u_0] \equiv (1,0)^\dagger \chi_s$+high order

$$V^{(a+b)}_{M_jB_j,M_iB_i} = -\frac{1}{32F_0^2}\sum_{V=K^*,\rho,\omega,\phi} C^V_{M_jB_j,M_iB_i}\frac{\mathring{M}_V^2}{\omega_V(q_1-q_2)}\left(\omega_{M_i}(q_1)+\omega_{M_j}(q_2)\right)$$
$$\times\left[\frac{1}{E-\omega_{B_i}(p_1)-\omega_V(q_1-q_2)-\omega_{M_j}(q_2)}+\frac{1}{E-\omega_{B_j}(p_2)-\omega_V(q_1-q_2)-\omega_{M_i}(q_1)}\right]$$

$$V^{(c)}_{M_jB_j,M_iB_i} = \frac{1}{4F_0^2}\sum_{B=N,\Lambda,\Sigma,\Xi} C^B_{M_jB_j,M_iB_i}\frac{m_B}{\omega_B(P)}\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{q}_2)(\boldsymbol{\sigma}\cdot\boldsymbol{q}_1)}{E-\omega_B(P)}.$$
$$V^{(d)}_{M_jB_j,M_iB_i} = \frac{1}{4F_0^2}\sum_{B=N,\Lambda,\Sigma,\Xi} \tilde{C}^B_{M_jB_j,M_iB_i}\frac{m_B}{\omega_B(K)}\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{q}_1)(\boldsymbol{\sigma}\cdot\boldsymbol{q}_2)}{E-\omega_{M_i}(q_1)-\omega_{M_j}(q_2)-\omega_B(K)}.$$

Subtractive renormalization

- LO potential: **one-baryon irreducible and reducible parts**

$$V_{\rm LO} = V_I\Big($$  $$\Big) + V_R\Big($$  $$\Big)$$

- LO T-matrix

$$T_{\rm LO} = V_{\rm LO} + V_{\rm LO}\, G\, T_{\rm LO} \;\Longrightarrow\; \begin{cases} T_{\rm LO} = T_I + (1+T_I\, G)\, T_R\, (1 + G\, T_I) \\ T_I = V_I + V_I\, G\, T_I \\ T_R = V_R + V_R\, G\,(1+T_I\, G)\, T_R \end{cases}$$

- Irreducible part: $T_I \xrightarrow{\Lambda\sim\infty}$ Finite
- Reducible part: $T_R \xrightarrow{\Lambda\sim\infty}$ Divergent
  - ✓ **Potential can be rewritten as separable form**

$$V_R(p',p;E) = \xi^T(p')\, C(E)\, \xi(p) \qquad \text{C(E): constant} \qquad \xi^T(q) := (1,q)$$

  - ✓ T_R can be rewritten as $T_R(p',p;E) = \xi^T(p')\,\chi(E)\,\xi(p) \qquad \chi(E) = \left[C^{-1} - \xi\, G\, \xi^T - \xi\, G\, T_I^S\, G\, \xi^T\right]^{-1}$

D.B.Kaplan, et al.,NPB478,629(1996); E. Epelbaum, et al.,EPJA51,71(2015)

  - ✓ Using **subtractive renormalization,** replacing Green function $G^{Rn} = G(E) - G(m_B)$

