Light QCD exotics

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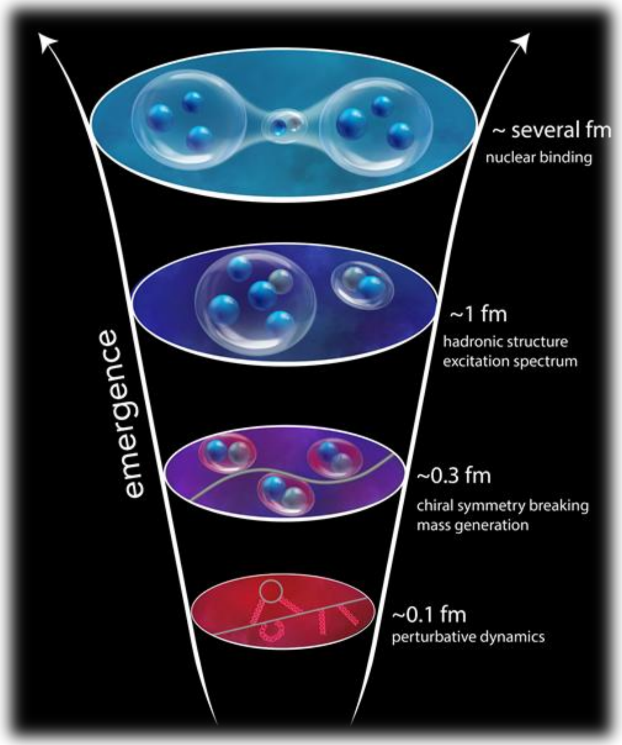
Mysterious gluons

- How does QCD give rise to hadrons?
  - What is the origin of confinement?
  - How is the mass generated in QCD?
- Role of gluons?
- Key thing to search for: hadrons explicitly manifest the gluonic degrees of freedom
  - Glueballs and hybrids

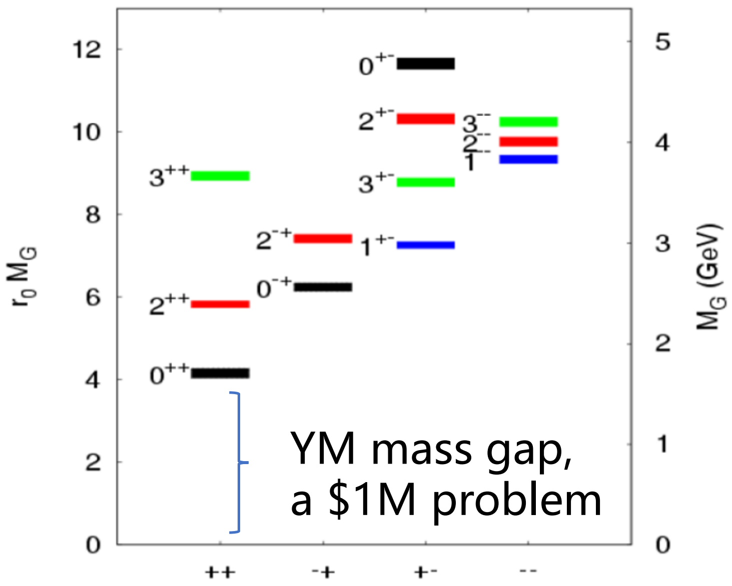


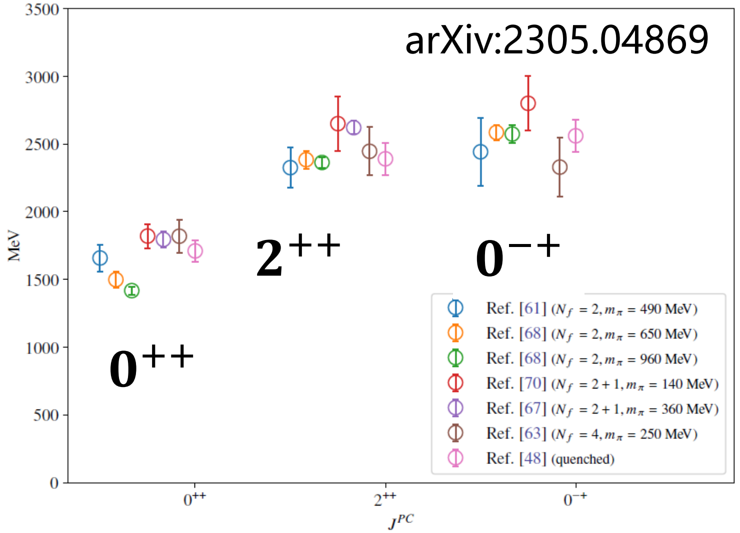
Eur.Phys.J.A 60 (2024) , 173

Glueballs

PRD 73 014516 (2006)

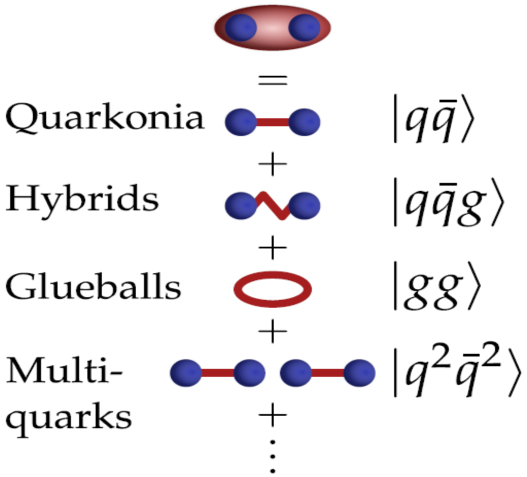
- Glueballs are the most direct prediction of QCD
  - Color singlets emerge as a consequence of the gluon self-interactions
  - Unique particles formed by gauge bosons (force)
- Essential for understanding of confinement and mass dynamical generation
- Theoretical predictions from lattice QCD and QCD-inspired models mostly consistent
  - Light-mass glueballs: $J^{PC} = 0^{++}, 2^{++},\ 0^{-+}$





Yang-Mills glueballs on lattice (quenched and unquenched)

non-$\mathbf{q\bar{q}}$ nature with ordinary quantum numbers is difficult to establish

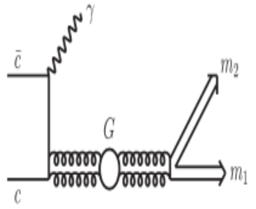


Glueball hunting for over 40 years

- **Supernumerary states** w.r.t. quark model
  - A priori, mixed with nearby $\mathbf{q\bar{q}}$
  - Assignment of some $q\bar{q}$ multiplets is difficult
- Detailed and accurate information about couplings to production and decay channels is required
- Strongly produced in **gluon-rich processes**
- Decay: **gluon is flavor-blind**
  - $\mathrm{SU(3)_{flavor}}$ symmetry expected, but differing quark masses leads exceptions
  - No rigorous predictions on decay patterns
  - Could be analogy to **OZI suppressed** decays of charmonium, as they all decay via gluons [PLB 380 189(1996), Commu. Theor. Phys. 24.373(1995)]

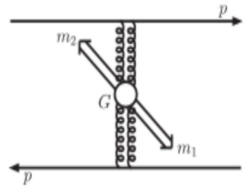
gluon-rich processes

[Phys. Rept. 454 1]



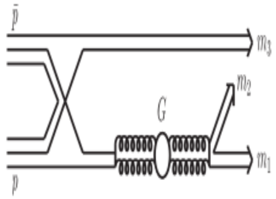
Charmonium decays:

BESIII, MRKIII…



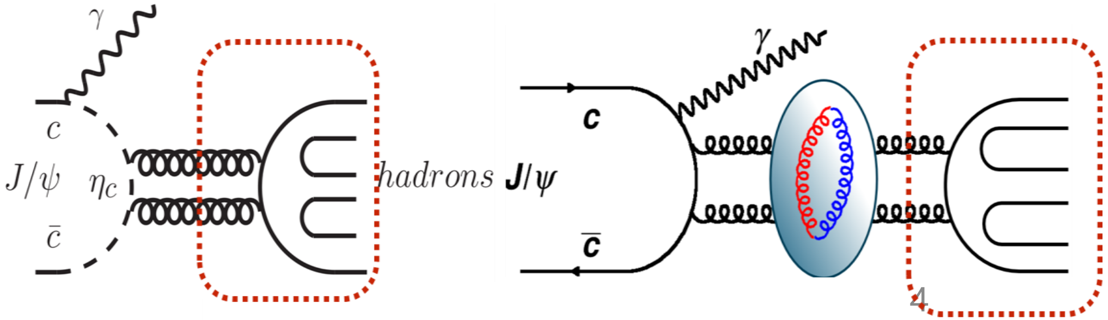
pp double-Pomeron exchange:

WA102, GAMS…



$\mathbf{p\bar{p}}$ annihilation:

Crystal barrel, OBELIX…

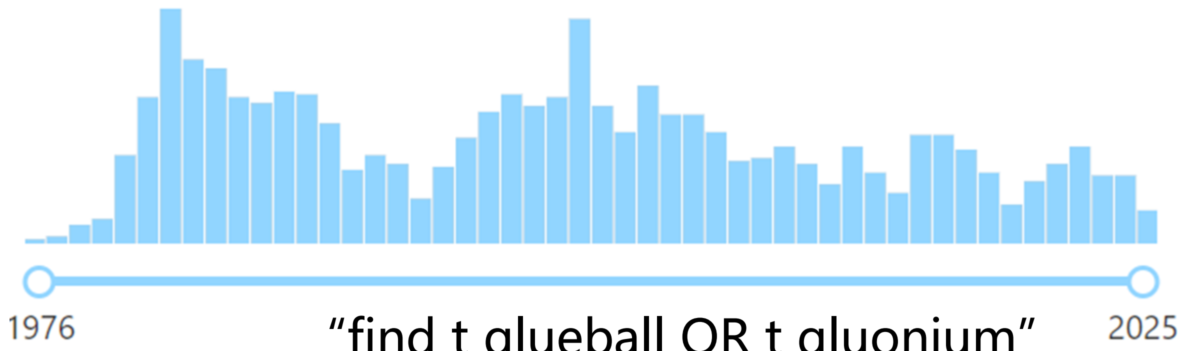


Some glueball candidates in the past

- The first glueball candidate, $\iota(1440)$, observed in J/ψ radiative decays in 1980s
- Scalar candidates $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ (MarkII in1980s, Crystal Barrel in 1990s)
- Narrow tensor glueball candidate $\xi(2230)$ (MarkIII in 1980s/BESI in 1990s)
  - Not confirmed by CLEO, BESII nor BESIII with much higher statistics

And,

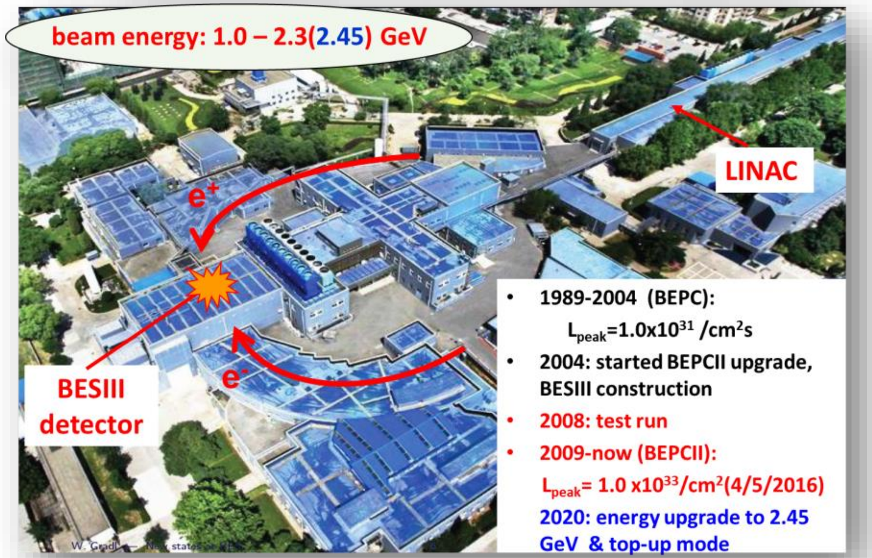
- Odderon (odd C-parity) from D0 and TOTEM (2021)

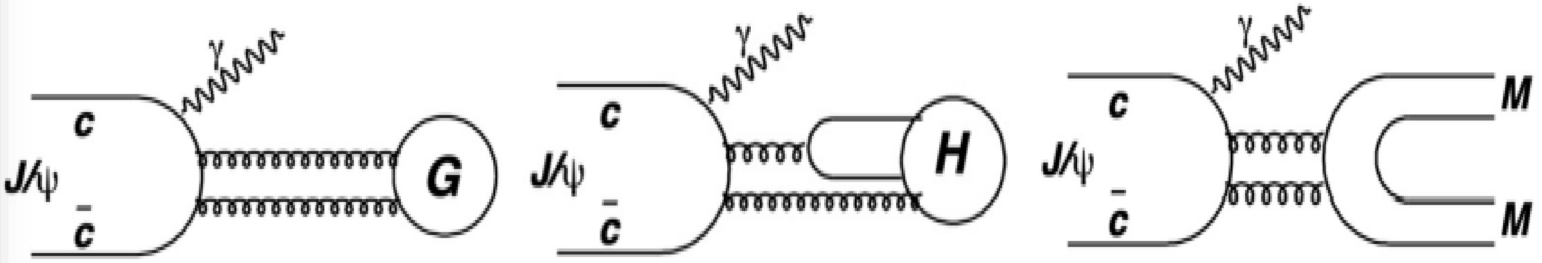


"find t glueball OR t gluonium"

"The Physics of Glueballs" Mathieu, Kochelev, and Vento, 2009
"The Status of Glueballs" Ochs, 2013
"Glueballs as the Ithaca of meson spectroscopy: From simple theory to challenging detection" Llanes-Estrada, 2021
"The Experimental Status of Glueballs" Crede and C. A.Meyer, 2009
…

Beijing Electron Positron Collider (BEPCII)





$\Gamma(J/\psi \to \gamma G) \sim O(\alpha\alpha_s^2)$ > $\Gamma(J/\psi \to \gamma H) \sim O(\alpha\alpha_s^3)$ > $\Gamma(J/\psi \to \gamma M) \sim O(\alpha\alpha_s^4)$

Charmonium decays provide an ideal lab for Gluonic Excitations

- **Gluon-rich process**
- **Well defined initial and final states**
  - **Kinematic constraints**
  - **Isospin and J^{PC} filters**
- **Clean high statistics data samples: 10×10^9 J/ψ and 2.7×10^9 ψ′ @ BESIII**
  - **High cross sections of $e^+e^- \to J/\psi,\ \psi'$**
  - **Low background**

Scalar glueball candidate

- **Supernumerary scalars** suggest additional degrees of freedom
  - However, mixing scenarios are controversial
- Measured $\mathrm{B}(\mathrm{J}/\psi \to \gamma \mathrm{f}_0(1710))$ is **x10 larger** than $\mathrm{f}_0(1500)$

  BESIII [PRD 87 092009, PRD 92 052003, PRD 98 072003]

  - LQCD: $\Gamma(\mathrm{J}/\psi \to \gamma \mathrm{G}_{0+})/\Gamma_{\mathrm{total}} = 3.8(9) \times 10^{-3}$ [PRL 110, 091601(2013)]

  ➢ **BESIII: $\mathbf{f_0(1710)}$ largely overlays with the scalar glueball**

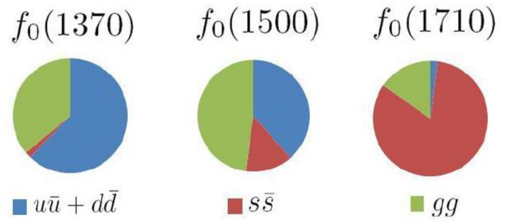
  ➢ **Identification of scalar glueball with coupled-channel analyses based on BESIII data**

  [PLB 816, 136227 (2021), EPJC 82, 80 (2022), PLB 826, 136906 (2022)]

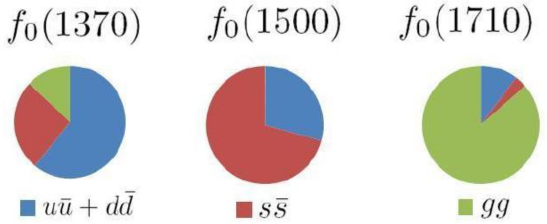
- **Further more, suppression of $\mathbf{f_0(1710) \to \eta\eta'}$ supports $\mathbf{f_0(1710)}$ has a large overlap with glueball**

