Double-charmonium scattering from lattice QCD

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Based on Y.M et al,EPJC 85,458(2025)

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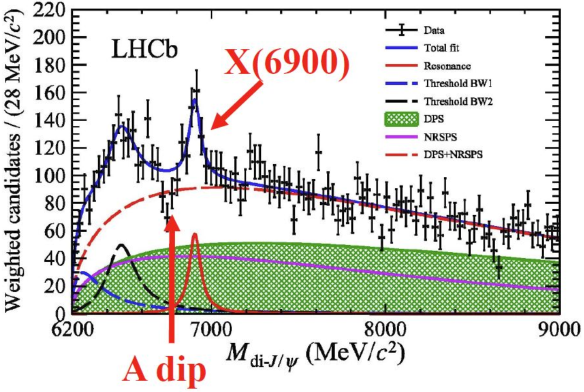
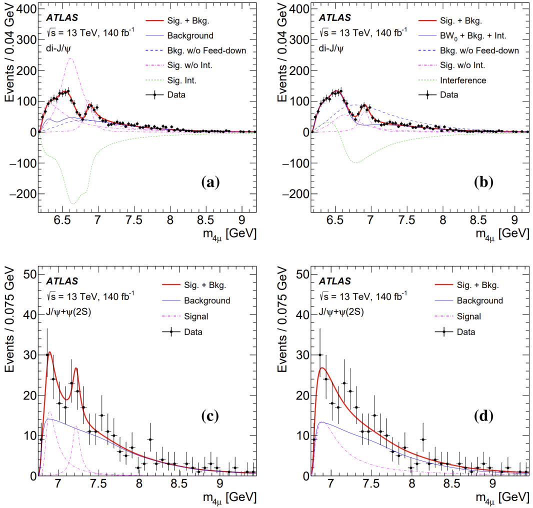
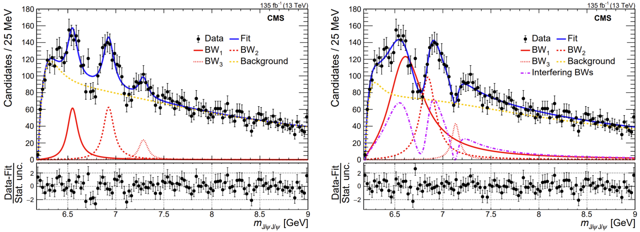
Motivation





- In 2020, LHCb first observed a narrow structure X(6900) in the di-J/ψ spectrum Sci.Bull.65 (2020) 23
- In 2023, ATLAS observed X(6900) in di-J/ψ channel and X(6900),X(7200) in $J/\psi + \psi(2S)$ channel PRL131,151902(2023)

Motivation



		BW_1	BW_2	BW_3
No interference	m (MeV)	$6552 \pm 10 \pm 12$	$6927 \pm 9 \pm 4$	$7287^{+20}_{-18} \pm 5$
	Γ (MeV)	$124^{+32}_{-26} \pm 33$	$122^{+24}_{-21} \pm 18$	$95^{+59}_{-40} \pm 19$
	N	470^{+120}_{-110}	492^{+78}_{-73}	156^{+64}_{-51}
Interference	m (MeV)	6638^{+43+16}_{-38-31}	6847^{+44+48}_{-28-20}	7134^{+48+41}_{-25-15}
	Γ (MeV)	$440^{+230+110}_{-200-240}$	191^{+66+25}_{-49-17}	97^{+40+29}_{-29-26}

CMS update(2025), Huzhen's talk on 10th XYZ Workshop

- Spin parity analysis $\Rightarrow J^{PC} = 2^{++}$
- With Run 3 data, first observation of interference

	BW1 (MeV)	BW2 (MeV)	BW3 (MeV)
m	6588 ± 19	6849 ± 12	7179 ± 10
Γ	454 ± 74	136 ± 18	67 ± 18

- In 2024, CMS found 3 significxant structures, named as X(6600),X(6900) and X(7200) PRL132,111901(2024)

Motivation

What is the nature of $X(6900)$?