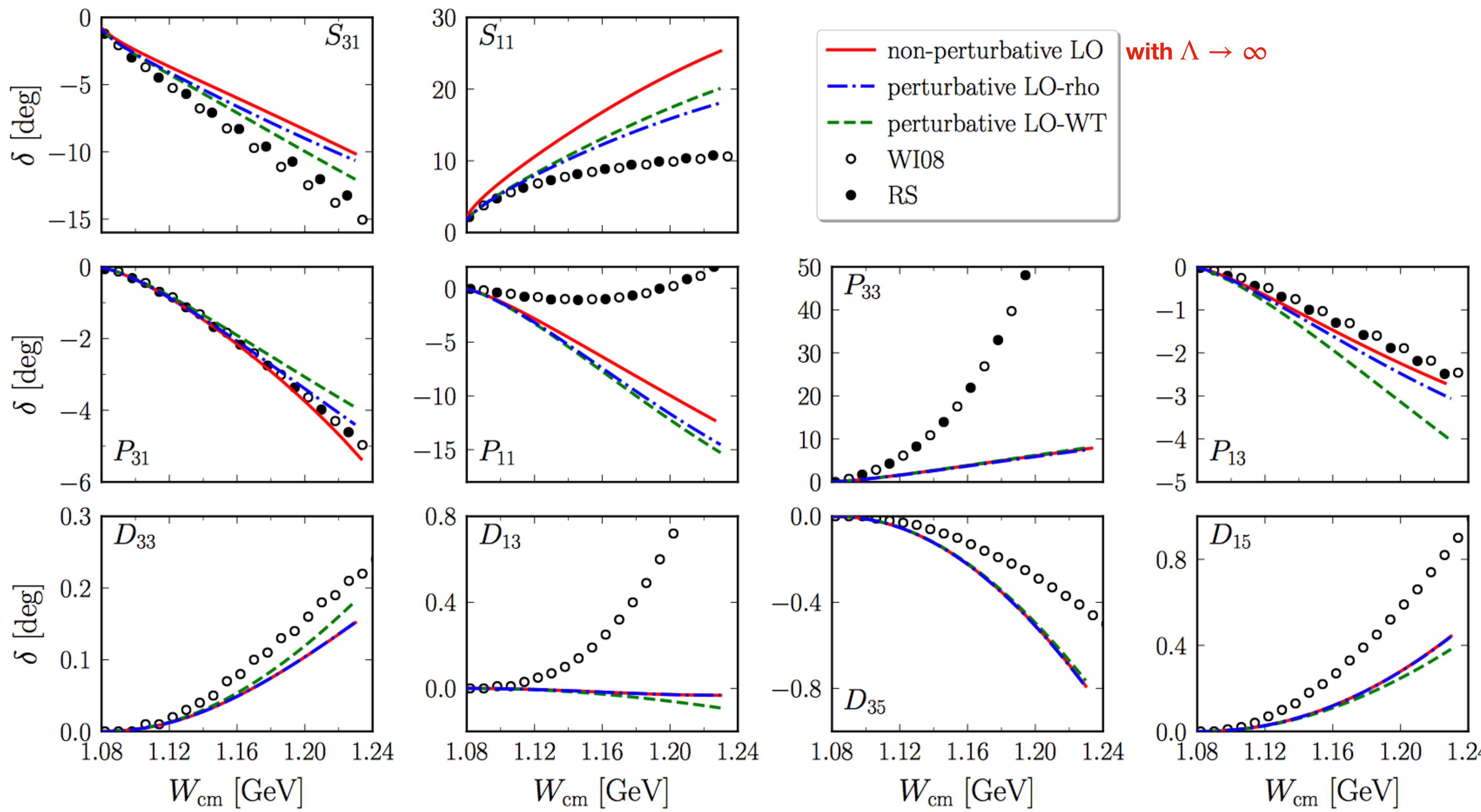
E. Epelbaum, et al., EPJA56(2020)152

Renormalized LO T-matrix

$$T_{\rm LO}^{Rn} = T_I + \left(\xi^T + T_I\, G^{Rn}\, \xi^T\right)\, \chi^{Rn}(E)\, \left(\xi + \xi\, G^{Rn}\, T_I\right)$$

Pion-Nucleon scattering

- Description phase shifts of pion-nucleon scattering



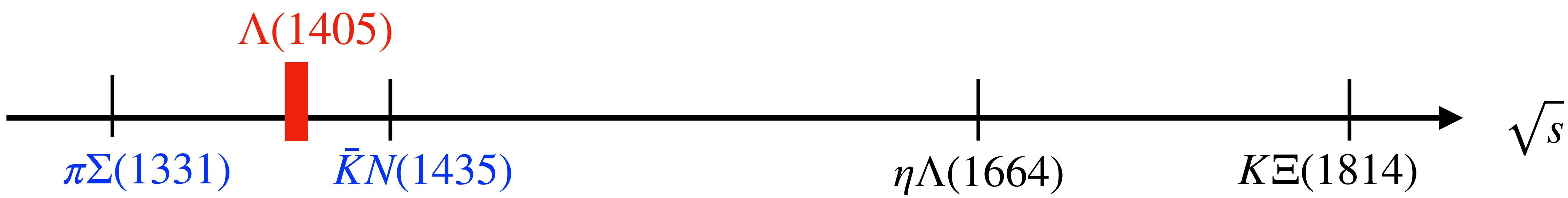
- Rho-meson-exchange contribution is similar as WT term
- Non-perturbative results are only slightly different from the ones of the perturbative approach

