



## 第四届强子与重味物理理论与实验联合研讨会

# Correlation function and the inverse problem for the N\*(1535) and Sigma\*(1430)

► Chu-Wen Xiao (肖楮文)

- Guangxi Normal University (广西师范大学)
- Collaborators: Eulogio Oset, Wei-Hong Liang, Raquel Molina Jia-Jun Wu, En Wang, Hai-Peng Li

arXiv: 2409.05787; 2310.12593

2025.3. Lanzhou



### Outline

Introduction
 Two-body interaction
 Inverse problem
 For the case of N\*(1535)
 Summary

### §1. Introduction



A  $\Sigma^*(1/2-)$  state with mass 1430 MeV near **KN** threshod was **predicted**: N. Kaiser, P. B. Siegel, and W. Weise, Nucl. Phys. A 594, 325 (1995) Isospin I = 1E. Oset and A. Ramos, Nucl. Phys. A 635, 99 (1998) J. A. Oller and U.-G. Meißner, Phys. Lett. B 500, 263 (2001) D. Jido, J. A. Oller, E. Oset, A. Ramos and U.-G. Meißner, Nucl. Phys. A 725, 181 (2003)  $\sim$  two-poles structure of  $\Lambda(1405)$  was found But, still NOT found yet..... A recent review on the  $\Sigma * (1/2)$  state: E. Wang, L.-S. Geng, J.-J. Wu, J.-J. Xie, and B.-S. Zou, Chin. Phys. Lett. 41 (2024) 10, 101401



There are some proposals to search for this **predicted** state:

Y.-H. Lyu, H. Zhang, N.-C. Wei, B.-C. Ke, E. Wang, and J.-J. Xie, Chin. Phys. C 47, 053108 (2023)

$$\gamma n \rightarrow K^+ \Sigma^{*-}_{1/2^-}$$

X.-L. Ren, E. Oset, L. Alvarez-Ruso, and M. J. Vicente Vacas, Phys. Rev. C 91, 045201 (2015) J.-J. Wu and B.-S. Zou, Few Body Syst. 56, 165 (2015)  $\overline{L} = 0$  L = 0

$$\bar{\nu}_l p \to l^+ \Phi B$$

E. Wang, J.-J. Xie, and E. Oset, Phys. Lett. B 753, 526 (2016)

$$\chi_{c0}(1P)\to \bar{\Sigma}\Sigma\pi$$



L.-J. Liu, E. Wang, J.-J. Xie, K.-L. Song, and J.-Y. Zhu, Phys. Rev. D 98, 114017 (2018)

$$\chi_{c0} \rightarrow \Lambda \Sigma \pi$$

J.-J. Xie and E. Oset, Phys. Lett. B 792, 450 (2019)



A recent evidence of the  $\Sigma * (1/2-)$  state:

Y. Ma et al. (Belle), Phys. Rev. Lett. 130, 151903 (2023)

But, from their analysis, they can NOT discriminate from the peak being due to

a resonance or to a cusp in the  $\overline{K}N$  threshold.



 $\Lambda_c^+ \to \Lambda \pi^+ \pi^+ \pi^-$ 

### §2. Two-body interaction



#### (1) Coupled channel interaction from the chiral unitary approach

$$\bar{K}^0 p, \pi^+ \Sigma^0, \pi^0 \Sigma^+, \pi^+ \Lambda, \text{ and } \eta \Sigma^+$$

