# Pion Photoproduction and Resonance Structure with Hamiltonian Effective Field Theory

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# CONTENTS

# $\gamma N \to \pi N$ and $N^*(1535)$ with Hamiltonian Effective Field Theory

# Nucleon Resonances

- Naive quark model predicts wrong mass order for $N^*(1440)$ & $N^*(1535)$.
- Nucleon Resonances are important for interpreting the scattering experimental data.
- Their properties are helpful to understand the nonperturbative behavior of QCD.

# Pion Photoproduction off Nucleon with Hamiltonian EFT

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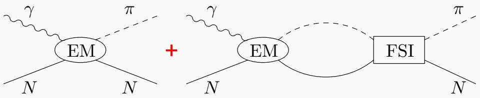

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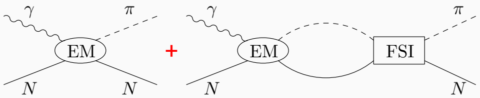


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\begin{aligned}
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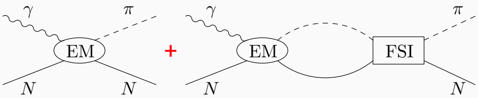


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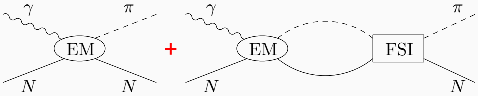


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  - understand the structure of $N(1535)$ and the interactions of $\pi N/\eta N$ at low energies and near the resonance

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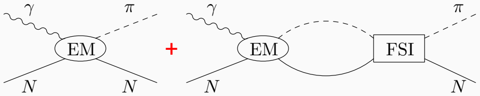


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  - Finite State Interaction (FSI) part can be determined independently
  - understand the structure of $N(1535)$ and the interactions of $\pi N/\eta N$ at low energies and near the resonance
  - necessities for the photon-nucleus investigation

1. $\pi N \to \pi N$
2. lattice QCD spectrum of $N^*$
3. $\gamma + N \to \pi + N$

# $N^*(1535)$ with $\pi N$ Scattering

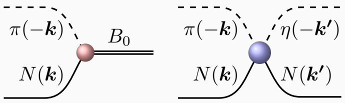
$N^*(1535)$ is the lowest resonance with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$.

- One needs to consider the interactions

  among the bare baryon $N_0^*$, $\pi N$ channel, and $\eta N$ channel.

$$G^2_{\pi N; N_0^*}(k) = \frac{3g^2_{\pi N; N_0^*}}{4\pi^2 f^2}\omega_\pi(k)$$

$$V^S_{\pi N, \pi N}(k, k') = \frac{3g^S_{\pi N}}{4\pi^2 f^2}\frac{m_\pi + \omega_\pi(k)}{\omega_\pi(k)}\frac{m_\pi + \omega_\pi(k')}{\omega_\pi(k')}$$

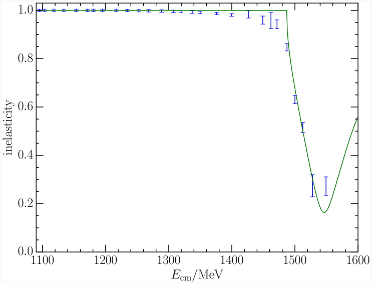


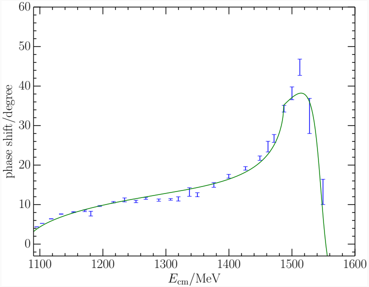

- Phase shifts and inelasticities

  are obtained by solving Bethe-Salpeter equation with the interactions.

$$T_{\alpha,\beta}(k, k'; E) = V_{\alpha,\beta}(k, k') + \sum_\gamma \int q^2 dq V_{\alpha,\gamma}(k, q)\frac{1}{E - \sqrt{m^2_{\gamma_1} + q^2} - \sqrt{m^2_{\gamma_2} + q^2} + i\epsilon}T_{\gamma,\beta}(q, k'; E)$$

# $N^*(1535)$ with $\pi N$ scattering at infinite volume




$\pi N$ Scattering with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$.