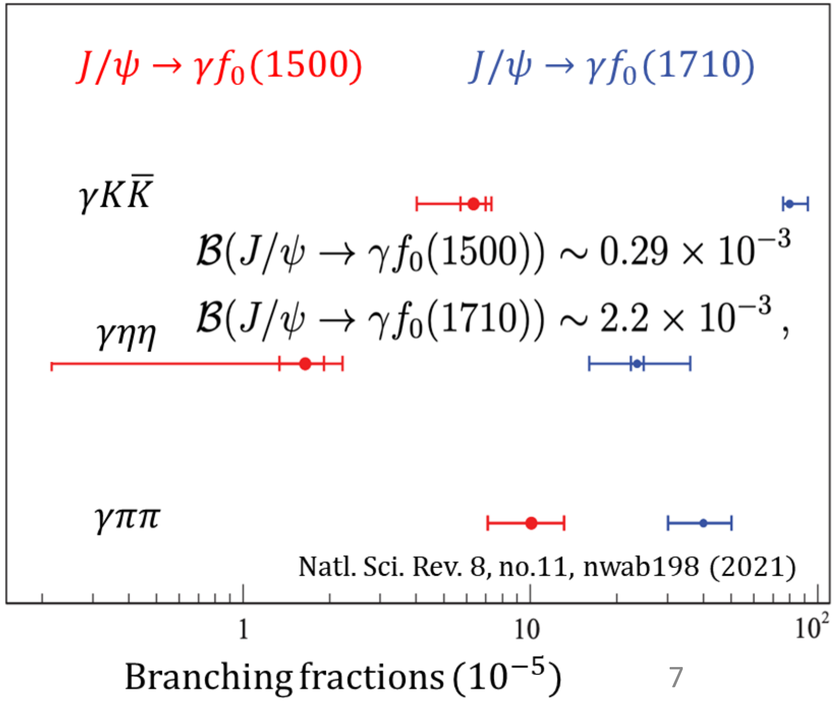
BESIII [PRD 106 072012(2022)]

Close and Kirk, PLB483 (2000) 345



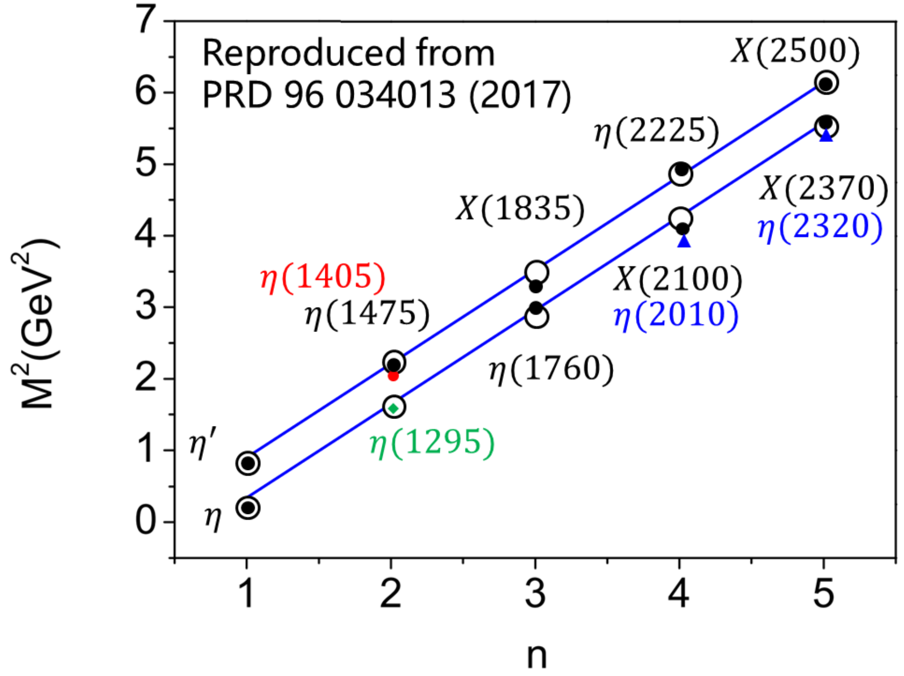
Cheng et al, Phys. Rev. D74 (2006) 094005





Where is the 0^{-+} glueball

- Pseudoscalar sector, a promising window
  - Only η, η′ (& radial excitations) from quark model
- Mass
  - LQCD： 0^{-+} glueball (2.3~2.6 GeV)
- Production
  - LQCD: $\Gamma(J/\psi \to \gamma G_{0-})/\Gamma_{\text{total}} = 2.31(80) \times 10^{-4}$, at the same level as 0^{-+} mesons [PRD.100.054511(2019)]
- **Decays**
  - **Possible guidance: OZI suppressed decays of η_c**
  - **3 pseudoscalar final state is a good place to look for** ($0^{-+} \to 2P$ is forbidden)



$\eta_c \to 3\,P$ in PDG

Decays involving hadronic resonances

Γ_1	$\eta'(958)\pi\pi$	(1.87 ± 0.26) %
Γ_2	$\eta'(958)K\overline{K}$	(1.61 ± 0.25) %

Decays into stable hadrons

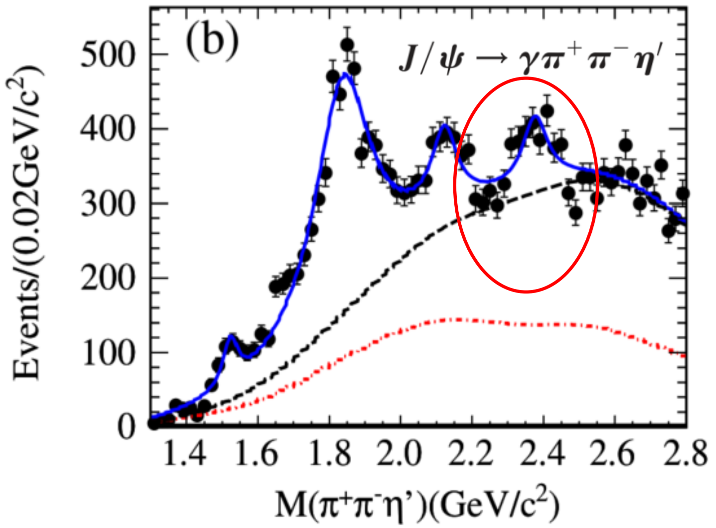
Γ_{34}	$K\overline{K}\pi$	(7.0 ± 0.4) %
Γ_{35}	$K\overline{K}\eta$	(1.32 ± 0.15) %
Γ_{36}	$\eta\pi^+\pi^-$	(1.7 ± 0.5) %

- No dominant decay
- Flavor symmetric

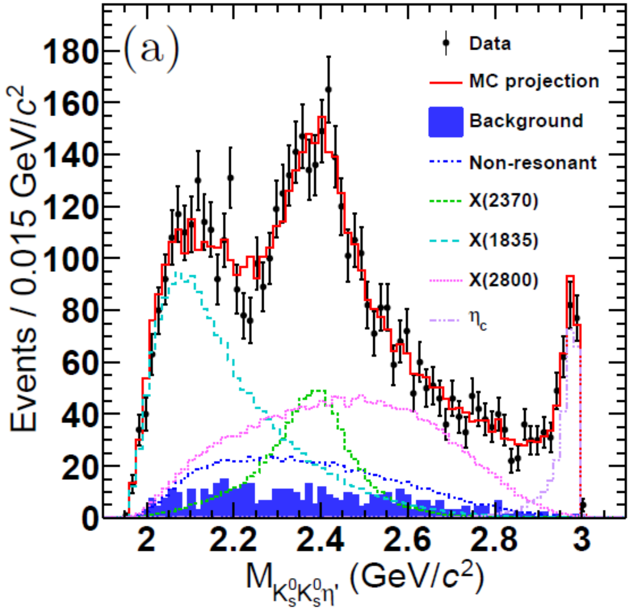
A glueball-like state $X(2370)$

- **Discovered by BESIII in $J/\psi \to \gamma\eta'\pi\pi$ in 2011**
- Confirmed by BESIII in $J/\psi \to \gamma\eta'\pi\pi, \gamma\eta' KK$
  - Not seen in $J/\psi \to \gamma\eta'\eta\eta$ [BESIII PRD 103 012009 (2021)], $J/\psi \to \gamma\gamma\phi$ [BESIII PRD.111.052011(2025)]. Upper limits of BF are well consistent with predictions of 0^{-+}glueball
- **Mass consistent with LQCD prediction for 0^{-+} glueball**
- **Spin-parity determined to be 0^{-+}** BESIII PRL **132, 181901(2024)**

$J/\psi \to \gamma\eta'\pi\pi$



BESIII PRL 106, 072002(2011), PRL 117, 042002 (2016)

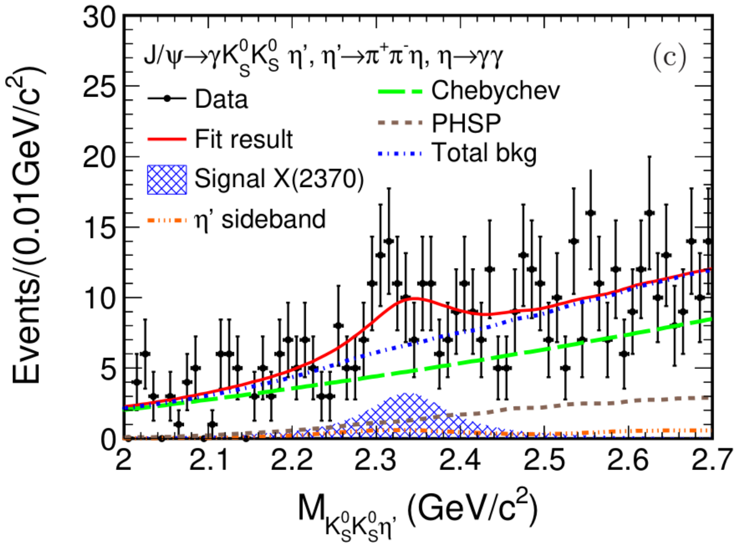


$J/\psi \to \gamma\eta' K^0_S K^0_S$

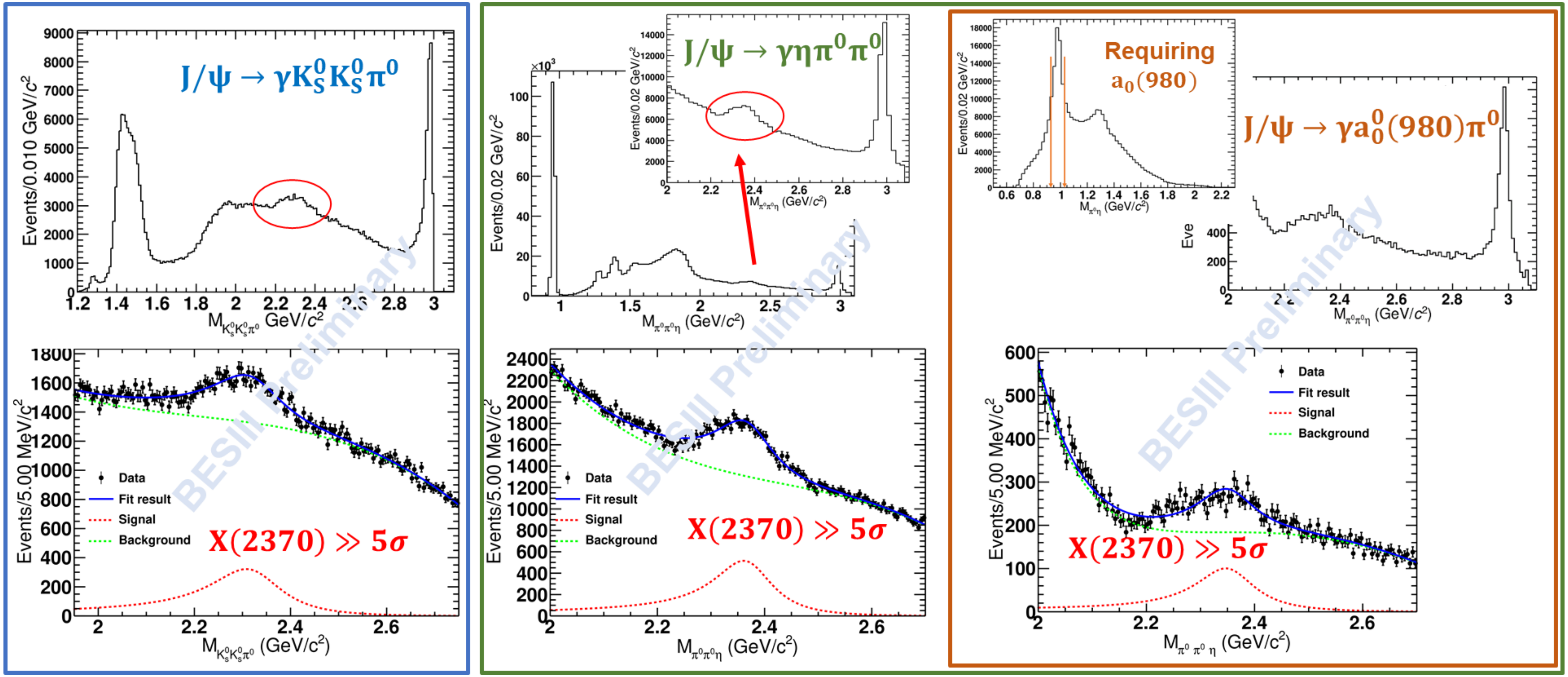
Highlights of ICHEP2024

J^{PC} = 0^{-+} with significance >9.8σ
M = 2395 ±11$^{+26}_{-94}$ MeV
Γ = 188$^{+18}_{-17}$$^{+124}_{-33}$ MeV
B(J/ψ→γX(2370))B(X(2370)→f_0(980)η')B(f_0(980)→$K^0_S K^0_S$) = 1.31 ± 0.22$^{+2.85}_{-0.84}$ ×10^{-5}

$J/\psi \to \gamma\eta' KK$



BESIII EPJC 80 746(2020)



New

$X(2370)$ observed in the gluon-rich J/ψ radiative decays

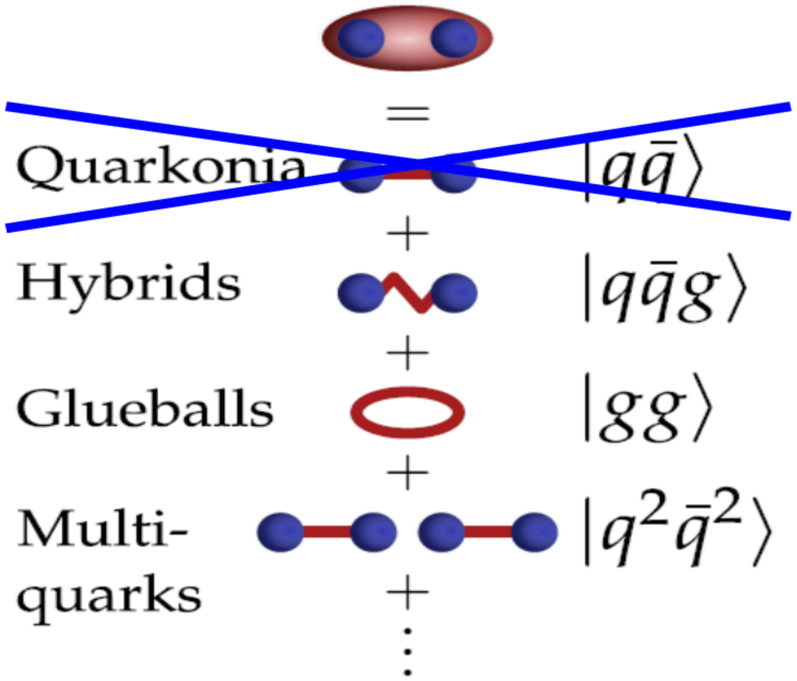
- **A first-time determination of $J^{PC} = 0^{-+}$**
- **Mass and production rate consistent with LQCD**
- **Decay modes $X(2370) \to \eta'\pi\pi, \eta' KK, K_S^0 K_S^0 \eta, K_S^0 K_S^0 \pi^0, \eta\pi^0\pi^0, a_0^0(980)\pi^0$, in analog to η_c** ($\eta(2320) \to \eta\eta\eta, \eta\pi\pi$ [PL B496 145(2000)] could be the same state)

Consistent with 0^{-+} glueball

Light hadrons with exotic quantum numbers

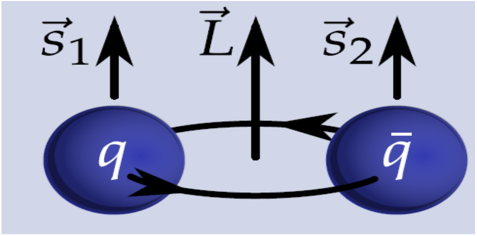
- **Finding unambiguous signature for exotics**
  - **Efforts concentrate on Spin-exotic**
    - **Forbidden for $q\bar{q}$:**

      $\mathbf{J^{PC} = 0^{--}, even^{+-}, odd^{-+}}$



Experiments:

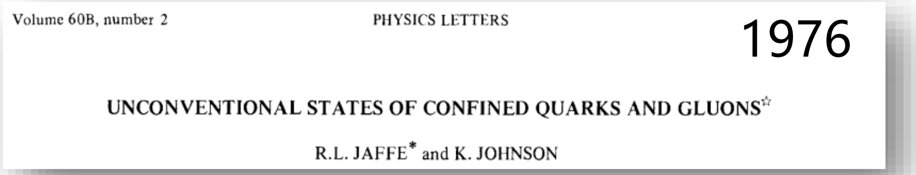
- **Hadroproduction:** GAMS, VES, E852, COMPASS
- **$p\bar{p}$ annihilation:** Crystal Barrel, OBELIX, PANDA(under construction)
- **Photoproduction:** GlueX(2017-), CLAS