✓ **Non-perturbative treatment is valid**, since ChPT has good convergence in SU(2) sector

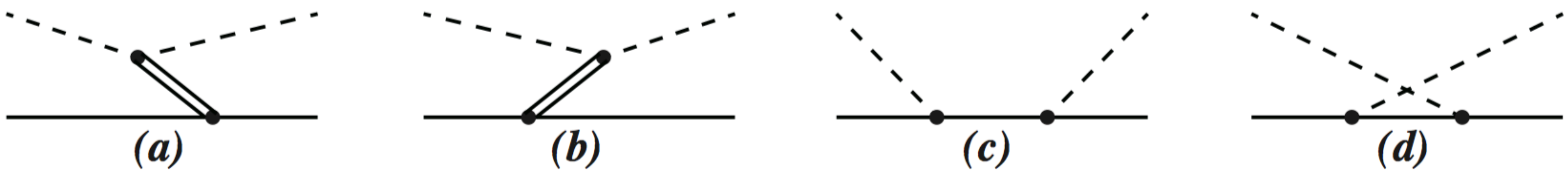
XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C80 (2020) 406

S=-1 meson-baryon scattering

- Four coupled channels $\bar{K}N, \pi\Sigma, \eta\Lambda, K\Xi$ in isospin limit



- Focus on the S-wave potential



  - Born term (p-wave) does not contribute
  - Crossed-Born term $\sim$5% of VME contribution
  - VME potential couplings

C^V	$\pi\Sigma$	$\bar{K}N$	$\eta\Lambda$	$K\Xi$
$\pi\Sigma$	$C^{\rho} = -16$	$C^{K^*} = 2\sqrt{6}$	0	$C^{K^*} = -2\sqrt{6}$
$\bar{K}N$	attractive	$C^{\{\rho,\omega,\phi\}} = \{-6, -2, -4\}$	$C^{K^*} = -6\sqrt{2}$	0
$\eta\Lambda$		attractive	0	$C^{K^*} = 6\sqrt{2}$
$K\Xi$				$C^{\{\rho,\omega,\phi\}} = \{-6, -2, -4\}$

S=-1 meson-baryon scattering

P. W. scattering equation

$$T^{LJ}_{M_jB_j,M_iB_i}(p',p) = V^{LJ}_{M_jB_j,M_iB_i}(p',p) + \sum_{MB}\int \frac{dkk^2}{(2\pi)^3} V^{LJ}_{M_jB_j,MB}(p',k)\frac{1}{2\,\omega_M\omega_B}\,\frac{m_B}{E-\omega_M-\omega_B+i\epsilon}\,T^{LJ}_{MB,M_iB_i}(k,p)$$

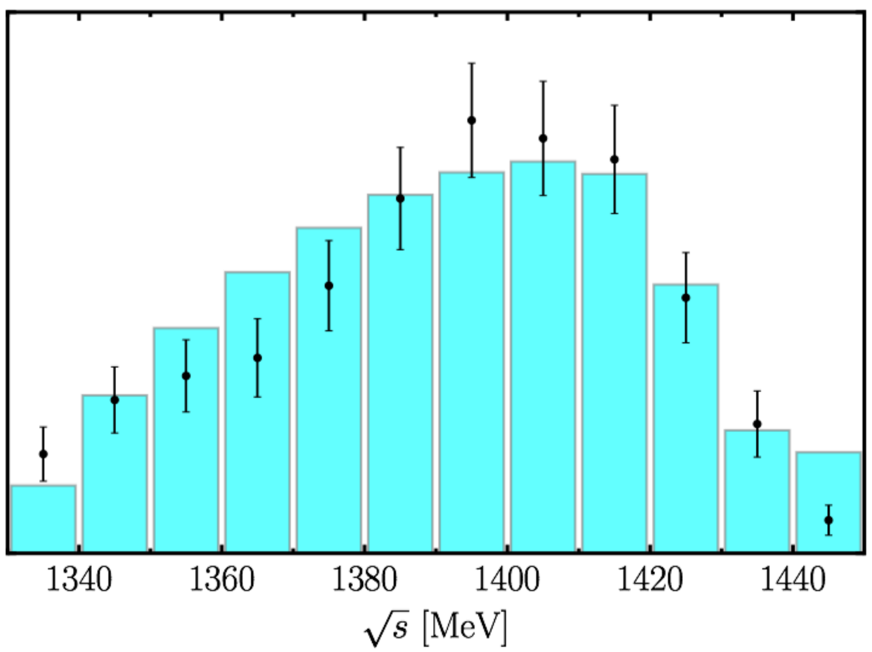
- Take into account **the off-shell effects of potential**
- Use **subtractive reormalization** to obtain the renormalized T-matrix
  - ➡ Cutoff-independent: $\Lambda \to \infty$