- Tetraquark, compact or molecular
- Dynamical effects PRL126,132001(2021)
- Gluonic tetracharm PLB,817,136339(2021)

Is there compact bound state below di-heavy-quarkonium threshold ?

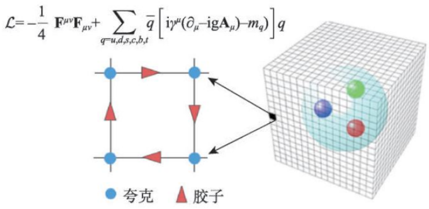
- Yes X.-K.Dong et al,Sci.Bull66,2462(2021) $\cdots$
- No PRD97,054505(2018)[lattice] $\cdots$

Role of lattice QCD

- $X(6900)$: very challenging for current lattice QCD almost impossible
  - Unknown internal structure
  - Very dense energy levels
  - Coupled-channel effect
- Basic information of di-charmonium scattering(Lattice QCD) $\rightarrow$ inputs for phenomenological studies
  - Constrain various phenomenological models
  - Improve the predictive power of phenomenology
- Experiments+phenomenology+lattice QCD $\rightarrow$ possible

Target : 0^+ $\eta_c\eta_c$ and 2^+ $J/\psi J/\psi$ scattering lengths

- K.G.Wilson, PRD 10, 2445(1974)
- Idea: put QCD on 4-d lattice
    - Quark field $\rightarrow$ site
    - Gauge filed $\rightarrow$ link



格点量子色动力学在中国[J].现代物理知识,2020,32(01):36-44.

- “格点场论既是世界观(非微扰的定义)又是方法论(非微扰的计算)” — 刘川,《格点量子色动力学导论》
    - 世界观 $\Rightarrow$ Non-perturbative definition of QCD,
      natural ultraviolet and infrared truncation
    - 方法论 $\Rightarrow$ Non-perturbative calculation of QCD,
      Monte-Carlo simulation

Ensembles

Ens	a(fm)	V/a^4	$a\mu_{sea}$	$N_{\rm conf} \times T_s$	m_π(MeV)
a98	0.098(3)	$24^3 \times 48$	0.0060	236×48	365
a85	0.085(2)	$24^3 \times 48$	0.0040	200×48	315
a67	0.0667(20)	$32^3 \times 64$	0.0030	200×64	300

- $N_f = 2$ twisted-mass guage configuration
- Dimensionless quantity $m_{J/\psi}a^\Gamma$ in the continuous limit
- Smeared stochastic Z_4-noise for the propagator
- Charm quark mass is tuned by physical J/ψ mass
- Same setup is used for $\eta \to 2\gamma$[Y.M et al, Sci.Bull68,1880(2023)] and $J/\psi \to \gamma\eta_c$[Y.M et al, PRD111,014508(2025)], which is verified by BESIII experiment [PRL134,181901(2025)]

Finite volume method

- Method: Lüscher finite volume formula

$$\delta E^{\Gamma} = -\frac{4\pi a^{\Gamma}}{mL^3}\left[1 + c_1\frac{a^{\Gamma}}{L} + c_2\left(\frac{a^{\Gamma}}{L}\right)^2 + \mathcal{O}(L^{-3})\right], \Gamma = A_1, E, T_2$$

