Neutrinoless double beta decay and underlying physics

Gui-Jun Ding

University of Science and Technology of China

Based on arXiv:2110.15347, JHEP 12 (2021) 169; arXiv:2301.02503, JHEP 03 (2023) 138; arXiv:2504.xxxxx, in collaboration with Ping-Tao Chen, Shi-Yu Li, Chang-Yuan Yao

第四届强子与重味物理理论与实验联合研讨会,兰州大学, 2025年3月21日—24日



Neutrinoless double beta (0vββ) decay: intersection of new physics, hadron physics and nuclear physics





Massive neutrinos: Dirac or Majorana?



0vββ decay is the most sensitive known experimental method to verify whether neutrinos are Majorana particles.

Ονββ-decays: signature and candidate nuclei

0vββ is potentially observable in certain even-even nuclei (9 isotopes including ⁴⁸Ca, ⁷⁶Ge, ¹⁰⁰Mo, ¹³⁰Te, ¹³⁶Xe) for which single beta decay is energetically forbidden. The decay rate is less than 1 event per ton and per year.





Standard mass mechanism of 0vßß decay

Mass mechanism: $0\nu\beta\beta$ decay is usually assumed to be dominantly mediated by light and massive Majorana neutrinos.

p

^u_d p

Ge Mo

Tonne-scale goal 10²⁸ yrs

50 100 150

Α

 W^{-}

KamLAND-Zen (¹³⁶Xe)

NH

 10^{-2}

m_{lightest} (eV)

 10^{-1}

 $\left< m_{\beta\beta} \right> (eV)$

 10^{-2}

 10^{-3}

 10^{-}

> The decay amplitude is proportional to the effective mass



- Unknown: lightest mass, hierarchy and Majorana phases
- Large theoretical uncertainties in the nuclear matrix elements

Possible BSM physics in 0vββ decay

The $0\nu\beta\beta$ decay can also be induced by other $\Delta L=2$ physics besides the Majorana neutrino mass. There are many possible scenarios:



Classification of 0vßß mechanisms from SMEFT

The amplitude of $0\nu\beta\beta$ decay can be generally divided into:



EFT:
$$\mathcal{L} = \mathcal{L}_{SM} + \frac{c_i^{(5)}}{\Lambda} \mathcal{O}_i^{MM} + \frac{c_i^{(7)}}{\Lambda^3} \mathcal{O}_i^{LR} + \frac{c_i^{(9)}}{\Lambda^5} \mathcal{O}_i^{SR} + \dots$$

$$\mathcal{O}_1^{MM} = \epsilon^{ik} \epsilon^{jl} (\overline{\ell_i^c} \ell_j) H_k H_l$$

$$\mathcal{O}_{1}^{LR} = \epsilon^{ik} \epsilon^{jl} (\overline{\ell_{i}^{c}} \ell_{j}) (\overline{d_{R}} Q_{k}) H_{l} ,$$

$$\mathcal{O}_{2}^{LR} = \epsilon^{ik} \epsilon^{jl} (\overline{\ell_{i}^{c}} \gamma^{\mu\nu} \ell_{j}) (\overline{d_{R}} \gamma_{\mu\nu} Q_{k}) H_{l} ,$$

$$\mathcal{O}_{3}^{LR} = \epsilon^{jk} (\overline{\ell_{i}^{c}} \ell_{j}) (\overline{Q}^{i} u_{R}) H_{k} ,$$

$$\mathcal{O}_{4}^{LR} = (\overline{\ell_{i}^{c}} \gamma^{\mu} e_{R}) (\overline{d_{R}} \gamma_{\mu} u_{R}) \epsilon^{ij} H_{j}$$