Our Pole: $1531 \pm 29 - i\ 88 \pm 2$ MeV.

Particle Data Group: $1510 \pm 20 - i\ 85 \pm 40$ MeV.

Z. W. Liu, W. Kamleh, D. B. Leinweber, F. M. Stokes, A. W. Thomas and J. J. Wu, Phys. Rev. Lett. 116 (2016) no.8, 082004

1. $\pi N \to \pi N$
2. **lattice QCD spectrum of $N^*$**
3. $\gamma + N \to \pi + N$

# Lattice QCD

- LQCD starts from the first principle of QCD
- model independent, reliable
- LQCD gives hadron spectra and quark distribution functions
  at finite volumes, large quark masses, discrete spaces
- not directly related to physical observables

# Connection between Scattering Data and Lattice QCD Data

## Lattice QCD

- large pion mass: extrapolation
- finite volume
- discrete space

## Lattice QCD Data $\rightarrow$ Physical Data

- Lüscher Formalisms and extensions:
  Model independent; efficient in single-channel problems
  Spectrum $\rightarrow$ Phaseshifts;
- Effective Field Theory (EFT), Models, etc
  with low-energy constants fitted by Lattice QCD data

## Physical Data $\rightarrow$ Lattice QCD Data

- EFT: discretization, analytic extension, Lagrangian modification
- various discretization: eg. discretize the momentum in the loop

# Lattice QCD and Effective Field Theory

Effective field theory deals with extrapolation powerfully.

Finite-volume effect can be studied by discretizing the EFT.

Discrete spacing effects can also be studied with EFT.

Scattering Data and Lattice QCD data are two important sources for studying resonances.

We should try to analyse them both at the same time.

# Hamiltonian Effective Field Theory

Hamiltonian Effective Field Theory (HEFT)

analyses both experimental data at infinite volume

and lattice QCD results at finite volume at the same time.

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  wavefunctions: analyse the structure of the eigenstates on the lattice

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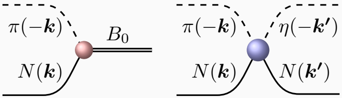
  wavefunctions: analyse the structure of the eigenstates on the lattice

- finite-volume and infinite-volume results are connected by the coupling constants etc.




$$\tilde{G}_i(k_n) = \sqrt{\frac{C_3(n)}{4\pi}}\left(\frac{2\pi}{L}\right)^{3/2} G_i(k_n),$$

$$\tilde{V}^S_{i,j}(k_n, k_m) = \frac{\sqrt{C_3(n)C_3(m)}}{4\pi}\left(\frac{2\pi}{L}\right)^3 V^S_{i,j}(k_n, k_m).$$

$C_3(n)$ represents the number of summing the squares of three integers to equal $n$.

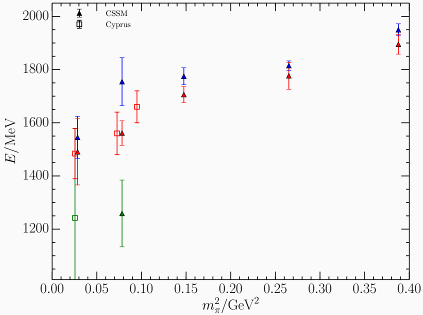
With the eigen-solution of the discretized Hamiltonian, one can obtain the mass spectrum and the components.

$$H_0 = \mathrm{diag}\{m^0_{N_1}, \omega_{\pi N}(k_0),\ \omega_{\eta N}(k_0), \omega_{\pi N}(k_1),\ \omega_{\eta N}(k_1), ...\},$$