Without the Coulomb interaction

$$V_{ij} = -\frac{1}{4f^2}C_{ij}(k_i^0 + k_j^0)$$

 $\begin{aligned} \left|\pi^{+}\Sigma^{0}\right\rangle &= -\frac{1}{\sqrt{2}} \big(\left|\pi\Sigma, I=2, I_{3}=1\right\rangle + \left|\pi\Sigma, I=1, I_{3}=1\right\rangle \big) \\ \left|\pi^{0}\Sigma^{+}\right\rangle &= -\frac{1}{\sqrt{2}} \big(\left|\pi\Sigma, I=2, I_{3}=1\right\rangle - \left|\pi\Sigma, I=1, I_{3}=1\right\rangle \big) \\ \left|\bar{K}^{0}p\right\rangle &= \left|\bar{K}N, I=1, I_{3}=1\right\rangle , \\ \left|\pi^{+}\Lambda\right\rangle &= -\left|\pi\Lambda, I=1, I_{3}=1\right\rangle \\ \left|n\Sigma^{+}\right\rangle &= -\left|n\Sigma, I=1, I_{3}=1\right\rangle \end{aligned}$ 

$C_{ij}$	$ar{K}^0 p$	$\pi^+\Sigma^0$	$\pi^0 \Sigma^+$	$\pi^+\Lambda$	$\eta \Sigma^+$
$ar{K}^0 p$	1	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{2}}$	$\sqrt{\frac{3}{2}}$
$\pi^+\Sigma^0$		0	-2	0	0
$\pi^0 \Sigma^+$			0	0	0
$\pi^+\Lambda$				0	0
$\eta \Sigma^+$					0

5



Coupled Channel Unitary Approach : solving Bethe-Salpeter equations, which take on-shell approximation for the loops.

$$T = V + V G T, T = [1 - V G]^{-1} V$$



where V matrix (potentials) can be evaluated from the interaction Lagrangians.

J. A. Oller and E. Oset, Nucl. Phys. A 620 (1997) 438 E. Oset and A. Ramos, Nucl. Phys. A 635 (1998) 99 J. A. Oller and U. G. Meißner, Phys. Lett. B 500 (2001) 263 G is a diagonal matrix with the loop functions of each channels:

$$G_{ll}(s) = i \int \frac{d^4q}{(2\pi)^4} \frac{2M_l}{(P-q)^2 - m_{l1}^2 + i\varepsilon} \frac{1}{q^2 - m_{l2}^2 + i\varepsilon}$$

The coupled channel scattering amplitudes **T matrix satisfy the unitary** :

$$\operatorname{Im} T_{ij} = T_{in} \,\sigma_{nn} \,T_{nj}^*$$
$$\sigma_{nn} \equiv \operatorname{Im} G_{nn} = -\frac{q_{cm}}{8\pi\sqrt{s}}\theta(s - (m_1 + m_2)^2))$$

To search the poles of the resonances, we should extrapolate the scattering amplitudes to the second Riemann sheets:

$$G_{ll}^{II}(s) = G_{ll}^{I}(s) + i \frac{i}{2\pi} \frac{M_l q_{cml}(s)}{\sqrt{s}}$$





#### **Correlation functions**





### §3. Inverse problem



 $p-\Xi^{-}$ 

 $p-\Omega^{2}$ 

300

200

k\* (MeV/c)

100

#### Why, we do the inverse problem? **a** 3.5 [' ALICE data 0 (1) What can we learn from the correlation functions? 3₽ Coulomb Coulomb + $p-\Xi^-$ HAL QCD 2.5<sup>1</sup> (<del>(</del>\*\*) Coulomb + $p-\Omega^-$ HAL QCD elastic Coulomb + $p-\Omega^-$ HAL QCD elastic + inelastic Interaction **Correlation Function** Emission source $S(r^*)$ Repulsive 1.5 C(K\* Attractive Attractive 0.5 1.5 0 $r^*$ (fm) Repulsive Schrödinger equation 150 50 100 200 Two-particle wave *k*\*(MeV/*c*) function $|\Psi(k^*, r^*)|$ C(k\*) С 200 $C(k^*) = \int S(r^*) |\Psi(k^*, r^*)|^2 d^3 r^* = \xi(k^*) \cdot \frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}$ 100 k\* (MeV/c)

