2025年轻强子专题研讨会
河南 安阳

Light single-gluon hybrid states with various exotic quantum numbers

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Outline

- Introduction
- Method of the QCD sum rules
- Numerical analyses
- Decay behavior
- Summary

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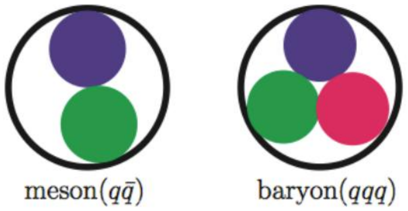
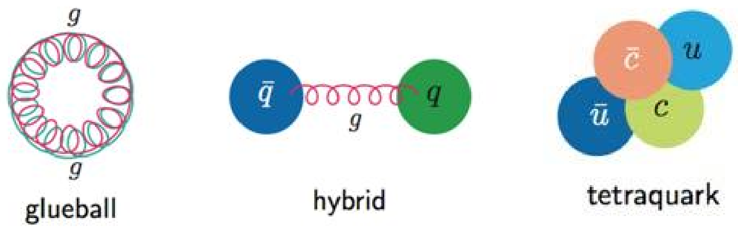
Introduction

- **Traditional quark model**



- **Exotic hadron: hybrid state, glueball, tetraquark, etc.**



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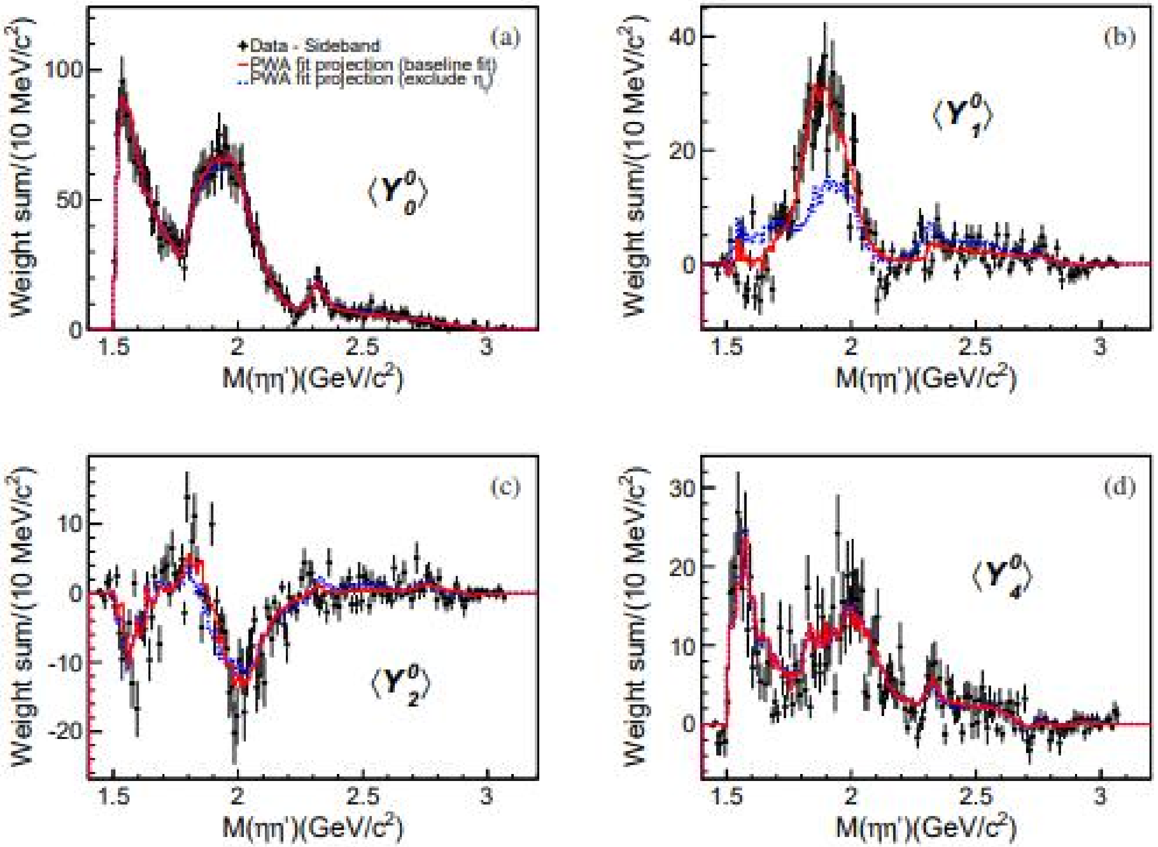
- **Exotic spin-parity quantum numbers**

$$0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$$

Introduction

BESⅢ



$$\eta_1(1855) \ : \ M = 1855 \pm 9^{+6}_{-1} \ \mathrm{MeV}/c^2 \, ,$$
$$\Gamma = 188 \pm 18^{+3}_{-8} \ \mathrm{MeV} \, .$$

PhysRevLett.129.192002.

Introduction

$$\eta_1(1855) \;:\; M = 1855 \pm 9^{+6}_{-1}\ \mathrm{MeV}/c^2\,,$$
$$\Gamma = 188 \pm 18^{+3}_{-8}\ \mathrm{MeV}\,.$$

$$\boxed{\begin{aligned} \pi_1(1400) \;:\; & M = 1354 \pm 25\ \mathrm{MeV}\,, \\ & \Gamma = 330 \pm 35\ \mathrm{MeV}\,; \\ \pi_1(1600) \;:\; & M = 1661^{+15}_{-11}\ \mathrm{MeV}\,, \\ & \Gamma = 240 \pm 50\ \mathrm{MeV}\,; \\ \pi_1(2015) \;:\; & M = 2014 \pm 20 \pm 16\ \mathrm{MeV}\,, \\ & \Gamma = 230 \pm 32 \pm 73\ \mathrm{MeV}\,. \end{aligned}}$$

Introduction

$\pi_1(1400)$

Phys. Lett. B 205 (1988) 397
Phys. Rev. Lett. 79 (1997) 1630–1633.
Phys. Lett. B 423 (1998) 175–184.
Phys. Rev. D 72 (2005) 114507.
Phys. Lett. B 314 (1993) 246–254.
AIP Conf. Proc. 619 (1) (2002)143–154.
Eur. Phys. J. C 80 (5) (2020) 453.