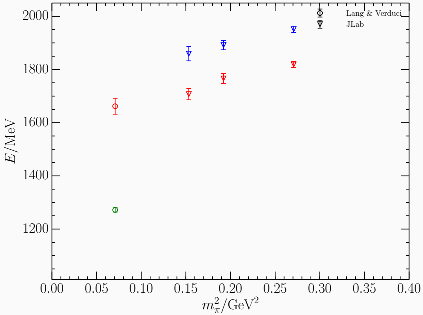
$$H_I = \begin{pmatrix} 0 & \tilde{G}_{\pi N}(k_0) & \tilde{G}_{\eta N}(k_0) & \tilde{G}_{\pi N}(k_1) & \tilde{G}_{\eta N}(k_1) & \cdots \\ \tilde{G}_{\pi N}(k_0) & \tilde{V}^S_{\pi N,\pi N}(k_0, k_0) & 0 & \tilde{V}^S_{\pi N,\pi N}(k_0, k_1) & 0 & \cdots \\ \tilde{G}_{\eta N}(k_0) & 0 & 0 & 0 & 0 & \cdots \\ \tilde{G}_{\pi N}(k_1) & \tilde{V}^S_{\pi N,\pi N}(k_1, k_0) & 0 & \tilde{V}^S_{\pi N,\pi N}(k_1, k_1) & 0 & \cdots \\ \tilde{G}_{\eta N}(k_1) & 0 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

# Spectra at Finite Volumes

3 sets of lattice QCD data at different pion masses and finite volumes




$L \approx 3$ fm




$L \approx 2$ fm

$N^*$ Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ at finite volumes

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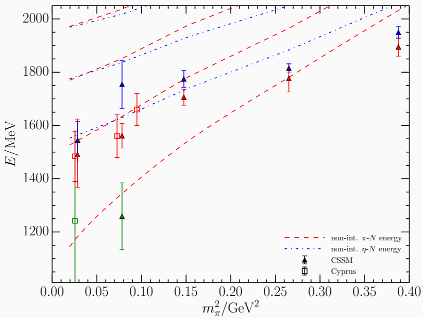
3 sets of lattice QCD data at different pion masses and finite volumes

Non-interacting energies of the two-particle channels

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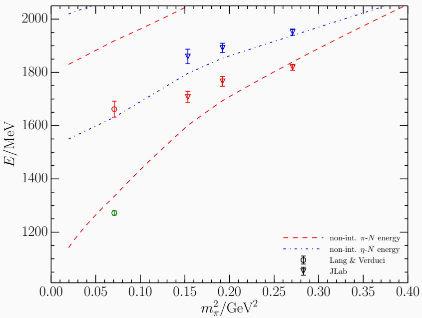


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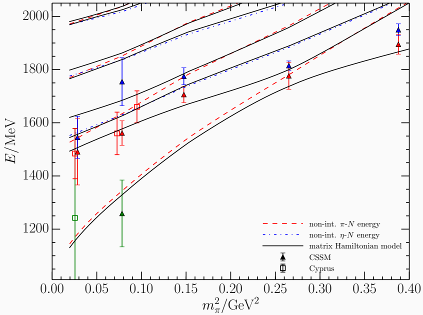
Eigenenergies of Hamiltonian effective field theory

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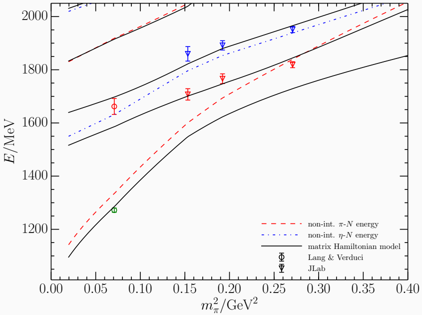


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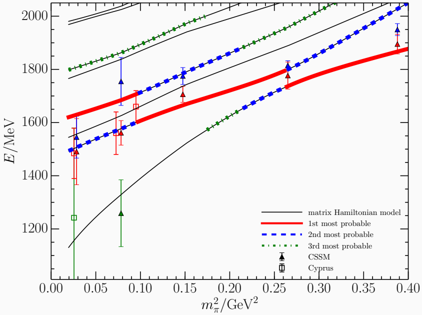
3 sets of lattice QCD data at different pion masses and finite volumes