[Pas, Hirsch, Klapdor, Kovalenko, hep-ph/0008182, hep-ph/9804374, PLB; Graesser, 1606.04549, JHEP]

$$\begin{split} \mathcal{O}_{1}^{SR} &= \epsilon_{ij}(\overline{Q}_{i}\gamma^{\mu}Q_{m})(\overline{u}_{R}\gamma_{\mu}d_{R})(\overline{\ell}_{j}\ell_{m}^{c}), \\ \mathcal{O}_{2}^{SR} &= \epsilon_{ij}(\overline{Q}_{i}\gamma^{\mu}\lambda^{A}Q_{m})(\overline{u}_{R}\gamma_{\mu}\lambda^{A}d_{R})(\overline{\ell}_{j}\ell_{m}^{c}), \\ \mathcal{O}_{3}^{SR} &= (\overline{u}_{R}Q_{i})(\overline{u}_{R}Q_{j})(\overline{\ell}_{i}\ell_{j}^{c}), \\ \mathcal{O}_{4}^{SR} &= (\overline{u}_{R}\lambda^{A}Q_{i})(\overline{u}_{R}\lambda^{A}Q_{j})(\overline{\ell}_{i}\ell_{j}^{c}), \\ \mathcal{O}_{5}^{SR} &= \epsilon_{ij}\epsilon_{mn}(\overline{Q}_{i}d_{R})(\overline{Q}_{m}d_{R})(\overline{\ell}_{j}\ell_{n}^{c}), \\ \mathcal{O}_{6}^{SR} &= \epsilon_{ij}\epsilon_{mn}(\overline{Q}_{i}\lambda^{A}d_{R})(\overline{Q}_{m}\lambda^{A}d_{R})(\overline{\ell}_{j}\ell_{n}^{c}), \\ \mathcal{O}_{7}^{SR} &= (\overline{u}_{R}\gamma^{\mu}d_{R})(\overline{u}_{R}\gamma_{\mu}d_{R})(\overline{e}_{R}e_{R}^{c}), \\ \mathcal{O}_{8}^{SR} &= \epsilon_{ij}(\overline{u}_{R}\gamma^{\mu}d_{R})(\overline{Q}_{i}d_{R})(\overline{\ell}_{j}\gamma_{\mu}e_{R}^{c}), \\ \mathcal{O}_{9}^{SR} &= \epsilon_{ij}(\overline{u}_{R}\gamma^{\mu}\lambda^{A}d_{R})(\overline{Q}_{i}\lambda^{A}d_{R})(\overline{\ell}_{j}\gamma_{\mu}e_{R}^{c}), \\ \mathcal{O}_{10}^{SR} &= (\overline{u}_{R}\gamma^{\mu}d_{R})(\overline{u}_{R}Q_{i})(\overline{\ell}_{i}\gamma_{\mu}e_{R}^{c}), \\ \mathcal{O}_{11}^{SR} &= (\overline{u}_{R}\gamma^{\mu}\lambda^{A}d_{R})(\overline{u}_{R}\lambda^{A}Q_{i})(\overline{\ell}_{i}\gamma_{\mu}e_{R}^{c}), \end{split}$$

Decomposing the short-range 0vßß operators

- 1 Topolopies: identify the L-loop connected topologies with 6 external legs
- 2 **Diagrams**: assign the fields of $0\nu\beta\beta$ operators to external lines, and specify the Lorentz nature (spinor or scalar) of each internal line.
- ③ **Models**: fix the $SU(3)_{c} \times SU(2)_{L} \times U(1)_{\gamma}$ quantum numbers of the internal fields by gauge invariance of each interaction vertex



Topologies for short-range 0vββ

Topolopies: Feynman diagrams where no property of fields is considered

(i) All connected topologies with 3- and 4- point vertices and 6 external legs

(ii) Remove tadpoles and self-energies (divergent)

(iii) Exclude non-renormalizable topologies





(iv) Discard topologies with 3-point loop vertices



any loop with 3 external legs can be compressible to a renormalizable vertex

2 tree + 6 one-loop renormalizable topologies



[Chen, Ding, Yao, 2110.15347, JHEP]

Field insertions: topologies \rightarrow diagrams

Focusing only on fermion and scalar boson mediators [Not considering gauge bosons]



Three kinds of renormalizable vertices
①fermion-fermion-scalar (FFS)