$\pi_1(1600)$

Phys. Rev. Lett. 81 (1998) 5760–5763.
Phys. Lett. B 563 (2003) 140–149.
Phys. Rev. D 84 (2011) 112009.
Nucl. Phys. A 663 (2000) 596–599.
Phys. Rev. Lett. 104 (2010) 241803.
Phys. Rev. D 68 (2003) 074505.

$\pi_1(2015)$

Phys. Lett. B 595 (2004) 109–117.
Nucl. Phys. B Proc. Suppl. 73 (1999) 264–266.
Phys. Rev. Lett. 94 (2005) 032002.

Introduction

- The QCD sum rule method has been widely applied to study the $J^{PC}=1^{-+}$ hybrid states.

Nucl. Phys. B 248 (1984) 1–18.Phys. Lett. B 485 (2000) 145–150
Eur.Phys. J. C 8 (1999) 465–471. Z.Phys. C 34 (1987) 347.
Phys. Rev. D 76(2007) 094001. Nucl. Phys. B 196 (1982) 125–146.
Phys. Lett.B 675 (2009) 319–325.

- This method has also been applied to study the $J^{PC}=0^{+-}$ and 2^{+-} hybrid states.

Phys. Rev. D 98 (9) (2018) 096020. Phys. Rev. D 108(2023), 114010

- Problem: other quantum numbers have not been well studied

Purpose: systematically investigate the single-gluon hybrid states through the QCD sum rule method.

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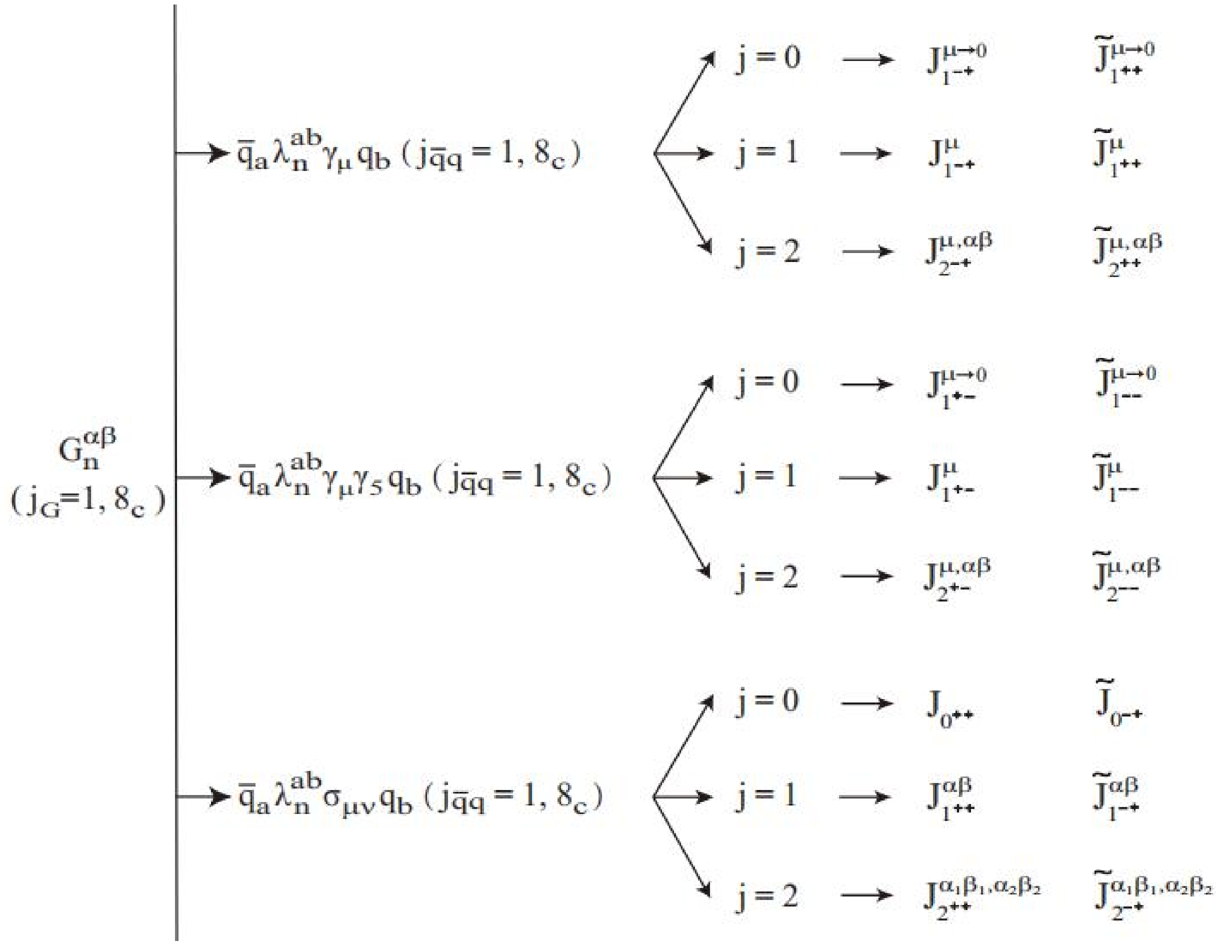
Numerical analyses

Interpolating currents:

$$J^{\mu}_{1^{-+}} = \bar{q}_a \lambda_n^{ab} \gamma_\beta q_b \; g_s G_n^{\mu\beta} \, ,$$