$\vec{J} = \vec{L} + \vec{S}$ $\quad P = (-1)^{L+1}$ $\quad C = (-1)^{L+S}$

Allowed J^{PC}: $0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 2^{++}, \ldots$

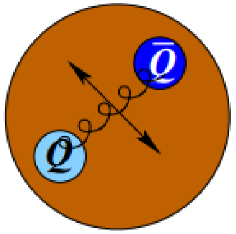
Predictions


Volume 60B, number 2 PHYSICS LETTERS

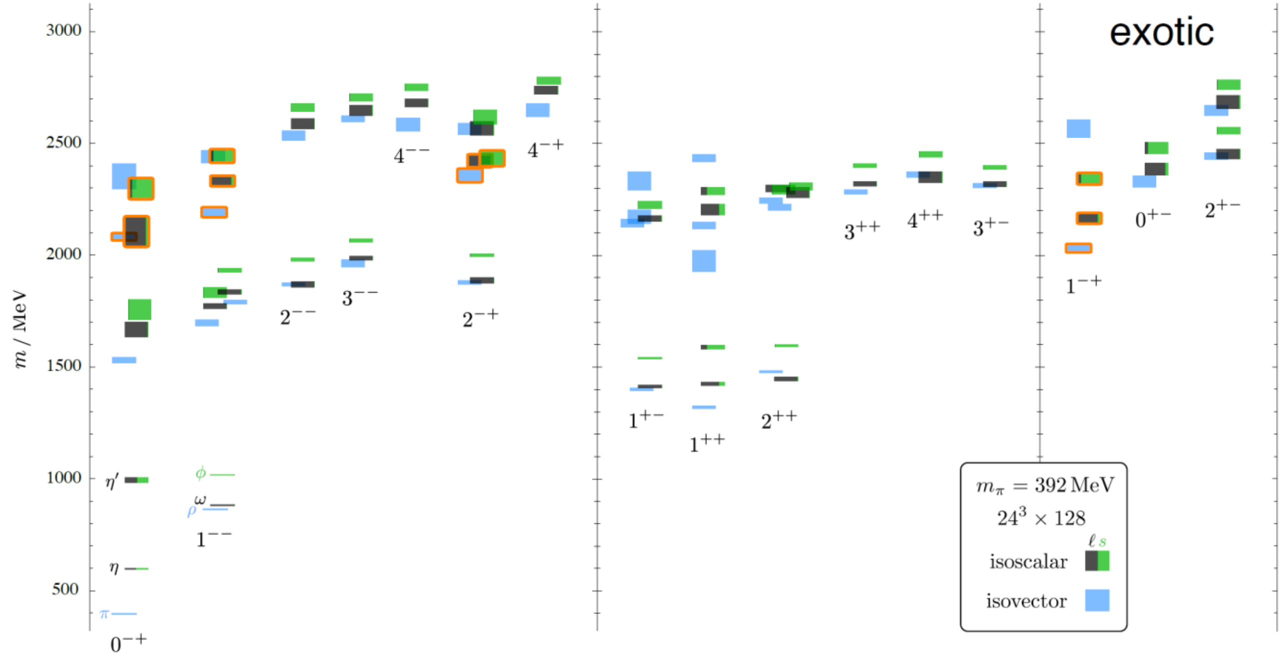
UNCONVENTIONAL STATES OF CONFINED QUARKS AND GLUONS☆

R.L. JAFFE* and K. JOHNSON

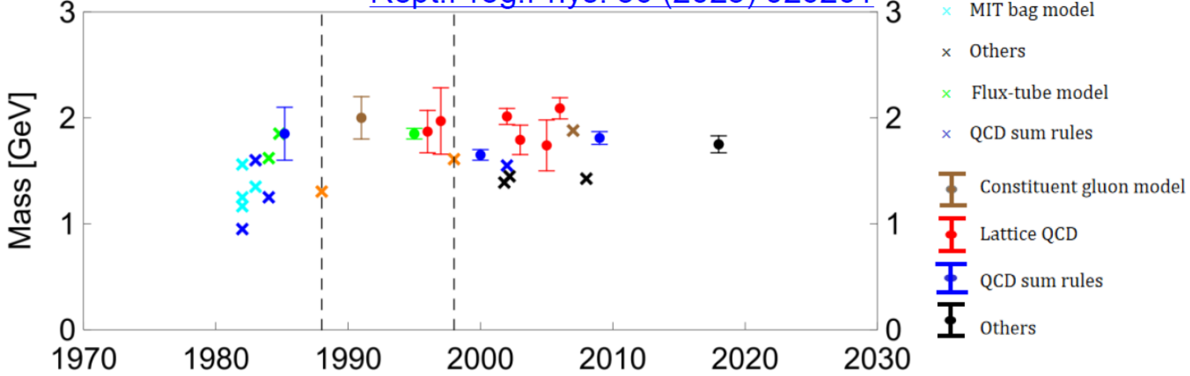
1976



PRD 88 094505(2013)

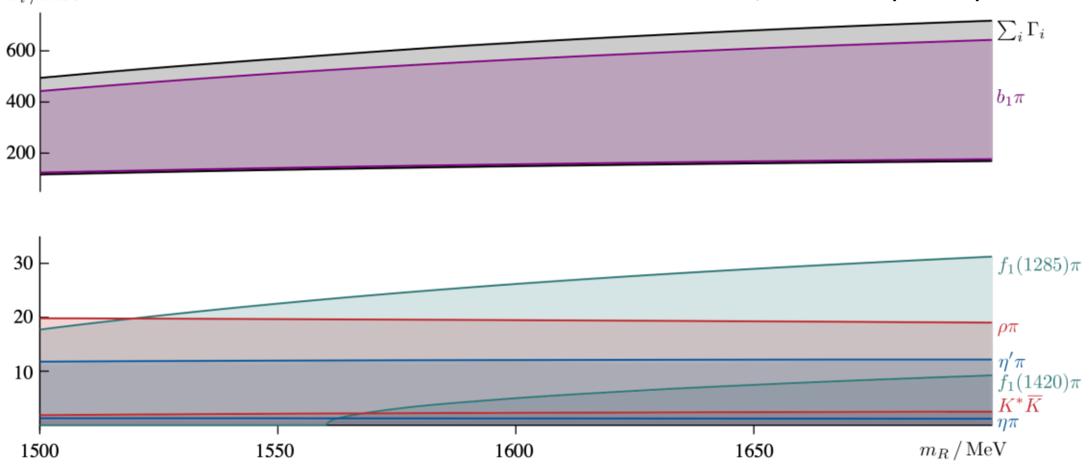


Rept.Prog.Phys. 86 (2023) 026201



Mass of $\mathbf{1^{-+}}$ hybrid

PRD 103, 054502(2021)

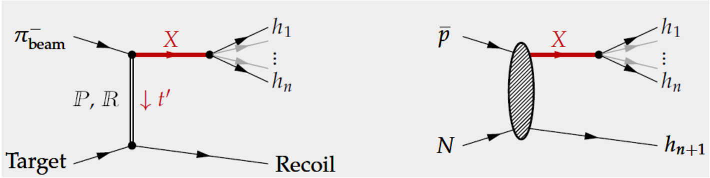


Decay width of $\mathbf{1^{-+}}$ hybrid π_1

On lattice,

- Meson excitations similar to quark model
- Hybrid supermultiplet: $\mathbf{0^{-+}}$, $\mathbf{1^{--}}$, $\mathbf{2^{-+}}$, $\mathbf{1^{-+}}$
- Lightest spin-exotic state in LQCD: $\mathbf{1^{-+}}$ hybrid

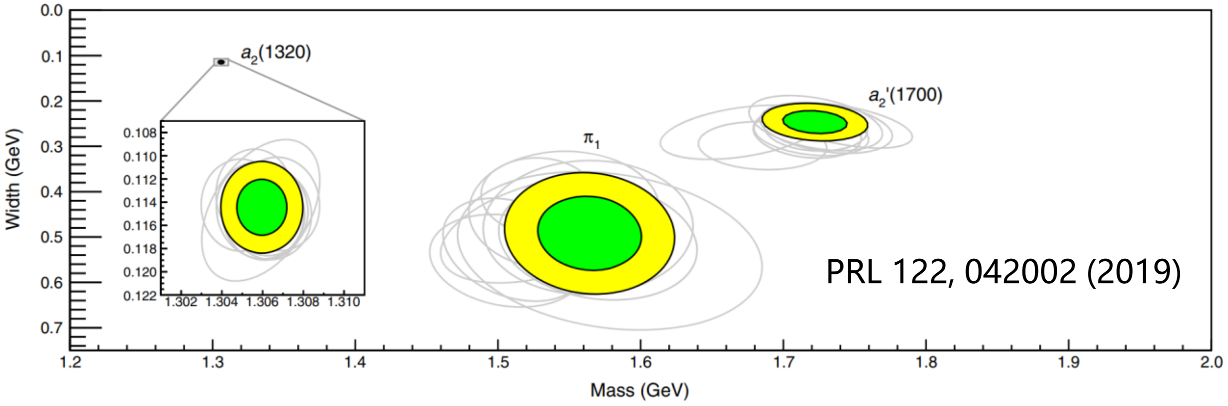
Detailed reviews:
PRC 82, 025208 (2010)， PPNP 82, 21 (2015)



Spin-exotic mesons

- **Over 3 decades, only 3 candidates so far:** **All 1^{-+} isovectors**
  - $\pi_1(1400)$: mostly in $\eta\pi$
  - $\pi_1(1600)$: seen in $\rho\pi$, $\eta'\pi$, $b_1\pi$, $f_1\pi$, but not $\eta\pi$
  - $\pi_1(2015)$: needs confirmation
- A big step forward: $\pi_1(1400)$ & $\pi_1(1600)$ can be one pole

[PRL 122, 042002 (2019), EPJ C 81, 1056 (2021)]

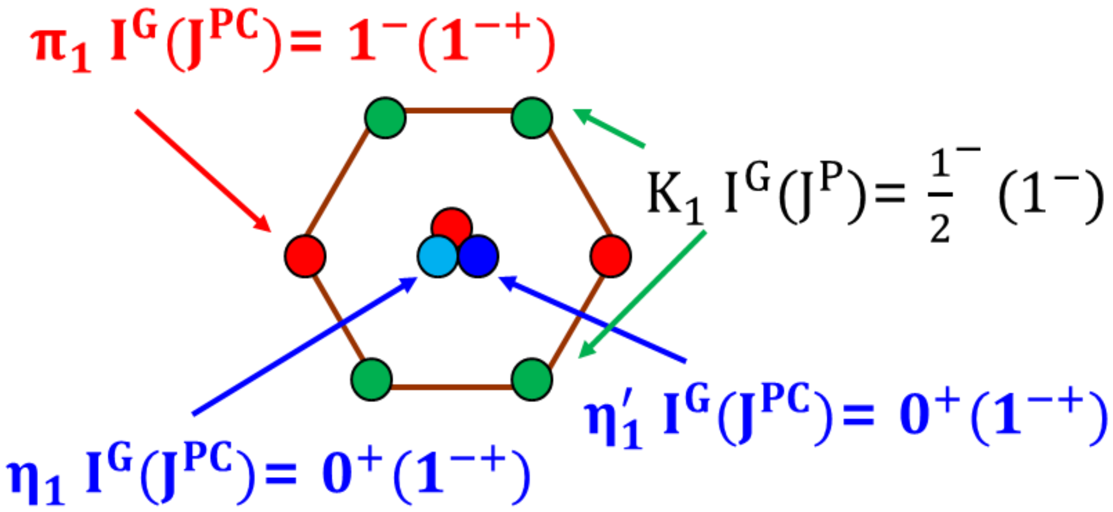


Coupled-channel analysis of COMPASS data

	Decay mode	Reaction	Experiment
$\pi_1(1400)$	$\eta\pi$	$\pi^-p \to \pi^-\eta p$	GAMS
		$\pi^-p \to \pi^0\eta n$	KEK
		$\pi^-p \to \pi^-\eta p$	E852
		$\pi^-p \to \pi^0\eta n$	E852
		$\bar{p}n \to \pi^-\pi^0\eta$	CBAR
		$\bar{p}p \to \pi^0\pi^0\eta$	CBAR
	$\rho\pi$	$\bar{p}p \to 2\pi^+2\pi^-$	Obelix
$\pi_1(1600)$	$\eta'\pi$	$\pi^-Be \to \eta'\pi^-\pi^0 Be$	VES
		$\pi^-p \to \pi^-\eta' p$	E852
	$b_1\pi$	$\pi^-Be \to \omega\pi^-\pi^0 Be$	VES
		$\bar{p}p \to \omega\pi^+\pi^-\pi^0$	CBAR
		$\pi^-p \to \omega\pi^-\pi^0 p$	E852
	$\rho\pi$	$\pi^-Pb \to \pi^+\pi^-\pi^- X$	COMPASS
		$\pi^-p \to \pi^+\pi^-\pi^- p$	E852
	$f_1\pi$	$\pi^-p \to p\eta\pi^+\pi^-\pi^-$	E852
		$\pi^-A \to \eta\pi^+\pi^-\pi^- A$	VES
$\pi_1(2015)$	$f_1\pi$	$\pi^-p \to \omega\pi^-\pi^0 p$	E852
	$b_1\pi$	$\pi^-p \to p\eta\pi^+\pi^-\pi^-$	

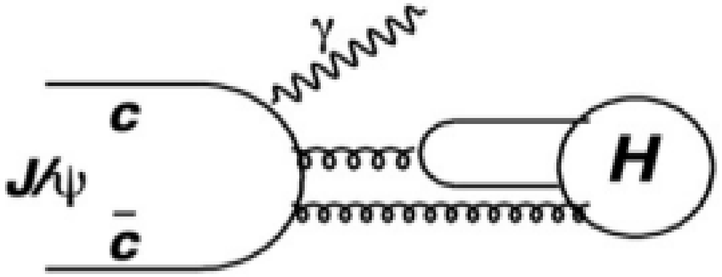
1^{-+} Hybrids

- **Isoscalar 1^{-+} is critical to establish the hybrid nonet**
  - **Can be produced in the gluon-rich charmonium decays**
  - **Can decay to $\eta\eta'$ in P-wave**



PRD 83,014021 (2011), PRD 83,014006 (2011), EPJ P135, 945(2020)

→ Search for $\eta_1\ (1^{-+})$ in $J/\psi \to \gamma\eta\eta'$



$$\Gamma(J/\psi \to \gamma H) \sim O(\alpha\alpha_s^3)$$

Observation of An Exotic 1^{-+} Isoscalar State $\eta_1(1855)$

PRL 129 192002(2022) , PRD 106 072012(2022)

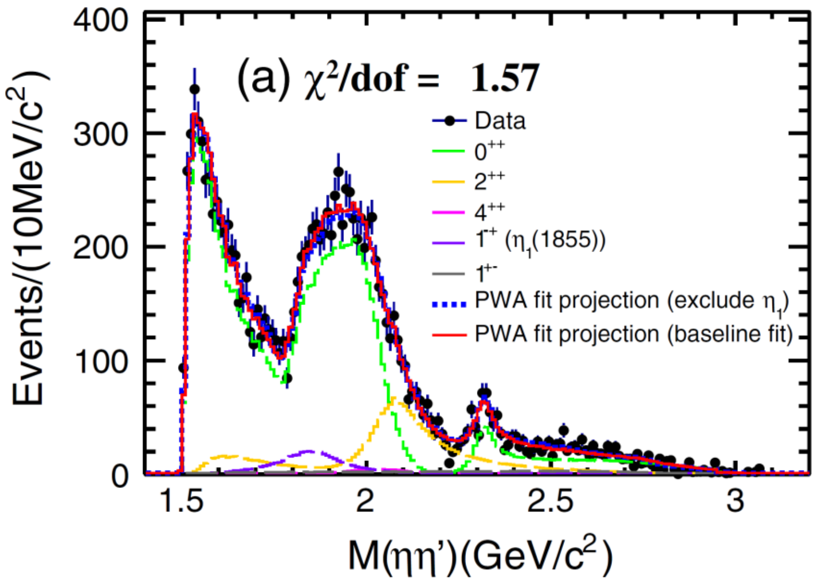
- **An isoscalar 1^{-+} , $\eta_1(1855)$, has been observed in $J/\psi \to \gamma\eta\eta'$ (>19σ)**