②scalar-scalar-scalar (SSS)

③scalar-scalar-scalar (SSSS)

[Chen, Ding, Yao, 2110.15347, JHEP]



> Diagrams continuum: attach external fields

Lorentz invariance fixes the mediator to be scalar or vector by chirality of external fermions



• A large number of possible diagrams

		$\begin{array}{c c} & \mathcal{O}_i^{SR} \\ \hline \text{TOPO} \end{array}$	N1	N2	N3	N4	N5	N6
Tree	{	N-0-1-1 N-0-2-1	$\frac{2}{11}$	$\frac{2}{6}$	5 18	$\frac{2}{6}$	2 11	$\frac{2}{11}$
		N-1-1-1	6	5	12	5	6	6
		N-1-2-1	96	30	54	30	96	96
		N-1-3-1	11	9	21	9	12	12
One leen		N-1-4-1	11	6	18	6	11	11
One-loop		N-1-4-2	11	6	18	6	11	11
		N-1-5-1	48	18	30	18	48	48
		N-1-5-2	48	18	30	18	48	48
		N-1-6-1	60	18	18	18	60	60

The redundant diagrams should be removed.

Classify 0vββ operators

Notation	0 uetaeta decay operators	External fields
N1	$\mathcal{O}_1^{SR}, \; \mathcal{O}_2^{SR}$	$\overline{Q}, Q, \bar{u}_R, d_R, \overline{\ell}, \ell^c$
N2	$\mathcal{O}^{SR}_3, \; \mathcal{O}^{SR}_4$	$Q,Q,\bar{u}_R,\bar{u}_R,\bar{\ell},\ell^c$
N3	$\mathcal{O}^{SR}_5, \; \mathcal{O}^{SR}_6$	$\overline{Q}, \overline{Q}, d_R, d_R, \overline{\ell}, \ell^c$
N4	\mathcal{O}_7^{SR}	$\bar{u}_R, \bar{u}_R, d_R, d_R, \bar{e}_R, e_R^c$
N5	$\mathcal{O}^{SR}_{8}, \; \mathcal{O}^{SR}_{9}$	$\overline{u}_R, \overline{Q}, d_R, d_R, \overline{\ell}, e_R^c$
N6	$\mathcal{O}^{SR}_{10}, \; \mathcal{O}^{SR}_{11}$	$\bar{u}_R, \bar{u}_R, Q, d_R, \bar{\ell}, e_R^c$

Determine quantum numbers of mediators: diagrams→models

The $SU(3)_{c} \times SU(2)_{L} \times U(1)_{\gamma}$ quantum numbers of the mediators fields are fixed by gauge invariance of each interaction vertex

• **3-point vertex:** $\overline{F}_1 F_2 S$, $S_1 S_2 S_3$

$$n_{\bar{F}_1} \otimes n_{F_2} \otimes n_S \supset \mathbf{1}, \quad Y_{\bar{F}_1} + Y_{F_2} + Y_S = 0$$
$$n_{S_1} \otimes n_{S_2} \otimes n_{S_3} \supset \mathbf{1}, \quad Y_{S_1} + Y_{S_2} + Y_{S_3} = 0$$

 n_X denotes the SU(2)_L or SU(3)_C representation of the field X

• **4-point vertex:** $S_1 S_2 S_3 S_4$

$$n_{S_1} \otimes n_{S_2} \otimes n_{S_3} \otimes n_{S_4} \supset \mathbf{1}, \qquad \sum_i Y_{S_i} = 0$$



GROUPMATH

Mathematica package **GroupMath** can help to determine the SM quantum numbers

Short-range contribution Versus mass mechanism

Black box theorem implies that the mass mechanism is always present in 0vββ decay. In 0vββ models, Majorana neutrino masses usually are generated at less than four-loop order.



The short-range contribution could dominate over the mass mechanism without fine-tuning in some parameter space, if the neutrino mass is generated at least at higher one-loop order.