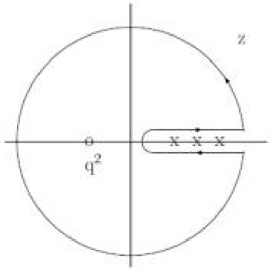
QCD sum rules

- In QCD sum rule analyses, we consider two-point correlation functions:

$$\begin{aligned} & \Pi^{\mu\nu}_{1^{-+}}(q^2) \\ \equiv\ & i \int d^4x e^{iqx} \langle 0 | \mathbf{T}[J^{\mu}_{1^{-+}}(x) J^{\nu\dagger}_{1^{-+}}(0)] | 0 \rangle \\ =\ & (g^{\mu\nu} - q^\mu q^\nu / q^2)\, \Pi_{1^{-+}}(q^2) + (q^\mu q^\nu / q^2)\, \Pi_{0^{++}}(q^2) \,. \end{aligned}$$

where J is the current which can couple to hadronic states.

- We use the dispersion relation to express $\prod_{1^{-+}}(q^2)$ as



$$\Pi_{1^{-+}}(q^2) = \int_{s_<}^{\infty} \frac{\rho_{1^{-+}}(s)}{s - q^2 - i\varepsilon} ds \,,$$

where $\rho_{1^{-+}}(s) \equiv \mathrm{Im} \prod_{1^{-+}}(s)/\pi$ is the spectral density, and $s_< = 4m_q^2$ is the physical threshold.

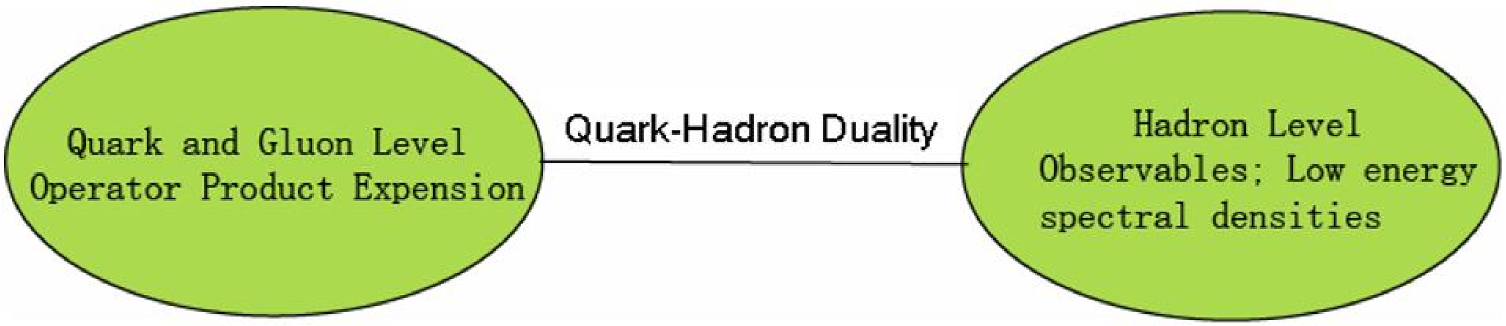
QCD sum rules

- At the hadron level, one pole dominance + continuum contribution:

$$
\begin{aligned}
&\rho^{\text{phen}}_{1^{-+}}(s) \times (g^{\mu\nu} - q^\mu q^\nu/q^2) \\
\equiv\ & \sum_n \delta(s - M_n^2) \langle 0 | J^\mu_{1^{-+}} | n \rangle \langle n | J^{\nu\dagger}_{1^{-+}} | 0 \rangle \\
=\ & f^2_{1^{-+}} \delta(s - M^2_{1^{-+}}) \times (g^{\mu\nu} - q^\mu q^\nu/q^2) + \text{continuum}\,.
\end{aligned}
$$

- At the quark-gluon level, operator product expansion (OPE). And Borel transformation at both the hadron and quark-gluon levels.

$$
\begin{aligned}
\Pi_{1^{-+}}(s_0, M_B^2) &\equiv f^2_{1^{-+}} e^{-M^2_{1^{-+}}/M_B^2} \\
&= \int_{s_<}^{s_0} e^{-s/M_B^2} \rho^{\text{OPE}}_{1^{-+}}(s) ds\,,
\end{aligned}
$$



QCD sum rules

$$M^2_{1^{-+}}(s_0, M_B) = \frac{\int_{s_<}^{s_0} e^{-s/M_B^2} s \rho^{\rm OPE}_{1^{-+}}(s) ds}{\int_{s_<}^{s_0} e^{-s/M_B^2} \rho^{\rm OPE}_{1^{-+}}(s) ds},$$

$$f^2_{1^{-+}}(s_0, M_B) = \Pi_{1^{-+}}(s_0, M_B^2) \times e^{M^2_{1^{-+}}/M_B^2}.$$

- Two parameters： M_B , s_0

- Criteria：

1. Positivity of spectral density
2. Convergence of OPE
3. Sufficient amount of pole contribution
4. The dependence of mass on parameters M_B , s_0

Numerical analyses

The OPE of current $J^{\mu}_{1^{-+}}$

$$\Pi^{\mu}_{1^{-+}}(M_B^2, s_0) = \int_{4m_s^2}^{s_0} \Big(\frac{s^3 \alpha_s}{60\pi^3} - \frac{m_s^2 s^2 \alpha_s}{3\pi^3} + s\big(\frac{\langle \alpha_s GG \rangle}{36\pi^2} + \frac{13 \langle \alpha_s GG \rangle \alpha_s}{432\pi^3} + \frac{8 m_s \langle \bar{s}s \rangle \alpha_s}{9\pi} \big)$$
$$+ \frac{\langle g_s^3 G^3 \rangle}{32\pi^2} - \frac{3 \langle \alpha_s GG \rangle m_s^2 \alpha_s}{64\pi^3} - \frac{3 m_s \langle g_s \bar{s}\sigma G s \rangle \alpha_s}{4\pi} \Big) \times e^{-s/M_B^2} ds$$
$$+ \Big(\frac{\langle \alpha_s GG \rangle^2}{3456\pi^2} - \frac{\langle g_s^3 G^3 \rangle m_s^2}{16\pi^2} - \frac{2}{9} \langle \alpha_s GG \rangle m_s \langle \bar{s}s \rangle + \frac{11}{9} \pi \langle \bar{s}s \rangle \langle g_s \bar{s}\sigma G s \rangle \alpha_s \Big),$$