Eigenenergies of Hamiltonian effective field theory

Coloured lines indicating most probable states observed in LQCD

We not only provide the mass but also analyze why some states are observed on the lattice

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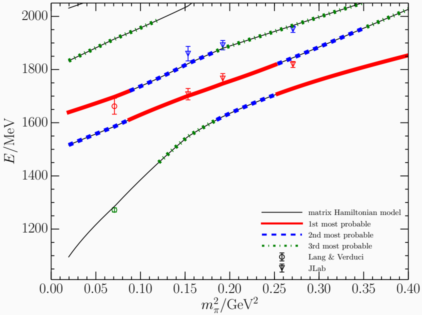


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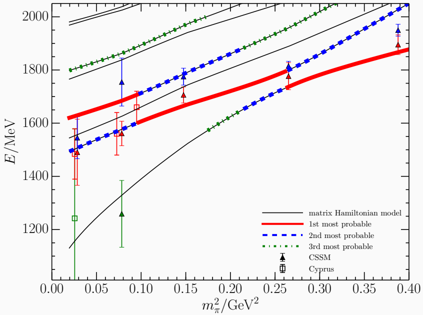
# Components of Eigenstates with $L \approx 3$ fm

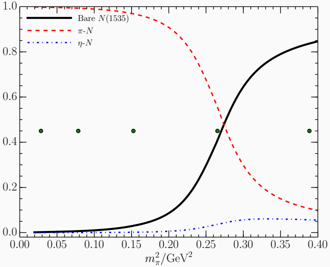



$N^*$ Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ and $L \approx 3$ fm

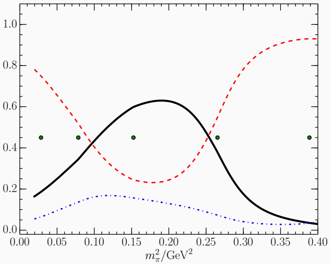
- The 1st eigenstate at light quark masses is mainly $\pi N$ scattering states.
- The most probable state at physical quark mass is the 4th eigenstate.

  It contains about 60% bare $N^*(1535)$, 20% $\pi N$ and 20% $\eta N$.

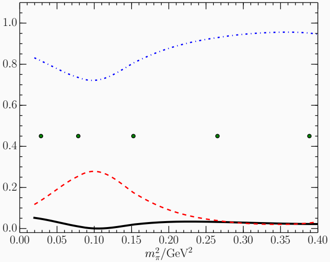
# Components of Eigenstates with $L \approx 3$ fm



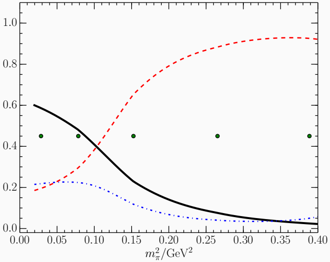

1st eigenstate




2nd eigenstate




3rd eigenstate




4th eigenstate

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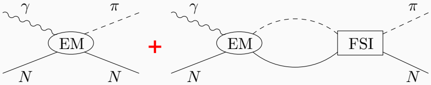
2. lattice QCD spectrum of $N^*$
3. $\gamma + N \to \pi + N$

# Electromagnetic Multipoles

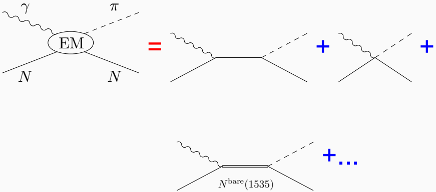
- $|\gamma N\rangle \to |\phi\,(\vec{k}),\ N(-\vec{k}, s_z'^N)\rangle,$
- $|\gamma N\rangle \to |\phi\, N; k, J, J_z, L\rangle\ ,$
- $|\gamma N\rangle \to |\phi\, N; k, J, J_z, \lambda_N'\rangle\ ,$

$k_x, k_y, k_z, s_z'^N$
$k,\ \ J,\ \ J_z,\ \ L$
$k,\ \ J,\ \ J_z, \lambda_N'$




Partial wave decomposition:

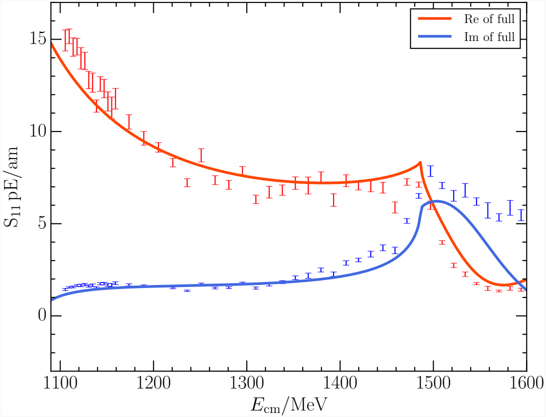
$$V_{\alpha,\gamma N}(J, \lambda_N', \lambda_\gamma, \lambda_N; k, q) = 2\pi \int_{-1}^{1} \mathrm{d}(\cos\theta) \sum_{s_z'^N} d^J_{\lambda_\gamma - \lambda_N, -\lambda_N'}(\theta) d^{1/2}_{s_z'^N, -\lambda_N'}(\theta)^* \mathcal{M}_{\alpha,\gamma N}(s_z'^N, \lambda_N, \lambda_\gamma; \vec{k}, \vec{q}),$$



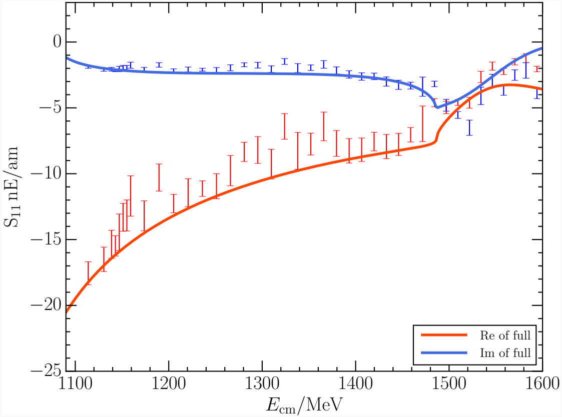

$$V^{JLS;\lambda_\gamma\lambda_N}_{\alpha,\gamma N}(k,q) \quad = \quad \sqrt{\frac{2L+1}{2J+1}} \sum_{\lambda_N'} \langle L, S, 0, -\lambda_N' | J, -\lambda_N'\rangle \times V_{\alpha,\gamma N}(J, \lambda_N', \lambda_\gamma, \lambda_N; k, q).$$

D. Guo and Z. W. Liu, Phys. Rev. D **105** (2022) no.11, 11

# Electric dipole amplitudes $E_{0+}$




$\gamma p \to \pi N$




$\gamma n \to \pi N$

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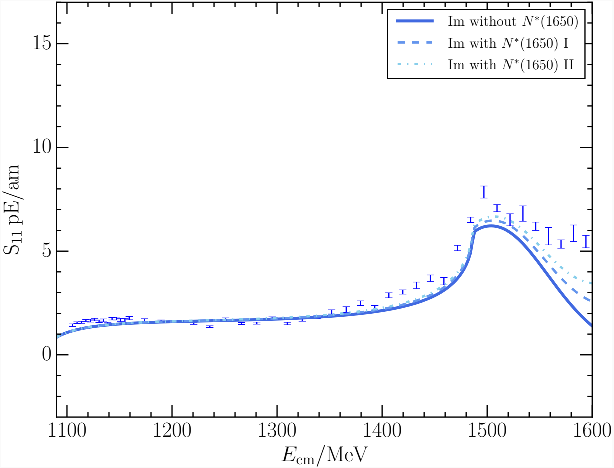
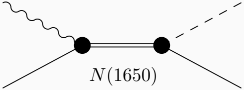
# Estimation of the $N^*(1650)$ contribution




Estimation:




+u channel

Couplings for the effective vertices are extracted from the decay widths of $N^*(1650)$.