Genuine models



Bird's-eye view of short-range 0vββ models



A large number of possible diagrams. For detail, see the attachment http://staff.ustc.edu.cn/~dinggj/supplementary_materials/Onbb.zip

Decomposing the long-range 0vßß operators

Long-range mechanism is not subject to helicity suppression!



$$\mathcal{O}_{1}^{LR} = \epsilon^{ik} \epsilon^{jl} (\overline{\ell_{i}^{c}} \ell_{j}) (\overline{d_{R}} Q_{k}) H_{l},$$

$$\mathcal{O}_{2}^{LR} = \epsilon^{ik} \epsilon^{jl} (\overline{\ell_{i}^{c}} \gamma^{\mu\nu} \ell_{j}) (\overline{d_{R}} \gamma_{\mu\nu} Q_{k}) H_{l},$$

$$\mathcal{O}_{3}^{LR} = \epsilon^{jk} (\overline{\ell_{i}^{c}} \ell_{j}) (\overline{Q}^{i} u_{R}) H_{k},$$

$$\mathcal{O}_{4}^{LR} = (\overline{\ell_{i}^{c}} \gamma^{\mu} e_{R}) (\overline{d_{R}} \gamma_{\mu} u_{R}) \epsilon^{ij} H_{j}$$

[Babu,Leung,hep-ph/0106054, NPB; Helo, Hirsch, Ota,1602.03362, JHEP; Lehman,1410.4193, PRD] topologies







[Chen, Ding, Yao, 2301.02503, JHEP]



Models: large variety of possible realizations accessible at high-energy colliders and high-intensity facilities, all genuine long-range 0vββ models up to 1-loop in the file http://staff.ustc.edu.cn/~dinggj/supplementary_materials/Long_range_Onbb.zip

> Black box theorem in long-range $0\nu\beta\beta: \Delta L = 2 \text{ operators} \rightarrow 0\nu\beta\beta \& \nu \text{ mass}$



Majorana neutrino masses are generated at least at the 2-loop order, regardless of long-range 0vββ operators

An example model of long-range 0vßß decay

> 3 new fields: two scalars S_1 , S_2 and a vector-like fermion F



Majorana neutrino masses generated at 3-loop





[Chen, Ding, Yao, 2301.02503, JHEP]

> Future ton-scale experiments impose strong constraint on the model and new physics contribution





Distinguishing different 0vββ mechanisms

> Comparison of the decay rates obtained using different isotopes

$$R^{\mathcal{O}_i}(^{\mathrm{A}}\mathrm{X}) \equiv \frac{T_{1/2}^{\mathcal{O}_i}(^{\mathrm{A}}\mathrm{X})}{T_{1/2}^{\mathcal{O}_i}(^{76}\mathrm{Ge})} = \frac{\sum_j |\mathcal{M}_j^{\mathcal{O}_i}(^{76}\mathrm{Ge})|^2 G_j^{\mathcal{O}_i}(^{76}\mathrm{Ge})}{\sum_k |\mathcal{M}_k^{\mathcal{O}_i}(^{\mathrm{A}}\mathrm{X})|^2 G_k^{\mathcal{O}_i}(^{\mathrm{A}}\mathrm{X})}$$

[Graf,Lindner,Scholer,2204.10845,PRD]



Combining 0vββ decay, collider measurements and cosmology

 $\mathcal{L} \supset \underline{g_Q}\overline{Q}Sd_R + \underline{g_L}\overline{L}(i\tau^2)S^*F + \lambda_{HS}(S^{\dagger}H)^2 + \text{h.c.}$



Collider signature: same-sign dilepton +2 jets + no E_T^{miss}

[Harz, Ramsey-Musolf, Shen, Urrutia-Quiroga, 2106.10838,PRD; Graesser,Li,Ramsey-Musolf, Shen,Urrutia-Quiroga, 2202.01237,JHEP]

Summary

Ovββ is the most sensitive probe to the Majorana nature of neutrinos. However, there are many possible sources divided into mass mechanism, long-range mechanism and shortrange mechanism.