The OPE with the quark-gluon content $\bar{q}qg$ ($q = u/d$) can be easily derived by replacing $m_s \to 0$, $\langle \bar{s}s \rangle \to \langle \bar{q}q \rangle$, and $\langle g\bar{s}\sigma Gs \rangle \to \langle g\bar{q}\sigma Gq \rangle$.

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Numerical analyses

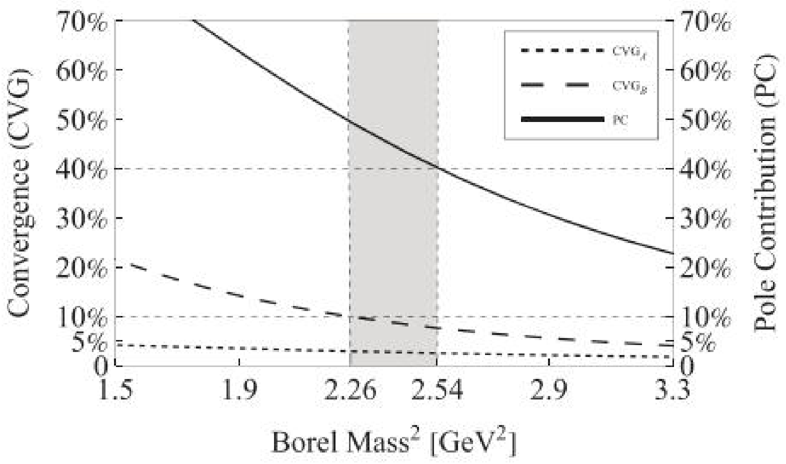
- **Convergence of OPE**

$$\mathrm{CVG}_A \equiv \left|\frac{\Pi^{g_s^{n=4}}(\infty, M_B^2)}{\Pi(\infty, M_B^2)}\right| \le 5\%\,,$$

$$\mathrm{CVG}_B \equiv \left|\frac{\Pi^{\mathrm{D}=6+8}(\infty, M_B^2)}{\Pi(\infty, M_B^2)}\right| \le 10\%\,.$$

- **Sufficient amout of pole contribution**

$$\mathrm{PC} \equiv \left|\frac{\Pi(s_0, M_B^2)}{\Pi(\infty, M_B^2)}\right| \ge 40\%\,.$$

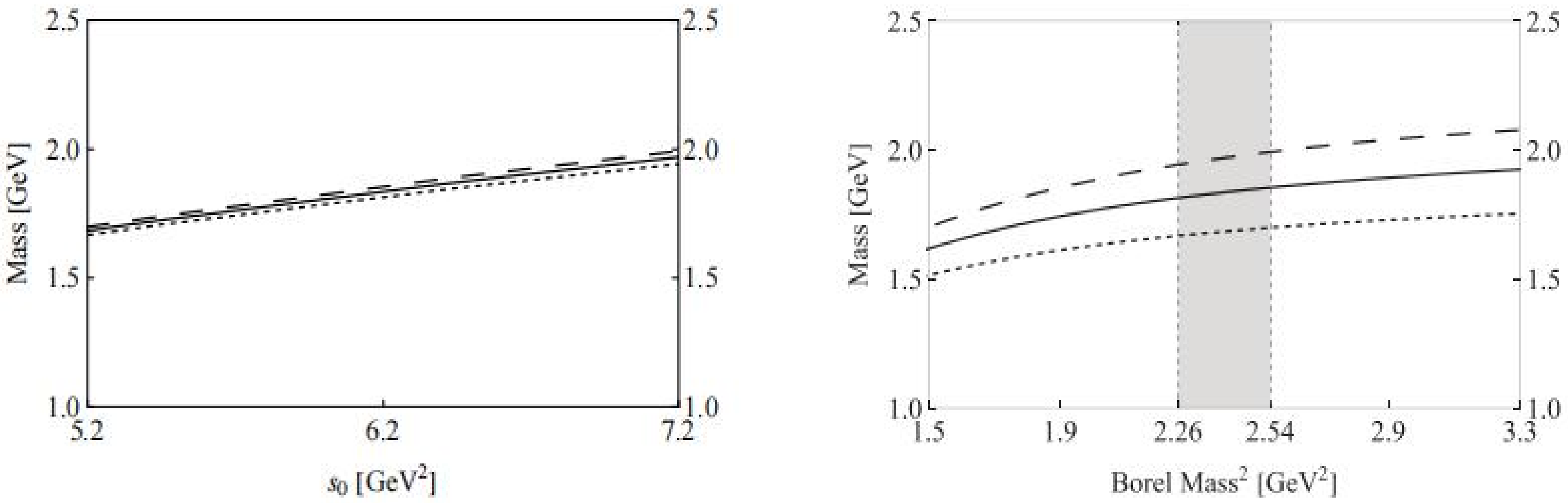


$$2.26\mathrm{GeV}^2 \le M_B^2 \le 2.54\mathrm{GeV}^2$$

Note that this Borel window is not so wide, and it may indicate that the understanding of this state as a particle has limitations