D. Guo and Z. W. Liu, Phys. Rev. D **105** (2022) no.11, 11

**Therefore, we updated our results by**

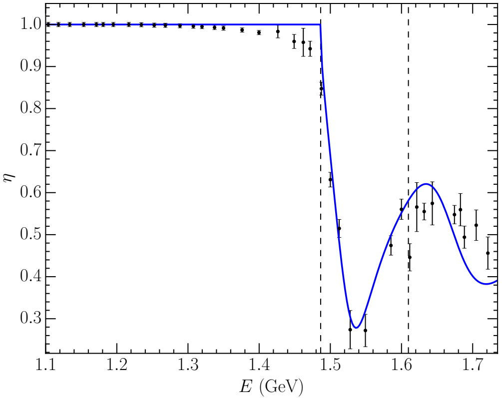
**Explicitly including $N^*(1650)$ as well as $N^*(1535)$**

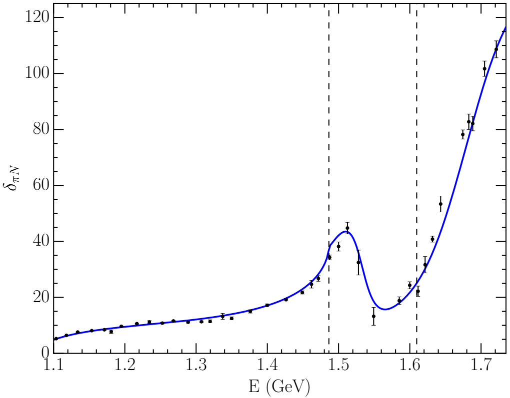
# $\gamma N \to \pi N$ and the Interference of $N^*(1535)$ and $N^*(1650)$

# Explicitly including $N^*(1650)$ as well as $N^*(1535)$

In Phys. Rev. D 108 (2023) 9, 094519, we consider

- two bare baryon states $N_1$ and $N_2$;
- $\pi N$, $\eta N$, and $K\Lambda$;
- more experimental data with larger energies (1.60, 1.75) GeV.



# Pole positions for $N^*(1535)$ and $N^*(1650)$

In the Particle Data Group (PDG) tables, the poles for the two low-lying odd-parity nucleon resonances are given as

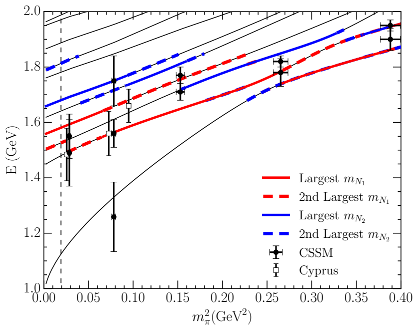
$$E_{N^*(1535)} = 1510 \pm 10 - (65 \pm 10)i \text{ MeV},$$
$$E_{N^*(1650)} = 1655 \pm 15 - (67 \pm 18)i \text{ MeV}.$$

Using HEFT, two poles for $N^*(1535)$ and $N^*(1650)$ in the second Riemann sheet are found at energies

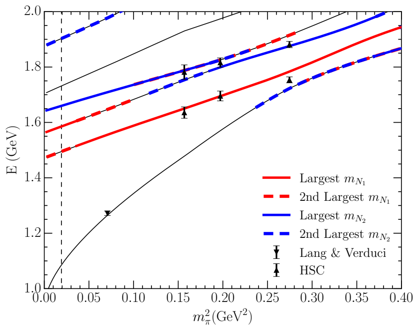
$$E_1 = 1500 - 50i \text{ MeV},$$
$$E_2 = 1658 - 56i \text{ MeV}.$$

Our results are in excellent agreement with the PDG pole positions.