A. Collaboration et al. (ALICE), Nature 588, 232 (2020)









#### (3) How to do the inverse problem?

#### Assume an energy dependence interaction potential

$$V_{ij} = -\frac{1}{4f^2} \tilde{C}_{ij} (k_i^0 + k_j^0)$$

Under the isospin constrain







#### Average of the fitted parameters

![](_page_17_Picture_1.jpeg)

$ ilde{C}_{11}$	$ ilde{C}_{12}$	$ ilde{C}_{14}$	$ ilde{C}_{15}$	$ ilde{C}_{22}$
$1.036\pm0.261$	$-0.985 \pm 0.138$	$-1.204 \pm 0.220$	$-0.829 \pm 0.406$	$1.924\pm0.147$
$ ilde{C}_{22}^{\prime}$	$ ilde{C}_{24}$	$ ilde{C}_{25}$	$ ilde{C}_{44}$	$ ilde{C}_{45}$
$-2.136 \pm 0.465$	$-0.057 \pm 0.342$	$-0.028 \pm 0.571$	$-0.053 \pm 0.141$	$-0.066 \pm 0.706$
$ ilde{C}_{55}$	$q_{ m max}({ m MeV})$	$R({ m fm})$		
$0.043 \pm 0.447$	$653.468 \pm 63.802$	$0.995 \pm 0.029$		

**Observables: the scattering length and effective range** 

$$\frac{1}{a_i} = \left. \frac{8\pi\sqrt{s}}{2M_i} (T_{ii})^{-1} \right|_{\sqrt{s}_{\mathrm{th},i}} \qquad r_i = \frac{1}{\mu_i} \frac{\partial}{\partial\sqrt{s}} \left[ \frac{-8\pi\sqrt{s}}{2M_i} (T_{ii})^{-1} + ik_i \right]_{\sqrt{s}_{\mathrm{th},i}}$$

 $\sqrt{s}_p = (1420 \pm 10) - i(101 \pm 19) \text{ MeV}$ 

J. A. Oller and U.-G. Meißner, Phys. Lett. B 500, 263 (2001) D. Jido, J. A. Oller, E. Oset, A. Ramos and U.-G. Meißner, Nucl. Phys. A 725, 181 (2003)

#### Average of the scattering lengths

$a_1$	$a_2$	$a_3$
$(0.468 \pm 0.088) - i(1.130 \pm 0.041)$	$-(0.148 \pm 0.010) - i(0.030 \pm 0.004)$	$-(0.113 \pm 0.010) - i(0.004 \pm 0.003)$
$a_4$	$a_5$	
$-(0.045\pm 0.008)$	$(0.083 \pm 0.010) - i(0.161 \pm 0.026)$	

#### Average of the effective ranges

$r_1$	$r_2$	$r_3$
$(0.025 \pm 0.150) - i(0.452 \pm 0.089)$	$-(38.019\pm 6.345) - i(16.534\pm 1.932)$	$-(75.053 \pm 17.150) + i(1.143 \pm 1.456)$
$r_4$	$r_5$	
$-(75.035 \pm 19.508)$	$(0.334 \pm 0.761) + i(0.380 \pm 0.947)$	

#### Consistent with the theoretical results we have before

#### A cusp- like structure

![](_page_19_Picture_1.jpeg)

![](_page_19_Figure_2.jpeg)

### §4. For the case of $N^*(1535)$

![](_page_20_Picture_1.jpeg)

#### N(1535) > N(1440): out of the simple quark model expectations

#### 1) molecular picture

J. Nieves and E. R. Arriola, Phys. Rev. D 64, 116008 (2001).
B. C. Liu and B. S. Zou, Phys. Rev. Lett. 96, 042002 (2006).
P. C. Bruns, M. Mai, and U. G. Meissner, Phys. Lett. B 697, 254 (2011).
P.C. Bruns and A. Cieply, Nucl. Phys. A992, 121630 (2019).
J. C. Nacher, A. Parreno, E. Oset, A. Ramos, A. Hosaka, and M. Oka, Nucl. Phys. A678, 187 (2000).