- > Systematic decomposition $0\nu\beta\beta$ operators: topologies > diagrams > models
- Many open problems: the 0vββ models in future colliders and LFV searches, implications in cosmology and leptogenesis....
- Both theoretical and experimental efforts are needed to fix the 0vββ signal and underling mechanism.

Thank you for your attention!

Backup

Current and future 0vßß experiments

Most stringent constraints on the half life:

- ¹³⁶Xe (KamLAND-Zen): T_{1/2}> 3.8x10²⁶ yrs [KamLAND-Zen Collaboration, 2406.11438]
- ⁷⁶Ge (GERDA): $T_{1/2} > 1.8x \ 10^{26} \text{ yrs}$ [GERDA collaboration, 2009.06079, PRL]
- ¹³⁰Te (CUORE): $T_{1/2}$ > 3.8x 10²⁵ yrs [CUORE collaboration, 2404.04453]

There are many $0\nu\beta\beta$ decay experiments in plan and construction



[Agostini, Benato, Detwiler, Menendez, Vissani, 2202.01787, Rev.Mod.Phys.]

Prospects of 0vββ experiments



Absolute neutrino masses from synergies of neutrino facilities



Next generation of ton-scale 0vββ experiments will cover the IO region.

Nuclear matrix elements



[Agostini, Benato, Detwiler, Menendez, Vissani, 2202.01787, Rev.Mod.Phys.] EDF: large NMEs **QRPA:** wider range **NSM: small NMEs** IMSRG ab initio 48Ca NME: quite small (no 2b currents)

Unfortunately, nuclear matrix elements can be calculated only based on some models which describe many-body interactions of nucleons in nuclei. Since different models focus on different aspects of nuclear physics, large uncertainties (a factor of 2 or 3) are unavoidable.

Determine 0vßß mechanisms by measuring electron

measure the angular and energy distributions of electron



[Ali, Borisov, Zhuridov, 0706.4165,PRD; SuperNEMO Collaborarion,1005.1241,EPJC]