# Finite-volume spectrum



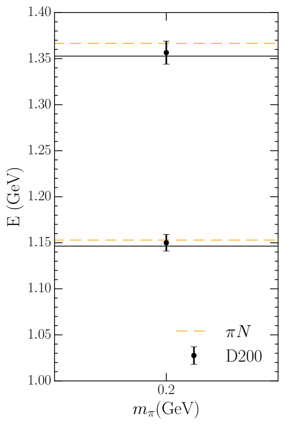

$L \sim 3$ fm




$L \sim 2$ fm

C. D. Abell, D. B. Leinweber, Z.-W. Liu, A. W. Thomas, J.-J. Wu, PRD 108 (2023) 9,094519

# Finite-volume Spectrum with larger Spatial Lattice Extent of $L = 4.05$ fm



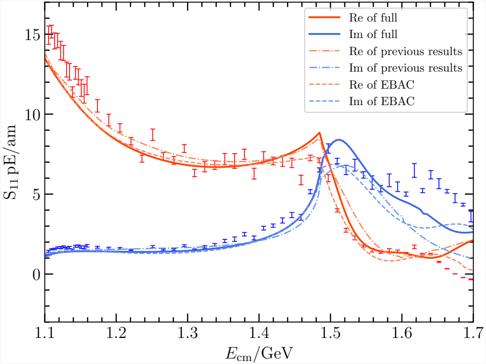

$L \sim 4$ fm

Comparison with the lattice QCD calculations from [J. Bulava, A. D. Hanlon, et. al. Nucl. Phys. B **987**, 116105 (2023).]

Dashed lines indicate the non-interacting two-particle $\pi N$ energies for $k = 0$ and $k = 1$.

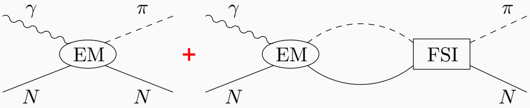
Solid lines are our HEFT results.

# Electric dipole amplitudes $E_{0+}$ with two bare states

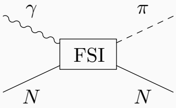



Yu Zhuge, Zhan-Wei Liu, Derek B. Leinweber, Anthony W. Thomas, Phys.Rev.D 110 (2024) 9, 094015.

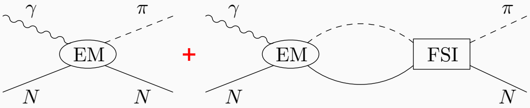
# The bare core in $N^*(1535)$



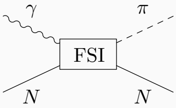

- If $N^*(1535)$ has no bare core, it would play roles ONLY in finite state interaction



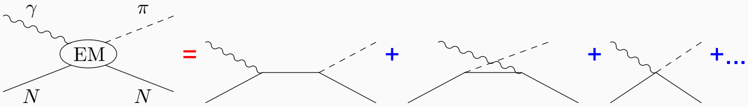
# The bare core in $N^*(1535)$



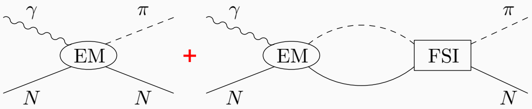

- If $N^*(1535)$ has no bare core, it would play roles ONLY in finite state interaction



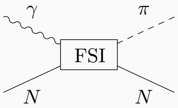

- If with bare core, $N^*(1535)$ also plays roles in electromagnetic potential



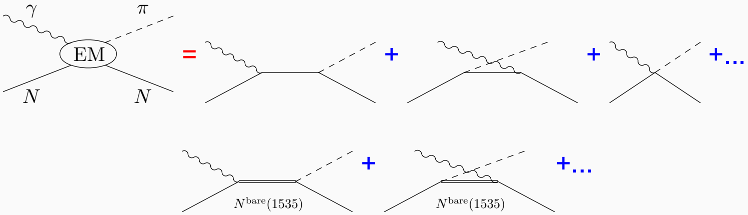
# The bare core in $N^*(1535)$



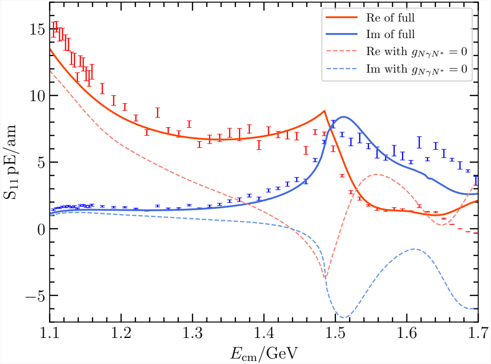

- If $N^*(1535)$ has no bare core, it would play roles ONLY in finite state interaction