2) three quark component mixed with some pentaquark configuration

L. Hannelius and D. O. Riska, Phys. Rev. C 62, 045204 (2000). B. S. Zou and D. O. Riska, Phys. Rev. Lett. 95, 072001 (2005).

#### 3) Molecule with s-sbar components

![](_page_21_Picture_1.jpeg)

J. J. Xie, B. S. Zou, and H. C. Chiang, Phys. Rev. C 77, 015206 (2008).
M. Doring, E. Oset, and B. S. Zou, Phys. Rev. C 78, 025207 (2008).
T. Mart, Phys. Rev. C 87, 042201(R) (2013).

4) Molecule with three-quark components

T. Hyodo, D. Jido, and A. Hosaka, Phys. Rev. C 78, 025203 (2008).

T. Sekihara, T. Arai, J. Yamagata-Sekihara, and S. Yasui, Phys. Rev. C 93, 035204 (2016).

T. Sekihara, T. Hyodo, and D. Jido, Prog. Theor. Exp. Phys. 2015, 063D04 (2015).
Z. W. Liu, W. Kamleh, D. B. Leinweber, F. M. Stokes, A. W. Thomas, and J. J. Wu, Phys. Rev. Lett. 116, 082004 (2016).

C. D. Abell, D. B. Leinweber, Z. W. Liu, A. W. Thomas, and J. J. Wu, Phys. Rev. D 108, 094519 (2023).

We bring the attention to a new source of information obtained about the relevance of the meson-baryon components of the N\*(1535).

#### (1) Coupled channel interaction

![](_page_22_Picture_1.jpeg)

$C_{ij}$	$K^0\Sigma^+$	$K^+\Sigma^0$	$K^+\Lambda$	$\pi^+ n$	$\pi^0 p$	ηp
$\overline{K^0\Sigma^+}$	1	$\sqrt{2}$	0	0	$\frac{1}{\sqrt{2}}$	$-\sqrt{2}$
$K^+\Sigma^0$		0	0	$\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
$K^+\Lambda$			0	$-\sqrt{\frac{3}{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{3}{2}$
$\pi^+ n$				v 2 1	$\sqrt{2}$	0
$\pi^0 p$					0	0
ηp						0
		1				

$$V_{ij} = -\frac{1}{4f^2} C_{ij} (k^0 + k'^0)$$

 $T = [1 - VG]^{-1}V$ 

#### (2) Correlation functions: Koonin-Pratt formula

![](_page_23_Picture_1.jpeg)