No free parameters needed to be fitted!

Two pole positions of $\Lambda(1405)$

		lower pole	higher pole
This work (LO)	$F_0 = F_\pi$	$1337.7 - i\,79.1$	$1430.9 - i\,8.0$
	$F_0 = 103.4$	$1348.2 - i\,120.2$	$1436.3 - i\,0.7$
NLO	*Y. Ikeda,NPA(2012)*	$1381^{+18}_{-6} - i81^{+19}_{-8}$	$1424^{+7}_{-23} - i26^{+3}_{-14}$
	Z.-H.Guo,PRC(2013)-Fit II	$1388^{+9}_{-9} - i114^{+24}_{-25}$	$1421^{+3}_{-2} - i19^{+8}_{-5}$
	M.Mai,EPJA(2015)-sol-2	$1330^{+4}_{-5} - i56^{+17}_{-11}$	$1434^{+2}_{-2} - i10^{+2}_{-1}$
	M.Mai,EPJA(2015)-sol-4	$1325^{+15}_{-15} - i90^{+12}_{-18}$	$1429^{+8}_{-7} - i12^{+2}_{-3}$

πΣ invariant mass spectrum



XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C81 (2021) 582

Comes to the unphysical quark mass region —>

Quark mass dependence of $\Lambda(1405)$

XLR, Phys. Lett. B 855 (2024) 138802

$\Lambda(1405)$ from Lattice QCD

- The first lattice study of $\Lambda(1405)$ pole positions

  Baryon Scattering Collaboration: PRL 132, 051901 (2024); PRD109,014511(2024)

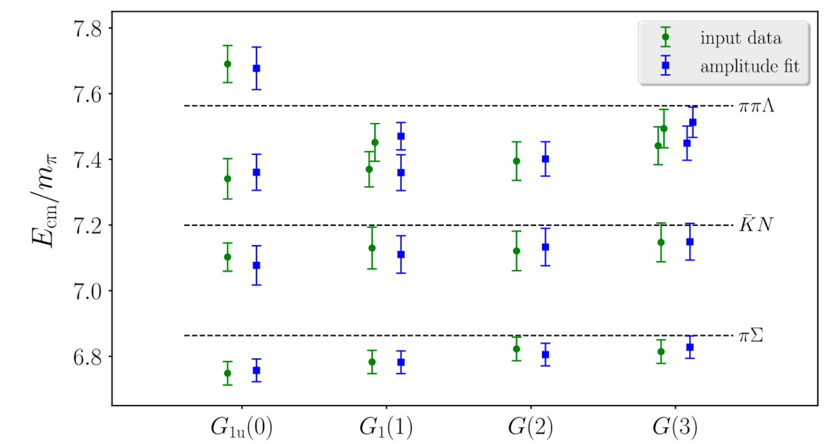
  - Focus on the $\pi\Sigma - \bar{K}N$ coupled channels (below $\pi\pi\Lambda$ threshold)
  - Pion and Kaon masses: $M_\pi \approx 200$ MeV, $M_K \approx 487$ MeV

Coordinated Lattice Simulations (CLS)

D200 ensemble

a (fm)	$(L/a)^3 \times T/a$	$m_\pi L$
0.0633(4)(6)	$64^3 \times 128$	4.181(16)