- If with bare core, $N^*(1535)$ also plays roles in electromagnetic potential



# The bare core in $N^*(1535)$ cannot be absent in pion photoproduction

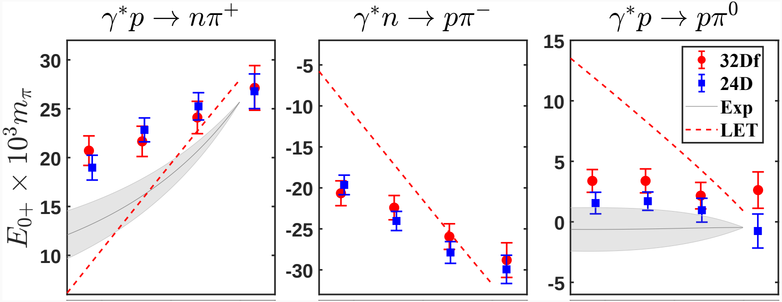



Yu Zhuge, Zhan-Wei Liu, Derek B. Leinweber, Anthony W. Thomas, Phys.Rev.D 110 (2024) 9, 094015.

# $\gamma^* N \to \pi N$ in Finite Volume and Comparison with Recent Lattice QCD Simulations

# Latest lattice QCD data on $E_0^+$

The lattice QCD results is very close to the partial wave analysis from the Jülich-Bonn-Washington collaboration.

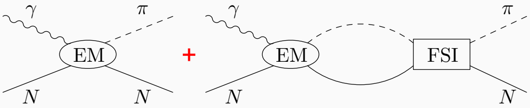



First lattice QCD simulation of pionproduction!

Gao, Yu-Sheng and Zhang, Zhao-Long and Feng, Xu and Jin, Lu-Chang and Liu, Chuan and Meißner, Ulf-G., Lattice QCD Study of Pion Electroproduction and Weak Production from a Nucleon, arXiv: 2502.12074

Please see Zhang Zhao-Long's talk in next session for more details.

# Direct extension of our previous work

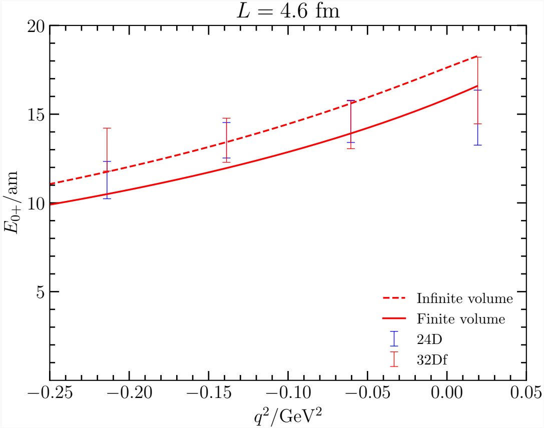



From the real photon ($q^2 = 0$) to the virtual spacelike photon ($q^2 < 0$), we

- do not adjust the previous parameters,
- add the form factors of neutrons and pion:
  - $F(q^2 = 0) = 1$,
  - $F(q^2 < 0) < 1$,
  - $F(q^2)$ is well determined by the experiment.

# Latest lattice QCD data and our preliminary results

The finite volume effect is at the order of the error bar of lattice QCD data.




Yu Zhuge, et. al. in preparation.

# Summary

# Summary

Combined with scattering data and lattice QCD simulations:

- $\pi N \to \pi N$ ,
- lattice QCD spectrum of $N^*$,
- $\gamma + N \to \pi + N$,
- lattice QCD simulation of pionproduction,

we have studied the properties of nucleon resonance and the relevant strong couplings. The triquark components are important for the $N^*(1535)$ and $N^*(1650)$.

With the lattice QCD simulations much more developed, some hadron puzzles will be solved out better compared to those with the traditional scattering experiments only.

## Thank you for your attention!