Tree-level decomposition of short-range 0vββ operators



		Long	Mediat	or $(U(1)_{em})$	$SU(3)_c$	
#	Decomposition	Range?	S or V_{ρ}	ψ	S' or V'_{ρ}	Models/Refs./Comments
1-i	$(\bar{u}d)(\bar{e})(\bar{e})(\bar{u}d)$	(a)	(+1, 1)	(0, 1)	(-1, 1)	Mass mechan., RPV [58]60], LR-symmetric models [39], Mass mechanism with ν_S [61], TeV scale seesaw, e.g., [62]63.
			(+1, 8)	(0.8)	(-1, 8)	64
1-ii-a	$(\bar{u}d)(\bar{u})(d)(\bar{e}\bar{e})$		(+1, 1)	(+5/3,3)	(+2, 1)	Provide Second
	(/(-/(/		(+1, 8)	(+5/3, 3)	(+2, 1)	
1-ii-b	$(\bar{u}d)(d)(\bar{u})(\bar{e}\bar{e})$		(+1, 1)	$(+4/3, \bar{3})$	(+2, 1)	
	0.0000000000000000000000000000000000000		(+1, 8)	$(+4/3, \bar{3})$	(+2, 1)	
2-i-a	$(\bar{u}d)(d)(\bar{e})(\bar{u}\bar{e})$		(+1, 1)	$(+4/3, \bar{3})$	(+1/3, 3)	
			(+1, 8)	(+4/3, 3)	$(+1/3, \bar{3})$	
2-i-b	$(\bar{u}d)(\bar{e})(d)(\bar{u}\bar{e})$	(b)	(+1, 1)	(0, 1)	$(+1/3, \bar{3})$	RPV 58 60, LQ 65 66
			(+1, 8)	(0, 8)	$(+1/3, \bar{3})$	
2-ii-a	$(\bar{u}d)(\bar{u})(\bar{e})(d\bar{e})$		(+1, 1)	(+5/3, 3)	(+2/3, 3)	
			(+1, 8)	(+5/3, 3)	(+2/3, 3)	
2-ii-b	$(\bar{u}d)(\bar{e})(\bar{u})(d\bar{e})$	(b)	(+1, 1)	(0, 1)	(+2/3, 3)	RPV 58 60, LQ 65 66
			(+1, 8)	(0, 8)	(+2/3, 3)	
2-iii-a	$(d\bar{e})(\bar{u})(d)(\bar{u}\bar{e})$	(c)	$(-2/3, \bar{3})$	(0, 1)	$(+1/3, \bar{3})$	RPV 58 60
			$(-2/3, \bar{3})$	(0, 8)	$(+1/3, \bar{3})$	RPV 58 60
2-iii-b	$(d\bar{e})(d)(\bar{u})(\bar{u}\bar{e})$		$(-2/3, \bar{3})$	(-1/3, 3)	$(+1/3, \bar{3})$	
			$(-2/3, \bar{3})$	$(-1/3, \overline{6})$	$(+1/3, \bar{3})$	
3-i	$(\bar{u}\bar{u})(\bar{e})(\bar{e})(dd)$		$(+4/3, \bar{3})$	(+1/3, 3)	$(-2/3, \bar{3})$	only with V_{ρ} and V'_{ρ}
			(+4/3, 6)	(+1/3, 6)	(-2/3, 6)	
3-ii	$(\bar{u}\bar{u})(d)(d)(\bar{e}\bar{e})$		$(+4/3, \bar{3})$	(+5/3, 3)	(+2, 1)	only with V_{ρ}
			(+4/3, 6)	(+5/3, 3)	(+2, 1)	
3-iii	$(dd)(\bar{u})(\bar{u})(\bar{e}\bar{e})$		(+2/3, 3)	(+4/3, 3)	(+2, 1)	only with V_{ρ}
			$(+2/3, \overline{6})$	(+4/3, 3)	(+2, 1)	
4-i	$(d\bar{e})(\bar{u})(\bar{u})(d\bar{e})$	(c)	(-2/3, 3)	(0, 1)	(+2/3, 3)	RPV 58-60
			(-2/3, 3)	(0, 8)	(+2/3, 3)	RPV 58 60
4-ii-a	$(\bar{u}\bar{u})(d)(\bar{e})(d\bar{e})$		$(+4/3, \bar{3})$	(+5/3, 3)	(+2/3, 3)	only with V_{ρ}
0.4255-0	100000000000000000000000000000000000000		(+4/3, 6)	(+5/3, 3)	(+2/3, 3)	see Sec. 🛄 (this work)
4-ii-b	$(\bar{u}\bar{u})(\bar{e})(d)(d\bar{e})$		(+4/3, 3)	(+1/3, 3)	(+2/3, 3)	only with V_{ρ}
			(+4/3, 6)	(+1/3, 6)	(+2/3, 3)	
5-i	$(\bar{u}\bar{e})(d)(d)(\bar{u}\bar{e})$	(c)	(-1/3, 3)	(0, 1)	(+1/3, 3)	RPV 58 60
-	111111111111111111		(-1/3, 3)	(0,8)	(+1/3, 3)	RPV 58160
5-11-a	$(\bar{u}\bar{e})(\bar{u})(\bar{e})(dd)$		(-1/3, 3)	(+1/3, 3)	(-2/3, 3)	only with V_{ρ}^{\prime}
			(-1/3, 3)	(+1/3, 6)	(-2/3, 6)	Della dell
5-ii-b	$(\bar{u}\bar{e})(\bar{e})(\bar{u})(dd)$		(-1/3, 3)	(-4/3, 3)	(-2/3, 3)	only with V'_{ρ}
			(-1/3, 3)	(-4/3, 3)	(-2/3, 6)	

[Bonnet, Hirsch, Ota, Winter, 1212.3045, JHEP]