✓ Extract FV energy spectrum



✓ Implement the Lüscher formalism

$$\det[\tilde{K}^{-1}(E_{cm}) - B^P(E_{cm})] = 0.$$

- Two poles of $\Lambda(1405)$

Virtual bound state

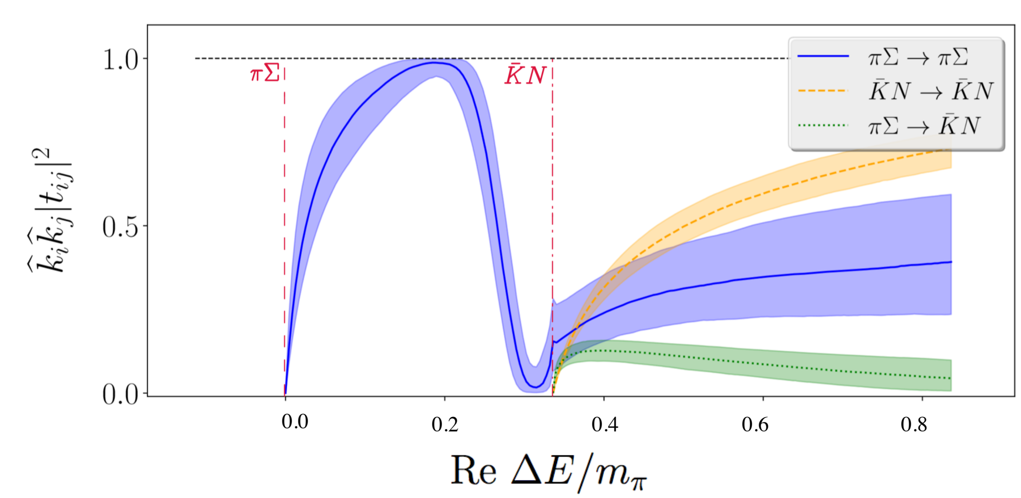
$$E_1 = 1392(9)_{stat}(2)_{model}(16)_a \text{MeV}$$

$$\left|\frac{c^{(1)}_{\pi\Sigma}}{c^{(1)}_{\bar{K}N}}\right| = 1.9(4)_{stat}(6)_{model}$$

Resonance

$$E_2 = [1455(13)_{stat}(2)_{model}(17)_a - i11.5(4.4)_{stat}(4.0)_{model}(0.1)_a] \text{MeV}$$

$$\left|\frac{c^{(2)}_{\pi\Sigma}}{c^{(2)}_{\bar{K}N}}\right| = 0.53(9)_{stat}(10)_{model}$$



Apply our framework to unphysical world

- BaSc results provide an ideal playground
  - **Check/verify the predictive power** of existing chiral unitary approaches
- We extend the calculation to the unphysical quark mass region
  - Use the same meson and baryon masses as the BaSc study

    ✓ $M_\pi = 203.7$ MeV, $M_K = 486.4$ MeV, $m_N = 979.8$ MeV, $m_\Sigma = 1193.9$ MeV