Numerical analyses

The dependence of mass on parameters



$$M_{|\bar{s}sg;0^+1^{-+}\rangle} = \boxed{1.84^{+0.14}_{-0.15}}\,\text{GeV}$$

Numerical analyses

Mass extracted from currents of $\bar{s}sg$

State $[J^{PC}]$	Current	s_0^{min} [GeV^2]	Working Regions		Pole [%]	Mass [GeV]	Decay Constant
			M_B^2 [GeV^2]	s_0 [GeV^2]			
$\vert\bar{s}sg;1^{--}\rangle$	$J^{\alpha\beta}_{1--}$	4.3	2.07–2.80	6.5	40–63	$1.94^{+0.20}_{-0.21}$	$0.054^{+0.013}_{-0.016}$ GeV^3
$\vert\bar{s}sg;1^{+-}\rangle$	$\tilde{J}^{\alpha\beta}_{1+-}$	16.2	3.60–5.40	20.0	40–65	$4.06^{+0.26}_{-0.16}$	$0.071^{+0.019}_{-0.020}$ GeV^3
$\vert\bar{s}sg;1^{+-}\rangle$	$J^{\alpha\beta}_{1+-}$	5.9	2.54–2.72	6.5	40–45	$2.01^{+0.17}_{-0.20}$	$0.050^{+0.005}_{-0.006}$ GeV^3
$\vert\bar{s}sg;1^{--}\rangle$	$\tilde{J}^{\alpha\beta}_{1--}$	16.9	3.73–5.30	20.0	40–61	$4.12^{+0.26}_{-0.13}$	$0.070^{+0.019}_{-0.020}$ GeV^3
$\vert\bar{s}sg;0^{++}\rangle$	$J^{\mu\to0}_{1-+}$	20.7	5.18–7.35	26.0	40–63	$4.50^{+0.23}_{-0.22}$	$0.136^{+0.030}_{-0.034}$ GeV^3
$\vert\bar{s}sg;0^{-+}\rangle$	$\tilde{J}^{\mu\to0}_{1++}$	7.2	3.45–4.08	9.5	40–53	$2.26^{+0.21}_{-0.24}$	$0.107^{+0.007}_{-0.005}$ GeV^3
$\vert\bar{s}sg;0^{--}\rangle$	$J^{\mu\to0}_{1+-}$	21.6	5.36–7.23	26.0	40–59	$4.57^{+0.22}_{-0.19}$	$0.134^{+0.031}_{-0.035}$ GeV^3
$\vert\bar{s}sg;0^{+-}\rangle$	$\tilde{J}^{\mu\to0}_{1--}$	7.5	3.41–3.98	9.5	40–52	$2.30^{+0.20}_{-0.24}$	$0.101^{+0.007}_{-0.006}$ GeV^3
$\vert\bar{s}sg;1^{-+}\rangle$	J^{μ}_{1-+}	5.1	2.26–2.54	6.2	40–49	$1.84^{+0.14}_{-0.15}$	$0.300^{+0.063}_{-0.058}$ GeV^4
$\vert\bar{s}sg;1^{++}\rangle$	$\tilde{J}^{\mu}_{1++}$	14.1	3.64–4.80	17.0	40–58	$3.65^{+0.17}_{-0.17}$	$1.678^{+0.530}_{-0.502}$ GeV^4
$\vert\bar{s}sg;1^{+-}\rangle$	J^{μ}_{1+-}	3.9	1.85–2.43	6.0	40–62	$1.82^{+0.13}_{-0.15}$	$0.278^{+0.059}_{-0.056}$ GeV^4
$\vert\bar{s}sg;1^{--}\rangle$	$\tilde{J}^{\mu}_{1--}$	13.8	3.50–4.80	17.0	40–61	$3.64^{+0.17}_{-0.17}$	$1.662^{+0.526}_{-0.498}$ GeV^4

Numerical analyses

State $[J^{PC}]$	Current	s_0^{min} [GeV^2]	Working Regions		Pole [%]	Mass [GeV]	Decay Constant
			M_B^2 [GeV^2]	s_0 [GeV^2]			
$\|\bar{s}sg;0^{++}\rangle$	J_{0++}	11.5	3.53–4.33	14.0	40–55	$3.11^{+0.22}_{-0.27}$	$3.535^{+1.338}_{-1.242}$ GeV^4
$\|\bar{s}sg;0^{-+}\rangle$	J_{0-+}	11.3	3.51–4.36	14.0	40–56	$3.08^{+0.23}_{-0.28}$	$3.509^{+1.328}_{-1.233}$ GeV^4
$\|\bar{s}sg;1^{++}\rangle$	$J^{\alpha\beta}_{1++}$	6.6	1.95–2.27	7.5	40–51	$2.34^{+0.14}_{-0.16}$	$0.061^{+0.012}_{-0.014}$ GeV^3
$\|\bar{s}sg;1^{-+}\rangle$	$\tilde{J}^{\alpha\beta}_{1-+}$	5.5	1.82–2.25	7.0	40–57	$2.08^{+0.18}_{-0.24}$	$0.061^{+0.010}_{-0.010}$ GeV^3
$\|\bar{s}sg;1^{-+}\rangle$	$J^{\alpha\beta}_{1-+}$	5.5	1.82–2.25	7.0	40–57	$2.08^{+0.18}_{-0.24}$	$0.061^{+0.010}_{-0.010}$ GeV^3
$\|\bar{s}sg;1^{++}\rangle$	$\tilde{J}^{\alpha\beta}_{1++}$	6.6	1.95–2.27	7.5	40–51	$2.34^{+0.14}_{-0.16}$	$0.061^{+0.012}_{-0.014}$ GeV^3
$\|\bar{s}sg;2^{++}\rangle$	$J^{\alpha_1\beta_1,\alpha_2\beta_2}_{2++}$	9.2	3.22–3.60	10.5	40–49	$2.59^{+0.19}_{-0.23}$	–
$\|\bar{s}sg;2^{-+}\rangle$	$\tilde{J}^{\alpha_1\beta_1,\alpha_2\beta_2}_{2-+}$	13.4	2.55–4.29	16.0	40–66	$3.72^{+0.72}_{-0.13}$	–
$\|\bar{s}sg;2^{-+}\rangle$	$J^{\alpha_1\beta_1,\alpha_2\beta_2}_{2-+}$	8.1	3.04–3.72	10.5	40–56	$2.51^{+0.20}_{-0.24}$	–
$\|\bar{s}sg;2^{++}\rangle$	$\tilde{J}^{\alpha_1\beta_1,\alpha_2\beta_2}_{2++}$	11.8	2.36–4.47	16.0	40–78	$3.54^{+0.42}_{-0.16}$	–