Tree-level decomposition of long-range 0vββ operators

#	Decompositions	Mediators		Projection to the basis ops.	m_{ν} @tree	m_{ν} @1loop	m_{ν} @2loop
#1 $(L_{\alpha}L_{\beta})(H)(\overline{d_R}Q)$		$S(1,1)_{+1}$	$S'(1,2)_{+rac{1}{2}}$	$-\mathcal{O}_{3a}(lpha,eta)$		$\mathrm{T} \nu \mathrm{I}$ -ii w. $\overline{\ell_R} L S'^{\dagger}$	$T2_4^{\rm B}(\alpha \neq \beta)$ $\mathcal{O}_3^7 \text{ in } [38]$
		$S(1, 3)_{+1}$	$S'(1,2)_{+\frac{1}{2}}$	$-\mathcal{O}_{3b}(lpha,eta)-\mathcal{O}_{3b}(eta,lpha)$	type II	_	
#2	$(L_{\alpha}Q)(H)(\overline{d_R}L_{\beta})$	$S(\overline{3},1)_{+rac{1}{3}}$	$S'(\overline{3},2)_{-rac{1}{6}}$	$rac{1}{2}\mathcal{O}_{3b}(lpha,eta)-rac{1}{2}\mathcal{O}_{3b}^{ ext{ten.}}(lpha,eta)$		$T\nu$ I-ii $\frac{53}{O_3^8}$ in $\frac{38}{38}$	[<u>14</u> , <u>68</u>]
		$S(\overline{3},3)_{+\frac{1}{3}}$	$S'(\overline{3},2)_{-rac{1}{6}}$	$ \frac{\frac{1}{2}\mathcal{O}_{3a}(\alpha,\beta) - \frac{1}{2}\mathcal{O}_{3a}^{\text{ten.}}(\alpha,\beta) }{-\frac{1}{2}\mathcal{O}_{3b}(\beta,\alpha) - \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\beta,\alpha) } $		ΤνΙ-ii <u>53]</u> O ₃ in <u>38</u>]	[14]
#3	$(L_{\alpha}L_{\beta})(Q)(\overline{d_R}H)$	$S(1,1)_{+1}$	$\psi_{L,R}(3,2)_{-rac{5}{6}}$	$-\mathcal{O}_{3a}(lpha,eta)$		$\begin{bmatrix} T\nu I-ii \\ w.S^{\dagger}HH' \end{bmatrix}$	$\begin{array}{c} T2_1^{\rm B}(\alpha \neq \beta) \\ \mathcal{O}_3^1 \text{ in } 38 \end{array}$
		$S(1, 3)_{+1}$	$\psi_{L,R}({f 3},{f 2})_{-rac{5}{6}}$	$-\mathcal{O}_{3b}(\alpha,\beta) - \mathcal{O}_{3b}(\beta,\alpha)$	type II		
#4	$(L_{\alpha}H)(Q)(\overline{d_R}L_{\beta})$	$\psi_R(1,1)_0$	$S(\overline{3}, 2)_{-\frac{1}{6}}$	$\frac{1}{2}\mathcal{O}_{3b}(eta,lpha)+\frac{1}{2}\mathcal{O}_{3b}^{ ext{ten.}}(eta,lpha)$	type I		
		$\psi_R(1,3)_0$	$S(\overline{3}, 2)_{-\frac{1}{6}}$	$\begin{aligned} &-\frac{1}{2}\mathcal{O}_{3a}(\alpha,\beta)+\frac{1}{2}\mathcal{O}_{3a}^{\text{ten.}}(\alpha,\beta)\\ &-\frac{1}{2}\mathcal{O}_{3b}(\alpha,\beta)+\frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\alpha,\beta)\end{aligned}$	type III		
#5	$(L_{\alpha}L_{\beta})(\overline{d_R})(QH)$	$S(1,1)_{+1}$	$\psi_{L,R}({f 3},{f 1})_{+rac{2}{3}}$	$\mathcal{O}_{3a}(lpha,eta)$		$\begin{bmatrix} \mathrm{T}\nu\mathrm{I-ii} \\ \mathrm{w.}S^{\dagger}HH' \end{bmatrix}$	$\begin{array}{ c c } T2_2^{\rm B}(\alpha \neq \beta) \\ \mathcal{O}_3^2 \text{ in } [38] \end{array}$