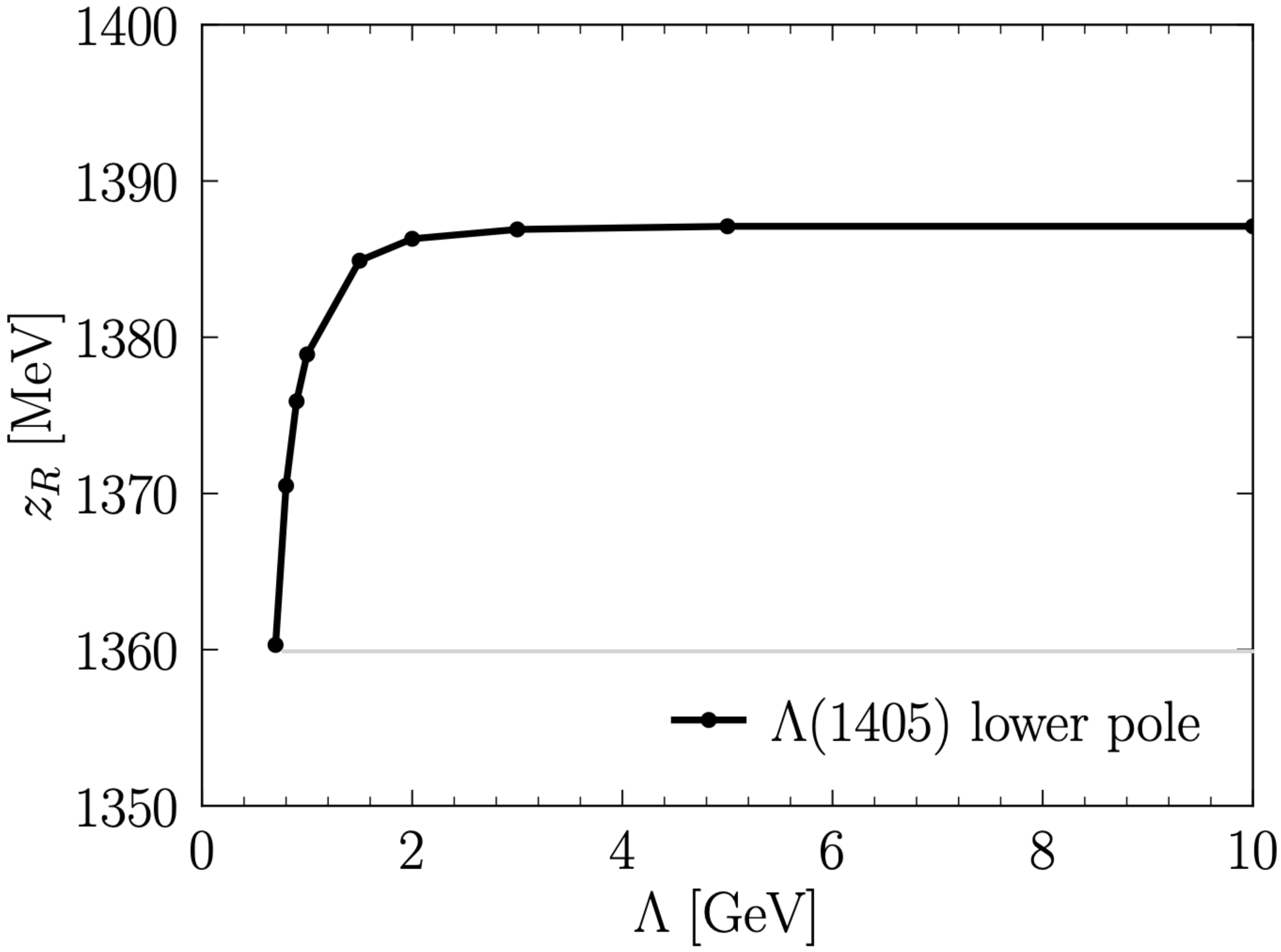
    ✓ $F_0 = F_\pi = 93.2$ MeV
  - Focus on the $\pi\Sigma - \bar{K}N$ coupled channels
  - **Consistent with the BaSc results** $\Lambda \to \infty$, no free parameters

	BaSc [PRL2024]	This work			
$\Lambda(1405)$	z_R [MeV]	z_R [MeV]	$g_{\pi\Sigma}$	$g_{\bar{K}N}$	$\lvert g_{\pi\Sigma}\rvert/\lvert g_{\bar{K}N}\rvert$
Lower pole	**1392(18)**	1387.14	$0.021 + i1.87$	$0.017 + i1.55$	1.21
Higher pole	**1455(21) − *i*11.5(6.0)**	$1469.86 - i4.71$	$0.038 + i0.98$	$1.51 - i1.22$	0.50