$$
\begin{aligned}
M_{|\bar{s}sg;0^+1^{-+}\rangle} &= 1.84^{+0.14}_{-0.15}\ \mathrm{GeV}\,,\\
f_{|\bar{s}sg;0^+1^{-+}\rangle} &= 0.300^{+0.063}_{-0.058}\ \mathrm{GeV}^4
\end{aligned}
$$

Mass extracted from currents of $\bar{q}qg(q = u/d)$

State $[J^{PC}]$	Current	s_0^{min} [GeV^2]	Working Regions M_B^2 [GeV^2]	Working Regions s_0 [GeV^2]	Pole [%]	Mass [GeV]	Decay Constant
$\|\bar{q}qg;1^{--}\rangle$	$J_{1--}^{\alpha\beta}$	4.2	2.03–2.48	5.5	40–54	$1.80^{+0.13}_{-0.16}$	$0.051^{+0.004}_{-0.004}$ GeV^3
$\|\bar{q}qg;1^{+-}\rangle$	$\tilde{J}_{1+-}^{\alpha\beta}$	16.2	3.61–4.58	18.0	40–53	$4.05^{+0.24}_{-0.12}$	$0.063^{+0.020}_{-0.020}$ GeV^3
$\|\bar{q}qg;1^{+-}\rangle$	$J_{1+-}^{\alpha\beta}$	5.0	2.29–2.45	5.5	40–45	$1.84^{+0.12}_{-0.14}$	$0.049^{+0.004}_{-0.004}$ GeV^3
$\|\bar{q}qg;1^{--}\rangle$	$\tilde{J}_{1--}^{\alpha\beta}$	16.3	3.52–4.56	18.0	40–53	$4.09^{+0.29}_{-0.14}$	$0.064^{+0.021}_{-0.020}$ GeV^3
$\|\bar{q}qg;0^{++}\rangle$	$J_{1-+}^{\mu\to 0}$	20.6	5.11–6.59	24.0	40–56	$4.45^{+0.22}_{-0.17}$	$0.124^{+0.032}_{-0.036}$ GeV^3
$\|\bar{q}qg;0^{-+}\rangle$	$\tilde{J}_{1++}^{\mu\to 0}$	7.7	3.58–3.81	8.5	40–45	$2.14^{+0.17}_{-0.19}$	$0.105^{+0.005}_{-0.004}$ GeV^3
$\|\bar{q}qg;0^{--}\rangle$	$J_{1+-}^{\mu\to 0}$	21.6	5.48–6.52	24.0	40–50	$4.49^{+0.21}_{-0.14}$	$0.123^{+0.032}_{-0.037}$ GeV^3
$\|\bar{q}qg;0^{+-}\rangle$	$\tilde{J}_{1--}^{\mu\to 0}$	7.1	3.32–3.73	8.5	40–49	$2.16^{+0.16}_{-0.19}$	$0.100^{+0.005}_{-0.005}$ GeV^3
$\|\bar{q}qg;1^{-+}\rangle$	J_{1-+}^{μ}	4.8	2.19–2.28	5.2	40–43	$1.67^{+0.15}_{-0.17}$	$0.243^{+0.057}_{-0.052}$ GeV^4
$\|\bar{q}qg;1^{++}\rangle$	$\tilde{J}_{1++}^{\mu}$	13.8	3.59–4.10	15.0	40–48	$3.54^{+0.16}_{-0.12}$	$1.370^{+0.494}_{-0.450}$ GeV^4
$\|\bar{q}qg;1^{+-}\rangle$	J_{1+-}^{μ}	4.6	2.10–2.27	5.2	40–46	$1.68^{+0.14}_{-0.16}$	$0.242^{+0.055}_{-0.051}$ GeV^4
$\|\bar{q}qg;1^{--}\rangle$	$\tilde{J}_{1--}^{\mu}$	13.7	3.57–4.10	15.0	40–49	$3.53^{+0.16}_{-0.12}$	$1.366^{+0.493}_{-0.450}$ GeV^4