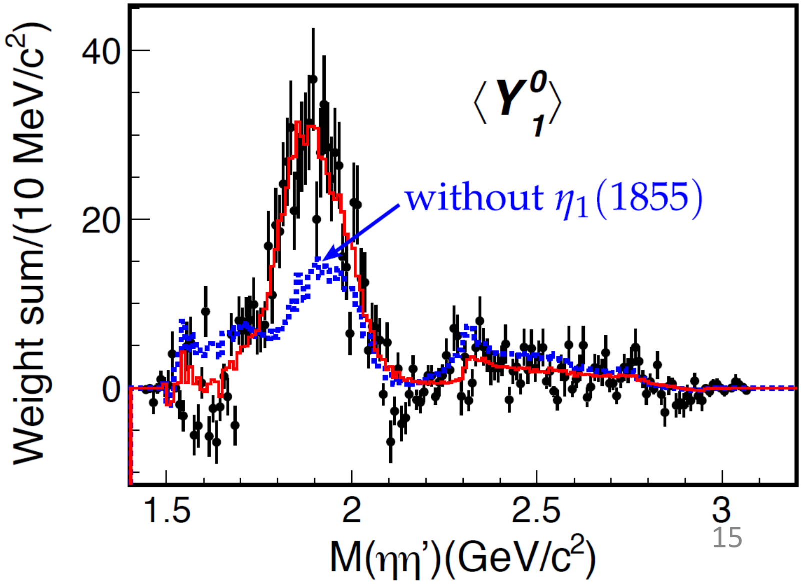
  $M = (1855 \pm 9^{+6}_{-1})\ \mathrm{MeV}/c^2,\ \Gamma = (188 \pm 18^{+3}_{-8})\ \mathrm{MeV}/c^2$

  $B(J/\psi \to \gamma\eta_1(1855) \to \gamma\eta\eta') = (2.70 \pm 0.41^{+0.16}_{-0.35}) \times 10^{-6}$

  - **Mass consistent with hybrid on LQCD**

- **$\eta\eta'$ in P-waves uniquely indicates 1^{-+} exotic quantum numbers**



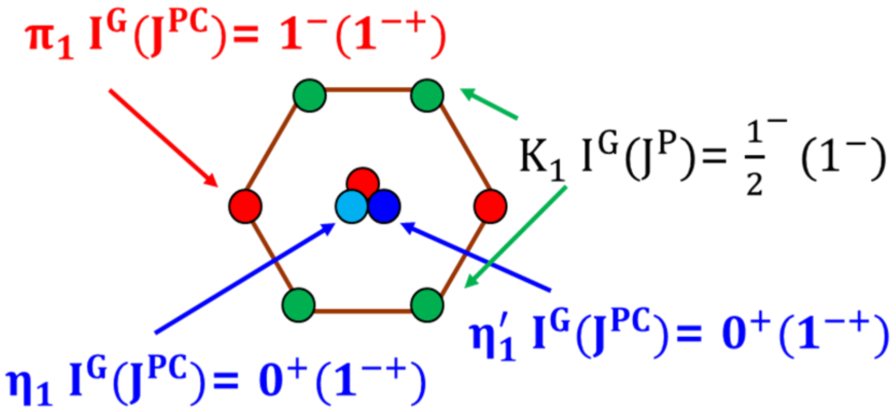


Observation of An Exotic 1^{-+} Isoscalar State $\eta_1(1855)$

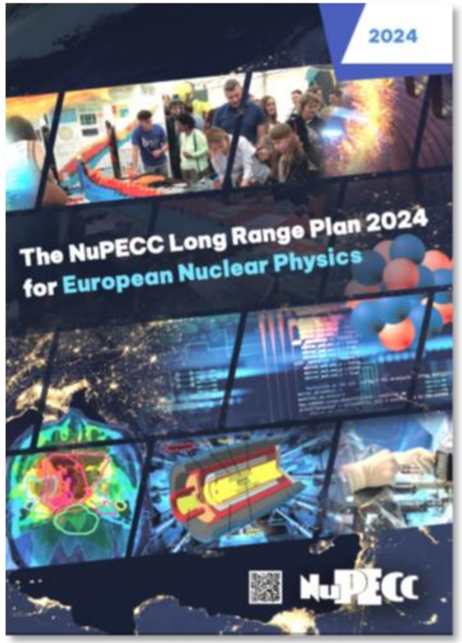
PRL 129 192002(2022) , PRD 106 072012(2022)

- Inspired many interpretations: Hybrid/$K\bar{K}_1$ Molecule/Tetraquark?

  NPA 1047 122874(2024); Rept.Prog.Phys. 86 (2023) 026201; PRD 107 (2023) 7, 074028; SCPMA 65 (2022) 6, 261011; CPC 46 , 051001(2022); CPL 39, 051201 (2022); PLB 834, 137478(2022); PRD 106 , 074003(2022); PRD 106, 036005(2022) ;...



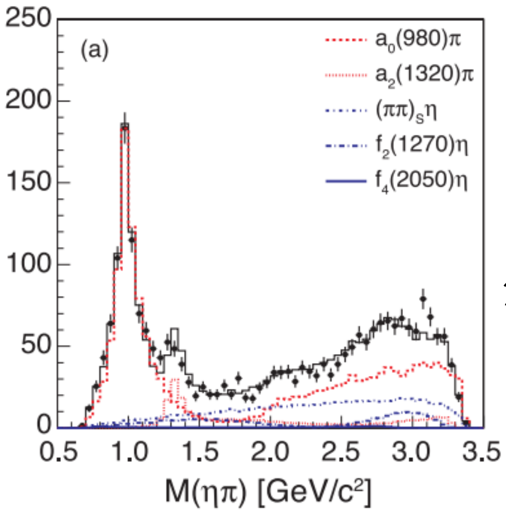
- Opens a new direction to completing the picture of spin-exotics
  - As a "recent achievements and highlights" in hadron spectroscopy in the NuPECC LRP
  - 50 years of QCD: Exotic mesons, "observation of an $\eta_1(1855)$ state could be a breakthrough" [EPJ.C 83 (2023) 1125]



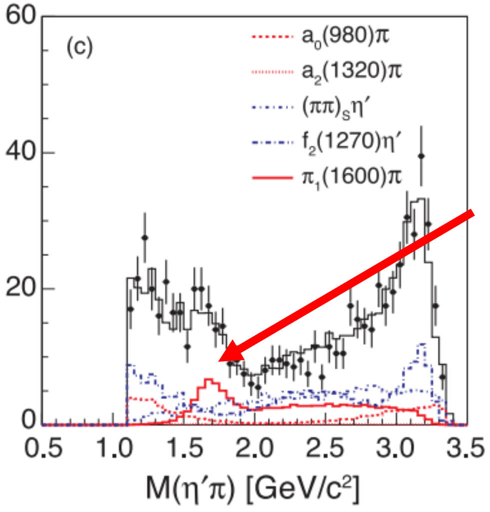
Studies of π_1 in $\chi_{c1} \to \pi^+\pi^-\eta^{(\prime)}$

PR D84 112009 (2011)

$\mathbf{2.6 \times 10^7}$ $\psi(3686)$@CLEO − c



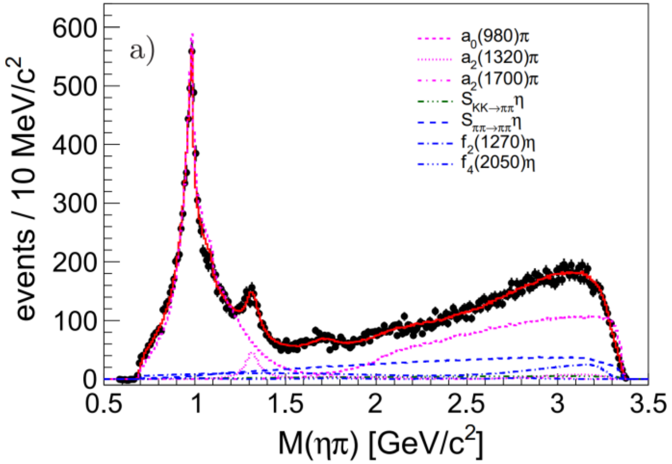
No evidence of $\pi_1 \to \eta\pi$



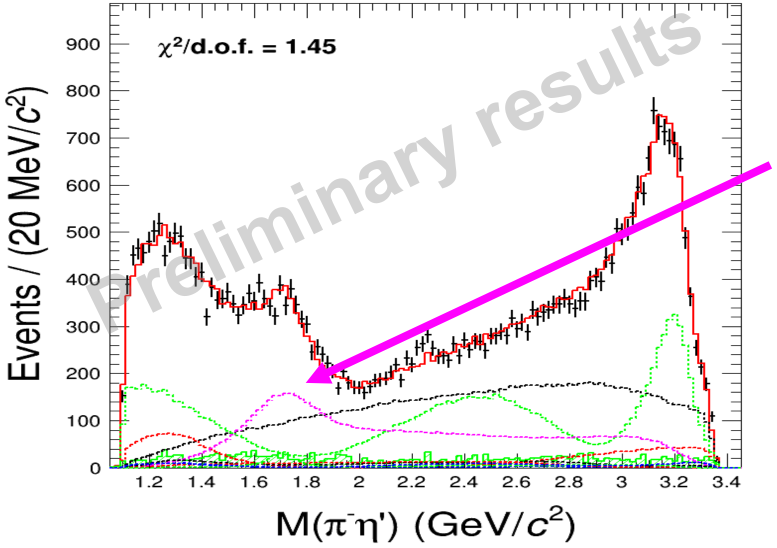
Evidence of $\pi_1 \to \eta'\pi$ (without significant BW phase motion)

PR D95 032002(2017)

44.8×10^7 $\psi(3686)$@BESIII



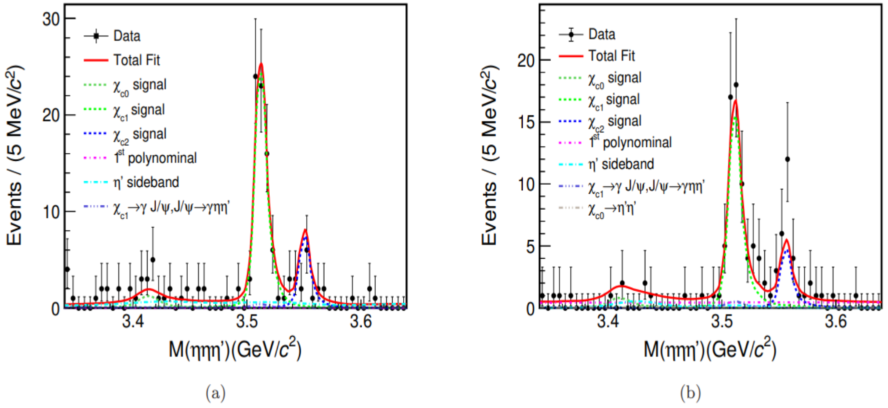
$\mathbf{2.7 \times 10^9}$ $\psi(3686)$@BESIII [New]

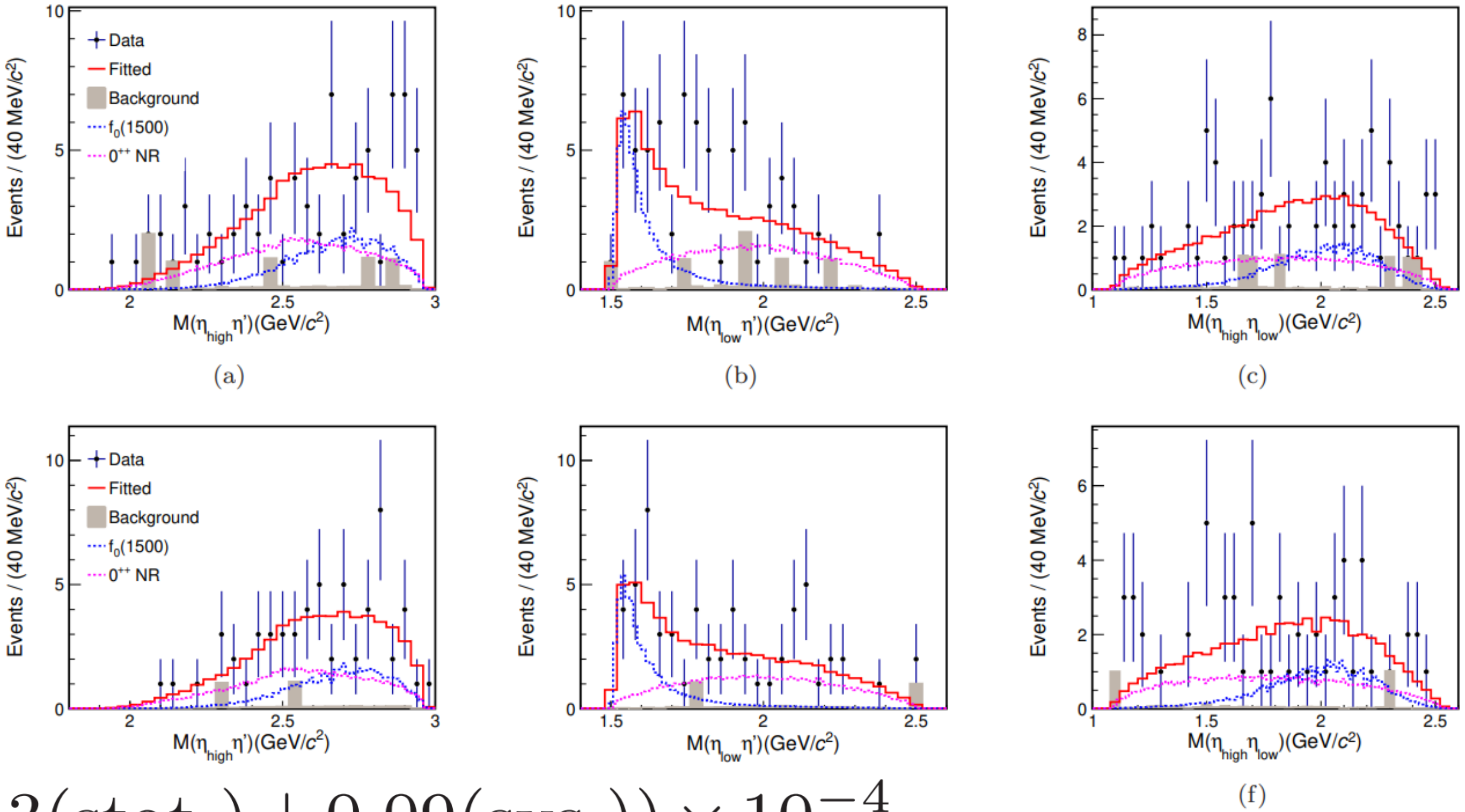


- **$\pi_1(1600)$ observed>10σ**
- **with a significant BW phase motion**
- **J^{PC}= 1^{-+}, better than other assignments well over 10σ**

Search for η_1 in $\chi_{c1} \to \eta\eta\eta'$

BESIII arXiv:2504.19087

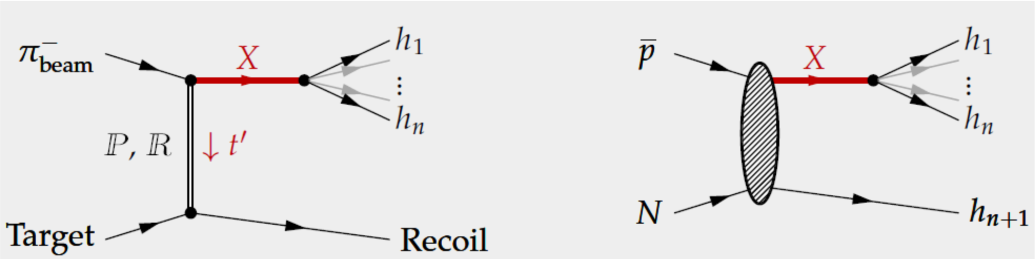


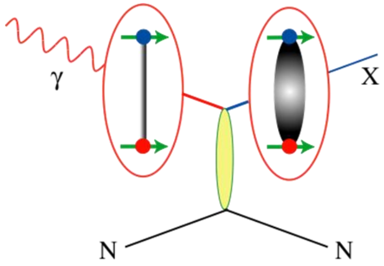


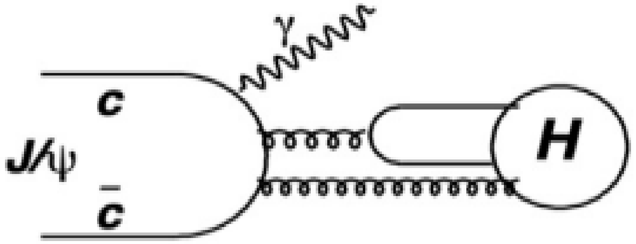
$$\mathcal{B}(\chi_{c1} \to \eta\eta\eta') = (1.39 \pm 0.13(\text{stat.}) \pm 0.09(\text{sys.})) \times 10^{-4}$$

$$\mathcal{B}(\chi_{c1} \to \eta_1(1855)\eta) \cdot \mathcal{B}(\eta_1(1855) \to \eta\eta') < 9.79 \times 10^{-5}.$$

Observations of π_1 and η_1 in charmonium decays provide a new path to study 1^{-+}







$\Gamma(J/\psi \to \gamma H) \sim O(\alpha\alpha_s^3)$

- **Measure the properties of $\pi_1(1600)$**
- **Confirm $\eta_1(1855)$ and measure its properties in more processes**
- **Identify the expected $\eta_1^{(\prime)}$ partner**
- **Other exotic quantum numbers: 0^{--} , 2^{+-} , ……**
- **Analog in $\bar{c}c$**