  - A full calculation with $\pi\Sigma, \bar{K}N, \eta\Lambda, K\Xi$ channels

    Lower pole : $z_R = 1389.05$ MeV   Higher pole : $z_R = 1464.55 - i9.44$ MeV

Cutoff independence of pole position



Finite-cutoff artifact
~ 30 MeV

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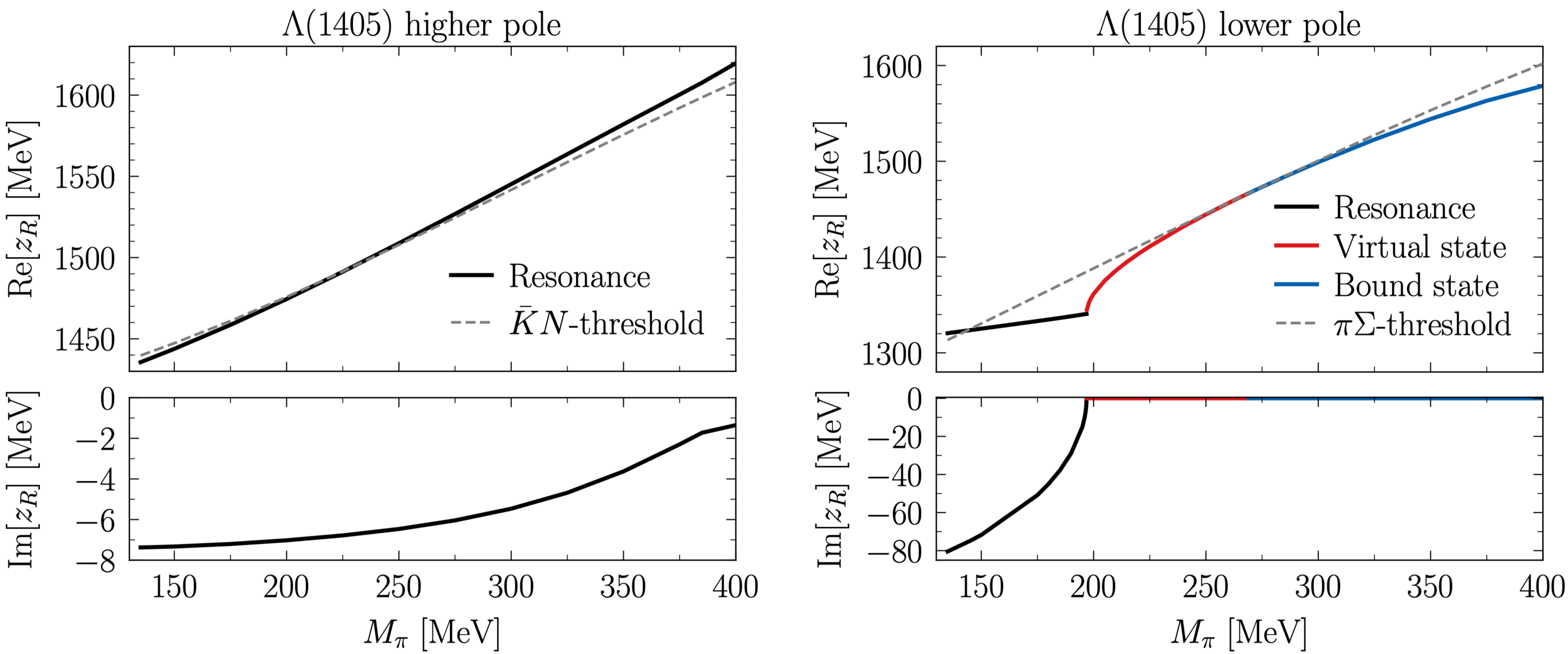
Light-quark mass dependence of $\Lambda(1405)$

- Full calculation with $\pi\Sigma, \bar{K}N, \eta\Lambda, K\Xi$ channels



With the increase of pion mass

- $\bar{K}N$ interaction gradually strengthens
- $\pi\Sigma$ interaction changes rapidly and is enhanced sufficiently

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Summary

- **A renormalized framework** for MB scattering is proposed
  - Time-ordered perturbation theory + Covariant chiral Lagrangians
  - Take into account the off-shell effects of potential
  - Employ the subtractive renormalization
    - ✓ **Achieve T-matrix is cutoff-independent**
- Leading order study
  - πN scattering; $\bar{K}N$ scattering with coupled channels
  - Obtain the **two-pole structure** of $\Lambda(1405)$
  - **Predictive power:** consistent with LQCD results
- Higher order studies in progress
  - Higher order corrections are perturbatively included
  - πN scattering: improve the description of phase shifts
  - Plan: extend to $\bar{K}N$ scattering, $\Lambda(1405)$, and other resonances

Thank you for your attention!