$$\begin{split} C_{K^{0}\Sigma^{+}}(p_{K^{0}}) &= 1 + 4\pi\theta(q_{\max} - p_{K^{0}}) \int drr^{2}S_{12}(r) \cdot \{|j_{0}(p_{K^{0}}r) + T_{K^{0}\Sigma^{+},K^{0}\Sigma^{+}}(E)\tilde{G}^{(K^{0}\Sigma^{+})}(r;E)|^{2} \\ &+ |T_{K^{+}\Sigma^{0},K^{0}\Sigma^{+}}(E)\tilde{G}^{(K^{+}\Sigma^{0})}(r;E)|^{2} + |T_{K^{+}\Lambda,K^{0}\Sigma^{+}}(E)\tilde{G}^{(K^{+}\Lambda)}(r;E)|^{2} \\ &+ |T_{\eta p,K^{0}\Sigma^{+}}(E)\tilde{G}^{(\eta p)}(r;E)|^{2} - j_{0}^{2}(p_{K^{0}}r)\}, \\ C_{K^{+}\Sigma^{0}}(p_{K^{+}}) &= 1 + 4\pi\theta(q_{\max} - p_{K^{+}}) \int drr^{2}S_{12}(r) \cdot \{|j_{0}(p_{K^{+}}r) + T_{K^{+}\Sigma^{0},K^{+}\Sigma^{0}}(E)\tilde{G}^{(K^{+}\Sigma^{0})}(r;E)|^{2} \\ &+ |T_{K^{0}\Sigma^{+},K^{+}\Sigma^{0}}(E)\tilde{G}^{(\eta p)}(r;E)|^{2} + |T_{K^{+}\Lambda,K^{+}\Sigma^{0}}(E)\tilde{G}^{(K^{+}\Lambda)}(r;E)|^{2} \\ &+ |T_{\eta p,K^{+}\Sigma^{0}}(E)\tilde{G}^{(\eta p)}(r;E)|^{2} - j_{0}^{2}(p_{K^{+}}r)\}, \\ C_{K^{+}\Lambda}(p_{K^{+}}) &= 1 + 4\pi\theta(q_{\max} - p_{K^{+}}) \int drr^{2}S_{12}(r) \cdot \{|j_{0}(p_{K^{+}}r) + T_{K^{+}\Lambda,K^{+}\Lambda}(E)\tilde{G}^{(K^{+}\Lambda)}(r;E)|^{2} \\ &+ |T_{K^{0}\Sigma^{+},K^{+}\Lambda}(E)\tilde{G}^{(K^{0}\Sigma^{+})}(r;E)|^{2} + |T_{K^{+}\Sigma^{0},K^{+}\Lambda}(E)\tilde{G}^{(K^{+}\Sigma^{0})}(r;E)|^{2} \\ &+ |T_{\eta p,K^{+}\Lambda}(E)\tilde{G}^{(\eta p)}(r;E)|^{2} - j_{0}^{2}(p_{K^{+}}r)\}, \end{split}$$

$$\begin{split} C_{\eta p}(p_{\eta}) &= 1 + 4\pi\theta(q_{\max} - p_{\eta}) \int dr r^{2} S_{12}(r) \cdot \{ |j_{0}(p_{\eta}r) + T_{\eta p,\eta p}(E) \tilde{G}^{(\eta p)}(r;E)|^{2} \\ &+ |T_{K^{0}\Sigma^{+},\eta p}(E) \tilde{G}^{(K^{0}\Sigma^{+})}(r;E)|^{2} + |T_{K^{+}\Sigma^{0},\eta p}(E) \tilde{G}^{(K^{+}\Sigma^{0})}(r;E)|^{2} + |T_{K^{+}\Lambda,\eta p}(E) \tilde{G}^{(K^{+}\Lambda)}(r;E)|^{2} - j_{0}^{2}(p_{\eta}r) \} \end{split}$$

#### (3) inverse problem

![](_page_24_Picture_1.jpeg)

$$V_{ij} = \begin{pmatrix} V_{11} & \sqrt{2}(V_{11} - V_{22}) & V_{13} & V_{14} \\ & V_{22} & \frac{1}{\sqrt{2}}V_{13} & \frac{1}{\sqrt{2}}V_{14} \\ & V_{33} & V_{34} \\ & & V_{44} \end{pmatrix}$$
$$V_{ij} = -\frac{1}{4f^2}\tilde{C}_{ij}(k^0 + k'^0)$$

Using the bootstrap method **Doing 50/100 fits** 

![](_page_25_Picture_0.jpeg)

![](_page_25_Figure_1.jpeg)

#### Average of the fitted parameters

![](_page_26_Picture_1.jpeg)

$\begin{array}{c c} C_{11} & & C_{22} \\ 1.10 \pm 0.20 & & -0.02 \pm 0.20 \end{array}$		$C_{33} \ 0.14 \pm 0.30$	$C_{44} \ 0.16 \pm 0.07$	$C_{13} \\ 0.13 \pm 0.20$
$\begin{array}{c} C_{14} & C_{34} \\ -1.10 \pm 0.20 & -1.37 \pm 0.16 \end{array}$		$q_{ m max} \left( { m MeV}  ight) \ 637 \pm 72$	$R ({ m fm}) \\ 1.02 \pm 0.02$	