Search for a 1^{-+} molecular state via $e^+e^- \to \gamma D_s^+ D_{s1}^-(2536) + c.c.$ — BESIII arXiv:2503.11015

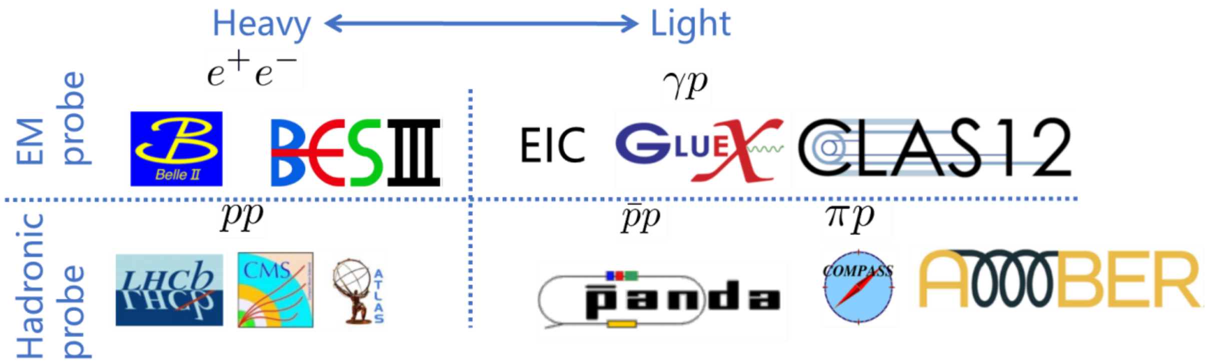
Search for 1^{-+} charmonium-like hybrid via $e^+e^- \to \gamma\eta^{(\prime)}\eta_c$ at center-of-mass energies between 4.258 and 4.681 GeV — BESIII arXiv:2504.13539

Outlook

- Having established the existence of new forms of hadrons is only the starting point
  - Measure their properties
  - Establish the multiplets

Underlying physics responsible for the rich spectrum

- Global efforts with various probes in running and planned experiments
  - Charmonium decays provide an ideal lab for glueballs and hybrids

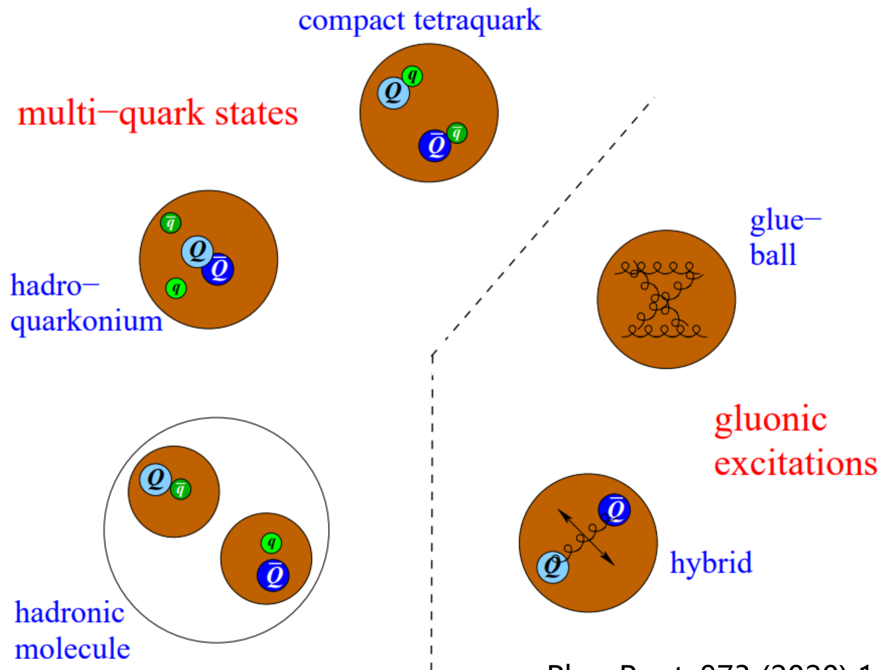


- Leveraging the high precision of current and future data requires
  - Advanced analysis techniques
  - Close theory-experiment collaboration

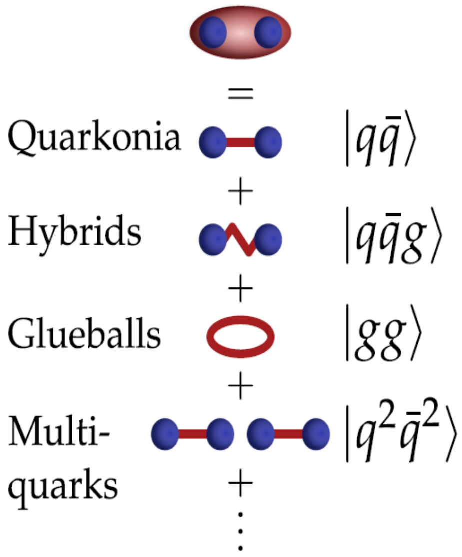
backups

Hadron spectroscopy

- **Quark model seems to work really well. Why?**
- **Compelling evidence for new hadrons**
  - Multi-quark candidates in heavy quark sector
    - **X(3872), $\mathbf{Z_c}$, $\mathbf{P_c}$, $\mathbf{T_{cc}}$, ⋯**
  - **Evidence for gluonic excitations remains sparse**



Phys.Rept. 873 (2020) 1



Physical meson

A linear superposition of all allowed color-singlet configurations

Identification is challenging

Manifestly exotic: with forbidden QN

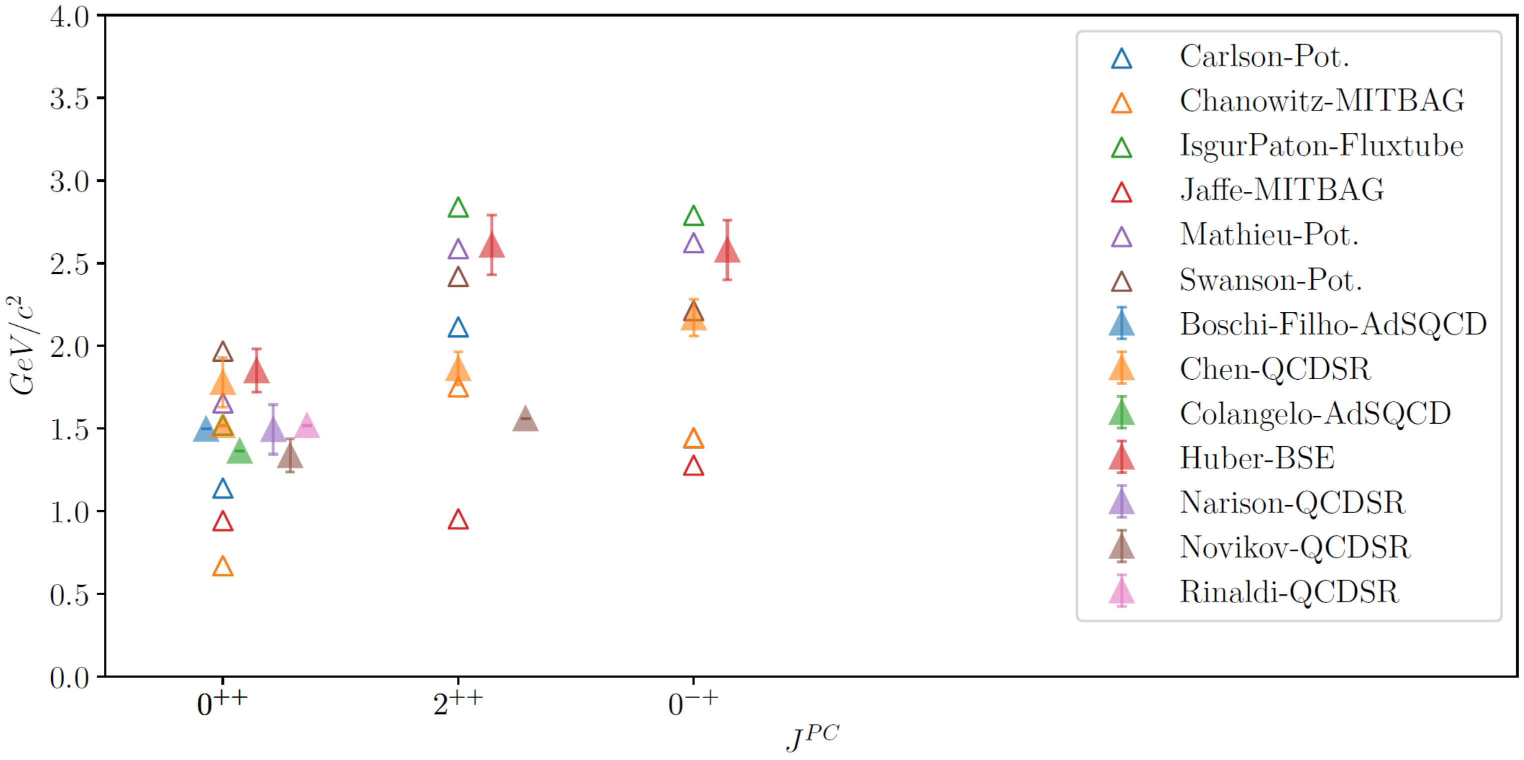
Flavor exotic: Z_c, T_{cc}, …

Spin exotic: $\mathbf{J^{PC} = 0^{--}, even^{+-}, odd^{-+}}$

Crypto exotic: with QN as $\mathbf{q\bar{q}}$

Supernumerary states
Abnormal properties
} w.r.t quark model

Glueballs in analytical and phenomenological models



What we have learned before

-- from MarkIII, BES, Crystal barrel, OBELIX, WA102, GAMS, E852, …

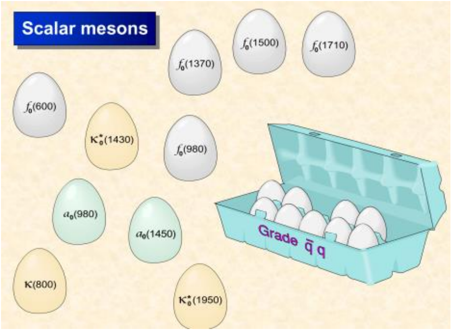
e^+e^- annihilation
$p\bar{p}$ annihilation
central exclusive production
charge-exchange reactions

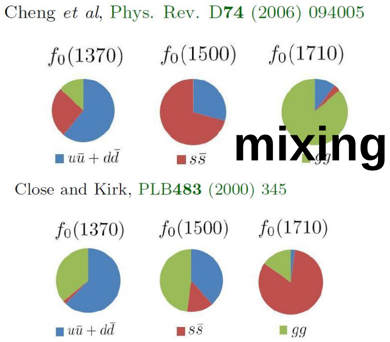
Scalar: 1 nonet in quark model, f_0 & f_0'

Exp: **overpopulation**

LQCD : ground state 0^+ glueball ~1.7 GeV;

$$\Gamma(J/\psi \to \gamma G_{0+})/\Gamma_{total} = 3.8(9) \times 10^{-3}$$





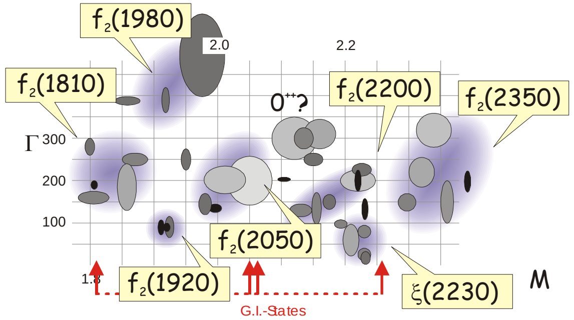
mixing schemes

Tensor: 2 nonets($^3P_2$,$^3F_2$), complicated

Exp: **large uncertainty**

LQCD: 2^{++}(2.3~2.4 GeV);

$$\Gamma(J/\psi \to \gamma G_{2+})/\Gamma_{total} = 1.1(2) \times 10^{-2}$$

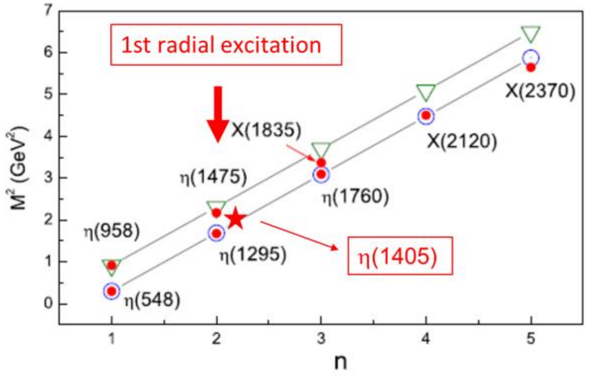


Pseudoscalar: η & η′ , “simple”

Exp: **lacking of info. above 2 GeV**; puzzles η(1295)? η(1405/1475)?

LQCD: 0^{-+}(2.3~2.6 GeV)

$$\Gamma(J/\psi \to \gamma G_{0-})/\Gamma_{total} = 2.31(80) \times 10^{-4}$$



Landscape of glueballs has been updated with BESIII's inputs

Scalar: 1 nonet in quark model, f_0 & f_0'

Exp: overpopulation

LQCD : ground state 0^+ glueball ~1.7 GeV;

$$\Gamma(J/\psi \to \gamma G_{0+})/\Gamma_{total} = 3.8(9) \times 10^{-3}$$

Tensor: 2 nonets($^3P_2$,$^3F_2$), complicated

Exp: large uncertainty

LQCD: 2^{++}(2.3~2.4 GeV);

$$\Gamma(J/\psi \to \gamma G_{2+})/\Gamma_{total} = 1.1(2) \times 10^{-2}$$

Pseudoscalar: η & η' , "simple"

Exp: lacking of info. above 2 GeV; puzzles $\eta(1295)$? $\eta(1405/1475)$?

LQCD: 0^{-+}(2.3~2.6 GeV)

$$\Gamma(J/\psi \to \gamma G_{0-})/\Gamma_{total} = 2.31(80) \times 10^{-4}$$

- ✓ **$f_0(1710)$ is largely overlapped with the scalar glueball, according to its production and decay properties**
- ✓ **Large production rate of $f_2(2340)$ in J/ψ radiative decays**
- ✓ **Non-observation of $\eta(1295)$**
- ✓ **Insights of $\eta(1405/1475)$**
- ✓ **$X(2370)$: a good candidate with analogy decay pattern as η_c**

Scalar glueball candidate: **decay properties**

Flavor-blindness of glueball decays

$$\boxed{\frac{1}{P.S.}\Gamma(G\to\pi\pi:K\overline{K}:\eta\eta:\eta\eta':\eta'\eta')=3:4:1:0:1}$$

*with chiral suppression

PRL 95 172001， PRL 98 149103

Expectation:

$$\Gamma(G\to\pi\pi)/\Gamma(G\to K\bar{K})\approx\frac{f_\pi^4}{f_K^4}\approx 0.48$$

Measured:

$$\frac{1}{P.S.}\Gamma(G\to\pi\pi:K\bar{K}:\eta\eta)\approx\underline{1.3:3.16}:1$$

New inputs from $\mathrm{J}/\psi\to\gamma\eta\eta'$

[BESIII PRL 129 192002(2022) , PRD 106 072012(2022)]

- Significant $\mathrm{f_0(1500)}$

$$\frac{\mathrm{B(f_0(1500)\to\eta\eta')}}{\mathrm{B(f_0(1500)\to\pi\pi)}}=\left(1.66^{+0.42}_{-0.40}\right)\times 10^{-1}$$

consistent with PDG

- **Absence of $\mathbf{f_0(1710)}$**

$$\frac{\mathrm{B(f_0(1710)\to\eta\eta')}}{\mathrm{B(f_0(1710)\to\pi\pi)}}<2.87\times10^{-3}\ @90\%\ \mathrm{C.L.}$$

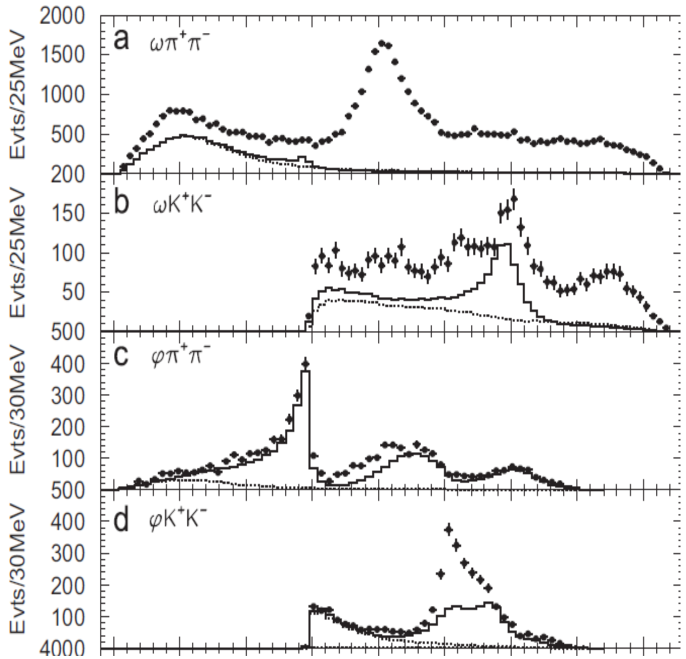
➢ Supports to the hypothesis that $\mathrm{f_0(1710)}$ overlaps with the ground state scalar glueball
  - Scalar glueball expected to be suppressed $\mathrm{B(G\to\eta\eta')/B(G\to\pi\pi)}<0.04$

[PR D 92, 121902; PR D 92, 114035]

Bottom line: Predictions on mixing scheme and decay property of glueball are model-dependent

More scalars

$f_0(1710)/f_0(1790)$?