Numerical analyses

State $[J^{PC}]$	Current	s_0^{min} [GeV^2]	Working Regions		Pole [%]	Mass [GeV]	Decay Constant
			M_B^2 [GeV^2]	s_0 [GeV^2]			
$\|\bar{q}qg;0^{++}\rangle$	J_{0++}	11.1	3.48–3.91	12.5	40–49	$2.94^{+0.20}_{-0.25}$	$2.893^{+1.029}_{-0.948}$ GeV^4
$\|\bar{q}qg;0^{-+}\rangle$	J_{0-+}	11.1	3.47–3.92	12.5	40–49	$2.93^{+0.20}_{-0.25}$	$2.882^{+1.026}_{-0.945}$ GeV^4
$\|\bar{q}qg;1^{++}\rangle$	$J^{\alpha\beta}_{1++}$	5.8	1.84–2.06	6.5	40–48	$2.11^{+0.17}_{-0.21}$	$0.056^{+0.012}_{-0.013}$ GeV^3
$\|\bar{q}qg;1^{-+}\rangle$	$\tilde{J}^{\alpha\beta}_{1-+}$	5.5	1.81–2.00	6.2	40–48	$2.00^{+0.13}_{-0.16}$	$0.055^{+0.007}_{-0.008}$ GeV^3
$\|\bar{q}qg;1^{-+}\rangle$	$J^{\alpha\beta}_{1-+}$	5.5	1.81–2.00	6.2	40–48	$2.00^{+0.13}_{-0.16}$	$0.055^{+0.007}_{-0.008}$ GeV^3
$\|\bar{q}qg;1^{++}\rangle$	$\tilde{J}^{\alpha\beta}_{1++}$	5.8	1.84–2.06	6.5	40–48	$2.11^{+0.17}_{-0.21}$	$0.056^{+0.012}_{-0.013}$ GeV^3
$\|\bar{q}qg;2^{++}\rangle$	$J^{\alpha_1\beta_1,\alpha_2\beta_2}_{2++}$	8.6	3.11–3.37	9.5	40–46	$2.44^{+0.20}_{-0.24}$	–
$\|\bar{q}qg;2^{-+}\rangle$	$\tilde{J}^{\alpha_1\beta_1,\alpha_2\beta_2}_{2-+}$	12.7	2.54–3.60	14.0	40–54	$3.68^{+0.62}_{-0.18}$	–
$\|\bar{q}qg;2^{-+}\rangle$	$J^{\alpha_1\beta_1,\alpha_2\beta_2}_{2-+}$	8.3	3.07–3.41	9.5	40–48	$2.40^{+0.21}_{-0.25}$	–
$\|\bar{q}qg;2^{++}\rangle$	$\tilde{J}^{\alpha_1\beta_1,\alpha_2\beta_2}_{2++}$	11.7	2.47–3.70	14.0	40–63	$3.46^{+0.27}_{-0.11}$	–

$$M_{|\bar{q}qg;1^-1^{-+}\rangle} = M_{|\bar{q}qg;0^+1^{-+}\rangle} = 1.67^{+0.15}_{-0.17}\ \text{GeV}\,,$$
$$f_{|\bar{q}qg;1^-1^{-+}\rangle} = f_{|\bar{q}qg;0^+1^{-+}\rangle} = 0.243^{+0.057}_{-0.052}\ \text{GeV}^4$$

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Decay behavior

A. Normal decay process

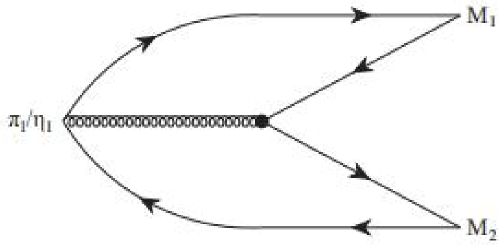
$$\pi_1 \equiv |\bar{q}qg; 1^-1^{-+}\rangle \to \rho\pi \,,$$

three-point correlation function:

$$T_{\mu\nu}(p,k,q) \;=\; \int d^4x d^4y e^{ikx} e^{iqy} \times \langle 0|\mathbb{T}[J_\nu^{\rho^-}(x) J_5^{\pi^+}(y) J_{1^{-+}}^{\mu\dagger}(0)]|0\rangle$$

select the isovector neutral-charged one

$$J^\mu_{1-+} \to \frac{1}{\sqrt{2}} \left(\bar{u}_a \lambda_n^{ab} \gamma_\beta u_b - \bar{d}_a \lambda_n^{ab} \gamma_\beta d_b\right) g_s G_n^{\mu\beta}$$



(a)

$$g_{\rho\pi} \;=\; 4.08^{+2.40}_{-1.83}\ \mathrm{GeV}^{-1} \,,$$
$$\Gamma_{\pi_1\to\rho\pi} \;=\; 242^{+310}_{-179}\ \mathrm{MeV} \,.$$

phenomenological side

$$T_{\mu\nu}(p,k,q) \;=\; g_{\rho\pi}\epsilon_{\mu\nu\alpha\beta} q^\alpha k^\beta \times \frac{f_{\pi_1} f_\rho m_\rho f'_\pi}{(m_{\pi_1}^2 - p^2)(m_\rho^2 - k^2)(m_\pi^2 - q^2)}$$

QCD side

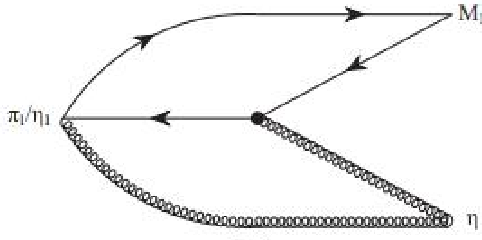
$$T_{\mu\nu}(p,k,q) = \frac{\epsilon_{\mu\nu\alpha\beta} q^\alpha k^\beta}{q^2} \times \left(\frac{\langle g_s \bar{q}\sigma G q\rangle}{6\sqrt{2}} \left(\frac{3}{p^2} + \frac{1}{k^2}\right) - \frac{\langle \bar{q}q\rangle\langle g_s^2 GG\rangle}{18\sqrt{2}} \left(\frac{1}{p^4} + \frac{1}{k^4}\right)\right) . \tag{65}$$

$$-g_{\rho\pi} \frac{f_{\pi_1} f_\rho m_\rho f'_\pi}{m_\rho^2 - m_{\pi_1}^2} \left(e^{-m_{\pi_1}^2/T^2} - e^{-m_\rho^2/T^2}\right)$$
$$= -\frac{2\langle g\bar{q}\sigma G q\rangle}{3\sqrt{2}} - \frac{\langle \bar{q}q\rangle\langle g_s^2 GG\rangle}{9\sqrt{2}} \frac{1}{T^2} \,.$$

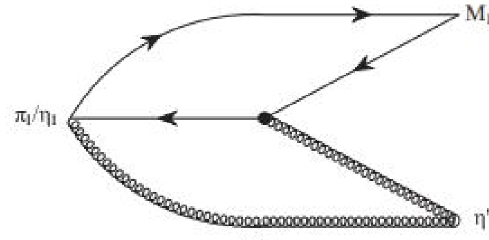
Decay behavior

B. Abnormal decay process

$$\eta_1 \equiv |\bar{s}sg; 0^+1^{-+}\rangle \to \eta\eta' \,,$$



(b)