		$S(1, 3)_{+1}$	$\psi_{L,R}(3,3)_{+\frac{2}{3}}$	$-\mathcal{O}_{3b}(lpha,eta)-\mathcal{O}_{3b}(eta,lpha)$	type II		
#6	$(L_{\alpha}Q)(\overline{d_R})(L_{\beta}H)$	$S(\bar{3},1)_{+\frac{1}{2}}$	$\psi_R(1,1)_0$	$-\frac{1}{2}\mathcal{O}_{3b}(\alpha,\beta)+\frac{1}{2}\mathcal{O}_{3b}^{\mathrm{ten.}}(\alpha,\beta)$	type I		
		$S(\overline{3},3)_{+\frac{1}{3}}$	$\psi_R(1,3)_0$	$\frac{\frac{1}{2}\mathcal{O}_{3a}(\alpha,\beta) - \frac{1}{2}\mathcal{O}_{3a}^{\text{ten.}}(\alpha,\beta)}{-\frac{1}{2}\mathcal{O}_{3b}(\beta,\alpha) - \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\beta,\alpha)}$	type III		
#7	$(L_{\alpha}Q)(L_{\beta})(\overline{d_R}H)$	$S(\overline{3},1)_{+rac{1}{3}}$	$\psi_{L,R}({f 3},{f 2})_{-rac{5}{6}}$	$\frac{1}{2}\mathcal{O}_{3b}(\alpha,\beta) - \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\alpha,\beta)$		$T\nu$ I-iii \mathcal{O}_3^4 in [38]	
		$S(\overline{3},3)_{+\frac{1}{3}}$	$\psi_{L,R}({f 3},{f 2})_{-rac{5}{6}}$	$\frac{\frac{1}{2}\mathcal{O}_{3a}(\alpha,\beta) - \frac{1}{2}\mathcal{O}_{3a}^{\text{ten.}}(\alpha,\beta)}{-\frac{1}{2}\mathcal{O}_{3b}(\beta,\alpha) - \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\beta,\alpha)}$		$T\nu$ I-iii \mathcal{O}_3^5 in [38]	
#8	$(\overline{d_R}L_{\alpha})(L_{\beta})(QH)$	$S(3,2)_{+\frac{1}{6}}$	$\psi_{L,R}(3,1)_{+rac{2}{3}}$	$-\frac{1}{2}\mathcal{O}_{3a}(\alpha,\beta)-\frac{1}{2}\mathcal{O}_{3a}^{\text{ten.}}(\alpha,\beta)$			$T2_{2}^{B}(m_{\nu})_{\alpha \neq \beta}$ $\mathcal{O}_{3}^{3} \text{ in } [38],$ [43]
		$S(3,2)_{+\frac{1}{6}}$	$\psi_{L,R}(3,3)_{+rac{2}{3}}$	$ \frac{\frac{1}{2}\mathcal{O}_{3b}(\alpha,\beta) + \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\alpha,\beta) }{+\frac{1}{2}\mathcal{O}_{3b}(\beta,\alpha) - \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\beta,\alpha) } $	_	$T\nu$ I-iii \mathcal{O}_3^6 in [38]	
# 9 ($(L_{\alpha}H)(L_{\beta})(\overline{d_R}Q)$	$\psi_R(1,1)_0$	$S(1,2)_{+\frac{1}{2}}$	$\mathcal{O}_{3b}(\beta, \alpha)$	type I		
		$\psi_R(1,3)_0$	$S(1,2)_{+\frac{1}{2}}$	$\mathcal{O}_{3a}(lpha,eta) + \mathcal{O}_{3b}(lpha,eta)$	type III		



đ

 $\langle H \rangle$





[Helo, Hirsch,Ota,

1602.03362,JHEP]