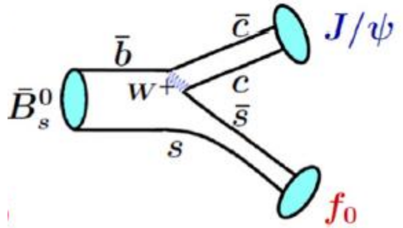


ωK^+K^- ➜ Peak around 1700 MeV/c^2 (OZI rule: $n\bar{n}$ structure)

$\phi\pi^+\pi^-$ ➜ Enhancement at 1790 MeV/c^2

ϕK^+K^- ➜ No peak around 1700 MeV/c^2

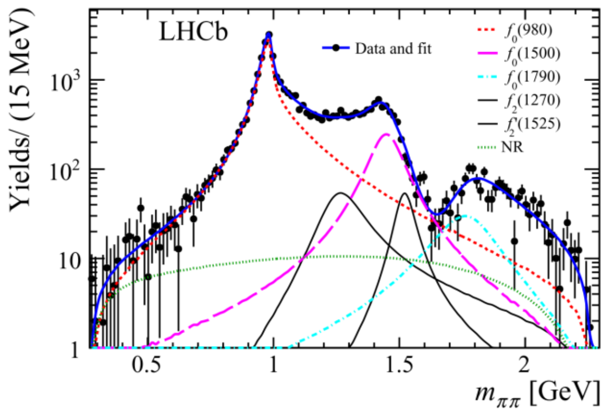
$f_0(1800)$

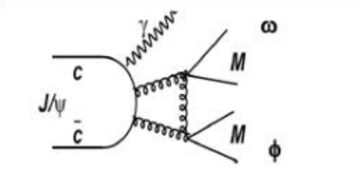


selective for $s\bar{s}$

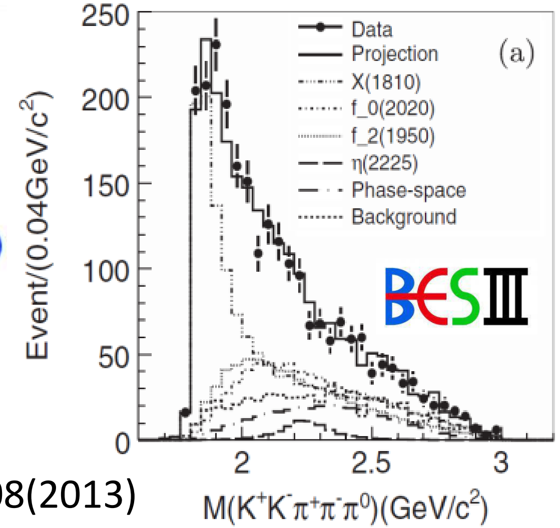
$B_s \to J/\psi f_0$

PLB 797 (2019) 134789





$J/\psi \to \gamma\omega\phi$ (DOZI)



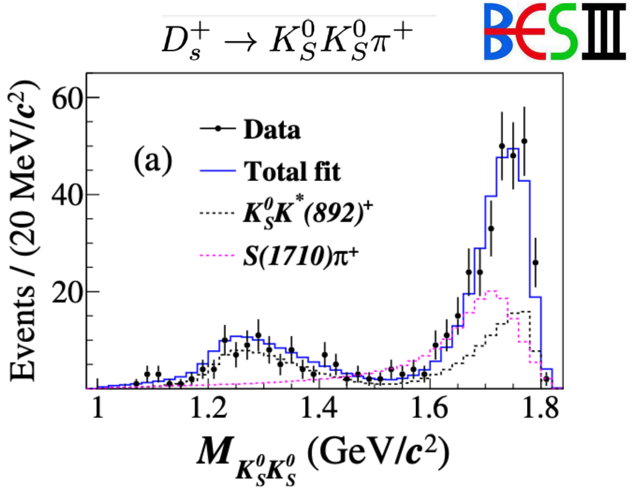
PRD 87, 032008(2013)

$a_0(1817)$

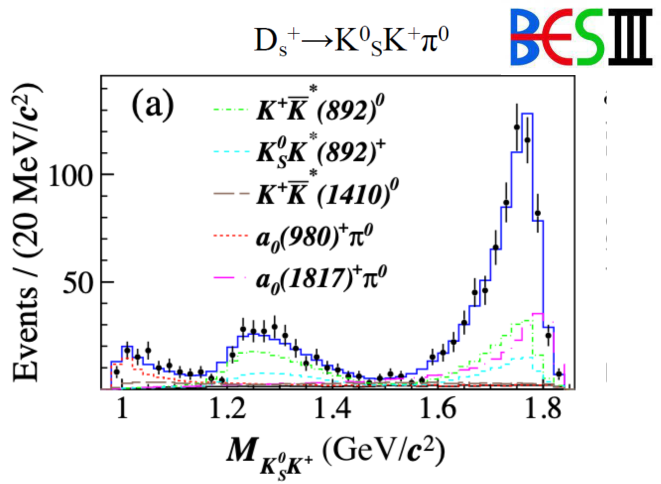
Isovector partner of $f_0(1800)$?

[Shulei 's talk]

PRD105, L051103 (2022)



PRL129, 182001 (2022)



Two photon couplings

$\gamma\gamma \to K_S K_S$

Belle PTEP 2013 (2013) 12, 123C01

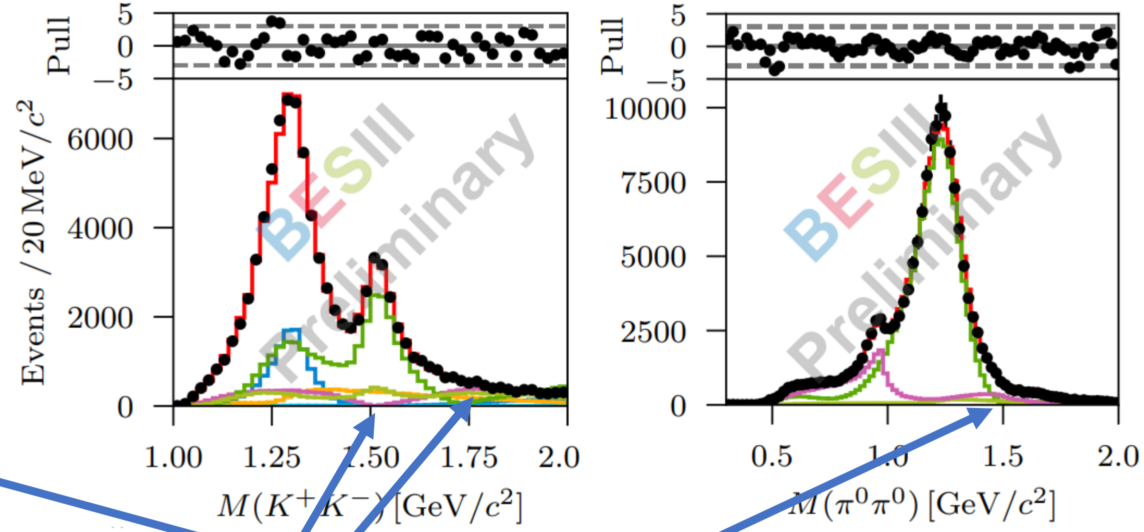
Parameter	$f_0(1710)$ fit			
	fit-H	fit-L	H,L combined	PDG
χ^2/ndf	694.2/585	701.6/585	–	–
Mass(f_J) (MeV/c^2)	1750^{+5+29}_{-6-18}	1749^{+5+31}_{-6-42}	1750^{+6+29}_{-7-18}	1720 ± 6
$\Gamma_{\text{tot}}(f_J)$ (MeV)	138^{+12+96}_{-11-50}	145^{+11+31}_{-10-54}	139^{+11+96}_{-12-50}	135 ± 6
$\Gamma_{\gamma\gamma}\mathcal{B}(K\bar{K})_{f_J}$ (eV)	12^{+3+227}_{-2-8}	21^{+6+38}_{-4-26}	12^{+3+227}_{-2-8}	unknown

$\gamma\gamma \to \pi^0\pi^0$

Belle PRD 78 (2008) 052004

Parameter	Nominal	$r_{02}=0$	No $f_0(Y)$	Unit
Mass($f_0(980)$)	982.2 ± 1.0	980.2 ± 1.0	$983.7^{+1.5}_{-1.0}$	MeV/c^2
$\Gamma_{\gamma\gamma}(f_0(980))$	$285.5^{+17.2}_{-17.1}$	$297.0^{+14.2}_{-13.7}$	$370.5^{+20.2}_{-18.7}$	eV
$g_{f_0(980)\pi\pi}$	1.82 ± 0.03	1.79 ± 0.03	1.89 ± 0.03	GeV
Mass($f_0(Y)$)	1469.7 ± 4.7	1466.8 ± 0.6	–	MeV/c^2
$\Gamma(f_0(Y))$	$89.7^{+8.1}_{-6.6}$	$422.4^{+18.4}_{-19.8}$	–	MeV
$\Gamma_{\gamma\gamma}\mathcal{B}(f_0(Y) \to \pi^0\pi^0)$	$11.2^{+5.0}_{-4.0}$	$6780.2^{+626.5}_{-574.7}$	0 (fixed)	eV

BESIII preliminary



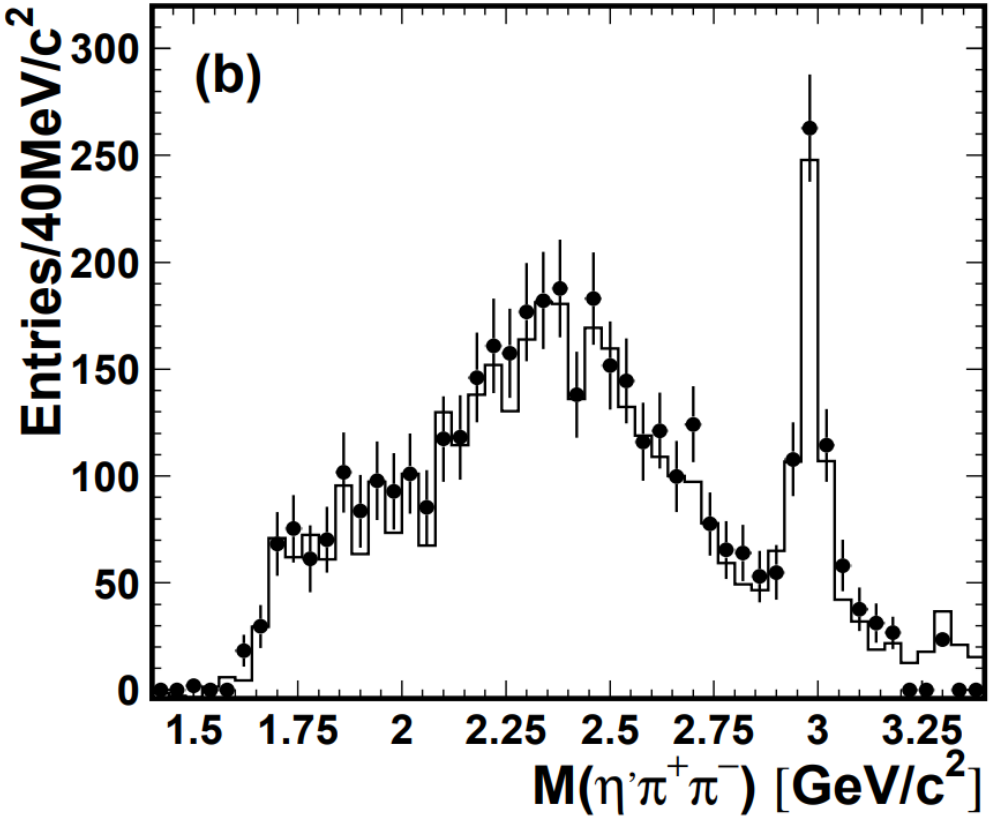
$f_0(1710)$? $f_0(1800)$?

$f_0(1500)$?

Proper assignment requires **more sophisticated model**

- $\gamma\gamma \to \eta'\pi^+\pi^-$

Belle PRD 86 052002(2012)



Amplitude analysis

Amplitude analysis is a key tool of hadron spectroscopy to disentangle contributions from individual resonances and to extract the resonance's spin-parity, mass, width and decay properties

$$Prob(\xi:\alpha) = \frac{\omega(\xi,\alpha)\epsilon(\xi)}{\int d\xi\omega(\xi,\alpha)\epsilon(\xi)}$$

$$\ln L = \sum_{\mathrm{n}=1}^{N_{data}} \ln(Prob(\xi,\alpha))$$

ξ (the four-momenta of the final-state particles),

$\omega(\xi,\alpha) = \frac{d\sigma}{d\Phi} = |\sum_i A_i|^2$ differential cross section,

$\epsilon(\xi)$ efficiency

Perform an un-binned log-likelihood fit (fit the data event-wise to high-dimensional distributions using complex weights) to make our model for ω agree with the experimental distribution by varying the α

For J/ψ radiative decays [Eur. Phys. J. A 16, 537]

$$A = \psi_\mu(m_1)e_\nu^*(m_2)A^{\mu\nu} = \psi_\mu(m_1)e_\nu^*(m_2)\sum_i \Lambda_i U_i^{\mu\nu}$$

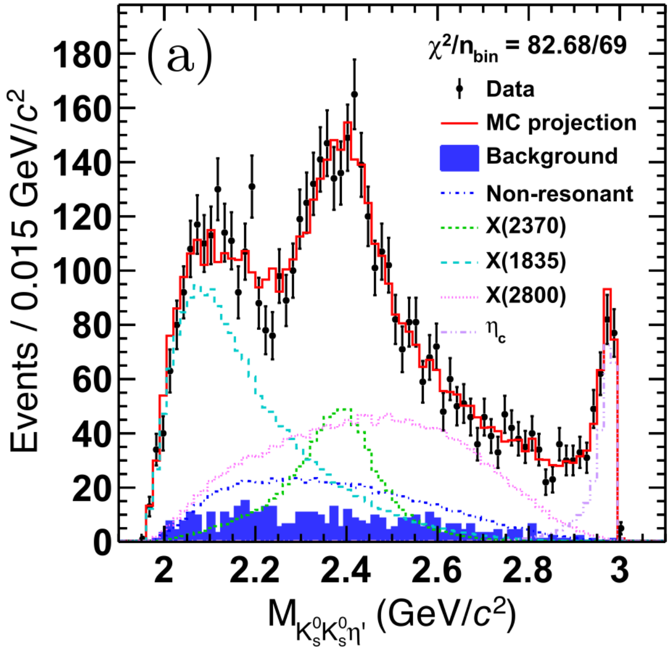
e.g. $\mathrm{J}/\psi \to \gamma 0^{-+}, 0^{-+} \to \mathrm{f}_0\eta, \mathrm{f}_0\pi\pi$

$$\langle\gamma 0^{-+}|(f_0\eta)1\rangle = S_{\mu\nu}B_1(Q_{\psi\gamma X})f_{(12)}^{(f_0)}$$

$$S_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}p_\psi^\alpha q^\beta$$

$B_1(Q_{\psi\gamma X})$ is Blatt-Weisskopf centrifugal barrier for $\mathrm{J}/\psi \to \gamma X$

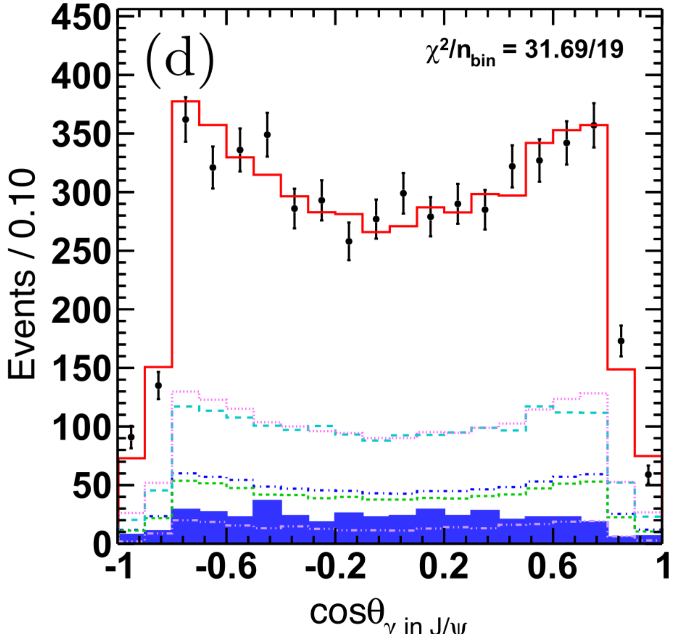
Spin-parity Determination of $X(2370)$ in $\mathrm{J}/\psi \to \gamma\eta' \mathrm{K_S^0 K_S^0}$