(c)

three-point correlation function:

$$T'_{\mu\nu}(p,k,q) = \int d^4x e^{-ikx} \langle 0|\mathbb{T}[J^{\mu}_{1-+}(0) J^{\eta\dagger}_{\nu}(x)]|\eta'\rangle \,,$$

$$J^{\mu}_{1-+} \to \bar{s}_a \lambda_n^{ab} \gamma_\beta s_b g_s G_n^{\mu\beta} \,.$$

phenomenological side

$$T'_{\mu\nu}(p,k,q) = g_{\eta\eta'} k_\mu k_\nu \frac{f_{\eta_1} g_\eta}{(m^2_{\eta_1} - p^2)(m^2_\eta - k^2)} \,,$$

QCD side

$$T'_{\mu\nu}(p,k,q) = \theta_s k_\mu k_\nu \left(-\frac{2m^2_{\eta'} f_{\eta'}}{3k^2} - \frac{4\pi^2 m^2_{\eta'} f_{\eta'} m_s \langle\bar{s}s\rangle}{3k^6} \right) \,,$$

$$g_{\eta\eta'} \frac{f_{\eta_1} g_\eta}{m^2_{\eta_1} - m^2_\eta} \left(e^{-m^2_\eta/M_B^2} - e^{-m^2_{\eta_1}/M_B^2} \right) = \frac{2\theta_s m^2_{\eta'} f_{\eta'}}{3} + \frac{2\pi^2 \theta_s m^2_{\eta'} f_{\eta'} m_s \langle\bar{s}s\rangle}{3} \frac{1}{M_B^4} \,.$$

$$\begin{aligned} g_{\eta\eta'} &= 3.08^{+1.30}_{-0.91} \text{ GeV}^{-1} \,, \\ \Gamma_{\eta_1 \to \eta\eta'} &= 5.0^{+4.6}_{-3.1} \text{ MeV} \,. \end{aligned}$$

Decay behavior

Channel	$\lvert \bar{q}qg; 1^{-}1^{-+}\rangle$ $M = 1.67^{+0.15}_{-0.17}$ GeV	$\lvert \bar{q}qg; 0^{+}1^{-+}\rangle$ $M = 1.67^{+0.15}_{-0.17}$ GeV	$\lvert \bar{s}sg; 0^{+}1^{-+}\rangle$ $M = 1.84^{+0.14}_{-0.15}$ GeV
$\pi_1/\eta_1 \to \rho\pi$	242^{+310}_{-179}	–	–
$\pi_1/\eta_1 \to b_1(1235)\pi$	$14.5^{+25.9}_{-13.9}$	–	–
$\pi_1/\eta_1 \to f_1(1285)\pi$	$35.9^{+53.9}_{-36.4}$	–	–
$\pi_1/\eta_1 \to \eta\pi$	$2.3^{+2.5}_{-1.2}$	–	–
$\pi_1/\eta_1 \xrightarrow{b} \eta\pi$	$57.8^{+65.0}_{-31.4}$	–	–
$\pi_1/\eta_1 \to \eta'\pi$	$0.43^{+0.50}_{-0.28}$	–	–
$\pi_1/\eta_1 \xrightarrow{c} \eta'\pi$	149^{+162}_{-78}	–	–
$\pi_1/\eta_1 \to a_1(1260)\pi$	–	$\boxed{79.5^{+112.4}_{-74.9}}$	–
$\pi_1/\eta_1 \xrightarrow{a} \eta\eta'$	–	$0.07^{+0.12}_{-0.07}$	$0.93^{+1.04}_{-0.69}$
$\pi_1/\eta_1 \xrightarrow{b} \eta\eta'$	–	$1.62^{+2.13}_{-1.61}$	$1.64^{+1.51}_{-1.01}$
$\pi_1/\eta_1 \xrightarrow{c} \eta\eta'$	–	$11.5^{+11.7}_{-11.5}$	$5.0^{+4.6}_{-3.1}$
$\pi_1/\eta_1 \to K^*(892)\bar{K} + c.c.$	$25.3^{+34.7}_{-24.7}$	$25.3^{+34.7}_{-24.7}$	$73.9^{+85.7}_{-58.0}$
$\pi_1/\eta_1 \to K_1(1270)\bar{K} + c.c.$	–	–	$14.6^{+19.8}_{-14.6}$
$\pi_1/\eta_1 \to K^*(892)\bar{K}^*(892)$	–	–	$0.08^{+0.39}_{-0.08}$
Sum	530^{+540}_{-330}	120^{+160}_{-110}	100^{+110}_{-80}

Outline

- Introduction
- Method of the QCD sum rules
- Numerical analyses
- Decay behavior
- **Summary**

Summary

- We calculate the masses of forty-four single-gluon hybrid states with the quark-gluon contents $\bar{q}qg$ ($q = u/d$) and $\bar{s}sg$.
- Our results support the interpretations of the $\pi_1(1600)$ and $\eta_1(1855)$ as the hybrid states $|\bar{q}qg;\ 1^-1^{-+}\rangle$ and $|\bar{s}sg;\ 0^+1^{-+}\rangle$, respectively.
- Considering the uncertainties, our results suggest that the $\pi_1(1600)$ and $\eta_1(1855)$ may also be interpreted as the hybrid states $|\bar{q}qg;\ 1^-1^{-+}\rangle$ and $|\bar{q}qg;\ 0^+1^{-+}\rangle$, respectively.
- To differentiate these two assignments and to verify whether they are hybrid states or not, we propose to examine the $a_1(1260)\pi$ decay channel in future experiments.

Thanks for your attention !