- Single-particle operator: $\mathcal{P}(t) = \bar{c}\gamma_5 c(t), \mathcal{V}_i(t) = \bar{c}\gamma_i c(t)$
- Two-particle operator

$$\begin{aligned}
\mathcal{O}^{A_1}(t) &= \mathcal{P}(t)\mathcal{P}(t) \\
\mathcal{O}^{E}(t) &= \left\{\frac{1}{\sqrt{2}}\left[\mathcal{V}_1(t)\mathcal{V}_1(t) - \mathcal{V}_2(t)\mathcal{V}_2(t)\right],\right. \\
&\quad \left.\frac{1}{\sqrt{2}}\left[\mathcal{V}_2(t)\mathcal{V}_2(t) - \mathcal{V}_3(t)\mathcal{V}_3(t)\right]\right\} \\
\mathcal{O}^{T_2}(t) &= \left\{\mathcal{V}_2(t)\mathcal{V}_3(t), \mathcal{V}_3(t)\mathcal{V}_1(t), \mathcal{V}_1(t)\mathcal{V}_2(t)\right\}
\end{aligned}$$

F.R.López, A.Rusetsky and C.Urbach,PRD**98**,014503(2018)

Energy shift

- Two-point function

$$C^{(2)}_{\eta_c}(t) = \frac{1}{T}\sum_{t_s}\langle \mathcal{P}(t+t_s)\mathcal{P}^\dagger(t_s)\rangle$$

$$C^{(2)}_{J/\psi}(t) = \frac{1}{T}\sum_{t_s}\langle \mathcal{V}_i(t+t_s)\mathcal{V}_i^\dagger(t_s)\rangle$$

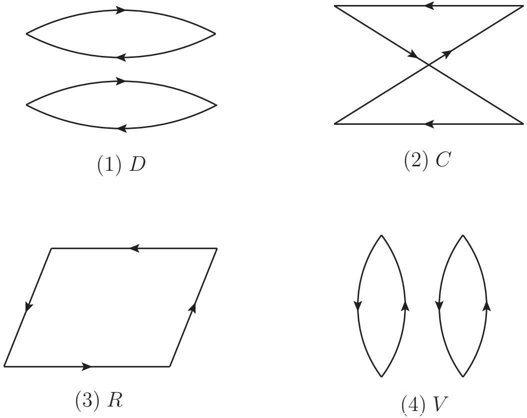
- Four-point function

$$C^{(4)}_{\Gamma}(t) = \frac{1}{T}\sum_{t_s}\langle \mathcal{O}^\Gamma(t+t_s)\left(\mathcal{O}^\Gamma(t_s)\right)^\dagger\rangle$$

- Ratio

$$R^\Gamma(t) = \frac{C^{(4)}_\Gamma(t) - C^{(4)}_\Gamma(t+1)}{(C^{(2)}_h(t))^2 - (C^{(2)}_h(t+1))^2}$$
$$\to A_R[\cosh(\delta E^\Gamma t') + \sinh(\delta E^\Gamma t')\coth(2m_h t')],\quad t' = t + 1/2 - T/2$$

Wick contraction

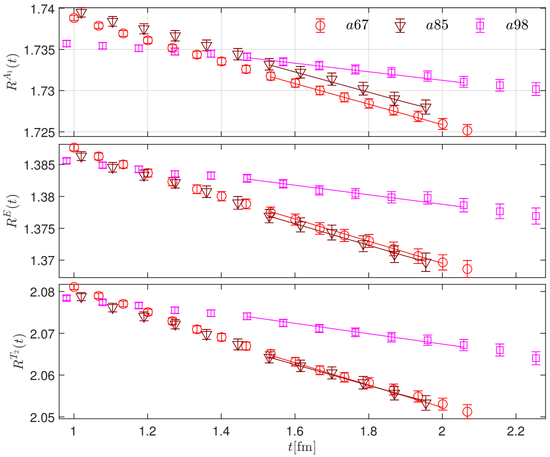


- V in $J/\psi J/\psi$: $\alpha_s^3(2m_c)$
- V in $\eta_c\eta_c$: $\alpha_s^2(2m_c)$