BESIII PRL 132 181901(2024)

Nominal fit solution

state	J^{PC}	Decay mode	Mass (MeV/c^2)	Width (MeV/c^2)	Significance
X(2370)	0^{-+}	$f_0(980)\eta'$	2395^{+11}_{-11}	188^{+18}_{-17}	14.9σ
X(1835)	0^{-+}	$f_0(980)\eta'$	1844	192	22.0σ
X(2800)	0^{-+}	$f_0(980)\eta'$	2799^{+52}_{-48}	660^{+180}_{-116}	16.4σ
η_c	0^{-+}	$f_0(980)\eta'$	2983.9	32.0	$> 20.0\sigma$
PHSP	0^{-+}	$\eta'(K_S^0 K_S^0)_{S-wave}$	− − −	− − −	9.0σ
		$\eta'(K_S^0 K_S^0)_{D-wave}$	− − −	− − −	16.3σ



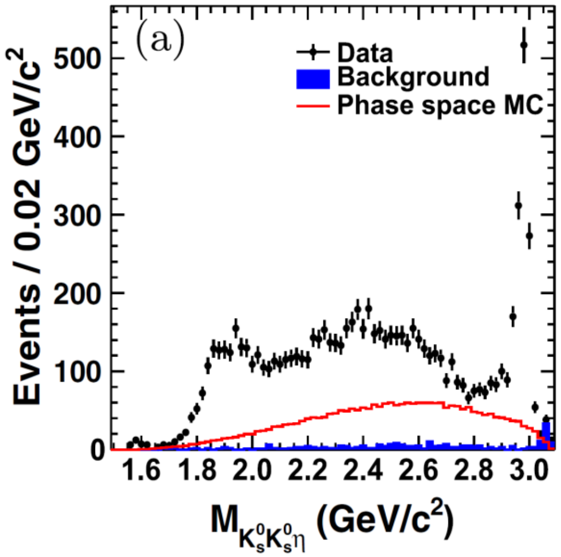
- **$\mathbf{X(2370)'s\ J^{PC} = 0^{-+}}$ with 9.8 $\boldsymbol{\sigma}$**
- Product branching fraction:

$$B(\mathrm{J}/\psi \to \gamma X(2370) B\left(X(2370) \to \eta' K_S^0 K_S^0\right) B\left(f_0(980) \to K_S^0 K_S^0\right)$$
$$= \left(1.31 \pm 0.22^{+2.85}_{-0.84}\right) \times 10^{-5}$$

Decay Modes for 0^{-+} Glueball Searches

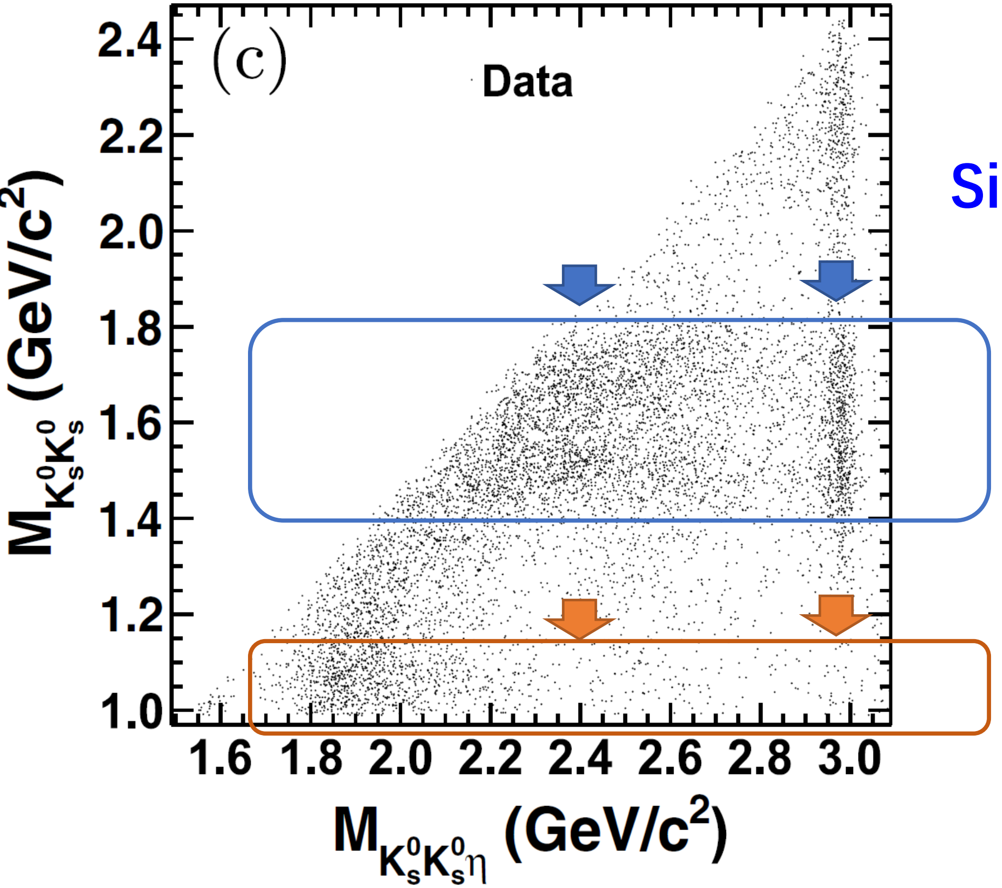
- Typically, PPP (3 pseudoscalar mesons, such as $\pi\pi\eta$, $\pi\pi\eta'$, $KK\pi$) modes are believed as golden decay modes in 0^{-+} glueball searches.
  - S wave decays for 0^{-+} mesons, no suppression factor, major decay modes
  - PPP modes are either forbidden or strongly suppressed in 0^{++}, 2^{++} mesons decays — spin-parity filter
- PP (2 pseudoscalar mesons) modes are forbidden for 0^{-+} mesons
- VV modes (2 vector mesons, such as $\omega\omega$, $\phi\phi$, $\rho\rho$, K^*K^*)
  - P wave decays for 0^{-+} mesons — suppressed decays, especially near mass threshold
  - All J^{PC} mesons allowed, not a spin-parity filter
- Baryon modes
  - All J^{PC} mesons allowed, not a spin-parity filter
- Multi-pion modes
  - All J^{PC} mesons allowed, not a spin-parity filter
  - 0-+ mesons decay mainly via 2 body sequential decays, i.e., mainly via f2(1270), a1(1260) pair intermediate states — 0-+ glueball mass may not be high enough, i.e., PS is not allowed.

$\mathbf{X(2370)}$ seen in $J/\psi \to \gamma K_S^0 K_S^0 \eta$



Observation and Spin-Parity Determination of the $X(1835)$ in $J/\psi \to \gamma K_S^0 K_S^0 \eta$

BESIII PRL 115 091803(2015)



Similar decay patterns of the $X(2370)$ and η_c

clear $X(2370)$ AND η_c signals

no $X(2370)$ OR η_c signal

Observation of **An Exotic 1^{-+} Isoscalar State $\eta_1(1855)$**

PRL 129 192002(2022) , PRD 106 072012(2022)

- Angular distribution as a function of $M(\eta\eta')$ expressed **model-independently**

$$\langle Y_l^0 \rangle \equiv \sum_{i=1}^{N_k} W_i Y_l^0(cos\theta_\eta^i)$$

- Related to the spin-0(S), spin-1(P), spin-2(D) amplitudes in $\eta\eta'$ by:

$$\sqrt{4\pi}\langle Y_0^0\rangle = S_0^2 + P_0^2 + P_1^2 + D_0^2 + D_1^2 + D_2^2,$$

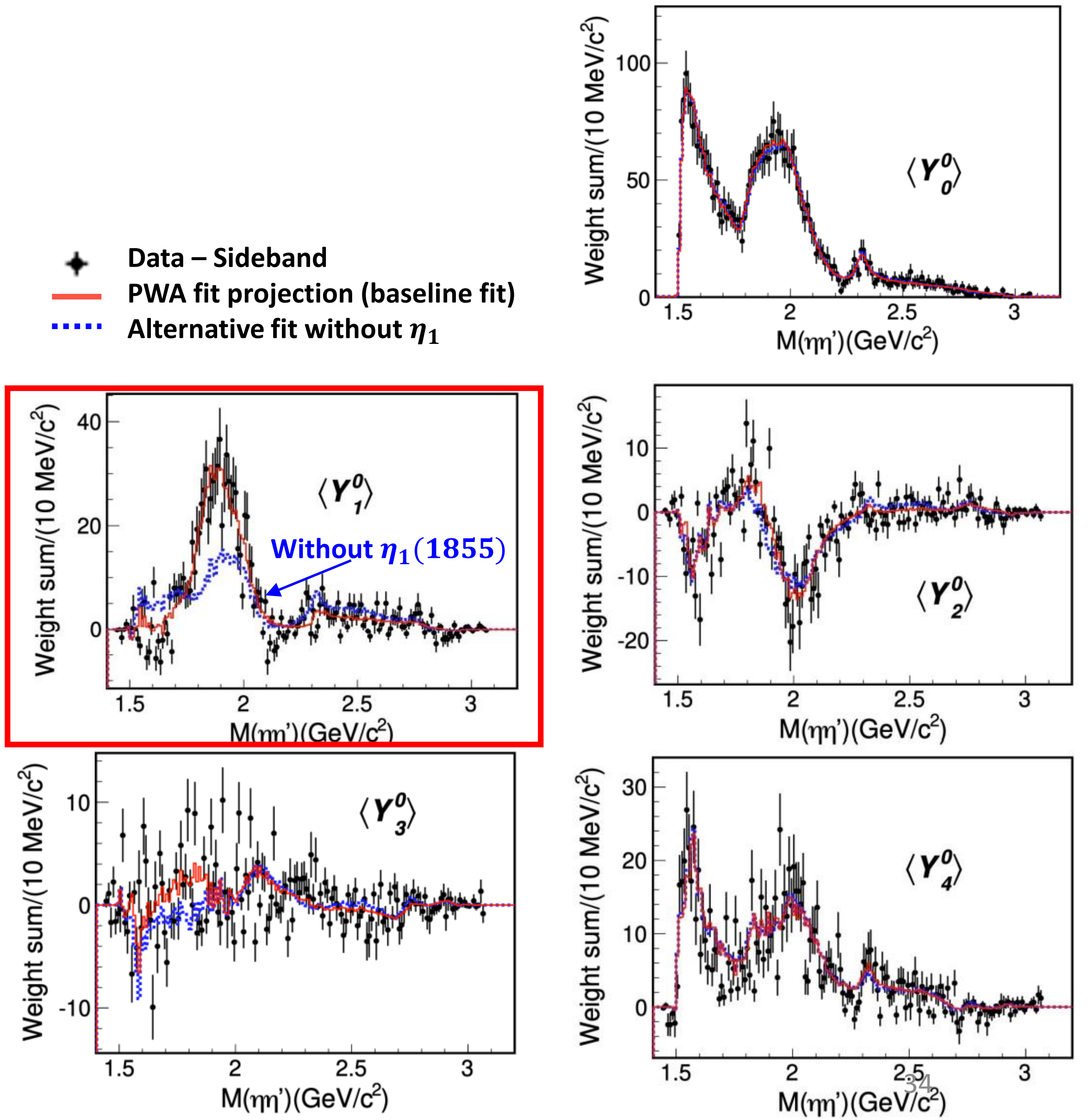
$$\sqrt{4\pi}\langle Y_1^0\rangle = 2S_0P_0\cos\phi_{P_0} + \frac{2}{\sqrt{5}}(2P_0D_0\cos(\phi_{P_0}-\phi_{D_0}) + \sqrt{3}P_1D_1\cos(\phi_{P_1}-\phi_{D_1})),$$

$$\sqrt{4\pi}\langle Y_2^0\rangle = \frac{1}{7\sqrt{5}}(14P_0^2 - 7P_1^2 + 10D_0^2 + 5D_1^2 - 10D_2^2) + 2S_0D_0\cos\phi_{D_0},$$

$$\sqrt{4\pi}\langle Y_3^0\rangle = \frac{6}{\sqrt{35}}(\sqrt{3}P_0D_0\cos(\phi_{P_0}-\phi_{D_0}) - P_1D_1\cos(\phi_{P_1}-\phi_{D_1})),$$

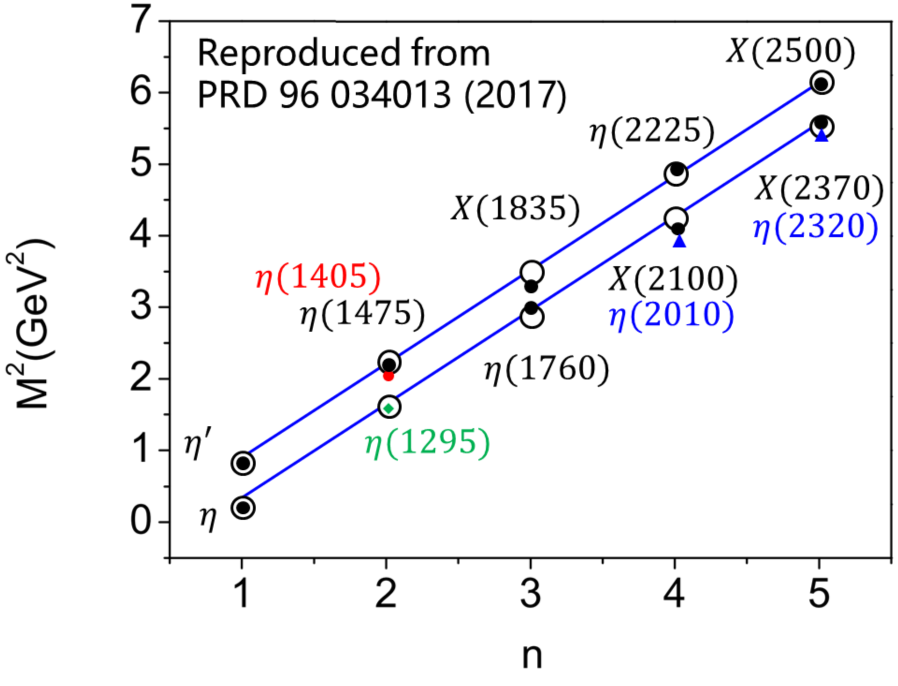
$$\sqrt{4\pi}\langle Y_4^0\rangle = \frac{1}{7}(6D_0^2 - 4D_1^2 + D_2^2).$$

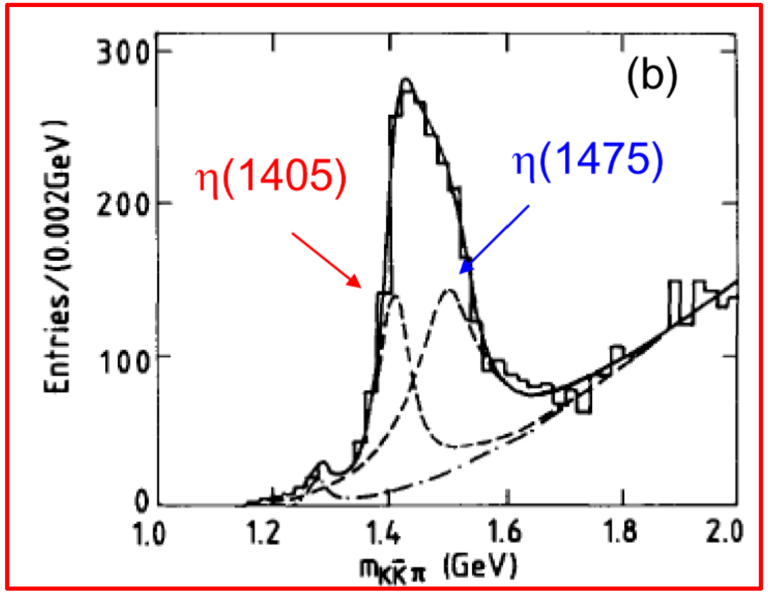
- **Narrow structure** in $\langle Y_1^0\rangle$
  - ➢**Cannot be described by resonances in $\gamma\eta(\eta')$**
  - **$\eta_1(1855) \to \eta\eta'$ needed**