#### **Observables:** the scattering length and effective range

/	$a_1 \\ (0.44 \pm 0.05) - (0.62 \pm 0.04)i$	$\begin{bmatrix} a_2 \\ (0.31 \pm 0.02) - (0.34 \pm 0.02)i \end{bmatrix}$
	$a_3 \\ (0.30 \pm 0.02) - (0.20 \pm 0.04)i$	$a_4 \\ -0.769 \pm 0.017$

$ \frac{a_1}{(0.46\pm0.04) - (0.64\pm0.03)i} $	$\begin{array}{c c} a_2 \\ (0.32 \pm 0.01) - (0.35 \pm 0.02)i \end{array}$
$a_3$ (0.30±0.02)-(0.22±0.04) <i>i</i>	$a_4 (-0.780 \pm 0.013) + (0 \pm 0)i$

				-		-			AL CARGE CONCELENCE
$\begin{array}{c} r_1 \\ (-1.2 \pm 0.3) - (2.7 \pm 0.2)i \end{array} \tag{-5.5 \pm}$		$ \begin{array}{c} r_2 \\ \pm 1.6) + (8.9 \pm 0.5)i \end{array} \qquad (-2.8 \pm 0.3) - (-2.8 \pm 0.3$		$(0.1 \pm 0.7)i$	-1.4	$r_4$ $1\pm 0.16$	1932		
	$r_1 \\ (-1.1 \pm 0.2) - (2.7 \pm$	= 0.2) <i>i</i>	$(-6.2 \pm 1.4) + (8.8 \pm$	± 0.5) <i>i</i>	(-2	$r_3 = (0.3 \pm 0.3) - (0.3 \pm 0.3)$	0.6) <i>i</i>	-1.48	$r_4 \\ 3 \pm 0.13$
$\overline{\frac{\sqrt{s_p}}{(1515\pm7)}}$	$(3.7 \pm 0.5) - (96 \pm 13)i$			$g_1 - (1.11 \pm 0.16)i$		(1	$\begin{array}{c} g_2 \\ (2.6\pm0.4) - (0.79\pm0.11)i \end{array}$		
			$g_3 (3.5 \pm 0.3) - (0.27 \pm 0.06)i$		$\begin{array}{ c c c c c c c c c c c c c c c c c c c$		$\pm 0.2)i$		
									=
	$\sqrt{\frac{s_p}{1515\pm 6}} - (89\pm$	: 9) <i>i</i>	$\begin{array}{c} g_1 \\ (3.7 \pm 0.3) - (1 \end{array}$	$.04 \pm 0.13)$	ì	$(2.6\pm0$	$g_2$ (0.2) - (0.7)	$4 \pm 0.10)i$	_
			$\begin{array}{c} g_{3} \\ (3.6 \pm 0.2) - (0 \end{array}$	$.28 \pm 0.05)$	) <i>i</i>	(-2.68 ±	$g_4 = 0.13) + ($	$(1.4 \pm 0.2)$	i

# §5. Summary

- We use the chiral unitary approach to dynamically generate the state
  - Taking the pseudo data from theory, we use the resampling method <sup>4</sup>
  - for the inverse problem in the fitting of the correlation functions.
  - The existing of this resonance can be tested by the information from the correlation functions.

Hope future experiments bring more clarifications on these issues.....

![](_page_28_Picture_7.jpeg)

![](_page_28_Picture_8.jpeg)

![](_page_28_Picture_9.jpeg)

![](_page_29_Picture_0.jpeg)

![](_page_29_Picture_1.jpeg)

Jul 11 – 15, 2025 Asia/Shanghai timezone

会议计划于2025年7月11日至15日在桂林市桂林宾馆召开, 其中11日报到注册。会议统一安排食宿,费用自理。会 议收取注册费,教师及博士后1500元/人,学生1000元/人。

会议网址: <u>https://indico.ihep.ac.cn/event/24044/</u>, 注册截止时间为6月20日。

![](_page_30_Picture_0.jpeg)

### Thanks for your attention!

感谢大家的聆听!