- R in $J/\psi J/\psi$: $\alpha_s(2m_c)$
- R in $\eta_c\eta_c$: $\alpha_s(2m_c)$

- Type-R and type-V are supposed to be highly suppressed

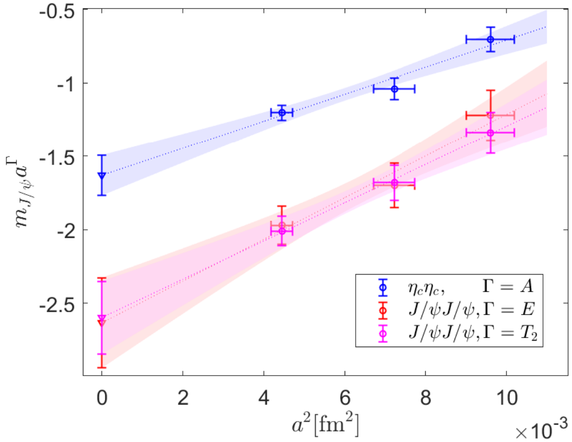
Energy shift



Ensemble	Γ	A_1	E	T_2
a98	δE^{Γ} [MeV]	0.59(07)	1.07(17)	1.18(14)
a85	δE^{Γ} [MeV]	1.40(11)	2.43(25)	2.39(20)
a67	δE^{Γ} [MeV]	1.42(07)	2.50(20)	2.57(15)
a98	$m_{J/\psi}a^{\Gamma}$	−0.705(81)	−1.22(17)	−1.34(14)
a85	$m_{J/\psi}a^{\Gamma}$	−1.042(72)	−1.70(15)	−1.68(12)
a67	$m_{J/\psi}a^{\Gamma}$	−1.202(51)	−1.97(13)	−2.01(10)
Cont.Limit	$m_{J/\psi}a^{\Gamma}$	−1.63(14)	−2.63(31)	−2.60(25)
Cont.Limit	a^{Γ} [fm]	−0.104(09)	−0.168(20)	−0.165(16)

- Asymptotic behavior: $T \gg t,\ \delta E/2m_{\bar{c}c} \ll 1,\ R^{\Gamma}(t) \propto \cosh(\delta E^{\Gamma} t') + \sinh(\delta E^{\Gamma} t') \coth(2m_h t') \to e^{-\delta E^{\Gamma} t}$

S-wave scattering length

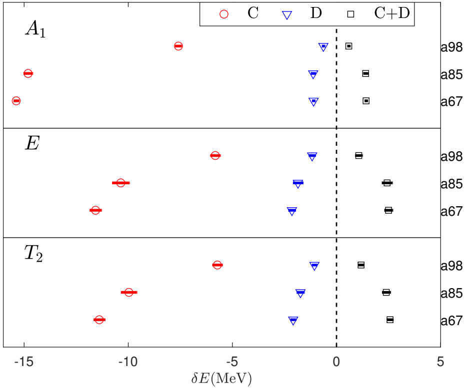


- S-wave scattering length

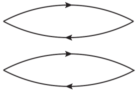
$$a^{0^+}_{\eta_c \eta_c} = -0.104(09)\ \mathrm{fm}, \ a^{2^+}_{J/\psi J/\psi} = -0.165(16)\ \mathrm{fm}$$

- No evidence of $\eta_c \eta_c$ and $J/\psi J/\psi$ with mass below the noninteracting thresholds in the 0^+ and 2^+ channels $[\delta E > 0]$

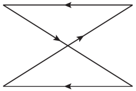
Individual wick contraction



Ensemble	δE[MeV]	A_1	E	T_2
a98	(D)	−0.63(07)	−1.17(16)	−1.06(12)
	(C)	−7.59(16)	−5.82(24)	−5.72(24)
	(D)+(C)	0.59(07)	1.07(17)	1.18(14)
a85	(D)	−1.12(11)	−1.85(24)	−1.73(18)
	(C)	−14.81(22)	−10.36(41)	−9.97(38)
	(D)+(C)	1.40(11)	2.43(25)	2.39(20)
a67	(D)	−1.10(07)	−2.14(17)	−2.08(13)
	(C)	−15.38(13)	−11.57(28)	−11.39(27)
	(D)+(C)	1.42(07)	2.50(20)	2.57(15)