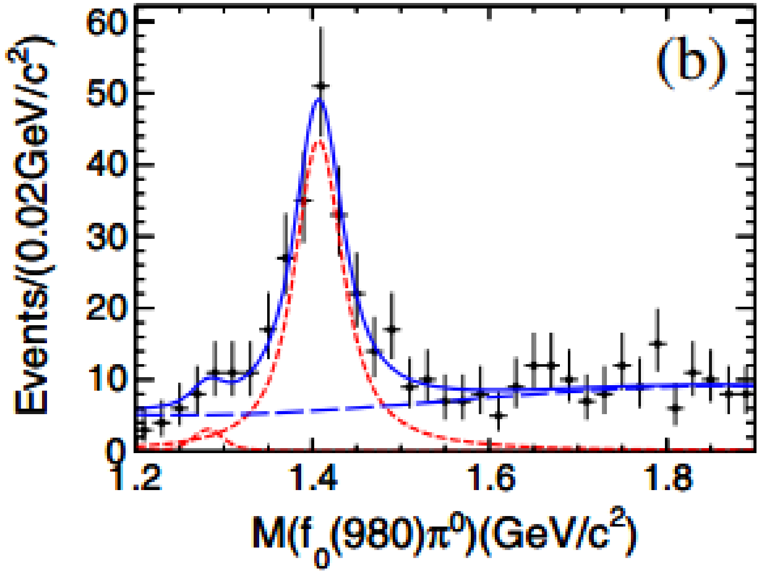
Where is the 0^{-+} glueball

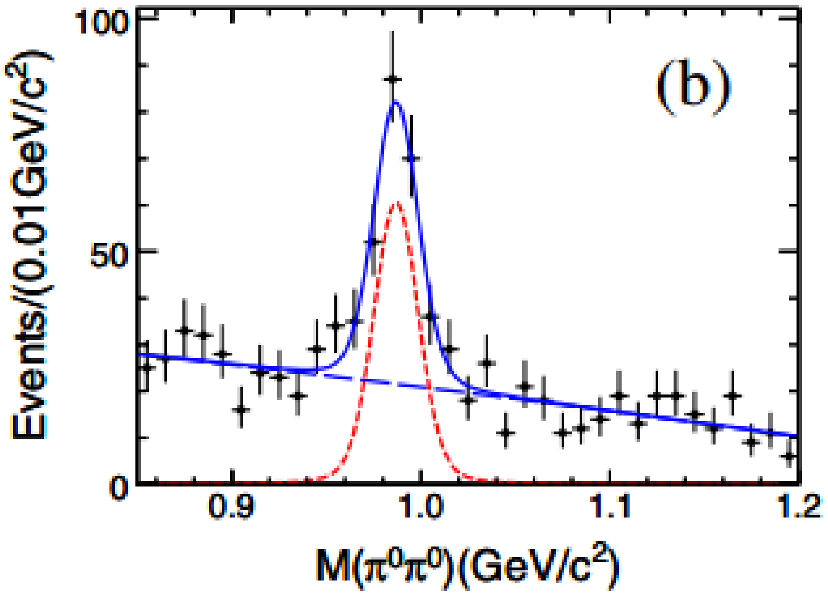
- Pseudoscalar sector, a promising window
  [Future Physics Programme of BESIII(2020)]
  - Only η, η′ (& radial excitations) from quark model
- Mass
  - LQCD: 0^{-+} glueball (2.3~2.6 GeV)
  - The first glueball candidate: ι(1440) (Split into η(1405) and η(1475))
    - Mass incompatible with LQCD
  - Little experimental information above 2 GeV

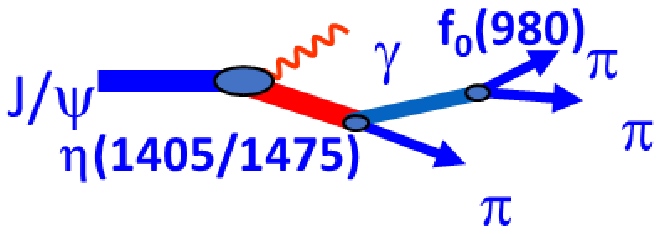




Isospin-violating decay of $\eta(1405) \rightarrow f_0(980)\pi^0$







BESIII PRL 108 182001(2012)

f0(980) is extremely narrow: $\Gamma \cong$ 10 MeV.

PDG: Γ(f0(980)) $\cong$ 40~100 MeV.

Triangle singularity mechanism has been proposed

- Manifested in many near-threshold structures

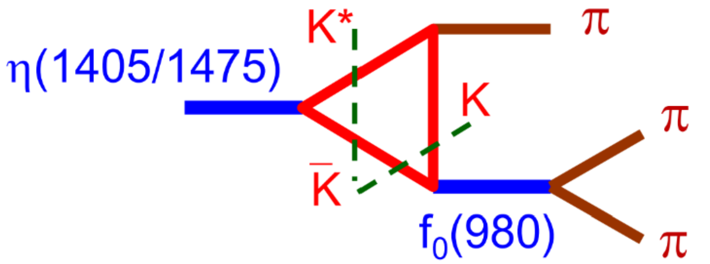
Anomalously large isospin violation:

$$\frac{Br(\eta(1405) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0)}{Br(\eta(1405) \rightarrow a_0^0(980)\pi^0 \rightarrow \eta\pi^0\pi^0)} \cong (17.9 \pm 4.2)\%$$

$$\xi_{af} = \frac{Br(\chi_{c1} \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0)}{Br(\chi_{c1} \rightarrow a_0(980)\pi^0 \rightarrow \eta\pi^0\pi^0)} < 1\%(90\%\ C.L.)$$

PRD, 83(2100)032003

PRL 108 081803 (2012)



Shed new lights on the $\eta(1405)/\eta(1475)$ puzzle

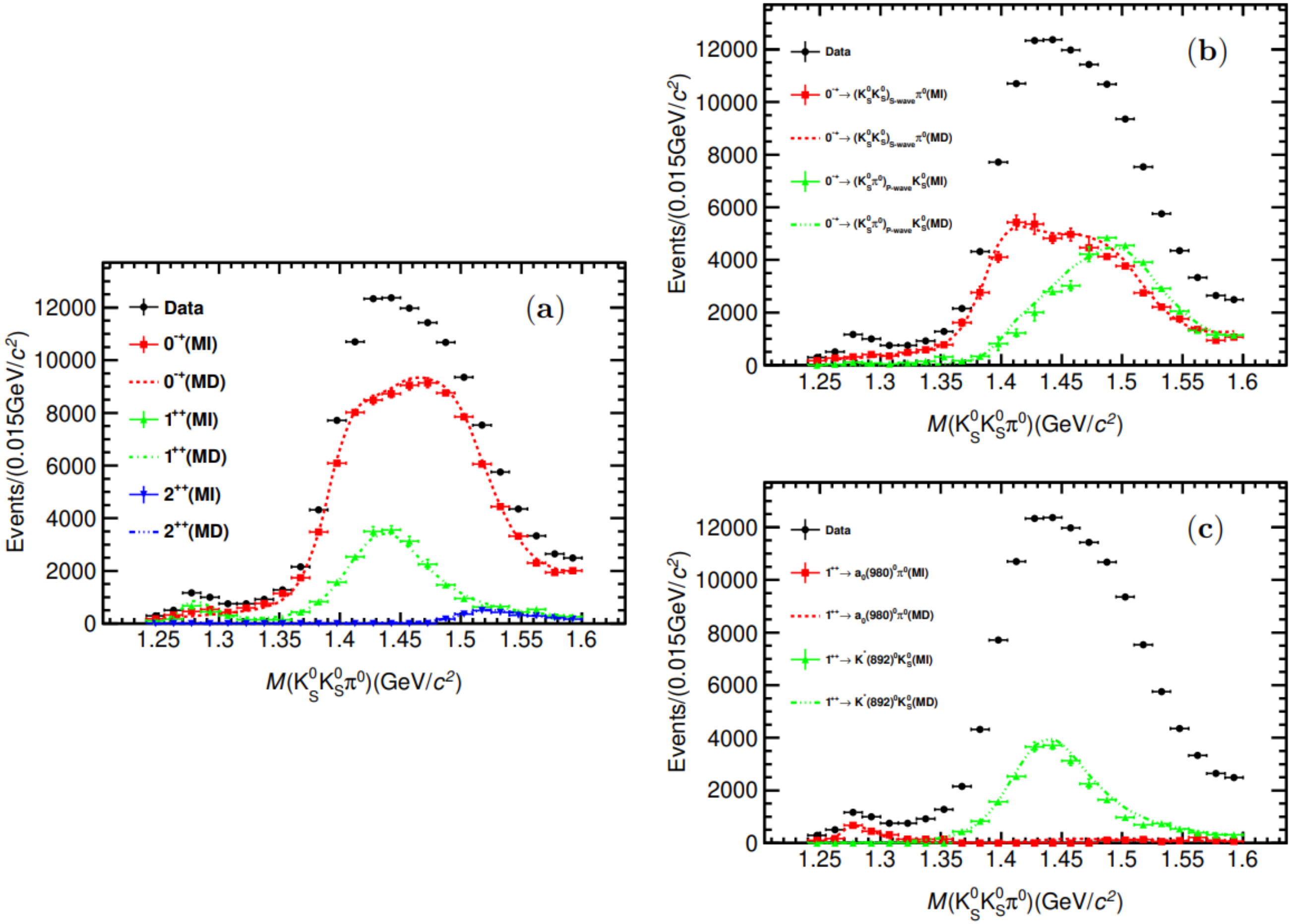
$$J/\psi \rightarrow \gamma K_S K_S \pi^0$$

BESIII JHEP 03 121(2023)

- **M**ass **I**ndependent PWA in bins of $M(K_S K_S \pi^0)$ to detangle J^{PC} components
  - **Valuable inputs to develop models**
- **M**ass **D**ependent PWA with BW to extract resonances
- **Consistency between MI and MD results**

- **Dominated by 0^{-+}**
  - **Two BWs around 1.4 GeV is needed**

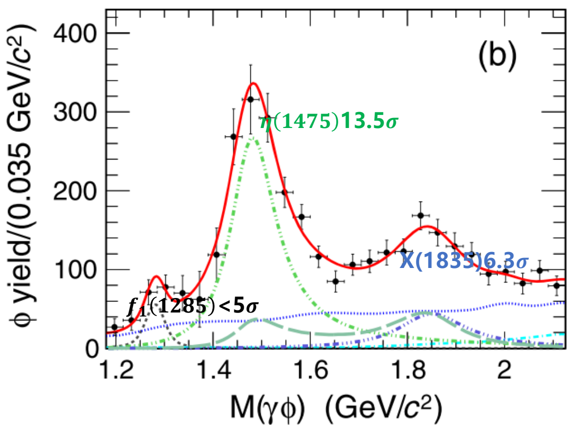
- Coupled-channel analysis
  - PRD 107, L091505 (2023) ; PRD 109, 014021 (2024); arXiv:2407.10234



$J/\psi \to \gamma\gamma\phi$, a $s\bar{s}$ flavor filter

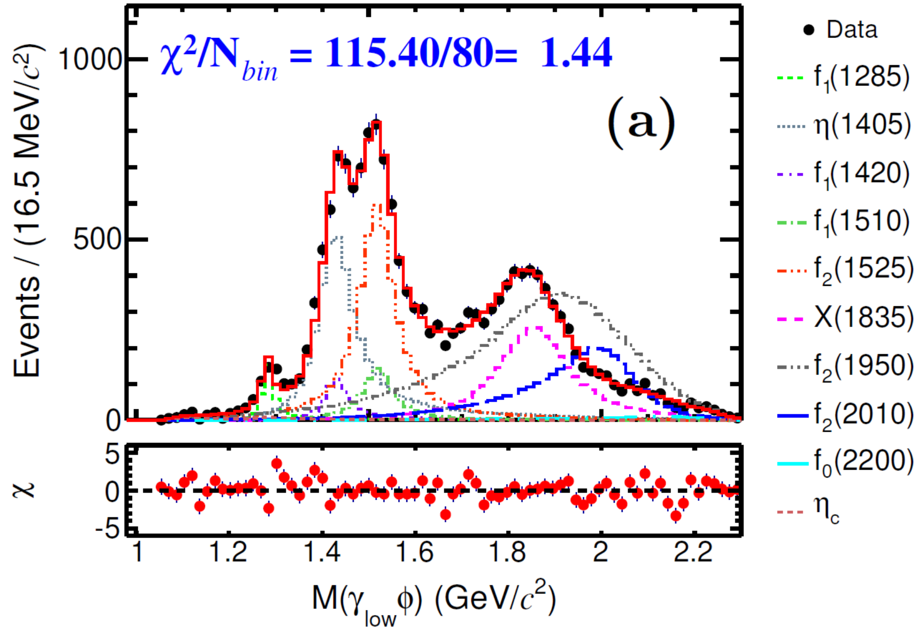
BESIII PhysRevD.111.052011(2025)

PR D97 051101 (2018)



Amplitude analysis with ML techniques for background subtraction

←Fit to mass spectrum



From the amplitude analysis,

- $\eta(1405)$ is observed, while $\eta(1475)$ can not be excluded
- $X(1835) \to \gamma\phi$ suggests its assignment of η' excitation
- $\eta_c \to \gamma\phi$ are observed. The very first radiative decay mode of η_c
- Observation of $f_2(1950)$ and $f_0(2200) \to \gamma\phi$ unfavored their glueball interpretations [PRD 108, 014023, Sci.China Phys.Mech.Astron. 67 (2024) 11, 111012]

Indications of tensor glueball

$$\boxed{\begin{aligned}&\Gamma(J/\psi \to \gamma G_{2^+}) = 1.01(22)keV\\ &\Gamma(J/\psi \to \gamma G_{2^+})/\Gamma_{tot} = 1.1 \times 10^{-2}\end{aligned}}$$

CLQCD, Phys. Rev. Lett. 111, 091601 (2013)

Experimental results

$$\mathbf{Br}(\mathrm{J}/\psi \to \gamma \mathrm{f}_2(2340) \to \gamma\eta\eta) = \left(3.8^{+0.62+2.37}_{-0.65-2.07}\right) \times 10^{-5}$$

BESIII PRD 87,092009 (2013)

$$\mathbf{Br}(\mathrm{J}/\psi \to \gamma \mathrm{f}_2(2340) \to \gamma\phi\phi) = \left(1.91 \pm 0.14^{+0.72}_{-0.73}\right) \times 10^{-4}$$

BESIII PRD 93, 112011 (2016)

$$\mathbf{Br}(\mathrm{J}/\psi \to \gamma \mathrm{f}_2(2340) \to \gamma \mathrm{K_s K_s}) = \left(5.54^{+0.34+3.82}_{-0.40-1.49}\right) \times 10^{-5}$$

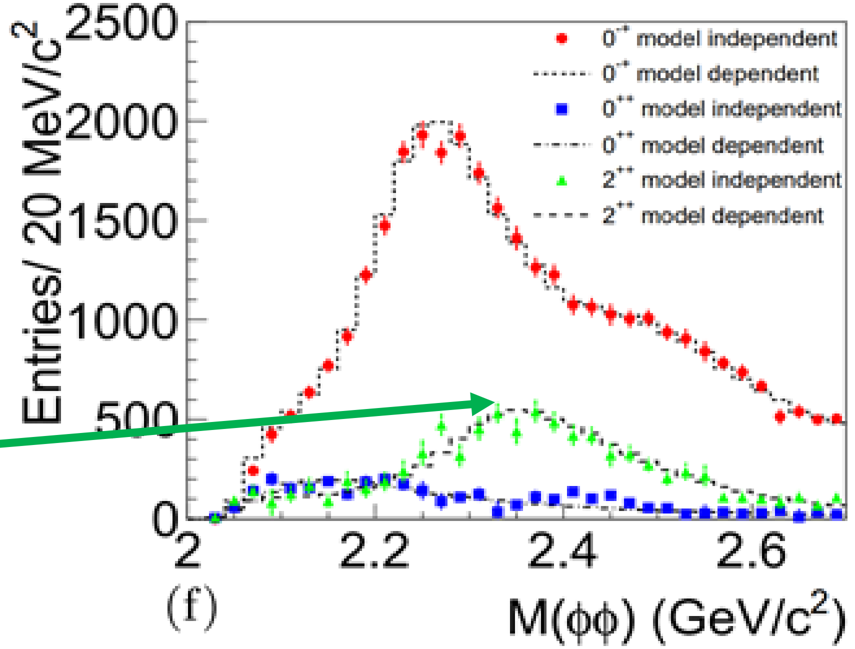
BESIII PRD 98,072003 (2018)

$$\mathbf{Br}(\mathrm{J}/\psi \to \gamma \mathrm{f}_2(2340) \to \gamma\eta'\eta') = \left(8.67 \pm 0.70^{+0.16}_{-1.67}\right) \times 10^{-6}$$

BESIII PRD 105,072002 (2022)

still desired to study more decay modes

BESIII $J/\psi \to \gamma\phi\phi$ [PRD 93, 112011 (2016)]



- $f_2(2010)$, $f_2(2300)$ and $f_2(2340)$ in πp reactions are all observed in $\mathbf{J}/\boldsymbol{\psi} \to \boldsymbol{\gamma\phi\phi}$ with a **strong production of $f_2(2340)$**
- Consistent with **double-Pomeron exchange** from WA102@CERN

More complicated due to the large number of tensor states