(1) D

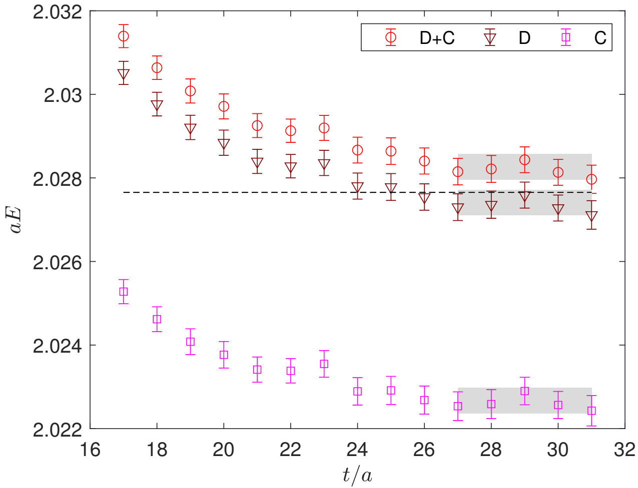


(2) C

Individual wick contraction: A_1 of a67

$$\tilde{C}^{(4)}(t) \equiv d\Big[\cosh\Big(E(t-\frac{T}{2})\Big) - \cosh\Big(E(t+1-\frac{T}{2})\Big)\Big]$$

	aE	d
D+C	$2.0283(3)$	$1.67(1)\times 10^{-23}$
D	$2.0274(3)$	$1.98(2)\times 10^{-23}$
C	$2.0227(3)$	$-0.307(2)\times 10^{-23}$

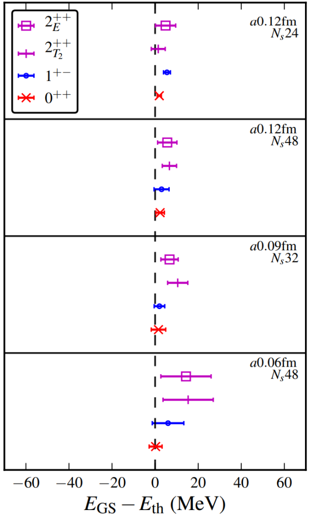


- The interference of the two diagrams leads to a repulsive interaction.

Previous lattice studies

- $\bar{b}\bar{b}bb$, soft gluon exchange and $\bar{q}q$ exchange PRD**97**,054505(2018)
  - 2+1+1 flavors from MILC collaboration,$a \sim [0.06, 0.12]\mathrm{fm}$
  - Repulsive interaction in any channel$(0^{++}, 1^{+-}, 2^{++})$
  - Individual diagram leads to attractive interaction

- $\Omega_{ccc}\Omega_{ccc}$ PKU&HALQCD,PRL**127**,072003(2021)
  - 2+1 flavor $O(a)$-improved Wilson action, $a \sim 0.085\mathrm{fm}$
  - $V(r)$ replusive at short range and attractive at midrange$(^1S_0)$
  - It supports a loosely bound state $[\delta E < 0]$



$\bar{b}\bar{b}bb$

Summary

- We present first-principle calculation on the scattering length of $\eta_c\eta_c$ and $J/\psi J/\psi$ in 0^+ and 2^+ channels
- No evidence of $\eta_c\eta_c$ and $J/\psi J/\psi$ with mass below the noninteracting thresholds in both channels
- We observe sizeable discretization effect, weak repulsive interaction in 0^+ $\eta_c\eta_c$ and 2^+ $J/\psi J/\psi$ systems
- The scattering lengths are obtained as

$$a^{0^+}_{\eta_c\eta_c} = -0.104(09)\ \mathrm{fm},\ a^{2^+}_{J/\psi J/\psi} = -0.165(16)\ \mathrm{fm}$$

Thank you!